Modelling and Predictive Control of Milk Pasteurisation in a Plate Heat Exchanger

Mohamed Tarek Khadir [1], Jacques Richalet [2], John Ringwood [1], Barry O'Connor [3]

[1] School of Electronic Engineering, Control System Group, Dublin City University, Dublin 9, Ireland
[2]ADERSA 100, rue de la Croix Martre 91873 Palaiseau Cedex France
[3]Food Engineering Department, University College Cork, Ireland
Fax +353-1-7045508, Tel +353-1-7045856, e-mail: khadirmt@eeng.dcu.ie

Abstract

This paper investigates the possible usage of Model Predictive Control (MPC) in a pasteurisation plant outlining the resulting benefits over classical control method, for instance Proportional Integral Derivative (PID) control. However MPC requires a plant model, a physical first principals model of the pasteuriser is developed and validated using data gathered on site at the 'Glanbia' milk production unit in Drogheda, Ireland. A PFC controller is then designed using this model, simulated results are presented and compared with the results given by the PID controller in operation.

1 Introduction

High temperature short time pasteurisation (HTST) is the most common used treatment in continuous time processes. The time/temperature combination varies according to the quality of the raw milk, the type of product treated and the country requirement. For milk pasteurisation the heating temperature is in general 72 -74^{o}C with a holding time of 15-20 seconds followed by cooling. Pasteurisation temperature in a milk plant is usually regulated using classical control methods such as PID see [1], although these methods give relatively satisfactory results a classical control can not occurs without a relative variance in the milk temperature due to disturbances. This phenomenon forces the control engineer to choose a set point higher than the one required for the pasteurisation to avoid any variance going below pasteurisation temperature. The results are a loss in energy, as we heat the milk more than needed as well as a possible alteration of milk characteristics.

2 Pasteurisation using the Clip 10-RM Plate Heat Exchanger

In our case the pasteuriser a Clip 10-RM a Plate Heat exchanger (PHE) from Alfa Laval is used. A PHE consists of a pack of stainless steel plates clamped in a frame. The plates are corrugated in a pattern designed to increase the flow turbulence of the medium and the product see [2]. A frame may contains several separate plate packs-sections for heating, regeneration or cooling. Liquids enter and leave through holes in the corners, varying patterns of open and blind holes route the liquid from a channel to another. In the Clip 10-RM the milk treatment is done as shown in Figure 1. First the raw milk at a concentration of 4.1% enters section S4 of the PHE at a temperature of 2.0^{o}C . It is then preheated to a temperature of 60.5^{o}C by the outgoing pasteurised milk, which on the other hand is brought to a temperature of 11.5^{o}C. Passing this section, the milk now at a temperature of 60.5^{o}C enters section S3 where its temperature increases to 64.5^o C by using hot water as a medium. The milk before reaching the next section is first separated from the fat then standardised and homogenised to a concentration of 3.5%. It then enters section S2 where it is preheated to a temperature of 72^{o}C using the already pasteurised milk as a medium. The milk is then brought to the pasteurisation temperature in section S1 (75.0^{o}C) using hot water at around 77.0^{o}C as a medium. After that the homogenised pasteurised milk is held at the pasteurisation temperature for 15 s in the holding tube section before being cooled using the incoming cold milk in section S4 and section S2. Finally the pasteurised milk enters the cooling section (section S5) at a temperature of 11.5^{o}C. The milk is chilled to a temperature of 1.0^{o}C using propylene glycol as a medium at a temperature of -0.5^{o}C. Note that the water for the heating sections S3 and S4 is brought to the adequate temperature in steam/water heater of type CB76 from Alfa Laval.

3 Modelling of the Plate Heat Exchanger

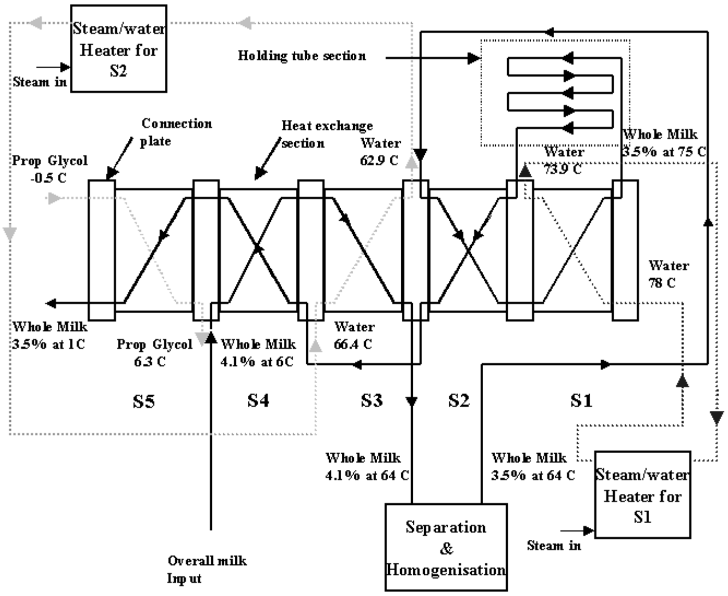


Figure 1: *General view of the pasteuriser Clip 10-RM*

Many PHE models can be found in the literature, for instance we can refer to the so-called "cinematic" model developed in [3]. In this model the effect of the plate is neglected, and the exchange in every channel (between two plates) is described by a differential equation. This leads to an n by n system, where n is the number of channels. In the same way, although a more complex representation based on a dispersion model is given by [4]. Another interesting approach is shown in [5], Where a second order system with time delay is proposed as a model for a counter current PHE, the model's parameters are determined by experimental sinosoidal and pulse testing. The sinosoidal test protocol is indeed difficult to achieve in real processes as the input will be the milk or water temperature. The two first models [3, 4] falls into the family of physical models as they are driven from the mathematical equations given by the system characteristics, where the model given by [5] is closer to a black box modelling approach. The aim of this paper is not to choose an approach over the other, nor to demonstrate which model is better than the other, as every method has its advantages and inconvenient. Moreover the validity of the model is in itself a relative concept, Valid for what purpose?. In this case the model will be used for the implementation of a predictive controller thus, a fairly complicated one will only increase the controller design complexity, explores some model's features that we may not need and probably increases computation time, adding to that the amount of time spent to establish such a model. [6] gives a good overview on modelling in general as well as modelling for MPC control strategies in particular. The modelling strategy adopted in this paper, starts by establishing a first principles model driven from the energy balance equation of the heat transfer in a PHE. Having that the model parameters are then tuned to give a minimum error between the process and the model responses. Before starting the mathematical analysis of a PHE, let us first emphasise the notion of a heat exchange through a wall or a plate illustrated by figure 2.

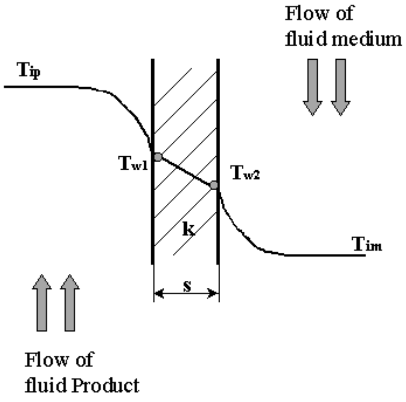


Figure 2: *Heat transfer through a wall*

The general heat flow is then given by equation (1).

$$Q = A.U.(T_{ip} - T_{im}) \tag{1}$$

On the other hand the same heat flow can be given by equations (2), (3) and (4):

$$Q = A.C_p.(T_{ip} - T_{w1}) \tag{2}$$

$$Q = A.\frac{K_{pa}}{s}.(T_{w1} - T_{w2}) \tag{3}$$

$$Q = A.C_m.(T_{w2} - T_{im}) \tag{4}$$

To obtain the temperature of the films we equalise (1) and (2) as well as (1) and (3). The results T_{w1} and T_{w2} are given by (4) and (5). This will be useful later on.

$$T_{w1} = T_{ip} - \frac{U}{C_p}(T_{ip} - T_{im}) \tag{5}$$

$$T_{w2} = T_{im} + \frac{U}{C_m}(T_{ip} - T_{im}) \tag{6}$$

Every section of the PHE is now considered as a single plate separating two channels. The thermal evolution of the product and medium temperature are given by (7) and (8) respectively:

$$\rho_p.C_p.V_p.\frac{dT_{sp}}{dt} = \rho_p.C_p.F_p.[T_{ip}(t) - T_{sp}(t)] + U_p.A.[T_{pa}(t) - T_{sp}(t)] \tag{7}$$

$$\rho_m.C_m.V_m.\frac{dT_{sm}}{dt} = \rho_m.C_m.F_m.[T_{im}(t) - T_{sm}(t)] + U_p.A.[T_{pa}(t) - T_{sm}(t)] \tag{8}$$

under the following assumptions.

- every fluid is ideally mixed in the direction of the flow
- the flow in the channels is equal to the input flow
- the heat conduction in the flow direction is negligible
- the heat is only transferred in one direction, perpendicular to the channel axis
- the effect of the temperature on the specific heat and density of the fluid is negligible
- there are no phase changes during the heat transfer (including no fooling)
- thermal loss are neglected

Replacing the plate temperature T_{pa} by the films temperature gathered in (4) and (5) we obtain the system given by differential equations in (9) and (10) as:

$$\tau_p.\frac{dT_{sp}}{dt} + T_{sp} = \lambda_p.T_{ip} + (1 - \lambda_p).T_{im} \tag{9}$$

$$\tau_m.\frac{dT_{sm}}{dt} + T_{sm} = \lambda_m.T_{im} + (1 - \lambda_m).T_{ip} \tag{10}$$

where:

$$\tau_p = \frac{\rho_p.C_p.V_p}{\rho_p.C_p.F_p + U_p.A}, \qquad \lambda_p = 1 - \frac{U}{C_p}.(\frac{U_p.A}{\rho_p.C_p.F_p + U_p.A})$$

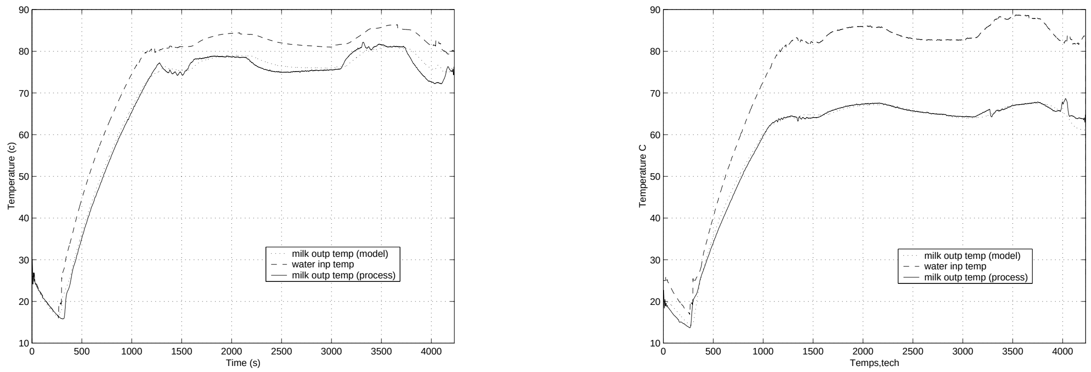
$$\tau_m = \frac{\rho_m.C_m.V_m}{\rho_m.C_m.F_m + U_m.A}, \qquad \lambda_m = 1 - \frac{U}{C_m}.(\frac{U_m.A}{\rho_m.C_m.F_m + U_m.A})$$

The physical model obtained in 9 and 10 is obviously linear, where the real process is most probably non linear taking into account the turbulent flow, the shape of the plates and their corrugation. According to that the physical model output will be able hopefully to "match" the process output only around a region, which better be the temperature set point region 64°C for S3 and 75°C for S1. Figure 3 shows the model and process responses (the milk temperature at the output each of section S1 and S3) to the same initial raw milk temperature at the input of section S4 Figure 1. The process responses signals have been collected via a 'SATTLINE' system operated by ABB Automation Ireland. The process data are collected during production. Which give a clear representation of the milk temperature variance when working around set point temperatures.

The modelling results shown in figure 3 do not derive directly from the discretised system equivalent to the one given in (9) and (10), the parameters τ_p, τ_m, λ_p and λ_m have all been multiplied by a factor. Nonetheless even at the first attempt the model response was not completely different from the process one, which emphasise the importance of first principle models.

4 Predictive controller design

Predictive control theory in general, achieve a process regulation by specifying the desired plant output at a particular instance or instances in the future and then calculating the controller action which minimises the predicted error. PFC still work this way, however it uses independent models, ie. the output is calculated using only the known measured process inputs. [7] Gives a general overview of what is MPC, as well as its developments through last decades. On the other hand [8] is more specific and deals mainly with PFC. In our case the simplest version of PFC is implemented in a controller, where:



(a) Section S1

(b) Section S3

Figure 3: *Response of the pasteuriser and the model to a test protocol*

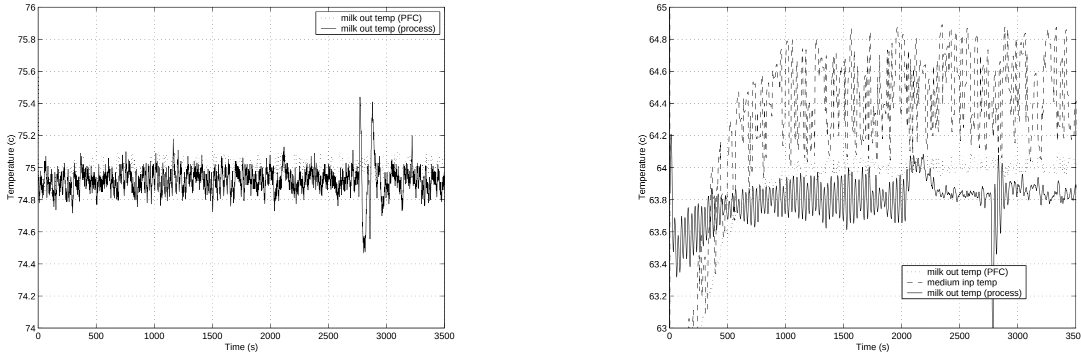
- the reference trajectory is an exponential which required only one initialisation point and gives responses without overload.
- the coincidence horizon H where the model and the process output are equal is brought to 1, single step prediction.
- the internal model chosen is a first order one of the form $\frac{k}{1+\tau.s}$
- the model response at time n+H, is given by $y_M = y_L + y_F$, where y_L is the free response of the model, while y_F is the forced response. The forced response is a function of the control signal u, projected on a base of functions. In the basic case $y_F(n+H) = u(n).OB_0(H)$, where OB_0 is the output of a a single basis element, ie. case where the control signal is structured by a single basis element (a step input to find).

Given the above conditions, at the coincidence point we obtain equation (11):

$$(r(n) - y_p(n)).(1 - \lambda^H) = y_L(n+H) + u(n).OB_0(H) - y_M(n) \tag{11}$$

The analytical solution of (11) gives the command variable in (12).

$$u(n) = \frac{(r(n) - y_p(n)).(1 - \lambda^H) - y_L(n+H) + y_M(n)}{SB_0(H)} \tag{12}$$



(a) Section S1

(b) Section S3

Figure 4: *PFC results when used for the regulation of the Clip 10-RM*

Assuming the above, a PFC controller has been designed for a control in temperature. Thus the medium (water) input temperature will be the controlled variable $u(n)$ for sections S3 and S1 of the PHE (the section where there is initially a PID control loop). Therefore, assuming that the medium temperature is fully controllable, the simulated results involving the designed predictive controllers

(PFC) on the Clip 10-RM PHE are given in figure 4. The graph shows the responses of the process during production time using the PID controller, as well as the PFC one around the set point temperatures, adding a pseudo random disturbance of 1^{o}C on the output signal for the PFC simulation. The raw milk temperature at the input of section S4 is varying around 2^{o}C.

5 Conclusions

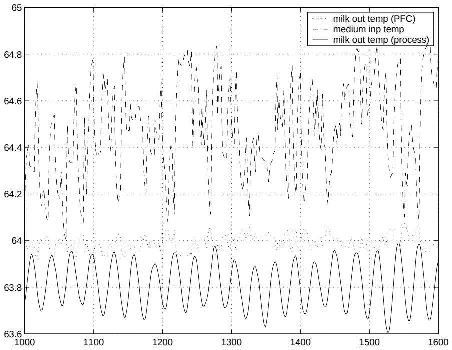


Figure 5: *Variance difference*

Once again, the model established in this paper is not the best we can have, and still perfectible. However from Figure 3 it can be seen that after tuning the parameters the model follows the behaviour of the system with an absolute error of 0.8^{o}C in section S3 and just over 1^{o}C for Section S3. Nevertheless the model still appropriate when used for MPC control design, see Figure 4 and a more complicated model may cause more inconvenience then improvement to the actual design. Concerning the advantage of using MPC (a PFC controller in our case), Figure 5 emphasise the variance difference of the output signal (the milk temperature) over the actual variance. Indeed the maximum variance given by the PFC when the steady state is reached, is around 0.18^{o}C, while the maximum variance of the on-line process is 2.42^{o}C. Knowing that the pasteurisation temperature is in fact just above 72^{o}C (75^{o}C is taking for security purposes see section 1), the pasteurisation set point could be brought between 73^o and 74^{o}C without the fear of going below the pasteurisation temperature due to a large variance. Achieving that a considerable amount of energy could be saved, on the other hand milk characteristics will not be altered due to overheating.

Nomenclature

A: Heat exchange area (m^2)
C_p: Specific heat coefficient of the product (j/kg/K)
C_m: Specific heat coefficient of the medium (j/kg/K)
F_p: Product flow rate (m^3/h)
F_m: Medium flow rate (m^3/h)
H: Coincidence point equal to unity
K_{pa}: Thermal conductivity of the wall W/(m k)
Q : Total heat flow (W)
s: Thickness of the wall (m)
SB_0: Output of 1st element of the basis function
T_{ip}: Input temperature of the fluid product (^{o}C)
T_{im}: Input temperature of the fluid medium (^{o}C)
T_{sm}: Output temperature of the fluid medium (^{o}C)
T_{sp}: Output temperature of the fluid product (^{o}C)
T_{pa}: Plate temperature (^{o}C)
T_{w1}: Temperature of the product film (^{o}C)
T_{w2}: Temperature of the medium film (^{o}C)
U: Total heat transfer coefficient (W/m^2/K)
U$_m$: Heat transfer coefficient plate/medium (W/m^2/K)
U$_p$: Heat transfer coefficient plate/product (W/m^2/K)
V_p: Volume of the product area (m^3)
V_m: Volume of the medium area (m^3)
y_M: Output given by the model
y_P: Output given by the process
y_L: Output given by the model free response
ρ_p: Product density (kg/m^3)
ρ_m: Medium density (kg/m^3)
λ: Decreasing parameter of the reference trajectory

References

[1] Robinson, R.K *Automation at the dairy, modern dairy technology*, Elseiver Applied Science 1993.
[2] Alpha Laval *Dairy Processing Handbook*, Tetra Pak, 1995.
[3] Lakshmanan, C.C, Potter, O.W *Dynamic Simulation of a Plate Heat Exchanger*, Int Journal of Heat and Mass Transfer Vol 33, N^o5, pp 995-1002 1990.
[4] Wilfried, R and Sarit, K *Hyperbolic axial dispersion model: concept and its application to a plate heat exchanger*, Int Journal of Heat and Mass Transfer Vol 38, N^o16, pp 3065-3076 1995.
[5] Khan,A.R, Baker, N.S and Wardle, A.P *The dynamic characteristics of a counter current plate heat exchanger*, Int Journal of Heat and Mass Transfer Vol 31, N^o6, pp 1269-1278 1988.
[6] Richalet,J, ADERSA *Pratique de l'Identification*, Traite des Nouvelles Technologies, Serie Automatique Hermes 1991.
[7] Morari, M. and Lee, J.H. *Model Predictive Control: Past, Present and Future*, Computers and Chemical Engineering, Vol. 23, 1999.
[8] Richalet, J. *Pratique de la Commande Predictive*, Traite des Nouvelles Technologies, Serie Automatique Hermes 1993.