

New Perspectives on Distortion Synthesis for VA Oscillators and Resonance Emulation

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Introduction

Distortion Synthesis is a collective name given to a number of correlate techniques developed for digital audio synthesis.

These include: FM, Discrete Summation Formulae, Non-linear Waveshaping, Phase Distortion, Phase-Aligned Formant and Split-Sideband Synthesis.

VA Models

The term "Virtual Analog" (VA) first appeared in the 1990s with the commercial introduction of digital synthesizer instruments that were intended to emulate the earlier analogue subtractive synthesizers.

VA models mainly involve two approaches:

1. Explicit digital modelling of analogue circuits.
2. Mimicking the output of an analogue system (by various means).

Distortion Synthesis in VA Models

Distortion Synthesis is used extensively in existing VA implementations of oscillators (even if largely unacknowledged):

- Lane's oscillator model: abs() waveshaping + filtering
- Smith & Stilson's BLIT: Summation Formulae + integration
- Valimaki's DPW: parabolic waveshaping of complex wave

New Approaches

We have investigated some new approaches to distortion synthesis for creating quasi-bandlimited (alias-suppressed) classic waveforms:

1. Hyperbolic tangent waveshaping
2. Modified FM synthesis

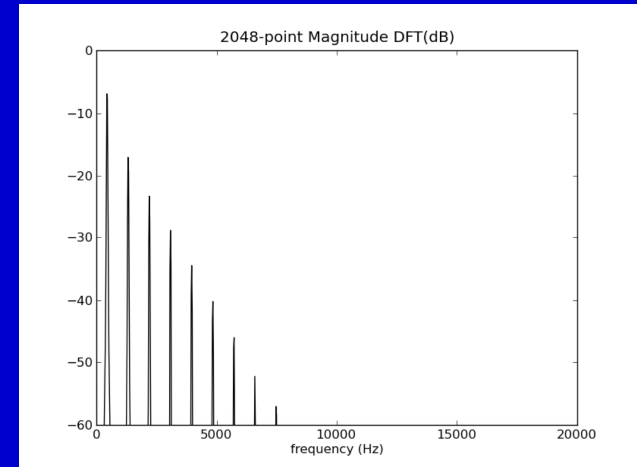
Hyperbolic Tangent Waveshaper

An interesting choice of waveshaper is the $\tanh()$ function, which will produce a odd-harmonic spectrum, which is alias-suppressed.

$$\begin{aligned} \tanh\left(\frac{\pi}{2} \sin(\omega)\right) &= \sum_{n=1}^{\infty} \frac{2^{2n}(2^{2n}-1)B_{2n}(\pi/2)^{2n-1}}{(2n)!} \sin^{2n-1}(\omega) = \\ &= \sum_{n=1}^{\infty} \frac{2^{2n}(2^{2n}-1)B_{2n}(\pi/2)^{2n-1}}{(2n)!} \left[\frac{2}{2^{2n-1}} \sum_{k=0}^{n-1} (-1)^{n-k-1} \binom{2n-1}{k} \sin([2n-2k-1]\omega) \right] = \\ &= \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} (-1)^{n-k-1} \frac{2B_{2n}(2^{2n}-1)(\pi/2)^{2n-1}}{n(k!)(2n-k-1)!} \sin([2n-2k-1]\omega) \end{aligned}$$

with B defined as

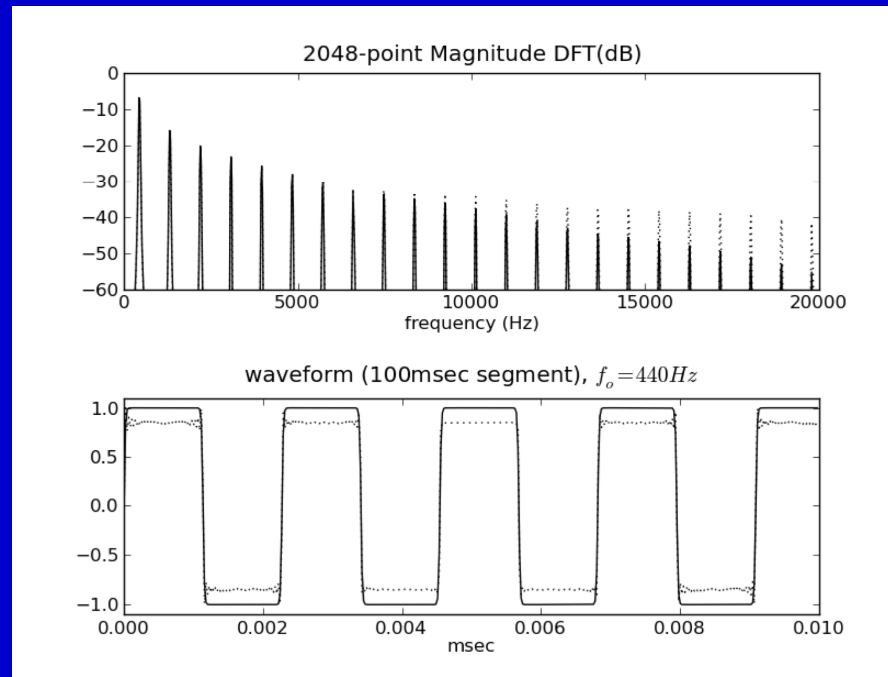
$$B_{2n} = (-1)^{n+1} \frac{2(2n)!}{(2\pi)^{2n}} \left[1 + \sum_{m=0}^{\infty} \frac{1}{m^{2n}} \right]$$



Square Wave Synthesis

However, it is important to drive the waveshaper a little harder to get a square wave, this is because

$$\tanh(kx(t)) \approx \text{sgn}(x(t)), \quad k \gg 0$$



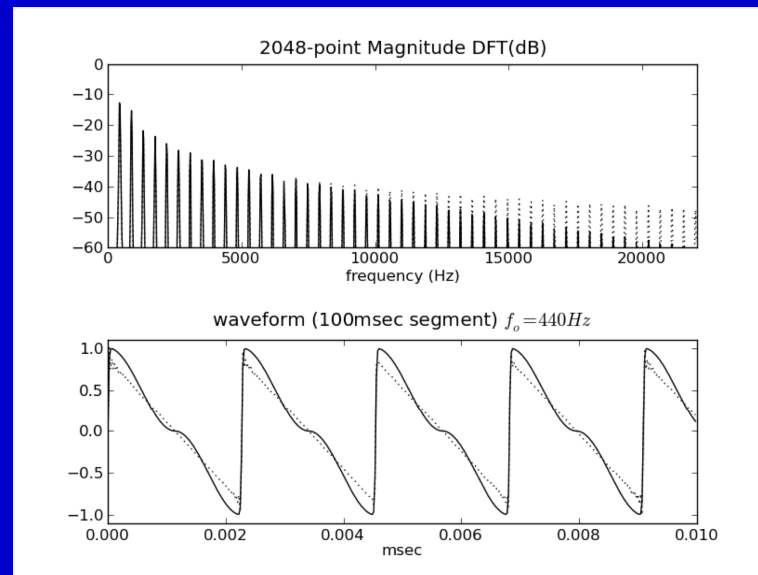
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Sawtooth Wave

Starting from a square wave is possible to approximate very closely a sawtooth spectrum and waveform, using some heterodyning

$$\begin{aligned} \text{saw}(t) &= \text{square}(\omega t)(\cos(\omega t) + 1) \\ &= \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin([2n+1]\omega t) + \frac{2n+2}{4n^2+8n+3} \sin(2[n+2]\omega t) \end{aligned}$$



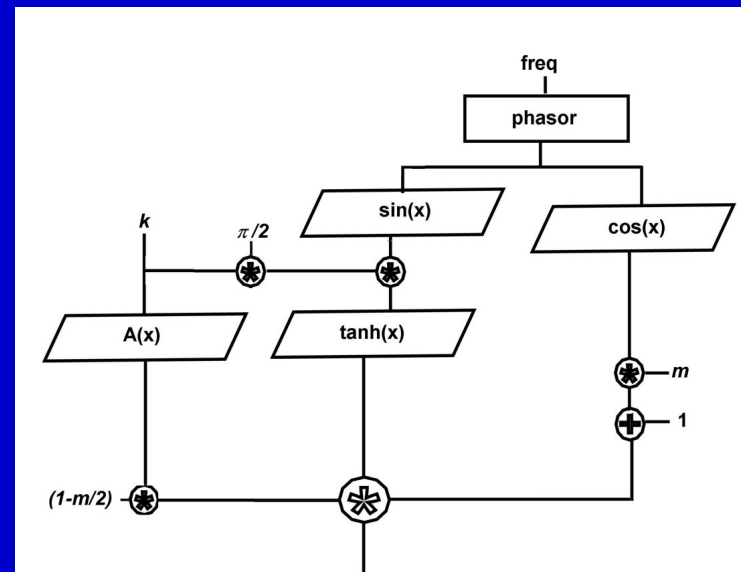
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Waveshaping alias-suppressed VA Oscillator

Combining the two we have a design for a waveshaping alias-suppressed oscillator, with a shape control (m). Alias-suppression is controlled by the distortion index k :

$$s(t) = A(k)\left(1 - \frac{m}{2}\right) \tanh\left(\frac{\pi k \sin(\omega t)}{2}\right) [1 + m \cos(\omega t)]$$



Modified FM

A technique derived from Classic FM, exhibiting Modified Bessel functions in its expansion.

FM

$$\begin{aligned}\Re\{e^{i[\omega_c + k \cos(\omega_m)]}\} &= \cos(\omega_c + k \cos(\omega_m)) = \cos(k \cos(\omega_m)) \cos(\omega_c) - \sin(k \cos(\omega_m)) \sin(\omega_c) \\ &= J_0(k) \cos(\omega_c) + \sum_{n=1}^{\infty} (-1)^{\text{int}(\frac{n}{2})} J_n(k) (\cos(\omega_c - n\omega_m) + (-1)^n \cos(\omega_c + n\omega_m))\end{aligned}$$

ModFM

$$\begin{aligned}\Re\{e^{i\omega_c + k \cos(\omega_m)}\} &= e^{k \cos(\omega_m)} \cos(\omega_c) \\ &= I_0(k) \cos(\omega_c) + \sum_{n=1}^{\infty} I_n(k) (\cos(\omega_c - n\omega_m) + \cos(\omega_c + n\omega_m))\end{aligned}$$

Scaling

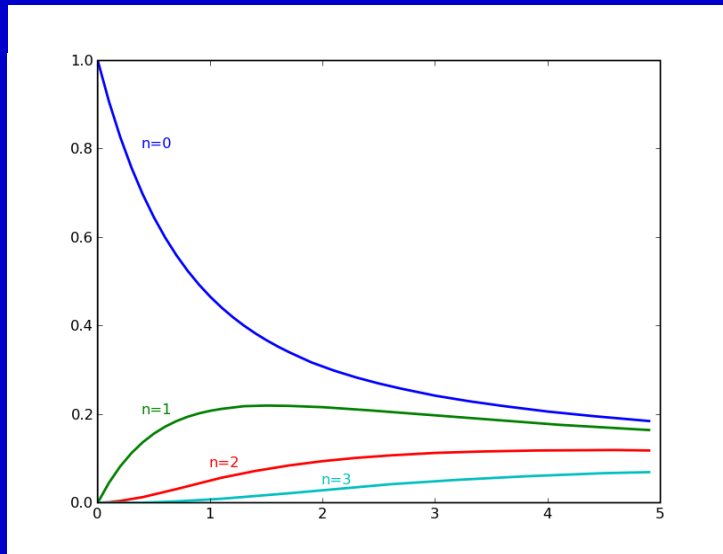
Given that the expansion of the ModFM synthesis expression is scaled by Modified Bessel coefficients, it requires suitable scaling for it to work with various modulation amounts:

$$\begin{aligned} s(t) &= e^{(k \cos(\omega_m t) - k)} \cos(\omega_c t) = \\ &= \frac{1}{e^k} \sum_{n=-\infty}^{\infty} I_n(k) \cos(\omega_c t + k \omega_m t) \end{aligned}$$

Advantages of ModFM

The comparison between FM and ModFM shows that the main difference is the presence of modified Bessel Functions

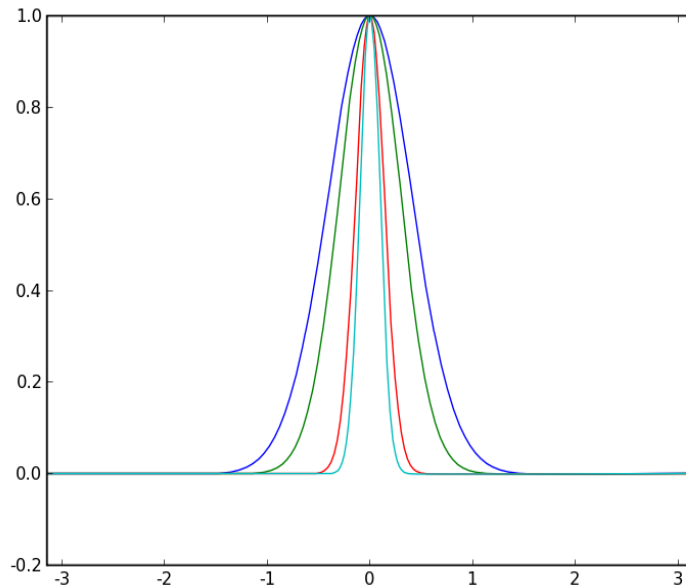
These, when appropriately scaled can produce more natural spectral evolutions with changes of modulation index



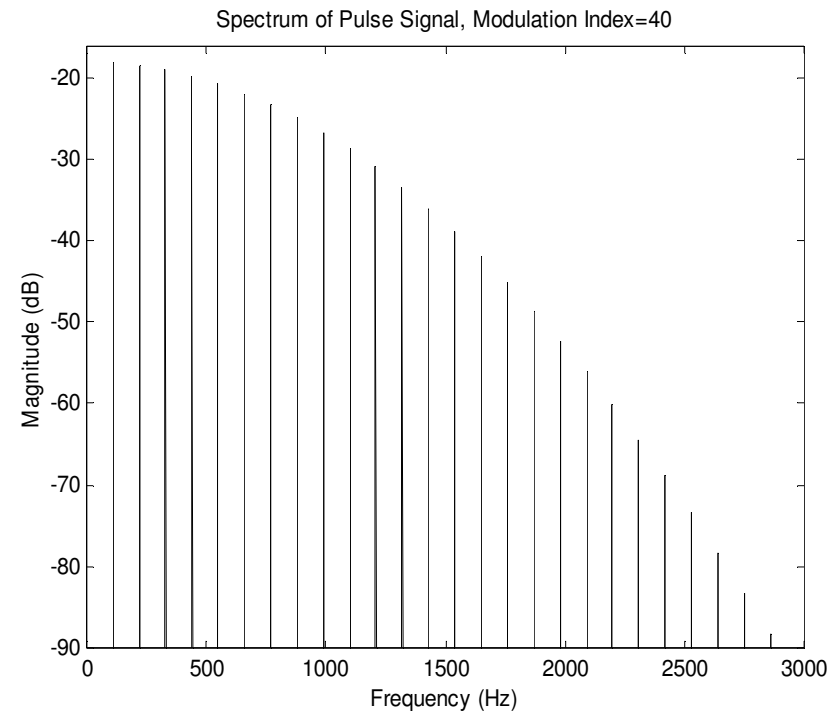
The plot of the scaling functions $2I'_n(k)e^{-k}$ with orders $n=0$ to 3

ModFM Pulse

With moderate to high values of k



ModFM pulse waveforms, for $k=5,10,50$ and 100

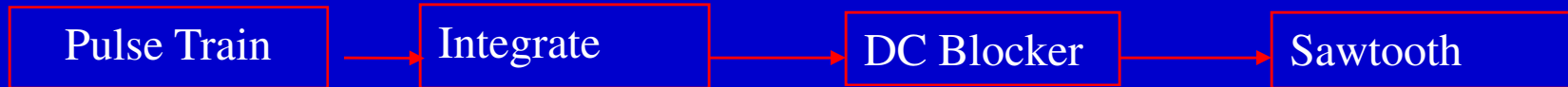


we can produce a pulse train signal

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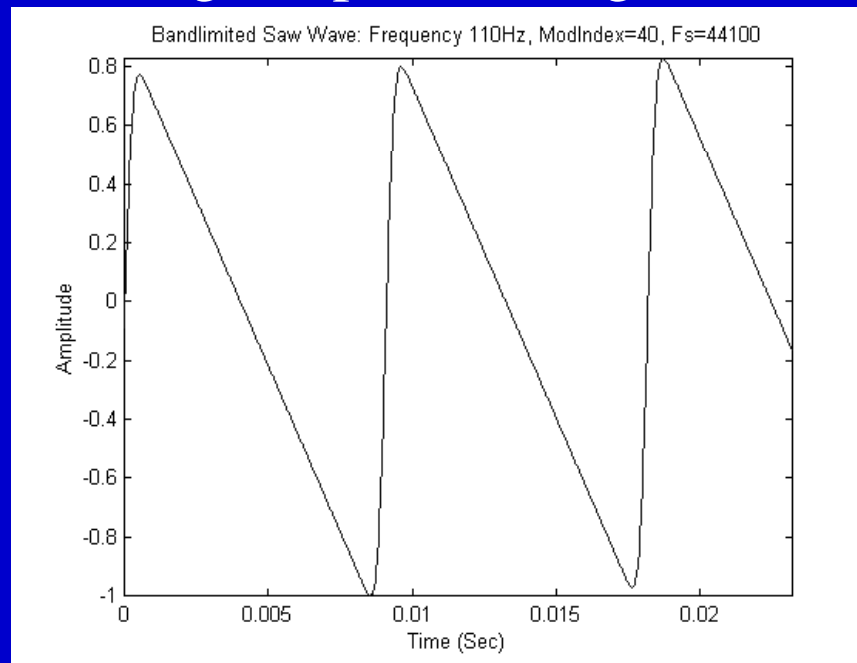
Bandlimited Sawtooth



From a bandlimited pulse, it is possible to generate a sawtooth wave by integration following the procedure given in (Stilson and Smith, 1996)

$$H(z) = \frac{1}{1 - z^{-1}}$$

integrator



Controlling the bandwidth

The expression for the spectrum of the modFM saw is

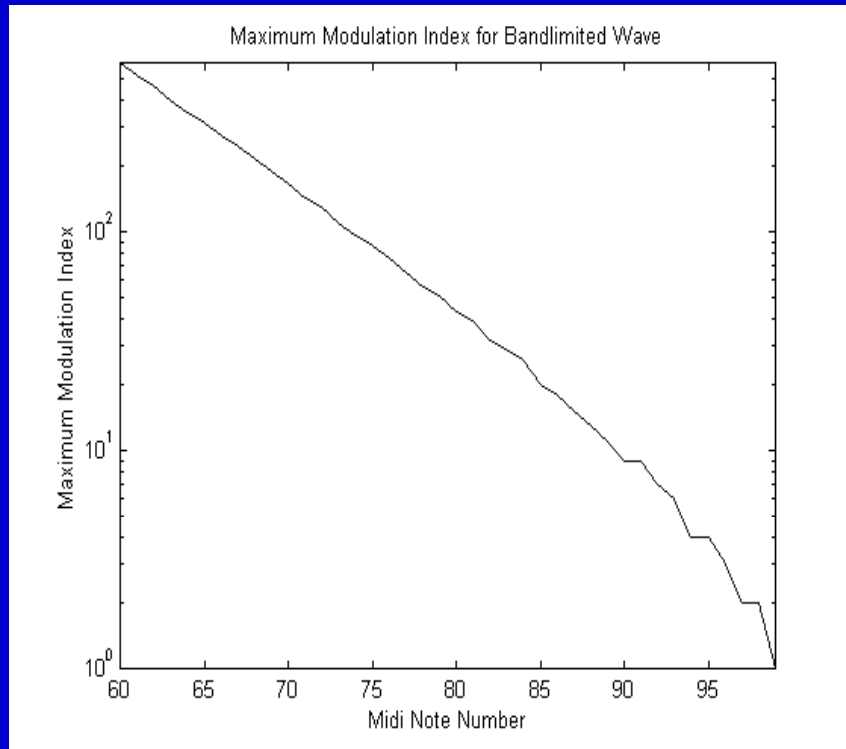
$$s_{saw}(t) = \frac{e^{-k}}{(n\omega t)} \sum_{n=1}^{\infty} I_n(k) \sin(n\omega t)$$

We can determine k so that it produces a bandwidth whose significant energy is contained within the digital baseband only.

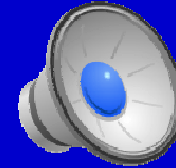
This amounts to finding a *max* k such that

$$\max_k \left\{ 20 \log_{10} \frac{I_{n+1}(k)(n+1)^{-1}}{I_1(k)} \right\} \leq -90 \text{dB}, \quad n = \left\lfloor \frac{sr}{2f_0} \right\rfloor$$

Index of Modulation



Max k plotted in relation to MIDI note numbers 60-127



A suitable optimisation routine was applied to derive these values.

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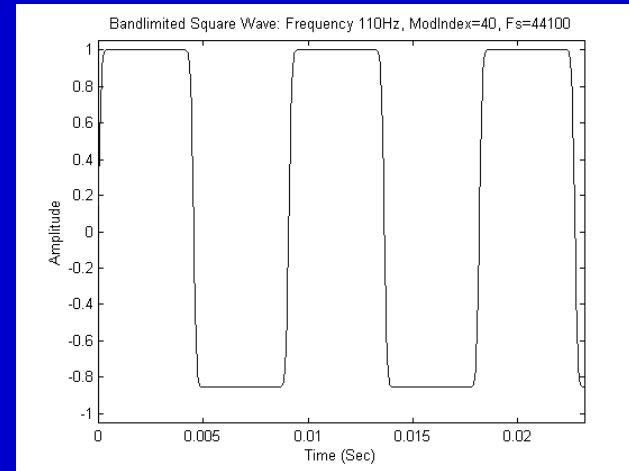
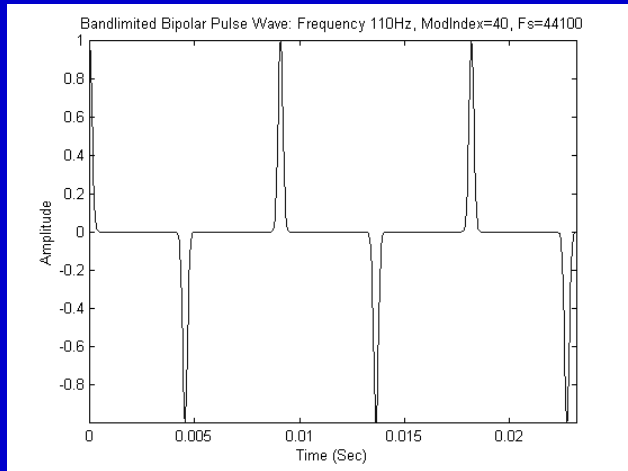
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Other waveforms

Square wave: a bipolar pulse can be generated by setting the $f_c:f_m$ ratio to 1:2

$$y(t) = e^{(k \cos(2\omega t) - k)} \cos(\omega t)$$

Integrating this expression yields a square wave



A triangle wave can be produced by a further stage of integration, followed by DC removal

Formant and Resonance Synthesis

We have also investigated the synthesis of resonance by means of distortion synthesis alone (without the use of IIR filters). ModFM can be a very efficient and useful method for this application.

For the synthesis of formants and resonance, we will use a *phase-synchronous* implementation of the ModFM equation. We will start by defining the carrier and modulator frequencies (f_c and f_m) based on a fundamental f_0 and a formant frequency f_f

$$f_m = f_0$$
$$f_c = nf_0 = \text{int}\left(\frac{f_f}{f_0}\right)f_0$$

Varying the Formant Frequency

In order to allow for a variable and sweep-able formant frequency, we will modify the original formula to use two carriers, tuned to adjacent harmonics in the formant region

$$e^{(k \cos(\omega_0 t) - k)} [(1 - a) \cos(n \omega_0 t) + a \cos([n + 1] \omega_0 t)]$$

$$a = \frac{f_f}{f_0} - n$$

these two carriers are linearly interpolated to generate the correctly-placed formant.

This expression defines our ModFM *formant operator*

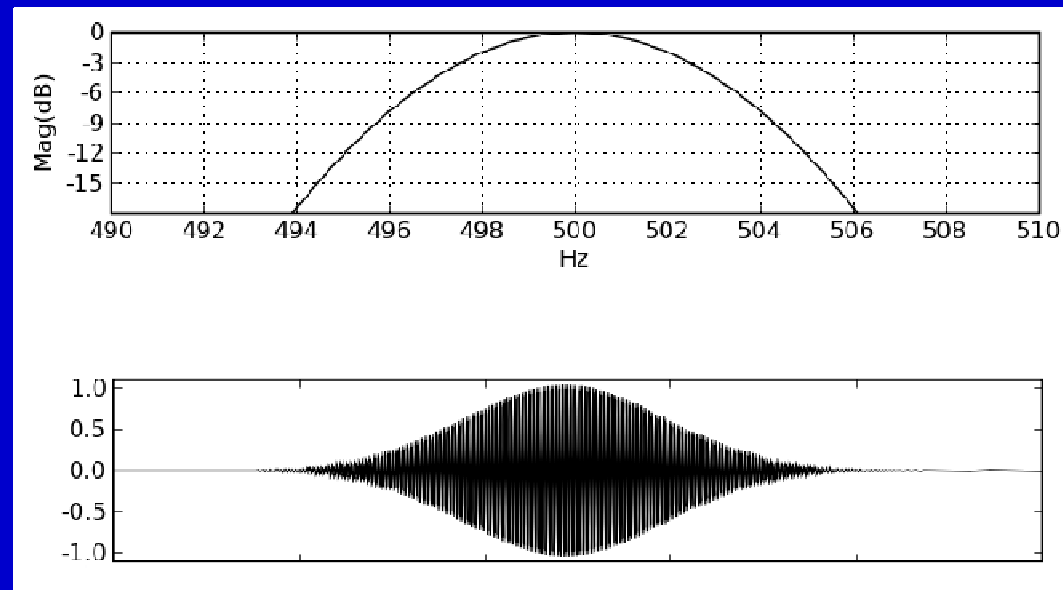
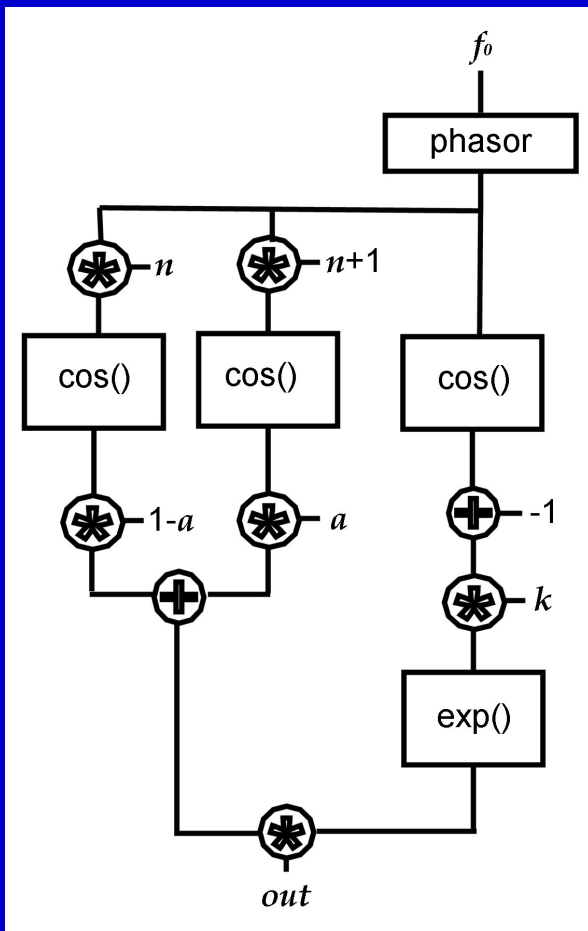
Bandwidth control

We can approximate the bandwidth, following the example set by Puckette in his PAF algorithm. Using an intermediary variable, we set the index of modulation k to

$$k = \frac{2\gamma}{(1-\gamma)^2}$$

The value of γ is approximated as a function of the bandwidth B and fundamental

$$\gamma \approx 2^{-\frac{f_0}{0.29B}}$$



A very narrow formant region
and a low-freq waveform plot

 female choir example

 resonant synth example

the ModFM formant operator

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Questions ?

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