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MODAL ANALYSIS OF THE QUASI-OPTICAL PERFORMANCE OF PHASE GRATINGS

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Abstract: This paper is concerned with the analysis of phase gratings as passive quasi-optical multiplexing devices. One important application of such components is in the local oscillator injection chain of heterodyne array receivers. Gaussian beam mode analysis can be applied as a powerful tool when modelling the optical performance of phase gratings in a real submillimeter system of finite throughput and bandwidth. In our experimental investigations we have concentrated on the Dammann Grating (DG) which is a binary optical component and thus straightforward to manufacture. A number of quartz gratings were fabricated and carefully tested to evaluate the practical limitations of such quasi-optical components. Because of its convenient refractive index quartz can be used to produce gratings with very low reflection losses. The results presented confirm DGs to be particularly suitable multiplexers for sparse arrays of finite bandwidth.

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1. Introduction

Phase gratings are useful as low loss multiplexer devices in submillimeter and terahertz quasi-optical systems, where there is a requirement for the generation of multiple images of a single input beam [1]. The development of transmission phase gratings is straightforward, as there are a number of dielectric materials with suitable mechanical and optical properties in the far infrared. At such wavelengths both transmission and reflection gratings are relatively easy to manufacture as the required tolerances are readily achieved in contrast to their visible-wavelength counterparts. In this paper we present the results of our investigation into the quasi-optical analysis of phase gratings and discuss some recent experimental measurements of Dammann phase gratings manufactured from fused quartz.

The Dammann Grating (DG) is a binary optical component (the optical path length through the grating taking on just two values ideally separated by half a wavelength) and consists of a regular arrangement of milled slots or recesses [2,3]. The use of DGs as quasi-optical multiplexers in heterodyne array receivers was proposed by Murphy *et al.* in [4], and they have since been incorporated into real systems, e.g. [5,6,7]. For quartz, which has a refractive index close to the ideal value of 2.0, it is possible, by adjusting the thickness of the grating, to obtain the resonant no-reflection condition for both the recessed slot and raised non-slot sections. This is because the extra thickness in the non-slot sections corresponds to one wavelength in the medium for $n = 2$.

The theoretical analysis of quasi-optical systems incorporating phase grating multiplexers is important in determining the practical limitations of such devices. For the local oscillator injection chain of a heterodyne array receiver, for example, it is important to achieve quasi-uniform power coupling across an array of mixer feed horns with high efficiency. The multiplexer may also be required to operate over some useful bandwidth (typically of the order of 10% for ground based astronomical systems covering a submillimeter atmospheric window). Gaussian Beam Mode Analysis (GBMA) was chosen as the quasi-optical analysis tool ideally suited to beam guide configurations of finite throughput [8,9]. An important point to note is that with a careful choice of the mode basis set the array of beams produced by the grating can be efficiently modelled with this technique [10]. Furthermore, the quasi-optical power coupling of the multiplexed source feed to an array of horn antennas can be easily calculated for an array system [11]. Gaussian beam mode analysis of the performance of a grating multiplexer in general is discussed in Section 2, while in Section 3 the DG is considered as an example.

In Section 4 of this paper we also report on experimental measurements made on transmission DGs. Three different gratings, designed to produce 2 × 2, 3 × 3 & 5 × 5 uniform two-dimensional arrays of images at 100GHz, were manufactured and tested as quasi-optical multiplexers. It was decided to test the principles involved at this frequency since both the manufacture and experimental measurements are easier. Such phase gratings can, of course, be designed for the submillimeter/terahertz wavebands.

2. Modal Analysis of Phase Grating Multiplexer Performance

Consider the case of a grating illuminated by a quasi-collimated incident field $E_i(x, y)$ in a simple Fourier 4-f optics set-up as illustrated in Figure 1. A regular phase grating consists of a two dimensional array of identical basis cells, with the transmission function of the basis cell being given by $t(x,y) = \exp(i\Phi(x, y))$. The transmission of an entire grating with rectangular symmetry is then given by a sum of the form: $\Sigma_{m,n}\, t(x-m\Delta x, y-n\Delta y)$, where Δx and Δy are the grating periods in the x and y directions, respectively. The grating then produces a strong corrugated modulation of the quasi-planar wavefront of the incident field, severely affecting the resulting beam pattern. The output plane of the 4-f system corresponds to the Fourier plane of the grating. An array of beams is produced with the beam intensities modulated by the Fourier Transform of a single cell $T(u,v)$ according to:

$$E_A(u,v) = \sum_{n,m=-\infty}^{\infty} T(m\Delta u, n\Delta v).E_O(u - m\Delta u, v - n\Delta v),$$

where $E_O(u, v)$ is the Fourier Transform $E_i(x, y)$ [12]. The spatial frequencies u and v are related to the co-ordinates in the output plane (x_O, y_O) through $u = x_O/\lambda f$ and $v = y_O/\lambda f$, with f being the focal length of the lens (see Figure 1). The inter-beam separations are given by: $\Delta u = \Delta x_O/\lambda f = 1/\Delta x$ and $\Delta v = \Delta y_O/\lambda f = 1/\Delta y$.

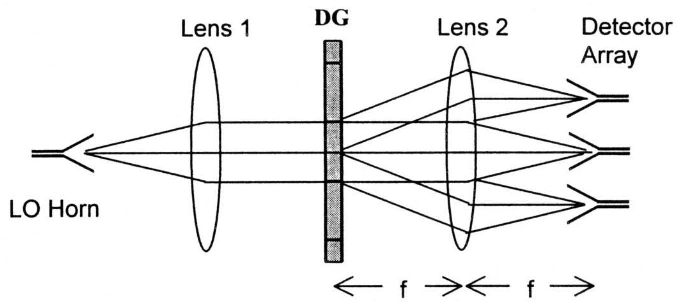

Figure 1 Optical set-up for beam multiplexing with a Dammann Grating.

This simple Fourier analysis however fails to take into account the finite size of the grating and any truncation in the optics between the grating and the output plane. A consequence of compact optics is the disruption of the spatial frequency spectrum of the beam in the optical train. Furthermore, what is actually important in determining the efficiency of a practical configuration is the coupling at the output plane to the subsequent quasi-optical system (which might be an array of horn antenna feeds, for example). GBMA is ideally suited for this kind of analysis.

From a multimode beam viewpoint the grating has the effect of seriously disturbing the lower order modes making up the incident beam, thus scattering significant power into very high order modes. Because of the Cartesian symmetry of the grating the fields can be most conveniently expressed in terms of Hermite-Gaussian beam modes, ψ_{mn}:

$$E(x,y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \psi_{mn}(x,y),$$

where $\psi_{mn}(x, y)$ are given by the usual expression [13]:

$$\psi_{mn}(x,y,z) = h_m(x;W)h_n(y;W)\exp(-ik(z+(x^2+y^2)/2R)+i\Delta\phi_{mn}),$$

with

$$h_m(s;W) = \left[2^m m!\sqrt{\pi W^2/2}\right]^{-1/2} H_m(\sqrt{2}s/W)\exp(-s^2/W^2),$$

and H_m, a Hermite polynomial of order m. The beam width parameter, W, phase radius of curvature, R, and phase slippage terms, $\Delta\phi_{mn}$, are all functions of position z [13].

Crucial to the efficiency of the GBMA approach in analysing the system is the choice of the optimum beam mode set. We require that the array of beams scattered by the grating can be described accurately by a finite sum of as few modal contributions as possible. As shown in [15] there exists a mode set of modest size in terms of which one can to a good approximation describe all of the off-axis beams with adequate accuracy. The beam waist radius for the best choice mode set at the output plane is given by:

$$W_o^2 \approx \sqrt{\frac{A}{\pi}}\frac{W_{oh}}{N},$$

where A is the area covered by the array of beams at the output plane, W_{oh} is the corresponding equivalent waist width of a simple Gaussian approximation to one of the beams on the output plane and N is the maximum "diffraction" order of the beams that we want to describe.

The mode coefficients for the source field are determined by performing the relevant overlap integral at the aperture of the source feed: $a_{mn} = \iint \psi_{mn}(x, y)^* E_s(x, y)\, dxdy$, e.g. [14]. The change in the form of the beam as it propagates away from this plane is determined by the evolution of the phase slippage term $\exp(i\Delta\phi_{mn}(z))$. This term does not depend on (x, y)

and can conveniently be included in the amplitude coefficient $A_{mn} = a_{mn}\exp(i\Delta\phi_{mn}(z))$.

As already noted the effect of the grating on the incident source beam $E_i(x, y)$ can be described in terms of the scattering of the power carried by the component modes into higher order modes because of the effect of the phase modulation introduced by the grating $\Phi(x, y)$. Thus,

$$E_i(x,y)\exp(i\Phi(x,y)) = \sum_{mn} A_{mn}\psi_{mn}\exp(i\Phi) = \sum_{mn}\sum_{m'n'} A_{mn}S_{m'm,n'n}\psi_{m'n'}$$

where

$$S_{m'm,n'n} = \iint_{GRATING} dxdy\,\psi_{m'n'}(x,y)^{*}\exp(i\phi(x,y))\,\psi_{mn}(x,y).$$

The mode coefficients for the transmitted (scattered) beam are given by $b_{m'n'} = \Sigma_{mn} S_{m'm,n'n} A_{mn}$. As the beam then propagates away from the grating the phase slippage terms $\exp(i\Delta\phi_{mn})$ further evolve. For propagation to the Fourier plane of the grating, $\Delta\phi_{mn} = (m+n+1)\pi/2$, so that on the output plane the amplitude coefficients are given by $B_{mn} = i^{(m+n+1)}b_{mn}$. However, the optics between the grating and the output plane will truncate the system of diffracted beams to some extent further scattering the modes.

To evaluate the performance of a grating as a multiplexer, one can calculate the coupling efficiency of the system of image beams to a test receiver horn moved about the output plane. We can conveniently represent the beam pattern of such a test receiver horn using the same mode set as for the multiplexer. Thus, $E_h = \Sigma c_{mn}\psi_{mn}$. For optimum coupling to the array of image beams the phase centre of the horn should be located at the image plane, and the beam width should closely match those of the array of images. The coupling of the horn antenna to the field is then given by: $\eta = |\,\Sigma_{mn} B_{mn}^{*} C_{mn}\,|^2$, where $C_{mn} = \exp(i\Delta\phi_{mn})c_{mn}$, takes into account the phase slippage between the horn aperture and its phase centre.

A phase grating will only operate correctly over a finite bandwidth and this may pose a limitation for certain applications. Two effects occur if the wavelength of the LO beam is not at the design wavelength of the grating: (i) The grating will only produce the required phase modulation of $\Phi(x,y)$ at the design frequency, since the phase delays for the grating are wavelength dependent, (ii) The actual inter-beam diffraction order separation in the output plane $\Delta x_O = \lambda f / \Delta x$ will change implying misalignment relative to any following optics. Realignment, however, can be achieved by designing in some variable magnification.

Gaussian beam mode analysis can again be applied to determine the deterioration in the power coupling characteristics across the band for a test horn placed in some fixed position in the output plane as the wavelength is varied. An example involving a Dammann grating is discussed in the next section.

3. Design and GBMA of Dammann Gratings

The DG is a binary optical component in which the modulated phase delay takes on just two values separated by π. The grating will usually possess rectangular symmetry, so that one can write the transmission $t(x,y) = t(x)t(y) = \pm 1$. In the example of the one-dimensional basic period of $t(x)$ shown in Figure 2, the free parameters are $\pm x_1$, $\pm x_2$, $\pm x_3$, etc. These are chosen so that as much of the power as possible is diffracted into a two dimensional array of uniformly intense non-overlapping images of the source beam of Figure 1.

Thus, if a $2M$+1 array of non-overlapping images in the x-direction is required, then $|T(m\Delta u)|=|T(0)|$ for $|m| \leq M$, and $T(m\Delta u) \approx 0$ for $|m| > M$. This is discussed in detail in [2], where various solutions are tabulated for different values of M. If an even number of output spots is required then neighbouring elements must be out of phase by π. This will cause the grating maxima to lie in the direction given by $(m + ½)\ \Delta u$, rather than $m\Delta u$, and an even number of equal intensity diffraction spots is obtained .

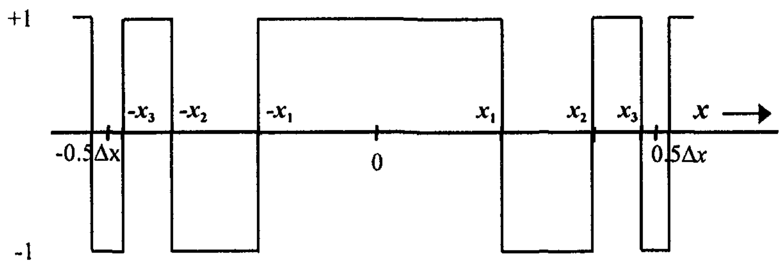

Figure 2 One-dimensional, symmetric binary function with period Δx.

The number of gratings cells illuminated by the source beam shown in Figure 1 determines the ratio of the output beam spacing to beam width. Assume for simplicity Gaussian field illumination of the grating, $E_i(x, y) \propto \exp(-(x^2+y^2)/W_G^2)$, where W_G is the usual Gaussian beam waist radius. At the output Fourier plane $E_O(x_O, y_O) \propto \exp(-(x_O^2+y_O^2)/W_F^2)$, where $W_F = \lambda f/\pi W_G$. This implies $W_G/\Delta x = [\pi(W_F/\Delta x_O)]^{-1}$, so that the incident beam width to cell size ratio at the grating is inversely proportional to the beam width to inter-beam spacing for the array of images at the output plane.

Consider as an example coupling the multiplexer output to an array of closely spaced diagonal horns. The best-fit Gaussian beam to the field at the mouth of a diagonal horn has a radius given by: $W = 0.43a$ [14]. For a horn of moderate length the beam waist radius at the horn phase centre will be somewhat less than this, typically: $W_{Oh} = 0.38a$. Thus, for optimum coupling to the array one requires $W_F = W_{Oh} = 0.38\Delta x_O$. The corresponding beam width, W_G, at the grating must be: $W_G = \lambda f / \pi W_F = 0.837\Delta x$. The Gaussian beam radius is of order the grating period! The implication of this is that we only need a *small number* of periods in the grating, reducing its size.

For the case where $W_G/\Delta x = 0.837$, the theoretically predicted resulting set of image Gaussian beams is shown in Figure 3. The grating is designed to produce a 5×5 array of images and the total number of periods in the DG equals 4×4. The slot edges of the basis cell in the x-

direction are at $x_1 = \pm 0.132\Delta x$ and $x_2 = \pm 0.480\Delta x$. For comparison, a set of 5 pure Gaussian beams is shown.

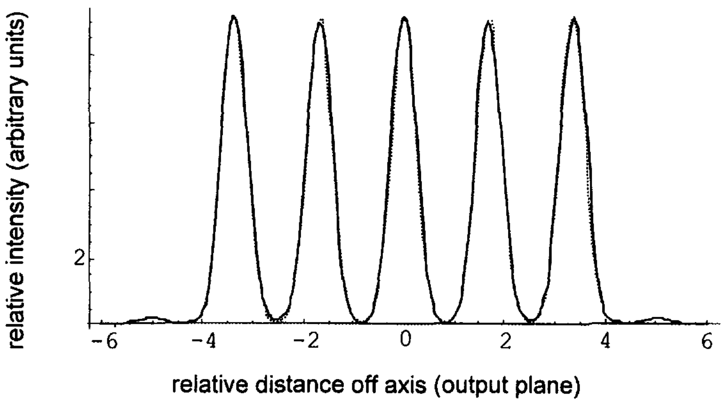

Figure 3 Array of Gaussian beams (dotted line) superimposed on DG beam array.

About 77.4% of the power is contained in the central orders for the one-dimensional case, the total coupling to a two dimensional array is of the order of 60%. This power is lost into diffraction orders higher than 2, and so some power spills round the side of the array of diagonal horns, and can be easily terminated using an absorbing microwave material around the array. It should also be possible to reduce the loss by having a basic grating cell with more grooves (more degrees of freedom).

We have also investigated a frequency detuning of a Dammann grating multiplexer using Gaussian beam mode analysis. These effects disturb the idealised coupling to an array of test horns placed in the output plane both in terms of the uniformity of the intensities and the displacement of the array of image beams with wavelength. The results of the analysis for a 5 × 5 grating are presented in Figure 4. It is worth noting that, in terms of local oscillator multiplexing applications, SIS mixer sensitivity to local oscillator power for optimised performance is usually not so critical that such a variation would cause a significant deterioration in performance across the array. Away from the design wavelength more power is proportionally coupled to the central beam.

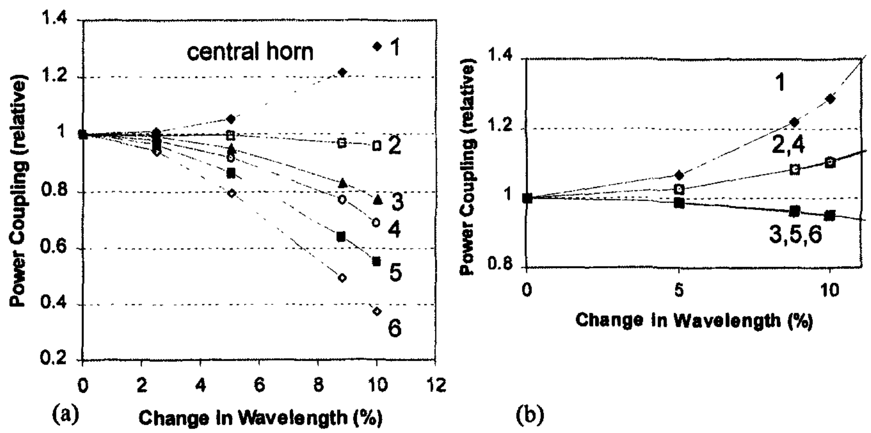

Figure 4 Power coupling to horns at six distances from the centre of a 5×5 array (b) with and (a) without variable magnification.

4. Experimental Measurements of Dammann Gratings

The practical feasibility of manufacturing and testing DGs was evaluated using a 100-GHz quasi-optical test facility. Gratings to produce 2×2, 3×3 and 5×5 beam arrays were designed (Figure 5 shows a cross-section along one of the axes for each grating). The gratings were manufactured from 112mm×112mm slabs of fused quartz by the Lithuanian optics company, Eksma.

The DGs were optically measured using the Fourier 4-*f* set-up shown in Figure 6. A 100-GHz Gunn oscillator was used with a conical horn-antenna feed as the source. The horn phase centre was placed at the focal point of an off-axis ellipsoidal mirror, M1, with an angle-of throw of 45°. The gratings were mounted on a Perspex holder at the common focal point of the two mirrors M1 and M2 where the source beam had a waist.

The mirror M2 was used to image the output of the grating onto the detector plane. The crystal detector was mounted on a computer-controlled XY raster-scanner centred on the focal point of M2. The scanner was capable of covering an area of 550 mm × 550 mm with a step resolution of 0.03 mm. Eccosorb was used to avoid unwanted reflections from component mountings.

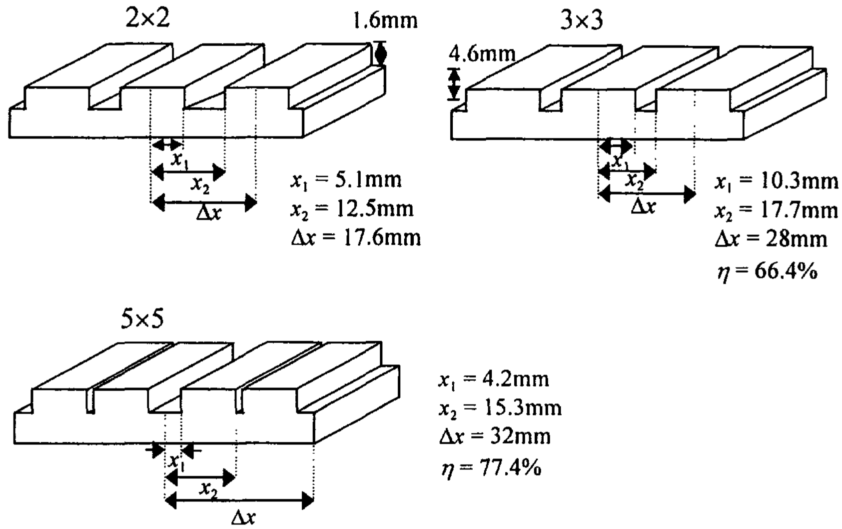

Figure 5 Designs for 2x2, 3x3 and 5×5 DGs. Designs for the 3x3 and 5×5 gratings were based on the calculations of Dammann & Klotz [5]. 6x6 grating periods were used in the 2x2 grating, and 4x4 periods in the other two. The efficiencies, η, refer to the 1-dimensional case. (The tolerance on each dimension was ±0.1mm).

The beam patterns obtained with the three gratings are presented in Figure 7. These are to be compared with the theoretical patterns predicted using the Gaussian beam mode analysis discussed previously and shown in Figure 8.

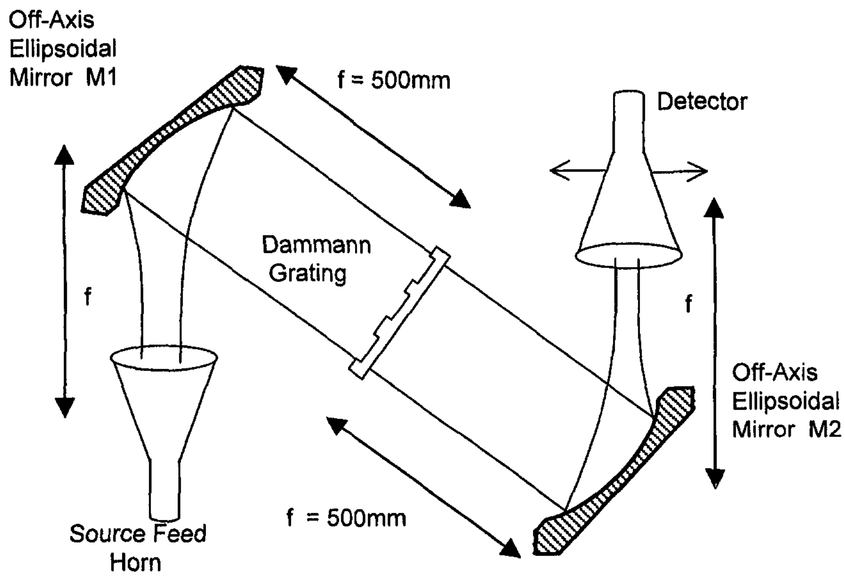

Figure 6 4-*f* set-up to test the Dammann gratings.

In all cases a 2-dimensional grid of beams is obtained. The long focal length of our mirrors meant that the beam waist at the grating was ~$2\Delta x$. From the previous discussion, the predicted beam-separation to beamwidth ratio at the output plane is then ~ 3:1, in agreement with what was found. For our particular choice of 3×3 grating both the theoretical and experimental plots show that the next highest order diffraction peaks are not completely suppressed. The output grid spacings are as expected for a square array reflected through 45° by an ellipsoidal mirror. The overall efficiency of the 2×2 and 5×5 gratings is higher and in these cases the central grid of peaks is far more intense than the higher orders.

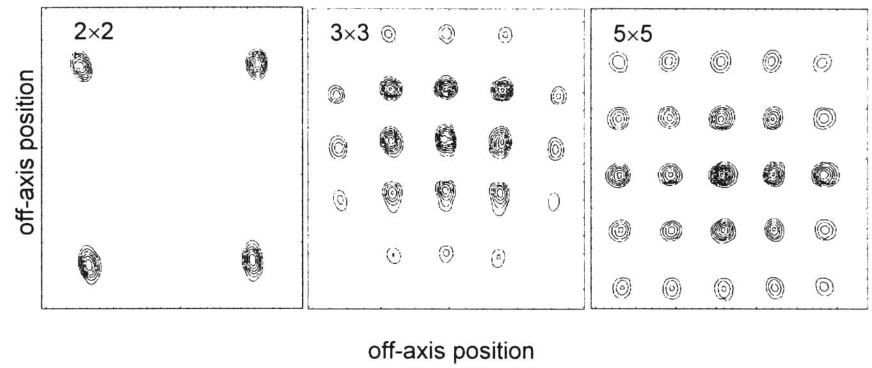

Figure 7 Output intensity patterns obtained with the 2×2, 3×3 and 5×5 gratings (contours are linear)

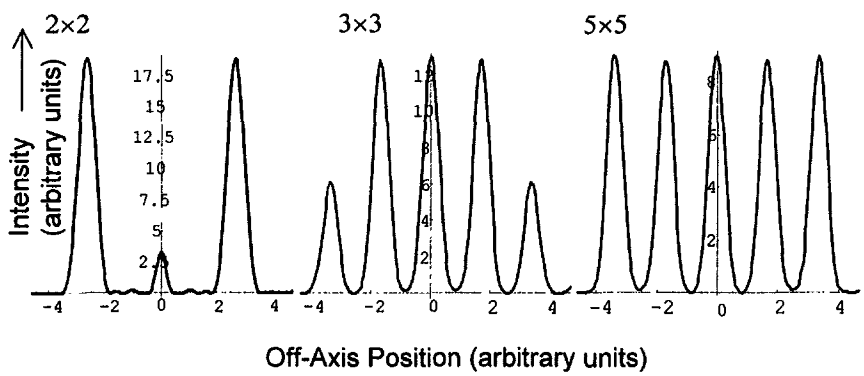

Figure 8 Cross section of the expected beam pattern from the 2×2, 3×3 and 5×5 gratings

In practice we found that for the 5×5 array, one row of 5 beams was consistently of a lower intensity than the other four rows, this we attribute to off-axis aberrations and beam clipping. For the 5×5 measurements, therefore, we replaced the mirrors M1 and M2 in Figure 6 with HDPE lenses (f = 250mm).

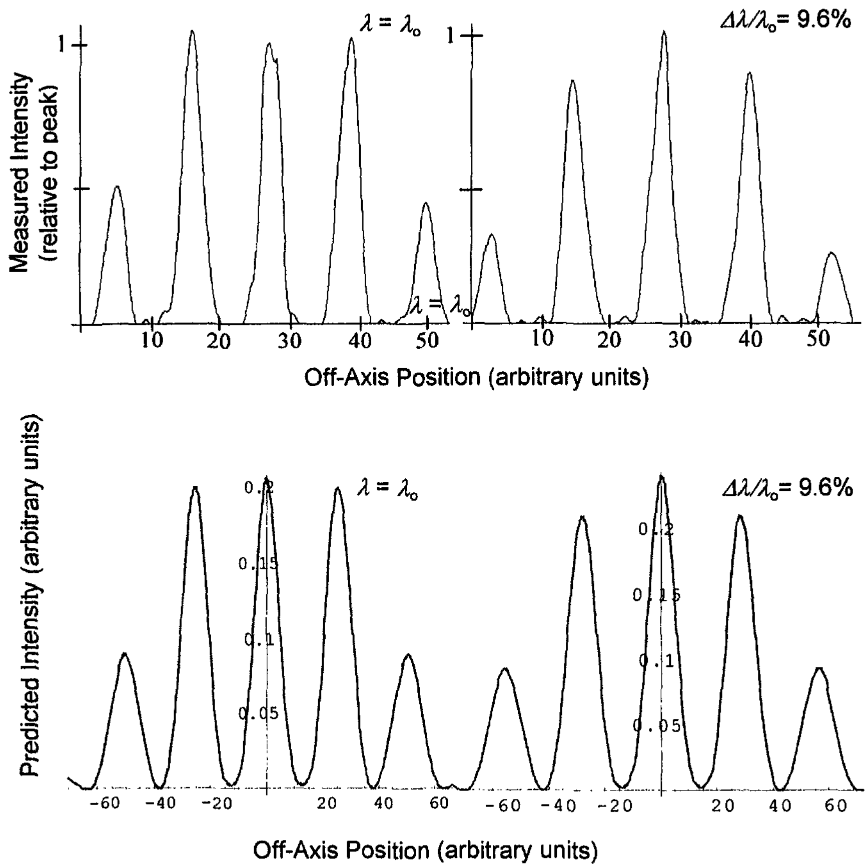

Figure 9 Cross-section of the measured and predicted beam patterns from the 3×3 grating. The output for the design wavelength and a 9.6% detuning are shown. Calculations were made using the Gaussian beam mode analysis discussed in this paper.

In order to investigate the bandwidth limitation of DGs, the 100-GHz Gunn oscillator was replaced with one whose frequency could be varied over the range 90 – 105 GHz. Measurements on the 3×3 grating at 90.4

GHz are presented along with the theoretical prediction in Figure 9. As expected the inter-beam spacing increases by a factor of $\Delta\lambda/\lambda_o$ and the intensity of the central beam increases at the expense of the other eight. The uniformity of the 2×2 and 5×5 beam patterns was also found to remain high over the bandwidth investigated. Figure 10 shows the output from the 5×5 grating at $\Delta\lambda/\lambda_o = 5\%$.

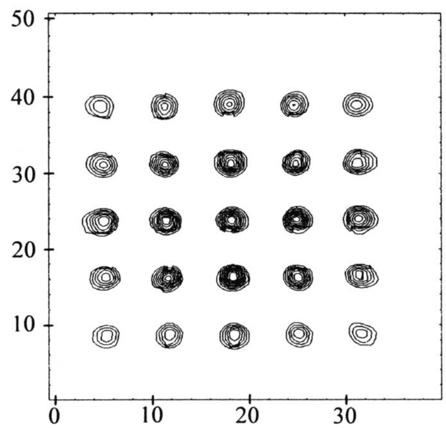

Figure 10 Output intensity patterns (linear contours) obtained with the 5×5 grating at 95.2 GHz.

5 Conclusions

In this paper we have discussed the analysis of a quasi-optical multiplexer based on phase gratings. It was shown how Gaussian beam modes could be used to efficiently describe the diffraction at the grating and subsequent propagation for compact quasi-optical systems of finite throughput.

As an example we have looked at Dammann Gratings (DGs) which are relatively simple structures. We have presented a study of the feasibility of using DGs for beam multiplexing at millimeter and submillimeter wavelengths. We have shown from a theoretical study that at the design wavelength we get good diffraction efficiencies of around 60% for a 5 ×

5 array, with the missing power being channelled into higher diffraction orders which can easily be terminated. The theoretical bandwidth of such a grating was found to be of the order of 15%, which is typical of astronomical array receivers operating in the submillimeter waveband. We also investigated the practical feasibility of producing a 2×2, 3×3 and 5×5 DGs, and presented measurements of prototype gratings.

In the case of the astronomical receiver, an LO multiplexing scheme would work best for the case where mixer characteristics are very similar, so that the LO requirement of individual mixers is the same. The scheme is particularly suitable for sparse arrays.

Acknowledgements

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