THE DOMAIN OF VALIDITY OF THE PUT-CALL PARITY

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Abstract. We give an example where the put-call parity does not hold, and we give the domain of validity of this formulae.

Consider a European call option and a European put option on the same non-dividend paying stock, with same maturity date T and same strike price $K > 0$. Let the current price of the stock be S_0, and denote by c (resp. p) the current price of the call (resp. put) option. Let also r denote the continuously compounded risk-free interest rate for an investment maturing in time T.

As described in Hull [1] p. 212, the put-call parity in this case states that

$$c + Ke^{-rT} = p + S_0. \tag{0.1}$$

We next give an example where Equation (0.1) is violated. Consider a stock whose initial price is \$120 and in each of two time steps may go up by \$10 or down by \$5. Each time the step is three months long. Consider a call and a put option written on this stock, with same maturity time of six months and same strike price $K = 140$.

It is straightforward to see that the call option will never be exercised, and thus we must have $c = 0$. The next figure represents the value of the put predicted by Eq. (0.1) as a function of the interest rate r.

Figure 1 shows that, for high enough interest rates, the predicted put price is negative. However, it is easy to see that for such high rates the put

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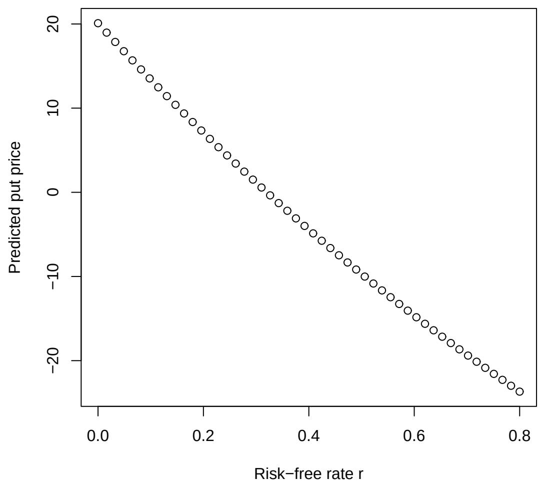

FIGURE 1. Predicted put prices as a function of r

option is dominated by the risk-free investment, and thus its price should be 0. Thus, the put-call parity misprices the put in this simple case.

Even if innocuous in appearance, our example has practical implications. First, it is easy to see that the put-call parity holds when K is chosen within the range of all possible values of the underlying asset at maturity time for continuity reasons (for instance, options prices can be calculated separately with a binomial tree algorithm, and the prices are consistent with Eq. (0.1)). Our counter-example is set so that K lies outside of this range. Thus, it is critical in practice to forecast accurately this range when issuing such options, for otherwise basic pricing methods will lead to severe mispricing.

References

[1] Hull, J. (2006) *Options, Futures and Other Derivatives,* 6th ed., Prentice Hall.

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