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Heterogeneous Noisy Beliefs and Dynamic Competition in Financial Markets*

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Abstract

This paper analyzes the competition of heterogeneously informed traders in a multi-auction setting. We obtain that the competition can take different forms depending on the number of traders, trading rounds and the noise in the information. When the number of traders is small and the number of trading rounds is large, traders may trade very aggressively at the opening and at the end of the trading day with lower trading intensity in between. Hence, we can explain volume patterns by the nature of the competition between traders rather than by pattern in the level of liquidity. We find that the noise in the signal may be beneficial for traders when the competition is strong as it gives them a monopolistic position on their private information. The amount of noise maximizing the trader's expected profit increases with the number of trading rounds as well as the number of traders. This implies that the value of information is closely related to the market where that information is subsequently being used.

JEL Classification: G14-G24-D43-D82

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1 Introduction

It is now commonly accepted that people hold divergent opinions on many subjects ranging from the future performance of a particular stock to the future growth rate of the economy. These diverging opinions may not be the result of any irrationality but the result of the information processing or the source of the information itself. This information or belief heterogeneity is thought to be the main driving force behind the large trading volume observed in financial markets (see Cochrane (2007)). The recent literature acknowledges the importance of the heterogeneity of beliefs and it has become a central assumption to very diverse analyses (see for instance He et al. (2009), De Kamps et al. (2014), Gollier (2007) and Verardo (2009)). Xiong (2013) provides an excellent literature review on the subject. Some of that literature focuses on the impact of belief heterogeneity on asset pricing. Gandhi and Serrano-Padial (2015) find that it affects returns and Anderson et al. (2005) find that it might explain the observed favourite-long shot bias. Ottaviani and Sørensen (2014) obtain that when traders are credit constrained the competitive equilibrium price underreacts to information and that this underreaction is larger the more heterogeneous beliefs are.

In our paper, we assume that traders have heterogeneous beliefs regarding the future value of a traded asset. We then study the competition between these traders. Given that framework, we derive the unique linear equilibrium in a multi-auction market where traders receive heterogenous signals.

Our article aims at answering several questions such as what dynamic strategies should informed market participants use to maximize their profits? How quickly does the price adjust to reflect private information? How are the insiders' profits affected by noisy private signals? Can informed traders reduce competition when they have noisy private signals i.e. can noisy information be profitable for informed traders? And as a natural extension to the previous is there an optimal level of noise that maximizes traders' profits?

When looking at the informed trader's behavior we obtain the following results which

depend on the level of noise in the trader's signal. When the trader's private information is precise, traders trade very aggressively on their private information. Insiders have very similar private information and try to exploit their private information very early during the trading day and increase their trading aggressiveness until the closure of the market. However, when the trader's private information is very noisy, traders can limit the size of their orders as they have a monopolistic position on the private information they have received. In that case, traders wait to exploit their informational advantage as, due to the noise in the signals, prices will take time to incorporate their private information. We find that the effect of the number of auctions, the number of traders and the level of noise do not have a straightforward effect on the competition between traders. Increasing the number of trading rounds leads to more aggressive traders if there are few traders, whereas if there are many traders they may trade aggressively at the beginning and at the end of the trading day. This leads to a pattern in the volume traded whereby the volume is high at the opening and at the closure of the market and lower between the two. This pattern is observed in financial markets. Our paper explains it as being a consequence of the number of trading rounds, the number of traders and the level of the noise.

We are not the first ones to analyze the strategic trading behavior of informed traders in a dynamic setting. Different frameworks have been used to perform that task. Kyle (1985) examines the trading behavior of a single perfectly informed trader and finds that the trader limits the size of his early trades in order not to reveal too much information too early. Information is then gradually incorporated into prices. That result depends critically on the presence of a single informed trader and also on the structure of the private information i.e. whether it is perfect or not. Holden and Subrahmanyam (1992) show, to the contrary of Kyle (1985), that the competition resulting from the presence of more than one informed trader with identical information results in almost all the private information to be revealed in the early auctions.[1] Foster and Viswanathan (1996) analyze the case of imperfect competition when the traders' information is correlated.

[1] Foster and Viswanathan (1993) find the same result.

Back et al. (2000) study the competition between strategic traders in continuous time. Both papers show that the competition between informed traders is very complex and depends critically on the initial correlation between the informed traders' signals. Those two papers are the closest to our analysis. They find that when the correlation is not too strong, the competition has two phases. Firstly, insiders trade very aggressively and release much of their private information in the earlier trading periods. This phase is known as the "rat race". Secondly, since the correlation between the residual private information of the informed traders evolves over time, after a number of auctions the insiders' residual information is negatively correlated between each other. This reflects a difference of opinion between the informed agents about the final value of the risky asset. The informed participants then become more reluctant to trade, since each insider could be on the wrong side of the market. Hence, the trading activity is less intense. This phase is known as the "waiting game". During that phase, insiders conceal their private information. This phenomenon leads to an adverse selection problem in the market at the end of the trading day. Hence, the competition between the insiders does not automatically lead to more efficient prices as one approaches the time of liquidation. Our result regarding the very intense competition when the level of noise is low is close in spirit to that rat race described in Holden and Subrahmanyam (1992), Foster and Viswanathan (1996) and Back et al. (2000).[2] However, when private information is very noisy we only obtain a waiting game. This result is in sharp contrast to Foster and Viswanathan (1996). Indeed, they find that the waiting game is followed by a rat race. In this case, the insiders limit their orders since their private information is noisy. The waiting game observed in our model is not due to a negative correlation between the signals as a consequence of trading. We show that the waiting game phase appears when the correlation between the signals of the traders is low - but positive. We also show that it is possible to have the reverse sequence of the two stages (first a rat race and then a waiting game).

[2]We show that the models of Kyle (1985) (discrete setting) and Holden and Subrahmanyam (1992) are encompassed in our model leading to the same results for some particular parameters values.

As a natural consequence of the previous results we find that when competition is strong (the number of informed traders and/or the number of auctions is large), increasing the noise in the traders' private information may lead to higher profits. In that case, traders can make more profits with noisy information than with perfect information. This can be explained by the fact that increasing noise gives a monopolistic position on the information received (increasing expected profits). However, adding too much noise in the traders' signals may decrease their profits as it leads traders to trade on noise too (this has a damaging impact on the expected profits). When the level of competition is not as strong, noise always reduces the profit of the informed traders. Whether one effect dominates the other one depends on the level of competition in the market as well as the level of noise. These results generalize the findings of Dridi and Germain (2009). This trade-off between noise and competition bears some similarities with the results put forward by Foster and Viswanathan (1996) and Back et al. (2000) regarding the level of correlation of the signals and the expected profits of the traders. Indeed, Foster and Viswanathan (1996) show that the expected profits of the traders are higher when there is some positive correlation compared to the case where the signals are uncorrelated.

Given the trade-off between competition and noise, we find the existence of an optimal level of noise (i.e. a level of noise maximizing the informed traders' expected profits). This optimal level of noise increases with the number of traders and with the number of auctions. As competition increases the noise maximizing expected profits increases. This result shows that the value of information is closely related to the market where that information is subsequently being used. In a highly competitive market, informed traders would be willing to pay a higher price for a noisier information. This would imply that companies specialized in the sale of information could introduce that in their pricing.

We model the heterogeneity of beliefs differently than Foster and Viswanathan (1996) and Back et al. (2000) but similarly to Kyle (1985) and most of the papers following that model. This enables us to study the effect of the heterogeneity of beliefs onto the competition between traders as in the two first papers cited. We are then able to analyze

the direct impact of noise on trading and study the trade-off between noise and competition in a dynamic setting highlighted in Dridi and Germain (2009).[3] This analysis has some relevance for the models of sale of information and more particularly for models of direct sale of information as per Admati and Pfleiderer (1988a). In a direct sale of information, the buyer of information observes the information and trades on it. Obviously an important aspect of the information is the noise embedded in it. One of our result leads to the fact that traders may actually be better off by buying noisy information for use in a very competitive market.

The heterogeneity of beliefs as we model it has also a theoretical appeal. Indeed, considering the effect of the variance of the noise on the traders' behavior (as we do) is not equivalent to considering the effect of the correlation between signals. A change in the correlation between signals only measures the degree to which signals are identical or not. A change in the variance of the noise in the traders' signals does not only affect the correlation between the traders' signals, but also the correlation of the traders' signals with the liquidation value of the asset. Then, changing the level of the noise simultaneously affects the correlation between the traders' signals and the correlation with the liquidation value of the asset. Indeed, a large variance of the noise leads to a lower correlation between signals. This implies a reduction of the level of competition by giving each trader a monopolistic position on his private information and thus prevents competition from destroying his profits. However, it also implies that informed agents are trading on noise which in turn reduces their expected profits. This effect is not captured by looking at the effect of the correlation between signals.

Ostrovsky (2012) highlights that in dynamics models the most important issue is the aggregation of information. In Foster and Viswanathan (1996) and Back, et al. (2000) the dispersed information forms a sufficient statistic and prices converge to the liquidation value. In our model, prices do not, in general, converge to the liquidation value of the

[3]The model of Dridi and Germain (2009) is a particular case of our model corresponding to a static setting.

risky asset. However, we obtain that prices tend to converge to the liquidation value when we increase the number of traders. Moreover, the rate of the increase for the price informativeness is lower in our model than in Holden and Subrahmanyam (1992), as noise slows down the revelation of information.

Some papers have empirically investigated the competition between informed traders taking place in financial markets. Ellison and Mullin (2007) find that the information is gradually incorporated into price confirming the result found in Kyle (1985). Cho (2007) analyses the behavior of stock prices ahead of earnings announcements. The paper finds evidence of informed trading. However, the evidence is more consistent with Foster and Viswanathan (1996) than with Kyle (1985). Our model predicts that changes in volume during the trading day can be explained by the presence of noise in the information of the traders when they compete in the market. [4]

The remainder of the paper is organized as follows. Section 2, presents the general setup. We show in section 3 the existence and the uniqueness of a linear equilibrium and characterize the different parameters at each auction. In section 4, we study the informativeness, the market depth and the expected profits according to the level of noise in the signals of the informed traders, the number of auctions and the number of traders. In section 5, we present some practical implications from the model developed. Finally, in section 6, we make some concluding remarks. All proofs are gathered in the Appendix.

2 The Model

We follow the notation of Kyle (1985) and Holden and Subrahmanyam (1992). We assume that a risky security is traded during N sequential auctions in a time interval which begins

[4] That competition has a direct impact on price efficiency i.e. on how prices reflect the information collected by traders. Hou and Li (2016) study information efficiency and show that the US market is more efficient in impounding information from other markets. Kim and Ryu (2015) study the speed of convergence of national stock index vis-à-vis the US Index. They find evidence of convergence for France, Germany and the UK but very limited evidence of convergence for Italy, Canada and Japan.

at $t = 0$ and ends at $t = 1$. Let Δt be the time interval between the nth auction and the previous auction, we assume that $\Delta t = \frac{1}{N}$. At $t = 1$, the liquidation value of the asset is revealed. This liquidation value is denoted by $\tilde{v}$, with $\tilde{v} \sim N(\bar{v}, \sigma_v^2)$. For simplicity and without loss of generality, we assume $\bar{v} = 0$. In each auction, the following market participants are present:

- M risk-neutral informed traders. At $t = 0$ each insider $i = 1, \ldots, M$ receives a signal $\tilde{S}_i = \tilde{v} + \tilde{\epsilon}_i$ about the liquidation value of the risky asset, where $\tilde{\epsilon}_i \sim N(0, \sigma_\epsilon^2)$, for $i = 1, \ldots, M$. Moreover, we assume that the error terms, $\tilde{\epsilon}_i$, are mutually independent and that they are independent of $\tilde{v}$. Informed participants receive heterogeneous signals as in Admati and Pfleiderer (1988b).

- Liquidity traders. They submit orders at each auction and do not possess any information about the fundamental value of the risky asset. We denote by $\Delta\tilde{u}_n$ their aggregate orders and we assume that $(\Delta\tilde{u}_n)$ are independently and identically normally distributed with zero mean and variance $\sigma_u^2 \Delta t$. Also, we assume that $\Delta\tilde{u}_n$ are independent of $\tilde{v}$ and $\tilde{\epsilon}_i$.

- Competitive risk-neutral market makers. They observe the aggregate orders, but do not know whether these orders are initiated by liquidity traders or insiders. They set the price p_n, at each auction n in a Bayesian way.

At the nth auction we denote $\Delta\tilde{X}_n$ as the aggregate order of all informed traders, and π_{in} the total expected profit of informed trader i, for $i = 1, \ldots, M$, from auction n to auction N.

Each risk neutral informed trader determines his optimal trading strategy by a process of backward induction in order to maximize his expected profits given his conjectures about the trading strategies of the other informed traders. We look for a linear equilibrium where each informed trader chooses an order which is linear in his private information and the previous public price.

Competition in market making drives the market makers' expected profits to zero, conditional on the aggregate submitted orders $\tilde{w_n} = \Delta\tilde{X}_n + \Delta\tilde{u}_n$. We also look for linear strategies for the market makers.

3 Equilibrium

We now introduce the equilibrium concept used in our model. To start, we define the conditions to be satisfied for a Bayesian Nash equilibrium. Then we restrict our search to linear Markov equilibrium and conjecture the equilibrium strategies for the market maker and informed traders.

Just before period n, the information of insider i consists of his own signal $\tilde{S}_i$, plus his own orders $(\tilde{x}_{i1}, \ldots, \tilde{x}_{in-1})$. In addition, all insiders know the past net trades $(\tilde{w}_1, \ldots, \tilde{w}_{n-1})$. Let

$$\begin{aligned} \tilde{x}_{in} &= X_{in}(\tilde{S}_i, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}, \tilde{w}_1, \ldots, \tilde{w}_{n-1}), \\ p_n &= P_n(\tilde{w}_1, \ldots, \tilde{w}_n), \end{aligned}$$

represent the optimal strategy of trader i and the optimal strategy of the market maker, respectively. Finally, let $X_i = (X_{i1}, \ldots, X_{iN})$ (for each i) and $P = (P_1, \ldots, P_N)$ represent the two vectors of strategy functions. Define the profit that accrues to informed trader i from period n on as:

$$\pi_{in}(X_1, \ldots, X_i, \ldots, X_M, P) = \sum_{k=n}^{N} (\tilde{v} - p_k)\tilde{x}_{ik}.$$

A Bayesian Nash equilibrium of the trading game is a $M + 1$ vector of strategies $(X_1, \ldots, X_M, P)$ such that:

- For any $i = 1, \ldots, M$, $n = 1, \ldots, N$ and for $X_i' = (X_{i1}', \ldots, X_{in}', \ldots, X_{iN}')$, we have:

$$\begin{aligned} & E[\pi_{in}(X_1, \ldots, X_i, \ldots, X_M, P) | \tilde{S}_i, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}, \tilde{w}_1, \ldots, \tilde{w}_{n-1}] \\ \geq\ & E[\pi_{in}(X_1, \ldots, X_i', \ldots, X_M, P) | \tilde{S}_i, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}, \tilde{w}_1, \ldots, \tilde{w}_{n-1}]. \end{aligned}$$

The optimal strategy function for informed trader i is best no matter which past strategies i may have played.

- Given the conditional expected profit of the market makers at trading round n ($E\left[(p_n - \tilde{v})\tilde{w}_n | \, \tilde{w}_1, \ldots, \tilde{w}_n\right]$) and perfect competition between market makers, for all $n = 1, \ldots, N$, we have:

$$p_n = E[\tilde{v}|\tilde{w}_1, \ldots, \tilde{w}_n]. \tag{3.1}$$

Also, we define the variance of the price error Σ_n, a measure of price informativeness, at auction n:

$$\Sigma_n = var[\tilde{v}|\tilde{w}_1, \ldots, \tilde{w}_n]. \tag{3.2}$$

We look for a linear Bayesian Nash equilibrium based on a dynamic programming argument. Note that the strategy of informed trader i at auction n is required to be the optimal strategy, not only when trader i plays his optimal strategy in the first $n-1$ periods. Furthermore, as in Foster and Viswanathan (1996), there are no off equilibrium observations of order flow by the other informed traders in the model as every order flow path is possible.

We now derive the following proposition which provides the different parameters of the equilibrium.

Proposition 1 *If $\Sigma_N > \frac{\sigma_\epsilon^2}{M}$ there exists a unique linear equilibrium with noisy private information in which the demand function of informed trader i at auction n and the price function at auction n are respectively equal to*[5,6]*:*

$$\hat{x}_{i,n} = \alpha_n \Delta t \tilde{S}_i + \beta_n \Delta t \hat{p}_{n-1}, \tag{3.3}$$

[5]Expressing equilibrium condition as a function of Σ_N is equivalent to express the same condition as a function of Σ_0. Indeed, we solve our equilibrium by a process of backward induction i.e. we set the value of Σ_N, then we compute $\Sigma_{N-1} \ldots \Sigma_0$. Hence, we can write Σ_0 as a bijective function of $= \Sigma_N$ -we observe numerically that Σ_0 is a strictly increasing function of Σ_N- and the equilibrium condition could be interpreted as a condition on Σ_0.

[6]If $\sigma_\epsilon^2 = 0$,we are in the Holden and Subrahmanyam (1992) model, the results in this case are presented in the appendix.

$$\hat{p}_n = \hat{p}_{n-1} + \lambda_n(\Delta X_n + \Delta u_n), \tag{3.4}$$

where the parameters are defined by the following equations

$$\alpha_n \Delta t = -\frac{2(k_{3n} - \frac{1}{2}k_{4n})\lambda_n - a_n}{(M+1)\lambda_n - 2M(k_{3n} - \frac{1}{2}k_{4n})\lambda_n^2} + \frac{(M-1)\psi_n}{M}\frac{(1-a_n)(1-2\lambda_n(k_{3n} - \frac{1}{2}k_{4n}))}{(M+1)\lambda_n - 2M(k_{3n} - \frac{1}{2}k_{4n})\lambda_n^2}, \tag{3.5}$$

$$\beta_n \Delta t = -\alpha_n \Delta t, \tag{3.6}$$

$$\lambda_n = \frac{M\alpha_n \Delta t \Sigma_{n-1}}{(\alpha_n \Delta t)^2 M^2 \Sigma_{n-1} + \sigma_u^2 \Delta t + M(\alpha_n \Delta t)^2 \sigma_\epsilon^2}, \tag{3.7}$$

$$\Sigma_n = var[\tilde{v}|\tilde{w_1}, \ldots, \tilde{w_n}] = \frac{\Sigma_{n-1}\left(\sigma_u^2 \Delta t + M(\alpha_n \Delta t)^2 \sigma_\epsilon^2\right)}{(\alpha_n \Delta t)^2 M^2 \Sigma_{n-1} + \sigma_u^2 \Delta t + M(\alpha_n \Delta t)^2 \sigma_\epsilon^2}, \tag{3.8}$$

with $a_n = \frac{\Sigma_n}{\Sigma_n + (1-\psi_n)\sigma_\varepsilon^2}$, $\psi_n = M\lambda_n \alpha_n \Delta t$ *and*

$$\delta_{n-1} = \delta_n + \lambda_n^2 k_{3n}[\sigma_u^2 \Delta t + (\alpha_n \Delta t)^2 (M-1)(1 + (M-1)a_n)\sigma_\epsilon^2]. \tag{3.9}$$

Trader i's value function is given by

$$E\left[\pi_{in} \left| \hat{p}_0, ..., \hat{p}_{n-1}, \tilde{S}_i \right.\right] = k_{1,n}\left(\tilde{S}_i - \hat{p}_{n-1}\right)^2 + \delta_n. \tag{3.10}$$

The coefficients k's are solving the following system of equations (fully defined in the Appendix)

$$k_{n-1} = Ak_n + C, \tag{3.11}$$

where k_{n-1}, k_n *are matrices of dimension* 6×1, A *is a matrix with dimension* 6×6 *and* C *is of dimension* 6×1. *All matrices are defined in the Appendix. The parameters are subject to the following boundary conditions*

$$\delta_N = k_{1,N} = k_{2,N} = k_{3,N} = 0, \tag{3.12}$$

$$\alpha_N \Delta t = \frac{a_N}{\lambda_N\left(2 + (M-1)\, a_N\right)}. \tag{3.13}$$

Proof: See Appendix.

The necessary condition is a learning process condition. It means that the trading process continues as long as informed traders still have some private information not yet incorporated in the market maker's information set. It also shows that the precision of the market maker's information is limited by the level of noise contained in the traders' signals. The market makers' estimate of the liquidation value can be written as:

$$\hat{v} = \frac{1}{M}\sum_{i=1}^{M} \tilde{S}_i = \tilde{v} + \frac{1}{M}\sum_{i=1}^{M} \tilde{\epsilon}_i. \tag{3.14}$$

Suppose that the traders' signals $(\tilde{S}_i)_{1\leq i\leq M}$ are in the information set of the market maker. In this case, the market maker is able to know the liquidation value $\tilde{v}$ with a precision measured by the inverse of the variance of the random variable $\frac{1}{M}\sum_{i=1}^{M} \tilde{\epsilon}_i$. That precision is then equal to $\frac{\sigma_\epsilon^2}{M}$ and represents the best precision of her estimate of the liquidation value $\tilde{v}$. The last error variance of price, Σ_N, is then greater than $\frac{\sigma_\varepsilon^2}{M}$. As a consequence, the level of noise, as measured by the variance of the noise σ_ε^2 cannot be too high.

Moreover, we numerically find that if we increase the frequency of trading N, Σ_N decreases for a given level of Σ_0. This is due to the fact that increasing the frequency of trading intensifies the competition between traders leading to more information being released. As a consequence, the frequency of trading is limited by the level of noise. The higher the frequency of trading, the lower the level of noise. Regarding the effect of the number of insiders M, it is not as clear. Increasing the number of insiders M decreases the lower bound of the necessary condition. However, it also intensifies the competition between traders and so lead to lower values of Σ_N. Hence, we cannot deduct analytically the effect of increasing the number of informed traders.

By proceeding by backward induction one determines the individual orders for each auction. There is then a link between the last error variance of price Σ_N and the initial one Σ_0 at the opening of the sequential auctions market. Choosing Σ_0 is therefore equivalent to setting Σ_N to a certain value. To illustrate the properties of our model, we compute

the linear equilibrium parameters for different settings.

All the results in the following sections are obtained numerically.

4 Numerical Results

We now illustrate our model with numerical simulations. In order to compare the different results we choose similar numerical settings to those of Holden and Subrahmanyam (1992), Foster and Viswanathan (1996) and Back et al. (2000). The results are simulated for a fixed initial value of Σ_0.

4.1 Informativeness and Liquidity

We are interested in how prices aggregate the different pieces of private information held by informed traders. In the next result, we study the informativeness of prices. We show that it is tightly linked to the noise in the traders' signals.

Numerical result 1 *The informativeness of prices ($\frac{1}{\Sigma_n}$) increases as the level of noise in the traders' signal decreases. Moreover, as the noise decreases the earlier the private information is incorporated into prices.*

In our model, the conditional correlation between the signals of the informed market participants cannot be negative. As a consequence, traders trade on the same side of the market. Nevertheless, that competition is softened as traders have noisy signals. We can compare our model to that of Foster and Viswanathan (1996) by looking at the correlation between the signals. The correlation between the informed agents' private signals, i and j at time n, is given by:

$$corr(\tilde{S}_i, \tilde{S}_j)_n = \frac{\Sigma_n}{\Sigma_n + \sigma_\epsilon^2} \quad for \quad i \neq j. \tag{4.1}$$

It can be seen from this expression that the value of σ_ϵ^2 impacts the correlation between two signals. This correlation affects the traders' behavior, which in turn impacts price informativeness. However, the variance of the noise also affects the traders' behavior

independently from its effect on the correlation. Noisier (more precise) signals also implies that informed agents, ceteris paribus, trade more (less) on noise reducing (increasing) price informativeness.

In Figures 1 and 2, we represent the error variance of prices over time for different values of the correlation between the signals of the informed traders (given by σ_ϵ^2).

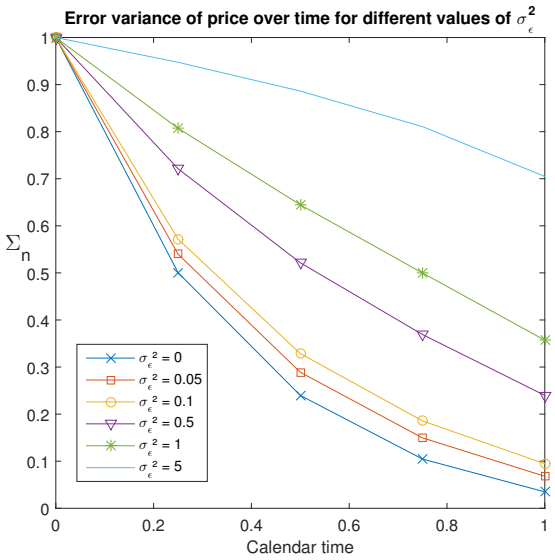

Figure 1: *The error variance of prices* (Σ_n) *over time,* $M = 2$, $N = 4$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

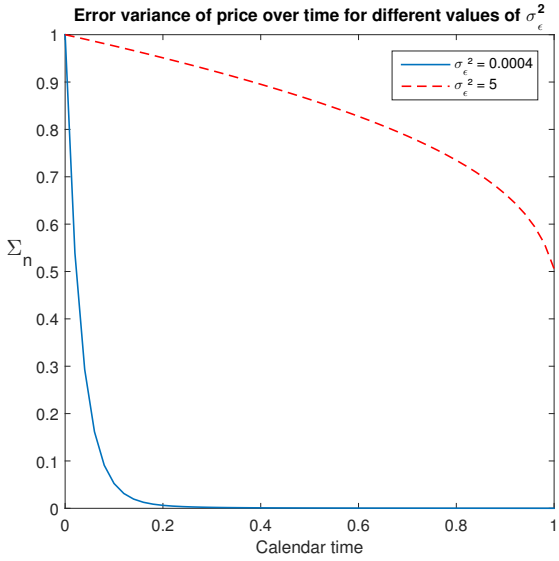

Figure 2: *The error variance of prices* (Σ_n) *over time,* $M = 2$, $N = 50$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

Figure 1 shows that the error variance of price Σ_n decreases more slowly when the variance of the noise, σ_ϵ^2, is large. In other words, the noisier the signal the less information is revealed during the periods of trading. Figure 1 also shows that, when the trader's private information is not too noisy, investors reveal more of their private information in the early auctions whereas the opposite is true when private information is very noisy. This result is consistent with Ottaviani and Sørensen (2006). Indeed, they show that insiders participate to the earliest bets on the basis of their common information as they do not wish to hide information that is already available to all. Therefore, information is quickly revealed to the market. Conversely, when the insiders' private information is very noisy, the trader/bettor tries to conceal this part by delaying his bets to the end of the betting session. In our

model, when traders have noisier signals the error variance of prices decreases more slowly. In this case, each market participant has some information that is unique.

In the next result, we study the market liquidity.

Numerical result 2 *The liquidity* ($\frac{1}{\lambda_n}$) *is non monotonic with* σ_ϵ^2. *It increases over time when the noise is small whereas it decreases when the noise is large.*

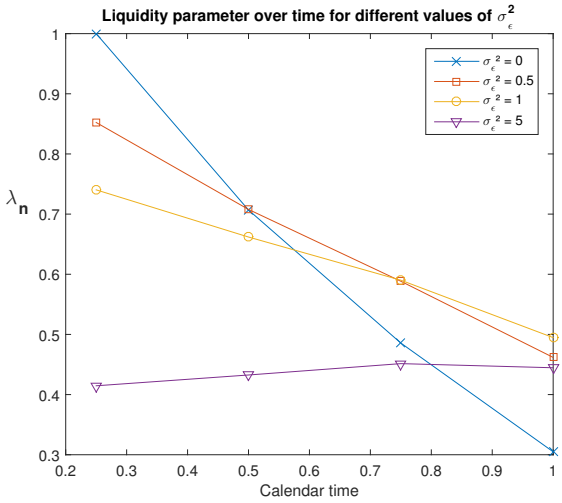

Figure 3: *The liquidity parameter* (λ_n) *over time,* $M = 2$, $N = 4$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

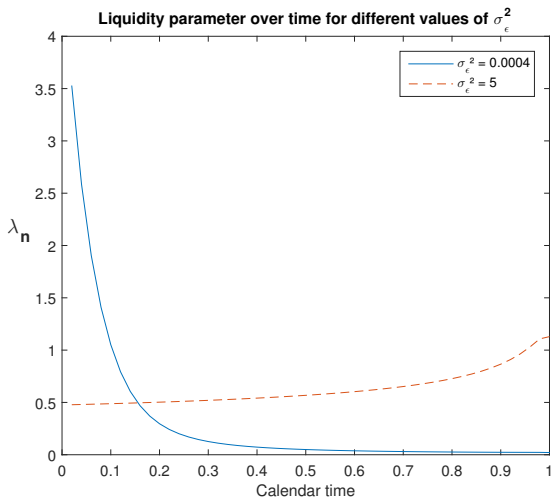

Figure 4: *The liquidity parameter* (λ_n) *over time,* $M = 2$, $N = 50$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

Figure 3 displays the dynamic of the liquidity parameter as a function of the noise. It shows that as more information is incorporated into prices, the less aggressively the market maker prices the asset. It also shows that, when there is a large level of noise in the private signals, the market maker's sensitivity to order flow, λ_n, increases slowly over time. This is due to the fact that informed traders delay their trades to the last auctions and do not reveal a large part of their private information. Hence, the market maker does not learn much about the liquidation value of the asset in the early periods of trading. She then reacts more aggressively to the order flow that appears in the last periods of trading. These results are shown numerically in Figures 3 and 4.

We now focus on the link between the informativeness of prices and the number of auctions or trading rounds. Figure 5 shows the informativeness of prices for different values of the number of auctions. In order to guarantee the existence of an equilibrium for the different parameter configurations, we take $\sigma_\epsilon^2 = 0.02$.

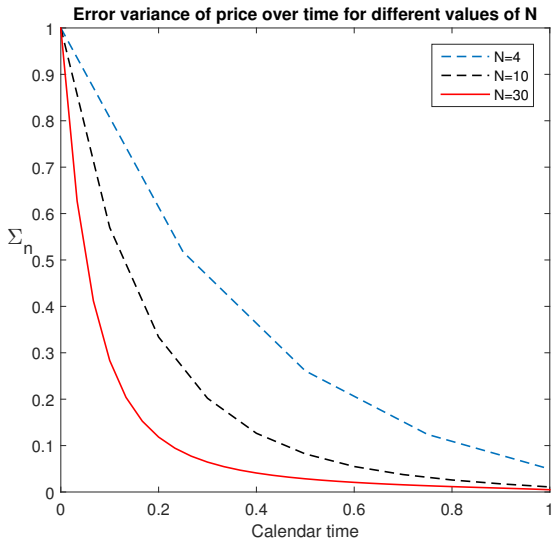

Figure 5: *The error variance of prices (Σ_n) for different values of N, $M = 2$, $\sigma_\epsilon^2 = 0.02$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

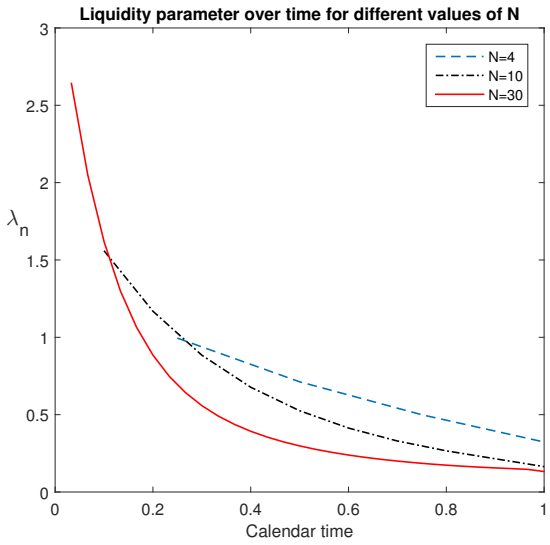

Figure 6: *The liquidity parameter (λ_n) for different values of N, $M = 2$, $\sigma_\epsilon^2 = 0.02$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

Hence, one can observe that for a fixed level of noise, σ_ϵ^2, the informativeness of prices increases with the number of trading rounds. Similarly, Figure 6 shows that the adverse selection problem (measured by the parameter λ_n) decreases with the number of auctions as one approaches the end of the trading day. It also shows that the larger the number of auctions, the higher the price is at the first trading round. These results are consistent with Vayanos (2001). Indeed, Vayanos (2001) shows that when the time between trades is small, in other words when the number of auctions is high, the insider trades aggressively at the earliest auctions in order to quickly achieve his optimal stock holdings. In his model, the insider trades for allocational motives. However, this case has some similarities with the informational motives of the insiders in our model. Indeed, in Vayanos (2001) the

strategic large trader knows that his sales lead to a drop in the price.

Figures 7 and 8 show the links between the informativeness of prices and the number of traders as well as the link between liquidity and the number of traders. One can see that for a fixed level of noise, the informativeness of prices increases with the number of traders. We also observe in Figure 8 that the liquidity parameter λ_n decreases with the number of traders. We can then conclude that increasing the number of traders and/or trading rounds boosts the competition between traders and so leads to the release of more information.

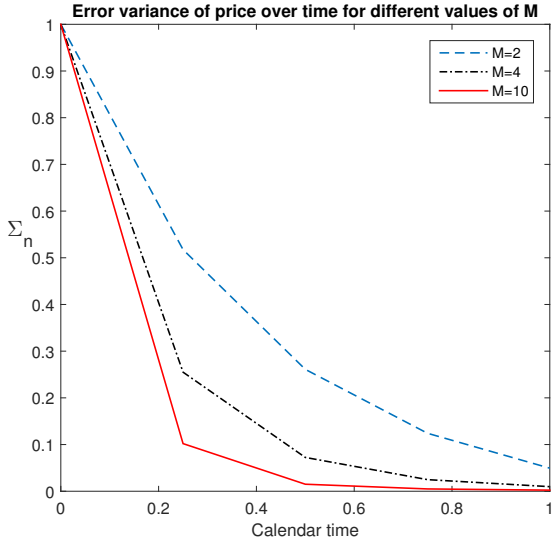

Figure 7: *The error variance of prices* (Σ_n) *for different values of* M, $N = 4$, $\sigma_\epsilon^2 = 0.02$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

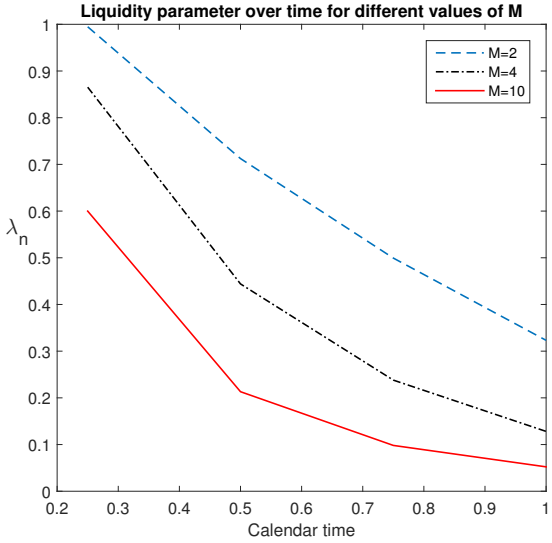

Figure 8: *The liquidity parameter* (λ_n) *for different values of* M, $N = 4$, $\sigma_\epsilon^2 = 0.02$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.

We now study the reaction of the traders to their private and public information, and present the different regimes of competition between traders.

4.2 Competition: The Rat Race and the Waiting Game

We are interested in the effect of the noise on the competition between informed traders. In the next result, we study the reaction of informed traders to their public and private

information. At this point it should be reminded that we have previously obtained in the proposition that $\beta_n \Delta t = -\alpha_n \Delta t$. As a result if the trading intensity on private information ($\alpha_n \Delta t$) increases or decreases the trading intensity on public information ($\beta_n \Delta t$) will do the same in absolute value.

Numerical result 3 *The informed market participants react more to their private information as time elapses. When the level of noise is low, the informed traders react aggressively to their private information and increase significantly their orders in the last periods of trading. The reaction of informed traders becomes less agressive when the level of noise increases.*

These results are shown in Figures 9 and 10.

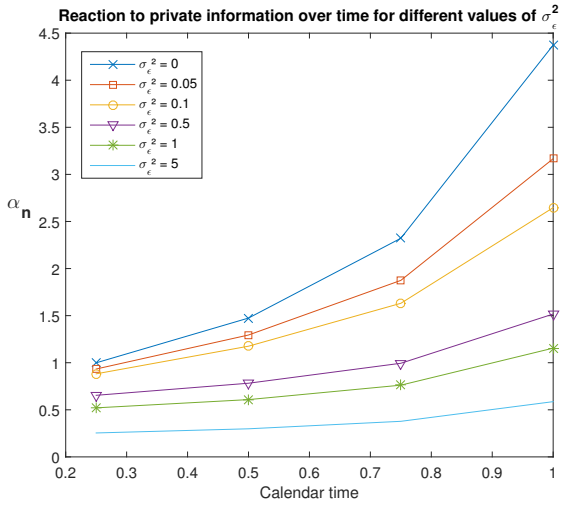

Figure 9: *The reaction of an informed investor to his private signal (α_n) over time, $M = 2$, $N = 4$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

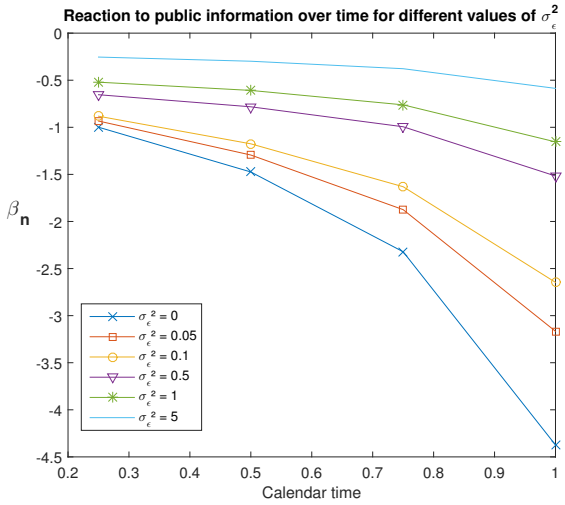

Figure 10: *The reaction of an informed investor to the prices (β_n) over time, $M = 2$, $N = 4$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

Figures 9 and 10 show that the informed traders trade gradually more aggressively on both their private and public information. In the explanations below we focus on how traders react to their private information. It can be seen that the more precise their signal the more aggressive the traders are. This aggressive trading is what we call a **rat race**.

When the level of noise is low, we observe this rat race during all the periods of trading with a greater intensity closer to the end of the trading day. As the level of noise increases, this trading aggressiveness decreases. When private information is very noisy, traders limit the size of their trades during the early periods of trading, we call that a **waiting game**. However, we still obtain that in the last periods of trading traders intensify their trading. This can be explained as follows. Firstly, as time gets closer to the end of the trading day informed traders have less scope to use their private information. Secondly, the impact of their trades has less long lasting effect on the liquidity. The intensity of the traders' trading decreases with σ_ε^2 (this also includes the traders' behavior at the late auctions). As said before, at the early auctions we observe that traders are not comparatively aggressive in their trading. As the noise in their information is not too high, traders refrain from trading too early as trading aggressively too early would lead to their private information being incorporated in the price early. However, when the level of noise is very high, the competition between informed traders is reduced (expression (4.1) is close to zero). Indeed, traders have more dispersed initial private information and traders enjoy a position close to a monopolistic one on their piece of information. They then trade accordingly on that information. This limits the competition between traders during all the periods of trading.

The waiting game observed in our model is not due to a negative correlation between the signals as a consequence of trading. We show that the waiting game phase appears when the correlation between the signals of the traders is low - but positive. In our model, increasing the level of noise in the traders' signals decreases the correlation between these signals ($Corr(\tilde{S}_i, \tilde{S}_j)_n \approx 0$). Hence, in this case, each trader considers that the information from other traders is completely uncorrelated to the true value of the asset. Therefore, he limits his orders during the early periods of trading in order to not reveal his private information and waits for the last periods to submit more significant orders.[7]

[7] In this case, we can compare our model to the one of Kyle (1985), since each informed trader considers other traders as noise traders, and so follows a strategy comparable to that observed in Kyle (1985).

Figures 11 and 12 show that, for a low level of noise, we only observe a rat race during all the periods of trading: we can see from these figures that the traders' reaction to their information increases rapidly during all the periods of trading. We also observe that the slope of the parameter α_n (which measures the intensity of competition) increases during all the periods of trading and more significantly at the last periods. This result generalizes the findings of Kyle (1985) and Holden and Subrahmanyam (1992) in the case of signals with low levels of noise. For an intermediate level of noise, we observe a waiting game that lasts most of the auctions, though the trading intensity dramatically increases in the last trading rounds. However, the intensity of that rat race decreases with the level of noise. Hence, for very high levels of noise, we only observe a waiting game that lasts for all the periods of trading.[8] One difference with Foster and Viswanathan (1996) is the order in which the two game stages can appear.

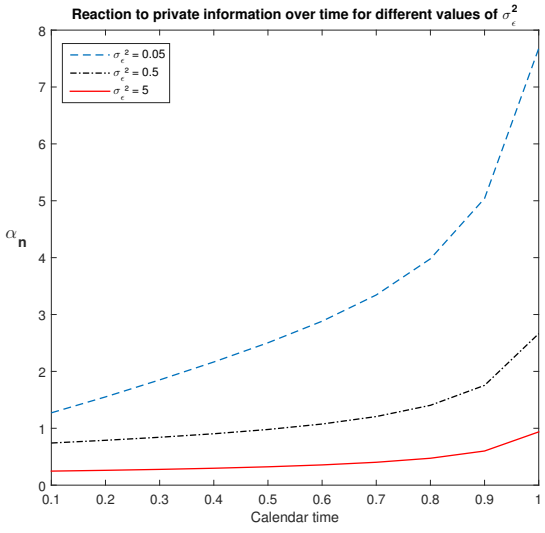

Figure 11: *Reaction of an informed investor to his private signal (α_n) over time, $M = 2$, $N = 10$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

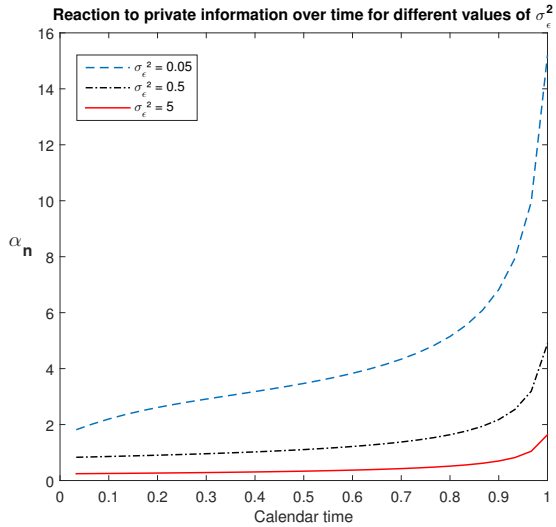

Figure 12: *Reaction of an informed investor to his private signal (α_n) over time, $M = 2$, $N = 30$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

[8] In fact, we always observe a rat race at the last auctions. However, the intensity of this rat race decreases with the level of noise and becomes difficult to observe when the level of noise is too high.

When we analyze the effect of increasing the number of auctions on the previous result, we obtain the following results. For a very low level of noise and when increasing the number of auctions, we still have a rat race during all the periods of trading.[9] The intensity of that rat race increases with the number of auctions.[10] When increasing the number of auctions for a high level of noise, a waiting game takes place for most of the auctions however with an increased trading intensity. We also obtain that increasing the number of auctions increases the intensity of the rat race observed in the last auctions.

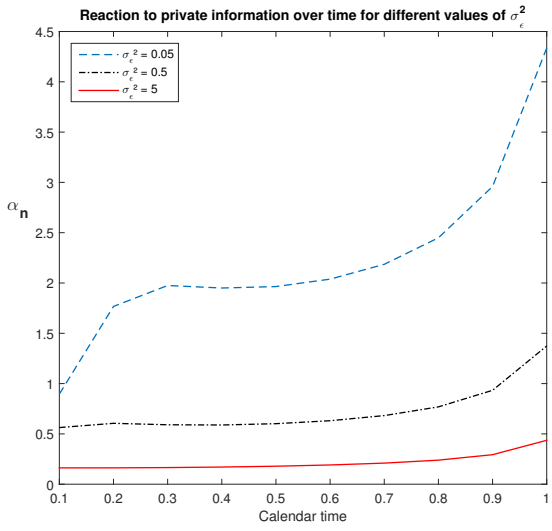

Figure 13: *Reaction of an informed investor to his private signal (α_n) over time, $M = 10$, $N = 10$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

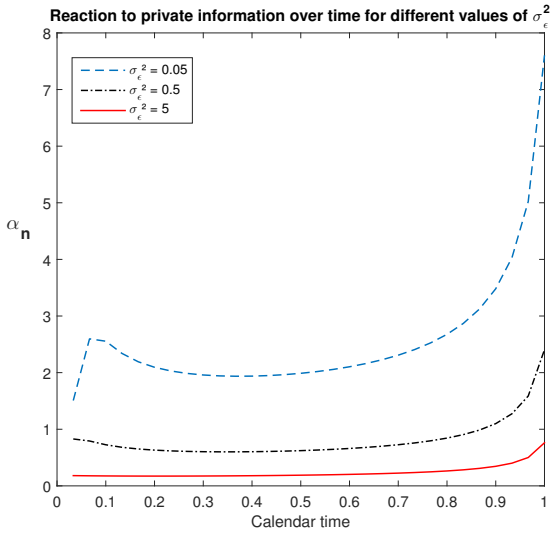

Figure 14: *Reaction of an informed investor to his private signal (α_n) over time, $M = 10$, $N = 30$, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

When we analyze the effect of increasing the number of insiders on the previous result, we obtain the following results when comparing Figure 11 with Figure 13 and Figure 12 with Figure 14. For a very low level of noise, we still have one phase only, i.e. the rat race.

[9] The simulations show that the range of σ_ϵ^2 for which this result is satisfied becomes smaller and closer to 0 when the number of periods increases.

[10] Numerically, we observe higher final values of α_n when we increase the number of auctions N, and keep σ_ϵ constant.

The results show that the intensity of this rat race increases with the number of insiders.[11] For a high level of noise, the competition is distorted as follows. We have an early and a late rat race with a waiting game occurring between those two rat races. The intensity of the early rate race increases with the number of insiders as well as with the number of auctions. It also increases when signals become more precise (see Figures 13 and 14). The intensity of the final rat race decreases with the number of traders.

4.3 Expected Profits

In this section, we are interested in understanding how the competition between the insiders affects their profits.

Numerical result 4 *The insiders' profits evolve as follows:*

1. *When the competition is low, measured by both M and N i.e. when $M = 2$ and $N < 7$, or $M = 3$ and $N < 3$, the introduction of the noise in the traders' signals reduces the profits.*

2. *When the level of competition is high ($M = 2$ and $N \geq 7$, or $M = 3$ and $N \geq 3$ or $M \geq 4$ and for any N), the traders' profits are non-monotonic with the level of noise. More precisely, the profits intially increase with low level of noise and then decrease with it for high value of the noise.*

These results can be explained as follows. Introducing noise in the traders' signals diminishes the intensity of the competition and so allows the traders to get greater profits (Figure 16). However, too much noise decreases the trader's trading intensity in such a way that traders switch to a waiting game and so diminishes the profits. Noise acts as a commitment not to trade. When the competition is low (measured by both N and M), only the negative effect of the noise is present (Figure 15).

[11] These results are obtained for a range of very low levels of noise, this range gets narrower when the number of insiders increases.

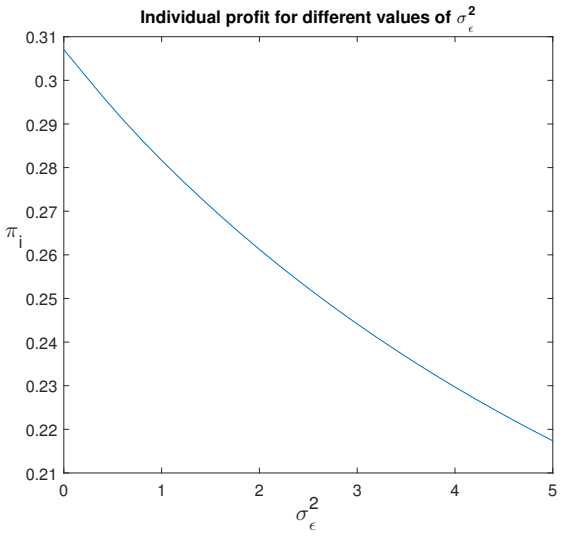

Figure 15: *The individual profit (π_i) as a function of σ_ε^2 for $M = 2$, the number of insiders, $N = 6$, the number of auctions, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

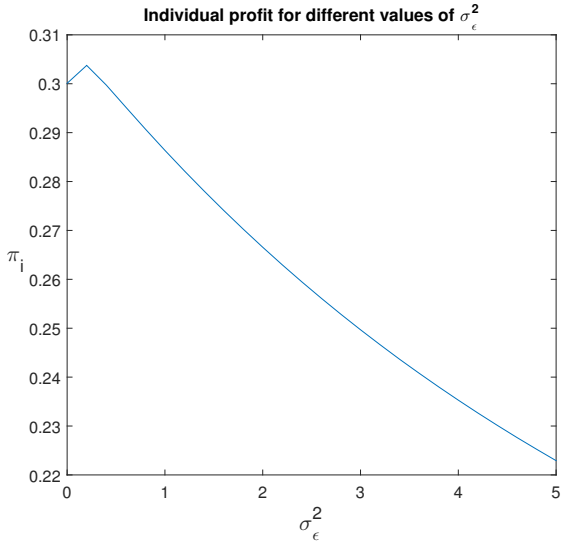

Figure 16: *The individual profit (π_i) as a function of σ_ε^2 for $M = 2$, the number of insiders, $N = 7$, the number of auctions, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

Generally speaking, introducing noise in the traders' signals may be a way to circumvent the Grossman and Stiglitz (1980) paradox. [12] For instance, Germain (2005) shows that when a very large number of sellers of information enter the market they can endogenize the amount of noise in such a way that they still not reveal their private information. Thus, Germain (2005) suggests that fund management activities create an endogenous noise on top of the noise from the liquidity trades.

4.4 Optimal Noise

In this section, we look at the optimal level of noise, i.e. the value of σ_ϵ^2 maximizing the informed traders' expected profit. Our previous results show that the presence of noise in

[12] Indeed, in efficient markets as prices reflect all information available, there is no incentive for agents to collect costly information. In this case one cannot recover the cost of acquiring information as prices are fully revealing. This leads to a paradox as investors will never collect information and prices will not be informative.

the private signals may reduce the level of competition between informed traders. This would lead to higher expected profits.

Numerical result 5 *When the number of insiders is relatively high, there exists an optimal level of noise maximizing the expected profits of the traders. This level increases with the number of insiders, and the optimal individual profit decreases with M.*

These results are shown in Figures 17 and 18.

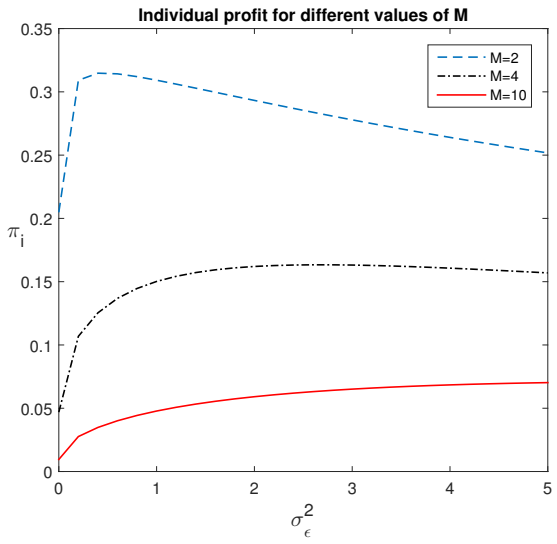

Figure 17: *The individual profit (π_i) as a function of σ^2_ε for $N = 20$, the number of auctions, $\sigma^2_u = 1$, $\Sigma_0 = 1$.*

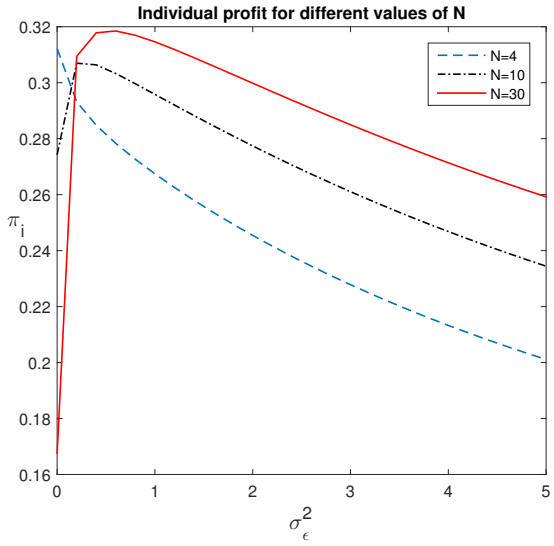

Figure 18: *The individual profit (π_i) as a function of σ^2_ε for $M = 2$, the number of insiders, $\sigma^2_u = 1$, $\Sigma_0 = 1$.*

Figures 17 and 18 show the individual profit for different values of the number of insiders. We obtain that the optimal level of noise, i.e. the level of noise maximizing the traders' expected profit, increases with the number of trading rounds. We also observe that the optimal individual profit, computed as the profit obtained from the first auction to the last and evaluated at the optimal noise, decreases with the number of insiders. This result is similar to the one obtained by Dridi and Germain (2009).

The next result looks at the effect of increasing the number of trading rounds and informed traders on the traders expected profits.

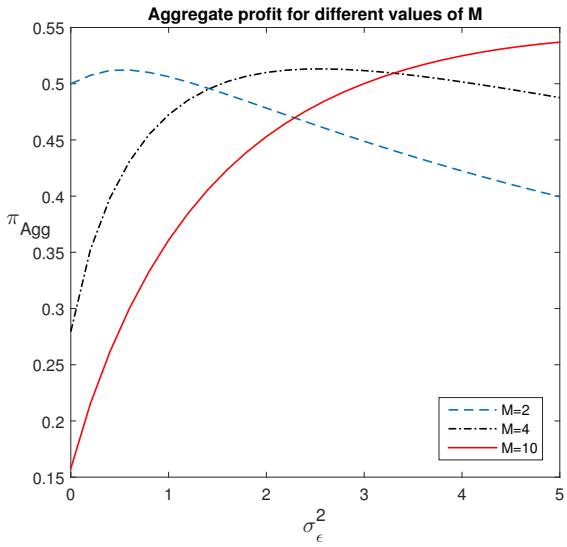

Figure 19: *The aggregate profit (π_{Agg}) as a function of σ_ε^2 for $N = 4$, the number of auctions, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

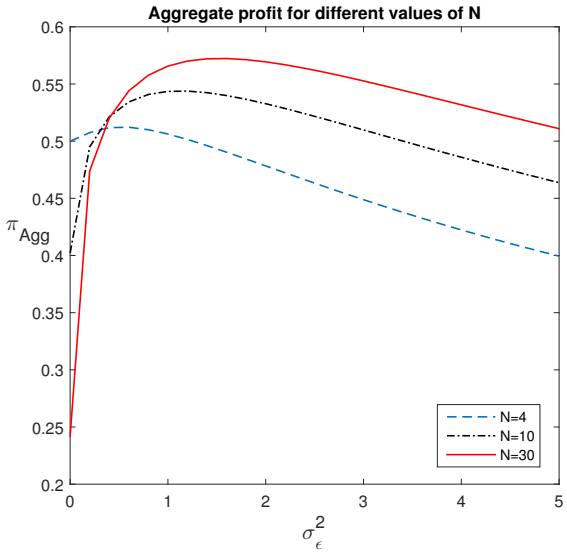

Figure 20: *The aggregate profit (π_{Agg}) as a function of σ_ε^2 for $M = 2$, the number of insiders, $\sigma_u^2 = 1$, $\Sigma_0 = 1$.*

Numerical result 6 *The optimal level of noise evolves as follows:*

1. *The optimal level of noise increases with the number of auctions. For a fixed level of noise, the individual profits decrease with the number of auctions N for low levels of noise, whereas it increases with the number of auctions for high levels of noise.*

2. *The optimal level of noise increases with the number of informed traders.*[13]

These results are shown numerically in Figures 19 and 20.

The previous results can be explained as follows: as we increase the number of auctions, the informed traders scope for positive profit increases. However, when the level of noise is low, the profit decreases with the number of auctions since, in this case, the traders' private information is quickly incorporated into the price which then converges quickly to the true value of the asset. When the level of noise is relatively high, less information is revealed to the market maker and so the price does not converge as quickly to the true

[13] We observe numerically that the optimal level of noise increases with N slower than with M.

value of the asset allowing the traders to obtain larger profits.

5 Practical Implications

Our paper has some policy implications as well as some empirical ones.

Numerical result 2 shows that the liquidity is increasing with the number of auctions. This result shows that if we compare across financial markets that differ in the number of trading rounds (for instance between batch auction market and continuous markets), markets offering traders more possibilities to trade (larger number of trading rounds) will display the highest liquidity. This is illustrated in Figure 6. Numerical result 2 also shows that price informativeness increases with the number of auctions. These two results imply that if the policy maker is interested in having a market with a high level of liquidity and that prices reveal private information, then an obvious choice would be to design a market close to a continuous market.

By looking at how informed traders trade on their private information, we obtain that the trader's response to private information is not smooth i.e. the trader's intensity may be high at the very beginning of the trading day then may decrease and stay low for quite a while and towards the end of the trading day may sharply increase again. The pattern described above occurs when the number of trading rounds and/or the number of informed traders are large and the private information held by traders is very precise. Our model predicts that this trading pattern results from the competition between traders. Some papers find that this trading behavior may result from changes in liquidity during the trading day. We prove that this behavior may not be due to changes in the liquidity but due to the numbers of competing traders, the numbers of trading rounds and the precision of private information. This change in the trading behavior implies pattern in the volume observed in the market. The relationship established in our paper could be empirically tested.

Finally in the section on optimal noise and more precisely in numerical result 6 we show

that the optimal noise (i.e. the noise in the private information maximizing the traders' expected profit) is a function of the number of trading rounds and informed traders. We prove that this optimal noise increases with the number of trading rounds and the number of traders. As competition increases the noise maximizing expected profits increases. This result shows that the value of information is closely related to the market where that information is subsequently being used. In a highly competitive market, informed traders would be willing to pay a higher price for a noisier information. Following that, companies specialized in the sale of information could introduce that in their pricing. This relationship between the value of information and the level of competition could be empirically investigated.

6 Conclusion

This article analyzes the introduction of heterogeneous noisy signals when strategic insiders compete in a multi-auction market. We derive the unique linear equilibrium and its properties. We find that the existence of an equilibrium is not always guaranteed. We show that its existence is tightly linked to the existence of private information not yet incorporated into prices. As such, the existence condition implies a negative relationship between the number of auctions and the noise in the traders' private signal. The existence of the equilibrium is guaranteed when the competition is limited through noisy signals.

Our model enables us to analyze the trade-off between noise and competition as in Dridi and Germain (2009). We show that when the competition is strong (the number of informed traders and/or the number of auctions is high), noisy information can reduce the intensity of the competition between insiders and can increase their expected profits. In that case noise acts as a commitment not to trade. When the number of informed traders and the number of trading rounds are low, the introduction of some noise in the traders' signals always leads to a drop in their expected profits.

As a consequence to the effect of the noise on competition, the model also investigates

the optimal level of noise i.e. the level of noise maximizing the expected profits of the traders. We obtain that this level increases with the number of traders and the number of auctions. This leads to the fact that traders would be willing to pay for a noisier private signal if they use that information in a more competitive market.

Furthermore, when the level of the noise is small leading to a strong correlation between signals, the competition between traders takes the form of a rat race during all the periods of trading. However, as we increase the level of noise, a waiting game phase appears during the early periods of trading, and the intensity of the rat race of the last auctions decreases. Hence, when the level of the noise is too large (implying that the correlation is weak) we only observe a waiting game. This result is in sharp contrast with Foster and Viswanathan (1996).

We also observe that when increasing the scope for traders to use their information (increasing the number of trading rounds), traders may trade very aggressively at the opening and closure of the trading day and decrease their trading intensity in between. This pattern is observed in financial markets. Our paper explains it as a consequence of the number of trading rounds, the number of traders and the level of the noise.

Our paper broadly agrees with the findings of Kyle (1985) where the traders gradually incorporates their information into price. We find that traders trade very aggressively at the last auction, as in Kyle (1985), however we do observe an auction where they refrain from trading and even decrease their trading intensity. One empirical prediction of this model is that changes in volume during the trading day can be explained by the presence of noise in the information of the traders when they compete in the market.

7 Appendix

Proof of Proposition 1

The proof involves four steps. We will start by resolving the dimensionality issue (we avoid the problem of increasing state history with time) when all traders follow their

optimal strategies. In the second step, we resolve the dimensionality issue when one trader deviates from his optimal strategy, conjecture the value function and then obtain the first order condition (FOC) that determines the equilibrium. In the third step, we show with a lemma that, at the equilibrium, the parameters of the demand function for insider i do not depend on i. In other words, at the equilibrium, all insiders have the same reaction to their private information (α_n) and to their public information (β_n). Finally, in the fourth step, we derive the insiders' backward induction program.

Step 1: The Dimensionality Issue

In this section we show how the dimensionality issue is resolved (i.e. we avoid the problem of increasing state history with time). As in Foster and Viswanathan (1996), we look at linear strategies for informed traders and learning by the market maker.

Consider trader i who is interested in forecasting the true value of the asset that is not predicted by the market after $n-1$ periods of trading, using his information $(\tilde{S}_i, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}, \tilde{w}_1, \ldots, \tilde{w}_{n-1})$. By equations (3.1) and (3.4), it can be shown that:

$$p_n = p_0 + \sum_{k=1}^{n} \lambda_k \tilde{w}_k.$$

Trader i's order for $r = 1, \ldots, n-1$ can be rewritten as:

$$\tilde{x}_{ir} = \alpha_{ir}\Delta t_r \tilde{S}_i + \beta_r \Delta t_r p_{r-1} = \alpha_{ir}\Delta t_r \tilde{S}_i + \beta_r \Delta t_r (p_0 + \sum_{k=1}^{r-1} \lambda_k \tilde{w}_k).$$

Following Foster and Viswanathan (1996), we obtain that trader i predicts $\tilde{v} - p_{n-1}$ as follows:

$$E[\tilde{v} - p_{n-1} | \tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] = a_n(\tilde{S}_i - p_{n-1}).$$

Because trader j submits an order of the form $\tilde{x}_{jn} = \alpha_{jn}\Delta t_n \tilde{S}_j + \beta_n \Delta t p_{n-1}$, trader i needs to predict $\tilde{S}_j$. This is done as follows: $E[\tilde{S}_j | \tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}] = a_n(\tilde{S}_i - p_{n-1}) + p_{n-1}$.

So $(\tilde{S}_i - p_{n-1}, p_{n-1})$ is a sufficient statistic for trader i to predict $\tilde{S}_j$, and there is no history-dependent hierarchy of forecasts. The dimensionality issue is resolved in our model when all traders submit their optimal orders. However, we must also consider deviations

from the optimal strategy by any one trader (keeping the behavior of other traders fixed). If trader i submits an arbitrary order sequence $(\tilde{x}_{i1}, \ldots, \tilde{x}_{in-1})$, which is different from the equilibrium orders (given by equation (3.3)), the sufficient statistics that we have computed need not be relevant.[14]

In the next step, we resolve the dimensionality issue when one trader deviates from his optimal strategy (keeping the strategies of other traders fixed) and find the necessary and sufficient conditions for the equilibrium. This finishes step 1 of the proof.

Step 2: Necessary and Sufficient Conditions For Equilibrium.

We proceed as in Foster and Viswanathan (1996) and get that

$$E[\tilde{v} - p_{n-1} | \tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] = a_n(\tilde{S}_i - \hat{p}^i_{n-1}) + \hat{p}^i_{n-1} - p_{n-1},$$

and

$$E[\tilde{S}_j | \tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] = a_n(\tilde{S}_i - \hat{p}^i_{n-1}) + \hat{p}^i_{n-1},$$

where $\hat{p}^i_n$ is the price that prevails at the n^{th} round of trading if trader i had followed the equilibrium strategy $(\hat{x}_{i0}, \ldots, \hat{x}_{in-1})$ at the first n periods of trading.

Hence, we find that $(\tilde{S}_i - \hat{p}^i_{n-1}, \hat{p}^i_{n-1} - p_{n-1}, \hat{p}^i_{n-1})$ is a sufficient statistic to forecast the liquidation value and the signals of other traders. We now conjecture the value function of trader i after $n-1$ to be:

$$\begin{aligned} E[\pi_{in} | p_1, \ldots, p_{n-1}, \tilde{S}_i] &= k_{1,n-1}\tilde{S}_i^2 + k_{2,n-1}\tilde{S}_i p_{n-1} + k_{3,n-1}p_{n-1}^2 + k_{4,n-1}p_{n-1}(\hat{p}^i_{n-1} - p_{n-1}) \\ &\quad + k_{5,n-1}\tilde{S}_i(\hat{p}^i_{n-1} - p_{n-1}) + k_{6,n-1}(\hat{p}^i_{n-1} - p_{n-1})^2 + \delta_{n-1}. \end{aligned}$$

We also conjecture the optimal strategy of a trader who has played an arbitrary strategy:

$$\tilde{x}_{ik} = \alpha_{ik}\Delta t_k \tilde{S}_i + \beta_k \Delta t_k p_{k-1} + \zeta_k \Delta t_k (\hat{p}^i_{k-1} - p_{k-1}).$$

[14] In particular $\tilde{S}_i - p_{n-1}$ is not orthogonal to $(\tilde{w}_1, \ldots, \tilde{w}_{n-1})$ since $p_{n-1} \neq E[\tilde{v} | \tilde{w}_1, \ldots, \tilde{w}_{n-1}]$ because trader i has not played his optimal strategy in the first $n-1$ rounds of trading.

One can consider the profit which is realized at the nth auction, and what remains to be gained from the next auction to the end of trading. This is given below:

$$E[\pi_{in}|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] = E[(\tilde{v}-p_n)\tilde{x}_{in}+\pi_{n+1}|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}].$$

The price at auction n is given by

$$p_n = p_{n-1} + \lambda_n(\Delta\tilde{X}_n + \Delta\tilde{u}_n),$$

with $\Delta\tilde{X}_n = \tilde{x}_{in} + \Delta X_n^*$ the aggregate order flow from the demand of the ith insider ($\tilde{x}_{in}$) and from the $M-1$ other informed participants (ΔX_n^*) at the nth auction.

Due to normality, we have the standard formula:

$$a_n = \frac{cov(\tilde{v}, \tilde{S}_i|\hat{w}_1^i, \ldots, \hat{w}_{n-1}^i)}{var(\tilde{S}_i|\hat{w}_1^i, \ldots, \hat{w}_{n-1}^i)} = \frac{cov(\tilde{v}, \tilde{v}+\tilde{\epsilon}_i|p_1, \ldots, p_{n-1})}{var(\tilde{v}+\tilde{\epsilon}_i|p_1, \ldots, p_{n-1})}.$$

Then, we obtain:

$$E[\tilde{v}|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] = \frac{\Sigma_{n-1}}{\Sigma_{n-1}+\sigma_\epsilon^2}(\tilde{S}_i-\hat{p}_{n-1}^i)+\hat{p}_{n-1}^i = a_n(\tilde{S}_i-\hat{p}_{n-1}^i)+\hat{p}_{n-1}^i,$$

with $a_n = \frac{\Sigma_{n-1}}{\Sigma_{n-1}+\sigma_\epsilon^2}$ and $\Sigma_{n-1} = var(\tilde{v}|\tilde{w}_1, \ldots, \tilde{w}_{n-1})$ being the error variance of price at the $(n-1)^{th}$ auction.

Knowing that the ith informed trader chooses his market order x_{in} that maximizes his future expected profit, we obtain the first order condition (FOC):

$$E[\tilde{v} - p_{n-1}|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] - \lambda_n E[\Delta X_n^*|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}]$$
$$E[k_{2,n}\lambda_n\tilde{S}_i + 2\lambda_n k_{3,n}p_n + k_{4,n}[\lambda_n(\hat{p}_n^i - p_n) - \lambda_n p_n] - \lambda_n k_{5,n}\tilde{S}_i - 2k_{6,n}\lambda_n(\hat{p}_n^i - p_n)$$
$$|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] - 2\lambda_n\tilde{x}_{in} = 0.$$

Moreover, we can directly derive the second order condition:

$$\lambda_n[1 - \lambda_n(k_{3n} - k_{4,n} + k_{6n})] > 0.$$

Given the linearity of the traders' market order, the aggregate order flow of the $j \neq i$ other informed participants is $\Delta X_n^* = \sum_{j\neq i}^M \alpha_{jn}\Delta t\tilde{S}_j + (M-1)\beta_n\Delta t p_{n-1}$, we have:

$$E[\textstyle\sum_{j\neq i}^M \alpha_{jn}\Delta t\tilde{S}_j + (M-1)\beta_n\Delta t p_{n-1}|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}] =$$
$$(a_n(\tilde{S}_i - \hat{p}_{n-1}^i) + \hat{p}_{n-1}^i)(\textstyle\sum_{j\neq i}^M \alpha_{jn}\Delta t) + (M-1)\beta_n\Delta t p_{n-1}.$$

This leads to the following expression for the FOC:

$$\tilde{S}_i[a_n - 2\lambda_n(1-\lambda_n(k_{3n}-\frac{1}{2}k_{4n}))\alpha_{in}\Delta t + \lambda_n(k_{2n}-k_{5n}) + \lambda_n a_n(2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1)$$
$$(\sum_{j\neq i}^{M}\alpha_{jn}\Delta t)] + p_{n-1}[-a_n + 2\lambda_n(k_{3n}-\frac{1}{2}k_{4n}) - (M+1-2M\lambda_n(k_{3n}-\frac{1}{2}k_{4n}))\lambda_n\beta_n\Delta t$$
$$-\lambda_n(1-a_n)(1-2\lambda_n(k_{3n}-\frac{1}{2}k_{4n}))(\sum_{j\neq i}^{M}\alpha_{jn}\Delta t)] + (\hat{p}^i_{n-1}-p_{n-1})[1-a_n$$
$$+(1-a_n)\lambda_n(2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1)(\sum_{j\neq i}^{M}\alpha_{jn}\Delta t) + 2\lambda_n(\beta_n+\zeta_n)\Delta t(\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1)$$
$$-2\lambda_n^2\beta_n\Delta t(M-1)(k_{6n}-\frac{1}{2}k_{4n}) + 2\lambda_n^2\zeta_n\Delta t(k_{6n}-\frac{1}{2}k_{4n}) - 2\lambda_n(k_{6n}-\frac{1}{2}k_{4n})] = 0.$$

By identification, we obtain the coefficients multiplied by $\tilde{S}_i$, p_{n-1} and $\hat{p}^i_{n-1}-p_{n-1}$ and solving for α_{in}, β_n, ζ_n we get

$$\alpha_{in}\Delta t = \frac{a_n+\lambda_n(k_{2n}-k_{5n})}{2\lambda_n[1-\lambda_n(k_{3n}-\frac{1}{2}k_{4n})]} + a_n\frac{2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1}{2[1-\lambda_n(k_{3n}-\frac{1}{2}k_{4n})]}(\sum_{j\neq i}^{M}\alpha_{jn}\Delta t), \tag{7.1}$$

$$\beta_n\Delta t = \frac{2(k_{3n}-\frac{1}{2}k_{4n})\lambda_n - a_n}{(M+1)\lambda_n - 2M(k_{3n}-\frac{1}{2}k_{4n})\lambda_n^2} - \frac{\lambda_n(1-a_n)[1-2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})]}{(M+1)\lambda_n - 2M(k_{3n}-\frac{1}{2}k_{4n})\lambda_n^2}\sum_{j\neq i}^{M}\alpha_{jn}\Delta t, \tag{7.2}$$

$$\zeta_n\Delta t = \frac{a_n - 1 + 2\lambda_n(k_{6n}-\frac{1}{2}k_{4n}) - (1-a_n)\lambda_n[2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1](\sum_{j\neq i}^{M}\alpha_{jn}\Delta t)}{2\lambda_n[\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1] + 2\lambda_n^2(k_{6n}-\frac{1}{2}k_{4n})} \tag{7.3}$$
$$-\frac{2\lambda_n[\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1] - 2\lambda_n^2(M-1)(k_{6n}-\frac{1}{2}k_{4n})}{2\lambda_n[\lambda_n(k_{3n}-\frac{1}{2}k_{4n})-1] + 2\lambda_n^2(k_{6n}-\frac{1}{2}k_{4n})}\beta_n\Delta t.$$

The first relationship needs to be solved for the α_{in} parameters. Let us define the following parameters

$$a' = \frac{a_n+\lambda_n(k_{2n}-k_{5n})}{2\lambda_n[1-\lambda_n(k_{3n}-\frac{1}{2}k_{4n})]}, \text{ and } b' = -\frac{a_n}{2}\frac{1-2\lambda_n(k_{3n}-\frac{1}{2}k_{4n})}{1-\lambda_n(k_{3n}-\frac{1}{2}k_{4n})}.$$

Given a' and b', between the parameters α_i can be rewritten as, for $i \neq j$

$$\alpha_i = a' + b'(\sum_{j\neq i}^{n}\alpha_j). \tag{7.4}$$

Step 3: The demand of the insiders.

The Lemma below gives the expression of the α_i parameters solving that relationship.

Lemma *Let a' and b' be two real numbers such as for $i \neq j$ the relationship* (7.4) *is verified then, if $b' \neq -1$ for $i = 1, \ldots, M$:*

$$\alpha_i = \frac{a'}{1 - b'(M-1)}.$$

Proof: We have the following M equalities

$$\begin{aligned}
\alpha_1 &= a' + b'(\alpha_2 + \ldots + \alpha_M), \\
\alpha_2 &= a' + b'(\alpha_1 + \alpha_3 + \ldots + \alpha_M), \\
&\vdots \\
\alpha_M &= a' + b'(\alpha_1 + \alpha_2 + \ldots + \alpha_{M-1}).
\end{aligned}$$

Let t be a real number such that $t = \sum_{i=1}^{M} \alpha_i$, by adding the M previous equalities, and solving for t we get:

$$t = \frac{Ma'}{1 - b'(M-1)}.$$

On the other hand, by considering the difference of the first two equalities we have:

$$\alpha_2 - \alpha_1 = b'(\alpha_1 - \alpha_2).$$

Hence, we obtain:

$$\alpha_2(1 + b') = \alpha_1(1 + b').$$

Then, if $b' \neq -1$, all the real numbers α_i are identical. Therefore, we can conclude for $i = 1, \ldots, M$:

$$\alpha_i = \frac{a'}{1 - b'(M-1)}.$$

It can be verified that the case where $b' = -1$ cannot happen due to the second order condition.

This ends the proof of the lemma.

By applying the lemma, we find the following expression of $\alpha_{in}\Delta t$ which is independent of i:

$$\alpha_{in}\Delta t = \alpha_n \Delta t = \frac{a_n + \lambda_n(k_{2n} - k_{5n})}{\lambda_n \left[2 + (M-1)a_n - 2\lambda_n(k_{3n} - \frac{1}{2}k_{4n})(1 + a_n(M-1))\right]}.$$

The expression of β_n is given by:

$$\beta_n \Delta t = \frac{2(k_{3n} - \frac{1}{2}k_{4n})\lambda_n - a_n}{(M+1)\lambda_n - 2M(k_{3n} - \frac{1}{2}k_{4n})\lambda_n^2} - \frac{M-1}{M}\frac{(1-a_n)(1-2\lambda_n(k_{3n} - \frac{1}{2}k_{4n}))}{(M+1)\lambda_n - 2M(k_{3n} - \frac{1}{2}k_{4n})\lambda_n^2}\psi_n,$$

with $\psi_n = M\lambda_n\alpha_n\Delta t$.

On the other hand, one obtains the relationship between the error variance of prices at the nth auction (Σ_n) and the error variance of prices at the $(n-1)th$ auction. Indeed:

$$\Sigma_n = var[\tilde{v}|\tilde{w}_1, \ldots, \tilde{w}_n] = var[\tilde{v}|\tilde{w}_1, \ldots, \tilde{w}_{n-1}] - \frac{cov(\tilde{v}, \tilde{w}_n)^2}{var(\tilde{w}_n)}.$$

Hence, one obtains:

$$\Sigma_n = \Sigma_{n-1} - \lambda_n cov(\tilde{v}, \tilde{w}_n) = (1 - M\lambda_n\alpha_n\Delta t)\Sigma_{n-1} = (1-\psi_n)\Sigma_{n-1}.$$

Since Σ_n is positive, we must have that $\psi_n < 1$.

The parameter a_n can be written as:

$$a_n = \frac{\Sigma_{n-1}}{\Sigma_{n-1} + \sigma_\epsilon^2} = \frac{\Sigma_n}{\Sigma_n + (1-\psi_n)\sigma_\epsilon^2}.$$

The error variance of the price at the nth auction, Σ_n, is equal to:

$$\Sigma_n = \frac{\Sigma_{n-1}(\sigma_u^2\Delta t + (\alpha_n\Delta t)^2 M\sigma_\epsilon^2)}{(\alpha_n\Delta t)^2 M^2\Sigma_{n-1} + \sigma_u^2\Delta t + (\alpha_n\Delta t)^2 M\sigma_\epsilon^2}.$$

The market efficiency condition implies that λ_n is the regression coefficient of $\tilde{v}$ on $\tilde{w}_n$, conditional on $\tilde{w}_1, \ldots, \tilde{w}_{n-1}$, then:

$$\lambda_n = \frac{\alpha_n\Delta t M\Sigma_{n-1}}{(\alpha_n\Delta t_n)^2 M^2\Sigma_{n-1} + \sigma_u^2\Delta t + (\alpha_n\Delta t_n)^2 M\sigma_\epsilon^2}.$$

This leads to the following expression

$$\frac{\lambda_n}{\Sigma_n} = \frac{M\alpha_n\Delta t}{M(\alpha_n\Delta t)^2\sigma_\epsilon^2 + \sigma_u^2\Delta t}.$$

Since $\alpha_n\Delta t = \frac{\psi_n}{M\lambda_n}$, one obtains:

$$\lambda_n^2 = \frac{\psi_n\Sigma_n - \frac{\psi_n^2\sigma_\epsilon^2}{M}}{\sigma_u^2\Delta t}.$$

That yields the following condition:

$$\Sigma_n > \psi_n \frac{\sigma_\epsilon^2}{M}.$$

Since $\Sigma_n \geq \Sigma_N$ and $\psi_n < 1$, the following condition is sufficient for an equilibrium

$$\Sigma_N > \frac{\sigma_\epsilon^2}{M}.$$

Step 4: The backward induction program of the insiders.

Each trader i, for $i = 1, \ldots, M$, maximizes his expected profit from the nth auction to the last one:

$$\begin{aligned} E[\pi_{in}|p_1, \ldots, p_{n-1}, \tilde{S}_i] &= k_{1,n-1}\tilde{S}_i^2 + k_{2,n-1}\tilde{S}_i p_{n-1} + k_{3,n-1}p_{n-1}^2 + k_{4,n-1}p_{n-1}(\hat{p}_{n-1}^i - p_{n-1}) \\ &\quad + k_{5,n-1}\tilde{S}_i(\hat{p}_{n-1}^i - p_{n-1}) + k_{6,n-1}(\hat{p}_{n-1}^i - p_{n-1})^2 + \delta_{n-1}. \end{aligned}$$

Since each trader uses a backward induction process, we have to find a recurrence relation between the different parameters:

$$\begin{pmatrix} k_{1,n-1} \\ k_{2,n-1} \\ k_{3,n-1} \\ k_{4,n-1} \\ k_{5,n-1} \\ k_{6,n-1} \end{pmatrix} = \begin{pmatrix} a_{11,n} & a_{12,n} & a_{13,n} & a_{14,n} & a_{15,n} & a_{16,n} \\ a_{21,n} & a_{22,n} & a_{23,n} & a_{24,n} & a_{25,n} & a_{26,n} \\ a_{31,n} & a_{32,n} & a_{33,n} & a_{34,n} & a_{35,n} & a_{36,n} \\ a_{41,n} & a_{42,n} & a_{43,n} & a_{44,n} & a_{45,n} & a_{46,n} \\ a_{51,n} & a_{52,n} & a_{53,n} & a_{54,n} & a_{55,n} & a_{56,n} \\ a_{61,n} & a_{62,n} & a_{63,n} & a_{64,n} & a_{65,n} & a_{66,n} \end{pmatrix} \begin{pmatrix} k_{1,n} \\ k_{2,n} \\ k_{3,n} \\ k_{4,n} \\ k_{5,n} \\ k_{6,n} \end{pmatrix} + \begin{pmatrix} c_{1,n} \\ c_{2,n} \\ c_{3,n} \\ c_{4,n} \\ c_{5,n} \\ c_{6,n} \end{pmatrix}. \quad (7.5)$$

By computing $E[\Delta X_n^*|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}]$ and $E[(\Delta X_n^*)^2|\tilde{S}_i, \tilde{w}_1, \ldots, \tilde{w}_{n-1}, \tilde{x}_{i1}, \ldots, \tilde{x}_{in-1}]$ and by substituting $\tilde{x}_{in} = \alpha_n \Delta t \tilde{S}_i + \beta_n \Delta t p_{n-1} + \zeta_n \Delta t(\hat{p}_{n-1}^i - p_{n-1})$ in the expression of the profit, and finally by identification we obtain the following[15]

$$\delta_{n-1} = \delta_n + \lambda_n^2 k_{3n} \left[\sigma_u^2 \Delta t_n + (M-1)(\alpha_n \Delta t)^2 (1 + (M-1)a_n)\sigma_\epsilon^2\right]. \quad (7.6)$$

[15]For the sake of space, the coefficients $a_{ij,n}$ for $i, j = 1, \ldots, 6$ of the matrix are omitted and are available upon request.

The coefficient of the reaction to private information at the *nth* auction are equal to:

$$\alpha_n \Delta t = \frac{a_n + \lambda_n(k_{2n} - k_{5n})}{\lambda_n \left[2 + (M-1)a_n - 2\lambda_n(k_{3n} - \frac{1}{2}k_{4n})(1 + a_n(M-1))\right]},$$

$$\psi_n = \frac{Ma_n + M\lambda_n(k_{2n} - k_{5n})}{2 + (M-1)a_n - 2\lambda_n(k_{3n} - \frac{1}{2}k_{4n})(1 + a_n(M-1))}.$$

By substituting $a_n = \frac{\Sigma_n}{\Sigma_n + (1-\psi_n)\sigma_\epsilon^2}$ and $\lambda_n^2 = \frac{\psi_n \Sigma_n - \frac{\psi_n^2 \sigma_\epsilon^2}{M}}{\sigma_u^2 \Delta t_n}$ and developing the previous equation, we get that ψ_n is the solution to the following equation of order six:

$$\frac{\sigma_\epsilon^2}{M}\gamma_{5n}\psi_n^6 + (\frac{\sigma_\epsilon^2}{M}\gamma_{4n} - \Sigma_n\gamma_{5n})\psi_n^5 + (\phi_{5n} + \frac{\sigma_\epsilon^2}{M}\gamma_{3n} - \Sigma_n\gamma_{4n})\psi_n^4 + (\phi_{4n} + \frac{\sigma_\epsilon^2}{M}\gamma_{2n} - \Sigma_n\gamma_{3n})\psi_n^3 + (\phi_{3n} + \frac{\sigma_\epsilon^2}{M}\gamma_{1n} - \Sigma_n\gamma_{2n})\psi_n^2 + (\phi_{2n} - \Sigma_n\gamma_{1n})\psi_n + \phi_{1n} = 0,$$

with

$$\begin{aligned}
\gamma_{1n} &= \frac{M^2(k_{2n}-k_{5n})^2(\Sigma_n+\sigma_\epsilon^2)^2}{\sigma_u^2\Delta t}, \\
\gamma_{2n} &= \frac{-2M^2(k_{2n}-k_{5n})^2(\sigma_\epsilon^2+\Sigma_n)\sigma_\epsilon^2 - 2(k_{3n}-\frac{1}{2}k_{4n})(M\Sigma_n+\sigma_\epsilon^2)}{\sigma_u^2\Delta t}, \\
\gamma_{3n} &= \frac{4(k_{3n}-\frac{1}{2}k_{4n})^2\sigma_\epsilon^4 + M^2(k_{2n}-k_{5n})^2\sigma_\epsilon^4 + 4M^2(k_{3n}-\frac{1}{2}k_{4n})^2\Sigma_n^2 - 4M(k_{2n}-k_{5n})(k_{3n}-\frac{1}{2}k_{4n})\Sigma_n\sigma_\epsilon^2}{\sigma_u^2\Delta t} \\
&\quad - \frac{8M(k_{2n}-k_{5n})\sigma_\epsilon^4(k_{3n}-\frac{1}{2}k_{4n}) + 4M^2(k_{2n}-k_{5n})\sigma_\epsilon^2(k_{3n}-\frac{1}{2}k_{4n})\Sigma_n - 8(k_{3n}-\frac{1}{2}k_{4n})^2M\Sigma_n\sigma_\epsilon^2}{\sigma_u^2\Delta t},
\end{aligned}$$

$$\begin{aligned}
\gamma_{4n} &= \frac{4M(k_{2n}-k_{5n})(k_{3n}-\frac{1}{2}k_{4n})\sigma_\epsilon^4 - 8(k_{3n}-\frac{1}{2}k_{4n})^2\sigma_\epsilon^4 - 8M(k_{3n}-\frac{1}{2}k_{4n})^2\Sigma_n\sigma_\epsilon^2}{\sigma_u^2\Delta t}, \\
\gamma_{5n} &= \frac{4(k_{3n}-\frac{1}{2}k_{4n})^2\sigma_\epsilon^4}{\sigma_u^2\Delta t},
\end{aligned}$$

$$\begin{aligned}
\phi_{1n} &= M^2\Sigma_n^2, \phi_{2n} = -2M\Sigma_n(2\sigma_\epsilon^2 + (M+1)\Sigma_n), \\
\phi_{3n} &= 4\sigma_\epsilon^4 + 4\sigma_\epsilon^2(M+1)\Sigma_n + (M+1)^2\Sigma_n^2 + 4M\sigma_\epsilon^2\Sigma_n, \\
\phi_{4n} &= -8\sigma_\epsilon^4 - 4\sigma_\epsilon^2\Sigma_n(M+1), \phi_{5n} = 4\sigma_\epsilon^4.
\end{aligned}$$

At the final auction there is no future profit, this implies that $k_{2N} = k_{3N} = 0$. After some computations and further simplifications the parameter ψ_N solves:

$$2\sigma_\epsilon^2\psi_N^2 - (2\sigma_\epsilon^2 + (M+1)\Sigma_N)\psi_N + M\Sigma_N = 0.$$

This ends the proof of proposition 1.

The case of perfect private information: $\tilde{S}_i = \tilde{v}$

It can be shown that by setting $\sigma_\epsilon^2 = 0$ in the proof of proposition 1, the results of Holden and Subrahmanyam (1992) are obtained.

The case of the static setting: $N = 1$

It can be shown that by setting $N = 1$ in the proof of proposition 1, the results of Dridi and Germain (2009) are obtained.

All other Propositions are obtained by numerical procedures.

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