

COORDINATION IN MARKETS WITH CONSUMPTION EXTERNALITIES: ADVERTISING AND PRODUCT QUALITY*

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In this paper we study advertising in markets with positive consumption externalities. In such markets, we show that firms may engage in advertising competition to coordinate consumer expectations on their own brand as long as they produce goods of similar quality. The firm with the lower-quality product has a greater incentive to advertise. Hence in equilibrium, the lower-quality product will often be more popular.

1 INTRODUCTION

In this paper we study advertising in markets with positive consumption externalities. In markets with consumption externalities, the value of the product to the consumer does not only depend on the characteristics of the product but it also depends on the purchasing decisions of other consumers either due to social interaction or due to network externalities. Reading a best seller gives the additional pleasure of having common conversation topics with other people. The decision about which software to purchase is probably not only determined by the relative prices or qualities of the competing packages. The choice also involves the expectations about which one of the software packages is going to be more widely adopted. The widely adopted packages have a greater selection of compatible products developed. The users of popular systems have more people around who can help with trouble shooting.

In such markets, Pastine and Pastine (2002) show that the firm has incentives to advertise to coordinate consumer expectations such that consumers believe that its brand will be the popular one. When a consumer observes an expensive advertising campaign, he/she realizes that the firm would not have advertised if it did not expect advertising to increase its sales. Hence it is in the best interest of the individual to purchase the heavily

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advertised brand, vindicating the advertising investment of the firm. Clark and Horstmann (2005) show that firms can use advertising to coordinate consumer purchases, even if advertising levels are imperfectly observable.

These papers analyze markets with either homogenous products or horizontally differentiated products. In this paper, we allow for vertically differentiated products. We investigate whether advertising can serve a coordinating function when goods are of different qualities. We then examine the nature of the advertising competition. We show that firms may advertise in order to coordinate consumer expectations on their own brand only if they produce goods of similar quality. The firm with the lower-quality product has a greater incentive to advertise and the lower-quality product will often be more popular.

2 FRAMEWORK

We would like to focus on coordinating advertising in markets with consumption externalities. Hence we construct a model that abstracts from all previously analyzed roles of advertising. To abstract from the ‘persuasive role’ of advertising,¹ we assume that consumers are rational and that their preferences are constant in the face of advertising. To abstract from the ‘informative role’ of advertising, either directly or indirectly through signaling, we assume common knowledge of the existence, prices and characteristics of the products.²

Consider a market with two brands and a unit continuum of consumers who differ in their taste (α) for the product $\alpha \sim U[0, \bar{\alpha}]$. Consumers’ preferences exhibit positive consumption externalities. The utility the consumer gets from the product is a function of the quality of the product (V) and it is increasing in the number of the people (q) who purchase the same brand. Consumers want at most one product from each brand. Thus, if this is the market for books, some consumers may prefer to buy more than one book, but no one will purchase the same book twice. The utility function exhibits diminishing marginal utility. Consumer i ’s indirect utility is given by

$$U_i = \begin{cases} 0 & \text{if } i \text{ makes no purchases} \\ (\alpha_i V_j + \beta q_j - P_j) & \text{if } i \text{ buys only product } j \\ (\alpha_i V_j + \beta q_j - P_j) + (\omega \alpha_i V_k + \beta q_k - P_k) & \text{if } i \text{ buys product } j \text{ and then } k \end{cases} \quad (1)$$

¹See Galbraith (1967), Solow (1967) and Dixit and Norman (1978).

²See Butters (1977), Grossman and Shapiro (1984), Kihlstrom and Riordan (1984), Meurer and Stahl (1994), Robert and Stahl (1993), Stahl (1994), Stegeman (1991), Milgrom and Roberts (1986), Nelson (1974) and Bagwell and Ramey (1994) for informative advertising.

for $j, k \in \{A, B\}$ and $j \neq k$, $\beta > 0$, $\omega \in (0, 1)$. P is the price, β is the consumption externality parameter and ω yields diminishing marginal utility.

For instance, this specification of consumers seems to be a reasonable approximation in the market for movies since people like to chat about movies. If a product with positive consumption externalities has high early sales, later consumers have more incentive to purchase that good and this snowball effect results in high total sales. This feature of markets with consumption externalities is consistent with the belief in the movie industry that a firm's ability to coordinate consumer expectations on its product during the opening weekend will be crucial to its success.³ In this paper we model purely coordinating advertising with no word-of-mouth communication about the popularity of the product: our consumers make their purchasing decisions after firms advertise but before observing the actions of other consumers. Thus in the movie industry interpretation, it is advertising aimed at creating high first-weekend sales that is examined.

Two firms (A, B) produce at a common, constant marginal cost, normalized to zero. The products are of potentially different qualities, V_A and V_B . Without loss of generality we assume throughout that $V_A \geq V_B$. Both A's and B's products sell at price P . The producer firm captures a percentage $\gamma \in (0, 1]$ of the sales receipts from its product.⁴

In the movie industry interpretation, at the retail level, a movie theater typically carries the products of more than one studio and sells tickets at the same price. That is, at a particular theater the tickets to a popular movie typically sell for the same price as the tickets to an unpopular one. We do not attempt to explain this phenomenon but take it as given. We will focus on the role of advertising abstracting from the interactions between advertising and retail price competition.⁵

In order to guarantee interior solutions to the consumers' problem we make two assumptions on parameter values.⁶ For some of the consumers the product is desirable enough to purchase both goods, even in the absence of consumption externalities: $\bar{\alpha} > P/\omega V_B$. And the consumption externality is relatively mild so that a consumer who does not care for the product ($\alpha = 0$) never purchases it just for the externality, even if everybody else is buying it, $\beta < P$. Together these two restrictions on parameter values yield a market that

³For example, see 'The Won and Lost Weekend', *The Economist*, 29 November 1997, p. 87. Studios spend up to three quarters of a movie's promotion budget in the four to five days before it opens.

⁴The typical contract between the movie studios and the theaters ties the producer's revenue directly to ticket sales. This risk-sharing arrangement helps to ensure that the studios have incentives to promote their movies.

⁵Pastine and Pastine (2002) have a dynamic model with homogeneous consumers where firms' advertising and price decisions are endogenously determined. Nevertheless, avoiding these interaction effects permits a much clearer understanding of coordinating advertising in markets with consumption externalities where goods are vertically differentiated.

⁶The analysis of the possible corner solutions in the absence of these assumptions is simple but creates numerous subcases without adding any intuition or altering the basic conclusions.

is fundamentally driven by product value. Consumption externalities are important in this market, but not to the extent that they eclipse the fundamental value of the product to consumers.

Before analyzing the case with vertically differentiated products (Section 4), we first examine a market with products of equal quality, $V_A = V_B \equiv V$. The insights gained about advertising competition in the homogenous goods case will then be extended to the vertically differentiated goods market.

3 EQUAL QUALITIES

Due to consumption externalities, there are multiple equilibria when $V_A = V_B$: consumers can coordinate on one of the products, or there might be no coordination.

3.1 Multiplicity of Equilibria

3.1.1 No Coordination. A and B are of the same quality and they have the same price. If firms do not engage in some activity to differentiate their products from each other, consumers may have no device to coordinate their expectations about the popularity of the products. Hence in equilibrium, consumers may expect that A and B will sell the same quantity $E(q_A) = E(q_B) = q$.

Since consumers differ in their taste for the product, while some may choose not to purchase either of the items, others may purchase one item and the consumers who have a high taste for the product may purchase both items. A consumer purchases a second item only if the additional utility from the second product is higher than the price, $\alpha_i \omega V + \beta q \geq P$. Hence people with high enough taste $\alpha \geq (P - \beta q) / \omega V$ will buy A and B. Since $\alpha \sim U[0, \bar{\alpha}]$, the proportion of people who buy both of the products is given by $1 - (P - \beta q) / \bar{\alpha} \omega V$.

A consumer will not buy either of the products if his/her marginal utility from the first purchase is less than the price, $\alpha_i V + \beta_j \leq P$. So consumers with a low taste for the good $\alpha < (P - \beta q) / V$ will not buy either of the products. Hence, the proportion of people that purchase a single product is $(P - \beta q) / \bar{\alpha} \omega V - (P - \beta q) / \bar{\alpha} V$.

People who decide to buy only one item are indifferent between A and B, so they split between the products. Hence the proportion of people who buy a particular product is given by the proportion of consumers who purchase both items plus half of the proportion of consumers who purchase one item. This yields the equilibrium number of people who purchase each good:

$$q = 1 - \frac{(P - \beta q)(1 + \omega)}{2\bar{\alpha}\omega V} \quad (2)$$

Solving for q ,

$$q = \frac{2\bar{\alpha}\omega V - P(1+\omega)}{2\bar{\alpha}\omega V - \beta(1+\omega)} \equiv \tilde{q} \quad (3)$$

$\tilde{q} < 1$, since $P > \beta$. And $\tilde{q} > 0$, since $\bar{\alpha} > P/\omega V$. As the price goes up, quantity demanded goes down. An increase in the consumption externality parameter β leads to an increase in the quantity demanded.

3.1.2 Coordinated Demand Equilibria. Suppose that consumers believe that firm j will sell more than firm k , $E(q_j) > E(q_k) \forall j, k \in \{A, B\}$ where $j \neq k$. Since all else is equal, consumers would prefer j 's product due to the consumption externalities. Consumers who purchase k are the ones who purchase both j and k . A consumer purchases the second item k if the additional utility from k is higher than the price, $\alpha_i\omega V + \beta q_k \geq P$. The proportion of people who buy k is then given by

$$q_k = 1 - \frac{P - \beta q_k}{\bar{\alpha}\omega V} \quad (4)$$

Solving for q_k yields

$$q_k = \frac{\bar{\alpha}\omega V - P}{\bar{\alpha}\omega V - \beta} \equiv \underline{q} \quad (5)$$

Consumers whose marginal utility from the first purchase is higher than the price buy j , $\alpha_i V + \beta q_j \geq P$. So people with $\alpha \geq (P - \beta q_j)/V$ purchase j . The proportion of the people who purchase j is

$$q_j = 1 - \frac{P - \beta q_j}{\bar{\alpha}V} \quad (6)$$

Solving for q_j yields

$$q_j = \frac{\bar{\alpha}V - P}{\bar{\alpha}V - \beta} \equiv \bar{q} \quad (7)$$

Inspection of (5) and (7) reveals that $\bar{q} > \underline{q}$ since $P > \beta$ and $\omega < 1$. The product that faces the coordinated demand sells more due to consumption externalities. Also notice that the proportion of people who buy j in the coordinated demand equilibrium is greater than the proportion of people who buy j when there is no coordination, $\bar{q} > \tilde{q}$. The proportion of people who buy k declines when consumers coordinate on j compared with the case where there is no coordination, $\underline{q} < \tilde{q}$.

3.2 The Role of Advertising

There is an equilibrium where consumers coordinate on A and there is also an equilibrium where consumers coordinate on B. Firms have preferences over

these possible equilibria. Thus, firms may try to differentiate themselves via advertising to coordinate consumer expectations on their own brand.

In light of the multiplicity of equilibria, imagine a consumer who observes an advertising campaign. The consumer could suppose either that the advertising was a mistake or that advertising will lead to increased sales. And in fact there are several equilibria so either set of expectations is perfectly rational. But they are not equally plausible as general predictions of consumer behavior. If a firm invested in advertising it must have believed that advertising would result in increased sales, and consumers are likely to take this into account when forming their own expectations. For consumers to ignore advertising in forming their expectations they must treat all observed advertising as costly mistakes by firms.⁷

3.3 *The Nature of Advertising Competition*

Let us momentarily postulate that the firm that advertises more heavily can coordinate consumers on its own brand, and find the equilibrium in the advertising competition. Using this we will then argue that this is likely to be the equilibrium outcome: it is indeed rational for the consumer to prefer to purchase the more heavily advertised product.

Abstracting from all other functions of advertising, when the only role of advertising is to coordinate consumer expectations, advertising competition has the same form as the first-price all-pay auction.⁸ In a first-price all-pay auction all participants must pay their bids regardless of whether they win the auction or not, and the highest bidder receives the prize. In our framework, the more heavily advertised brand (the firm with the highest bid in the form of advertising expense) can capture the popular brand position. The value of the auction for a firm is the difference in the profits the firm would get if consumers coordinate on its product versus if consumers coordinate on the rival's product. If a firm out-advertises its rival its quantity sold is given by (7) and if its competitor out-advertises it, its quantity sold are given by (5). Therefore the value of 'winning' the advertising competition (Γ) to each firm over and above the value of losing is given by the difference between these quantities, \bar{q} and \underline{q} , times the price of the product, times the share of sales which goes to the producer:

⁷Firms in markets with significant consumption externalities often try to encourage this type of reasoning. In the early months of 1995, during the height of its advertising campaign for OS/2, IBM frequently cited its \$500 million investment in the operating system. When Windows 95 was launched in August 1995, Microsoft widely publicized that it was spending \$200 million on advertising alone. See, for example, Panettieri (1995) and Rebello and Kuntz (1995).

⁸All-pay auctions have been analyzed by Tullock (1980), Hillman and Riley (1989) and Baye *et al.* (1993, 1996). The proofs used here are adapted from Ellingsen (1991). General statements on expected pay-offs can be found in Baye *et al.* (1993) and Siegel (2009). All-pay auctions have been used by many authors to model lobbying for monopoly rents, R&D races, political campaigns, tournaments and job promotion.

$$\Gamma_A = \Gamma_B = \frac{\gamma P(P - \beta)\bar{\omega}V(1 - \omega)}{(\bar{\omega}V - \beta)(\bar{\omega}V - \beta)} \quad (8)$$

Since the firms' problems are symmetric, the value of winning the advertising competition is the same for both producers.

In this section we will derive the properties of the equilibrium that will be useful in the discussion. See Hillman and Riley (1989) or Ellingsen (1991) for the derivation of the full equilibrium.⁹

Lemma 1: Considering $\Gamma_A \leq \Gamma_B$, neither firm will put a probability mass point on any advertising level greater than zero.

Proof: Define a range A_i as $(0, x)$ where x is an arbitrary number greater than Γ_B . Define a'_i as the supremum of A_i . Suppose the lowest mass point of firm i in A_i is given by $a_i^* \in A_i$. Then firm j would not put any probability at $a_j = a_i^*$ as a slight increase in its advertising would result in a discrete increase in the probability of winning. As there is no probability of $a_j = a_i^*$, firm i could lower its advertising slightly without changing its probability of winning. ■

Lemma 2: Considering $\Gamma_A \leq \Gamma_B$, both firms have infimum advertising levels of zero. The expected value of the game to firm A is zero.

Proof: Firm i 's infimum advertising level must be less than Γ_A since there can be no probability mass at Γ_A by Lemma 1. Suppose that firm i has an infimum advertising level of $a_i^{\text{inf}} \in (0, \Gamma_A)$. Then firm j would never choose $0 < a_j \leq a_i^{\text{inf}}$. If it did it would be paying a positive amount and would lose for sure, since by Lemma 1, the probability of firm i choosing exactly a_i^{inf} is zero in this range. Therefore firm i could lower its infimum advertising level without changing the probability of winning.

If firm A had an expected value from the game greater than zero, it would have to have a non-zero probability of winning at its infimum advertising level since all advertising levels in the support of the mixed strategy must yield the same expected pay-off. From Lemma 1 this would imply that firm B would be putting a probability mass at zero advertising. If so firm A would never advertise zero since it could gain a discrete increase in its probability of winning at negligible cost by advertising just slightly more than zero. Thus firm B would lose with certainty with zero advertising, yielding a pay-off of zero. In this candidate equilibrium firm B would be earning zero expected profits and A would be earning positive expected profits so its

⁹The literature on all-pay auctions is extensive. With more than two players there may not be a unique equilibrium, but the result on the costs for the two players with the highest valuation can be obtained across all equilibria; see Baye *et al.* (1993).

supremum advertising level would have to be strictly below Γ_A . In that case firm B could raise its advertising to just above Γ_A and make positive expected profits. Hence in equilibrium the expected value of the game to firm A must be zero. ■

Lemma 3: Considering $\Gamma_A \leq \Gamma_B$, both firms have supremum advertising levels of Γ_A . The expected value of the game to firm B is $\Gamma_B - \Gamma_A$.

Proof: If firm B had a supremum level of advertising of $a_B^{\text{sup}} < \Gamma_A$, firm A could win for sure with that level of advertising since by Lemma 1 the probability of firm B choosing exactly a_B^{sup} is zero. However this would result in a positive pay-off for A, a contradiction of Lemma 2.

Suppose that firm A had a supremum advertising level of $a_A^{\text{sup}} < \Gamma_A$. Then firm B would never set $a_B > a_A^{\text{sup}}$ as it can win for sure with that level of advertising since by Lemma 1 the probability of firm A choosing exactly a_A^{sup} is zero. This presents a contradiction of the first part of the proof of this lemma.

Firm B wins for sure with its supremum advertising level since by Lemma 1 the probability of firm A choosing exactly that level is zero. Since Γ_A is in the support of its mixed strategy and it wins for sure with that advertising level, the expected pay-off for firm B is $\Gamma_B - \Gamma_A$. ■

Lemma 4: Considering $\Gamma_A \leq \Gamma_B$, for each firm advertising levels almost everywhere on $\alpha_i \in (0, \Gamma_A]$ must have positive probability.

Proof: Suppose there were an interval (t, s) in where firm i had zero probability of advertising. Then firm j would have zero probability of advertising in that interval since it could lower its advertising to t and have the same chance of winning. But in this case firm i would never bid $s + \varepsilon$ as it could lower its advertising to t , saving $s + \varepsilon - t$ in advertising costs and losing only the probability that j advertises in the range $[s, s + \varepsilon]$. By Lemma 1 the loss in probability is negligible for small ε . So if there is an interval of zero probability it must go up to the top of the range, which contradicts Lemma 3. ■

Proposition 1: No pure-strategy equilibrium exists. The mixed-strategy equilibrium involves almost everywhere continuous mixed strategies where both firms mix in the range $[0, \Gamma_A]$. The advertising competition results in the expected dissipation of all gains to advertising.

Proof: From Lemma 1 there can be no pure-strategy equilibrium with advertising levels greater than zero. If firm A is playing a pure strategy of zero advertising then firm B would play a pure strategy of advertising just above zero, winning for sure with negligible advertising expenses. This would con-

tradict Lemma 1, hence there can be no pure-strategy equilibrium. Both firms have continuous mixed strategies almost everywhere on $[0, \Gamma_A]$ by Lemma 4. Expected benefits of the advertising competition for each firm are zero by Lemmas 2 and 3.

Ex ante each firm is indifferent between advertising and simply allowing the other to capture the coordinated demand. While the actualized profit of the firm depends on the outcome of the mixed-strategy equilibrium in advertising competition, the expected profit is simply equal to the profit that the firm would get if it simply passively allowed its rival to coordinate consumer expectations on its product. In this case, the firm's quantity sold would be given by \underline{q} which is strictly less than the quantity sold the firm would get if neither firm advertised, \tilde{q} . Thus both firms would be better off if they could agree not to advertise. However, each would have an incentive to cheat on such an agreement. If the rival were not to advertise then the firm could coordinate consumer expectations on its product with a minimal advertising expense and capture sales of \bar{q} which are strictly higher than the quantity it could sell if it abided by the agreement.

Due to advertising more people buy at least one product (a business creation effect). When consumers coordinate their expectations on a brand, the positive consumption externality creates an additional incentive for the consumers to buy that brand. In the movie interpretation this means that even people who do not enjoy movies very much go to the movies, since they expect to socialize with more people. Without coordination, only people with $\alpha \geq (P - \beta\tilde{q})/V$ purchase the good. With coordination all the people with $\alpha \geq (P - \beta\bar{q})/V$ go to the movies, where $(P - \beta\bar{q})/V < (P - \beta\tilde{q})/V$ since $\bar{q} > \tilde{q}$.

Due to advertising, fewer people buy the less advertised product (a business stealing effect). Without advertising consumers who were going to purchase one item would split between the products. With advertising the firm with lower advertising no longer gets these customers. It only gets consumers who value the good enough to purchase both. Hence, the incentive to purchase the second item declines. From (3) and (5), $\underline{q} < \tilde{q}$.

Due to advertising, total quantity sold goes down (a business dissipation effect). Total sales when there is no advertising is the summation of the demand for A and demand for B, given by $2\tilde{q}$. When one firm advertises more than the other, one gets \bar{q} and the other gets \underline{q} . There is a reduction in the total sales:

$$\underline{q} + \bar{q} - 2\tilde{q} = -\frac{(P - \beta)(1 - \omega)^2 \beta \bar{\alpha} V}{(\bar{\alpha} V - \beta)(\bar{\alpha} \omega V - \beta)[2\bar{\alpha} \omega V - \beta(1 - \omega)]} < 0 \quad (9)$$

with coordination, one firm has high demand and the other has low demand. Hence the incentive for a consumer to buy his/her first product increases, and the incentive to buy his/her second decreases. But since the consumer has diminishing marginal utility from the consumption of the product, the intrinsic

sic value of the second purchase is less important relative to the consumption externality than it is for the first purchase. Hence coordination increases the sales of highly advertised product less than it decreases the sales for the less advertised product.

Due to the stochastic nature of the advertising competition, *ex post* the firm that is out-advertised will have regrets. But the consumer will realize that firms advertised only because they expected advertising to increase their sales. So the individual consumer will realize that firms believe that advertising is being used as a coordination device, and that firms are confident enough about this belief to invest large sums of money. Hence it will in fact be in the best interest of a consumer to buy the more heavily advertised brand. In aggregate, this leads to higher sales, higher consumption externalities and hence greater consumer surplus from the more heavily advertised product.

4 VERTICAL DIFFERENTIATION

In markets with vertical differentiation consumers can learn about the qualities of the products based on trusted critics. Computer magazines rate the qualities to software packages. There are consumer reports on the reliability of different brand-name automobiles. For instance, Reinstein and Snyder (2005) find that movie reviews have a large and significant effect on consumers' expectation of film quality. We assume that qualities of the products are common knowledge and $V_A > V_B$. We first examine whether multiple equilibria exist when one product is of superior quality. We then discuss the role of advertising as a coordination device in markets with vertically differentiated goods.

4.1 Coordinate on A

Suppose that consumers believe that more people will purchase A. Then the consumer prefers A, for two reasons. First, he/she expects to enjoy A more due to its higher quality. Second, he/she expects to derive higher consumption externalities from A. The only consumers that end up buying B are the ones who purchase both of the products. The second item¹⁰ yields the additional surplus equal to $(\alpha\omega V_B + \beta q_B - P)$. Therefore, consumers with $\alpha \geq (P - \beta q_B)/\omega V_B$ buy B. Since $\alpha \sim U[0, \bar{\alpha}]$, solving for the equilibrium q_B ,

$$q_B = \frac{\bar{\alpha}\omega V_B - P}{\bar{\alpha}\omega V_B - \beta} \quad (10)$$

Since $P > \beta$, $0 < q_B < 1$.

Everyone with a positive consumer surplus from A, with α such that $\alpha_i V_A + \beta q_A \geq P$, buys A. Solving for the equilibrium q_A ,

¹⁰An individual who is purchasing both goods will always purchase A first since $V_A + \omega V_B > V_B + \omega V_A$.

$$q_A = \frac{\bar{\alpha}V_A - P}{\bar{\alpha}V_A - \beta} \tag{11}$$

since $V_A > V_B$, sales of A are greater than the sales of B, $q_A > q_B$.

4.2 Coordinate on B

Can it be rational for consumers to believe that the lower-quality product B will be more popular? Below we will show that $E(q_A) < E(q_B)$ is only rational when A and B are of similar qualities. If the quality difference between the two products is too large, then the expectation that $q_B > q_A$ is not rational, and hence there is no multiplicity of equilibria. Only when products are of similar qualities advertising may have a coordinating role to play in this market.

If consumers believe that the lower-quality product will sell more, then the product that consumer i prefers to buy depends on that consumer's taste for the intrinsic value of the good. For high α_i such that $\alpha_i V_A + \beta q_A > \alpha_i V_B + \beta q_B$, consumer i would prefer the high-quality product A. Define $\tilde{\alpha}$ as the critical level where a consumer is indifferent between A and B if he/she is buying his/her first item:

$$\tilde{\alpha} = \frac{\beta(q_B - q_A)}{V_A - V_B} \tag{12}$$

Consumers with $\alpha \geq \tilde{\alpha}$ prefer A over B, due to its high quality, and consumers with $\alpha < \tilde{\alpha}$ prefer B over A, due to the consumption externality.

Define α' such that if α_i is low ($\alpha_i < \alpha'$), consumer i will not buy either of the products. Define α'' such that if α_i is high ($\alpha_i > \alpha''$), he/she buys both of the products. The critical values α' and α'' will be derived shortly. If α_i takes an intermediate value, $\alpha' < \alpha_i \leq \alpha''$, he/she will buy one, and the one he/she chooses will depend on the relationship between α_i and $\tilde{\alpha}$. There are three potential cases $\tilde{\alpha} \leq \alpha'$, $\alpha' < \tilde{\alpha} \leq \alpha''$ and $\tilde{\alpha} > \alpha''$. We will show that the only possibility for rational expectations of $q_B > q_A$ is in the third case where the parameter values are such that $\tilde{\alpha} > \alpha''$.

The first case ($\tilde{\alpha} \leq \alpha'$) directly implies that $q_B < q_A$ and so the consumers' expectations that $q_B > q_A$ are not rational. The second case ($\alpha' < \tilde{\alpha} \leq \alpha''$) is less obvious, but is also not possible in equilibrium. To see this notice that in this case everyone with $\alpha_i \geq \tilde{\alpha}$ will buy A so $q_A = 1 - \beta(q_B - q_A) / [\bar{\alpha}(V_A - V_B)]$. Solving for the quantity firm A sells, $q_A = [\bar{\alpha}(V_A - V_B) - \beta q_B] / [\bar{\alpha}(V_A - V_B) - \beta]$. This yields $q_A \geq 1$, so if the parameters are such that $\tilde{\alpha} \leq \alpha''$ consumer expectations of $q_B > q_A$ are not rational.

Thus the only possibility for rational expectations of $q_B > q_A$ is if $\tilde{\alpha} > \alpha''$. As before, α'' is where the consumer's additional utility from buying two items instead of one is just equal to the price of the product. If the consumer buys one

item only, he/she will buy B. The utility from purchasing one item is therefore $\alpha_i V_B + \beta q_B - P$. The utility from two items is equal to $[\alpha_i V_A + \beta q_A - P + \alpha_i \omega V_B + \beta q_B - P]$. Hence the additional utility from buying the second item is equal to $[\alpha_i V_A - \alpha_i(1 - \omega)V_B + \beta q_A - P]$. The additional utility from purchasing the second item is zero for the consumer i with $\alpha_i = \alpha''$:

$$\alpha'' = \frac{P - \beta q_A}{V_A - (1 - \omega)V_B} \quad (13)$$

The only consumers who buy A are those who value the good so much that they buy both A and B ($\alpha_i > \alpha''$). Therefore q_A is given by

$$q_A = 1 - \frac{P - \beta q_A}{\bar{\alpha}V_A - \bar{\alpha}(1 - \omega)V_B} \quad (14)$$

solving for q_A ,

$$q_A = \frac{[\bar{\alpha}V_A - \bar{\alpha}(1 - \omega)V_B] - P}{[\bar{\alpha}V_A - \bar{\alpha}(1 - \omega)V_B] - \beta} \quad (15)$$

$q_A \in (0, 1)$ since $P > \beta$ and $\bar{\alpha}\omega V_A > P$.

Consumers whose utility from purchasing B is positive, $\alpha_i V_B + \beta q_B - P > 0$ buy B. This yields the critical value α' :

$$\alpha' = \frac{P - \beta q_B}{V_B} \quad (16)$$

All consumers with $\alpha > \alpha'$ purchase B, therefore q_B is given by

$$q_B = 1 - \frac{P - \beta q_B}{\bar{\alpha}V_B} \quad (17)$$

solving for q_B ,

$$q_B = \frac{\bar{\alpha}V_B - P}{\bar{\alpha}V_B - \beta} \quad (18)$$

$q_B \in (0, 1)$ since $P > \beta$, and $\bar{\alpha}\omega V_B > P$. Thus,

$$q_B - q_A = \frac{\bar{\alpha}(P - \beta)[(1 - \omega)V_B - (V_A - V_B)]}{(\bar{\alpha}V_B - \beta)[\bar{\alpha}(V_A - V_B) + \bar{\alpha}\omega V_B - \beta]} \quad (19)$$

$q_B > q_A$ if $(1 - \omega)V_B > (V_A - V_B)$.

These results also require that $\alpha'' < \bar{\alpha}$ which imposes the condition,

$$\frac{P - \beta q_A}{V_A - (1 - \omega)V_B} < \frac{\beta(q_B - q_A)}{V_A - V_B} \quad (20)$$

Substituting in q_A from (15) and $(q_B - q_A)$ from (19) and rearranging (20) yields

$$V_A - V_B < \frac{(1-\omega)\beta}{\bar{\alpha}} \quad (21)$$

Condition (21) is the necessary and sufficient condition for the existence of an equilibrium where consumers coordinate on the low-quality good. If the quality difference between the two products is too large, then the expectation that $q_B > q_A$ is not rational, and advertising will not have a coordinating role to play in this market.

Proposition 2: If quality differences between the products are too large we will not observe advertising for coordination purposes.

Proof: If the qualities of the two goods are very different, loosely speaking, one movie receives two thumbs up, the other two thumbs down, formally $V_A - V_B \geq [(1-\omega)\beta]/\bar{\alpha}$, there is only one equilibrium and it involves consumers coordinating on the high-quality good. In this case, everyone who buys a product purchases the higher-quality one, and only those people who really like the product will purchase the lower-quality one as well. Suppose B is a very low-quality brand but that firm B did advertise. Because B is so poor, there is no rational expectations equilibrium where consumers coordinate on B. In this situation, purchasing the higher-quality product is the only reasonable behavior, and consumers will coordinate on the higher-quality product, even if faced with advertising for the low-quality product. Hence rational firms will choose not to advertise when the quality difference between the products is too large. ■

When there are significant quality differences, there is no multiplicity of equilibria and firms cannot coordinate consumer expectations using advertising. Hence the quality assessments, such as movie reviews, are serving an informative role, but also, implicitly, they are serving a coordinating function.

From (21), notice that advertising may serve a coordinating role when the consumption externality (β) is large. If the consumption externality is small, the quality of the products becomes relatively more important in the purchasing decisions of the consumers, and the coordinating function is served by quality differences. If the importance of quality versus externality is high, advertising will not coordinate consumer expectations.

When the quality difference between the products is not too drastic, i.e. for parameter values where (21) holds, coordination via product quality may be difficult to achieve. Of course, if all else is equal it is quite natural for consumers to coordinate on the high-quality good. However, the producer of the low-quality good has strong incentives to ensure that all else is not equal. The fundamental problem is that there are two equilibria, and each individual consumer must try to predict which equilibrium the other consumers are going to play.

There is a subgame-perfect Nash equilibrium where consumers coordinate on the high-quality brand and ignore all advertising. In this equilibrium firms would not advertise. Notice that, for this equilibrium to survive, the off-equilibrium beliefs of the consumer must be such that the consumer interprets any positive level of advertising as irrational behavior of the firm. Now suppose that the producer of the lower-quality product did actually mount a costly advertising campaign. An individual consumer could suppose that the firm engaged in the advertising campaign because it had information that consumers in general were going to use advertising to coordinate their expectations on one of the equilibria. In this case, a consumer who was only going to buy one product would prefer B, firm B's sales would increase, vindicating its investment in advertising. Therefore, when the qualities of the products are not too dissimilar, information on product quality may not be the deciding factor in coordinating consumer expectations. However, it will have a very significant effect on the incentives that the two firms face in their advertising competition.

Proposition 3: The value of winning the advertising competition is higher for the firm with the low-quality product.

Proof: The value of winning the advertising competition for a firm is proportional to the difference in sales when it coordinates consumers on its brand versus when the rival captures the coordinated demand. So for A the value of winning the advertising competition is proportional to the difference between (11) and (15):

$$\Gamma_A = \frac{\gamma P \bar{\alpha} (P - \beta) (1 - \omega) V_B}{(\bar{\alpha} V_A - \beta) [(\bar{\alpha} \omega V_B - \beta) + \bar{\alpha} (V_A - V_B)]} \quad (22)$$

For B, the value of winning the advertising competition is proportional to the difference between (10) and (18):

$$\Gamma_B = \frac{\gamma P \bar{\alpha} (P - \beta) (1 - \omega) V_B}{(\bar{\alpha} V_B - \beta) (\bar{\alpha} \omega V_B - \beta)} \quad (23)$$

The numerators of (22) and (23) are the same but the denominator of (23) is small since $V_A > V_B$. Hence the value of winning the advertising competition is higher for firm B. ■

A is the higher-quality product. Even when it does not coordinate consumers on its brand it still sells well due to its high quality. If A coordinates demand, B only gets a low level of sales due to its low quality. Moving from this low demand to the coordinated demand yields a big change in sales for B. Hence the value of winning the advertising competition is higher for the low-quality product firm.

Coordinating advertising competition when firms have vertically differentiated products has the same form as an all-pay auction where bidders have asymmetric valuations.

Proposition 4: No pure-strategy equilibrium exists. The mixed-strategy equilibrium involves almost everywhere continuous mixed strategies where both firms mix in the range $[0, \Gamma_A]$. The advertising competition results in the expected dissipation of all gains to advertising for the high-quality firm and expected value of the competition is equal to $\Gamma_B - \Gamma_A > 0$ for the low-quality firm.

Proof: From Lemma 1 there can be no pure-strategy equilibrium with advertising levels greater than zero. If firm A is playing a pure strategy of zero advertising then firm B would play a pure strategy of advertising just above zero, winning for sure with negligible advertising expenses. This would contradict Lemma 1, hence there can be no pure-strategy equilibrium. Both firms have continuous mixed strategies almost everywhere on $[0, \Gamma_A]$ by Lemma 4. Expected benefits of the advertising competition for firm A are zero by Lemma 2. Expected benefits of the advertising competition for firm B are $\Gamma_B - \Gamma_A$ by Lemma 3. This expression is positive by Proposition 3. ■

Firm A would never bid higher than the value of the auction Γ_A . Firm B can always win the competition by bidding slightly higher than Γ_A and collect the prize. This leaves B with an advantage in the advertising competition equal to the difference in valuations, so in equilibrium it is able to capture an expected profit from the advertising competition equal to $\Gamma_B - \Gamma_A$.

While the firm with the low-quality product strictly prefers to engage in advertising competition, the expected profit created by entering the advertising competition for the firm with the high-quality product is equal to zero. The firm with the high-quality product is indifferent between entering the competition or not. However it would not be an equilibrium strategy for A not to enter the competition: any announcement suggesting that A will not advertise would not be credible. If A were not to advertise, B would capture the coordinated demand with a small advertising expense. Then it would no longer be optimal for A not to advertise. It would rather out-advertise B with negligible advertising expenses. Hence in equilibrium both firms will advertise. The outcome of the advertising competition will be stochastic. Sometimes we will observe the producer of the high-quality product advertising more, and other times we will observe the producer of the low-quality product advertising more.

Proposition 5: The firm with the low-quality product is more likely to win the advertising competition. As long as quality difference is not too drastic consumers will more often coordinate on the low-quality product.

Proof: Define $G_A(a)$ as the probability of firm A having an advertising level less than or equal to 'a'. And $G_B(a)$ represents firm B's cumulative probability distribution. The expected value of the advertising competition to firm A is zero by the above argument. So when firm A advertises 'a', the expected value created due to advertising competition for firm A is given by

$$0 = G_B(a)(\Gamma_A - a) + (1 - G_B(a))(-a) \quad (24)$$

Solving for $G_B(a)$ yields

$$G_B(a) = \frac{a}{\Gamma_A} \quad (25)$$

The expected value of the advertising competition to firm B is given by $\Gamma_B - \Gamma_A$. When firm B advertises 'a', the expected value created due to advertising competition for firm B is given by

$$\Gamma_B - \Gamma_A = G_A(a)(\Gamma_B - a) + (1 - G_A(a))(-a) \quad (26)$$

Solving for $G_A(a)$ yields

$$G_A(a) = \frac{\Gamma_B - \Gamma_A + a}{\Gamma_B} \quad (27)$$

The probability of firm A winning the advertising competition is given by the probability that A advertises 'a' times the probability that B advertises less than 'a', integrated over the support of the mixed strategy:

$$\int_0^{\Gamma_A} G_B(x) \frac{\partial G_A(x)}{\partial x} dx = \frac{\Gamma_A}{2\Gamma_B} \quad (28)$$

Similarly, the probability of coordinating on B is given by

$$\int_0^{\Gamma_A} G_A(x) \frac{\partial G_B(x)}{\partial x} dx = \frac{2\Gamma_B - \Gamma_A}{2\Gamma_B} \quad (29)$$

Since $\Gamma_B > \Gamma_A$, the probability that consumers coordinate on the low-quality product is higher than the probability that consumers coordinate on the high-quality product. ■

This for instance implies that consumers will watch many mediocre movies just for the consumption externality. Nevertheless, the movies with the highest sales will be the high-quality movies that are highly publicized. If the quality difference is significant, the high-quality product will get the coordinated demand and in equilibrium we will not observe coordinating advertising.

5 CONCLUDING REMARKS

In markets with consumption externalities, firms might engage in advertising competition not only to inform the consumers, or to persuade them that one brand is somehow intrinsically different from other brands, but also to convince them that others will be buying it as well. It is plausible, for example, that the large advertising campaign for Windows 95 was not merely an attempt to inform consumers of the existence and price of the product, but also a device to coordinate consumer expectations on the brand.

The paper predicts that in markets with consumption externalities, when quality differences are not too great, consumers may often knowingly purchase the lower-quality product. When Windows 95 and OS/2 were being introduced to the market, most of the software reviewers agreed that OS/2 was of slightly higher quality than Windows 95. However Microsoft managed to coordinate consumer expectations on Windows 95. IBM complained about having lost market share to Microsoft despite their higher quality. This paper suggests that OS/2 would have not lost the battle if it was of significantly superior quality. The quality difference between these products must have been small enough that consumers chose to enjoy consumption externalities at the cost of a slight decline in the product quality.

In the models of asymmetric information of Kihlstrom and Riordan (1984) and Ippolito (1990), the theoretically predicted correlation between quality and advertising is positive. In a repeat purchase framework, the firm with higher quality has higher incentives to advertise, since the potential buyer would engage in repeat purchase of the high-quality good. However, empirically Bagwell (2007) reports in his survey article that studies do not offer strong evidence for the hypothesis of a positive association between quality and advertising. We show that the theoretically predicted correlation between advertising and quality is weakly negative when advertising serves as a coordination device in markets with consumption externalities. The equilibrium of the advertising competition is in mixed strategies. While there will be times where the high-quality product has higher advertising, more frequently it will be the low-quality product that will capture the higher advertising position.

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