Performance Requirements for Analog-to-Digital Converters in Wideband Reconfigurable Radios

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ABSTRACT

With the current trend towards software defined radio, several candidate architectures for the analog receiver front-end have been presented. A common proposal for software defined reconfigurable radio is to develop a wideband ADC and utilise this for capturing a large segment of the spectrum. This would enable the subsequent signal processing operations of channel selection and data extraction to be carried out by a digital processor. This would allow the radio to be reconfigured by simply changing the software.

In analysis of these systems, powerful neighbouring signals, or blockers, are considered but it has been conveniently assumed that suitable dynamic range will be available at the ADC. This is an acceptable assumption in narrowband systems where automatic gain control and analogue channel select filters can be used, but is not appropriate for a wideband system. In this paper we present an analysis based on bit-error-rates (BER) which shows the effect of blockers in a wideband architecture on the performance of the communication link and on the dynamic range requirements of the ADC.

We consider, as a representative example of a real world situation, the use of a wideband receiver on a Rayleigh fading channel. In any analysis of wideband receivers, the behaviour of the channel must also be included as the performance of the communication link is a combination of noise sources from both the channel and the electronics. The effect of high power interferers and blockers on the quantisation noise from the ADC will be mathematically modelled and the BER rates for the communication system will be presented. Given these results, it is possible to determine the minimum required resolution and dynamic range for an ADC in a wideband system given the spectral environment at the frequencies of interest.

Keywords: BER, ADC, quantization, wideband

1. INTRODUCTION

Bit error rate (BER) is an important measure of performance in any digital communication system. In any given system, the uncoded BER is directly related to the signal-to-noise ratio (SNR) observed at the decision device. The noise in the system comes from many sources (e.g. thermal noise) and is generally modelled as a white Gaussian process. A noise source which is often excluded from initial calculations is the quantization noise introduced due to the necessary use of an analog-to-digital converter (ADC) in the receiver.[1] A reason for this is that the noise introduced by quantization is modelled as uniformly distributed rather than Gaussian. Also when a sufficient number of bits of resolution are available quantization noise may have a small effect on performance. Using automatic gain control (AGC) and an analogue channel-select filter for reception of a narrowband signal, it is possible to use the full range of bits associated with the receive ADC for detection of the signal-of-interest and thereby easily characterize the noise contribution. In a wideband receiver architecture however, it is possible (and indeed likely) that there exists a set of two signals with vastly differing receive signal power in the receive band. In this case, the ADC must accommodate the strongest signal otherwise non-linear signal distortion will occur, causing power to transfer from one frequency to another possibly interfering with the signal of interest. Consequently, a signal of lower receive power is effectively ‘blocked’ by being subject to high quantization noise (relative to its own power).

In this paper, we model a frequency-selective wideband channel and therefore we expect each channel-of-interest to be received with a different power. This negatively affects the implicit resolution available at the ADC for low power signals. A lack of resolution, due to some channels having low power relative to other channels or the overall signal average, often causes reception of weaker signals to be extremely difficult and in some cases impossible. This blocking effect presents a significant problem for the implementation of wideband receivers where the overall receive signal power is often significantly larger than the power of the signal of interest. A mathematical model is derived which allows the performance of the ADC to be specified given known blocking signals and a Rayleigh fading channel. The minimum number of bits to achieve a desired SNR is derived in terms of the channel and the ADC parameters. An extension to the relation between ADC parameters and the BER of coherent PSK and FSK is also developed.

2. STATEMENT OF PROBLEM

2.1. System model

The system under consideration is that of a wideband receiver for a frequency-selective fading channel. The receiver model, which is shown as a block diagram in fig. 1, consists of a wideband receive antenna, a wideband bandpass filter, a low-noise amplifier (LNA), a mixer to move the band of interest to a low-IF which is completely captured by the ADC, an ADC, a digital channel selection filter to extract only the signal of interest and the non-wideband digital reception circuitry.

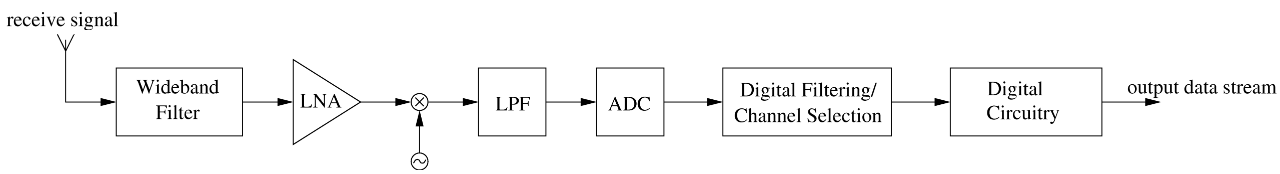

Figure 1. Block diagram of wideband receiver

The signal received by the antenna is assumed to consist of multiple signals corresponding to channels we may wish to resolve. We here make the assumption that each of these signals is subject to an independent, slowly fading, flat channel. Mathematically, the receive signal at time t may be written

$$r(t) = \sum_{k=1}^{N} \alpha_k e^{j\phi_k} x_k(t) \tag{1}$$

where $x_k(t)$ are the individual signals received by the wideband receiver, $r(t)$ is the received signal itself and α_k and ϕ_k are the phase and gain associated with each of the channels. We assume that each channel is varying sufficiently slowly that after the signal has been extracted, the phase shift can be accurately estimated. Therefore, for simplicity of presentation, ϕ_k is omitted in the development below. The value of α_k is appropriately scaled such that

$$E\left\{x_k^2(t)\right\} = 1 \quad \forall\, k = 1, 2, \ldots, N \tag{2}$$

i.e. each transmitted signal $x_k(t)$ is modelled with unit power.

This received signal is wideband filtered, amplified through an LNA, shifted to a low-IF and then both quantized and sampled by the ADC to give the digital signal. Electronic noise is introduced by each of the above components and quantization noise is introduced by the ADC. The noisy signal as seen at the input to the ADC is

$$r(t) = \sum_{k=1}^{N} \alpha_k x_k(t) + \eta(t) \tag{3}$$

where the noise $\eta(t)$ is additive white Gaussian noise (AWGN) of spectral density N_0 W/Hz.

3. NOISE MODELLING

The noise which is present at the input to the ADC is noise due to the front-end electronics (such as the antenna, the low-noise amplifier). The noise at the output of the ADC is the combination of this noise with electronic noise of the ADC and the quantization noise of the ADC. We examine the properties of the noise, below.

3.1. Quantization noise

Since an AGC is present at the input of the receiver, the quantization noise due to the ADC may modelled as a uniform process, assuming the presence of a significant number of bits of resolution.[2] In the absence of an AGC at the input, the full range of the ADC may not be used and the model developed below may not be appropriate. The use of the digital channel selection filter immediately after the ADC means that the statistics of the noise at the ADC output are not needed. This is because the quantization noise samples, as modelled above, comprise a set of independent identically distributed random variables, which when passed through a sufficiently long filter approach a Gaussian probability distribution (by the Central Limit Theorem).

Consider an ADC with range, $-\frac{K}{2} \rightarrow \frac{K}{2}$ and a uniform level spacing over the full range. Using N_{bits} bits in the quantizer implies a total of $2^{N_{bits}}$ distinct quantization levels. Given a level spacing of Δ, the highest quantization point should be $\frac{K}{2} - \frac{\Delta}{2}$ and the lowest at $-\frac{K}{2} + \frac{\Delta}{2}$. The level spacing is thus

$$\Delta = \frac{\left(\frac{K}{2} - \frac{\Delta}{2}\right) - \left(-\frac{K}{2} + \frac{\Delta}{2}\right)}{2^{N_{bits}}} \tag{4}$$

which by simple re-arrangement gives

$$\Delta = \frac{K}{2^{N_{bits}} - 1} \tag{5}$$

This is the full range over which the ADC operates and is such that the signal of highest receive power may be successfully resolved. Assuming that the quantization noise is uniform in nature,[3] it is well known that quantization noise power corresponding to a uniform distributed process has variance:

$$\sigma^2 = \frac{\Delta^2}{12} \tag{6}$$

3.2. Noise statistics and power

In the wideband receiver, the received signal is digitally filtered immediately after the ADC. The first digital filter is a channel selection filter used to reject signals which occur outside the band of the signal of interest. This filter has an additional effect on the noise beyond rejection of out-of-band noise. The noise at the input to the channel selection filter is the sum of thermal noise, which is Gaussian distributed, and the quantization noise which is uniformly distributed. The noise at the output of the channel selection filter is Gaussian distributed. This is because the quantization noise, as modelled above, is independent. In this way, all the noise in the system will be Gaussian distributed. Also, since the channel selection filter has a flat response in the band of interest, the noise is also white when the channel is downconverted to the appropriate symbol rate.

The noise power at the output of the channel selection filter is dependent on its associated bandwidth. We assume that the channel selection filter has a 0 dB gain in the passband and infinite attenuation outside. The quantization noise at the output of this channel selection filter thus has power related to the passband bandwidth, i.e.

$$N_q = W\sigma^2 = \frac{W\Delta^2}{12} \tag{7}$$

where W is the bandwidth of the channel of interest, as selected by the channel selection filter, normalized by the half-sampling frequency, $\frac{f_s}{2}$. The thermal noise with related noise spectral density N_0 is

$$N_{th} = W f_s N_0 \tag{8}$$

The total noise power is thus additive white Gaussian with average power

$$P_{noise} = N_q + N_{th} = W\left(\frac{\Delta^2}{12} + N_0 f_s\right) \tag{9}$$

3.3. Receive signal power

Given the simultaneous reception of multiple signals, $x_k(t)$, each of which has a given power which is Rayleigh distributed, then, the average receive power may be written directly as

$$P_{av} = E\left\{|r(t)|^2\right\} = E\left\{\left(\sum_{k=1}^{N} \alpha_k x_k(t)\right)^2\right\} \tag{10}$$

which, because the receive signals are each independent and zero-mean and the factors and under the assumption that α_k is constant over the period of interest, becomes

$$P_{av} = \sum_{k=1}^{N} \alpha_k^2 E\left\{x_k^2(t)\right\} \tag{11}$$

which by (2) becomes

$$P_{av} = \sum_{k=1}^{N} \alpha_k^2 \tag{12}$$

Arbitrarily defining the channel of interest as the having gain α, the ratio between the power of the desired signal and that to which the AGC is adjusted is the power ratio which is simply

$$\frac{\alpha^2}{\sum_{k=1}^{N} \alpha_k^2} \tag{13}$$

Thus, the parameter α^2 is the fraction of the total receive signal power corresponding to the signal of interest, i.e.

$$P = \alpha^2 P_{av} \tag{14}$$

3.4. Signal-to-noise ratio

The signal-to-noise ratio of the channel-of-interest at the output of the channel select filter is

$$\mathrm{SNR} = 10\log_{10}\left[\frac{\alpha^2 P_{av}}{P_{noise}}\right] \tag{15}$$

Assuming that the AGC at the frontend of the receiver normalizes the incoming signal such that the average receive signal power $P_{av} = 1$, the signal-to-noise ratio at the output of the channel select filter is

$$\begin{aligned} \mathrm{SNR} &= 10\log_{10}\left[\frac{\alpha^2}{P_{noise}}\right] \quad \text{dB} && (16) \\ &= 10\log_{10}\left[\alpha^2\right] - 10\log_{10}\left[P_{noise}\right] && (17) \end{aligned}$$

The blocking effect on the SNR is apparent from the above equation. The first term refers solely to a weaker received signal's poor SNR compared to a stronger signal. The second term, however, is the noise effect due to both the thermal noise and the quantization noise. The expected output SNR may thus be determined based on the thermal noise, blocking characteristics, and dynamic range of the ADC under the assumption that the channel selection filter allows robust selection of the channel of interest.

Given a specific number of bits of resolution, the SNR may be determined, by substitution of (9), as

$$\begin{aligned} \mathrm{SNR} &= 10\log_{10}\left[\frac{\alpha^2}{W\left(\frac{\Delta^2}{12} + N_0 f_s\right)}\right] && (18) \\ &= 10\log_{10}\left[\frac{12\alpha^2}{W\left(\frac{K^2}{\left(2^{N_{bits}}-1\right)^2} + 12 N_0 f_s\right)}\right] && (19) \end{aligned}$$

Conversely, for a given SNR to be achieved in a wideband receiver, the number of bits of resolution may be determined from the above since

$$\frac{\alpha^2}{P_{noise}} = 10^{\frac{\text{SNR}}{10}} \tag{20}$$

which by re-arrangement of (19) becomes

$$\alpha^2 10^{\frac{-\text{SNR}}{10}} = \frac{W}{12}\left(12N_0 f_s + \frac{K^2}{\left(2^{N_{bits}} - 1\right)^2}\right) \tag{21}$$

$$\Longrightarrow N_{bits} = \log_2\left(1 + K\sqrt{\frac{W}{12\alpha^2 10^{\frac{-\text{SNR}}{10}} - 12WN_0 f_s}}\right) \tag{22}$$

This equation relates the number of bits of resolution to the power of the signal-of-interest relative to the total received power and the desired SNR. Note that when

$$\text{SNR} > 10\log_{10}\left(\frac{\alpha^2}{WN_0 f_s}\right) \tag{23}$$

then there is no possible resolution on the ADC which can obtain the desired SNR. This is because this corresponds to the SNR of the system without quantization and the quantization process can only decrease the SNR.

3.5. Bit-error rate

The BER performance of a flat channel may be quantified in terms of the SNR-per-bit, which is related to the SNR in the above section as

$$\gamma = \frac{E_b}{N_0} = \frac{Wf_s}{2R}(\text{SNR}) \tag{24}$$

$$= \frac{6\alpha^2 f_s}{R\left(\frac{K^2}{\left(2^{N_{bits}-1}\right)^2} + 12N_0 f_s\right)} \tag{25}$$

where R is the transmission rate in bits/second.

For a given modulation type, the instantaneous bit-error rate (BER) may be written in terms of the associated SNR-per-bit, which in turn is a function of the ratio of input-to-desired power level. The average BER over time for any channel may be calculated as

$$\int_0^\infty P_e(\gamma)p(\gamma)\, d\gamma \tag{26}$$

where $P_e(\gamma)$ is the probability of bit-error given an SNR-per-bit of γ and p_γ is the pdf associated with the SNR-per-bit corresponding to the channel-of-interest. In the case of the wideband receiver, we have shown above that, instantaneously, the BER corresponds to the BER associated with an additive white Gaussian noise channel (AWGN) channel. BER may be calculated directly in this case.

When the channel is Rayleigh fading, the receive gain, α, is Rayleigh distributed and thus the SNR-per-bit is chi-squared distributed. It is well-known[4] that this probability of error in a Rayleigh fading channel may be expressed in terms of the average value of the SNR-per-bit in the specific case of PSK and FSK. From (25) the average SNR-per-bit may be written as

$$\bar{\gamma} = \left[\frac{6f_s}{R\left(\frac{K^2}{\left(2^{N_{bits}-1}\right)^2} + 12N_0 f_s\right)}\right] E\left\{\alpha^2\right\} \tag{27}$$

The probability of bit-error for coherently detected binary PSK in a Rayleigh flat fading channel is known to be related to the average SNR-per-bit as

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) \tag{28}$$

and for coherently detected binary FSK in a Rayleigh flat fading channel is known to be

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}\right) \tag{29}$$

These equations allow the BER of both PSK and MSK to be directly related to the ADC parameters of sampling rate and bits-of-resolution given a known level of blocking in the system. For example, for coherently detected binary PSK the probability of bit-error is related to the ADC parameters through the equation

$$P_e = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{6f_s}{R\left(\frac{K^2}{\left(2^{N_{bits}-1}\right)^2}+12N_0f_s\right)}E\left\{\alpha^2\right\}}{1+\frac{6f_s}{R\left(\frac{K^2}{\left(2^{N_{bits}-1}\right)^2}+12N_0f_s\right)}E\left\{\alpha^2\right\}}}\right) \tag{30}$$

Thus, given a signal subject to Rayleigh fading with an average receive power relative to the overall receive signal power $E\left\{\alpha^2\right\}$, the number of bits required for a desired probability of bit-error is given by

$$N_{bits} = \log_2\left[1+\sqrt{\frac{K^2R(1-2P_e)^2}{6E\left\{\alpha^2\right\}f_s[1-(1-2P_e)^2]-12N_0f_sR(1-2P_e)^2}}\right] \tag{31}$$

Thus, given the presence of blockers in the wideband at the input of the ADC, the number of bits of resolution required to obtain a given BER target may be calculated.

4. IMPLEMENTATION

In this section, an illustrative example is provided to convince the reader of the validity of the mathematical model presented in this paper. In this example, the input to the ADC is modelled as a sum of modulated signals at different frequencies. The channel selection filter is implemented as a linear phase FIR filter designed using the constrained least-squares method. In the simulations given below, the channel selection filter is of 201-tap FIR and provides > 70 dB out-of-band rejection (see figure 2). In the case of more severe blocking signals, it is necessary to either use a more aggressive analog receive filter or a more aggressive channel selection filter.

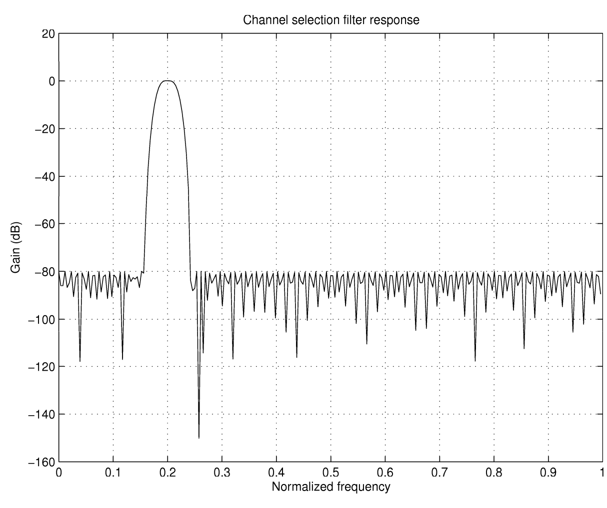

Figure 2. Illustrative Frequency response of a channel select filter

The sampled, quantized signal is digitally demodulated and the uncoded bit-error rate is determined. It is assumed that timing recovery is perfectly achieved in each case. The simulated data transmitted is generated as a random bit-stream.

As an example, the case of detecting a BPSK modulated signal in the presence of other BPSK signals in the wideband range of the ADC is investigated. In particular, the effect of receiving 4 BPSK signals was examined with the signal-of-interest having an average receive power of -64 dBm in the presence of 4 blocking signals, each of power -4 dBm. The total receive signal power is 1 mW. The signal of interest is at 1960 MHz, has a bandwidth of 36 MHz and is subject to a flat, Rayleigh fading channel. The blocking signals are each at 1780, 2140, and 2320 MHz and of bandwidth 36 MHz. In this case the ADC is used to digitize the band from 1600 MHz to 2500 MHz, requiring a sampling rate $f_s = 1800$ MHz. The thermal noise power at the input to the ADC was -60 dBm.

To illustrate the effect of the blocking signals, we perform the following simulations and compare the results to those theoretically obtained

Simulations

1) Blocking signals - no fading

   In this example, the signal-of-interest is considered to have a constant average power over the time interval examined. A graph showing both the theoretical and simulated BERs for various number of bits of resolution is shown in figure 3.

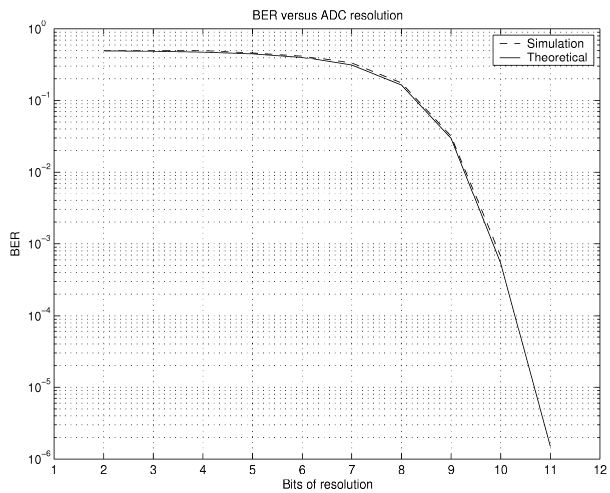

Figure 3. Graph of BER vs ADC resolution (flat channel)

   It is clear from this graph is that the theoretical values are almost indistinguishable from the values obtained by simulation, but the simulation BERs are slightly higher for the same number of bits of resolution. This small discrepancy may be explained due to the non-ideality of the channel select filter which introduces greater noise than suggested by the mathematics and also the deviation from Gaussianity of the noise due to the ADC. Noting that a small overhead is necessary, the mathematical model may be successfully used to determine the number of bits-of-resolution required to reach a desired specification.

   Note that the theoretical probability of error for this same signal when quantization noise is omitted is $P_e \approx 1.5 \times 10^{-16}$.

2) Blocking signals - Rayleigh flat fading

   In this case, we assume that the AGC is too slow to track the fading of the blocking signals. Thus, the parameters of the ADC do not change over the course of the estimation. The channel-of-interest is modelled as a Rayleigh fading channel whose average power is as in the first example. In this case, the mathematical relation between BER and number of bits is given in (30). A graph of both the theoretical and simulated BERs for different numbers of bits of resolution for this case is given in figure 4.

   This graphs exhibits the same properties as observed for the flat channel in blocking. The theoretical values are close to those obtained by simulation, but the simulation BER values are slightly higher for the same number of bits of

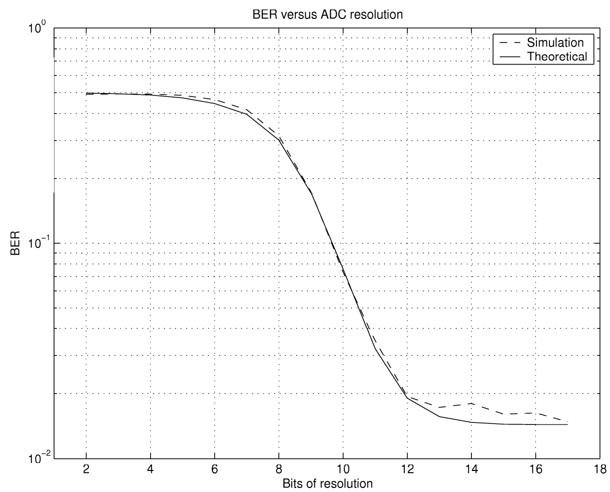

Figure 4. Graph of BER vs ADC resolution (fading channel)

resolution. In this case, the huge difference between BER at consecutive numbers of bits is sufficiently high that such a small difference should not affect our ability to choose the ADC resolution required.

5. COMMENTS

The above simulations for binary PSK show that the BER calculations for a fading channel in the presence of high power blocking signals and thermal noise are correct. We can also deduce from this that the mathematical results for SNR and SNR-per-bit on which these calculations were based are also accurate. Although the BER calculations and simulations were performed for the case of binary PSK, in the more general case of an arbitrary modulation scheme, the SNR values may be used in conjunction with requirement on standard of interest to accurately determine the resolution required of an ADC in a low-IF wideband radio frontend for successful signal recovery.

6. CONCLUSIONS

In this paper, we examined the particular effect of ADC quantization noise in a wideband receiver on the ability to recover a signal in the presence of higher power blocking signals. An implicit advantage to the analysis of the wideband receiver is that the quantization noise becomes Gaussian distributed due to the presence of a channel selection filter. This observation allowed a theoretical formulation to be presented which allowed for the total number of bits to be determined in order to meet a given SNR requirement and known sampling rate (both of which are typically specified by a given standard of interest). This formulation made the assumption that the digital channel selection filters present were sufficiently well designed that the blockers could be removed in the digital domain after the sampling and quantization had taken place. This formulation is valid in general. Given an SNR value, BER values for various modulation schemes may be derived. In particular, a theoretical formulation was also presented for the bit-error performance of coherent PSK and FSK in the presence of blocking signals in a wideband receiver was presented. This formulation was based on the SNR calculation of the first part. Simulation examples were presented which validate the BER formulae for the case of the flat fading Rayleigh channel.

REFERENCES

1. B. Widrow, "A study of rough amplitude quantization by means of nyquist sampling theory," *IRE Trans. Circuit Theory* **CT-3**, 1956.
2. A. B. Sripad and D. L. Snyder, "A necessary and sufficent condition for quantization errors to be uniform and white," *IRE Trans. Circuit Theory* **ASSP-25**, 1977.
3. W. R. Bennet, "Spectra of quantised signals," *The Bell System Technical Journal* **27**, 1948.
4. J. G. Proakis, *Digital Communications*, McGraw-Hill, 4 ed., 1995.