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Investment timing under uncertainty in oligopoly: Symmetry or leadership?

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Abstract

This paper examines firms' investment-timing decisions in an oligopolistic set-up. Facing demand uncertainty, firms decide whether to invest early or wait until uncertainty has been resolved. We show that the precise form that investment commitment takes matters for the investment-timing outcomes that emerge. When firms can commit immediately to the final investment level, investment leadership may occur and early investment is referred to as being primarily "aggressive". By contrast, the presence of a time lag between *when* and *how much* firms invest yields symmetric investment-timing outcomes only; we argue that early investment is mainly "defensive" in that case.

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1. Introduction

Almost all forms of investment have three features in common: investment decisions are made under uncertainty, are at least partially irreversible and the timing of the investment is crucial. On the one hand, there is a flexibility gain associated with delaying investment in the face of an uncertain future.² On the other hand, in oligopolistic industries there is also a potential benefit of committing in advance to investment in order to affect the strategic environment in which future

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² See Dixit and Pindyck (1994) for a comprehensive discussion of the option value of delaying irreversible investment.

competition takes place.³ This offsetting gain from commitment generates a trade-off between investing early and delaying investment.

There is a small literature on the commitment-flexibility trade-off in output setting games.⁴ Maggi (1996) extends this literature to look at a model of strategic investment under uncertainty. The main finding in his paper is that, in a duopoly setting with ex ante identical firms, one of the firms commits to early investment and the other adopts a wait-and-see strategy and becomes an investment follower. A key assumption in Maggi (1996) is that investment is immediately in place; investing early then implies that firms select the capital *level* to which they are then committed.⁵ Maggi's result suggests that early investment commitment is primarily "aggressive": firms commit early to capture the benefits of investment leadership. There are several real-world examples confirming this motive underlying early investment.⁶ However, empirical evidence from case studies indicates that this does not always need to be the case. In a case study of the bulk chemical industry, Ghemawat (1984) points out how the presence of significant construction lags in the industry can allow "defensive" commitment: firms invest early to avoid being in the strategically disadvantaged position of investment follower. He cites (p. 157) how one firm, Kerr-McGee, prevented its rival, Du Pont, from obtaining a first-mover advantage: by introducing its own investment plans before Du Pont's expansion had fully materialised, Kerr-McGee forced Du Pont to revise its initial capacity plans. In other words, while the construction process was ongoing, the final capital level was not yet irrevocably fixed.⁷ In this example, it is the construction lag in investment that prevents firms from sticking to their initial choice of capital. There are, however, other types of investment that take place over time. One important example is R&D. After starting an R&D project, a firm is still able to deviate (to some degree) from the initially specified R&D expenditure level and indeed may wish to do so once its rivals' investment plans have been observed or leaked. In this paper, we investigate whether investment commitment with gestation lags – such as those described above – also generates investment leadership by ex ante symmetric firms (as it does when investment is immediately in place) or whether it leads to ex post symmetric roles. In fact, we show that this modification of the assumption regarding the form of investment commitment leads to equilibria in which firms adopt symmetric roles: either both firms invest early or both adopt a wait-and-see approach.

We set-up a model of investment under oligopoly that combines the three key features of investment: uncertainty, irreversibility and timing. We consider two different investment-timing games. In the first game, investment is immediately in place; investing early implies that firms select the investment *level* to which they are then committed. In the second game, firms first commit

 $^{^{3}}$ As argued first by Spence (1977) and Dixit (1980), firms have an incentive to undertake actions that will improve their future strategic position relative to their competitors. Tirole (1988) provides a textbook treatment of this issue.

⁴ Examples are Appelbaum and Lim (1985), Spencer and Brander (1992) and Sadanand and Sadanand (1996). Note that the flexibility in timing discussed in these papers is quite different from the technological flexibility discussed in, for instance, Vives (1989) and Boyer and Moreaux (1997). In those models there need not be a trade-off between flexibility and strategic commitment.

⁵ This is, for instance, the case when a factory is bought rather than being built from scratch, or when a new technology is adopted.

⁶ For instance, Sony gained an investment first-mover advantage over Philips when the latter decided to set-up a CD-plant in the US only after Sony's plant was fully operational (see McGahan, 1993).

⁷ Just as Rome was not built in a day, factories are not constructed overnight. Industries in which investment typically are characterised by significant construction lags are, for instance, Aerospace and Pharmaceuticals (Pindyck, 1991). Ghemawat (1984) reports that, for a typical plant in the titanium dioxide industry, there is a lag of at least 4 years between the decision to construct and the actual start-up date.

to the timing of their investment but cannot yet credibly set the actual capital level. After having chosen *when* to invest, firms observe the timing decision made by their rival before finalising their investment levels. This alternative two-step form of commitment is more appropriate to capture the investment decision process when investment involves gestation lags. Adopting the terminology for endogenous timing games introduced by Hamilton and Slutsky (1990), the former game will be referred to as the "action commitment" game and the latter will be labelled the "observable delay" game.⁸

In Section 2, we discuss the set-up in which two rival firms choose capital and output for a market characterised by demand uncertainty. In Section 3, the investment-timing pattern that emerges with "action commitment" is compared to the investment timing with "observable delay". In Section 4 some welfare issues are addressed. Section 5 concludes.

2. The model

This section outlines the basic model, which will be used to study strategic behaviour both in the game in which firms can immediately commit to an investment level, the "action commitment" (AC) game, and in the game in which they commit to the investment *timing* before they can fix the investment *level*, the "observable delay" (OD) game. In order to put the subtle differences between the two games in sharp relief, we first outline what is common to both games in Section 2.1 and then highlight the differences in Section 2.2.

2.1. Investment with demand uncertainty

Consider a Cournot duopoly in which firms produce identical products. Output of firm *i* is denoted by q_i (*i* = 1, 2). Firms face uncertainty about demand, captured by the following inverse demand function:

$$p = a - Q + u \tag{1}$$

with p denoting the market price and $Q = q_1 + q_2$ industry output; $u \in [\underline{u}, \overline{u}]$ is the stochastic demand component with zero mean and variance $\sigma^{2,9}$

Firms invest in capital k_i (*i* = 1, 2). Firm *i*'s total costs, TC^{*i*}, are given by:

$$TC^{i} = (c_{0} - k_{i})q_{i} + \frac{k_{i}^{2}}{2\eta}$$
 with $TC^{i}_{k_{i}} = -q_{i} + \frac{k_{i}}{\eta}$ and $TC^{i}_{q_{i}} = c_{0} - k_{i}$ (2)

The positive parameter η is inversely related to the cost of capital and c_0 is a positive constant. TC^{*i*}_{*k_i*} and TC^{*i*}_{*q_i*} are defined respectively as the marginal cost of capital and the marginal production cost. Investing in capital reduces the marginal cost of production for firm *i*. We assume $c_0 > k_i$. This cost function is commonly used in oligopoly models with strategic investment.¹⁰ It captures

⁸ Hamilton and Slutsky (1990) restrict attention to price and output games and do not look at investment decisions. Furthermore, since they assume certainty, they are not concerned with the trade-off between commitment and flexibility.

 $^{^{9}}$ These are the only restrictions on the probability distribution that are required. A straightforward – but by no means the only – candidate is the uniform distribution.

¹⁰ See, for example, Grossman and Maggi (1998). We rule out the case in which $c_0 \le k_i$. In their strategic trade model, Grossman and Maggi include $c_0 \le k_i$; the marginal cost curve then remains constant at c_0 for levels of k_i beyond c_0 . For an in-depth exploration, we refer to their work.



Fig. 1. (a) The action commitment game and (b) the observable delay game.

in a very simple way that a firm can choose to have lower marginal costs in exchange for higher fixed costs. Profits of firm *i* are given by:

$$\pi_i = pq_i - \mathrm{TC}^i, \quad i = 1, 2 \tag{3}$$

In both games, there are two periods (see Fig. 1a and b). Firms face uncertainty about demand in period 1, which disappears at the start of period 2. In our model a firm can commit to some but not all of its factors of production long-term: firms can commit to capital, but not to output. Outputs are always chosen simultaneously in period 2, that is, after uncertainty has been resolved. A firm's first-order condition for output is¹¹:

$$A - 2q_i - q_j + k_i + u = 0 (4)$$

with $A \equiv a - c_0$ and $i, j = 1, 2; i \neq j$.

Hence, the equilibrium output for firm *i* is:

$$q_i = \frac{1}{2}(A + 2k_i - k_j + u) \tag{5}$$

Firms decide *when* to invest in period 1. For simplicity, we assume throughout that firms are risk neutral. Hence, their investment-timing decisions follow from maximising expected profits. Firms

¹¹ We restrict attention to interior solutions.

either commit to their capital in the first period or postpone investment.¹² When capital is chosen in period 1, it is set before output. But, if chosen in period 2, it is determined simultaneously with output. If a firm invests early, it determines its capital in period 1 by maximising expected profits (max_{ki} $E\pi_i$). On the one hand, commitment to capital in the first period enables firms to choose capital strategically, allowing them to influence future outputs to their advantage. On the other hand, by setting capital levels in period 1 (before uncertainty has been resolved), a firm reduces its output flexibility compared to when it delays investment. When capital is chosen in period 2, it will be set in line with actual demand (i.e., $k_i = k_i(u)$ with $(\partial k_i/\partial u) > 0$). This investment flexibility in turn enhances the firm's output flexibility. Hence, our model captures the stylised fact that, in practice, investors who value flexibility have an incentive to delay investment when they face significant uncertainty.¹³

2.2. Two forms of commitment

Firms face a trade-off between flexibility and commitment, both in the AC- and in the ODgame. The AC-game is a two-stage game, illustrated in Fig. 1a. In stage 1 of the AC-game, firms either choose capital levels or delay investment; with commitment, the timing of investment and the choice of capital level are thus compressed into a single action. In stage 2, outputs are chosen and firms that delayed investment set capital levels.

The nature of commitment with investment that is not immediately in place is different and is more appropriately captured by the OD-game. This game consists of three stages (see Fig. 1b). Firms first decide *when* to invest (stage 1), but because investment takes time, the actual level of capital is not immediately fixed.¹⁴ Only in a later stage, after the timing decisions have been made and observed by both parties, do firms fix their capital levels. So, a firm that chooses to commit and thus invests early, sets its capital level after the investment-timing choices are made, but before uncertainty has been resolved (stage 2). A firm that delayed investment, sets its capital level at the same time it chooses output (stage 3).¹⁵ Although the distinction between the two games seems quite subtle, we will show that it makes a considerable difference to the investment-timing pattern that emerges. As argued in Section 1, in many cases this two-step form of investment commitment – or, the OD-game – is a more accurate description of the investment process.

Note that, in Fig. 1a and b and throughout the text, superscripts 'c' and 'd' denote commitment and delay, respectively. The first superscript on the variables refers to firm 1 and the second to firm 2.

¹² There exists a literature that addresses the issue of partial commitment to output in a multi-period set-up (see, for instance, Pal (1996)). However, partial commitment is probably less realistic with capital commitment, since the costs of adding capital to an existing level may be prohibitively high in the short run, due to the existence of indivisibilities and incompatibility in technologies.

¹³ Despite being risk neutral, firms value flexibility. This follows from the fact that expected profit is increasing in the variance of demand. The positive effect of the variance on ex ante expected profits is due to the fact that the actual ex post realisation of profits is convex in u. Due to the indirect effect of capital on output, the positive effect of the variance on expected profits is larger under investment flexibility than under commitment, implying that a rise in uncertainty increases the value of investment delay. This feature is consistent with option value theory in finance. Note that risk aversion (which complicates the analysis considerably) simply strengthens the gains from remaining flexible but yields no additional insights.

¹⁴ Firms' timing decisions, once made, are assumed too costly to be reversed.

¹⁵ In the OD-game delaying implies that the investing project can only be completed in period 2.

	C_1, C_2	C_1, D_2	D_1, C_2	D_1, D_2
<i>k</i> ₁	$k_1^{\rm cc} = \frac{4}{3}\eta E q_1^{\rm cc}$	$k_1^{\rm cd} = \frac{2(2-\eta)}{3-2\eta} \eta E q_1^{\rm cd}$	$k_1^{\rm dc}(u) = \eta q_1^{\rm dc}(u)$	$k_1^{\rm dd}(u) = \eta q_1^{\rm dd}(u)$
<i>k</i> ₂	$k_2^{\rm cc} = \frac{4}{3} \eta E q_2^{\rm cc}$	$k_2^{\rm cd}(u) = \eta q_2^{\rm cd}(u)$	$k_2^{\rm dc} = \frac{2(2-\eta)}{3-2\eta} \eta E q_2^{\rm dc}$	$k_2^{\rm dd}(u) = \eta q_2^{\rm dd}(u)$

Capital levels for the different candidate equilibria under action commitment and observable delay

3. Investment timing

In each game, the equilibria that emerge depend on the level of uncertainty. However, at a given level of uncertainty the two games may generate *different* outcomes. We proceed in two steps. In Section 3.1, we discuss all the possible *candidate* equilibria of the two games.¹⁶ Then, in Sections 3.2 and 3.3, we will show how the *actual* equilibrium investment-timing pattern in each game depends on the precise form of commitment.

3.1. Candidate timing equilibria

In both games, there obviously are four possible timing combinations: $(C_1, C_2), (C_1, D_2), (D_1, C_2)$ and (D_1, D_2) , where C_i and D_i (with i = 1, 2) refer to commitment and delay, respectively. It follows directly that these combinations are the only *candidate* timing equilibria under OD. Under AC, there is also a candidate equilibrium corresponding to each of the four possible investment-timing combinations. Again, $(C_1, C_2), (C_1, D_2), (D_1, C_2)$ and (D_1, D_2) are the only possible equilibria, but "commitment" (*C*) now refers to the immediate choice of a particular capital level (k).¹⁷

3.1.1. Equilibrium capital levels

The capital levels for each of the candidate equilibria under AC are the same as those at the corresponding equilibria under OD and are reported in Table 1.¹⁸ In (C_1 , C_2) and (D_1 , D_2), firms choose their capital simultaneously; firms' capital levels per unit output are symmetric, but larger for investment commitment than for delay (i.e., $(k_i^{cc}/Eq_i^{cc}) > (k_i^{dd}(u)/q_i^{dd}(u))$). Commitment allows a firm to credibly increase its capital level, reducing marginal costs (see Eq. (2)) and in turn pushing its output reaction function to the right. As a result, it increases its market share, while the rival produces less and has lower profits. In the terminology of Fudenberg and Tirole (1984), firms that commit to investment adopt a "top-dog" business strategy. Furthermore, capital levels are strategic substitutes. Hence, a firm that sets its capital level prior to its rival, has an additional incentive to invest more, thus reducing both the capital and the output level of its rival. In candidate equilibria (C_1 , D_2) and (D_1 , C_2), only one firm invests early and hence leads in investment. The committed capital level per unit output chosen by the leader is larger than that chosen by either firm when both firms commit ($(k_1^{cd}/Eq_1^{cd}) = (k_2^{dc}/Eq_2^{dc}) > (k_i^{cc}/Eq_i^{cc})$).

Table 1

¹⁶ We consider only pure strategies.

 $^{^{17}}$ For convenience and where it will not cause confusion, we will use *C* to represent commitment under AC even though commitment then always refers to a capital *level*.

¹⁸ Expected profits in each candidate equilibrium are given in Table A.1.



Fig. 2. Capital reaction functions and candidate equilibria.

3.1.2. The trade-off between commitment and flexibility in the AC- and OD-games

We now compare the sustainability of the different investment-timing combinations under AC and OD. This is the key to understanding the differences between the two games. Fig. 2 proves helpful in explaining these differences intuitively. For the remainder of this section and without loss of generality, we will adopt the perspective of firm 1.

Fig. 2 shows firm 1's capital reaction function when it commits, denoted by $r_1^c(k_2)$, and the expected capital reaction function when it delays, $Er_1^d(k_2)$. To avoid overburdening the diagram, firm 2's capital reaction function is drawn for commitment only and denoted in Fig. 2 by $r_2^c(k_1)$. A committed firm's capital reaction function, $r_i^c(k_j)$, traces its first-period best response to capital levels of a committed rival; a delaying firm's (expected second-period) capital reaction function, $Er_i^d(k_j)$, traces its (expected) best response to capital levels of a committed or a delaying rival. These reaction functions are the same under AC and under OD. Note that the $r_1^c(k_2)$ -reaction function is to the right of $Er_1^d(k_2)$, reflecting the more aggressive choice of capital under commitment. The candidate equilibrium with investment leadership by firm 2 occurs at point A, while the candidate equilibrium in which both firms commit occurs at point B, the intersection of $r_1^c(k_2)$ and $r_2^c(k_1)$ (the remaining two candidate equilibria are not illustrated). The level of uncertainty together with the type of commitment (AC or OD) will determine the actual equilibrium outcomes.

Comparing the sustainability of the candidate equilibria under AC and OD, we first consider (C_1, C_2) . In both games, as uncertainty rises, the value of flexibility increases and this equilibrium eventually breaks down as players wish to deviate from commitment to delay given rival commitment. We define the critical uncertainty threshold above which firm 1 prefers delay to commitment and hence above which (C_1, C_2) breaks down under AC as σ_m^2 (see Table 2). At σ_m^2 , we have $E\pi_1(D_1, k_2^{cc}) = E\pi_1(C_1, k_2^{cc})$.¹⁹ The corresponding threshold under OD is defined

¹⁹ Here, C_1 refers to a particular capital level, $k_1 = r_1^c (k_2^{cc})$ (see Footnote 17).

Threshold	Definition	Applies in
$\overline{\sigma_x^2}$	$E\pi_1(D_1, C_2) = E\pi_1(C_1, C_2)$	OD
σ_h^2	$E\pi_1(D_1, D_2) = E\pi_1(C_1, D_2)$	AC, OD
σ_m^2	$E\pi_1(D_1, k_2^{\rm cc}) = E\pi_1(C_1, k_2^{\rm cc})$	AC
σ_l^2	$E\pi_1(D_1, k_2^{\rm dc}) = E\pi_1(C_1, k_2^{\rm dc})$	AC

The uncertainty thresholds under action commitment and observable delay^a

^a These are always ranked as follows: $\sigma_x^2 > \sigma_h^2 > \sigma_m^2 > \sigma_l^2$.

as σ_x^2 at which $E\pi_1(D_1, C_2) = E\pi_1(C_1, C_2)$. It requires a higher level of uncertainty for (C_1, C_2) to break down under OD than under AC, i.e., $\sigma_x^2 > \sigma_m^2$. We explain the intuition for this with the aid of Fig. 2. Under AC, firm 2 has committed to capital *level* k_2^{cc} in (C_1, C_2) . Taking k_2^{cc} as given, a deviation by firm 1 from commitment to delay is represented by a jump from points B to B'. In contrast, under OD at (C_1, C_2) , if firm 1 were to deviate from commitment, it would take account of the fact that, in stage 2, its rival (having observed that firm 1 delays) would *adjust* to a *higher* capital level $(k_2^{dc}$ instead of $k_2^{cc})$. In Fig. 2, this deviation is represented by a movement from points B to A. As firm 1's expected profits are lower at A than at B', deviating from the (C_1, C_2) -equilibrium is less attractive under OD than under AC; hence, $\sigma_x^2 > \sigma_m^2$.

Continuing our approach of adopting the perspective of firm 1, next consider the candidate equilibrium (D_1, C_2) (point A in Fig. 2), in which firm 1 is the investment follower. Under AC, with firm 2 committed to k_2^{dc} , the uncertainty threshold below which (D_1, C_2) breaks down will be referred to as σ_l^2 , at which $E\pi_1(D_1, k_2^{dc}) = E\pi_1(C_1, k_2^{dc})$.²⁰ Under OD, (D_1, C_2) breaks down below σ_x^2 . There exists a range of uncertainty levels for which the investment follower is happy to delay under AC, but would want to deviate to commitment under OD, i.e., $\sigma_x^2 > \sigma_l^2$. Fig. 2 is again used to explain the intuition. In AC, if firm 1 were to deviate from delay to commitment, it could not affect the rival's high capital level k_2^{dc} . In the figure, such a deviation corresponds to a movement from points A to A'. By contrast, a deviation by firm 1 from delay to commitment in OD would force the committed rival firm to adjust its capital downward to a less aggressive level, k_2^{cc} (shown by a movement follower's incentive to deviate from delay to commitment is stronger than under AC.

Next, consider the candidate equilibrium (C_1, D_2) , in which firm 1 is the investment leader. When uncertainty becomes sufficiently high, firm 1 eventually wishes to deviate to delay. Importantly, *both* in the AC- and the OD-game, if firm 1 were to deviate from commitment, it would take account of the fact that its rival would observe this delay and would choose a *higher* capital level in period 2 (k_2^{dd} instead of k_2^{cd}). Hence, this equilibrium breaks down when uncertainty exceeds the same threshold level under OD and AC. We define this threshold as σ_h^2 , at which $E\pi_1(D_1, D_2) = E\pi_1(C_1, D_2)$.

Finally, consider (D_1, D_2) (this is not shown in Fig. 2) under AC and OD. In both games, this equilibrium eventually breaks down as uncertainty falls, inducing players to deviate from delay to commitment given rival delay. Under both OD and AC, this equilibrium breaks down when uncertainty falls below σ_h^2 , which also forms the upper bound for the (C_1, D_2) -equilibrium.

Table 2

²⁰ Here, C_1 refers to the capital level $k_1 = r_1^c(k_2^{dc})$ (see Footnote 17).

The definitions of the uncertainty thresholds in Table 2 will be useful in the next two sections, in which we derive the *actual* equilibria that emerge in each game.

3.2. Solving the AC-game

In this section we solve the benchmark AC-game, which captures the case in which the investment level is immediately in place (see Fig. 1a). In stage 1, each firm chooses *either* k_i , $k_i \in R_+$, if it commits, or the discrete action D, if it delays.

Ex ante symmetry allows us to look at the problem from the perspective of firm 1 without loss of generality. If firm 2 commits, its only actions that can form part of an equilibrium are k_2^{cc} and k_2^{dc} . As discussed in the previous section, the uncertainty threshold above which firm 1 delays if firm 2 commits to k_2^{cc} is σ_m^2 . If firm 2 commits to k_2^{dc} , the threshold above which firm 1 delays is σ_l^2 . If the rival firm delays, firm 1 will delay rather than commit above σ_h^2 . We show in Appendix B that $0 < \sigma_l^2 < \sigma_m^2 < \sigma_h^2$. To understand the intuition for this ranking of thresholds, first note that, ceteris paribus, the benefit of commitment to a firm increases in the gap between the price and its marginal production cost.²¹ However, the price-cost margin enjoyed by a firm is itself negatively related to the output level of its rival. Thus, higher k_2 -values are associated with higher q_2 -values and a reduced price-cost margin for firm 1, which in turn weakens the incentive for firm 1 to commit. Hence, the lower the capital level of firm 2, the higher the level of uncertainty at which firm 1 prefers commitment to delay. Next, note that $k_2^{dc} > k_2^{cc}$ and that both these capital levels are larger than the expected capital levels of firm 2 when it delays. This explains the ranking of the thresholds. Given this ranking, the equilibrium outcomes at different uncertainty levels can be established. These are given in the following proposition.

Proposition 1. The equilibria in the AC-game depend on the level of uncertainty:

- (i) For $\sigma^2 < \sigma_l^2$, both firms commit; (C₁, C₂) is the unique equilibrium.
- (ii) For $\sigma_l^2 \le \sigma^2 \le \sigma_m^2$, both firms commit, or, one firm commits and the other delays; (C_1, C_2) , (ii) For $\sigma_1^2 = \sigma_1^2 - \sigma_m^2$, and (D_1, C_2) are the equilibria. (iii) For $\sigma_m^2 < \sigma^2 < \sigma_h^2$, one firm commits and the other delays; (C_1, D_2) and (D_1, C_2) are the
- equilibria.
- (iv) For $\sigma^2 = \sigma_h^2$, one firm commits and the other delays, or, both firms delay; $(C_1, D_2), (D_1, C_2)$ and (D_1, D_2) are the equilibria.
- (v) For $\sigma^2 > \sigma_h^2$, both firms delay; (D_1, D_2) is the unique equilibrium.

Proof. See Appendix C.

Fig. 3 illustrates the outcomes of the AC-game. It depicts the uncertainty thresholds as functions of η . As shown in the figure, the *ranking* of the thresholds is independent of the value for η . The uncertainty thresholds demarcate different regions. Areas I, III and V are the main regions (region II is merely a narrow boundary²² between I and III, while IV represents a knife-edge between III and V). In areas I and V, firms invest at the same time, while investment leadership prevails in

²¹ Compared to delay, commitment implies a larger capital and hence output level, and the value of a higher output level increases in the per-unit operating profit.

²² For instance, at $\eta = 0.15$, region II is only 0.00060 wide in terms of σ^2 , while this distance narrows down even further to a σ^2 -range of 0.00005 at n = 0.05.



^bIn the narrow area II, the equilibria are (C_1, D_2) , (D_1, C_2) and (C_1, C_2) .

Fig. 3. Investment timing with action commitment.

area III. The occurrence of investment leadership is a key feature of the AC-game, to which we will return in Section 3.4.

Note that, for $\sigma_l^2 \leq \sigma^2 \leq \sigma_h^2$, there are multiple pure-strategy equilibria. For instance, for $\sigma_m^2 < \sigma^2 < \sigma_h^2$, there are two leadership equilibria (that are the mirror image of each other).²³ With equilibrium multiplicity, Pareto dominance is sometimes applied to select an equilibrium. But here, since both firms prefer leading to following, this equilibrium selection method will not solve the question of which firm will be assigned with the leadership role. There is, however, also a symmetric mixed-strategy equilibrium in which each firm chooses the probability with which it will commit. The equilibrium probability – which is the same for each firm – is low at high levels of uncertainty and high at low values of σ^2 . Naturally, the actual timing outcome remains the result of a randomisation over the pure strategies *C* and *D*. To avoid the paper becoming too taxonomical, we do not analyse mixed-strategy equilibria in detail.

3.3. Solving the OD-game

In this subsection we solve the OD-game, which captures a two-step form of investment commitment (see Fig. 1b). Now, because firms *observe* each other's investment timing *before* determining their actual capital level, the nature of commitment is less "sticky" than with the quick-in-place investment in the AC-game. This feature has important implications for the outcomes of the game.

Again, because of ex ante symmetry we need only examine the problem from the perspective of firm 1. If firm 2 delays, firm 1 prefers to commit below σ_h^2 , the same threshold as in the

 $^{^{23}}$ Such leadership equilibria also occur in the continuous-time model of technology adoption by Fudenberg and Tirole (1985) (for an extension of that model to a set-up with uncertainty, see Huisman and Kort (1999)).



Fig. 4. Investment timing with observable delay.

AC-game. However, *if* firm 2 commits, firm 1's behaviour under OD differs from that under AC. A firm that contemplates deviation from (C_1, C_2) must take account of the fact that its rival will react aggressively by setting a higher capital level (under AC a firm that contemplates deviation from (C_1, C_2) takes k_2^{cc} as fixed). For this reason, as explained in detail in Section 3.1, (C_1, C_2) is easier to sustain as an equilibrium under OD than under AC. As a result, the threshold level of uncertainty below which firm 1 commits given rival commitment is higher in the OD-game than the corresponding thresholds in the AC-game $(\sigma_x^2 > \sigma_m^2 > \sigma_l^2)^{.24}$ It is now even above the threshold given rival delay, i.e., $\sigma_x^2 > \sigma_h^2$ (see Appendix B). The outcomes for different ranges of uncertainty are given in the following proposition.

Proposition 2. The equilibria in the OD-game depend on the level of uncertainty:

- (i) For $\sigma^2 < \sigma_h^2$, both firms commit; (C_1, C_2) is the unique equilibrium. (ii) For $\sigma_h^2 \le \sigma^2 \le \sigma_x^2$, both firms commit, or, both firms delay; (C_1, C_2) and (D_1, D_2) are the
- (iii) For $\sigma^2 > \sigma_x^2$, both firms delay; (D_1, D_2) is the unique equilibrium.

Proof. See Appendix C. \Box

Fig. 4 depicts investment timing in the OD-game. As in the AC-game, the ranking of the σ^2 thresholds is independent of η . In area I, both firms invest early, whereas both delay investment

 $^{^{24}}$ Unlike with AC, commitment (C) now simply means "investing early" and is therefore a discrete action. Hence, there just is a single threshold at which firm 1 is indifferent given rival commitment. Given that firm 2 chooses to commit but can only fix its capital level in stage 2, firm 1 will compare its expected profits from also committing, $E\pi_1(C_1, C_2)$, to those from delaying investment, $E\pi_1(D_1, C_2)$, knowing that its investment-timing choice will affect the optimal level of firm 2's capital in stage 2 (k_2^{cc} if C_1 and k_2^{dc} if D_1).

	AC-game	OD-game
$\overline{\sigma_2^2 > \sigma_x^2}$	(D_1, D_2)	(D_1, D_2)
$\sigma_h^2 < \sigma^2 < \sigma_x^2$	(D_1, D_2)	$(C_1, C_2); (D_1, D_2)$
$\sigma_m^2 < \sigma^2 < \sigma_h^2$	$(C_1, D_2); (D_1, C_2)$	(C_1, C_2)
$\sigma_l^2 < \sigma^2 < \sigma_m^2$	$(C_1, C_2); (C_1, D_2); (D_1, C_2)$	(C_1, C_2)
$\sigma^2 < \sigma_l^2$	(C_1, C_2)	(C_1, C_2)

Table 3 Investment timing under action commitment and observable delay

in area III. At intermediate levels of uncertainty (area II) a firm only invests early if its rival does so as well.

Note that, like in the AC-game, there is a range of uncertainty $(\sigma_h^2 \le \sigma^2 \le \sigma_x^2)$ in which multiple equilibria occur. However, unlike in the AC-game, there is now a Pareto dominant equilibrium, (D_1, D_2) , which is the "natural focal point" in this range of uncertainty.²⁵

3.4. Aggressive and defensive motive for investment commitment

Table 3 gives a synoptic overview of the investment-timing patterns in AC and OD for different bands of uncertainty. A clear difference in outcomes of the AC- and the OD-game is that leader-follower equilibria occur in the former, but never exist in the latter (see Propositions 1 and 2). To understand this difference, it is helpful to define formally the "aggressive motive" and the "defensive motive" for commitment. We will say that the motive for commitment is "aggressive", if it is optimal to commit given delay by the rival. In both games the aggressive motive for commitment holds for uncertainty levels up to the σ_h^2 -threshold. The motive for commitment is called "defensive", if the firm chooses to commit given commitment by the rival. The uncertainty range over which the defensive motive for commitment exists is different in each game. In the OD-game, defensive commitment continues to hold at higher levels of uncertainty (i.e., up to $\sigma_x^2 > \sigma_h^2$) than aggressive commitment does.²⁶ This is not true in the AC-game, in which the defensive motive for commitment is much weaker. In fact, defensive commitment ceases to occur at much lower uncertainty levels than the maximum uncertainty threshold for aggressive commitment. This suggests that under OD firms tend to commit more for defensive than for aggressive reasons, that is, more to avoid ending up as the follower than to gain a first-mover advantage. Under AC, the opposite is true: the relative strength of the aggressive commitment motive gives rise to investment leadership equilibria.

A related difference between the outcomes in the two games is formally stated in the following proposition.

Proposition 3. Under observable delay (C_1, C_2) continues to be an equilibrium at higher levels of uncertainty than under action commitment.

Proof. See Appendix C. \Box

 $^{^{25}}$ As pointed out by a referee, in an infinitely repeated version of this game – and of the AC-game – a tacit collusion equilibrium, characterised by delay by both firms and sustained by punishment strategies, is likely to emerge.

²⁶ In the OD-game, the motive for commitment is both aggressive and defensive for $\sigma^2 < \sigma_h^2$ (thus, commitment is a dominant strategy in that range of uncertainty).

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In other words, with OD, *both* firms will, at relatively high levels of uncertainty, invest early, whereas this is not the case with AC. With OD, unlike under AC, a firm may still want to commit at relatively high levels of uncertainty provided that its rival does so too, whereas it would delay if its rival decides to delay investment. Hence, the relative strength of the defensive commitment motive explains the symmetry in investment-timing outcomes in the OD-game.

3.5. Profits

For low enough levels of uncertainty ($\sigma^2 < \sigma_h^2$), the expected profit ranking in both games is given by $E\pi_1^{cd} > E\pi_1^{cd} > E\pi_1^{cc} > E\pi_1^{dc}$ for firm 1 (and, similarly, by $E\pi_2^{dc} > E\pi_2^{dd} > E\pi_2^{cc} > E\pi_2^{cd}$ for firm 2). However, as the level of uncertainty increases, the ranking changes since the relative benefits of investment delay become larger: $E\pi_1^{dd} > E\pi_1^{cd}$ and $E\pi_2^{dd} > E\pi_2^{dc}$ for $\sigma^2 > \sigma_h^2$, and $E\pi_1^{dc} > E\pi_1^{cc}$ and $E\pi_2^{cd} > E\pi_2^{cc}$ for $\sigma^2 > \sigma_x^2$. So, are firms better off under AC or under OD? The answer depends on the level of uncertainty to P.

So, are firms better off under AC or under OD? The answer depends on the level of uncertainty that prevails. When the investment-timing outcome with AC is the same as the one with OD (i.e., for $\sigma^2 < \sigma_l^2$ and for $\sigma^2 > \sigma_x^2$), firms' profits are also the same. At intermediate levels of uncertainty ($\sigma_l^2 < \sigma^2 < \sigma_h^2$), a firm is better off under AC than under OD provided that it emerges as the investment leader ($E\pi_1^{cd} > E\pi_1^{cc}$), but is worse off if it ends up being the follower in investment ($E\pi_1^{dc} < E\pi_1^{cc}$). At relatively high levels of uncertainty ($\sigma_h^2 < \sigma^2 < \sigma_x^2$), (D_1, D_2) is guaranteed only in the AC-game, while it is merely one of two possible outcomes in the OD-game. Hence, since both firms prefer (D_1, D_2) to (C_1, C_2), firms will be at least as well off under AC as under OD in this range of uncertainty.

4. Welfare

We now ask which of the four investment-timing equilibria is preferred from a social perspective. We then examine whether these outcomes differ from those generated by the market in the AC- and in the OD-game. Define expected welfare (EW) as the expected sum of consumer surplus (CS) and industry profits:

$$EW = E(CS + \pi_1 + \pi_2) \tag{6}$$

with $CS = Q^2/2$. We have already discussed the commitment-flexibility trade-off from the firm's perspective. From a social perspective, there also exists a trade-off between commitment and delay. Commitment leads firms to strategically overinvest. The capital–output ratio under commitment exceeds the cost-minimising level (obtained by $TC_{k_i}^i = 0$), which is (from expression (2)) given by:

$$\frac{k_i}{q_i} = \eta \tag{7}$$

Although raising the social costs of investment, commitment has positive implications for consumers: higher investment leads to a higher industry output and hence to a lower price, thus increasing expected consumer surplus. However, consumers also benefit from flexibility, which is greatest when both firms delay²⁷ and which becomes more important as uncertainty rises.

²⁷ This follows from the fact that, like profit, consumer surplus is convex in u.







Fig. 5. (a) Market outcomes versus socially preferred outcomes with action commitment. (b) Market outcomes versus socially preferred outcomes with observable delay.

In Fig. 5a and b, the actual investment-timing outcomes are indicated by superscript 'm', while the socially preferred outcomes are superscripted by 's' (σ_w^2 is defined as the uncertainty threshold at which EW(C_1 , C_2) = EW(D_1 , D_2)).²⁸ A shaded label highlights the areas in which the market outcome differs from the socially preferred investment timing. Under AC (Fig. 5a), the market produces too much commitment in area III (and possibly in area IV), but too little commitment in area IIB (and in the leadership equilibria in area IIA). In contrast, under OD (Fig. 5b), the market will *never* produce too little but may generate too much commitment.

This brief positive welfare analysis indicates which investment-timing equilibrium yields the highest welfare for different ranges of uncertainty and allows us to compare these with those generated by the market. However, it is clear that policy intervention could improve welfare since the oligopoly distortion implies that firms are producing too little. In addition, when firms are investing strategically, their capital exceeds the socially efficient level. The standard instruments to deal with these distortions are production subsidies to raise output and capital taxes to reduce the capital-output ratio. However, a package of policies including production subsidies and capital taxes may not be sufficient to reach the first-best. With the first two distortions dealt with, the first-best also requires flexibility at all levels of uncertainty. When uncertainty is high enough, policies to guarantee flexibility will not be needed as firms will have an incentive to maintain flexibility themselves. But, at lower levels of uncertainty the government would need an instrument to ensure flexibility.²⁹ There will be a greater range of σ^2 over which the government will need to adopt "commitment deterrence" policies when firms can observe each other's investment timing before fixing investment levels.³⁰

5. Conclusion

Standard investment theory finds that an increase in uncertainty causes delay. However, under oligopoly with quantity-setting firms, there are also incentives to invest early. This paper analysed investment-timing decisions in an oligopolistic set-up in which firms decide whether to invest early or wait until uncertainty has been resolved. An important finding in the existing literature is that ex ante symmetric firms will end up adopting asymmetric roles ex post: one firm assumes the role of investment leader and the other follows in investment. We re-examined this result and showed that the investment leadership outcome is sensitive to the form that investment commitment takes. The assumption that firms can commit immediately to a specific capital level when they invest early, proves to be crucial for investment leadership to arise. We contrasted a game in which firms commit in one step to an investment level with an alternative game in which investment timing is observed before the investment level is finalised. The latter set-up is consistent with the stylised fact that there is a gestation lag associated with most investment projects. We found that, with this small – but, in our view, realistic – modification in the nature of investment commitment, outcomes are symmetric with either both firms investing early or both adopting a wait-and-see

²⁸ This is the only relevant threshold from a social perspective, since either (C_1, C_2) or (D_1, D_2) will be socially preferred to (C_1, D_2) and (D_1, C_2) .

²⁹ Although expected welfare rises in σ^2 , it would be quite inappropriate to draw policy conclusions from this. What matters for the purpose of this paper is that the coefficient of σ^2 is larger when firms are flexible than when they have committed to their investment at an early stage. This is the case in our model, even under the simplifying assumption of risk neutrality. An assessment of policies that influence uncertainty would, however, require modelling agents' attitudes to risk and stretches beyond the scope of this paper.

³⁰ Commitment deterrence policies are discussed (in an open-economy setting) in Dewit and Leahy (2004).

approach. Crucially, in this case investment leadership no longer emerges as an equilibrium. In addition to contrasting investment-timing outcomes, profits and welfare in the two games were also compared.

We explained that the reason why a subtle change in the assumptions regarding the form of commitment has this effect on outcomes lies in differences in the defensive incentive to commit in the two cases. When firms observe the timing of investment before finalising their capital levels, they have a greater defensive incentive to commit. Case studies such as Ghemawat's analysis of the bulk chemical industry (cited in Section 1) demonstrate that defensive commitment, facilitated by investment gestation lags, is far more than just a theoretical possibility.

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Appendix A. Expected profits in the candidate equilibria

For expected profits in the candidate equilibria under action commitment and observable delay, see Table A.1.

Table A.1

Expected profits in the candidate equilibria under action commitment and observable delay^a

	C_1, C_2	C_1, D_2	D_1, C_2	D_1, D_2
$E\pi_1$	$\gamma [Eq_1^{\rm cc}]^2 + \frac{1}{9}\sigma^2$	$\zeta [Eq_1^{\rm cd}]^2 + \left(\frac{1-\eta}{3-2\eta}\right)^2 \sigma^2$	$\varphi[Eq_1^{ m dc}]^2 + rac{1-\eta/2}{(3-2\eta)^2}\sigma^2$	$\varphi [Eq_1^{\rm dd}]^2 + \frac{1 - \eta/2}{(3 - \eta)^2} \sigma^2$
$E\pi_2$	$\gamma [Eq_2^{\rm cc}]^2 + \frac{1}{9}\sigma^2$	$\varphi[Eq_2^{\rm cd}]^2 + \frac{1 - \eta/2}{(3 - 2\eta)^2}\sigma^2$	$\zeta \left[Eq_2^{ m dc} ight]^2 + \left(rac{1-\eta}{3-2\eta} ight)^2 \sigma^2$	$\varphi[Eq_2^{\rm dd}]^2 + \frac{1 - \eta/2}{\left(3 - \eta\right)^2}\sigma^2$
$a Eq$ $\zeta \equiv 1 - $	$\stackrel{\text{cc}}{_{i}} \equiv Eq_{i}(k_{1}^{\text{cc}}, k_{2}^{\text{cc}}), \\ -2\eta((2-\eta)/(3-2\eta))^{2}$	$Eq_i^{cd} \equiv Eq_i(k_1^{cd}, k_2^{cd}), Eq_i^{dc} \equiv dq_i q \equiv 1 - \eta/2.$	$\equiv Eq_i(k_1^{\rm dc}, k_2^{\rm dc}), Eq_i^{\rm dd} \equiv Eq_i(k_1^{\rm dc}),$	$k_{1}^{\rm dd}, k_{2}^{\rm dd}), \gamma \equiv 1 - (8/9)\eta,$

Appendix B. Derivation of the crucial uncertainty thresholds

Because firms are symmetric, we need only derive the crucial uncertainty thresholds for firm 1. Each threshold is defined as the level of uncertainty, σ_k^2 (with k = l, m, h, x), at which firm 1 is indifferent between commitment and delay, given firm 2's strategy choice (see Table 2).

(i) Under action commitment

Under AC, firm 2 has three possible first-stage actions that can be part of a candidate equilibrium: D_2 , k_2^{cc} and k_2^{dc} . If firm 2 chooses D_2 , the threshold at which firm 1 is indifferent

between commitment and delay is given by σ_h^2 . From the definition of σ_h^2 and using the relevant expressions for $E\pi_1$ in Table A.1, we obtain:

$$\varphi[Eq_1^{\rm dd}]^2 + \frac{1 - \eta/2}{(3 - \eta)^2}\sigma_h^2 = \zeta[Eq_1^{\rm cd}]^2 + \left(\frac{1 - \eta}{3 - 2\eta}\right)^2\sigma_h^2$$

Rearranging terms then yields:

$$\sigma_h^2 = \frac{\zeta [Eq_1^{\rm cd}]^2 - \varphi [Eq_1^{\rm dd}]^2}{\frac{1 - \eta/2}{(3 - \eta)^2} - \left(\frac{1 - \eta}{3 - 2\eta}\right)^2}$$
(B.1)

If firm 2 chooses k_2^{cc} , the uncertainty threshold at which firm 1 is indifferent between commitment and delay is given by σ_m^2 . From the definition of σ_m^2 , using the relevant expressions for $E\pi_1$ in Table A.1 and with

$$E\pi_1(D_1, k_2^{cc}) = \varphi[Eq_1(Er_1^{d}(k_2^{cc}), k_2^{cc})]^2 + \frac{1 - \eta/2}{(3 - 2\eta)^2}\sigma^2$$

we obtain (after some rearranging of terms):

$$\sigma_m^2 = \frac{\gamma [Eq_1^{cc}]^2 - \varphi [Eq_1(Er_1^d(k_2^{cc}), k_2^{cc})]^2}{\frac{1 - \eta/2}{(3 - 2\eta)^2} - \frac{1}{9}}$$
(B.2)

If firm 2 chooses k_2^{dc} , the uncertainty threshold at which firm 1 is indifferent between commitment and delay is given by σ_l^2 . From the definition of σ_l^2 , using the relevant expressions for $E\pi_1$ in Table A.1 and with $E\pi_1(C_1, k_2^{dc}) = \gamma [Eq_1(r_1^c(k_2^{dc}), k_2^{dc})]^2 + (1/9)\sigma^2$, we obtain:

$$\sigma_l^2 = \frac{\gamma [Eq_1(r_1^c(k_2^{dc}), k_2^{dc})]^2 - \varphi [Eq_1^{dc}]^2}{\frac{1 - \eta/2}{(3 - 2\eta)^2} - \frac{1}{9}}$$
(B.3)

From expression (5) and using the expressions given in Table 1 in (B.1)–(B.3), we obtain $0 < \sigma_l^2 < \sigma_m^2 < \sigma_h^2$.

(ii) Under observable delay

Under OD, firm 2 has two possible timing choices: D_2 and C_2 . If firm 2 chooses D_2 , the threshold at which firm 1 is indifferent between commitment and delay is σ_h^2 , which is given by (B.1). If firm 2 chooses C_2 , the uncertainty threshold at which firm 1 is indifferent between commitment and delay is given by σ_x^2 . From the definition of σ_x^2 and using the relevant expressions in Table A.1, we obtain:

$$\sigma_x^2 = \frac{\gamma [Eq_1^{\rm cc}]^2 - \varphi [Eq_1^{\rm dc}]^2}{\frac{1 - \eta/2}{(3 - 2\eta)^2} - \frac{1}{9}}$$
(B.4)

From expression (5) and using the expressions of Table 1 in (B.1) and (B.4), we obtain $\sigma_h^2 < \sigma_x^2$.

Appendix C. Proofs of propositions

Proof of Proposition 1. For each candidate equilibrium, we determine the uncertainty range over which it actually is an equilibrium. Without loss of generality (as firms are symmetric), we take the perspective of firm 1.

(a) The candidate equilibrium (C_1, C_2) :

This is an equilibrium at $\sigma^2 = 0$, when there are no flexibility advantages of delaying and both firms commit, regardless of the timing choice of their rival. For $\sigma^2 \le \sigma_m^2$, given k_2^{cc} firm 1 will not wish to deviate from k_1^{cc} (by symmetry, firm 2 will not want to deviate from k_2^{cc} in that region). Thus, (C_1, C_2) will be an equilibrium for $\sigma^2 \le \sigma_m^2$. When $\sigma^2 > \sigma_m^2$, (C_1, C_2) cannot be an equilibrium since we have $E\pi_1(C_1, k_2^{cc}) < E\pi_1(D_1, k_2^{cc})$, implying that firm 1 wants to delay.

(b) The candidate equilibrium (D_1, D_2) :

For $\sigma^2 < \sigma_h^2, E\pi_1(D_1, D_2) < E\pi_1(C_1, D_2)$, implying that (D_1, D_2) cannot be an equilibrium. For $\sigma^2 \ge \sigma_h^2, E\pi_1(D_1, D_2) \ge E\pi_1(C_1, D_2)$, implying that firm 1 does not wish to deviate from delay and hence (D_1, D_2) is an equilibrium.

(c) The candidate equilibria (C_1, D_2) and (D_1, C_2) :

Threshold σ_h^2 also demarcates the maximum uncertainty upper limit for the leader–follower equilibria, (C_1, D_2) and (D_1, C_2) . In other words, (C_1, D_2) (and by symmetry (D_1, C_2)) cannot be an equilibrium when $\sigma^2 > \sigma_h^2$, since then $E\pi_1(C_1, D_2) < E\pi_1(D_1, D_2)$. The lowest of the thresholds, σ_l^2 , provides the lower bound for leader–follower equilibria. When $\sigma^2 < \sigma_l^2$, we have $E\pi_1(C_1, k_2^{dc}) > E\pi_1(D_1, k_2^{dc})$, and hence firm 1 will wish to deviate from delay given that firm 2 commits to the investment leadership capital level, k_2^{dc} . Given $\sigma_h^2 > \sigma_m^2 > \sigma_l^2 > 0$, Proposition 1 (i)–(v) follow from (a)–(c).

Proof of Proposition 2. The investment-timing equilibria under OD are determined by a method similar to the one described in Proof of Proposition 1, but now using the thresholds σ_x^2 and σ_h^2 , with $\sigma_x^2 > \sigma_h^2 > 0$.

(a) The candidate equilibria (D_1, D_2) and (C_1, C_2) : Since $\sigma_x^2 > \sigma_h^2$, when $\sigma^2 > \sigma_x^2$ each firm will play delay, regardless of its rival's timing choice. Hence, (D_1, D_2) is the unique equilibrium when $\sigma^2 > \sigma_x^2$. For $\sigma^2 < \sigma_h^2$, commitment will be played by each firm, regardless of its rival's timing choice. Hence, (C_1, C_2) is the unique equilibrium for $\sigma^2 < \sigma_h^2$. At $\sigma_h^2 \le \sigma^2 \le \sigma_x^2$, commitment is the best response to rival commitment, while delay is the best response to rival delay. Therefore, both (D_1, D_2) and (C_1, C_2) are equilibria for $\sigma_h^2 \le \sigma^2 \le \sigma_x^2$.

(b) The candidate equilibria (C_1, D_2) and (D_1, C_2) :

When commitment is an optimal response to delay $(\sigma^2 \le \sigma_h^2)$, delay is not an optimal response to commitment, and, when delay is an optimal response to commitment ($\sigma^2 \ge \sigma_x^2$), commitment is not an optimal response to delay. Hence, (C_1, D_2) and (D_1, C_2) are never equilibria.

Proof of Proposition 3. From part (a) of Proof of Proposition 2, it follows that with OD (C_1 , C_2) is an equilibrium for $\sigma^2 \le \sigma_x^2$. From part (a) of Proof of Proposition 1, it follows that with AC (C_1, C_2) is an equilibrium for $\sigma^2 \le \sigma_m^2$. Since $\sigma_m^2 < \sigma_x^2$ (from Appendix B), Proposition 3 must be true. \Box

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