CFO and Channel Estimation Techniques for GFDM

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***Abstract*—Carrier frequency offset (CFO) caused by the misalignment of the transmitter and receiver local oscillators can adversely affect the performance of any multicarrier system if not accurately estimated and corrected. Thus, in this paper, we propose a CFO and channel estimation technique based on the maximum-likelihood (ML) criterion for generalized frequency division multiplexing (GFDM). Our proposed CFO estimator does not have any limitation on the CFO acquisition range while providing an accurate estimate. We propose a preamble block containing two frequency domain ZC (Zadoff-Chu) sequences for training which leads to a low complexity implementation of the CFO estimator. Compared with the existing solution in the literature with the largest CFO estimation range and precision, our technique brings around two orders of magnitude complexity reduction without any performance penalty. We also evaluate the performance of our proposed technique through numerical simulations while showing its superiority to the existing literature.**

***Index Terms*—GFDM, CFO, Channel, Estimation, ML.**

I. Introduction

Future mobile networks need to address a wide range of challenges associated with new use-cases and applications such as ultra high definition video streaming, interactive gaming, virtual and augmented reality. These applications necessitate the need for the new signalling techniques with higher reliability and data rates than the existing 3GPP LTE systems. Additionally, the vast amount of spectrum available in millimeter-wave (mmWave) frequency bands has been discussed as an enabler for such media-rich high rate applications during the last couple of years, [1]. Multicarrier transmission techniques are potential candidates that can be deployed for the physical layer of mmWave systems [1]. Generalized frequency division multiplexing (GFDM) has recently been proposed as a candidate for mmWave communications [2].

One of the most challenging issues in multicarrier systems is their high sensitivity to the carrier frequency offset (CFO) that is mainly due to the local oscillator (LO) misalignments. In typical wireless systems, LO accuracy is usually in the order of parts-per-million of the carrier frequency. Hence, the CFO that is imposed by the LO misalignments can become considerable in mmWave systems. CFO if not accurately estimated and corrected, can adversely affect the performance of GFDM [3]. Current solutions in the literature such as the one in [4], have an acquisition CFO range limited to only half of a subcarrier spacing. Thus, one of the goals of this paper is to develop a CFO estimation technique for GFDM without any limitation on the CFO acquisition range.

Additionally, accurate channel estimation is of a paramount importance in any practical communication system. To this end, the authors in [5] propose a channel estimation method using a number of scattered pilots in a GFDM block through least squares (LS) criterion. In [6], the authors propose a joint CFO and channel estimation method which can be straightforwardly extended to GFDM. To the best of our knowledge this is the only available solution in the literature applicable to GFDM without any limitation on the CFO range. However, its implementation imposes a substantial amount of computational burden to the system. To address this issue, in this paper, we propose a joint CFO and channel estimation technique based on the maximum-likelihood (ML) criterion with a low computational complexity. Our technique has no limitation on the CFO acquisition range. We propose deployment of a preamble GFDM block containing only two similar frequency domain Zadoff-Chu (FD-ZC) sequences to shorten the training overhead. This is also the key to the development of our proposed low complexity CFO estimator. Based on our complexity analysis and numerical results, our proposed technique leads to around two orders of magnitude complexity reduction without any performance penalty compared to the solution in [6]. We have analyzed and compared the performance of our proposed technique with [6] through numerical simulations. Our results show that the performance of the solution in [6] highly depends on the training sequence that is deployed. This is while the performance of our solution is independent of the training sequence that is utilized.

II. System model

We consider a GFDM system transmitting a block of data symbols with the length MN including the total number of N subcarriers and M overlapping symbols in time. Hence, the $MN \times 1$ received signal vector $\mathbf{r}$ in the presence of CFO and multipath channel, after discarding the cyclic prefix (CP) at the receiver can be written as

$$\mathbf{r} = \mathbf{E}\mathbf{X}\mathbf{h} + \boldsymbol{\nu}, \tag{1}$$

where $\boldsymbol{\nu} \sim \mathcal{CN}\left(0, \sigma_v^2 \mathbf{I}_{MN}\right)$ is the complex additive white Gaussian noise (AWGN) vector with the variance σ_v^2. $\mathbf{h}$ is the $L \times 1$ channel vector where L is the length of the multipath channel, and $\mathbf{E} = \mathrm{diag}(\boldsymbol{\varphi})$ is the diagonal CFO matrix whose diagonal elements include the elements of the vector $\boldsymbol{\varphi} = [1, e^{\frac{j2\pi\varepsilon}{N}}, \ldots, e^{\frac{j2\pi\varepsilon(MN-1)}{N}}]^{\mathrm{T}}$ where ε is the normalized CFO to the subcarrier spacing. Additionally, the $MN \times L$ matrix $\mathbf{X}$ contains the first L columns of a circulant matrix

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whose first column includes the GFDM transmit signal $\mathbf{x}$. Multiplication of $\mathbf{X}$ to the channel vector $\mathbf{h}$ in (1) realizes the circular convolution of the GFDM transmit signal $\mathbf{x}$ with the multipath channel $\mathbf{h}$. According to the frequency spreading GFDM transmitter structure, $\mathbf{x}$ can be obtained as [8]

$$\mathbf{x}=\sum_{m=0}^{M-1} \mathrm{circshift}(\mathbf{x}_m, mN), \tag{2}$$

where $\mathrm{circshift}(\cdot, \ell)$ denotes the downward circular shift operation with ℓ positions. $\mathbf{x}_m = \boldsymbol{\mathcal{F}}_{MN}^{\mathrm{H}} \mathbf{C}\mathbf{d}_{\mathrm{e}}[m]$. The $MN \times MN$ matrix $\mathbf{C}$ is circulant with the first column $\mathbf{c} = [c_0, c_1, \ldots, c_{M-1}, 0, \ldots, 0, c_{M-1}, \ldots, c_1]^{\mathrm{T}}$ which contains the $2M-1$ nonzero frequency domain coefficients of the prototype filter, and the $MN \times 1$ vector $\mathbf{d}_{\mathrm{e}}[m]$ is the M-fold expanded version of the vector $\mathbf{d}[m] = [d_{0,m}, \ldots, d_{N-1,m}]^{\mathrm{T}}$, with the entries $d_{n,m}$ corresponding to the data symbols to be transmitted on the nth subcarrier and the mth time slot. Finally, $\boldsymbol{\mathcal{F}}_{MN}$ is the MN-point normalized discrete Fourier transform (DFT) matrix with the elements $[\boldsymbol{\mathcal{F}}_{MN}]_{k\ell} = \frac{1}{\sqrt{MN}} e^{-j\frac{2\pi k \ell}{MN}}$. Noting that circular shift in time translates into phase shift in the frequency domain, (2) can be rearranged as

$$\mathbf{x} = \boldsymbol{\mathcal{F}}_{MN}^{\mathrm{H}} \left(\mathbf{C}\mathbf{d}_{\mathrm{e}}[0] + \ldots + \mathrm{diag}(\mathbf{a}_{M-1}) \mathbf{C}\mathbf{d}_{\mathrm{e}}[M-1] \right), \tag{3}$$

where $\mathbf{a}_m = [1, e^{-\frac{j2\pi m}{M}}, \ldots, e^{-\frac{j2\pi m}{M}(MN-1)}]^{\mathrm{T}}$ represents the phase shifts in the frequency domain. It is worth to note that the derived expansion in equation (3) is the key to simplification of our proposed estimation procedure in Section III.

III. Proposed Joint CFO and Channel Estimation

In this section, we propose a preamble-based joint CFO and channel estimation method through the ML criterion. Using (1), channel coefficients can be estimated as

$$\widehat{\mathbf{h}} = \left(\mathbf{X}^{\mathrm{H}}\mathbf{X}\right)^{-1} \mathbf{X}^{\mathrm{H}}\mathbf{E}^{\mathrm{H}}\mathbf{r}. \tag{4}$$

Substituting (4) into (1), the CFO can be estimated as

$$\hat{\varepsilon} = \underset{\tilde{\varepsilon}}{\mathrm{argmax}} \left\{ \mathbf{r}^{\mathrm{H}} \widetilde{\mathbf{E}} \mathbf{X} \left(\mathbf{X}^{\mathrm{H}}\mathbf{X}\right)^{-1} \mathbf{X}^{\mathrm{H}} \widetilde{\mathbf{E}}^{\mathrm{H}} \mathbf{r} \right\}, \tag{5}$$

where $\widetilde{\mathbf{E}}$ is obtained in the same way as $\mathbf{E}$ by substitution of $\tilde{\varepsilon}$ into $\mathbf{E}$ rather than ε. To lower the computational complexity of the CFO estimation, we propose utilization of the FD-ZC training sequence to make the matrix $\mathbf{X}^{\mathrm{H}}\mathbf{X}$ diagonal. For this purpose, a ZC sequence with the length N is defined as $\mathbf{z} = \frac{1}{\sqrt{N}}[1, e^{\frac{j\beta\pi}{N}}, \ldots, e^{\frac{j\beta\pi}{N}(N-1)^2}]^{\mathrm{T}}$, where β is an integer parameter relatively prime with respect to N. Thus, the FD-ZC sequence can be obtained as $\boldsymbol{\psi} = \boldsymbol{\mathcal{F}}_N \mathbf{z}$. It is known that cyclically shifted versions of the ZC sequence constitute a set of orthogonal basis vectors, [7]. To reduce the training overhead, we consider $M = 2$ for the preamble. Setting $\mathbf{d}_{\mathrm{e}}[0] = \mathbf{d}_{\mathrm{e}}[1] = \boldsymbol{\psi}_{\mathrm{e}}$ in (3), where $\boldsymbol{\psi}_{\mathrm{e}}$ is the twofold expanded version of $\boldsymbol{\psi}$, we have

$$\begin{aligned} \mathbf{x} &= \boldsymbol{\mathcal{F}}_{2N}^{\mathrm{H}} \left(\mathbf{I}_{2N} + \mathrm{diag}(\mathbf{a}_1) \right) \mathbf{C}\boldsymbol{\psi}_{\mathrm{e}} \\ &= \boldsymbol{\mathcal{F}}_{2N}^{\mathrm{H}} \mathrm{diag}\left([2 \quad 0 \cdots 2 \quad 0] \right) \mathbf{C}\boldsymbol{\psi}_{\mathrm{e}} \\ &= \frac{2}{\sqrt{2}} c_0 [(\boldsymbol{\mathcal{F}}_N^{\mathrm{H}}\boldsymbol{\psi})^{\mathrm{T}}, (\boldsymbol{\mathcal{F}}_N^{\mathrm{H}}\boldsymbol{\psi})^{\mathrm{T}}]^{\mathrm{T}} = \frac{2}{\sqrt{2}} c_0 [\mathbf{z}^{\mathrm{T}}, \mathbf{z}^{\mathrm{T}}]^{\mathrm{T}}. \end{aligned} \tag{6}$$

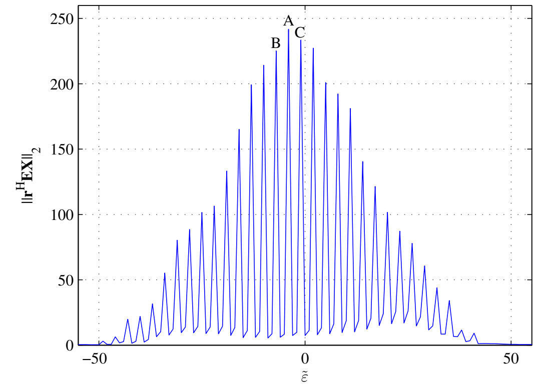

Fig. 1. Typical cost function for (7) with $\varepsilon = -4.1283$, $N = 128$, $L = 16$.

Accordingly, $\mathbf{x}^{\mathrm{H}}\mathbf{x} = 2c_0^2$ and thus $\mathbf{X}^{\mathrm{H}}\mathbf{X} = 2c_0^2\mathbf{I}_L$. This shows that the orthogonal property of the ZC sequence is preserved after GFDM modulation and (5) is simplified to

$$\hat{\varepsilon} = \underset{\tilde{\varepsilon}}{\mathrm{argmax}} \left\{ \left|\left| \mathbf{r}^{\mathrm{H}} \widetilde{\mathbf{E}} \mathbf{X} \right|\right|_2 \right\}. \tag{7}$$

Finally, the channel can be estimated through substitution of (7) into (4).

IV. Low Complexity Implementation of The Proposed CFO Estimation Method

The CFO estimation methods proposed in the literature for GFDM have an acquisition CFO range limited to half of subcarrier spacing. In contrast, similar to [6], our proposed method in this paper does not have any limitation in terms of the CFO range. This is while our proposed estimation method is simpler to implement than [6]. Additionally, as it is shown in Section V, opposed to the estimation method in [6], our proposed estimator does not have any limitations to the training sequence that is deployed. To address the complexity issue, in our method, we use the properties of the cost function in (7), when the FD-ZC sequence is utilized. In this case, as shown in Fig. 1, the cost function (7) has a set of local peaks. Our study shows that it is sufficient to find only three of the local peaks ('A', 'B' and 'C') to achieve an accurate CFO estimation with a low computational burden. Hence, by setting the search step-size to $\gamma = 1$, in the first stage, the local peaks 'A', 'B' and 'C' in Fig. 1 are acquired. In the next stage, we first set the step-size to 0.1 and search locally only in the proximity of each individual candidate peak within the neighbourhood of one subcarrier spacing to find the value of $\tilde{\varepsilon}$ that maximizes the cost function in (7) for all the peaks. Then, we search the proximity of this peak with the step-size 0.01 in the neighbourhood of 0.1 subcarrier spacing, and finally, we repeat the search for the step-size 0.001 in the neighbourhood of 0.01 subcarrier spacing to find an accurate CFO estimate.

The proposed ML estimation in (5) with conventional search method results in a very high computational complexity. Using FD-ZC sequence reduces $\mathbf{X}^{\mathrm{H}}\mathbf{X}$ to a diagonal matrix. Furthermore, our proposed search algorithm substantially reduces the computational complexity of (7). Based on the discussion above and equation (7), computational complexity of our

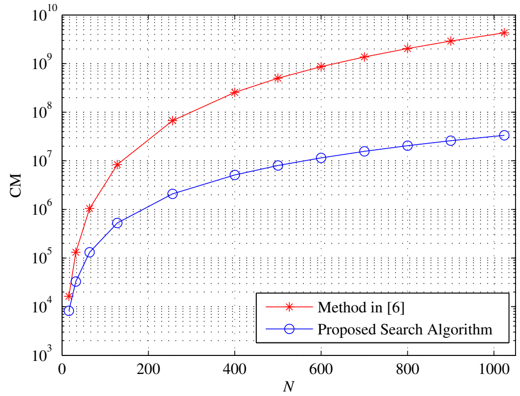

Fig. 2. Computational complexity comparison.

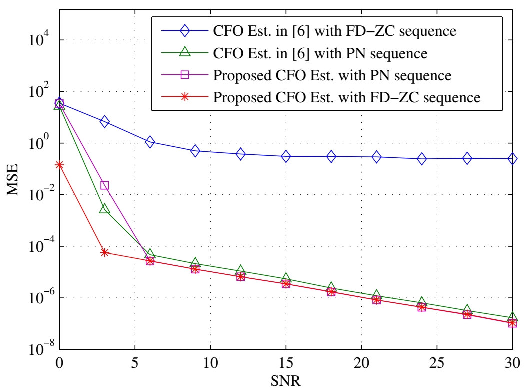

Fig. 3. MSE of the CFO estimation.

proposed search algorithm, in terms of the number of complex multiplications, is in the order of $\mathcal{O}(2LN^2)$. In comparison, the complexity of the method in [6] is in the order of $\mathcal{O}(4N^3)$. Fig. 2 compares the computational complexity of the method in [6] with our proposed algorithm for $L = 16$. As the figure shows, our method leads to around two orders of magnitude complexity reduction for the large number of subcarriers.

V. Simulation Results

In this section, we evaluate the performance of our proposed estimation techniques presented in Sections III and IV through numerical results. We assume $N = 128$, $M = 2$, for the preamble and use the extended typical urban channel model (ETU), i.e., an LTE channel model. The CP length is $\lfloor 0.1N \rfloor$, i.e., long enough to accommodate the wireless channel delay spread. We use a root-raised cosine prototype filter with the roll-off factor of $\alpha = 0.1$, in all the simulations.

Fig. 3 and 4 show the MSE performance of our proposed joint CFO and channel estimation techniques where the CFO values are randomly chosen in the range $\varepsilon \in [-N/2, N/2)$ from the uniform distribution. In these figures, we compare and show the MSE performance of our proposed joint CFO and channel estimation technique with the proposed method in [6] when the pseudo-random noise (PN) sequence and FD-ZC training sequences are deployed. As shown in these figures, the MSE performance of our proposed estimators improve as SNR increases. Also the MSE performance of the method in [6]

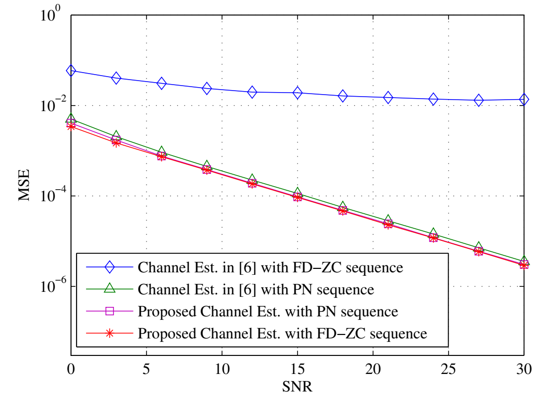

Fig. 4. MSE of the Channel estimation.

with PN sequence follows our proposed method for the SNRs higher than 6 dB. This is while the method in [6] suffers from a poor performance when FD-ZC training sequence is utilized. According to our results, opposed to [6], the performance of our proposed CFO and channel estimation technique is independent of the training sequence that is deployed.

VI. Conclusion

In this paper, we proposed an ML-based joint CFO and channel estimation technique for GFDM. Our proposed CFO estimation technique does not have any limitation on the CFO range. To reduce the computational complexity, we suggested utilization of two frequency domain ZC sequences in the GFDM preamble block. This leads to a substantial amount of savings in computations without any performance penalty, i.e. around two orders of magnitude compared with the existing solution with the largest CFO estimation range and precision. Finally, we have confirmed the above points through numerical simulations while showing the superiority of our technique through comparisons with the existing literature.

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