ABSOLUTE TILT FROM A LASER GUIDE STAR: A FIRST EXPERIMENT

S. ESPOSITO[1*], R. RAGAZZONI[2], A. RICCARDI[1], C. O'SULLIVAN[3], N. AGEORGES[3], M. REDFERN[3] and R. DAVIES[4]
[1] *Osservatorio Astrofisico di Arcetri, Fermi, Italia*
[2] *Osservatorio Astronomico di Padova, Vicolo dell'Osservatorio, Italia*
[3] *Department of Physics, University College Galway, Galway, Ireland*
[4] *Max-Planck-Institut fur extraterrestrishe Physik, Garching, Germany*
[*] *author for all correspondence, e-mail: esposito@arcetri.astro.it*

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Abstract. Absolute tip–tilt recovery using a tilt signal measured on a Laser Guide Star is a central problem in the framework of the development of Adaptive Optics Systems reaching full sky coverage down to visible wavelengths. In the past few years, various techniques aimed at solving this problem have been proposed. However only a couple of these has been recently tested in practice. We report about an experiment aimed at evaluating the performance of one of these techniques called the 'Elongation Perspective' technique. Our experiment has been performed using the ALFA system in Calar-Alto (Spain) and involves the simultaneous operation of the 3.6 m and the 2.2 m telescopes at the Observatory. This article describes the telescope configuration used, as well as the data reduction process carried out in order to estimate the scientific object tilt. The technique performances are discussed in terms of the residual tilt error variance and related correlation coefficient. The analysis shows that, despite the low SNR of our measurements, the atmospheric tilt variance is reduced to 80% of its initial value corresponding to a correlation coefficient of about 0.6. To get a better estimate of the performance achievable using this technique, the tilt error variance due to photon noise in the laser measurement is estimated and removed from the obtained tilt error variance. When this correction is done, this variance is reduced to about 50% of its initial value, showing that the use of this technique can give rise to a significant reduction of the scientific object image motion.

Keywords: adaptive optics, laser guide star

1. Introduction

In the recent years, Adaptive Optics Systems (AOS) using a Laser Guide Star (LGS) as reference source have been developed and installed on 4 m class telescope, such as, for example, the MPIA and MPE AOS called 'Adaptive optics system with a Laser For Astronomy' (ALFA) (Quirrenbach et al., 1997). More recently, systems for 8 m class telescopes are under development and should be in operation in a short time. VLT, Keck and Gemini, among the others, plan to have a LGS facility for routine operations.

However, these systems need a Natural Guide Star (NGS) located close to the scientific object. This is done in order to retrieve the atmospheric tilt perturbation

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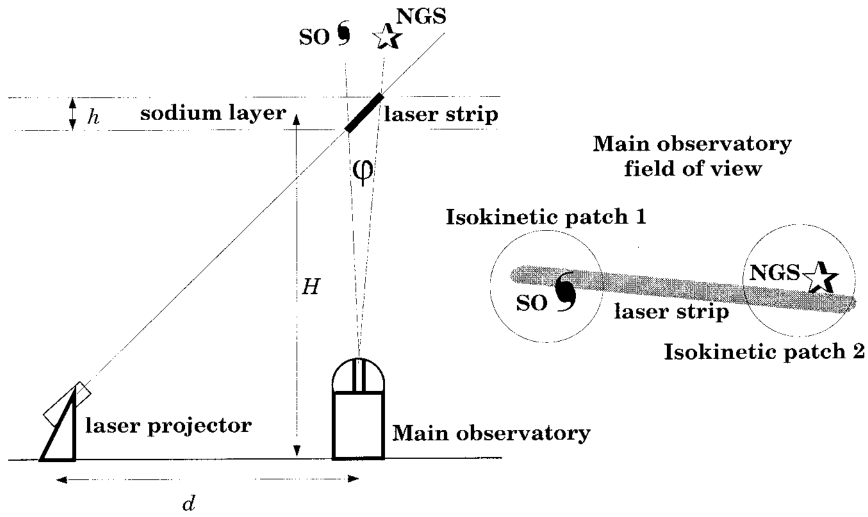

Figure 1. The laser projector and observing telescope configuration.

(Rigaut and Gendron, 1992; Oliver et al., 1993; Parenti and Sasiela, 1994; Sandler et al., 1994) that is not properly sensed by the laser beacon. This requirement reduces the fraction of the sky where the AOS can be properly operated considerably. Several schemes have been proposed in the literature (Ragazzoni and Esposito, 1997; Esposito, 1998) to estimate the scientific object tilt using LGS tilt measurements. Recently one of these techniques has been tested at the Starfire Optical Range Observatory (Belenkii et al., 1999).

Using the ALFA system installed at the German 3.6 m telescope together with the 2.2 m telescope, located in Calar Alto (Spain), we have performed some observations devoted to obtaining the absolute tilt using one of the proposed techniques, namely the 'Elongation Perspective' technique (Ragazzoni, 1997). Our experiments are aimed at performing some early experimental verification of theoretical predictions.

Although this experiment suffer from some limitations (e.g. photon noise), it constitutes a first step toward more exhaustive investigations, which could lead within a decade to the full demonstration and implementation of this method and provide the long sought-after solution to the tip-tilt problem mentioned above.

2. The natural guide star perspective auxiliary projector technique

We briefly present the concept of the technique here. A more detailed description can be found in Ragazzoni (1997). In this technique we use two auxiliary laser projectors to measure the two orthogonal tilt components. The two auxiliary projectors are located at a certain distance from the main telescope so that the laser source beam is seen from the main observatory as an elongated strip. In this case the auxiliary projector is pointed so that one edge of the laser strip is located, as seen from the main telescope, at the same position as the Scientific Object (SO) while, another part of the laser strip is located inside the isokinetic patch of a NGS. Note that there are no fundamental restrictions to the distance between the SO and the considered NGS. In this situation, as sketched in Figure 1, it is possible to obtain the downward tilt of the SO itself, in the direction perpendicular to the projected laser strip. In fact, it is easy to show that

$$T_{L1} - T_{SO} + \Delta T_1^{fa} = T_{L2} - T_{NGS} + \Delta T_2^{fa} = T_{\mathrm{LASER}}^{up} , \tag{1}$$

where T_{LASER}^{up} is the upward tilt of the laser beam and ΔT^{fa} is an error contribution due to focus anisokinetism (Esposito et al., 1996; Neyman, 1996) that is different at the two different locations of the natural guide stars. Neglecting focus anisokinetism effects the SO tilt is given by

$$T_{SO} = T_{L1} - (T_{L2} - T_{NGS}) , \tag{2}$$

where the quantities T_{L1}, T_{L2} and T_{NGS} are the measured tilts of the laser strip patch one, laser strip patch two and of the natural star as reported in Figure 1. Still referring to Figure 1, it is possible to show that the angular length of the laser strip ϕ as seen from the observatory is given by

$$\phi = dh/H^2 . \tag{3}$$

Assuming a telescope-projector distance of $\approx$ 300 m (as is the case in Calar Alto) we obtain a projected length of about 70 arcseconds so that the natural star can be located well outside the isokinetic patch of the SO.

3. The telescopes arrangement and measure configuration

In the framework of the European Community funded TMR network known as *Laser Guide Star for 8 m class telescopes* (Foy, 1998) we set out to test this technique experimentally. This has been done using the ALFA laser Adaptive Optics system located in Calar Alto (Spain). In our experiment, the laser beam was projected by the 3.6 m telescope and observed by the 2.2 m telescope. The two telescopes are about three hundred meters apart so that the laser beam appears in the smaller telescope field of view as an elongated strip about 70 arcseconds long. By appropriately pointing the laser it is possible to obtain that in the field of view of the 2.2 m telescope the laser strip appears to have two natural stars close to its edges, at either end. Using this configuration, which is similar to the one reported in Figure 1, it is relatively easy to test the technique performances. This is done by measuring the tilt of the two NGSs T_{N1} and T_{N2}, and the tilts of the laser strip portions T_{L1} and T_{L2} located close to the NGSs themselves. Using the measured

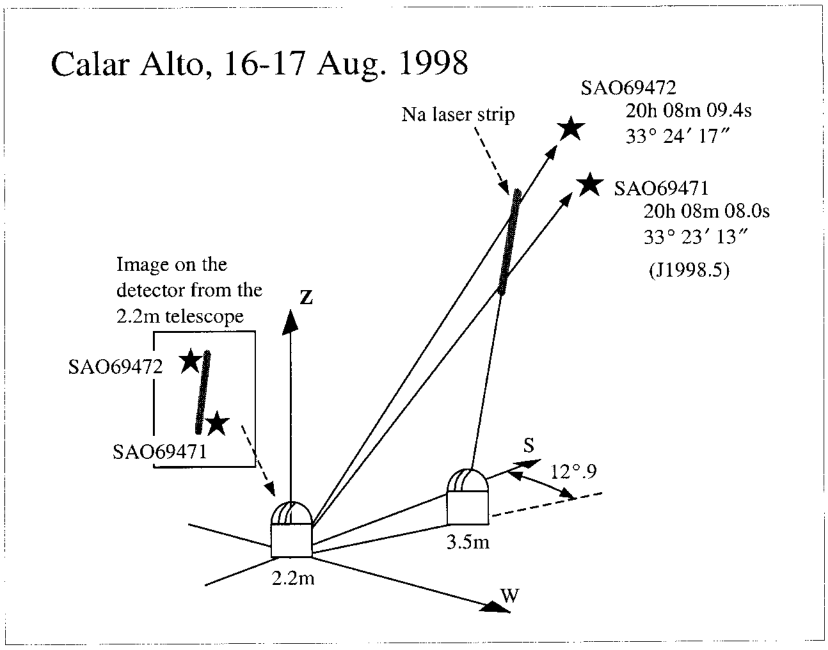

Figure 2. Arrangements of the laser projector (3.5 m telescope) and observing telescope (2.2 m telescope) in the measurement configurations in Calar Alto.

data we can compare the tilt of the NGSs with their estimates obtained considering Equation (2). In the following analysis, we attempt to estimate the tilt of the NGS1. During the experiment we identified two stars not too far from the zenith lying in the described configuration when the laser was propagated from the 3.6 m telescope in the correct direction. The stars chosen were, respectively, SAO69471 and SAO69472. The observation was performed in the geometrical arrangement shown in Figure 2. The 2.2 m telescope was equipped with a MAMA (Timothy and Bybee, 1986) detector that gave information about the arrival time and the position of each photon detected with a time resolution of one microsecond and spatial resolution of about 0.3 arcsecond per pixel. A long exposure (10 s) frame is shown in Figure 3 as an example. To get the highest number of photons per pixel from the LGS we focussed the 2.2 m telescope on the laser strip. Consequently the natural stars are strongly defocussed. This is shown in Figure 3 where the natural star images clearly present the pupil central obscuration.

4. Technique verification and tilt estimation

To verify the performance achieved with this technique we had to measure the four tilts contained in Equation (2), but with the SO tilt replaced by the natural guide tilt

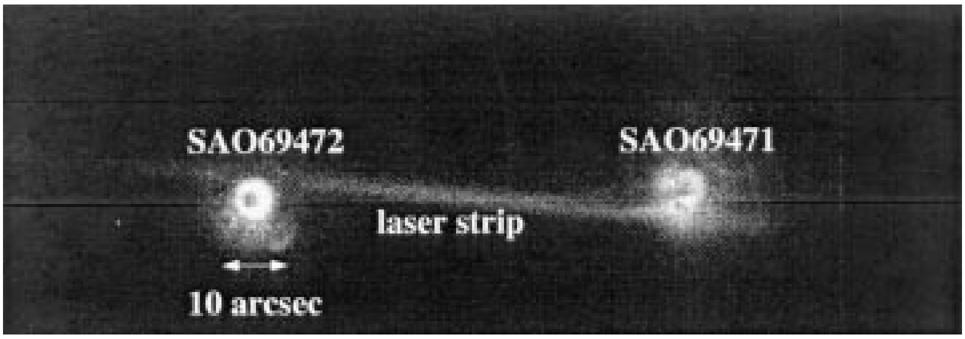

Figure 3. A MAMA subframe of 512×180 pixels with an integration time of 10 sec. Scale is 0.3 arcseconds per pixel.

T_{N1}.[a] In order to obtain the required tilt, a first consideration is important. In our images the laser strip and the two natural guide stars are somewhat superimposed which made it difficult to measure the tilt of these objects. However, the ratio between the number of photons received from each star and the corresponding laser strip portion could be estimated from our data and it turns out to be about 5%. Therefore we calculate the tilt of a subframe centered on the two NGSs, neglecting the contribution from the laser strip photons. Furthermore we need to indirectly evaluate the tilts T_{L1} and T_{L2} of the two laser strip portions located under the NGSs themselves. We obtained these two tilts by interpolating the tilt values measured for the two laser strip portions located on the left and right side of each of the NGS subframes. This situation is presented in Figure 4 where we show a sketch of the various subframes used to identify the subimages needed to compute the various tilts. For each subimage we obtained the tilt by considering the one-dimensional profile in the y direction.[b] Referring to the symbols introduced in Figure 4, our estimate of the the laser strip tilts T_{L1} and T_{L2} results

$$\tilde{T}_{L1} = \left(T^r_{L1} + T^l_{L1}\right)/2 \tag{4}$$
$$\tilde{T}_{L2} = \left(T^r_{L2} + T^l_{L2}\right)/2\,. \tag{5}$$

Furthermore, as will be clarified in the next section, this interpolation approach reduces the error on the estimated laser tilt with respect to the case where only one of the tilts located beside the considered NGS directly as an estimate of the central patch laser tilt.

As an estimator of the image displacement we chose the energy distribution median which is relatively less sensitive to photon noise errors than baricenter

[a] Hereafter, we refer to ‘tilt’ as the tilt component in the y direction of the detector pixel coordinate system that is, to a first approximation, orthogonal to the observed laser strip.

[b] Going from 2D images to 1D profiles in calculating the tilt does not significantly affect the tilt determination process and speeds up the calculations (Stone, 1989).

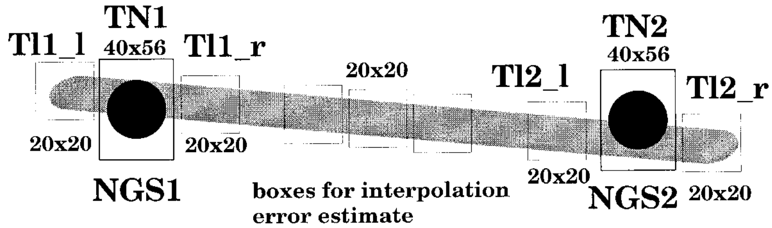

Figure 4. A sketch of the various boxes considered to determine the needed tilt signals. The numbers close to the boxes represent their dimension in pixels.

measurements, as reported in Stone (1989). Image displacement signals are obtained by rebinning the MAMA data with a minimum temporal binwidth of 0.1 s giving a trade-off between photon noise errors and the atmospheric tilt signal attenuation. Furthermore in the following data analysis we calculate the tilt signals T_{N1}, T_{N2}, T_{L1} and T_{L2} by considering a running average of time width ranging between 0.2 and 2 s. The technique performance is evaluated by calculating the variances σ_1^2, σ_2^2 and σ_3^2 of the signals S_1, S_2 and S_3 defined as follows:

$$\begin{aligned} S_1 &= T_{N1} \\ S_2 &= T_{N1} - T_{N2} \\ S_3 &= T_{N1} - \tilde{T}_{N1} \,, \end{aligned} \tag{6}$$

where $\tilde{T}_{N1}$ represent our estimate of T_{N1} and is given by

$$\tilde{T}_{N1} = \tilde{T}_{L1} - (\tilde{T}_{L2} - T_{N2}) \,. \tag{7}$$

The introduced variances allow us to obtain the correlation coefficients between the two coupled tilts that forms the quantities S_2 and S_3. This is done considering that the correlation coefficient γ between two signals a and b having the same standard deviation σ is such that

$$\left\langle (a-b)^2 \right\rangle = 2\,\sigma^2 (1-\gamma) \,, \tag{8}$$

where γ has the standard definition

$$\gamma = \frac{< ab >}{\sigma^2} \,. \tag{9}$$

Results obtained for the variances σ_1^2 ,σ_2^2 and σ_3^2 and the two correlation coefficients γ_2 and γ_3 are reported in Figure 5 as a function of the width of the running average window.

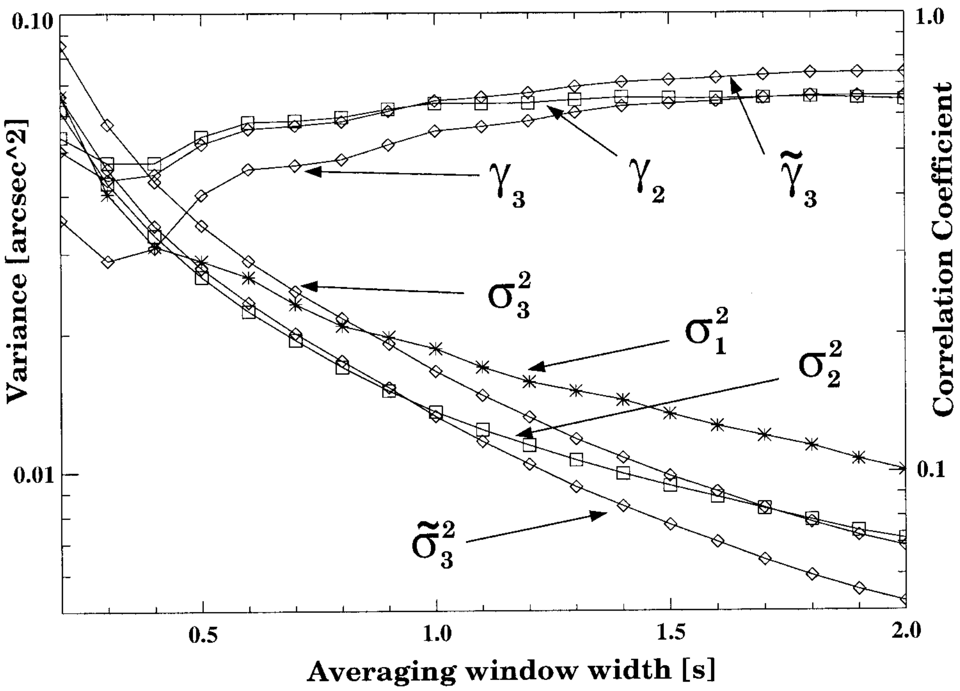

Figure 5. Behavior of the signals variance versus the running average window width. The upper lines represent the correlation coefficients.

5. Results discussion and photon noise error

Considering Figure 5 we can check if our tilt estimate $\tilde{T}_{N1}$ is useful as absolute tilt correction signal. The first condition for this is that $\sigma_3^2 < \sigma_1^2$ or $\gamma_3 > 0.5$. This condition is satisfied for a running average window larger than 1.0s. As an example at 1.5 s using the estimate signal to correct the tilt T_{N1} gives a residual tilt variance σ_3^2 of about 80% of the initial tilt variance. For comparison purpose we plot variance σ_2^2 that corresponds using the tilt T_{N2} to correct T_{N1}. This variance is, in our data, greater than σ_3^2. However σ_3^2 approaches σ_2^2 when the width of the averaging window is increased. This suggest that photon noise limits our tilt measurement $\tilde{T}_{N1}$. As seen from Equation (2) the variance σ_3^2 contains a contribution due to photon noise in the laser tilt measurements T_{L1} and T_{L2}. In our measurements this term is not negligible because of the low detector Q.E ($<$ 3%). To estimate the technique performance when a better SNR on the laser measurements is available we quantify this contribution in the following analysis in order to subtract it from σ_3^2. For reasons of clarity we identify a particular measurement of a quantity T_a with the symbol θ_a. Furthermore we use, as before, the symbol $\tilde{T}_a$ to refer to our approximation of a quantity T_a. Following these conventions and using Equation (2) where focus anisokinetism error is neglected the variance σ_3^2 is given by the expression

$$\sigma_3^2 = <(\theta_{N1} - \tilde{T}_{N1})^2> = <(\theta_{N1} - [\theta_{N2} - (\theta_{L1} - \theta_{L2})])^2> = \\ = \sigma_{N1}^2 + \sigma_{N2}^2 + \sigma_{L1}^2 + \sigma_{N2}^2 , \qquad (10)$$

where $< \ >$ indicates the expectation value and σ_{N1}^2, σ_{N2}^2, σ_{L1}^2 and σ_{L2}^2 represent the measurement error variances, supposed uncorrelated, of the quantities T_{N1}, T_{N2}, T_{L1}, T_{L2}, respectively. Considering Equation (5) we can obtain an expression for σ_{L1}^2 and σ_{L2}^2. To this end we write an expression for a particular measurement of T_{L1}^l and T_{L1}^r

$$\theta_{L1}^l = T_{L1} + \Delta T_1^l + \Delta n_1^l \qquad (11)$$

$$\theta_{L1}^r = T_{L1} + \Delta T_1^r + \Delta n_1^r , \qquad (12)$$

where ΔT and Δn represent the tilt error term due to tilt angular decorrelation and photon noise, respectively. With these notations the error variances σ_{L1}^2 and σ_{L2}^2 of our estimations T_{L1} and T_{L2} are given by

$$\sigma_{L1}^2 = \frac{<(\Delta T_1^l)^2> + <(\Delta T_1^l)^2> + <(\Delta n_1^l)^2> + <(\Delta n_1^l)^2>}{4} \qquad (13)$$

$$\sigma_{L2}^2 = \frac{<(\Delta T_2^l)^2> + <(\Delta T_2^l)^2> + <(\Delta n_2^l)^2> + <(\Delta n_2^l)^2>}{4} . \qquad (14)$$

The experimental data allow us to estimate the two different error contributions $<(\Delta T)^2>$ and $<(\Delta n)^2>$. To do this we consider the tilt signals T_{S1}, T_{S2} and T_{S3} of the three central laser patches shown in Figure 4. Following notation of Equations (11) and (12) a particular measurement of these three quantities has the expression

$$\theta_{S1} = T_{S2} + \Delta T_1 + \Delta n_1 \qquad (15)$$

$$\theta_{S2} = T_{S2} + \Delta n_2 \qquad (16)$$

$$\theta_{S3} = T_{S2} + \Delta T_3 + \Delta n_3 . \qquad (17)$$

Our experimental data allow to calculate the quantity

$$\sigma^2 = \left\langle \left[\theta_{S2} - \frac{1}{2} (\theta_{S1} + \theta_{S3}) \right]^2 \right\rangle , \qquad (18)$$

that using Equations (15), (16) and (17) gives

$$\sigma^2 = <(\Delta n_2)^2> + \\ + \frac{<(\Delta n_1)^2> + <(\Delta n_3)^2> + <(\Delta T_1)^2> + <(\Delta T_2)^2>}{4} . \qquad (19)$$

As in Sandler et al. (1994) we assume that the tilt error variance due to photon noise is inversely proportional to the number of photons used to calculate the tilt signal. We locate the three laser patches in the middle of the laser strip so that each patch receive approximately (within 10%) the same number of photons N_s per integration time. Using this condition we have

$$\sigma^2 = \frac{3}{2} < (\Delta n)^2 > + \frac{1}{4}\left[< (\Delta T_1)^2 > + < (\Delta T_2)^2 >\right] = \sigma_n^2 + \sigma_T^2 . \tag{20}$$

The last error variance σ_T^2, due to tilt angular anisoplanatism, has an analytical expression given in Sandler et al. (1994). We assume, as an overestimation of variance σ_T^2, a linear relationship between angular anisoplanatism tilt error variance and the angular separation of the considered objects. In this situation we can rescale the measured variance σ_2^2 to the angular separation of the considered laser patches. This allows us to evaluate the quantity σ_T^2 experimentally. Our calculations show that $\sigma^2 \gg \sigma_T^2$ so that we obtain

$$< (\Delta n)^2 >= \frac{2}{3}\sigma^2 . \tag{21}$$

This estimation of the photon noise error for a given portion of the laser strip can be used to calculate the two quantities

$$\sigma_{TL1}^2 = \frac{1}{4}\left[\left\langle(\Delta n_{L1}^l)^2\right\rangle + \left\langle(\Delta n_{L1}^r)^2\right\rangle\right] \tag{22}$$

$$\sigma_{TL2}^2 = \frac{1}{4}\left[\left\langle(\Delta n_{L2}^l)^2\right\rangle + \left\langle(\Delta n_{L2}^r)^2\right\rangle\right] , \tag{23}$$

that gives the photon noise error due to laser measurements contained in variance σ_3^2. This is done again by considering the photon noise as being inversely proportional to the received photons. Using this condition we obtain $< (\Delta n_{L1}^l)^2 >$, $< (\Delta n_{L1}^l)^2 >$, $< (\Delta n_{L1}^l)^2 >$ and $< (\Delta n_{L1}^l)^2 >$ by rescaling the error variance $< (\Delta n)^2 >$, previously determined according to Equation (21), to the number of photons received in the four laser patches used to calculate T_{L1} and T_{L2}. Subtracting the two quantities σ_{TL1}^2 and σ_{TL2}^2 from σ_3^2 gives a new variance $\tilde{\sigma}_3^2$ where laser photon noise effect is removed. This quantity, together with the related correlation coefficient $\tilde{\gamma}_3$ is shown in Figure 5. The residual variance $\tilde{\sigma}_3^2$ is about σ_2^2 for a binning window shorter than 1.0s and is lower for longer windows. In this case the correlation coefficient $\tilde{\gamma}_3$ stay well above 0.6 and reach a maximum value of about 0.75. This value correspond to a tilt variance reduction of 50%. This shows that, if laser measurements with a better SNR were used, the technique could achieve a significant accuracy in estimating the tilt T_{N1}. Finally we note that similar results on the correlation coefficient of the tilt estimate signals have been obtained in Belenkii et al. (1999).

6. Conclusion

This article describes a first experimental evaluation of the so called ‘Elongation Perspective’ technique. The data analysis has shown that a reduction of the tilt variance of about 30% of the initial value can be obtained using the considered technique. However, the principal limitation of the performed experiment is the low SNR of the laser measurements due to the low quantum efficiency of the MAMA detector ($< 3\%$) used. When photon noise error is numerically removed in the data analysis a significant reduction of the tilt variance of about 50% is achieved.

To get better SNR the experimental configuration could be improved in various ways, for example, independent focusing of the LGS and NGS and dichroic filters to select the light from these two sources, to cite the more evident ones. Finally experiments with higher SNR are needed in order to demonstrate that using techniques like this Adaptive Optics Systems can be operated in sky regions not containing bright NGSs.

Acknowledgments

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