A REACTIVE APPROACH FOR MINING PROJECT EVALUATION UNDER PRICE UNCERTAINTY

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ABSTRACT

Projects in the mining industry are undertaken with the objective of maximizing economic value, which is near-universally measured by the Net Present Value (NPV), considering all capital expenditure and operating cash flow. This industry is usually considered high risk because of historically volatile commodity prices (directly impacting revenues) and the fact that very large capital expenditures are required upfront for the construction of processing, mining and transport infrastructure. In order to optimize overall investment returns in a mining project, it is thus essential to use the best possible project valuation method so that the sizing of initial capital expenditures are appropriate to the expected returns and risks. A commonly used method for mining project evaluation calculates the expected NPV of a mine plan based upon the expected future commodity price given the current spot price. This method often undervalues a mining project since it ignores future price uncertainty and does not allow for managerial flexibility or optionality. This paper presents an alternate approach to mining project evaluation – the "reactive" approach. This "reactive" approach emulates a strategy that a real operating mine would undertake over its mine life by modifying the mine plan in each new period in response to the latest information on metal price. This paper also demonstrates that the "reactive" approach can estimate the mine project value more accurately by recognizing additional value due to the presence of management flexibility and optionality.

INTRODUCTION

It is common practice in the mining industry to evaluate a project by assuming that the forward commodity price is known over the life-of-asset. Software tools are then used to develop life-of asset mining plans. These plans encapsulate decisions such as which material to excavate and when (the mining schedule), where to send the material (the waste dump *vs.* processing plants) and the sizing of mine infrastructure (truck and excavator fleets and process plant). Nowadays, it is becoming more common to develop these long term strategic plans using software optimization tools, such as MinMax planner, Whittle strategic mine planning, Earthworks multimine scheduler (References 5, 9 and 12) and proprietary tools such as Blasor (Stone *et al.*, 2004), which allow the mine planner to maximize an objective such as net discounted operational cash flow (a "chunky economics" proxy for Net Present Value, NPV). An obvious problem with this approach is that the value objective is highly sensitive to the assumed deterministic forward commodity price which, in most cases, historical data demonstrates to be quite volatile. This raises two questions for the mine planner—the first is whether this valuation approach can be used to properly rank different investment options (*viz*: proposed mine A against proposed mine B) and the second is whether the mine plan which is optimal for the assumed forward commodity price will also be optimal, or more reasonably near-optimal, when price uncertainty is accounted for.

This paper primarily addresses the first of these issues, that of accurate life-of-asset valuation. In seeking to apply a better valuation method, we conclude that the best mine planning strategy is one that is updated throughout the life-of-asset according to the evolving commodity spot price and the assumed forward price model. This is the so-called "reactive" mining project evaluation method. Whilst this method does not deliver a single static go-forward mine plan, as traditional planning approaches do, it does provide the mine planner with everything she/he needs – a method for making the best strategic mining decisions depending upon spot price at any stage of the asset's life.

In the remainder of the paper, we will first introduce the mathematical model that we have applied to determine an optimal mine plan for a given price scenario. In the following section, we will introduce a log-normal mean reverting price model used in our research to describe the stochastic forward commodity price. We will then present how the expected NPV of a mining project is calculated based upon the expected price scenario given the current spot price. Next, we will present the "reactive" mining project evaluation approach and then how an upper bound on the achievable expected NPV of the mining project can be obtained assuming perfect knowledge of future price. A case study will then be presented to illustrate that by using only the expected price in estimating the value of a project (through calculation and valuation of a static forward mine plan) the project value is underestimated whereas using a reactive approach to estimate the project value, a higher and more accurate valuation (which encapsulates the value of real options exercisable by mine management in response to price variability) is obtained. Finally, we will draw conclusions.

MILP MODEL FOR MINE PLAN OPTIMIZATION UNDER DETERMINISTIC COMMODITY PRICE

The NPV of a mining project is commonly maximized by simultaneous optimization of the material extraction sequence and cut-off grades (COG) for a single orebody block model. In recent years, many mathematical techniques have been applied to optimize the material extraction sequence and COG using deterministic forward metal prices (Akaike & Dagdelen, 1999; Knowles, 1999; Caccetta & Hill, 2003). In this research, we use a mixed integer linear programming (MILP) formulation

(Menabde *et al*., 2004) as the mathematical model to maximize the NPV for the business. The objective function of the MILP model is:

$$\text{maximize} \sum_{i}^{M}\sum_{j}^{G}\sum_{t}^{T} V_{i,j,t}\, x_{i,j,t}\, d_t \tag{1}$$

subject to constraints:

$$\sum_{i,j,t} R_i x_{i,j,t} \le R_t \,, \qquad \textit{for all } t \tag{2}$$

$$\sum_{i,j,t} Q_{i.j} x_{i,j,t} \le Q_t \,, \qquad \textit{for all } t \tag{3}$$

$$\sum_{\tau=1}^{t}\sum_{j}^{G} x_{i,j,\tau} \le \sum_{\tau=1}^{t}\sum_{j}^{G} x_{k,j,\tau} \,, \qquad \textit{for all } i, t \textit{ and } k \subset S_i \tag{4}$$

$$\sum_{j=1}^{G} \delta_{j,t} = 1 \,, \qquad \textit{for all } t \tag{5}$$

$$x_{i,j,t} \le \delta_{j,t} \,, \qquad \textit{for all } i, j \textit{ and } t \tag{6}$$

where

- T is the number of planning periods;
- M is the number of panels;
- G is the number of all possible cut-off grades;
- R_i is the total rock tonnage in panel *i*;
- $Q_{i,j}$ is the total ore tonnage in panel *i* when mined and processed with COG *j*;
- R_t is the maximum mining capacity in period *t*;
- Q_t is the maximum processing capacity in period *t*;
- S_i is the set of panels that must be removed before starting panel *i*;
- $V_{i,j,t}$ is the value of panel *i* when mined and processed with COG *j*;
- d_t is the discount factor in period *t*;
- $x_{i,j,t}$ is the fraction of panel *i* extracted with COG *j* in period *t*;
- $\delta_{j,t}$ is a binary variable controlling the selection of the COG applied in period *t*.

MODELING COMMODITY PRICE UNCERTAINTY

The modeling of commodity price uncertainty has attracted a great deal of attention in the mathematical finance literature. Even though recent multi-factor models are very promising for explaining commodity price behavior (Schwartz, 1997; Cortazar et al., 1999), for simplicity, we will use a model where the commodity spot price is assumed to follow the stochastic process (Schwartz, 1997):

$$dp = \eta(\ln \overline{p} - \ln p)\,pdt + \sigma\, pdz \tag{7}$$

Let $y = \ln p$, applying Ito's Lemma allows characterization of the log price by an Ornstein-Uhlenbeck stochastic mean reverting process:

$$dy = \eta(\overline{y} - y)dt + \sigma dz \tag{8}$$

with

$$\overline{y} = \ln\left(\overline{p}\right) - \frac{\sigma^2}{2\eta} \tag{9}$$

where

$\overline{p}$ is the long-run equilibrium commodity price;

η measures the speed of mean reversion to the long run mean log price $\overline{y}$;

dz is an increment to a standard Brownian motion;

σ refers to the price volatility rate.

Using the properties of the log-normal distribution, the expectation of the forward price given the current spot price p_0 is given by

$$E_t(p_0) = \exp\left\{\ln(p_0)e^{-\eta t\Delta t} + \left[\ln\left(\overline{p}\right) - \frac{\sigma^2}{2\eta}\right](1 - e^{-\eta t\Delta t}) + \sigma^2(1 - e^{-2\eta t\Delta t})/4\eta\right\} \tag{10}$$

where

p_t is the spot price at time t;

Δt is the fixed time interval from time t to t +1;

The correct discrete-time format for the continuous-time process of mean-reversion is the stationary first-order autoregressive process (Dixit & Pindyck, 1994), so the sample path simulation equation for y_t is performed by using the exact discrete-time expression:

$$y_t = y_{t-1}e^{-\eta\Delta t} + \overline{y}(1 - e^{-\eta\Delta t}) + N(0,1)\sigma\sqrt{(1 - e^{-2\eta\Delta t})/2\eta} \tag{11}$$

where

$N(0,1)$ is the normally distributed random variable.

By substituting Equation 11 to $p = e^y$, we have the exact discrete-time equation for p_t given by

$$p_t = \exp\left\{\ln(p_{t-1})e^{-\eta\Delta t} + \left[\ln\left(\overline{p}\right) - \frac{\sigma^2}{2\eta}\right](1 - e^{-\eta\Delta t}) + N(0,1)\sigma\sqrt{(1 - e^{-2\eta\Delta t})/2\eta}\right\} \tag{12}$$

Let $P = \{p_t, t = 0,...,T\}$ denote a price scenario with spot prices p_t , where p_t is determined by Equation 12. Let $\boldsymbol{P_0} = \{E_t(p_0), t = 0,...,T\}$ denote the expected price scenario given the current spot price p_0 , where $E_t(p_0)$ is determined by Equation 10. Figure 1 presents a sample path of copper price simulated using the above price model together with a path of the expected price scenario $\boldsymbol{P_0}$ and the actual historical price path from which the sample price path is constructed.

MINE PLAN VALUATION USING THE CONDITIONAL EXPECTATION OF FUTURE PRICE

We propose N possible forward price paths based on the current spot price and each price path represents an equally likely reality of future price. We seek to determine a schedule that gives a NPV which is highest on average when applied to all price realizations. To emphasize that this

NPV is obtained using the expected forward price conditional on the current spot price ($\boldsymbol{P_0}$), we refer to it as “the expected NPV conditional on the current spot price”.

We maximize the NPV of the mining project using $\boldsymbol{P_0}$ to get a single schedule s^* that will be optimal on average over all N price realizations. If we use $NPV(\boldsymbol{P_0}, s^*)$ to denote this NPV, we have

$$NPV(\boldsymbol{P_0}, s^*) = \max_{i,j,t} \sum_{i}^{M} \sum_{j}^{G} \sum_{t}^{T} V_{i,j,t}^{\boldsymbol{P_0}} x_{i,j,t}^{\boldsymbol{P_0}} d_t \quad (13)$$

However, the conditionally expected price scenario cannot represent the future price precisely in reality. If we implement the schedule s^* according to the real uncertain future price, we may get a NPV lower or higher than that obtained using the conditionally expected price scenario. Therefore, to evaluate a mining project using the conditional expectation of future price, we will compute the NPV when the schedule s^* is applied with a given price realization. We denote this expected NPV with $NPV_{conditional}$ and it is given by

$$NPV_{conditional} = E[NPV(P_k, s^*)] \quad (14)$$

where $NPV(P_k, s^*)$ represents the NPV obtained by valuating the schedule s^* using price realization P_k.

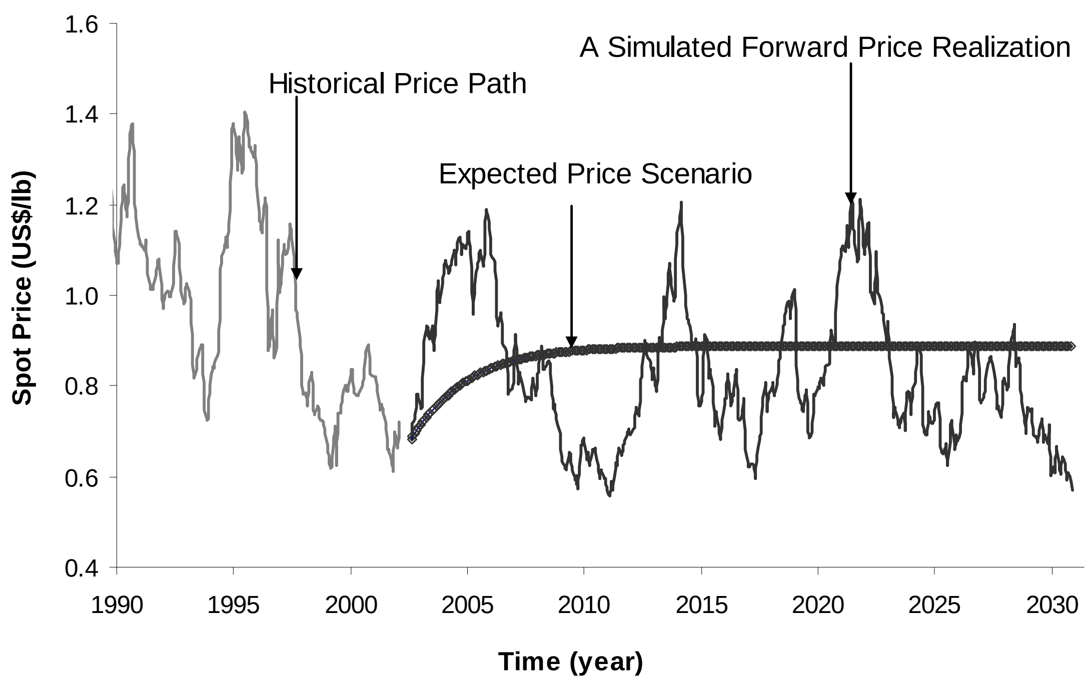

Figure 1: Simulation of the copper price using a log-normal mean reverting price model (Schwartz, 1997)

MINE PLAN VALUATION USING A REACTIVE APPROACH

The method using the conditional expectation of future price calculates a single static go-forward life-of-mine plan based upon the expected price scenario given the current spot price, regardless of expected volatility in forward price. Hence, the mine plan may be undervalued without accounting for the implicit flexibility or optionality that mine management may exercise. In a real operation, the management of a mine can improve or protect the NPV in response to future price fluctuation by modifying the mine plan based on the latest information on commodity price. Emulating this strategy, we propose a "reactive" approach for mining project evaluation.

Given the N equally likely price realizations of future price, for each realization, the "reactive" approach iteratively updates the mine plan according to the new information on commodity spot price at each time t. At time t = 0, we determine the schedule by maximizing the NPV using the conditionally expected price realization $\boldsymbol{P_0}$ and then implement the schedule only for the first time period until we have the new price available at time t = 1 (here, we assume the price keeps constant within one time period). We then re-optimize the schedule for the remaining material using the expected prices of the new spot price p_1 and once again only implement the new schedule for the first time period until the price is updated at time t = 2. This process goes on until the number of time periods reaches a planning horizon *T*. In this way, we can obtain a NPV based on an updated schedule according to the evolving commodity spot price under a given price realization and then compute the expected NPV over N price realizations. We use $NPV_{reactive}$ to denote the expected NPV obtained by the "reactive" approach. To express this method mathematically, we have equations 15-17:

$$NPV(\boldsymbol{P}_{k,\tau}, s^*_{k,\tau}) = \max_x \sum_{t=1}^{T-\tau} \sum_j \sum_i V_{i,j,t}^{\boldsymbol{P}_{k,\tau}} x_{i,j,t}^{\boldsymbol{P}_{k,\tau}} d_t \tag{15}$$

$$NPV(P_k, s_{reactive}(P_k)) = \sum_{\tau=0}^{T-1} V_{i,j,1}^{\boldsymbol{P}_{k,\tau}} x_{i,j,1}^{\boldsymbol{P}_{k,\tau}} d_{\tau+1} \tag{16}$$

$$NPV_{reactive} = E[NPV(P_k, s_{reactive}(P_k))] \tag{17}$$

where $\boldsymbol{P}_{k,\tau}$ represents the expected price scenario conditional on the spot price $p_{k,\tau}$ at time τ under price realization P_k; $NPV(\boldsymbol{P}_{k,\tau}, s^*_{k,\tau})$ represents the NPV at time τ with the optimal schedule $s^*_{k,\tau}$ under the conditionally expected price scenario $\boldsymbol{P}_{k,\tau}$ and $NPV(P_k, s_{reactive}(P_k))$ represents the NPV with the schedule $s_{reactive}(P_k)$ obtained by the "reactive" approach under price realization P_k. Thus the reactive approach calculates an evolving optimal mine plan for each of the price realizations. The value of the mining project is the average of the valuations of the optimal evolving schedules over all price realizations.

MINE PLAN VALUATION WITH PERFECT KNOWLEDGE OF FUTURE PRICE

In the above sections, we have introduced a method which calculates the expected NPV conditional on the current spot price. We have also presented a "reactive" approach which calculates the expected NPV based on a mine plan initially obtained according to the expected price scenario given the current spot price and then updated throughout the life-of-asset in response to the new commodity spot price at each new period. Both methods are based on the fact that no one can accurately predict the future commodity price. However, if we assume that we know the future price perfectly well, we will tailor our schedule to the real future price. Given the N possible future price realizations, for each of them we calculate the optimal mine plan by maximizing the NPV. Each of the N realizations is equally likely to forecast the actual future price but we don't know which one is real Therefore, we compute the expected value over N price realizations and refer to the expected NPV obtained in this way as "the expected NPV with perfect knowledge of future price".

Let $NPV(P_k, s^*(P_k))$ denote the NPV maximized by an optimal schedule $s^*(P_k)$ tailored to the price realization P_k. The expected value of $NPV(P_k, s^*(P_k))$, denoted as $NPV_{perfect}$ representing the expected NPV of a mine plan assuming the perfect knowledge of future price is known prior to producing a schedule, is given by:

$$NPV_{perfect} = E[NPV(P_k, s^*(P_k))] \qquad (18)$$

Note that $s^*(P_k)$ in Equation 18 is the optimal schedule for the individual realization P_k. s^* in Equation 14 is the optimal schedule which maximizes the NPV under the conditionally expected price scenario $\boldsymbol{P_0}$ and $s_{reactive}(P_k)$ in Equation 17 is the "reactive" schedule for price realization P_k. Therefore, we always have

$$NPV(P_k, s^*(P_k)) \geq NPV(P_k, s^*) \qquad (19)$$

and

$$NPV(P_k, s^*(P_k)) \geq NPV(P_k, s_{reactive}(P_k)) \qquad (20)$$

It is straightforward that

$$NPV_{perfect} \geq NPV_{conditional} \qquad (21)$$

and

$$NPV_{perfect} \geq NPV_{reactive} \qquad (22)$$

Equations 21 and 22 show $NPV_{perfect}$ is an upper bound of the achievable expected NPV of a mining project given price uncertainty. The concept of using the assumption of perfect knowledge of the future to obtain an upper bound on the expected NPV of a mining operation is first proposed by Froyland *et al*. (2004), accounting for geological uncertainty of the orebody in the valuation of a mining project.

CASE STUDY

In this section, we will use the three methods described above to value an example copper mine. This copper mine contains about 16,000 blocks with a total rock tonnage of 962 million tonnes. We use Monte-Carlo simulations to obtain 25 equally likely copper price realizations based on the log-normal mean reverting price model introduced earlier in this paper. The initial spot price p_0 and the equilibrium price $\overline{p}$ used in the price model are \$0.25/lb and \$0.45/lb, respectively. The mean reversion rate η and volatility rate σ are estimated from the historical price data using an autoregressive parameter identification algorithm.

The expected NPV computed using the three different methods introduced in the above sections are tabulated in Table 1. Some headers in Table 1 are explained in Table 2. We can see that the valuation from the "reactive" approach is $52 million greater than "the expected NPV conditional on the current spot price".

Table 1: The expected NPV obtained by using three mining project evaluation methods under price uncertainty

$NPV_{conditional}$ *(million $)*	$NPV_{reactive}$ *(million $)*	$NPV_{perfect}$ *(million $)*	$E(\%NPV_{conditional/perfect})$ *(%)*	$E(\%NPV_{reactive/perfect})$ *(%)*	$\%\Delta NPV$
303	358	370	79.3	96.8	82%

Table 2: Explanation of Notation in Table 1

Notation	Explanation
$E(\%NPV_{conditional/perfect})$	The expected value of $\%NPV_{conditional/perfect}$. $\%NPV_{conditional/perfect}$ represents the ratio of the NPV obtained by implementing the schedule determined using the conditionally expected price scenario $\boldsymbol{P_0}$ over the optimal NPV under price scenario P_k, given by $\frac{NPV(P_k, s^*)}{NPV(P_k, s^*(P_k))} * 100\%$.
$E(\%NPV_{reactive/perfect})$	The expected value of $\%NPV_{reactive/perfect}$. $\%NPV_{reactive/perfect}$ represents the ratio of the NPV obtained by the "reactive" valuation approach over the optimal NPV under price scenario P_k, given by $\frac{NPV(P_k, s_{reactive}(P_k))}{NPV(P_k, s^*(P_k))} * 100\%$.
$\%\Delta NPV$	The percentage improvement in the expected NPV by the reactive approach over the gap between $NPV_{perfect}$ and $NPV_{conditional}$, given by $\frac{NPV_{perfet} - NPV_{reactive}}{NPV_{perfect} - NPV_{conditional}} * 100\%$.

In Figure 2, each vertical column corresponds to a single price realization P_k. The dots are 25 values of $NPV(P_k, s^*)$, *i.e.* the NPV from the application of the optimal schedule based upon the conditionally expected price $\boldsymbol{P_0}$, under price realization P_k. The squares are 25 values of $NPV(P_k, s_{reactive}(P_k))$ obtained by the "reactive" approach for price realization P_k. The triangles are 25 values of $NPV(P_k, s^*(P_k))$, *i.e.* the NPV from the optimal schedule tailored for price realization P_k. Thus these triangles appear on the top of vertical spread of points. It is obvious from Figure 2 that the "reactive" approach produces higher NPV than the method using conditional (on current spot price) expectation of future price. For price realizations 4, 12 and 24, the squares almost completely overlap the triangles. This means that for these cases the NPV valuation using the "reactive" approach is very close to that of the optimal (but unrealizable) schedule for that price realization.

Figures 3 (a) and (b) show the histograms of the NPV for each price realization using the method with expected forward price conditional on current spot price and the "reactive" approach, respectively. Those figures imply that using a "reactive" valuation method, the ability of mine operators to mitigate loss from poor (unfortunate) price outcomes, is properly valued. This means that the spread of low-side outcomes predicted by approach using the expected price scenario conditional on the current spot price are unrealistic and lead to a project valuation which is inaccurate and too low.

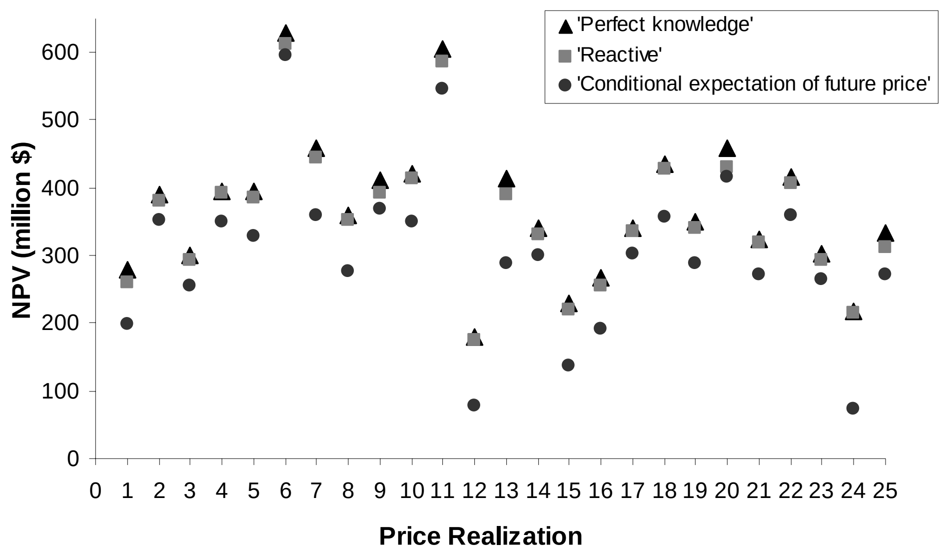

Figure 2: NPV for each price realization using three mining project evaluation methods

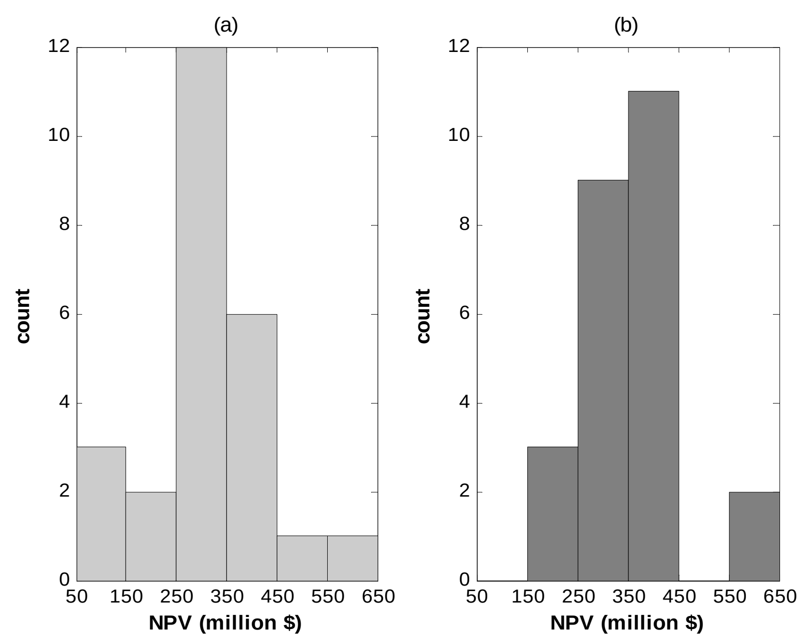

Figure 3: Histograms of NPV for each price realization using the method with expected price conditional on the current spot price (a) and the reactive approach (b)

To take a close look at the behavior of the "reactive" approach and the method using the expected price scenario conditional on the current spot price under different price realizations, we present Figures 4 and 5 below. Figures 4 (a) and (b) show the plots of price realizations 24 and 6 respectively. Price realization 24 has the largest difference between the NPV from the "reactive" approach and "the expected NPV conditional on current spot price" whilst price realization 6 shows the smallest difference. Figures 5 (a) and (b) show the accumulated NPV at each time period from implementing the static schedule based on the conditionally expected price scenario $\boldsymbol{P_0}$ and the "reactive" schedule according to the evolving spot prices under price realizations 24 and 6, respectively.

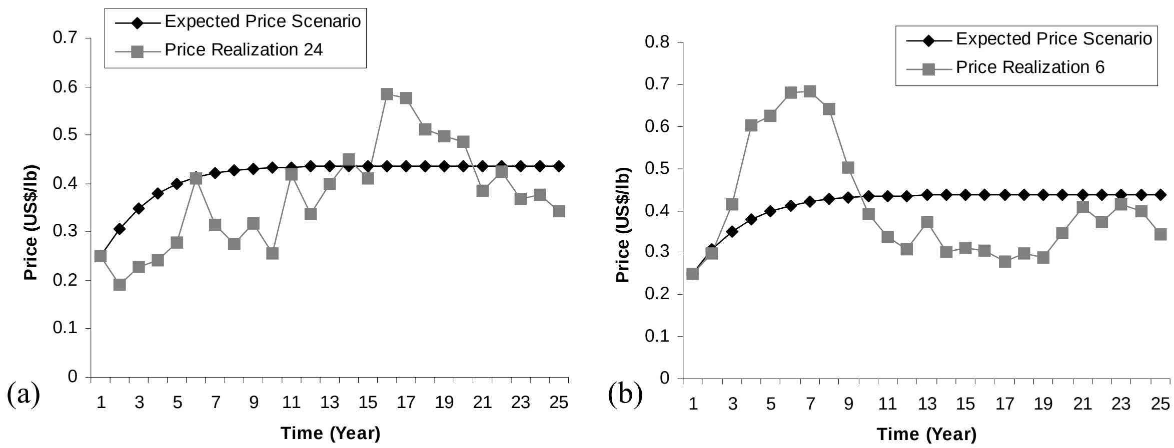

Figure 4: Plots for price realization 24 (a) and price realization 6 (b)

The conditionally expected price path in Figure 4 shows an expected "reversion to mean" behavior from a lower initial price to a higher long term equilibrium price. In the optimal mine plan based on this conditionally expected price scenario, the mining operation starts in years 3 and ends in year 13. This is because in the first two years the price is relatively low and is expected to increase in the future. Therefore the mining operation defers and waits for the price to go up. If we implement this mine plan under real future price, low grade material may still be processed at a low price resulting in a profit loss and sometimes this may cause a negative NPV (*e.g.* years 3 to 5 under price realization 24 in Figure 5(a)), whereas for other price realizations (*e.g.* price scenario 6) we may still obtain a positive NPV (Figure 5(b)).

The "reactive" approach emulates a mine planning strategy which reacts to the commodity price changes and accordingly updates the mine plan. Therefore, in the "reactive" mine plan under price realization 24 (Figure 5 (a)), the mining operation starts in year 6 but stops in year 7 and then resumes in year 11 (the accumulated NPV remains unchanged during years 7 and year 10) to protect the possible profit loss due to the low copper price during year 1 to year 6 and year 7 to year 11. The "reactive" mine plan under price realization 6 (Figure 5 (b)) also reacts to the low price during year 13 and year 21 by stopping the mining operation from year 14 to year 20. Note that the "stop then resume" behavior of mining operation, characteristic of a "reactive" approach, mimics exercising the mothballing option without considering mothballing and mine reopening costs. Therefore, we argue that the "reactive" valuation approach can recognize additional value of a mining project by incorporating managerial flexibility and optionality.

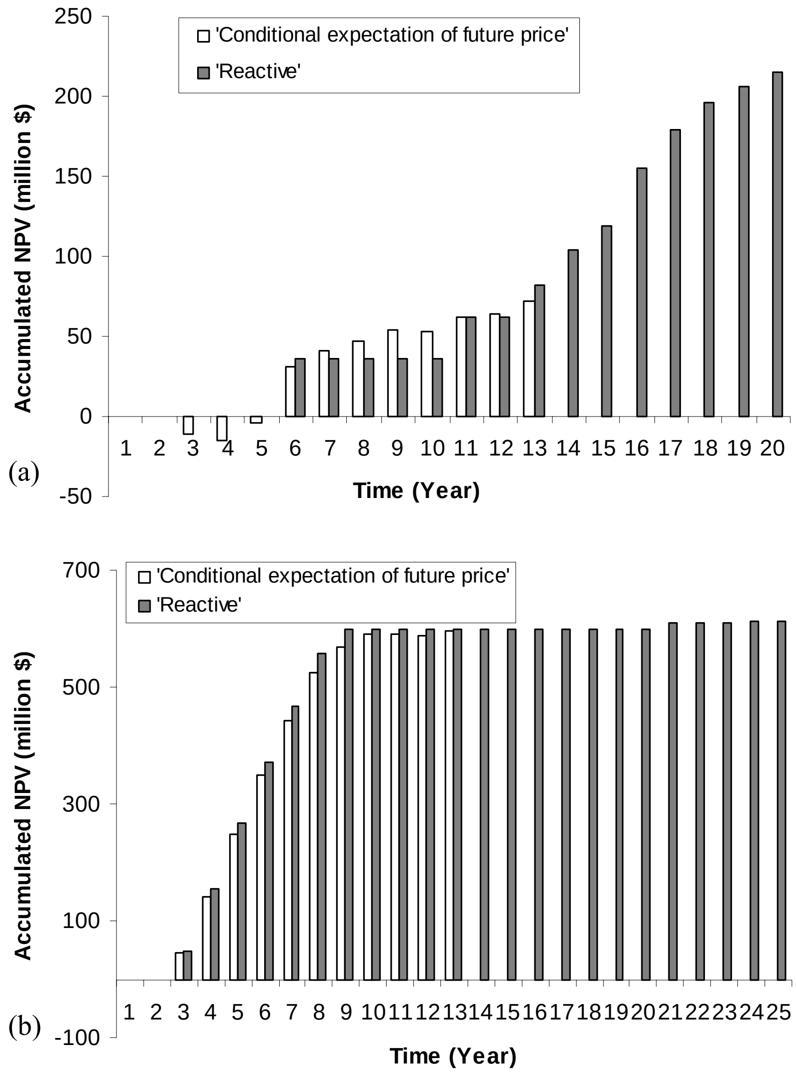

Figure 5: Accumulated NPV from implementing the static schedule based on conditional expectation of future price and the reactive schedule according to the evolving spot price under price scenario 24 (a) and price scenario 6 (b)

CONCLUSIONS

This paper has presented an alternate life-of-mine evaluation method under price uncertainty – the "reactive" approach. This "reactive" approach calculates the expected NPV based on mine plans that are updated according to the evolving commodity spot price and assumed forward price model. Compared with the commonly used evaluation method which is based on a static forward mine plan obtained according to the expected price scenario given the current spot price, the "reactive" approach has led to a higher project valuation, which is close to the absolute upper bound on the achievable expected NPV of the mining project, in the example studied in this paper. This higher valuation arises from the capability of the "reactive" approach to incorporate managerial flexibility and optionality in mining project evaluation.

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