

# Review of Twin Reduction and Twin Removal Techniques in Holography

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**Abstract**—In this paper we review the major contributions over the past sixty years to the subject of twin reduction and twin removal in holography. We show that this collective work may be broken down into a number of categories including the well known techniques of off-axis holography and phase retrieval.

**Index Terms**—Holography, In-Line, On- Axis, Twin Reduction

## I. INTRODUCTION

Holography was invented by Gabor in 1948<sup>1,2</sup>. In his initial experiments, involving electron microscopy, an object was irradiated with a radiation beam of strong coherence. The waves weakly scattered by the object interfered with the background wave on a photographic film where this interference pattern was recorded. This recorded intensity distribution contains information about the amplitude and the phase of the incident object wavefield. The limits to the Gabor experiment are the resolution of the material film and the coherence of the radiation source. Gabor also showed how it was possible to reconstruct the original object wavefield by illuminating the recorded film transparency with the original background wave. However, the image of the reconstructed object is marred by the presence of a twin image, an inherent artifact of the method. After the invention of the laser methods would later be invented to cleverly evade this twin image by using new experimental architectures, but this would impose greater restrictions on the system, in particular on the resolution of the film. However for certain radiation sources these new experimental architectures have no physical implementation or only crude and expensive equivalents. In these cases one must rely upon the initial Gabor architecture and the twin image remains a problem that must be dealt with. Such is true for many cases including x-ray holography, gamma-ray holography and electron holography. In the case of digital (optical) holography the Gabor like in-line architectures also offer advantage as discussed below. Crystallographic structure is often determined using diffraction methods. While electron emission holography is useful for studying surface structure, it's anisotropic nature prevents the study of internal structure. The more isotropic

scatter of x-rays overcomes this limitation. Improvements in x-ray detectors have enabled the application of x-ray holography for crystallography<sup>3</sup> which is especially useful because the recovered phase information offers an improvement over traditional diffraction techniques. As in the case of gamma-ray holography<sup>4</sup> and electron emission holography<sup>5-10</sup>, the in-line architecture is used and the short distance between the source and the object implies that the twin image will be located very close to the reconstructed atom image. This creates a detrimental and unavoidable source of noise. Low voltage and high voltage electron holography are particularly useful in the imaging of weakly scattering objects such as DNA molecules<sup>7-8</sup>. The severe aberrations caused by lenses in electron imaging makes the in-line set-up the method of choice. Another advantage in this case is that the phase sensitivity of in-line electron holography is particularly high.

## II. TWIN REDUCTION BY SUBTRACTION

In 1951, soon after Gabor's invention and many years before any off-axis architecture would be developed Bragg and Rogers developed an innovative solution to the twin image problem<sup>11-12</sup>. The basic idea is that the disturbance caused by the unwanted twin image on the wanted plane is, in fact, its own hologram. If a second hologram is taken from the original object, which accurately reproduces the disturbance that is due to the unwanted image at the wanted plane, the first reconstruction can be corrected by subtraction. Using a collimated light source we must take the second hologram at twice the distance of the first, since the wanted and unwanted images are formed at equal distances from the hologram on either side of it. The divergent case is somewhat more complicated. It is notable that the method works well with both phase-contrast and amplitude-contrast. This method fell into obscurity until later advances in digital technology allowed for a simplified subtraction process<sup>12</sup>. A similar subtraction based technique is reported in 1956<sup>13</sup>, where two holograms must be recorded and the object must be repositioned for the second recording enabling a canceling of the twin image term. Xiao et. Al. showed<sup>14</sup> how the Bragg and Rogers method could be applied in x-ray holography in real time by recording two holograms while taking advantage of the penetration property of x-ray radiation.

## III. TWIN ELIMINATION BY OFF AXIS HOLOGRAPHY

After the invention of the Laser, Leith and Upatnieks proposed<sup>15</sup> in 1963 a new experimental optical architecture

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that enables the complete separation of the twin image term and the zero order term from the wanted image. In their method the reference beam was incident on the hologram plane at some angle relative to the normal. In this way the twin images were modulated on well-separated carrier spatial frequencies. The range of separation of the terms is dependent on the angle of the reference beam. The significantly increased bandwidth of the hologram places a much greater requirement on the resolution of the recording material. Furthermore the architecture imposes the need for numerous optical elements, in particular a beam splitter, which are not readily available in many areas of holography. The increased bandwidth of the hologram for this set-up poses a problem for digital holography. In DH, the pixilated recording cameras have resolutions an order of magnitude less than commercial photo materials. Thus the bandwidth is already severely limited and the use of an off-axis architecture will only serve to limit it further. Nevertheless Cuche et. al. proposed<sup>16</sup> and experimentally validated the off axis technique for real time digital holographic recording. In this case it is possible to digitally spatially filter the hologram to completely remove the DC term and the twin image. In 1966 an alternative to the off-axis method was proposed<sup>17</sup> for recording in the Optical Fourier Transform domain. In this case the reference beam, a point source in the object plane, can be placed adjacent to the object to effectively create an off axis hologram in the Fourier domain and to spatially separate the twin images.

#### IV. TWIN REDUCTION BY FRAUNHOFER

In 1966 another twin reduction method was proposed<sup>18</sup> by effectively recording in the far field of the object. When replayed in the far field the image of the object will appear but the twin image will be so spread out that it appears as a DC term and is therefore effectively removed. A year later the method was reviewed and applied to particle analysis<sup>19</sup> and this was followed by a further reassessment almost a decade later<sup>20</sup>. The method was also applied to in-line electron holography to view undecagold cluster. The method is found to work well with the phase contrast technique. In [22] Garcia et. al. extensively review lensless in-line holographic microscopy and show that the twin image is of no consequence in the reconstructions. They comment that this is because of the diverging reference beam. While the reconstruction of the object image converges upon reconstruction, the out of focus twin image continues to diverge and effectively forms the Fraunhofer case where it is a constant DC term in the ‘far-field.’

#### V. TWIN REDUCTION BY SINGLE SIDEBAND ELIMINATION

In 1968 Bryngdahl and Lohmann developed<sup>23</sup> a method to suppress the twin image. The method consists of filtering out half of the spatial frequency spectrum from the transmitted signal during the recording of the hologram and then filtering out half of the spatial frequency spectrum from the signal during the reconstruction process. The authors suggest that the result of this process will be that “each point scatterer in the object will cause only one half of a zone-plate pattern in the hologram plane.” The reconstructed signal will have the twin

images on separate sides of its Fourier spectrum and they can be easily isolated by spatial filtering.

#### VI. TWIN REMOVAL BY PHASE RETRIEVAL

The 1970s and 1980s saw the development of a new field of research for recovering the phase of a wave field. These methods do not require interference and are collectively known under the name of “phase retrieval algorithms.” We can divide these phase retrieval schemes into two subsets; (i) deterministic<sup>24</sup> and (ii) iterative<sup>25-27</sup>. Deterministic phase retrieval algorithms are based on analysing the propagation of an intensity diffraction pattern. Iterative methods on the other hand rely on recursive ‘ping pong’ algorithms that converge over time based on some constraints that are imposed in every iteration of the algorithm. They are often highly reliant on some initial condition set at the outset of the iteration process. These ping pong algorithms often require two recorded diffraction patterns but some have been developed that can work with only a single recorded intensity and a suitable constraint. While both deterministic and iterative phase retrieval algorithms have both been shown to work with some success, the iterative class has received by far the greatest attention in the literature. In [28] Gerchberg demonstrated how phase retrieval could be used with electron microscopy to recover the phase of the wave field. Despite the initial promise of phase retrieval algorithms, they have never managed to achieve results on a par with holographic methods. However there has been considerable interest in their usefulness in removing the twin image for in-line holography<sup>29-41</sup>.

Twin removal has been successfully accomplished with both deterministic phase retrieval<sup>29</sup> and iterative phase retrieval<sup>30-41</sup>. In [29] deterministic phase retrieval is combined with numerical simulations of light propagation to solve the twin image problem. In [30] and [31] Liu and Scott performed the first investigation of using iterative phase retrieval for removing the twin image. The authors note that “retrieval of phase permits separation of real-object distributions from the twin-image interference that accompanies conventional optical reconstruction.” However, this algorithm is limited to purely absorbing objects and cannot recover phase shifts caused by transmissive objects. Unfortunately the same can be said for many similar algorithms that followed. Liu continued to improve upon these algorithms by incorporating into them a noise constraint based on a model of additive noise<sup>32</sup>. Koren et. al. developed<sup>33-34</sup> a new constraint for these iterative twin removal algorithms. They noticed that the out of focus image was considerably larger in area than it is in focus counterpart and they unitized this fact to form their constraint. They showed their algorithm to work for complex objects as well as absorbing objects. These phase retrieval twin removal algorithms were soon successfully applied to in-line x-ray holography<sup>35-36</sup>. In [37] another constraint is developed for these ping-pong algorithms, this time to remove the twin image from electron holographic images. In the object plane the phase is replaced with a parabolic phase (similar to the expected shape of the object surface) and in the image plane the intensity is replaced with the measured intensity. The method only works well with pure amplitude objects.

Advances were made in understanding the sampling requirements of these phase retrieval algorithms in [38]. For real absorbing objects another iterative algorithm has been developed<sup>39</sup> to remove the twin image, this time utilizing both the Gerchberg-Saxton algorithm and the Fraunhofer technique. The iterative phase retrieval algorithm was extended for the case of multiple recordings of different in-line holograms in [40]. Very recently<sup>41</sup> improvements have been made on the iterative technique by using a better model of the object. This improved method works well for phase objects as well as for pure absorbing objects.

#### VII. TWIN REDUCTION BY LINEAR FILTERING/DIGITAL DECODING

The first digital signal processing technique for the removal of the twin image<sup>42</sup> appeared in 1979 but provided poor results and received little interest. Improved DSP based algorithms were developed some years later by Onural and Scott<sup>43-45</sup>. They described linear filtering operations to decode the information contained in the holograms. The filter is a series expansion of the inverse of that operator that maps object opacity function to hologram intensity. However their work did not allow for phase objects. Further advances in linear filtering for twin reduction that did allow in some cases for phase objects.<sup>46</sup> Maleki and Devaney [47] have proposed a deconvolution algorithm to remove the twin but the method is plagued with the singularity problem, and the calculation is complex. Spence et. al. described another non iterative method<sup>48</sup> but this too is limited to non phase objects. Yang et. al. have developed an algorithm<sup>49</sup> that seems quite similar to previous work on subtraction holography discussed earlier. They devise a DSP method that relies on multiple reconstructions and subtractions. A similar method is developed in [50], seemingly unaware of work in [49], however their algorithm requires two in-line holograms to be captured.

#### VIII. TWIN REMOVAL BY PHASE SHIFTING HOLOGRAPHY OR SOME FORM OF PHASE MODULATION

In 1997 Yamaguchi and Zhang developed a new method<sup>51</sup> for recording digital holograms free from the twin image, known as 'phase shifting digital holography'. The method allowed for the use of the in-line architecture but required a number of separate interferograms to be captured in which a phase shift is introduced to one of the interfering wavefields between capture. These phase shifts are usually effected by rotation of a quarter or half wave plate or through the minute vibration of a mirror. A similar method is presented in [52]. Chen et. al. have presented a method<sup>53</sup> that allows for the phase-shifting technique to be applied with an arbitrary phase shift and just two captures. Kim et. al. have proposed a method<sup>54</sup> for removing the twin image from a white light real time holographic system by utilizing polarization optics and the addition of images. However their method is based on the triangular interferometer that has numerous disadvantages. For complex gamma ray holography a phase shifting technique<sup>55</sup> has also been applied based on changing the phase of the nuclear scattering amplitude by detuning from the

resonance. Gabor and Goss also implemented an early technique<sup>56</sup> based on phase shifting in which two holograms were captured with a quarter wave phase shift was used between captures. Reconstruction was optical and required the use of a "quadrature prism" to combine the previous captures and remove the virtual image. In [57] a number of these phase modulation twin removal methods are reviewed. In the case of optical scanning holography, techniques have been proposed<sup>58-60</sup> to remove the twin was proposed involving simultaneous acquisition of sine and cosine Fresnel zone-lens plate coded images and adding the two holograms.

#### IX. TWIN REDUCTION BY ADDING IMAGES AT DIFFERENT FOCUS

A number of related techniques have appeared in the literature relating to twin removal in inline electron holography<sup>9,10, 62,63</sup>. It appears that one of the most successful methods for twin reduction in electron holography is by recording multiple holograms of the same objects with different wavelengths and then superimposing the intensities of the reconstruction. The out of focus twin image will change for each wavelength and average out. A similar technique has also been developed based on distance instead of wavelength<sup>62,63</sup>. By recording a series of holograms of the same objects but at different distances implies that the out of focus images will be different in each reconstruction and will integrate to approximate a constant value if a sufficient number of holograms are recorded and reconstructed.

#### X. TWIN REMOVAL BY SPATIAL FILTERING OF RECONSTRUCTION PLANES

Pedrini et. al. propose the first instance<sup>64</sup> of the spatial filtering of reconstruction planes of digital holograms. This involves cutting out the wanted digitally reconstructed image from its surrounding pixels. However this area still contained considerable noise from the unwanted twin image. In [16] traditional spatial filtering in the Fourier domain was applied to an off axis digital hologram. Denis et. al. have proposed a novel method<sup>65</sup> of spatially filtering the reconstruction domain. It was shown that by cutting out the reconstructed focused unwanted twin and returning to the plane of the virtual wanted image by numerical propagation one could free oneself of the unwanted noise. The method was proposed only in the area particle holography and the removal of the twin images was a manual operation. In this paper we propose the use of a similar technique for macroscopic objects. We discuss for the first time the relative spreading of the unwanted twin image and the wanted image and how one might manage this spreading in the numerical reconstruction techniques.

#### XI. DC REMOVAL

Throughout this discussion we have paid little attention to the zero order term – i.e. the intensity terms that appear as a by product of the holographic process. In some cases this artifact is far noisier than the unwanted twin. Many of the methods discussed above will remove this term in addition to removing the unwanted twin. A number of methods have been

developed in the literature to remove this term alone to augment those methods that do not. These methods are based on spatial filtering of the hologram<sup>66</sup>, subtracting stochastically different holograms<sup>67</sup>, phase-shifting<sup>68</sup> and by subtracting the numerical generated intensity of the object and reference waves from the digital hologram<sup>69</sup>.

## XII. CONCLUSION

In this paper we have categorically reviewed the subject of twin reduction and twin elimination in holography. This subject is of paramount importance in interference imaging due to the presence of the twin as a source of noise in the reconstructed image. We have reviewed over sixty years of research on this area. This paper will serve as a valuable reference to those interested in this subject.

## REFERENCES

- [1] D. Gabor, "A new microscopic principle," *Nature (London)* **161**, 777 (1948).
- [2] D. Gabor, "Microscopy by reconstructed wavefronts," *Proc. R. Soc. A* **197**, 454 (1949).
- [3] M. Tegze and J. Faigl, "X-ray holography with atomic resolution," *Nature (London)* **380**, 49 (1996).
- [4] P. Korecki and J. Korecki, "γ-Ray Holography – Three-Dimensional Imaging of a Local Atomic Structure" *Hyperfine Interact.* **144**, 85 (2002).
- [5] J. F. Arocena, T. A. Rotwell, and M. R. A. Shegelski, "Iterative reconstruction of in-line electron holograms," *Micron* **36**, 23 (2005)
- [6] J.C.H. Spence, "STEM and Shadow-imaging of Biomolecules at 6eV Beam Energy," *Micron* **28**, 116, 1997
- [7] H. -W. Fink, H. Schmid, E. Ermantraut, and T. Schulz, "Electron holography of individual DNA molecules," *J. Opt. Soc. Am. A* **14**, 2168-2172 (1997)
- [8] T. Matsumoto, T. Tanji and A. Tonomura, "Visualization of DNA in Solution by Fraunhofer In-Line Electron Holography: II. Experiments." *Optik* **100** (No.2) (1995) 71-74
- [9] S.Y. Tong, H. Hua Li, and H. Huang, "Energy extension in three-dimensional atomic imaging by electron emission holography," *Phys. Rev. Lett.* **67**, 3102 (1991).
- [10] J. J. Barton, "Removing multiple scattering and twin images from holographic images" *Phys. Rev. Lett.* **67**, 3106 (1991).
- [11] W. L. Bragg and G. L. Roger, Elimination of the Unwanted Image in Diffraction Microscopy *Nature (London)* **167**, 190 (1951).
- [12] G. L. Rogers, "In-line soft-x-ray holography: the unwanted image," *Opt. Lett.* **19**, 67- (1994)
- [13] P. Kirkpatrick and H. M. A. El-Sum, "Image formation by reconstructed wave fronts. I. Physical principles and methods of refinement," *J. Opt. Soc. Am.* **46**, 825- (1956)
- [14] T. Xiao; H. Xu; Y. Zhang; J. Chen; Z. Xu, "Digital image decoding for in-line X-ray holography using two holograms," *Journal of Modern Optics*, **45:2**, 343 – 353
- [15] E. Leith and J. Upatnieks, *J. Opt. Soc. Am.* **53**, 1377 (1963).
- [16] E. Cuche, P. Marquet and C. Depeursinge, "Spatial filtering for zero-order and twin-image elimination in digital off-axis holography," *Appl. Opt.* **39**, 4070 (2000).
- [17] G. W. Stroke, D. Brumm, A. Funkhouser, A. Labeyrie and R. C. Restrict, "on the absence of phase-recording or 'twin-image' separation problems in 'Gabor' (in-line) holograms" *BRIT, J. APPL. PHYS.*, 1966, VOL. 17, 497
- [18] J. B. DeVelis, G. B. Parrent Jr., and B. J. Thompson, "Image Reconstruction with Fraunhofer Holograms," *J. Opt. Soc. Am.* **56**, 423- (1966)
- [19] B. J. Thompson, J. H. Ward, and W. R. Zinky, "Application of hologram techniques for particle size analysis," *Appl. Opt.* **6**, 519- (1967)
- [20] G. A. Tyler and B. T. Thompson, "Fraunhofer holography applied to particle size analysis: a reassessment," *Opt. Acta* **23**, 685-700 (1976).
- [21] T. Matsumoto, T. Tanji, A. Tonomura, "Phase-contrast visualization of an undecagold cluster by in-line electron holography" *Ultramicroscopy* **54** 317 (1994)
- [22] J. Garcia-Sucerquia, W. Xu, S. K. Jericho, P. Klages, M. H. Jericho, and H. J. Kreuzer, "Digital in-line holographic microscopy," *Appl. Opt.* **45**, 836-850 (2006)
- [23] O. Bryngdahl and A. Lohmann, "Single-Sideband Holography," *J. Opt. Soc. Am.* **58**, 620- (1968)
- [24] M. R. Teague, "Deterministic phase retrieval: a Green's function solution," *J. Opt. Soc. Am.* **73**, 1434- (1983)
- [25] R. W. Gerchberg and W. O. Saxton, "A practical algorithm for the determination of phase from image and diffraction plane pictures," *Optik* **35**, 227-246 (1972).
- [26] J. R. Fienup, "Reconstruction of an object from the modulus of its Fourier transform," *Opt. Lett.* **3**, 27- (1978)
- [27] J. R. Fienup, "Phase retrieval algorithms: a comparison," *Appl. Opt.* **21**, 2758- (1982)
- [28] R.W. Gerchberg, "Holography without fringes in the electron microscope" *Nature* **240** (1972), p. 404.
- [29] J.B. Tiller, A. Barty, D. Paganin, K.A. Nugent, "The holographic twin image problem: a deterministic phase solution," *Optics Communications* **183** 2000.7-14
- [30] G. Liu and P. D. Scott, "Phase retrieval for in line holograms," in *Proceedings of the Nineteenth Annual Conference on Information Sciences and Systems* (Johns Hopkins U. Press, Baltimore, Md., 1985), pp. 237-241.
- [31] G. Liu and P. D. Scott, "Phase retrieval and twin-image elimination for in-line Fresnel holograms," *J. Opt. Soc. Am. A* **4**, 159- (1987)
- [32] G. Liu, "Object reconstruction from noisy holograms," *Optical Engineering* **29**(01), pp.19-24 (1990)
- [33] G. Koren, D. Joyeux, and F. Polack, "Twin-image elimination in in-line holography of finite-support complex objects," *Opt. Lett.* **16**, 1979- (1991)
- [34] G. Koren, F. Polack, and D. Joyeux, "Iterative algorithms for twin-image elimination in in-line holography using finite-support constraints," *J. Opt. Soc. Am. A* **10**, 423- (1993)
- [35] S. Lindaas, M. Howells, C. Jacobsen, and A. Kalinovsky, "X-ray holographic microscopy by means of photoresist recording and atomic-force microscope readout," *J. Opt. Soc. Am. A* **13**, 1788-1800 (1996)
- [36] Kodama, M. Yamaguchi, N. Ohyama, T. Honda, K. Shinohara, A. Ito, T. Matsumura, K. Kinoshita, K. Yada, "Image reconstruction from an in-line X-ray hologram with intensity distribution constraint," *Opt. Commun.* **125**, 36-42, (1996)
- [37] L. Bleloch, A. Howie, and E. M. James, "Amplitude recovery in Fresnel projection microscopy," *Appl. Surf. Sci.* **111**, 180 (1997).
- [38] J. Miao, D. Sayre, and H. N. Chapman, "Phase retrieval from the magnitude of the Fourier transforms of nonperiodic objects," *J. Opt. Soc. Am. A* **15**, 1662-1669 (1998)
- [39] X. M. H. Huang, J. M. Zuo, and J. C. H. Spence, "Wavefront reconstruction for in-line holograms formed by pure amplitude objects" *Appl. Surf. Sci.* **148**, 229 (1999).
- [40] Y. Zhang, G. Pedrini, W. Osten, and H. Tiziani, "Whole optical wave field reconstruction from double or multi in-line holograms by phase retrieval algorithm," *Opt. Express* **11**, 3234-3241 (2003)
- [41] T. Litychevskaia and H.-W. Fink, "Solution to the Twin Image Problem in Holography," *Phys. Rev. Lett.* **98**, 233901 (2007).
- [42] Marie, K.H.S., Bennett, J.C., Anderson, A.P. Digital processing technique for suppressing the interfering outputs in the image from an inline hologram, *Electron Lett*, **15** (1979) 241-243
- [43] L. Onural and P. D. Scott, "A digital filtering system for decoding in-line holograms," in *Proceedings of the 1985 IEEE Conference on Acoustics, Speech, and Signal Processing* (Institute of Electrical and Electronics Engineers, New York, 1985), pp. 708-711.
- [44] L. Onural, "Digital decoding of in-line holograms," Ph.D. dissertation (State University of New York at Buffalo, Amherst, New York, 1985).
- [45] L. Onural and P.D. Scott, Digital decoding of in-line holograms. *Opt Eng* **26**, pp. 1124-1132. (1987)
- [46] K. A. Nugent, "Twin-image elimination in Gabor holography," *Opt. Commun.* **78**, 293 (1990).
- [47] M. H. Maleki and A. J. Devaney, "Noniterative reconstruction of complex-valued objects from two intensity measurements," *Optical Engineering* **33**(10), pp. 3243-3253, 1994.

- [48] J.C.H. Spence, X. Zhang and W. Qian, in: *Electron Holography*, eds. A. Tonamura, L.F. Allard, G. Pozzi, D.C. Joy and Y.A. Ono (Elsevier Science, 1995) pp. 267-276.
- [49] S. Yang, X. Xie, Y. Zhao, C. Jia, "Reconstruction of near-field in-line hologram," *Optics Communications* **159** 1999.29–31
- [50] A. Zhang and X. Zhang, "Reconstruction of a complex object from two in-line holograms," *Opt. Express* **11**, 572-578 (2003)
- [51] Yamaguchi and T. Zhang, "Phase-shifting digital holography," *Opt. Lett.* **22**, 1268-1270 (1997)
- [52] S. Lai, B. King and M. A. Neifeld, "Wave front reconstruction by means of phase-shifting digital in-line holography," *Opt. Commun.* **173**, 155-160 (2000)
- [53] G. L. Chen, C. Y. Lin, H. F. Yau, M. K. Kuo, and C. C. Chang, "Wave-front reconstruction without twin-image blurring by two arbitrary step digital holograms," *Opt. Express* **15**, 11601-11607 (2007)
- [54] S.-G. Kim, B. Lee, and E.-S. Kim, "Removal of bias and the conjugate image in incoherent on-axis triangular holography and real-time reconstruction of the complex hologram," *Appl. Opt.* **36**, 4784–4791 (1997).
- [55] P. Korecki, G. Materlik, and J. Korecki, "Complex gamma-ray hologram: solution to twin images problem in atomic resolution imaging," *Phys. Rev. Lett.* **86**, 1534–1537 (2001).
- [56] D. Gabor and W. P. Goss, "Interference Microscope with Total Wavefront Reconstruction," *J. Opt. Soc. Am.* **56**, 849- (1966)
- [57] Y. Takaki, H. Kawai, and H. Ohzu, "Hybrid Holographic Microscopy Free of Conjugate and Zero-Order Images," *Appl. Opt.* **38**, 4990-4996 (1999)
- [58] K. Doh, T. -C. Poon, M. H. Wu, K. Shinoda and Y. Suzuki, "Twin-image elimination in optical scanning holography," *Optics & Laser Technology*, **28**, 135-141 (1996)
- [59] P. Sun and J.-H. Xie, "Method for reduction of background artifacts of images in scanning holography with a Fresnelzone-plate coded aperture," *Appl. Opt.* **43**, 4214–4218 (2004).
- [60] T. -C. Poon, T. Kim, G. Indebetouw, B. W. Schilling, M. H. Wu, K. Shinoda, and Y. Suzuki, "Twin-image elimination experiments for three-dimensional images in optical scanning holography," *Opt. Lett.* **25**, 215-217 (2000)
- [61] K. Doh, T.-C. Poon, and G. Indebetouw, "Twin-image noise in optical scanning holography," *Opt. Eng.* **35**, 1550–1555 (1996).
- [62] Weierstall, U., Huang, X. and Spence, J., Twin image suppression and forward focussing in low voltage in-line electron holography. *Proc. MSA*, ed. G. Bailey. Jones & Begell, New York. 1997
- [63] Lin, J. A. and Cowley. J. M., Reconstruction from in-line electron holograms by digital processing. *Ultramicroscopy*. **19**, 179 189, 1986
- [64] Pedrini, P. Fröning, H. Fessler, and H. J. Tiziani, "In-Line Digital Holographic Interferometry," *Appl. Opt.* **37**, 6262-6269 (1998).
- [65] Loic Denis, Corinne Fournier, Thierry Fournel, Christophe Ducottet, "Twin-image noise reduction by phase retrieval in in-line digital holography," *Proceedings of SPIE Vol.* **5914**, 59140J-1 (2005)
- [66] T. Kreis and W. P. O. Juptner, "Suppression of the dc term in digital holography," *Opt. Eng.* **36**, 2357 (1997).
- [67] N. Demoli, J. Mestrovic, and I. Sovic, "Subtraction Digital Holography," *Appl. Opt.* **42**, 798-804 (2003)
- [68] Y. Zhang, Q. Lü and B. Ge, "Elimination of zero-order diffraction in digital off-axis holography," *Opt. Commun.* **240**, 261-267, (2004).
- [69] G. -L. Chen, C. -Y. Lin, M. -K. Kuo, and C. -C. Chang, "Numerical suppression of zero-order image in digital holography," *Opt. Express* **15**, 8851-8856 (2007)