Distortion Minimization via Multiple Sensors under Energy Harvesting Constraints

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***Abstract*—We consider a wireless sensor network equipped with energy harvesting technology. It contains M sensors that observe a random process and transmit an amplified uncoded analog version of the observed signal through fading wireless channels to a remote station. The remote station, often called the fusion center, estimates the realization of the random process by using a best linear unbiased estimator. In this paper, we consider the optimal energy allocation policy that minimizes total distortion over a finite time horizon subject to energy harvesting constraints at the sensors. We focus on two types of available side information at the sensor, i.e. (1) causal side information involving the present and previous channel states and the previous values of the harvested energy and (2) full (non-causal) side information, under both finite and infinite energy storage capacity at each sensor's battery. The derivations and some structural properties of the optimal energy allocation schemes are discussed, and numerical results presented.**

I. Introduction

Recent developments in wireless communications and electronics have enriched various practical applications of inexpensive, compact and versatile wireless sensor networks (WSNs), such as in military, civil engineering or healthcare [1]. In a WSN, each sensor in the network measures the quantity of interest, creates a local signal and then conveys it to the fusion center (FC), where the received signal is processed and the final estimation of the observed quantity computed accordingly. In order to achieve the optimal estimation performance under limited resources, it is crucial to wisely design energy management strategies for each specific WSN. Energy allocation and management issues in WSN have been studied quite extensively. For example, [2] considered a WSN with an orthogonal multiple-access scheme from sensors to the FC where the best linear unbiased estimator (BLUE) [3] is implemented and derived optimal power allocation policy that minimizes total distortion subject to sum power constraint at the sensors. In [4], the authors considered the same multiple-access scheme as [2] but studied the effect of spatial source correlation and determined the optimal power allocation that minimizes total transmission power under distortion constraints. In the past, most such studies focused on WSN environments where the sensors were equipped with fixed batteries which are hard to replace in general, leading to multiple works on maximizing lifetime, or minimizing power or energy consumption with a constraint on the quality of estimation at the fusion centre. In many applications, e.g. biomedical sensors implanted within the human body, the battery lifetime can be prolonged by integrating *energy harvesting techniques* that can harvest solar, magnetic, piezoelectric or vibrational energy. As the harvested energy arrival process in inherently random, energy management issues become extremely important for such applications where the performance of the WSN is evaluated over a time horizon. Several previous works have considered throughput maximization problems in wireless communication networks under *energy harvesting constraints* (EHC) via sophisticated dynamic programming [5] techniques. In [6], throughput optimal and mean delay optimal energy allocation policies over an infinite time horizon in a single sensor node were studied. In [7], the optimal energy allocation policies that maximize the mutual information of a wireless link were derived under either causal or full side information available at the transmitter. In [8], the authors considered optimal energy allocation problems - (i) maximizing the throughput by a deadline and (ii) minimizing transmission completion time of the communication session, over static and fading channels. Related work on minimizing distortion or estimation error for a remote estimation problem can be found in [9], where the authors consider the problem of finding a communication scheduling strategy for the sensor and an estimation strategy for the estimator that jointly minimize an expected sum of communication and distortion costs over a finite time horizon. See also [10] where state estimation with energy harvesting sensors is considered with fairness control.

In this paper we aim to design optimal energy allocation policies to minimize the total distortion or estimation error of a random Gaussian source measured by multiple sensors over a finite time horizon. Specifically, we consider a WSN with M sensors employing an analog transmission system, where the noisy sensor observations of a remote Gaussian source are amplified and forwarded to the FC over fading wireless channels, and each sensor has a battery that can be replenished by randomly harvested energy. We assume that the sensors are sampling measurements at a uniform interval over the entire time horizon and their data queues are always full. We consider two types of side information (SI) at each sensor similar to [7]: (i) *causal SI* which consists of past and present channel conditions and past values of harvested energy, and (ii) *full SI* which consists of past, present and future channel conditions and values of harvested energy. Our novel contributions can be summarized as follows: (i) Section III presents the optimal energy allocation policy for causal SI by using a dynamic programming technique and also obtains some structural properties of the optimal solution which help

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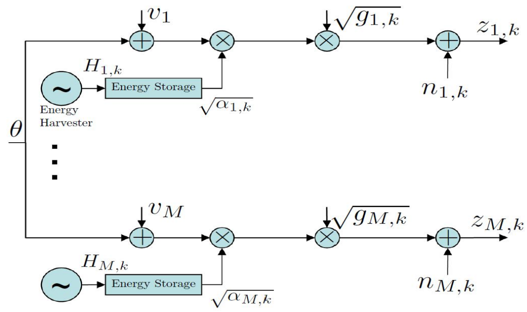

Fig. 1. System model

simplify the numerical search for the optimal policy, (ii) in Section IV, the optimal energy allocation solution for full SI is derived and is presented in closed form when the horizon length $K = 2$ for a single sensor case, and (iii) finally, we look the case of unlimited energy storage at the battery with full SI in Section V and derive the closed form optimal energy allocation scheme that minimizes the total distortion for any finite horizon length.

The remainder of this paper is organized as follows. Section II describes the system model. Section VI illustrates distortion performances from the derived energy allocation policies under various SI assumptions and battery capacities. Section VII presents some concluding remarks.

Notations: Boldface letters represent vectors. The value of parameter X at time slot k is denoted by X_k. Define $\mathbf{X}_{(:,k)} = [X_{1,k}, \ldots, X_{M,k}]^T$, $\mathbf{X}_{n,[k]} = [X_{n,1}, \ldots, X_{n,k}]^T$ and $\mathbf{X}_{[:,[k]]} = \left[\mathbf{X}_{1,[k]}^T, \ldots, \mathbf{X}_{M,[k]}^T\right]^T$. $\mathsf{p}_Y(.\mid z)$ represents the probability density function (pdf) of a random variable Y given z. $\mathsf{E}[\cdot]$ denotes expectation. $\delta(.)$ denotes Dirac delta function. $\frac{\partial \mathcal{L}}{\partial x^*}$ represents the partial differentiation of $\mathcal{L}$ with respect to x evaluated at $x = x^*$. Also, $\succeq$ and $\preceq$ represent componentwise inequalities.

II. System Model

We consider a wireless sensor network (WSN) with M sensors using an orthogonal multiple-access scheme as shown in Fig. 1. The observation $x_m(t)$ from sensor $m \in \mathcal{M} = \{1, \ldots, M\}$ within any time slot is represented as $x_m(t) = \theta(t) + v_m(t)$, where the random process $\theta(t)$ denotes the phenomenon of interest and $v_m(t)$ denotes the measurement noise. We assume that $\theta(t)$ and $v_m(t)$ are independent and identically distributed (i.i.d.) random processes and have zero mean and variances σ_θ^2 and σ_m^2, respectively. The sensors transmit their measurements via orthogonal fading channels to a fusion centre (FC) where $\theta(t)$ is estimated.

In this paper we assume that the transmitter adopts an analog amplify and forward uncoded strategy [11]. Thus, the transmitted signal from the m-th sensor is an amplified version of the signal $x_m(t)$ with a power amplifying factor $\alpha_{m,k}$ at the time slot $k \in \mathcal{K} = \{1, \ldots, K\}$. Without loss of generality, we presume that each of the K time slots are of duration 1. The energy consumption in time slot k can be modelled as $E_{m,k} = \alpha_{m,k}\left(\sigma_\theta^2 + \sigma_m^2\right)$ where $\sigma_\theta^2 + \sigma_m^2$ is the average power of $x_m(t)$ per symbol. At the FC, an estimate $\hat{\theta}(t)$ of $\theta(t)$ is obtained from the received signals $z_{m,k}(t) = \sqrt{\alpha_{m,k} g_{m,k}}\, x_m(t) + n_{m,k}(t)$, $m = 1, 2, \ldots, M$ where $\sqrt{g_{m,k}}$ is the channel power gain between the m-th sensor and the FC in slot k and $n_{m,k}(t)$ denotes i.i.d. additive white Gaussian noise (AWGN) with variance ξ_m^2. For simplicity, the channel noise variances are assumed to be the same for all M sensors in this paper, i.e. $\xi_m^2 = \xi^2$.

A. Distortion measure

We presume that BLUE is utilized at the FC due to its universality and simplicity (See [2] and references therein). The achievable distortion at the receiver in the k-th time slot is given by

$$D_k\left(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)}\right) = \begin{cases} \sigma_\theta^2 & , \mathbf{E}_{(:,k)} = \mathbf{0} \\ \sigma_\theta^2 \left[\sum\limits_{m=1}^{M} d_{m,k}\left(E_{m,k}, s_{m,k}\right)\right]^{-1} & , \text{Otherwise} \end{cases} \quad (1)$$

where $d_{m,k}\left(E_{m,k}, s_{m,k}\right) = \frac{E_{m,k} s_{m,k}}{1+\gamma_m^{-1} E_{m,k} s_{m,k}}$, $s_{m,k} = \frac{g_{m,k}}{\xi_m^2\left(\sigma_\theta^2+\sigma_m^2\right)}$ and $\gamma_m = \frac{\sigma_m^2}{\sigma_\theta^2}$.

Remark 1: It should be noted that the total distortion $D_k = \sigma_\theta^2\left[\sum\limits_{m=1}^{M} d_{m,k}\left(E_{m,k}, s_{m,k}\right)\right]^{-1}$ when $\mathbf{E}_{(:,k)} \succ \mathbf{0}$ [3]. However when $\mathbf{E}_{(:,k)} = \mathbf{0}$, the best estimate of $\theta(t)$ is simply $E[\theta(t)] = 0$, leading to the maximum distortion $D_k = \sigma_\theta^2$. It can be easily shown that $\sigma_\theta^2\left[\sum\limits_{m=1}^{M} d_{m,k}(.,.)\right]^{-1}$ is a convex function over $\mathbf{E}_{(:,k)}$ for a given $\mathbf{s}_{(:,k)}$ when $\mathbf{E}_{(:,k)} \succ \mathbf{0}$. The discontinuity of the distortion function at $\mathbf{E}_{(:,k)} = \mathbf{0}$ is not a problem as convex functions can be discontinuous at boundary points. In fact, using the property that the distortion function is a decreasing function of $\mathbf{E}_{(:,k)}$ and also that it attains its maximum value at $\mathbf{E}_{(:,k)} = \mathbf{0}$, it can be shown that $D_k\left(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)}\right)$ is convex for $\mathbf{E}_{(:,k)} \succeq \mathbf{0}$.

B. Energy storage

Assume that each sensor consumes energy $E_{m,k}$ at time slot k from the energy storage or battery. The battery energy of sensor m from slot 1 to k is given by $\mathbf{B}_{m,[k]} \succeq \mathbf{0}$. During time slot k, the energy harvester collects an amount of energy $H_{m,k} \geq 0$, which is then stored in the battery which has a maximum storage capacity $\tilde{B}_m$. At the time instant $k+1$, the energy in battery is typically assumed to follow a linear model [7],[8], given by,

$$B_{m,k+1} = \min\left\{\tilde{B}_m, B_{m,k} + H_{m,k} - E_{m,k}\right\} \quad , \forall k, \forall m \quad (2)$$

Note that the harvested energy $H_{m,k}$ is stored in time slot k and thus cannot be used in time slots 1 to k. Let $\tilde{\mathbf{B}} = \left[\tilde{B}_1, \ldots, \tilde{B}_M\right]^T$ be the vector of maximum storage energy of M batteries. In this article, the channel power gain $g_{m,k}$ and the harvested energy $H_{m,k}$ are assumed to be constant in slot k and vary from slot to slot in an i.i.d. manner. We also assume that harvested energy and channel power gains across the M sensors are also independent. We define $\mathbf{c}_{m,k} = \left(g_{m,k}, H_{m,k-1}, B_{m,k}\right)$ as the system state (SI) in slot k at sensor m, for $k = 1, 2, \ldots, K$ and $m = 1, 2, \ldots, M$.

Remark 2: Note that it is quite common to consider a more general finite-state Markov chain model for the fading channels and the harvested energy [7]. The techniques in our paper can be easily extended to this general case by using a Markov decision process (MDP) approach where the optimal transmission energy allocation problem can be solved as a stochastic control problem. This approach will be considered in an extended version of this paper where structural properties of the optimal energy allocation policy derived later in this paper will be used to prove computationally simple policies such as a threshold policy in the case of a binary action space (two possible choices for the transmission energy) [12].

III. Causal Side Information

In this section, we assume that in the k-th time slot, the FC knows the direct channel power gains of all sensors, $\mathbf{g}_{[:,[k]]}$ (due to availability of receiver side channel state information), the initial battery energy $B_{:,1}$ of all sensors and the harvested energy $\mathbf{H}_{[:,[k-1]]}$ of all sensors for time slots $l = 1, 2, \ldots, k-1$. The information $H_{m,k-1}$ (in the k-th slot) and $B_{m,1}$ (at the onset) can be communicated from the m-th sensor to the FC via a separate control channel. We aim to find the optimal energy allocation strategy for the sensors that minimizes the expected total distortion measure over K time slots by exploiting causal SI. To this end, the FC, being equipped with more computational and communication capability, can compute the optimal energy level $E^*_{m,k}$ offline and then store it in a lookup table. In real time, the FC, based on its causal SI, can access the appropriate optimal transmission energy and instruct the sensors to use the same via feedback. The optimization problem in this part can be stated as follows:

$$\underset{\mathbf{E}_{[K]}\succeq 0}{\min.}\; D_{\text{causal}} = \sum_{k=1}^{K} \mathsf{E}_{\mathbf{c}_k}\left[D_k\left(\mathbf{E}_{(:,k)}(\mathbf{c}_{(:,k)}), \mathbf{s}_{(:,k)}\right)\right] \quad (3)$$

subject to (2).

Note that the expectation of the distortion in the k-th slot (3) is computed over all random variables for a given initial storage energy $\mathbf{B}_{(:,1)}$ and it represents the expected distortion in time slot k. It is worth mentioning that the optimal $E_{m,k}$ cannot be computed independently since it is constrained by $B_{m,k}$ which is also the function of $E_{m,k-1}$. We can use dynamic programming techniques to solve this problem, as described below in Section III-A.

A. Optimal solution via dynamic programming

For a given initial state, the optimization problem (3) can be solved by using Lemma 3.1.

Lemma 3.1: Given $\mathbf{c}_{(:,1)} = \tilde{\mathbf{c}}_{(:,1)}$, the minimum distortion, $D^*_{\text{causal}} = Q_1(\tilde{\mathbf{c}}_{(:,1)})$, can be determined recursively through Bellman's equations, from $k = K$ to $k = 1$ as follows:

$$\begin{aligned}&Q_K(\mathbf{g}_{(:,K)}, \mathbf{H}_{(:,K-1)}, \mathbf{B}_{(:,K)})\\ &= \underset{\mathbf{0}\preceq\mathbf{E}_{(:,K)}\preceq\mathbf{B}_{(:,K)}}{\min.} D_K(\mathbf{E}_{(:,K)}, \mathbf{s}_{(:,K)}) \;= D_K(\mathbf{B}_{(:,K)}, \mathbf{s}_{(:,K)})\end{aligned} \quad (4)$$

$$\begin{aligned}Q_k(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)}) &= \underset{\mathbf{0}\preceq\mathbf{E}_{(:,k)}\preceq\mathbf{B}_{(:,k)}}{\min.} D_k(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)})\\ &+\bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)} - \mathbf{E}_{(:,k)})\end{aligned} \quad (5)$$

where $s_{m,k} = \frac{g_{m,k}}{\xi_m^2(\sigma_\theta^2+\sigma_m^2)}$ and $\bar{Q}_{k+1}$ is given by

$$\begin{aligned}&\bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{y}_{(:,k)})\\ &= \mathsf{E}_{\mathbf{g}_{(:,k+1)},\mathbf{H}_{(:,k)}}\big[Q_{k+1}(\mathbf{g}_{(:,k+1)}, \mathbf{H}_{(:,k)},\\ &\qquad \min\left\{\tilde{\mathbf{B}}, \mathbf{y}_{(:,k)} + \mathbf{H}_{(:,k)}\right\}) \mid \mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}\big]\end{aligned} \quad (6)$$

Given $\mathbf{g}_{(:,k)}$ in the present time slot and $\mathbf{H}_{(:,k-1)}$ in the past time slot, the expectation in (6) is computed over $\mathbf{g}_{(:,k+1)}$ and $\mathbf{H}_{(:,k)}$ which represent the channel power gain in the next slot and the harvested energy for the present slot, respectively. □

Proof: Similar to [7], one can apply Bellman's equations [5] and use (2) to obtain the above equations. ■

Intuitively, (4) implies that each sensor spends the remaining energy to obtain the minimum distortion in the last time slot (slot K). However, when $1 \leq k \leq K-1$, the sensor has to consider the tradeoff between the distortion in the current slot k and the expected value of the total distortion in the future slots.

B. Structural properties of the optimal solution

In this part, we will utilize Theorems 1 and 2 to show that the search of the optimal is in systematic manner.

Theorem 1: Given $\mathbf{g}_{(:,k)}$ and $\mathbf{H}_{(:,k-1)}$, the functions $Q_k(\cdot,\cdot,\mathbf{B}_{(:,k)})$ and $\bar{Q}_k(\cdot,\cdot,\mathbf{B}_{(:,k)})$ are convex in $\mathbf{B}_{(:,k)}$ for all k. □

Theorem 2: Given $\mathbf{g}_{(:,k)}$ and $\mathbf{H}_{(:,k-1)}$, the optimal energy allocation $E^*_{m,k}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)})$, which solves (4) and (5), is non-decreasing in $B_{m,k}$ regardless of $E^*_{n,k}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)})$, for all $k \in \mathcal{K}$, $m \in \mathcal{M}$ and $n \neq m$. □

Proofs of Theorem 1 follows from basic properties of convex analysis (see [13] for more details) and proof of Theorem 2 is provided in the Appendix.

Intuitively, Theorem 1 reveals the convexity property of the problem and Theorem 2 implies that searching for the optimal $E^*_{m,k}$ can be done systematically as $E^*_{m,k}(B)$ is non-decreasing in $B_{m,k}$ and the search can be carried out in one direction for a given $B_{m,k}$. To obtain $E^*_{m,k}$, one can first solve an unconstrained minimization (without EHC), i.e. $T^*_{m,k} = \arg \underset{T_{m,k}\geq 0}{\min.} \mathcal{D}_k(T_{m,k})$ where $\mathcal{D}_k(\mathbf{T}_{(:,k)}) = D_k(\mathbf{T}_{(:,k)}, \mathbf{s}_{(:,k)}) + \bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)} - \mathbf{T}_{(:,k)})$ for given $\mathbf{g}_{(:,k)}$ and $\mathbf{g}_{(:,k-1)}$. Then, the optimal $\mathbf{T}^*_{(:,k)}$ is given by solving $\frac{\partial \mathcal{D}_k(\mathbf{T}_{(:,k)})}{\partial T_{m,k}} = 0$, for all m. Hence, the optimal solution can finally be written as

$$E^*_{m,k}(B) = \begin{cases} 0 & , T^*_{m,k} \leq 0 \\ T^*_{m,k} & , 0 \leq T^*_{m,k} \leq B_{m,k} \\ B_{m,k} & , T^*_{m,k} \geq B_{m,k} \end{cases} \quad (7)$$

Note that since $E^*_{m,k}$ is a function of $E^*_{n,k}, n \neq m$, so the computation of $E^*_{m,k}$ cannot be carried out independently for each m.

IV. Full Side Information with Arbitrary $\tilde{B}$

In this section, we relax the problem by assuming that the sensor has prior (non-causal) knowledge of the initial energy $\mathbf{B}_{(:,1)}$, entire channel state $\mathbf{g}_{[:,[K]]}$ and harvested energy state $\mathbf{H}_{[:,[K-1]]}$ in any transmission slot for all k. Thus, the side information in this case can be treated as deterministic.

Lemma 4.1: Given $\mathbf{g}_{[:,[K]]}$ and $\mathbf{H}_{[:,[K-1]]}$, the minimum distortion, $D^*_{\text{full}} = Q_1(\mathbf{B}_{(:,1)})$, can be determined recursively through Bellman's equations, from $k = K$ to $k = 1$ as follows:

$$\begin{aligned}Q_K(\mathbf{B}_{(:,K)}) &= \underset{\mathbf{0}\leq\mathbf{E}_{(:,K)}\leq\mathbf{B}_{(:,K)}}{\min.} D_K(\mathbf{E}_{(:,K)}, \mathbf{s}_{(:,K)})\\ &= D_K(\mathbf{B}_{(:,K)}, \mathbf{s}_{(:,K)})\end{aligned} \quad (8)$$

$$\begin{aligned}Q_k(\mathbf{B}_{(:,k)}) &= \underset{\mathbf{0}\leq\mathbf{E}_{(:,K)}\leq\mathbf{B}_{(:,K)}}{\min.} \big[D_k(\mathbf{B}_{(:,k)}, \mathbf{s}_{(:,k)})\\ &+Q_{k+1}(\min\left\{\tilde{\mathbf{B}}, \mathbf{B}_{(:,k)} - \mathbf{E}_{(:,k)} + \mathbf{H}_{(:,k)}\right\})\big]\end{aligned} \quad (9)$$

for $k = 1, \ldots, K-1$. □

To gain some insight into the solution, we derive the optimal energy allocation strategy in the simple case of a single sensor when $K = 2$. Note that we drop the subscript m in this case for obvious reasons. Given full SI (B_1, H_1, g_1, g_2), the optimal transmission energy per slot, $0 \le E_1^*(B_1) \le B_1$ is given by

$$E_1^* = \begin{cases} 0 & , H_1 > \tilde{B} \text{ and } \mathcal{W}(B_1) \le \mathcal{W}(0) \\ B_1 & , H_1 > \tilde{B} \text{ and } \mathcal{W}(B_1) > \mathcal{W}(0) \\ 0 & , H_1 \le \tilde{B} \text{ and } \mathcal{W}(T_1^*) \le \mathcal{W}(0) \\ B_1 & , H_1 \le \tilde{B},\ \mathcal{W}(B_1) > \mathcal{W}(0) \\ & \quad \text{and } 0 < T_1^* \le B_1 + H_1 - \tilde{B} \\ T_1^* & , H_1 \le \tilde{B},\ \mathcal{W}(T_1^*) > \mathcal{W}(0), \\ & \quad \text{and } B_1 + H_1 - \tilde{B} \le T_1^* < B_1 \end{cases} \tag{10}$$

where $T_1^* = \frac{\sqrt{g_2}}{\sqrt{g_1}+\sqrt{g_2}}(B_1 + H_1)$ and $\mathcal{W}(E_1) = D_1(E_1, \frac{g_1}{\sigma_\theta^2+\sigma^2}) + D_2(\min\left\{\tilde{B}, B_1 + H_1 - E_1\right\}, \frac{g_2}{\sigma_\theta^2+\sigma^2})$. □
The above solution implies that the sensor can decide to turn off in slot 1 if the distortion from BLUE in the first slot is more than σ_θ^2. Otherwise, $E_1^* > 0$. Next, if the energy in time slot 2 exceeds the maximum energy storage capacity, the sensor will consume all available energy B_1. We compute the optimal energy $E_1^* = \frac{\sqrt{g_2}}{\sqrt{g_1}+\sqrt{g_2}}(B_1+H_1)$. Clearly, the sensor tends to spend more energy in the first slot if it knows that the channel gain $\sqrt{g_2}$ is high. Thus, the amount of remaining energy $\frac{\sqrt{g_1}}{\sqrt{g_1}+\sqrt{g_2}}(B_1+H_1)$ in slot 2 is sufficient to obtain the satisfactory level of total distortion.

V. Full Side Information with Infinite $\tilde{B}$

Finally, we derive the optimal energy allocation strategy of the distortion minimization problem with full SI available at each sensor and $\tilde{B}_m \to \infty$. Clearly, this is the most idealistic but impractical scenario. However, the performance achieved in this setting can be used as the benchmark for the smallest achievable distortion.

As each sensor has an infinite energy storage capacity with an initial stored energy of $B_{m,1}$, the finite horizon distortion minimizing problem solved in Lemma 4.1 can be formulated as follows:

$$\begin{aligned} \min_{\mathbf{E}_{(:,k)} \succeq 0,\ \forall k \in \mathcal{K}} . \ D_\infty \quad &= \sum_{k=1}^{K} D_k \\ \text{subject to } \sum_{l=1}^{k} E_{m,l} - B_{m,1} - \sum_{l=1}^{k-1} H_{m,l} \le 0, &\quad \forall m \text{ and } \forall k. \end{aligned} \tag{11}$$

Let $\lambda_{m,k}$ denote the Lagrange multiplier of the EHC of sensor m in k-th time slot. Thus, we can write the associated Lagrangian as

$$\begin{aligned} \mathcal{L} &= \sum_{k=1}^{K} \mathcal{L}_k(\mathbf{E}_{(:,k)}) \\ &= \sum_{k=1}^{K} \left(D_k + \sum_{m=1}^{M} \lambda_{m,k} \left[\sum_{l=1}^{k} E_{m,l} - B_{m,1} - \sum_{l=1}^{k-1} H_{m,l} \right] \right) \end{aligned} \tag{12}$$

From Karush-Kuhn-Tucker (KKT) necessary conditions, we have

$$\frac{\partial \mathcal{L}}{\partial E_{m,k}^*} \begin{cases} \ge 0 & , E_{m,k}^* = 0 \\ = 0 & , 0 < E_{m,k}^* < B_{m,1} + \sum_{l=1}^{k-1} H_{m,l} - \sum_{l=1}^{k-1} E_{m,l}^* \\ \le 0 & , E_{m,k}^* = B_{m,1} + \sum_{l=1}^{k-1} H_{m,l} - \sum_{l=1}^{k-1} E_{m,l}^* \end{cases} \tag{13}$$

Define $U_{m,k}^* = B_{m,1} + \sum_{l=1}^{k-1} H_{m,l} - \sum_{l=1}^{k-1} E_{m,l}^*$ which represents the largest amount of energy that the m-th sensor can use in slot k. Then, applying (13), the optimal energy allocation strategy can be expressed as given in Lemma 5.1 below.

Lemma 5.1: Suppose that each sensor has full SI and the energy capacity of the battery at its transmitter is infinite. The optimal energy allocation in slot k, $E_{m,k}^*$, is given by

$$E_{m,k}^* = \begin{cases} 0 & , \left[\sum_{m=1}^{M} d_{m,k}\left(\Omega_{m,k}^*, s_{m,k}\right) \right]^{-1} \ge 1 \\ \Omega_{m,k}^* & , \left[\sum_{m=1}^{M} d_{m,k}\left(\Omega_{m,k}^*, s_{m,k}\right) \right]^{-1} < 1 \end{cases} \tag{14}$$

where $\Omega_{m,k}^*$ is given by

$$\Omega_{m,k}^* = \begin{cases} 0 & , \Omega_{m,k} \le 0 \\ \Omega_{m,k} & , 0 < \Omega_{m,k} < U_{m,k}^* \\ U_{m,k}^* & , \Omega_{m,k} \ge U_{m,k}^* \end{cases} \tag{15}$$

where $\Omega_{m,k} = \frac{1}{\gamma_m^{-1}\sqrt{s_{m,k}}} \left(\frac{D_k}{\sigma_\theta} \sqrt{\nu_{m,k}} - \frac{1}{\sqrt{s_{m,k}}} \right)$ and $\nu_{m,k} = \left(\sum_{l=k}^{K} \lambda_{m,l}^* \right)^{-1}$. □
See [13] for a Proof of Lemma 5.1. We can also characterize the following property of $\nu_{m,K}$:

$$0 \le \nu_{m,1} \le \ldots \le \nu_{m,K} < \infty \tag{16}$$

Proof of all but the last inequality follows from the definition of $\nu_{m,k}$ above. The fact that $\nu_{m,K} < \infty$ can be shown by contradiction. This non-decreasing property of $\nu_{m,k}$ over k is helpful for the computation of the optimal solution. This property of $\nu_{m,k}$ can be also used to verify that the closed-form optimal solutions in Section IV and in this section are the same when $K = 2$ for the single-sensor scenario. (See Remark 3 in [13])

VI. Numerical Results

We present some numerical results on the performance of the optimal energy allocation strategies for both causal and full SI. We assume that the channel power gain $g_{m,k}$ and harvested energy $H_{m,k}$ are i.i.d. across the sensors and over the time slots. The channel power gain is assumed to be exponentially distributed (Rayleigh fading) with mean $\mathsf{E}[g_{m,k}] = \bar{g}$. The battery at each sensor is presumed to be identical, i.e. the maximum storage energy capacity $\tilde{B}_m = \tilde{B}$ for all m. The initial energy $B_{m,1}$ and the harvested energy $H_{m,k}$ are assumed to take a value in $\{0, 1, 2, 3\}$ (in millijoules) with equal probability. We use $\sigma_\theta^2 = 1$ W., $\xi_m^2 = 10^{-2}$ W. and $\gamma_m = 10^{-2}$. A comparative performance evaluation of various schemes is illustrated in Fig. 2 by plotting average total distortion per slot against the average channel gain $\bar{g}$. The results are generated by Monte Carlo simulations averaged over 10^4 independent random channel realizations.

Given a fixed $\tilde{B}$, average distortion per slot performances for both causal and full SI case are obviously the same when $K = 1$, since sensors cannot exploit the side information of the future slot. For $K > 1$ and $M = 2$, the distortion performance is improved when the number of total time slots K increases for both causal and full SI. As K increases, the average total distortion is reduced significantly when K is small, but the decrement becomes less substantial when K

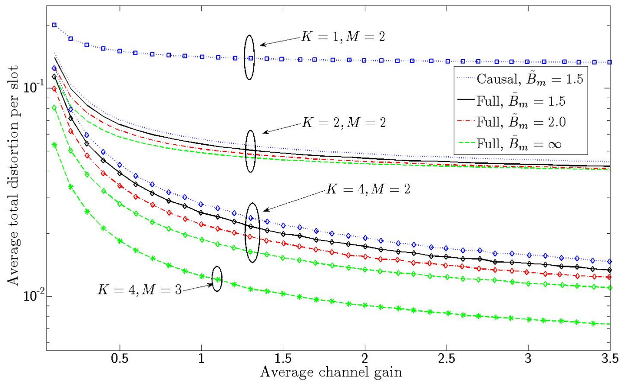

Fig. 2. Average total distortion per slot against average channel gain under various energy allocation schemes

is large for both causal and full SI cases. Further, the performance improvement is more significant as K grows when full SI is exploited as sensors can gain advantage from side information of the future slots. Given a fixed $K > 1$ and M, Fig. 2 reveals that, as the maximum energy storage capacity $\tilde{B}$ increases, the average total distortion per slot performance is also improved as the sensors have more flexibility in choosing transmission power levels. Finally, Fig. 2 also illustrates the advantage of multiple sensors as the distortion performance can be improved by increasing the number of sensors, which is well known.

VII. Conclusion

We considered the problem of minimizing the distortion incurred in estimating a random source via multiple sensors with energy harvesting capability over a finite time horizon. In this article, several energy allocation policies were derived based on the available side information (causal or full) and the size of energy storage (finite or infinite). We also discussed some structural properties of the optimal solutions. Numerical results were presented to illustrate the distortion performances corresponding to these various scenarios. Future work will consider the use of MDP based optimal control formulation over a finite or infinite horizon and derivation of computationally simple energy allocation policies such as threshold type policies based on similar structural properties as derived in this paper.

Appendix

Proof of Theorem 2

To facilitate the proof of Theorem 2, we use Lemma A.1.

Lemma A.1: Let $E_k^*(B) = \arg\max F(B, E_k)$, where the maximization is over the interval $[E_l(B), E_u(B)]$ and $E_l(B)$ and $E_u(B)$ are non-decreasing in B. Provided $F(\cdot)$ has non-decreasing differences in (B, E_k), i.e. $\forall E' \geq E$, $B' \geq B$,

$$F(B', E') - F(B, E') \geq F(B', E) - F(B, E) \qquad (17)$$

, the maximal and minimal selections of $E_k^*(B)$ denoted as $\bar{E}(B)$ and $\underline{E}(B)$, are non-decreasing.

Proof: Please refer to Theorem 2 in [14] ■

For given $\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}$, we consider $E_{m,k}$ and $B_{m,k}$ by fixing any arbitrary $E_{n,k}$ and $B_{n,k}$, $\forall n \neq m$. Then, the optimal $E^*_{m,k}(\mathbf{B}_{(:,k)})$ can be rewritten as

$$\begin{aligned} &E^*_{m,k}(\mathbf{B}_{(:,k)}) = E^*_{m,k}(B_{m,k}) \\ &= \arg \max_{0 \leq E_{m,k} \leq B_{m,k}} \big[-D_k(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)}) \\ &\quad -\bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)} - \mathbf{E}_{(:,k)})\big] \end{aligned} \qquad (18)$$

For arbitrary $E_{n,k}$, (18) suggests that $E_{m,k}$ is only constrained by energy storage in the battery $B_{m,k}$ which allows us to apply Lemma A.1. Let $\mathcal{F}_m(B_{m,k}, E_{m,k}) = -D_k(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)}) - \bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,k-1)}, \mathbf{B}_{(:,k)} - \mathbf{E}_{(:,k)})$ for fixed $E_{n,k}$ and $B_{n,k}$. It is obvious that the optimal $E_{m,k}$ is bounded by $E^l_{m,k}(B_{m,k}) = 0 \leq E_{m,k} \leq B_{m,k} = E^u_{m,k}(B_{m,k})$ and these bounds are non-decreasing in $B_{m,k}$. Now, we have to show that $\mathcal{F}_m(B_{m,k}, E_{m,k})$ satisfies Lemma A.1 by three following steps: (1) $-D_k(\mathbf{E}_{(:,k)}, \mathbf{s}_{(:,k)})$ is independent of $B_{m,k}$ and obviously has non-decreasing differences in $(B_{m,k}, E_{m,k})$. (2) Due to convexity of $\bar{Q}_{k+1}(\mathbf{g}_{(:,k)}, \mathbf{H}_{(:,\underline{k}-1)}, \mathbf{y}_{(:,k)})$ in $\mathbf{y}_{(:,k)}$ from Theorem 1, $G(y_{m,k}) = -\bar{Q}_{k+1}(\cdot,\cdot,\cdot)$ is concave in $y_{m,k}$ for fixed $y_{n,k}$. Thus for $y_{m,k} \leq z_{m,k}$ and $\epsilon_{m,k} \geq 0$, we have $\mathcal{G}(z_{m,k} + \epsilon_{m,k}) - \mathcal{G}(z_{m,k}) \leq \mathcal{G}(y_{m,k} + \epsilon_{m,k}) - \mathcal{G}(y_{m,k})$. By replacing $y_{m,k} = B_{m,k} - E'_{m,k}$, $z = B_{m,k} - E_{m,k}$ and $\epsilon_{m,k} = B'_{m,k} - B_{m,k}$ in $\mathcal{G}(.)$, we can use Lemma A.1 to show that $-\bar{Q}_{k+1}$ has non-decreasing differences in $(B_{m,k}, E_{m,k})$ for arbitrary given $E_{n,k}$ and $B_{n,k}$. (3) Since $\mathcal{F}_m(B_{m,k}, E_{m,k})$ has non-decreasing differences in $(B_{m,k}, E_{m,k})$ for any m, the proof of Theorem 2 is completed.

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