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Outage-Based Power Control for Generalized Multiuser Fading Channels

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Abstract— We consider an uplink power control problem with constraints on outage probability, for cellular CDMA systems where allocation decisions are made on a slow time-scale. A generalized framework to solve such problems for a wide range of fading distributions is proposed, including an extension that couples power control with a minimum outage probability multiuser receiver. The resulting algorithms are simple and iterative in nature that yield the optimal minimum sum-power solution. Deriving a general upper bound on outage probability, we map these problems to equivalent, sub-optimal and computationally efficient iterative algorithms. We give numerical results to validate the methods developed for a variety of Nakagami-*m* fading figures.

I. INTRODUCTION

DVANCES in wireless services are continually relying on higher bit rates and more stringent quality of service (QoS) guarantees from carriers. The networks that support them are constrained by the limited wireless spectrum and so increased utilization is of paramount importance. Power control is one powerful technique to help achieve these conflicting goals resulting in plentiful research over the years. In interference limited systems such as CDMA, multiuser detection (MUD) may also be employed to significantly improve the situation.

Only in recent years has *joint* power control and MUD become of interest. Power control aims to intelligently balance the received powers of all users such that no user creates excessive interference to others. This is especially important in the CDMA uplink as the *near-far effect* can significantly degrade performance, however there are other benefits, such as longer battery life of mobile devices as each user needs only expend sufficient power to meet their QoS requirements. MUD is similar, in the sense that it exploits the structure of the multiple-access interference to suppress it [1], [2]. By jointly optimizing both user powers and multiuser receivers, we may gain the performance advantage of both [3], [4].

The classic papers on power control such as [5]–[10] specify constraints on the signal-to-interference ratio (SIR) to quantify QoS requirements. Resultant algorithms run at the same timescale as fast fades, potentially having a high computational cost and associated DSP power penalty in battery powered mobile devices. Such schemes may also incur a reduction in capacity as there is significant exchange between the base station and mobile device under fast closed loop power control.

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This work deals with situations when it is not feasible or not desirable to follow fast-fades directly. Rather than demanding that users achieve a target SIR, we relax the constraints and ask that each user maintains its SIR above some prescribed threshold with high probability: we thus consider constraints on outage probability rather than SIR.

The problem of power control with constraints on outage probability is considered in [11] where interior point optimization methods are employed to find the solution. In [12] we developed a simple iterative algorithm to minimize total transmit power subject to outage constraints. Both of these papers dealt only with Rayleigh fading.

Work to date assumes a specific fading model and analysis inherently follows from that selection. A more general framework is needed, so that one may simply "plug in" a desired fading distribution to cater for different cell landscapes. For example, one may use a Rician model for indoor cells with strong line-of-sight components, or a Nakagami-*m* model for outdoor situations lacking strong specular signal components.

In this work, we specifically address the problem of jointly optimizing user power and multiuser receivers so that each user's outage outage probability constraint is met with minimal power expended. We do this with only modest conditions on the associated fading distribution. This is a significant paradigm shift from prior work dealing with joint power and multiuser detection having SIR constraints [3].

The main contributions of this paper are:

- A framework for developing conceptually simple, iterative algorithms that determine user uplink transmit powers such that their sum is minimized, subject to outage constraints. Modest assumptions are made on the fading distribution yielding a generality whereby various fading models can be "plugged in" with ease.
- 2) An extension to the framework for joint optimization over user transmit powers and linear multiuser receivers.
- 3) A general procedure to obtain an upper bound on outage probability. This enables a mapping from outage probability to constraints on average SIR, and thus allows a simple and computationally efficient algorithm to solve the above problems sub-optimally [3], [12].

Simulation results are provided to validate the methods numerically for the Nakagami-m fading distribution with various fading figures.

II. SYSTEM MODEL

We consider the uplink of a synchronous direct sequence CDMA communications system with K users and a processing gain of N.

A. Received Signal Model

We assume BPSK modulation and chip-matched filtering. The received signal at the base station (BS) assigned to user i can be written as an N-dimensional signal vector for each symbol interval, given by

$$\mathbf{r}_i = \sum_{j=1}^K \sqrt{G_{ij} F_{ij} P_j} b_j \mathbf{s}_j + \mathbf{n}_i$$

where P_j is the transmit power of user j, b_j are data bits taking on values of ± 1 with equal probability, \mathbf{s}_j is the *N*dimensional spreading sequence of user j and \mathbf{n}_i is AWGN with zero mean and covariance $\sigma^2 \mathbf{I}$. We assume fixed spreading sequences, with elements of \mathbf{s}_j taking values $\pm 1/\sqrt{N}$.

As with prior work, we assume G_{ij} to be the positive *slowly-varying path gain* of user *j* to the assigned BS of user *i*. All analysis to follow assumes fixed G_{ij} terms. In practice, this assumption implies that all results are valid only over a finite time duration where factors affecting these gains do not vary significantly.

The terms F_{ij} model *fast time-scale fading*, all being *i.i.d.* non-negative random variables with $E[F_{ij}] = 1$ for all i, j. We assume further that the cumulative distribution function (CDF) of F_{ij} , denoted $\mathcal{F}_F(\cdot)$, is continuous and strictly increasing on \mathbb{R}_+ . This is the case for most fading distributions of interest. e.g. Nakagami-*m*. We will denote by \mathbf{F}_{-i} the collection of random variables $\{F_{ij} : j \neq i\}$.

Let \mathbf{c}_i denote the linear receiver filter coefficients for user i at its assigned BS and $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_K]$. The filter output of user i at its assigned BS is given by

$$\mathbf{y}_i = \mathbf{c}_i^\top \mathbf{r}_i = \sum_{j=1}^K \sqrt{G_{ij} F_{ij} P_j} (\mathbf{c}_i^\top \mathbf{s}_j) b_j + \tilde{n}_i$$

where $\tilde{n}_i = \mathbf{c}_i^\top \mathbf{n}_i$ is $N(0, \sigma^2 \mathbf{c}_i^\top \mathbf{c}_i)$.

B. SIR and Outage Probability

The SIR γ of the *i*-th user is given by

$$\gamma_i = \frac{G_{ii}(\mathbf{c}_i^{\top} \mathbf{s}_i)^2 F_{ii} P_i}{\sum\limits_{j \neq i} G_{ij}(\mathbf{c}_i^{\top} \mathbf{s}_j)^2 F_{ij} P_j + \sigma^2(\mathbf{c}_i^{\top} \mathbf{c}_i)}$$
(1)

where we treat receiver noise as interference.

The corresponding outage probability of user *i* is defined as the proportion of time that some positive SIR threshold γ_i^{th} is not met for sufficient reception at the BS receiver. The outage probability for user *i* is given by

$$O_i = Pr\{\gamma_i \le \gamma_i^{th}\}.$$
(2)

An alternative expression is obtained by writing

$$O_{i} = Pr\{F_{ii} \leq \Psi_{i}\}$$

= $E\left[\mathbf{1}_{\{F_{ii} \leq \Psi_{i}\}}\right]$
= $E\left[E\left[\mathbf{1}_{\{F_{ii} \leq \Psi_{i}\}} \mid \mathbf{F}_{-i}\right]\right]$
= $E\left[Pr\{F_{ii} \leq \Psi_{i}\} \mid \mathbf{F}_{-i}\right]$
= $E\left[\mathcal{F}_{F}(\Psi_{i})\right]$ (3)

where

$$\Psi_i = \gamma_i^{th} \frac{\sum_{j \neq i} G_{ij}(\mathbf{c}_i^{\top} \mathbf{s}_j)^2 F_{ij} P_j + \sigma^2(\mathbf{c}_i^{\top} \mathbf{c}_i)}{G_{ii}(\mathbf{c}_i^{\top} \mathbf{s}_i)^2 P_i}$$

and $\mathbf{1}_A$ is an indicator function of the event A.

III. USER POWER OPTIMIZATION WITH OUTAGE CONSTRAINTS

We first consider the simplified problem of optimizing user powers subject to outage constraints, where linear receivers are fixed. Without a loss of generality, we can drop the fixed linear receiver filter terms $(\mathbf{c}_i^{\top}\mathbf{s}_j)^2$, since we can absorb them into the fixed G_{ij} terms.

A. Problem Definition

We wish to find each user's power $\mathbf{P} = [P_1, \dots, P_K]$ such that the total power transmitted by all users is minimized, while meeting all outage probability constraints. We have,

$$\min_{\mathbf{P}} \sum_{i=1}^{K} P_i$$

s.t. $O_i \le \Omega_i, \quad P_i \ge 0, \quad i = 1, \dots, K$

where $\mathbf{\Omega} = [\Omega_1, \dots, \Omega_K]$ specifies all users' outage constraints with $\Omega_i \in (0, 1)$.

B. Outage Probability – Monotonicity

The following general results for outage probability are crucial to the formulation of an algorithm to solve the above optimization problem.

Result 1:
$$O_i(\mathbf{P})$$
 is strictly decreasing in P_i .
i.e. Let $\beta > \alpha$, then $O_i(\mathbf{P}|_{P_i=\alpha}) > O_i(\mathbf{P}|_{P_i=\beta})$.
Result 2: $O_i(\mathbf{P})$ is increasing in $P_j, j \neq i$.

i.e. Let $\beta > \alpha$, then $O_i\left(\mathbf{P}\big|_{P_j=\alpha}\right) \le O_i\left(\mathbf{P}\big|_{P_j=\beta}\right), i \ne j$. These results follow directly from the assumed properties

of the CDF, $\mathcal{F}_F(\cdot)$. The proofs can be found in [13] and are omitted due to space limitations.

C. Optimal Solution

Assuming a non-empty feasible set, we have the following results relating to the optimal solution:

Lemma 1: The optimal solution \mathbf{P}^* will have outage constraints satisfied with equality (a generalization of the observation made in [11] for the Rayleigh fading environment). That is: $O_i(\mathbf{P}^*) = \Omega_i \ \forall i$.

Proof: Suppose **P** is a power vector in the feasible set and that there exists a user with $O_i(\mathbf{P}) < \Omega_i$. Using Results 1 and 2 we see that we can lower the power of user *i*

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while keeping all outage constraints satisfied. This means that there is a feasible set of powers with smaller sum than \mathbf{P} so that \mathbf{P} is clearly not optimal.

Lemma 2: With P_j fixed for all $j \neq i$, the equation $O_i(\mathbf{P}) = \Omega_i$ has a unique positive solution P_i^* .

Proof: Immediate from Result 1.

Denote the unique solution of Lemma 2 as

$$P_i^* = I_i(\mathbf{P}_{-i}),\tag{4}$$

where \mathbf{P}_{-i} are (K-1)-length vectors, having the same form as the full length power vector with the *i*-th element omitted.

The function $I_i(\cdot)$ specifies the power required by user *i* to meet its outage constraint when interfering users have fixed powers \mathbf{P}_{-i} . Envisage an algorithm where, starting from some initial power vector, each user independently updates its power to meet its outage constraint — assuming that the other users powers are fixed. This leaves us with a new set of powers which form the starting point for the next iteration. This is an intuitively pleasing algorithm but will it converge to the solution of our optimization problem?

With the above algorithm in mind define

$$\mathbf{I}(\mathbf{P}) = [I_1(\mathbf{P}_{-1}), \dots, I_K(\mathbf{P}_{-K})].$$
(5)

We shall refer to (5) as the *interference function*, to maintain consistency with the framework in [14] and prior work [12].

We propose a new power control algorithm (PCA) having the standard form

$$\mathbf{P}^{n+1} = \mathbf{I}(\mathbf{P}^n) \tag{6}$$

where *n* denotes the iteration step. The algorithm is initialized with powers set to the receiver noise power: $P_i^0 = \sigma^2$, $\forall i$.

Theorem 1: $I(\mathbf{P})$ is a standard interference function.

The proof involves showing that $I(\cdot)$ satisfies the three properties required of a standard interference function given in [14] (positivity, monotonicity and scalability) and is deferred to [13] due to space limitations.

Since $I(\mathbf{P})$ is a standard interference function, the PCA (6) converges to the final optimal solution $\mathbf{P}^* = I(\mathbf{P}^*)$. This solution is the minimum power required to meet all users' outage constraints.

Remark: While the original problem involved solving a coupled system of K nonlinear equations in K unknowns, each step of the proposed algorithm requires the separate solution of K equations, each in one variable.

IV. JOINT POWER AND MUD OPTIMIZATION WITH OUTAGE CONSTRAINTS

We now consider the situation where we have choice over linear receivers and no-longer neglect the terms $(\mathbf{c}_i^{\top} \mathbf{s}_j)^2$.

A. Revised Problem Definition

We wish to jointly find each user's power $\mathbf{P} = [P_1, \dots, P_K]$ and linear receivers $\mathbf{C} = [\mathbf{c}_1, \dots, \mathbf{c}_K]$ such that the total power transmitted by all users is minimized, while meeting all outage probability constraints. We now have,

$$\min_{\mathbf{P},\mathbf{C}} \sum_{i=1}^{K} P_i$$

s.t. $O_i \le \Omega_i, \quad P_i \ge 0, \quad \|\mathbf{c}_i\|_2 = 1, \quad i = 1, \dots, K$
bat is equivalent to:

that is equivalent to:

$$\min_{\mathbf{P}} \sum_{i=1}^{K} P_i$$

s.t.
$$\min_{\mathbf{c}_i, \|\mathbf{c}_i\|_2 = 1} O_i \le \Omega_i, \quad P_i \ge 0, \qquad i = 1, \dots, K.$$

See [3] and [12] for similar refinements.

B. Optimal Solution

Define a new function

$$\mathbf{J}(\mathbf{P}) = [J_1(\mathbf{P}_{-1}), \dots, J_K(\mathbf{P}_{-K})]$$
(7)

where each component is given by

$$J_{i}\left(\mathbf{P}_{-i}\right) = \min_{\mathbf{c}_{i}, \|\mathbf{c}_{i}\|_{2}=1} I_{i}(\mathbf{P}_{-i}, \mathbf{c}_{i})$$
(8)

and where $I_i(\mathbf{P}_{-i}, \mathbf{c}_i)$ was defined in (4) with a fixed \mathbf{c}_i .

Mirroring the development in Section III, we propose a new PCA having the standard form

$$\mathbf{P}^{n+1} = \mathbf{J}(\mathbf{P}^n) \tag{9}$$

where n denotes the iteration step. The algorithm is again initialized with powers set to the receiver noise power.

Theorem 2: $\mathbf{J}(\mathbf{P})$ is a standard interference function.

The proof is straightforward (see [13]) and is based heavily on the fact that $I(\cdot)$ is a standard interference function.

Since $\mathbf{J}(\mathbf{P})$ is a standard interference function, the PCA (9) converges to the optimal solution $\mathbf{P}^* = \mathbf{J}(\mathbf{P}^*)$. This solution is the minimum power required to meet all user outage constraints, with linear receivers having converged to the Minimum Outage Probability (MOP) receiver [12].

We refer to this algorithm as the MOP-PCA.

V. BOUNDS ON OUTAGE PROBABILITY

For a Rayleigh distribution, it has been shown that the outage expression above can be bounded by an expression dealing with *average* SIR, consequently allowing the power control problem to be transformed to a more familiar one involving *average* SIR constraints [12]. The algorithm that solved such a transformed problem utilized the closed form MMSE receiver, relieving the need for an expensive *N*-dimensional minimization over a complicated non-linear function as required by the MOP-PCA above.

We now develop an upper bound on outage probability for a wide variety of distributions. When the bound applies, the optimizations above can be solved efficiently using simple iterative methods, albeit sub-optimally. Large system approximations can also be applied for decentralized solutions with a single iteration [12]. We show through numerical studies in Section VI that even when the bound does not apply, the resulting algorithms give excellent approximations to the optimal solution.

A. Certainty Equivalence Margin (CEM)

The CEM represents a margin of error for average SIR when representing a system by its certainty-equivalent form, ignoring all statistical variation in the instantaneous signal and noise power, and replacing such terms with their expected values [11], [12]. We write,

$$CEM_{i}^{\sigma} = \frac{\overline{SIR}_{i}}{\gamma_{i}^{th}} = \frac{1}{\gamma_{i}^{th}} \frac{G_{ii}(\mathbf{c}_{i}^{\top}\mathbf{s}_{i})^{2}P_{i}}{\sum_{j\neq i} G_{ij}(\mathbf{c}_{i}^{\top}\mathbf{s}_{j})^{2}P_{j} + \sigma^{2}(\mathbf{c}_{i}^{\top}\mathbf{c}_{i})} \qquad (10)$$

where the average SIR is defined as

$$\overline{SIR}_{i} = \frac{G_{ii}(\mathbf{c}_{i}^{\top}\mathbf{s}_{i})^{2}P_{i}}{\sum_{j\neq i}G_{ij}(\mathbf{c}_{i}^{\top}\mathbf{s}_{j})^{2}P_{j} + \sigma^{2}(\mathbf{c}_{i}^{\top}\mathbf{c}_{i})}.$$
(11)

B. Upper Bound on Outage Probability

Suppose the fading CDF, $\mathcal{F}_F(\cdot)$, is concave on \mathbb{R}_+ . Then by Jensen's inequality, we have

$$O_{i} = E \left[\mathcal{F}_{F} \left(\Psi_{i} \right) \right]$$

$$\leq \mathcal{F}_{F} \left(E \left[\Psi_{i} \right] \right)$$

$$= \mathcal{F}_{F} \left(\gamma_{i}^{th} \frac{\sum_{j \neq i} G_{ij} (\mathbf{c}_{i}^{\top} \mathbf{s}_{j})^{2} E[F_{ij}] P_{j} + \sigma^{2} (\mathbf{c}_{i}^{\top} \mathbf{c}_{i})}{G_{ii} (\mathbf{c}_{i}^{\top} \mathbf{s}_{i})^{2} P_{i}} \right)$$

$$= \mathcal{F}_{F} \left(\frac{1}{CEM_{i}^{\sigma}} \right).$$
(12)

This bound allows us to map outage constraints to constraints on CEM or average SIR; constraints that are much easier to deal with.

C. Example: Nakagami-m Fading Distribution

1) m = 1 (*Rayleigh*): A received signal having power y with a Rayleigh distributed envelope has a cumulative power distribution

$$\mathcal{F}_Y(y) = 1 - e^{-y}, \quad y \ge 0 \tag{13}$$

when E[y] = 1.

It is trivial to show that (13) is concave for all $y \ge 0$. Utilizing (12), we have the upper bound on outage

$$O_i \leq 1 - e^{-1/\operatorname{CEM}_i^o}$$

and is exactly the expression found previously via different means [12].

2) $m \neq 1$: The received signal power y for a general $m \neq 1$ has cumulative power distribution

$$\mathcal{F}_Z(y) = \frac{1}{\Gamma(m)} m^m \int_0^y z^{m-1} e^{-mz} dz, \quad y \ge 0$$

when E[y] = 1.

We can differentiate twice to yield

$$\frac{d^2 \mathcal{F}_Z}{dy^2}(y) = \frac{1}{\Gamma(m)} m^m y^{m-2} e^{-my} \left[(m-1) - my \right]$$

Observe that $\frac{d^2\mathcal{F}_Z}{dy^2}(y) < 0$ for all y > 0 provided $m \leq 1$. Thus an upper bound applies for $m \leq 1$:

$$O_{i} \leq \mathfrak{F}_{Z} \left(\frac{1}{CEM_{i}^{\sigma}}\right)$$
$$= \frac{1}{\Gamma(m)} m^{m} \int_{0}^{\frac{1}{CEM_{i}^{\sigma}}} z^{m-1} e^{-mz} dz$$
$$= \frac{1}{\Gamma(m)} \int_{0}^{\frac{m}{CEM_{i}^{\sigma}}} t^{m-1} e^{-t} dz$$
$$= \Gamma \left(\frac{m}{CEM_{i}^{\sigma}}, m\right)$$

where

$$\Gamma(x,a) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt$$

is an incomplete Gamma function.

D. Mapping from Outage to Average SIR Constraints

Assume the upper bound (12) holds. We can further bound this quantity by the outage constraint of user *i*:

$$O_i \leq \mathcal{F}_F\left(\frac{1}{CEM_i^{\sigma}}\right) \leq \Omega_i.$$
 (14)

By doing so, the right-hand inequality defines a new constraint on CEM_i^{σ} . By formulating and solving a new problem considering only these new constraints rather than the originals, we guarantee that the original outage constraints are also met. However, such an approach is sub-optimal with the error dependent on the tightness of the left-hand inequality.

In effect, we are *mapping* outage constraints to average SIR constraints. Taking the right-hand inequality from (14) yields

$$CEM_{i}^{\sigma} \geq \frac{1}{\mathcal{F}_{F}^{-1}(\Omega_{i})}$$
$$\overline{SIR}_{i} \geq \frac{\gamma_{i}^{th}}{\mathcal{F}_{F}^{-1}(\Omega_{i})}$$
$$\overline{SIR}_{i} \geq \Gamma_{i}^{th}$$

where $\mathcal{F}_{F}^{-1}(\cdot)$ denotes the inverse of the appropriate CDF, and we have redefined the outage-mapped average SIR threshold

$$\Gamma_i^{th} = \frac{\gamma_i^{th}}{\mathcal{F}_F^{-1}(\Omega_i)} \tag{15}$$

for a general fading distribution, which was first introduced in [12] for the Rayleigh fading environment. The inverse CDF can be found numerically with a simple line search when it cannot be found easily in closed form.

The mapped problem can be formulated as

$$\min_{\mathbf{P}} \sum_{i=1}^{K} P_i$$

s.t.
$$\max_{\mathbf{c}_i, \|\mathbf{c}_i\|_2 = 1} \overline{SIR}_i \ge \Gamma_i^{th}, \quad P_i \ge 0, \quad i = 1, \dots, K$$

and there exists a known iterative algorithm to find the optimal solution to this new problem [3], [12].

We refer to this algorithm as the MMSE-PCA, since the inner optimization utilizes the MMSE receiver to maximize the average SIR.

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	Algorithm	K			
m		4	8	16	32
1	MOP	1.1140	1.5369	2.4541	430.8579
2	MMSE	1.1142	1.5408	2.4589	430.9965
1	MOP	1.1118	1.5199	2.4031	286.4167
1	MMSE	1.1118	1.5199	2.4031	286.5411
0	MOP	1.1086	1.4901	2.3206	108.1565
2	MMSE	1.1078	1.4900	2.3197	107.4755
0	MOP	1.1067	1.4732	2.2733	56.7039
3	MMSE	1.1066	1.4730	2.2728	56.0825

TABLE I Average Normalized User Powers for Nakagami-m

VI. SIMULATION RESULTS

We consider an isolated circular cell having radius 1km and uniform locations of users within. Slowly varying gains G_{ij} are modeled by a distance dependent loss (exponent 4) superimposed by log-normal shadowing (zero mean, 8dB variance), and are fixed once chosen. We select processing gain N = 32 and AWGN power $\sigma^2 = 10^{-13}$ corresponding to approximately a 1MHz bandwidth.

We define three QoS classes, each having outage probability and SIR threshold pairs as $\{(5\%, 9dB), (10\%, 8dB), (20\%, 7dB)\}$ and assign 25% of users to the first class, 50% to the second and the remaining to the third. User signatures are selected randomly and fixed once chosen; initial filter coefficients are set to the matched filter and initial user powers to the AWGN power. We utilize the results from [15] to compute the outage probability.

Figure 1 shows the total sum power of all users as a function of the iteration step for K = 32 users. In all scenarios, the converged results of the MOP- (shown dotted) and mapped MMSE-PCAs (solid) are almost indistinguishable. For $m \le 1$, this is due to the tightness of the bound. Where the bound does not apply, we still see an almost indistinguishable result. This is important, as it validates the use of (12) as an approximation of the outage probability.

Table I lists average normalized user powers at convergence in more detail as we vary $K = \{4, 8, 16, 32\}$ users. Firstly, each user's transmit power is normalized by the power required to meet its outage constraint with equality in a single-user situation having the now optimal matched-filter. Secondly, an average is taken over these normalized powers in each scenario to produce the tabulated results.

Results for the MMSE-PCA are very close to the optimal powers resulting from the MOP-PCA, even when $m = \{2, 3\}$ and the bound does not apply. In these two cases, the outage probabilities resulting from the MMSE-PCA are just above the corresponding outage constraints Ω_i , however for practical engineering purposes we may still consider the solution to be acceptable: user outage constraints are met with equality to four significant figures.

VII. CONCLUSION

This paper introduced a new framework to solve power control problems incorporating outage probability constraints for a wide range of fading distributions. Solutions were presented as iterative algorithms that optimize user powers



Fig. 1. Total transmission power for the MOP- and MMSE-PCAs (K = 32).

and linear receivers to the jointly optimum values. Utilizing an upper bound on outage, a mapping to a sub-optimal and computationally efficient algorithm was demonstrated having exceptionally small error for those simulations considered.

REFERENCES

- S. Verdú, *Multiuser detection*. New York, USA: Cambridge University Press, 1998.
- [2] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. on Comm.*, vol. 42, no. 12, pp. 3178–3188, 1994.
- [3] S. Ulukus and R. D. Yates, "Adaptive power control and MMSE interference suppression," *Wireless Networks, Special Issue on Multiuser Detection in Wireless Communications*, vol. 4, no. 6, pp. 489–496, 1997.
- [4] A. F. Almutairi, S. L. Miller, H. A. Latchman, and T. F. Wong, "Power control algorithm for MMSE receiver based CDMA systems," *IEEE Comm. Letters*, vol. 4, no. 11, pp. 346–348, 2000.
- [5] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. on Vehic. Tech.*, pp. 541–646, 1993.
- [6] J. M. Aein, "Power balancing in system employing frequency reuse," COMSAT Technical Review, vol. 3, no. 2, pp. 277–300, 1973.
- [7] R. W. Nettleton and H. Alavi, "Power control for a spread spectrum radio system," in *Vehicular Technology Conf.*, pp. 242–246, 1983.
- [8] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. on Vehic. Tech.*, vol. 41, pp. 57–62, Feb. 1992.
- [9] S. A. Grandhi, R. Vijayan, D. J. Goodman, and J. Zander, "Centralized power control in cellular radio systems," *IEEE Trans. on Vehic. Tech.*, vol. 42, pp. 466–468, Nov. 1993.
- [10] S. V. Hanly, "An algorithm of combined cell-site selection and power control to maximize cellular spread spectrum capacity," *IEEE Journal* on Selected Areas in Comm., vol. 13, pp. 1332–1340, Sept. 1995.
- [11] S. Kandukuri and S. Boyd, "Optimal power control in interferencelimited fading wireless channels with outage-probability specifications," *IEEE Trans. on Wireless Comm.*, vol. 1, no. 1, pp. 46–55, 2002.
- [12] J. Papandriopoulos, J. Evans, and S. Dey, "Optimal power control for Rayleigh-faded multiuser systems with outage constraints," *IEEE Trans.* on Wireless Comm., 2003, accepted.
- [13] J. Papandriopoulos, J. Evans, and S. Dey, "Outage-based optimal power control for generalized multiuser fading channels," *IEEE Trans. on Comm.*, 2003, submitted (see http://jpap.yi.org/research.phtml).
- [14] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE Journal on Selected Areas in Comm.*, vol. 13, no. 7, pp. 1341–1347, 1995.
- [15] C. T. A. Annamalai and V. K. Bhargava, "Simple and accurate methods for outage analysis in cellular mobile radio systems – A unified approach," *IEEE Trans. on Comm.*, vol. 49, pp. 303–316, Feb. 2001.