Nonlinear time series analysis of human alpha rhythm

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Abstract

Nonlinearity is often deduced by showing that a dataset signi£cantly deviates from its phase randomized versions, *i.e.* surrogate data. For real data, however, non-stationarities like artifacts and onsets and offsets of rhythmic activity will cause false positives. We propose a new test which detects dynamical nonlinearity by measuring time-asymmetry, using surrogate data merely to estimate the standard deviation of the process.
The method is applied to multi-channel MEG measurements of ongoing alpha-band activity modulated by a simple visual memory task involving motor activity. The signal to noise ratio was enhanced using ICA, and the analysis was performed on a single separated source. We found that, if the peak at 10 Hz is accompanied by a substantial higher harmonic, time asymmetry can be detected signi£cantly in virtually any epoch of 3 second duration. Finally, we applied our recently proposed method to estimate correlation dimension for noisy data. We found very satisfactory scaling plots with dimension around 1.5. As a byproduct, we showed that the nondeterministic fraction can be explained almost completely by external noise.

1 Introduction

We £rst propose and apply a new method that detects nonlinear dynamics in a time series. The main goal of the construction is to avoid false positives from non-stationarities. In contrast to the more common tests for inconsistency with colored noise [1], which are based on comparison of the data with phase randomized versions ('surrogate data') and are sensitive *e.g.* to spiky artifacts or activity onset/offset, our test is based on the detection of time asymmetry, as proposed by [2] and others. A severe problem is that without reference to surrogate data, the statistical analysis implicitly assumes mutual independence of all analyzed data segments. Enforcing this [2] leads (for systems with long autocorrelation, as in the alpha rhythm) to a disastrous reduction of usable data. Apart from the actual de£nition of the discriminating statistic, which de£nes 'difference', our new approach is to refer to surrogate data only to estimate its standard deviation, while still measuring its absolute difference (to zero) and not its relative difference (to the corresponding value for the surrogate data). This method is applied to human alpha activity present in two MEG datasets.
The second goal is estimation of the correlation dimension (D_2) of the alpha rhythm, which in turn is an estimate of the number of degrees of freedom of the underlying dynamical system. In practice, the noisy signals obtained from real-world systems make a meaningful estimate of the correlation dimension extremely dif£cult to achieve. Here, we apply a recently proposed method [3] to remove noise bias from the scaling plots.

2 Measurements and pre-processing

In 8 MEG measurements (from a visual stimulation paradigm designed for other purposes, sampled at 500 Hz) of neural responses to visual stimulation, the underlying generators were separated using Independent Component Analysis [4]. Separated sources with alpha activity (de£ned by a 10 Hz spectral peak) could be clearly identi£ed. We found two subjects with a substantial energy in the 20 Hz harmonic. For the others, the energy at 20 Hz was detectable but too low for further analysis. Since a single frequency is always consistent with linear dynamics, only these two respective time series were further analyzed, after being passed through a 0.5 Hz–40 Hz bandpass £lter.
Dataset A was measured using a 49-channel planar MEG system, where the sensor array was located tangentially over position T6 of the 10–20 electrode position standard. The alpha generator was localized to the occipital lobe, and the rhythm is hence identi£ed as 'occipital alpha.' We found the rhythm to be very stable, and active about 70% of the time. For the dimension analysis below we picked a single time window of 16 seconds of stable alpha activity.
Dataset B was measured using an 80-channel whole head MEG system. The alpha generator localized to the central region, and was suppressed by £nger movement. It was hence associated with rolandic mu rhythm [5]. Due to the motor-related suppression we found only a few relatively short, but still strong, bursts of alpha activity. Since a dimension analysis requires a relatively long time series, we concatenated the four bursts into a single 11.3 second time series.

3 Detecting nonlinearity

We will detect nonlinearity within the framework of correlation sums and correlation dimensions. Our £rst step is to embed a single valued time series $x = (x_1, \cdots, x_n)$ into an m-dimensional space by constructing vectors $\vec{x}_i = (x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\tau})^T$ The time delay, set here to $\tau = 6$, should be chosen suf£ciently large to avoid essential redundancies but otherwise as small as possible. From these vectors a correlation sum $C(\epsilon, m)$ at scale ϵ is de£ned by

$$C(\epsilon, m) = \frac{1}{N} \sum_{i<j-W} f(||\vec{x}_i - \vec{x}_j||/\epsilon) \tag{1}$$

where we take f to be a gaussian kernel $f(x) = \exp(-x^2/4)$. N is the total number of terms to be summed over, $||\cdot||$ is the euclidian norm, and W is a parameter set to exclude pairs of vectors which are too close in time to be regarded as independent [1]. The correlation dimension can be estimated from

$$d(\epsilon, m) = \frac{d \ln C(\epsilon, m)}{d \ln \epsilon} \tag{2}$$

If $d(\epsilon, m)$ is constant as a function of both ϵ (scaling) and m (saturation) in suf£ciently large regions, this constant is called the correlation dimension D_2 of the underlying dynamical system.

To detect dynamical nonlinearities, we test whether the correlation dimension is invariant under local reversal of time as de£ned by replacing $\vec{x}_j$, the second vector in $f(||\vec{x}_i - \vec{x}_j||/\epsilon)$, by its ¤ipped version. As a measure of time-asymmetry we choose a weighted average of the difference of $d_1(\epsilon, m)$ and $d_2(\epsilon, m)$, calculated for the original and the locally time reverted time series, respectively:

$$\Delta(x) = \sum_{m=1}^{M} \int d(\ln \epsilon) \frac{d_1(\epsilon, m) - d_2(\epsilon, m)}{S(\epsilon, m)} \frac{\epsilon^2}{\epsilon^2 + \sigma_0^2} \tag{3}$$

where $S(\epsilon, m) = (C_1^{-1}(\epsilon, m) + C_2^{-1}(\epsilon, m))^{1/2}$ is a (rough) estimate of the standard deviation of $d_1(\epsilon, m) - d_2(\epsilon, m)$, the maximal embedding dimension is $M = 10$, and $\epsilon^2/(\epsilon^2 + \sigma_0^2)$ is an additional heuristic weighting factor with a free parameter σ_0. We found in simulations that after normalizing all time series to unit standard deviation the setting $\sigma_0 = 1$ generally improved the results. The standard deviation of $\Delta(x)$ is then estimated as

$$\sigma(x) = \left(\frac{1}{K} \sum_{k=1}^{K} \left(\Delta(x_k^P)\right)^2 \right)^{1/2} \tag{4}$$

where x_k^P is the k-th version of the phase randomized data, and we set $K = 20$. The number of standard deviations by which x deviates from the null hypothesis (being time symmetric) is estimated as the ratio of $\Delta(x)/s(x)$. Note that in the de£nition of $\sigma(x)$ the mean was not subtracted.

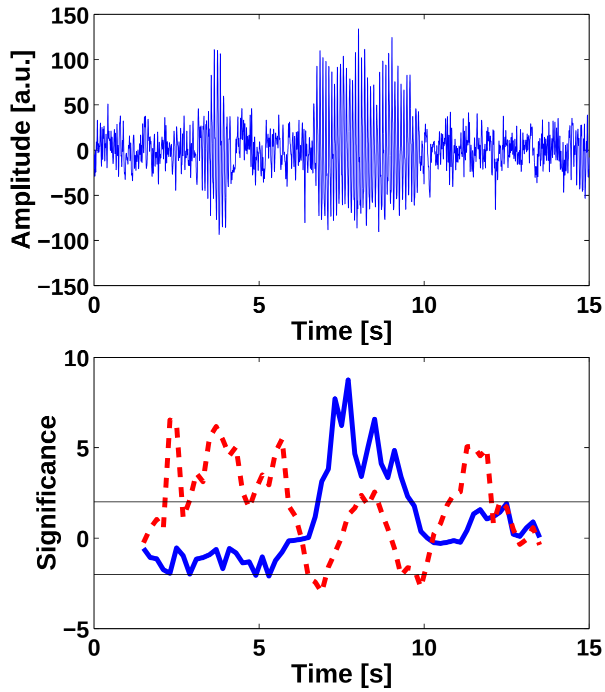

Figure 1: Top: selected time window with a short and a long alpha burst. Bottom: signi£cance of time asymmetry (solid) and deviation from colored noise (dotted) as a function of the center of a 3 second window.

This accounts for the possibility that remnants of autocorrelation can bias $\Delta(x)$, which is compensated for by a corresponding increase of the estimated standard deviation.

For comparison we also calculate the signi£cance value of the 'pure phase randomization test' similar to Theiler's [1]. Let $\Delta^P(x, x_k^P)$ be the dimension difference (3) of x and the k-th realization of the phase randomized data. Signi£cance is then given by the ratio of the mean and the standard deviation of $\Delta^P(x, x_k^P)$.

We tested this method on simulated data and found that, in contrast to the pure phase randomization test, a strong deviation from stationarity (here: a sudden increase of the amplitude of gaussian white noise by a factor of 3 in the middle of the epoch) did not lead to false positives, while our test was more sensitive to true nonlinearities (here: the Henon-map).

For the real data, we measured nonlinearity in the two time series (A and B) over a sliding 3 second window and set $W = 500$. For dataset A we found signi£cant time asymmetry in 71% of the epochs, which approximately coincide with those epochs containing alpha rhythm. Data set B consists mainly of noise, with a few strong alpha bursts. Fig. 1 shows the signi£cance, as a function of the center of the moving window, for both this new test and the conventional one. The new test gives clear detection of time-asymmetry over the long alpha burst, while the conventional test is almost uninterpretable. Note that for the short burst any detection of signi£cant time asymmetry

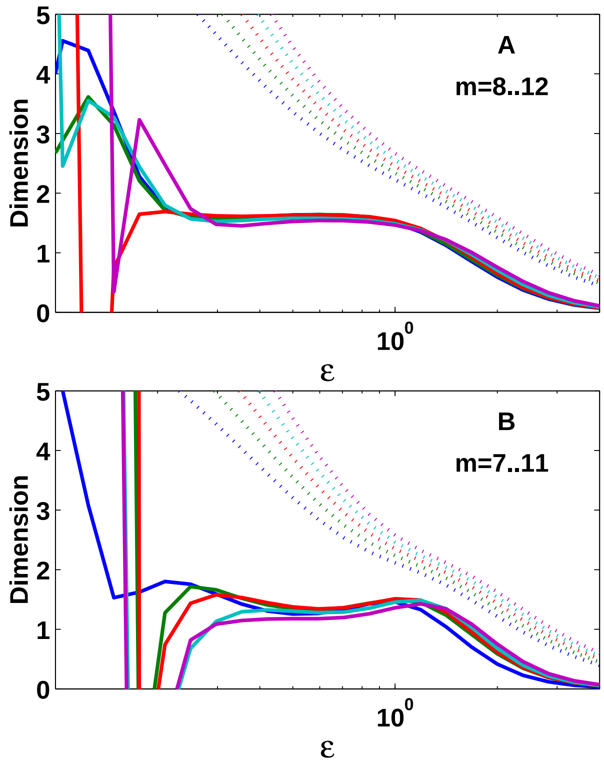

Figure 2: Estimated dimension as a function of scale ϵ for various embedding dimensions m for datasets A and B. In contrast to the uncorrected estimates (dotted) we observe very satisfactory scaling and saturation for the corrected estimates (solid) calculated with (5).

could not be interpreted as a property of the alpha dynamics, since the burst is shorter than the minimum delay W.

4 Dimension and noise level

In [3] we showed that

$$d'(\epsilon, m) = \frac{d(\epsilon, m) - m(d(\epsilon, m+1) - d(\epsilon, m))}{1 - (d(\epsilon, m+1) - d(\epsilon, m))} \quad (5)$$

is a noise-robust estimator of the correlation dimension when the correlation sums are calculated using gaussian kernels.

Fig. 2 shows scaling plots within a selected range of embedding dimensions for data sets A and B, for both the uncorrected and corrected dimension estimates. For both data sets the uncorrected dimensions neither scale nor saturate to any non-trivial value. For dataset A we see very clear scaling for the corrected estimate over about a factor of 4 in scale, and we can identify a correlation dimension of $D_2 \approx 1.6$. The scaling plot for data set B is less convincing, but after noise correction a scaling region can again be clearly identi£ed, giving an estimated correlation dimension of $D_2 \approx 1.4$.

In [3] we also presented a similar method to estimate the noise level. Normalizing the standard deviations of the time series to one we found a 'total noise level' of 0.34 and 0.38 for A and B, respectively. An independent estimate of the additive noise is available by comparing time windows with and without alpha, leading to noise level estimates of 0.30 and 0.34 for A nd B, respectively. Hence, the 'total noise level' can be almost completely explained by additive noise independent of alpha, implying that the alpha generator appears to be very deterministic.

5 Conclusion

We propose a new method to detect dynamical nonlinearity in a time series. In contrast to the commonly used comparison of the data to phase randomized versions, which tests whether the data are inconsistent with colored but *stationary* noise, this test was based on detecting asymmetry with respect to local time reversal. The important new idea is that, in order to estimate statistical properties, we still refer to phase randomized data, but use them only to estimate the standard deviation of the measure. As a consequence, the test is extremely robust against nonstationarities and, surprisingly, was found to be even more sensitive to true nonlinear dynamics. Application to MEG data of human alpha rhythm with a substantial 20 Hz harmonic resulted in signi£cant time-asymmetry in virtually all epochs which contain alpha.

We also applied a new method to remove the noise bias from scaling plots in the estimation of the correlation dimension of a dynamical system. After correction we found excellent scaling properties at a correlation dimension $D_2 \approx 1.5$. Comparison of 'total' and additive noise showed that the alpha generator is essentially deterministic.

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