

## Gluons, quarks and deconfinement at high density

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We compute gluon and quark propagators in 2-colour QCD at large baryon chemical potential  $\mu$ . The gluon propagator is found to be antiscreened in the superfluid, confined phase and screened in the large- $\mu$ , deconfined phase. We present the first attempt to determine corresponding electric and magnetic gluon masses. The quark propagator undergoes dramatic modifications in the superfluid region as a result of the formation of a superfluid gap. These modifications include the appearance of zero crossings in the vector part of the (normal) quark propagator, a large suppression of the scalar part, and the emergence of anomalous propagation.

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## 1. Introduction

Determining the phase diagram of QCD at large baryon density and small temperatures remains one of the outstanding problems of strong interaction physics. This problem is of both theoretical and phenomenological interest: on the theoretical side, an exceptionally rich phase structure may be present, while the phenomenological interest is spurred by the possibility that some of these phases may be present in compact stars, and may have observable consequences.

Direct lattice simulations of QCD at high density and low temperature are hindered by the sign problem, so alternative approaches are required. One such approach is to study QCD-like theories which may be simulated on the lattice, and apply the lessons learnt from these theories to the case of real QCD. Foremost among these theories is QCD with gauge group SU(2) (QC<sub>2</sub>D).

Medium modifications of quark and gluon propagators is one topic where QC<sub>2</sub>D may directly inform real QCD calculations. The gluon propagator is used as input into the gap equation for the superfluid gap at high density, but the propagator that is used is usually based either on (resummed) perturbation theory or on simple generalisations of the vacuum propagator. Nontrivial medium modifications or nonperturbative effects may thus significantly alter the results. The quark propagator encodes information about effective quark masses and gap parameters, while first-principles results for gluon and quark propagators together can be used to check the assumptions going into dense QCD calculations in the Dyson–Schwinger equation framework [1, 2].

## 2. Formulation

We will be using  $N_f = 2$  degenerate flavours of Wilson fermion, with a diquark source  $j$  included to lift low-lying eigenvalues and study diquark condensation without uncontrolled approximations. The fermion action can be written

$$S_F = \left( \bar{\Psi}_1 \ \Psi_2^T \right) \begin{pmatrix} M(\mu) & j\gamma_5 \\ -j\gamma_5 & M(-\mu) \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \bar{\Psi}_2^T \end{pmatrix} \equiv \bar{\Psi} \mathcal{M}(\mu) \Psi, \quad (2.1)$$

where  $M(\mu)$  is the usual Wilson fermion matrix with chemical potential  $\mu$ . It satisfies the symmetries

$$KM(\mu)K^{-1} = M^*(\mu), \quad \gamma_5 M^\dagger(\mu) \gamma_5 = M(-\mu), \quad (2.2)$$

with  $K = C\gamma_5\tau_2$ . The first of these is the Pauli–Gürsey symmetry. The inverse of  $\mathcal{M}$  is the Gor'kov propagator,

$$\mathcal{G}(x, y) = \mathcal{M}^{-1} = \begin{pmatrix} \langle \Psi_1(x) \bar{\Psi}_1(y) \rangle & \langle \Psi_1(x) \Psi_1^T(y) \rangle \\ \langle \bar{\Psi}_2^T(x) \bar{\Psi}_1(y) \rangle & \langle \bar{\Psi}_2^T(x) \Psi_1^T(y) \rangle \end{pmatrix} = \begin{pmatrix} S(x, y) & T(x, y) \\ \bar{T}(x, y) & \bar{S}(x, y) \end{pmatrix}. \quad (2.3)$$

The components  $S$  and  $T$  denote normal and anomalous propagation respectively. The Gor'kov propagator has the symmetry properties

$$K\mathcal{G}K^{-1} = \begin{pmatrix} S^* & -T^* \\ -\bar{T}^* & \bar{S}^* \end{pmatrix}, \quad (2.4)$$

$$\bar{S}(x, y) = -S(y, x)^T, \quad T(x, y) = T(y, x)^T, \quad \bar{T}(x, y) = \bar{T}(y, x)^T. \quad (2.5)$$

We will also write the inverse propagator as

$$\mathcal{G}^{-1} = \begin{pmatrix} N & \Delta \\ \bar{\Delta} & \bar{N} \end{pmatrix}, \quad (2.6)$$

which has the same symmetry properties as  $\mathcal{G}$ .

The normal propagator  $S$  can in general be written in terms of four momentum-space form factors,

$$S(p) = \vec{p} S_a(\vec{p}^2, p_4) + S_b(\vec{p}^2, p_4) + \gamma_4(p_4 - i\mu)S_c(\vec{p}, p_4) + i\gamma_4 \vec{p} S_d(\vec{p}^2, p_4). \quad (2.7)$$

In QC<sub>2</sub>D the Pauli–Gürsey symmetry ensures that all form factors are purely real. The structure of the anomalous propagator depends on the pattern of diquark condensation. Assuming that the condensation occurs in the colour singlet channel with quarks of unequal flavour, the anomalous propagator can be written as  $T(p) = T'(p)C\Gamma\tau_2$  (and similarly for the anomalous part  $\Delta(p)$  of the inverse propagator), where  $\Gamma = \gamma_5$  for condensation in the scalar ( $0^+$ ) channel. Spin-1 condensation leads to more complicated structures, but is energetically disfavoured compared to spin-0 condensation and will not be considered here. The remaining spin structure can be written in terms of form factors  $T_a, T_b, T_c, T_d$  analogous to (2.7), ie

$$T'(p) = \vec{p} T_a(\vec{p}^2, p_4) + T_b(\vec{p}^2, p_4) + \gamma_4(p_4 - i\mu)T_c(\vec{p}, p_4) + i\gamma_4 \vec{p} T_d(\vec{p}^2, p_4). \quad (2.8)$$

Similarly, the inverse propagator can be written in terms of form factors  $A, B, C$  and  $D$  for the normal part  $N$ , and  $\phi_a, \phi_b, \phi_c, \phi_d$  for the anomalous part  $\Delta'(p)$ . The form factors  $\phi_i$  are the gap functions.

The gluon propagator in presence of a chemical potential in Landau gauge may be decomposed into an magnetic and electric form factor,

$$D_{\mu\nu}(\vec{q}, q_0) = P_{\mu\nu}^T D_M(\vec{q}^2, q_4^2) + P_{\mu\nu}^E D_E(\vec{q}^2, q_4^2). \quad (2.9)$$

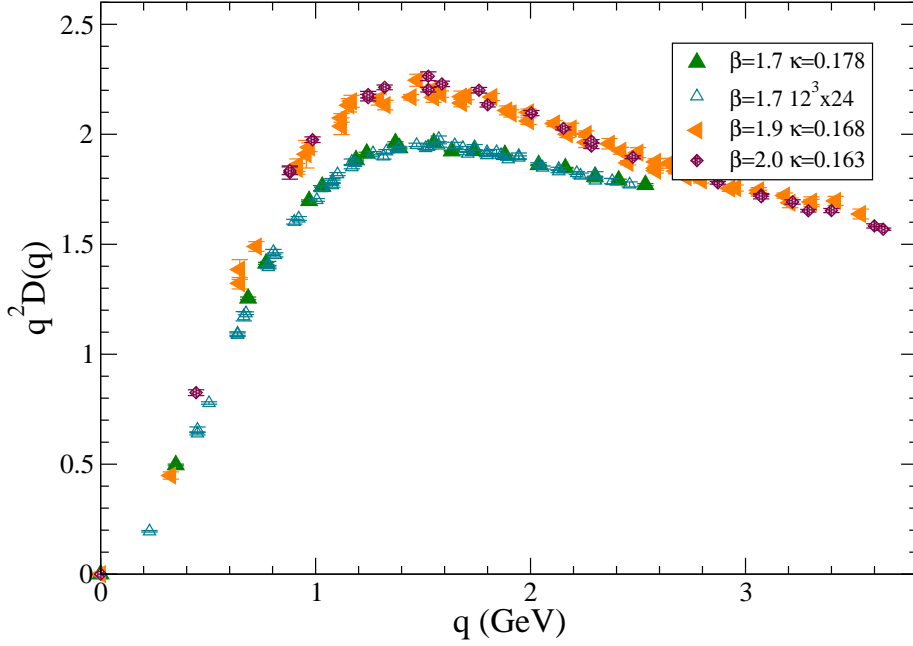
The projectors  $P_{\mu\nu}^T(q), P_{\mu\nu}^E(q)$  are both 4-dimensionally transverse, and are spatially transverse and longitudinal respectively.

### 3. Results

We have generated gauge configurations on two lattices: a “coarse” lattice with  $\beta = 1.7, \kappa = 0.178, V = 8^3 \times 16$ , and a “fine” lattice with  $\beta = 1.9, \kappa = 0.168, V = 12^3 \times 24$ . The lattice spacings are 0.23fm and 0.18fm respectively, while  $m_\pi/m_\rho = 0.8$  in both cases. A range of chemical potentials  $\mu$  were used with diquark source  $aj = 0.04$ , while additional configurations were generated with  $aj = 0.02, 0.06$  for selected values of  $\mu$ . In addition to this, we have also generated configurations at  $\mu = 0$  for two “finer” lattices, with  $\beta = 2.0, \kappa = 0.162$  (“heavy”) and  $\kappa = 0.163$  (“light”), both with volumes  $V = 12^3 \times 24$ .

#### 3.1 Gluons

Results for the gluon propagator on the coarse lattice have been presented in [3]; we will supplement those here with results from the fine lattice. On both lattices, an onset transition to a



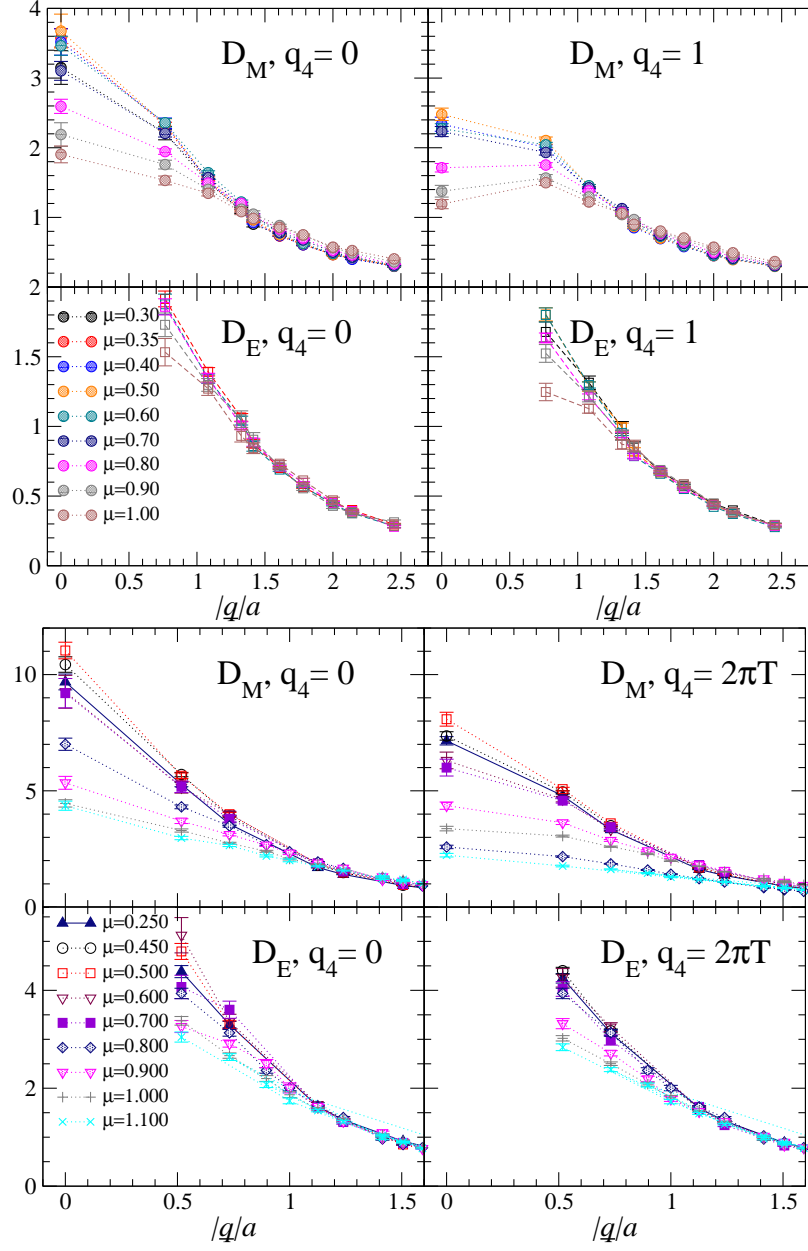
**Figure 1:** The gluon dressing function at zero chemical potential, for different lattice spacings and volumes.

phase with nonzero baryon density and diquark condensate was found at  $\mu_o \approx m_\pi/2$ , while BCS-like scaling of energy density, baryon density and diquark condensate was found at higher  $\mu$ . On the coarse lattice the crossover to BCS-like scaling was associated with a nonvanishing Polyakov loop  $L$ , indicating a coincident deconfinement transition [3]. On the fine lattice, these two transitions are separate, with the deconfining transition occurring at considerably larger  $\mu$  [4].

First of all, we investigate the scaling behaviour of the gluon propagator in the vacuum ( $\mu = 0$ ). Figure 1 shows the gluon dressing function  $q^2 D(q)$  for three of our four different lattices. For the coarse lattice parameters, we also have data for two different volumes. The data have all been cylinder cut [5] to select the points with smallest lattice artefacts. Since the lattice spacing for the finer lattice has not yet been independently determined, the matching procedure described in [5] has been used to find the ratio of lattice spacings  $a_f/a_{ff}$  that gives the best match for the gluon propagator on the fine ( $f$ ) and finer ( $ff$ ) lattices.

We see that finite volume effects are small for the momenta considered here, but scaling violations (finite lattice spacing effects) are very large between the coarse and the two finer lattices. The good scaling observed between the two finer lattices may be somewhat misleading, since the matching procedure used in setting the scale for the finer lattice assumes we are in the scaling régime. Nonetheless, the good agreement over a wide range of momenta indicates that lattice artefacts here are not too large.

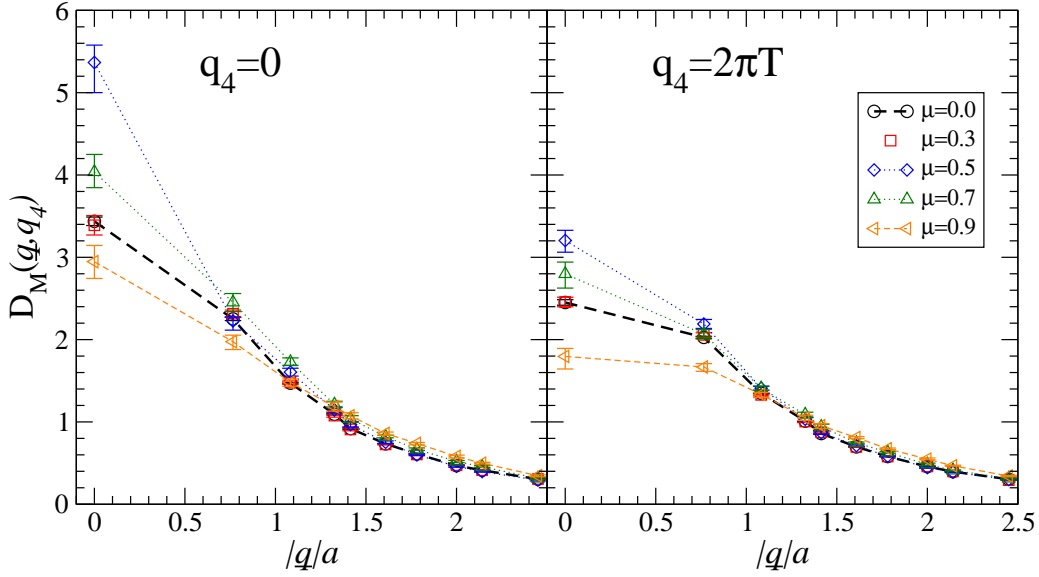
Figure 2 shows the two lowest Matsubara modes of the unrenormalised gluon propagator as a function of spatial momentum  $|\vec{q}|$  for a range of chemical potentials, on both lattices. In all cases, the propagator at the lowest chemical potential  $\mu$  shown is consistent with the vacuum propagator. On the coarse lattice both magnetic and electric propagator are strongly screened at large  $\mu$ , while they are enhanced at low momentum in the intermediate-density region. The static ( $q_0 = 0$ ) mag-



**Figure 2:** The unrenormalised gluon propagator on the coarse lattice (top) and on the fine lattice (bottom), for various chemical potentials  $a\mu = 0.25 - 1.10$ .

netic gluon propagator turns out to have a surprisingly strong dependence on the diquark source, which counteracts the infrared suppression at large  $\mu$  as  $j \rightarrow 0$ , but does not remove it completely. This is demonstrated in fig. 3, which shows the magnetic gluon propagator for the two lowest Matsubara frequencies, extrapolated to zero diquark source. We clearly see a strong infrared enhancement at intermediate  $\mu$ , but at  $a\mu = 0.9$  ( $\mu = 0.78\text{GeV}$ ) both the static and non-static modes are suppressed in the infrared.

The same qualitative picture can be seen on the fine lattice, but in this case the infrared sup-



**Figure 3:** Magnetic gluon propagator on the coarse lattice, extrapolated to zero diquark source  $j$ . The left-hand plot shows the lowest Matsubara mode ( $q_4 = 0$ ), while the right-hand plot shows the first nonzero Matsubara mode.

pression sets in at much larger  $\mu$  (around  $a\mu = 0.8$  or  $\mu = 0.9\text{GeV}$ ). This is consistent with the hypothesis that the screening effect is linked with the deconfinement transition, ie that it is a result of the gluons being screened by coloured quark degrees of freedom.

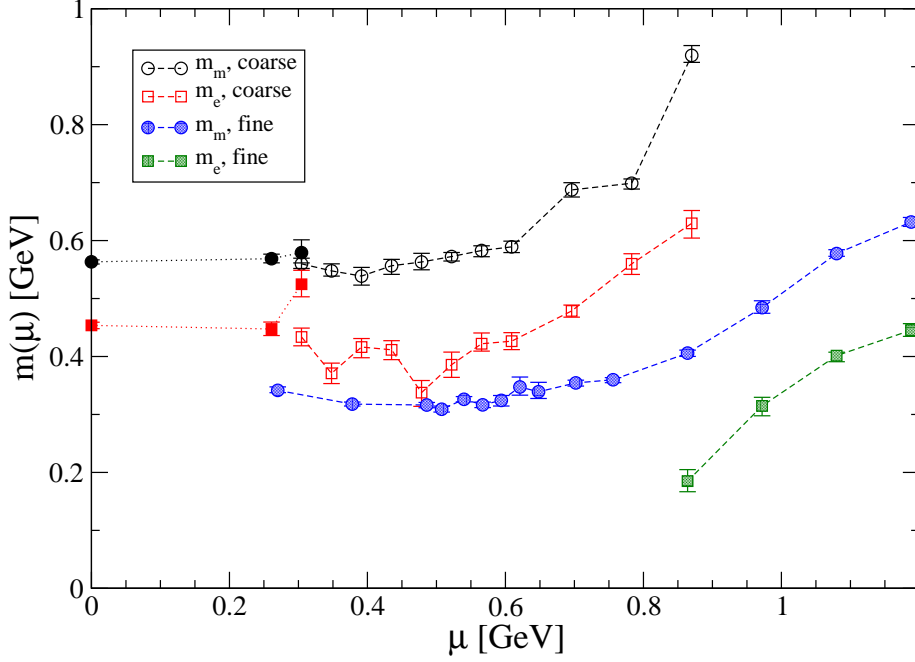
It is worth pointing out that the enhancement resp. screening noted here is in comparison to the vacuum gluon propagator, which is known to be infrared suppressed due to nonperturbative effects (as discussed at length in other contributions to this conference). It seems reasonable to assume that although the static magnetic gluon is unscreened to all orders in perturbation theory, nonperturbative effects may be responsible for the additional screening observed here in the deconfined phase.

We have attempted to fit the gluon propagator to a simple massive form,

$$D_{E,M}(\vec{q}, q_4; \mu) = \frac{Z_{e,m}}{\vec{q}^2 + q_4^2 + m_{e,m}^2(\mu)}. \quad (3.1)$$

The resulting electric and magnetic gluon masses  $m_{e,m}$  are shown as functions of  $\mu$  in figure 4. It is worth noting that the quality of these fits is quite poor. This is expected, as it is known that at  $\mu = 0$  it is not possible to describe the gluon propagator by a simple, momentum-independent mass, while at large  $\mu$  one should reproduce the results of hard-dense-loop (HDL) resummed perturbation theory, which also has a more complicated functional form. A form which interpolates between HDL and available results for  $\mu = 0$  [5, 6, 7] is likely to yield better results. A further technical complication is that we have defined  $D_E$  only at nonzero spatial momenta, while the fits to  $D_M$  include the  $\vec{q} = 0$  point. This is the reason for the discrepancy between  $m_e$  and  $m_m$  at  $\mu = 0$ , where they should be equal. This also tends to yield lower values for  $m_e$  throughout.

With these provisos, we can see that both the electric and magnetic gluon masses remain roughly constant for small and intermediate  $\mu$ , before increasing at large  $\mu$ , corresponding roughly



**Figure 4:** The electric and magnetic gluon mass as a function of chemical potential  $\mu$ , determined from a fit to a simple massive propagator on each lattice. For the coarse lattice, the filled symbols denotes fits to data with zero diquark source  $j$ , while the open symbols are from  $ja = 0.04$ . For the fine lattice all data are for  $ja = 0.04$ . It was not possible to get any fit for the electric gluon on the fine lattice for  $a\mu < 0.7$ .

to the deconfined phase. We see, however, that there is a large difference between the mass values from the two lattices, indicating that scaling violations are still very large at these lattice spacings.

### 3.2 Quarks

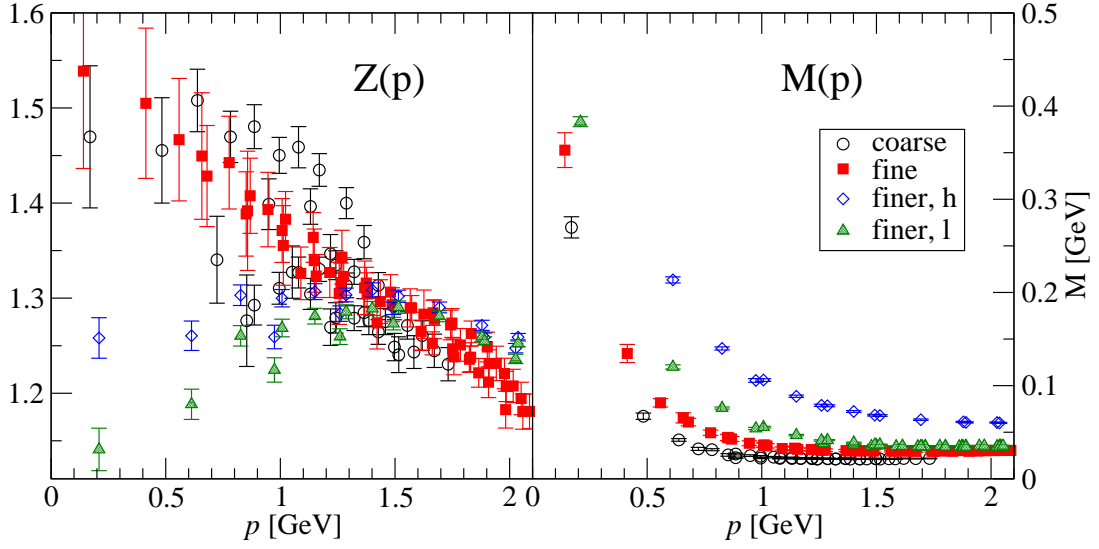
In the vacuum, there are only two independent tensor components of the quark propagator, which is conventionally written as

$$S(p) = \frac{Z(p)}{i\not{p} + M(p)}, \quad (3.2)$$

where  $M$  is the mass function and  $Z$  the renormalisation function. These are shown in fig. 5, for the different lattice spacings and quark masses available. Both  $Z(p)$  and  $M(p)$  have been multiplicatively tree-level corrected [8]; however, since the critical quark mass is not yet known, the tree-level correction of  $M(p)$  is not yet properly carried out.

We immediately see that there are large scaling violations in both form factors, and large violations of rotational symmetry in  $Z(p)$ . In particular, we note that  $Z(p)$  increases in the infrared for the coarser lattices, whereas it is usually found to be infrared suppressed. We see that this suppression appears to be recovered as we move towards the continuum limit. A careful continuum extrapolation will be needed to obtain quantitative results.

At nonzero chemical potential, we find that the form factors  $S_a, S_b$  and  $S_c$  (spatial-vector, scalar and temporal-vector) of the normal quark propagator and the form factors  $T_b$  and  $T_d$  (scalar and



**Figure 5:** The quark propagator renormalisation function (left) and mass function (right) at zero chemical potential, for different lattice spacings.

tensor) of the anomalous propagator are nonzero, while the remaining components are zero. Results for the coarse lattice were shown in [9]; here we will show results for the fine lattice only.

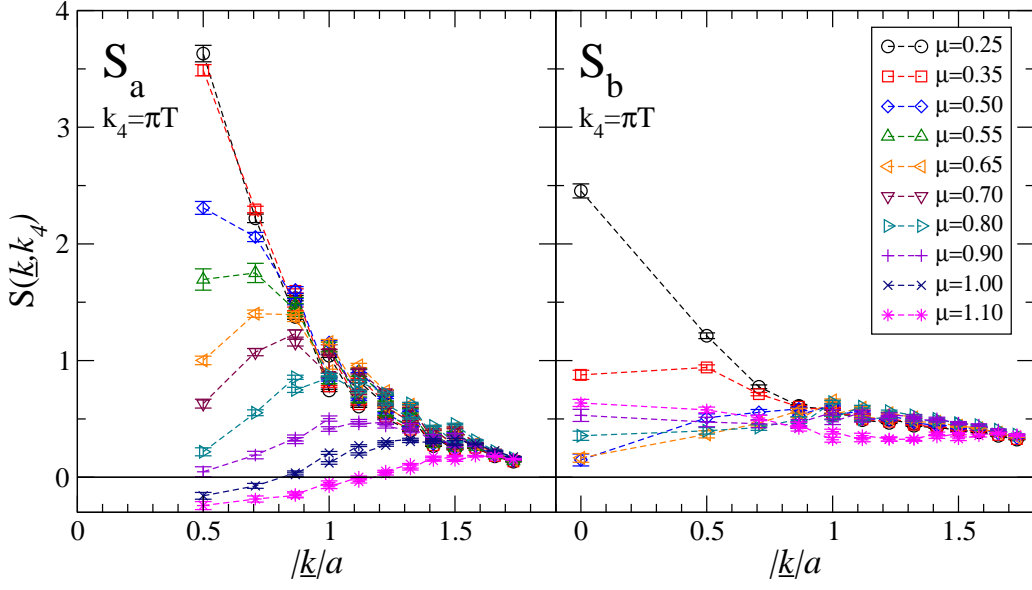
Figure 6 shows the spatial-vector part  $S_a$  and scalar part  $S_b$  of the normal quark propagator for a range of chemical potentials  $a\mu = 0.25 - 1.1$ . These both exhibit dramatic medium modifications. The scalar propagator  $S_b$  is strongly suppressed in the superfluid phase, suggesting a drastic reduction in the in-medium effective quark mass. This is linked to the appearance of the diquark condensate: the chiral condensate rotates into the diquark condensate in the superfluid phase [10]. We would therefore expect to find the missing strength in the anomalous propagator. The change in behaviour is sudden and takes place around  $\mu_o = m_\pi/2$ , while for larger  $\mu$  there is little change.

The spatial-vector propagator  $S_a$  is also infrared suppressed at large  $\mu$ , but this suppression happens gradually as a function of  $\mu$ , and sets in only above  $\mu_o$ . At the largest densities we see that  $S_a(\vec{k}, k_4 = \pi T)$  becomes negative for small spatial momentum  $|\vec{k}|$ .

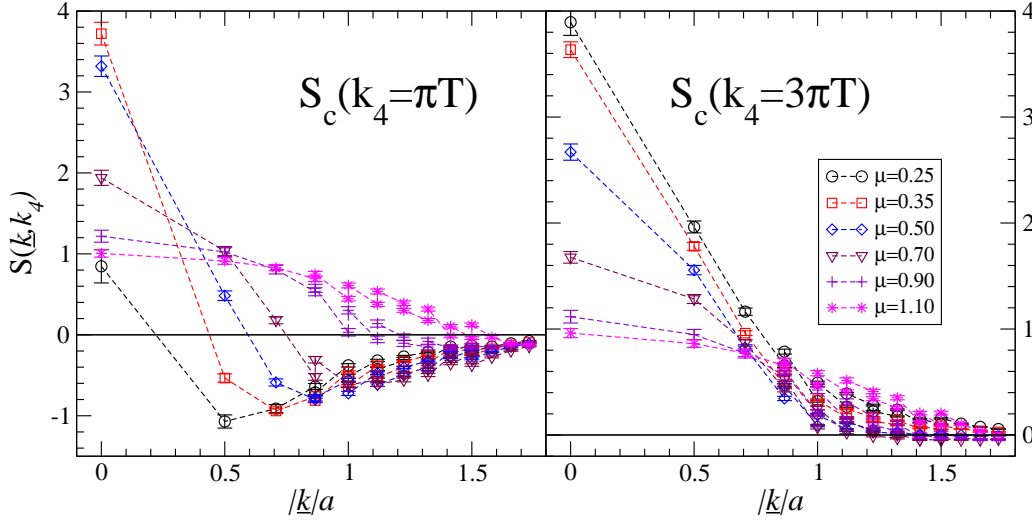
The two lowest Matsubara modes of the temporal-vector propagator  $S_c$  are shown in 7. We see that the lowest Matsubara mode ( $k_4 = \pi T$ ) becomes negative at intermediate momenta, approaching zero from below at high momenta. This is a dramatic change compared to the vacuum propagator, which stays positive at all momenta, and indicates the formation of a superfluid gap. The location of the zero crossing in the  $k_4 \rightarrow 0$  limit corresponds to the Fermi momentum  $k_F$ . In accordance with this, the zero crossing moves to larger  $|\vec{k}|$  as  $\mu$  increases. On closer inspection, we find that the second Matsubara mode ( $k_4 = 3\pi T$ ) also becomes negative for large  $\mu$  (at large spatial momentum). It would therefore in principle be possible to extrapolate this zero crossing to  $k_4 = 0$  and thus find  $k_F$  as a function of  $\mu$ .

Figure 8 shows the nonzero components of the anomalous Gor'kov propagator. The dominant part is, as expected, the scalar part  $T_b$ , but a clear signal is also found for the tensor part  $T_d$ , in accordance with what was found on the coarse lattice [9]. We find that the lattice artefacts in





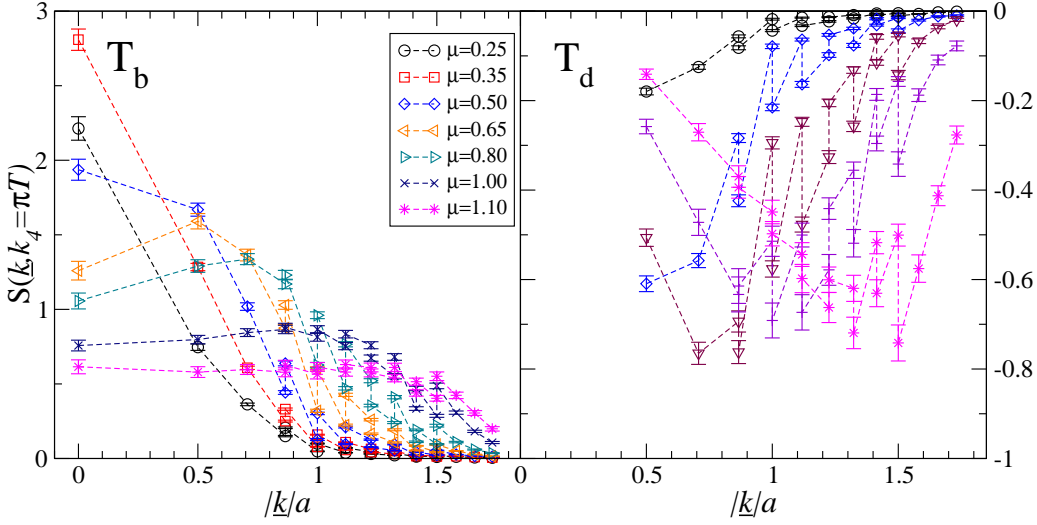
**Figure 6:** The lowest Matsubara frequency of the spatial-vector (left) and scalar (right) part of the normal quark propagator, on the fine lattice, for different chemical potentials  $\mu$ .



**Figure 7:** The temporal-vector part of the normal quark propagator, on the fine lattice, for different chemical potentials  $\mu$ .

the scalar part is substantially reduced compared to the coarse lattice, while the tensor part is still subject to very large violations of rotational symmetry. It may therefore be open to question whether this component will survive the continuum limit.

The scalar anomalous propagator shows a clear change in behaviour as one goes from small to large chemical potential. Firstly, we note that it increases between  $a\mu = 0.25$  and  $0.35$ . The former point is below the superfluid transition, but anomalous propagation is present due to the explicit



**Figure 8:** The scalar (left) and tensor (right) part of the anomalous quark propagator, on the fine lattice.

diquark source. We expect that  $T_b$  (and  $T_d$ ) will vanish in the  $j \rightarrow 0$  limit for  $\mu < \mu_o$ . As  $\mu$  increases above  $\mu_o$ ,  $T_b$  develops a plateau at low momentum, which extends to larger  $|\vec{k}|$  with increasing  $\mu$ . At large  $\mu$ ,  $T_b$  thus becomes approximately constant, suggesting that anomalous propagation may be described by a momentum-independent diquark gap  $\Delta$ .

#### 4. Discussion and outlook

We have found substantial modifications of both gluon and quark propagators in the dense medium. In the superfluid, confined phase, the electric and magnetic gluon propagators are both enhanced in the infrared compared to the vacuum. In the deconfined phase, they are both screened (infrared suppressed). This screening is evident even in the static magnetic gluon, which is unscreened to all orders in perturbation theory. If these results carry over to SU(3) they would invalidate the use of an unscreened static-magnetic gluon propagator in the gap equation at large  $\mu$ . A careful analysis at different volumes and lattice spacings is however necessary to draw quantitative conclusions.

The dramatic modifications seen in the quark propagator are directly related to the appearance of a diquark gap. Our next step will be to compute the form factors, including the diquark gap and mass function, by inverting the quark propagator. Further quantitative studies will include determining the Fermi momentum  $p_F$  by extrapolating the zero crossing in the temporal-vector propagator  $S_c$  to  $k_4 = 0$ , and determining the size of Cooper pairs from the anomalous propagator, to study the BEC–BCS crossover in more detail.

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