R&D spillovers and the case for industrial policy in an open economy

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In this paper we consider the case for subsidies towards firms which generate R&D spillovers in open economies. We show that in the presence of strategic behaviour by firms many expected results are overturned. Local R&D spillovers to other domestic firms may justify an R&D tax rather than a subsidy; R&D cooperation by local firms over-internalises the externality and also justifies an R&D tax; and international spillovers which benefit foreign firms may justify a subsidy, even though the government cares only about the profits of home firms.

1. Introduction

The now-conventional view of R&D spillovers in the 1990s is that they are pervasive, quantitatively important and a justification for industrial policy. (See for example, *The Economist*, 1996). But these views were also held by Marshall in the 1890s and they were made explicit by Pigou in his 1912 *Economics of Welfare*. In this paper we ask: what have we learned in the intervening hundred years? More specifically, what additional arguments for industrial policy towards industries characterised by R&D spillovers are provided by recent work in the theory of international trade policy?[1]

Most recent discussions of R&D spillovers in open economies, such as the work on endogenous growth of Grossman and Helpman (1991), have assumed they occur in industries characterised by monopolistic competition. The combination of free entry (so long-run profits are competed away) and no strategic interdependence between firms, leads to models which, while complicated in other respects, have very simple implications for policy. R&D spillovers towards other domestic firms generate an externality which should be subsidised. In this paper we explore a different approach to R&D subsidies, which combines them with the other half of the 'new' trade theory, the theory of strategic trade policy. We show that even

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[1] Throughout this paper we use 'industrial policy' as a synonym for R&D subsidies. As a referee has pointed out, this ignores many other dimensions to industrial policy, such as the provision of information and the encouragement of R&D cooperation. In the latter context, we show in Leahy and Neary (1997) that oligopolistic firms in a closed economy will always have a private incentive to engage in R&D cooperation, so the relevant question for public policy is when this should be discouraged.

simple oligopoly models, where profits persist and firms behave strategically, have surprising implications for optimal policy in the presence of R&D spillovers.

Most previous work on R&D spillovers and strategic behaviour has focused on intra-industry spillovers in a closed economy.[2] This issue was addressed by Spence (1984) who showed that spillovers tend to weaken the strategic incentive to invest in R&D. d'Aspremont and Jacquemin (1988) demonstrated in a linear duopoly model that R&D cooperation can internalise this externality. Their work was extended and generalised by Suzumura (1992), who considered many firms and general demands and by Kamien *et al.* (1992) and Ziss (1994) who also considered price competition. Leahy and Neary (1997) synthesise this literature, disentangling the effects of cooperation from those of strategic behaviour, and consider the implications for industrial policy in a closed economy.

R&D spillovers in an open economy raise additional issues which are inherently more complex. This is because the assumption of symmetry between firms which is normally made in closed economy models cannot be maintained. In the first place, a nationalistic government cares only about the profits of home firms. In the second place, the nature of spillovers is more complicated. They may be either local or international,[3] and either inter- or intra-industry. Intuitively we might expect that local spillovers justify subsidising R&D and international spillovers justify taxing it. For example, Spencer (1986) argues that a domestic industry will be a better candidate for R&D subsidies the lower are spillovers of new technology to foreign firms. However, we shall see that strategic behaviour by firms may reverse these expected results.

The other issue raised by R&D spillovers in an open economy is whether they can be internalised by cooperation between domestic firms, thus rendering industrial policy redundant. Once again, it turns out that this presumption is overturned if firms behave strategically.

The plan of the paper is as follows. Section 2 sets up the model, which builds on Spencer and Brander (1983) and Leahy and Neary (1996). Like these papers we examine trade and industrial policy in a two-period duopoly model in which firms choose R&D and output. However, these earlier papers did not discuss R&D spillovers. In Section 3 we explore how strategic behaviour and R&D cooperation affect optimal industrial policy in a model when spillovers are local and inter-industry. In Section 4 we explore strategic effects in the presence of international spillovers. In Section 5 we examine the linear case in order to obtain more definite results. Finally, Section 6 concludes with a summary of results.

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[2] International R&D spillovers between firms in the same industry have been examined by Cheng (1987), Motta (1996), and Neary and O'Sullivan (1997). Inter-industry spillovers in a closed economy have been examined by Katsoulacos and Ulph (1998).

[3] The distinction between local and international spillovers is studied in a different context by Rivera-Batiz and Oliva (1996). Keller (1998) presents empirical evidence suggesting that the parameter for local inter-industry spillovers (β in our notation) is between 20% and 50%, while that for international intra-industry spillovers (ε in our notation) is between 50% and 95%.

2. The model

Consider a model in which there are two oligopolistic industries.[4] In each industry a different domestic firm and a foreign firm export a homogeneous commodity to a third market. The third-country demand function facing industry i is

$$p_i = p_i(q_i + q_i^*) \qquad -b_i = p_i'(q_i + q_i^*) < 0 \qquad i = 1, 2 \tag{1}$$

where q_i represents home exports and q_i^* represents foreign exports in sector i. (An asterisk will often be used to represent a foreign variable.) We also define $r_i = (q_i + q_i^*)b_i'/b_i$, the elasticity of the slope of the industry i demand function. This will prove useful later. We distinguish two time periods. Period 1 is the pre-market R&D phase and period 2 is the output phase.

The home and foreign firms in each industry choose R&D levels x_i and x_i^* respectively for period 1 and outputs for period 2. Each firm's marginal production costs are constant in output but are declining in its own R&D expenditure and (through spillover effects) in the R&D expenditures of home firms in other industries and of foreign firms in the same industry. Thus in the general case there are both international intra-industry R&D spillovers and local inter-industry spillovers. The marginal costs of a typical home firm can thus be written as

$$c_i = c_i(x_i, x_j, x_i^*), j \neq i \qquad \frac{\partial c_i}{\partial x_i} = -\theta_i < 0 \tag{2}$$

where θ_i measures the cost-reducing effect of R&D. Similarly, a typical foreign firm faces marginal production costs

$$c_i^* = c_i^*(x_i^*, x_i) \qquad \frac{\partial c_i^*}{\partial x_i^*} = -\theta_i^* < 0 \tag{3}$$

where we ignore the possibility of inter-industry spillovers between foreign firms. Local inter-industry spillovers are captured by the parameter β_i

$$\beta_i \equiv -\frac{\partial c_i/\partial x_j}{\theta_i} \qquad 0 \leq \beta_i \leq 1 \tag{4}$$

and international intra-industry spillovers are represented by the parameters ε_i and ε_i^*

$$\varepsilon_i \equiv -\frac{\partial c_i/\partial x_i^*}{\theta_i} \qquad \varepsilon_i^* \equiv -\frac{\partial c_i^*/\partial x_i}{\theta_i^*} \qquad 0 \leq \varepsilon_i, \varepsilon_i^* \leq 1 \tag{5}$$

In practice, we only consider the cases where spillovers are either intra- or inter-industry. Finally, in period 1 the home and foreign firms incur R&D costs of $\Gamma_i(x_i)$ and $\Gamma_i^*(x_i^*)$ respectively.

The typical home firm chooses its levels of R&D and output to maximise profits

$$\pi^i = R^i(q_i, q_i^*) - (c_i - s_i)q_i - \Gamma_i(x_i) + \sigma_i x_i \tag{6}$$

where $R^i = p_i q_i$ is total revenue (depending, from (1), on q_i and q_i^*); s_i is the per unit export subsidy; and σ_i is the per unit R&D subsidy. The first-order condition for R&D depends on assumptions about move order and inter-firm cooperation, to

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[4] Extension to n industries is straightforward and adds nothing important to the analysis.

be considered below. However, in all the equilibria considered the firms choose their outputs simultaneously given the export subsidy and R&D levels. The home first-order condition for output is therefore

$$\frac{\partial \pi_i}{\partial q_i} = R_q^i - c_i + s_i = 0 \tag{7}$$

This first-order condition defines the home firm's output reaction function conditional on the export subsidy and the R&D levels. The typical foreign firm faces a similar problem, except that we make the simplifying assumption that it does not receive subsidies. Its profits therefore equal $R^{*i} - c_i^* q_i^* - \Gamma_i^*(x_i^*)$ and its first-order condition for output is

$$\frac{\partial \pi_i^*}{\partial q_i^*} = R_{q*}^{*i} - c_i^* = 0 \tag{8}$$

Next, consider government behaviour. We assume that the home government (the only one which is policy active) maximises a welfare function equal to the sum of profits net of subsidy payments in the two sectors

$$W = \sum_{i=1}^{2} \{\pi_i - s_i q_i - \sigma_i x_i\} = \sum_{i=1}^{2} \{R^i - c_i q_i - \Gamma_i(x_i)\} \tag{9}$$

Totally differentiating (9) and using the first-order condition (7) to simplify yields an expression for welfare change which will prove useful later

$$dW = \sum_{i=1}^{2} \left\{ -s_i dq_i + R_{q*}^i dq_i^* - q_i dc_i - \Gamma_i' dx_i \right\} \tag{10}$$

This shows the proximate determinants of welfare in the model. A rise in home exports raises the deadweight loss of subsidies and so lowers welfare; a rise in foreign exports lowers home profits ($R_{q*}^i < 0$) and so lowers welfare; finally, welfare rises with any fall in unit production costs but falls with extra R&D to the extent that additional direct costs are incurred.

The primary focus of the paper is on the implications for optimal industrial policy of strategic behaviour by firms. To investigate this, we contrast two equilibria, which differ in their assumptions about the ability of firms to commit. First, the case in which firms can commit in period 1 to future outputs will be referred to as a 'Full Commitment Equilibrium' (FCE). Second, the case in which firms cannot commit to future output will be called a 'Government-Only Commitment Equilibrium' (GCE). In FCE, firms simultaneously choose their R&D and output levels at the start of period 1 and cannot use R&D strategically to affect rival output. By contrast, in GCE firms choose their R&D before output and therefore have a strategic incentive to vary their R&D in order to manipulate the output of their rival. In both FCE and GCE we assume that the government can choose both its R&D subsidy and its export subsidy before firms choose their actions.[5]

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[5] For models in which governments cannot commit to their future second-period intervention see Leahy and Neary (1996, 1997, 1999) and Neary and Leahy (1996).

A second issue with which the paper is concerned is the effect of inter-industry R&D cooperation on optimal policy. To examine this we compare the case in which home firms cooperate on their R&D levels with the case in which they choose their R&D non-cooperatively.[6] With local spillovers there are therefore four different combinations of assumptions (FCE or GCE, with or without R&D cooperation) and in each case we assume that the equilibrium is subgame perfect.

3. Local inter-industry spillovers

3.1 The non-strategic and non-cooperative benchmark

In this section we consider the case in which there are positive spillovers between domestic firms but no international spillovers. We will first consider the benchmark case in which firms do not cooperate on their R&D levels and do not choose their R&D strategically. This is the non-cooperative FCE case in which the government chooses its subsidies in the first stage and the firms choose R&D levels and output levels simultaneously in the second stage. The typical home firm's first-order condition for R&D is thus

$$\frac{\partial \pi_i}{\partial x_i} = \theta_i q_i - \Gamma_i' + \sigma_i = 0 \tag{11}$$

and each foreign firm's first-order condition for R&D is

$$\frac{\partial \pi_i^*}{\partial x_i^*} = \theta_i^* q_i^* - \Gamma_i^{*\prime} = 0 \tag{12}$$

In the first stage of the game the government sets its two policy variables, the export subsidy and the R&D subsidy, in each of the two markets. In effect the government uses these four policy instruments to choose the home firms' R&D and output levels before the foreign firms choose their actions. The foreign R&D and outputs then respond to the home actions according to generalised reaction functions which are derived by combining the two foreign first-order conditions (8) and (12). These generalised reaction functions can be written as

$$q_i^* = Q_i^{FL}(q_i) \qquad x_i^* = X_i^{FL}(q_i) \tag{13}$$

where the superscript FL denotes FCE with local spillovers. As shown in the Appendix these functions are independent of home R&D in either industry.

We are now ready to see how changes in welfare are related to changes in output and R&D levels. Substitute in (10), the expression for welfare change, from: the home firms' first-order condition for R&D, (11); the total derivative of q_i^* from (13);[7] and the change in marginal production costs (2), which in this case is $dc_i = -\theta_i(dx_i + \beta_i dx_j)$, with the term $\beta_i \theta_i dx_j (i, j = 1, 2; j \neq i)$ representing the inter-industry spillover. After some manipulations we obtain

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[6] International cooperation with spillovers is discussed in Neary and O'Sullivan (1997).

[7] The derivatives of (13) are examined in the Appendix.

$$dW = \sum_{i=1}^{2} \left\{ \left(-s_i + R_{q*}^i \frac{dQ_i^{FL}}{dq_i} \right) dq_i - (\sigma_i - \beta_j \theta_j q_j) dx_i \right\} \tag{14}$$

This gives the change in welfare as a function of changes in the four variables chosen by domestic firms. However, since the government has four instruments at its disposal (two subsidies in each industry) it can be viewed as controlling all four right-hand side variables directly. Thus the optimal export subsidies are obtained from (14) by setting the coefficients of dq_i equal to zero for each i

$$s_i^{FL} = R_{q*}^i \frac{dQ_i^{FL}}{dq_i} \tag{15}$$

where $R_{q*}^i = -b_i q_i < 0$. The right-hand side of (15) captures the standard Brander–Spencer (1985) rent-shifting effect. This export subsidy must be positive when foreign output is a strategic substitute for home output.[8] In that case the subsidy, by committing the home firm to larger output, reduces the output of the foreign firm in that industry which in turn raises home profits and welfare. Unlike in the standard Brander–Spencer case the export subsidy is being used here not just to reduce foreign output directly but also to reduce foreign R&D, which has an additional indirect effect tending to reduce foreign output. The derivative dQ_i^{FL}/dq_i captures both the direct effect of home output and the indirect effect that works through changes in foreign R&D.

The optimal R&D subsidies are obtained from (14) by setting the coefficients of dx_i equal to zero for each i to give

$$\sigma_i^{FL} = \beta_j \theta_j q_j \tag{16}$$

These subsidies are positive since they correct for the positive R&D externalities within the economy. This confirms the presumption that R&D spillovers justify a subsidy.

There is a clear division of labour between the export subsidy and the R&D subsidy in this case. The export subsidy plays a purely rent-shifting role while the R&D subsidy is targeted towards ensuring that R&D is chosen at the socially cost-minimising level; i.e. the level at which the marginal social benefit of R&D, $\theta_i q_i + \beta_j \theta_j q_j$, equals its marginal social cost Γ_i'. This may be confirmed by substituting from (16) into (11).

3.2 The effect of strategic behaviour without R&D cooperation

We will now begin our analysis of the effect of strategic behaviour on optimal policy. We first consider the government-only commitment equilibrium in the absence of R&D cooperation. In the GCE case firms play strategically taking account of the effect of their R&D on the output of the rival firm. This does not

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[8] The relationship between strategic substitutability and the sign of dQ_i^{FL}/dq_i is discussed in the Appendix. Foreign output is a strategic substitute for home output if the marginal profitability of foreign output falls in home output; i.e. if R_{q*q}^{*i} is negative. This may be considered the normal case under Cournot competition.

affect the first-order conditions for home and foreign output which are, as in all cases, given by (7) and (8) above. However, the first-order condition for R&D for a typical home firm now becomes, instead of (11)

$$\frac{d\pi_i}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i^*} \frac{dq_i^*}{dx_i} = 0 \tag{17}$$

where

$$\frac{\partial \pi_i}{\partial q_i^*} = R_{q*}^i \qquad \frac{dq_i^*}{dx_i} = -\frac{\theta_i E_i}{b_i} \qquad E_i \equiv \frac{1 + \alpha_i^* r_i}{3 + r_i}$$

Here $\alpha_i = q_i/(q_i + q_i^*)$ is the market share of the home firm in industry i and $\alpha_i^* = 1 - \alpha_i$ is that of the foreign firm in industry i. The parameter E_i is positive when foreign output is a strategic substitute for home output.[9] Equation (17) simplifies to

$$(1 + E_i)\theta_i q_i - \Gamma_i' + \sigma_i = 0 \tag{18}$$

Comparing this with eq. (11) for the FCE case, we see that, provided outputs are strategic substitutes, the home firms strategically over-invest in R&D in the manner of Spence (1977), Dixit (1980), and Brander and Spencer (1983). The same is true of the foreign firms, whose first-order condition can similarly be written as

$$\frac{d\pi_i^*}{dx_i^*} = \frac{\partial \pi_i^*}{\partial x_i^*} + \frac{\partial \pi_i^*}{\partial q_i} \frac{dq_i}{dx_i^*} = 0 \tag{19}$$

In the first stage the government sets its policy variables anticipating that foreign R&D and output will respond according to generalised reaction functions. In the GCE case these reaction functions are derived by combining the foreign firms' first-order conditions, (8) and (19), and can be written as

$$q_i^* = Q_i^{GL}(q_i) \qquad x_i^* = X_i^{GL}(q_i) \tag{20}$$

where the superscript GL denotes GCE with local spillovers. As in the FCE case these reaction functions are independent of home R&D. Proceed by substituting the home firm's first-order condition for R&D, (11), and the derivatives of (20) in the expression for welfare change, (10). The resulting optimal export subsidies have the same form as those represented in (15) for the FCE case. In the GCE case the optimal export subsidy to sector i can be written as

$$s_i^{GL} = R_{q*}^i \frac{dQ_i^{GL}}{dq_i} \tag{21}$$

As in the FCE case the export subsidy is used to shift rent from foreigners to the home country. As shown in the Appendix, strategic substitutability still works towards a positive optimal subsidy though it is no longer sufficient to ensure it.

The optimal R&D subsidy is

$$\sigma_i^{GL} = \beta_j \theta_j q_j - E_i \theta_i q_i \tag{22}$$

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[9] To see this note that the numerator $1 + \alpha_i^* r_i$ equals $-R_{q*q}^{*i}/b_i$; and as shown in the Appendix the denominator $3 + r_i$ must be positive to ensure stability.

The first term on the right-hand side of (22) can be interpreted in the same manner as the corresponding term in (16). The government encourages investment in R&D because there is a positive externality. The second term, as in Spencer and Brander (1983), corrects for strategic over-investment in R&D by the home firm. As in FCE the R&D subsidy ensures that home R&D is at the socially cost minimising level. This can be seen by using (22) to eliminate the R&D subsidy in (18). Summarising:

Proposition 1 The optimal R&D subsidy in GCE is the sum of a positive term which corrects for the firm's failure to internalise the R&D externality and (given strategic substitutability) a negative term which corrects for the firm's strategic over-investment in R&D.

Export subsidies and R&D subsidies under GCE cannot be directly compared with those under FCE as they are evaluated at different levels of output and R&D. However, the additional term in the GCE R&D subsidy makes it presumptively smaller than its counterpart under FCE when foreign output is a strategic substitute for home output. The optimal policy towards R&D may even be a tax if the strategic effect outweighs the externality effect. In the special case of no externalities ($\beta_j = 0$) considered by Spencer and Brander (1983) R&D should definitely be taxed.

3.3 The effect of R&D cooperation without strategic behaviour

We begin our analysis of the effect of R&D cooperation on the optimal industrial policy by considering the benchmark FCE case in which the firms do not choose R&D strategically. Given R&D levels the industries are unlinked so cooperation between home firms in their choice of output is not an issue. As before, each firm chooses its output to maximise its own profits and the resulting first-order conditions are given by (7) and (8) above. Since there are no inter-industry spillovers among foreign firms they have nothing to gain by cooperating with each other and foreign R&D first-order conditions are given by (12) above. Hence only the R&D behaviour of home firms is affected by cooperation. We will assume that the levels of home R&D are chosen to maximise the home firms' joint profits but that the decision to cooperate does not itself affect the spillover parameters.[10] In the case of cooperative FCE each home firm chooses its R&D to maximise their joint profit function

$$\Pi = \sum_{i=1}^{2} \left\{ R^i - (c_i - s_i)q_i - \Gamma_i + \sigma_i x_i \right\} \tag{23}$$

This implies the first-order condition

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[10] Of course the decision to cooperate in general changes the levels of R&D, which alters the spillover parameters unless c_i in (2) and c_i^* in (3) are linear functions. For an alternative account in which the decision to cooperate itself affects the spillover parameters see Katz (1986), Kamien *et al.* (1992), Motta (1996), and Katsoulacos and Ulph (1998).

$$\frac{\partial \Pi}{\partial x_i} = \theta_i q_i + \beta_j \theta_j q_j - \Gamma_i' + \sigma_i = 0 \tag{24}$$

Compared to (11) this has an additional term in the spillover parameter β_j. The cooperative internalises the R&D externality and sets the marginal social benefit of R&D equal to its marginal cost inclusive of subsidies. As in the non-cooperative case the home government effectively chooses the two R&D levels and the two output levels to maximise welfare with foreign R&D and output reacting according to (13). Since the behaviour of the foreign firms is unchanged by the decision of the home firms to cooperate, the foreign generalised reaction functions are also unchanged. The optimal subsidies are obtained by substituting (24) and the total derivatives of (13) in the expression for welfare change, (10). The optimal export subsidy is identical both in form and magnitude to that under non-cooperative FCE, as given in equation (15).[11] The optimal R&D subsidy is

$$\sigma_i^{FLC} = 0 \tag{25}$$

The superscript C indicates R&D cooperation. Compared to the R&D subsidy under non-cooperative FCE the term in β_j disappears because the firms now internalise the externality so there is no need for government intervention.

Proposition 2 When firms do not behave strategically (i.e. in FCE), the optimal R&D subsidy is zero with cooperation on R&D, and hence is lower than the optimal R&D subsidy with no cooperation on R&D for any positive spillover parameter β_j.

Another way of expressing this result is that the Coase Theorem applies in this case: R&D cooperation is a perfect substitute for industrial policy.

3.4 The effect of strategic behaviour with R&D cooperation

We now consider how strategic behaviour interacts with R&D cooperation and how this affects optimal policy. In the GCE case the home cooperative takes account of how its R&D affects the output of the foreign firm. The first-order condition for home R&D is

$$\frac{d\Pi}{dx_i} = \frac{\partial \Pi}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i^*} \frac{dq_i^*}{dx_i} + \frac{\partial \pi_j}{\partial q_j^*} \frac{dq_j^*}{dx_i} \qquad \text{where} \qquad \frac{dq_j^*}{dx_i} = -\frac{\beta_j \theta_j E_j}{b_j} \tag{26}$$

As before E_j is positive if outputs are strategic substitutes in industry j. This first-order condition can be rewritten as

$$(1 + E_i)\theta_i q_i + (1 + E_j)\beta_j \theta_j q_j - \Gamma_i' + \sigma_i = 0 \tag{27}$$

In the first stage the government chooses its subsidies, anticipating that the foreign firms will react according to the GCE reaction functions in (20). The resulting optimal export subsidies are identical to those under non-cooperative GCE and are given in (21). The optimal R&D subsidy is

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[11] The optimal subsidies can be directly compared because the real equilibria with optimal policy are the same whether firms cooperate or not.

$$\sigma_i^{GLC} = -(E_i\theta_i q_i + \beta_j E_j \theta_j q_j) \tag{28}$$

While in the no-cooperation case higher spillovers contribute to a higher R&D subsidy, in this case they work surprisingly in the opposite direction. To find the effect on optimal policy of R&D cooperation when firms play strategically we can subtract the cooperative R&D subsidy from the non-cooperative R&D subsidy to get[12]

$$\sigma_i^{GL} - \sigma_i^{GLC} = \beta_j \theta_j q_j (1 + E_j) \geq 0 \tag{29}$$

Unlike in the non-strategic FCE case, R&D cooperation under GCE does not render industrial policy redundant.

Proposition 3 When firms behave strategically (i.e. in GCE), the optimal R&D subsidy to firm i is negative with R&D cooperation (given strategic substitutability), and it is smaller than the optimal subsidy without R&D cooperation for any positive spillover parameter β_j.

The Coase Theorem does not apply in this case because the cooperative 'over-internalises' the externality. It over-invests in R&D in both industries to obtain a strategic advantage over both foreign firms and taxes on both R&D levels are required to restrain it from this socially wasteful activity.

4. International spillovers

4.1 The non-strategic benchmark

In this section we turn to examine the case in which there are international spillovers between firms. To keep the analysis from becoming excessively complicated we will assume no inter-industry spillovers between firms: $\beta_i = 0$. In all cases the firms' first-order conditions for output are given in (7) and (8) above and under FCE the firms' first-order conditions for R&D are given in (11) and (12). The only difference is that now the marginal costs and the θ's depend on the R&D levels of foreign rivals as well as on those of the firms themselves. This is due to the presence of international R&D spillovers. As demonstrated in the Appendix this difference implies that the foreign generalised reaction functions for R&D and output depend not just on home output as in the case without international spillovers but on the levels of home R&D. Intuitively this is because home R&D now directly affects the foreign first-order condition through the spillover effects. The resulting reaction functions under FCE with international spillovers can be written as

$$q^* = Q^{FI}(q, x) \qquad x^* = X^{FI}(q, x) \tag{30}$$

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[12] As in the previous sub-section, the optimal subsidies can be directly compared because the real equilibria in GCE are the same whether firms cooperate or not.

where the superscript I indicates international spillovers. We can drop industry subscripts in this section because there are no links between industries in the absence of local spillovers and hence each industry can be treated separately.

Proceed as before by using the home firms' first-order condition for R&D, (11), and the total derivatives of (30) in the expression for welfare change, (10), to obtain

$$dW = \left(-s + R_{q*} \frac{\partial Q^{FI}}{\partial q} + \varepsilon \theta q \frac{\partial X^{FI}}{\partial q} \right) dq + \left(-\sigma + R_{q*} \frac{\partial Q^{FI}}{\partial x} + \varepsilon \theta q \frac{\partial X^{FI}}{\partial x} \right) dx \quad (31)$$

Setting the coefficient of dq equal to zero yields the optimal export subsidy

$$s^{FI} = R_{q*} \frac{\partial Q^{FI}}{\partial q} + \varepsilon \theta q \frac{\partial X^{FI}}{\partial q} \quad (32)$$

The first term on the right-hand side is the usual rent-shifting effect. We show in the Appendix that this must be positive if foreign output is a strategic substitute for home output. We also show in the Appendix that if foreign output is a strategic substitute for home output then the second term on the right-hand side is negative. By raising home output a subsidy would reduce foreign R&D and thus reduce the beneficial spillover enjoyed by the home firm. Hence with strategic substitutes this effect works against a positive export subsidy.

The optimal R&D subsidy is also obtained from (31) by setting the coefficient of dx equal to zero

$$\sigma^{FI} = R_{q*} \frac{\partial Q^{FI}}{\partial x} + \varepsilon \theta q \frac{\partial X^{FI}}{\partial x} \quad (33)$$

Unlike in the local spillovers case there is now a rent-shifting role for the R&D subsidy. The government can reduce foreign output thus shifting rent to the home country by taxing R&D and thus reducing the beneficial spillovers to foreigners. The first term on the right hand side captures this intertemporal rent-shifting effect of R&D. This term is negative because q^* is directly increasing in x.

The second term on the right-hand side captures the welfare effect of more home R&D via its effect on the level of foreign R&D. As shown in the Appendix, an increase in home R&D leads to more foreign R&D and thus increases beneficial spillovers to home firms. Hence this term, which we call the 'spillback' effect, works towards a positive R&D subsidy.

Proposition 4 The optimal R&D subsidy under FCE with international spillovers is negative if and only if the negative rent-shifting effect outweighs the positive spillback effect.

Note the apparent paradox: the fact that R&D spillovers benefit the foreign firm provides a motive for subsidisation, even though foreign profits are of no concern to the home government and even without strategic behaviour.

4.2 The effect of strategic behaviour

We consider next the GCE case with international spillovers in which the firms choose their R&D strategically. The first-order condition for home R&D is

$$\frac{d\pi}{dx} = \frac{\partial\pi}{\partial x} + \frac{\partial\pi}{\partial q^*}\frac{dq^*}{dx} = [1 + A(\overline{\varepsilon^*} - \varepsilon^*)]\theta q - \Gamma' + \sigma = 0 \tag{34}$$

where

$$A \equiv \left(\frac{2 + \alpha r}{3 + r}\right)\frac{\theta^*}{\theta} > 0 \qquad \text{and} \qquad \overline{\varepsilon^*} \equiv \left(\frac{1 + \alpha^* r}{2 + \alpha r}\right)\frac{\theta}{\theta^*} \tag{35}$$

By comparison with (11), the home firm strategically over-invests in R&D when the spillover to the foreign firm is below the threshold level $\overline{\varepsilon^*}$. As in d'Aspremont and Jacquemin (1988), the threshold equals $\frac{1}{2}$ when demands are linear ($r = 0$) and R&D is equally efficient in both firms ($\theta = \theta^*$). The foreign first-order condition can similarly be written as

$$\frac{d\pi^*}{dx^*} = [1 + A^*(\overline{\varepsilon} - \varepsilon)]\theta^* q^* - \Gamma^{*'} = 0 \tag{36}$$

where

$$A^* \equiv \left(\frac{2 + \alpha^* r}{3 + r}\right)\frac{\theta}{\theta^*} > 0 \qquad \text{and} \qquad \overline{\varepsilon} \equiv \left(\frac{1 + \alpha r}{2 + \alpha^* r}\right)\frac{\theta^*}{\theta} \tag{37}$$

The government sets both the R&D subsidy and export subsidy anticipating that the foreign firm responds according to the generalised reaction functions which are derived by combining (8) and (36). These generalised reaction functions can be written as

$$q^* = Q^{GI}(q, x) \quad x^* = X^{GI}(q, x) \tag{38}$$

The optimal export subsidy has the same form as (32) in the FCE case

$$s^{GI} = R_{q*}\frac{\partial Q^{GI}}{\partial q} + \varepsilon\theta q\frac{\partial X^{GI}}{\partial q} \tag{39}$$

These terms have the same interpretation as in the FCE case although, as shown in the Appendix, it is somewhat more difficult to sign them. Finally, the optimal R&D subsidy is

$$\sigma^{GI} = R_{q*}\frac{\partial Q^{GI}}{\partial x} + \varepsilon\theta q\frac{\partial X^{GI}}{\partial x} - A(\overline{\varepsilon^*} - \varepsilon^*)\theta q \tag{40}$$

The first two terms on the right-hand side of (40) can be interpreted in the same manner as the corresponding terms in (33). The final term corrects for strategic over-investment or under-investment in R&D by the home firm: it is negative for low spillovers but positive for high spillovers ($\varepsilon^* > \overline{\varepsilon^*}$).

Proposition 5 In GCE with international spillovers, the optimal R&D subsidy has the same form as in FCE except for an extra term (negative for low spillovers and positive for high spillovers) which offsets the strategic behaviour of the home firm.

Since the sign of the optimal R&D subsidy in this case is inherently ambiguous, it is desirable to examine its magnitude under special functional forms and we turn to this in the next section.

Table 1 Optimal subsidies to R&D in the linear model

Type of spillover	FCE	GCE
Local; no R&D cooperation	$\beta_j\theta_j q_j > 0$	$\beta_j\theta_j q_j - \frac{1}{3}\theta_i q_i \gtreqless 0$
Local; with R&D cooperation	0	$-\frac{1}{3}(\beta_j\theta_j q_j + \theta_i q_i) < 0$
International	$-\varepsilon^*\theta^* q\,\dfrac{1 - \varepsilon\frac{\theta}{\theta^*}\eta^*}{2 - \eta^*} < 0$	$-\dfrac{\theta q}{3}\left[1 - 2\varepsilon^*\frac{\theta^*}{\theta}\right] - \varepsilon^*\theta^* q\,\dfrac{3 - 2\varepsilon\frac{\theta}{\theta^*}\eta^*\left(2 - \varepsilon\frac{\theta}{\theta^*}\right)}{3\left(2 - \frac{4}{3}\eta^* + \frac{2}{3}\varepsilon\frac{\theta}{\theta^*}\eta^*\right)}$

5. The linear-quadratic case

We can illustrate more forcefully some of the general results of previous sections if we specialise to specific functional forms. In this section we adopt a simple linear specification of demand and assume that R&D affects marginal costs in a linear fashion and is itself subject to quadratic costs. The linear inverse demand function is given by

$$p_i = a_i - b_i(q_i + q_i^*) \tag{41}$$

where a_i and b_i are constants. Marginal costs for the home and foreign firm are given by

$$c_i = \overline{c_i} - \theta_i(x_i + \beta_i x_j + \varepsilon_i x_i^*) \qquad \text{and} \qquad c_i^* = \overline{c_i^*} - \theta_i^*(x_i^* + \varepsilon_i^* x_i) \tag{42}$$

where all the parameters are constant. Finally, the costs of R&D are given by $\gamma_i x_i^2/2$ and $\gamma_i^*(x_i^*)^2/2$ for the home and foreign firm respectively, where γ_i and γ_i^* are constant. It proves useful to define the composite parameters $\eta_i = \theta_i^2/b_i\gamma_i$ and $\eta_i^* = \theta_i^{*2}/b_i\gamma_i^*$, which can be interpreted as the relative return to R&D for a typical home and foreign firm respectively.[13]

Under these assumptions, the optimal export subsidies are positive in all cases: the ambiguities noted in some cases under general demands vanish. We therefore concentrate on the R&D subsidies, which are shown in Table 1. In the absence of strategic behaviour (i.e. in FCE), the signs of the optimal R&D subsidies accord with simple intuition. R&D should be subsidised when spillovers are local, though not when firms cooperate on R&D (when no intervention is warranted); and it should be taxed when spillovers are international. (The latter result shows that the ambiguity found in (33) for the general case is resolved: the rent-shifting effect dominates the spillback effect.)

However, when firms behave strategically the results are much less clear-cut. With local spillovers, the GCE results confirm eqs (22) and (28). In particular, the optimal subsidy without R&D cooperation is increasing in the spillover parameter

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[13] Strictly, η_i equals the return to R&D, measured by its effect in reducing unit production cost (θ_i), relative to its cost, measured by the induced fall in price in the absence of strategic behaviour and spillovers ($p_i' dq_i/dx_i$, which from the total differential of (11) equals $b_i\gamma_i/\theta_i$).
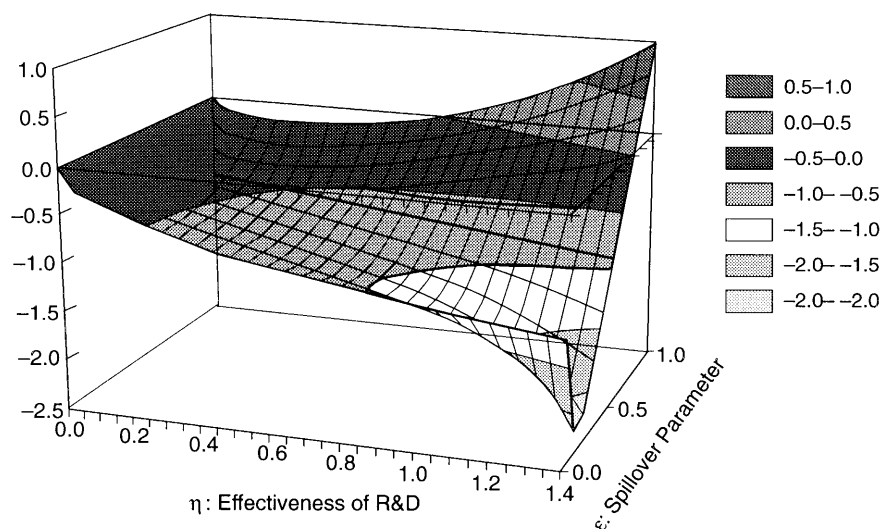
Fig. 1. Optimal R&D subsidy per unit output in the presence of international spillovers

β_j whereas with cooperation it is decreasing in β_j. This shows clearly that the cooperative over-internalises the externality when it plays strategically.

Finally, the case of international spillovers when firms behave strategically (eq. (40) specialised to the linear-quadratic case) is the most complex of all and this subsidy may even be positive. Figure 1 shows the optimal R&D subsidy as a function of η and ε under the additional assumption that the firms are symmetric (so $\theta = \theta^*$, $\gamma = \gamma^*$, $\varepsilon = \varepsilon^*$ and $\eta = \eta^*$). (Inspection of the bottom right-hand entry in Table 1 shows that in this case the sign of the optimal subsidy depends only on the two parameters η and ε. The figure also adopts the normalisation $b\gamma = 1$.) As can be seen from the figure the R&D subsidy is falling in η, the relative effectiveness of R&D, at $\varepsilon = 0$. With zero spillovers the government's only motive for intervention is to correct for strategic overproduction on the part of the home firm. As ε becomes positive the other two motives for intervention, the rent-shifting effect which works towards a tax and the spillback effect which works towards a subsidy, come into play. In addition as ε gets larger the need to correct for the home firm's strategic overproduction is reduced. For most values of η and ε the per-unit R&D subsidy is increasing in the spillover parameter and at high values of both η and ε it turns positive.

6. Summary and conclusion

In this paper we have explored the implications for trade and industrial policy in open economies of R&D spillovers between oligopolistic firms. Specifically, we have investigated a strategic trade model which allows for local R&D spillovers between

Table 2 Motives for industrial policy and their implications for the sign of the optimal R&D subsidy

Motive for intervention	Type of spillover	
	Local (β)	International (ε)
Encourage externalities	$+^*$	$+\dagger$
Shift rents	0	$-$
Offset wasteful strategic behaviour$\ddagger$	$-$	$-$for low ε
		$+$for high ε

* Zero when local firms cooperate on R&D.

$\dagger$ The spillback effect.

$\ddagger$ This motive only arises in GCE, not in FCE.

firms in different industries and intra-industry R&D spillovers between firms in different countries.

Our analysis has identified three distinct motives for R&D policy, whose qualitative implications are summarised in Table 2. First, there is the standard Pigovian motive: R&D should be encouraged because it yields externalities which cannot be captured by home firms acting alone. This naturally mandates a subsidy when the spillovers are local. More surprisingly, and contrary to the suggestion of Spencer (1986), it also justifies a subsidy when the spillovers are international. This is because of what we call the spillback effect which arises from the reciprocal nature of the R&D spillovers. The home firm benefits from the additional R&D undertaken by the foreign firm as a result of an increase in its own R&D.

The second motive for R&D policy is to shift rents, or, more accurately, to exercise the government's superior commitment power in order to move the home firm to the position it would adopt on its own if it were a Stackelberg leader. In the presence of international spillovers, changes in home R&D affect foreign output directly and thus affect the level of home profits. This mandates an R&D tax to reduce foreign output and so shift rents to the home country. Note that this is true even though an export subsidy is simultaneously imposed to shift rents: with international spillovers there is no clear division of labour between the two policy instruments, contrary to what we might expect from the targeting principle.

Finally, when firms behave strategically there is a third motive for intervention. Such strategic behaviour involves over- or under-investment in R&D relative to the social-cost-minimising level. This is socially wasteful, and so intervention to offset it is justified. This motive for intervention has highly counter-intuitive implications. In the absence of intra-industry spillovers (and irrespective of whether or not there are local inter-industry spillovers), a home firm tends to over-invest strategically in R&D and so an R&D tax is justified. However, the greater the degree of intra-industry spillovers, the more the home firm tends to under-invest in R&D (since it anticipates that some of the benefits will accrue to its foreign rival). Hence, paradoxically, stronger international spillovers strengthen the case for an R&D

subsidy, even though the spillovers accrue to a foreign firm, whose profits are of no concern to the home government.

The other issue considered in the paper is whether R&D cooperation by firms can internalise inter-industry R&D spillovers, thus rendering industrial policy redundant. We have shown that this does indeed happen when strategic behaviour is absent. However, when firms cooperate and behave strategically they will typically over-internalise the externality and engage in too much R&D, thus mandating an offsetting R&D tax.

Our model has naturally simplified in many respects. We have confined our analysis to the canonical third-country paradigm pioneered by Brander and Spencer, ignoring home consumption and foreign retaliation. The qualifications which must be made to the results when we relax these assumptions have been extensively examined elsewhere. (See Brander and Spencer, 1985, and Brander, 1995, for example.) We have assumed that there is only one firm in each domestic industry. Relaxing this would provide a strategic motive for R&D subsidies; but it would also dilute the rent-shifting motive for subsidising exports and the net effect is uncertain. Allowing for additional firms would also enrich the possibilities for R&D cooperation, requiring an analysis of endogenous R&D coalition formation. (We are indebted to a referee for pointing this out). We have considered only the first-best case, where the R&D subsidy is supplemented by an optimal export subsidy. In the alternative case, where an export subsidy is not available, there is an additional second-best motive for subsidising R&D, as in Spencer and Brander (1983). Finally, we have concentrated on the Cournot case of output competition in the second stage. As is well known, many of the conclusions are likely to be reversed if instead firms compete on price in a Bertrand manner.

It would be desirable in future work to relax many of these assumptions. However, doing so is unlikely to overturn our basic point, which has been obscured by the concentration on studying R&D spillovers in models of monopolistic competition. When strategic motives for investing in R&D are taken into account, the case for subsidising local spillovers and taxing international spillovers is much less clear-cut.

Acknowledgements

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Appendix

A.1 Strategic effects in GCE

To calculate the strategic effects in GCE, we totally differentiate the home and foreign firms' first-order conditions for output (7) and (8) to get

$$R^i_{qq}dq_i + R^i_{qq^*}dq^*_i + \theta_i\left(dx_i + \varepsilon_i dx^*_i + \beta_i dx_j\right) + ds_i = 0 \tag{43}$$

$$R^{*i}_{q^*q^*}dq^*_i + R^{*i}_{q^*q}dq_i + \theta^*_i(dx^*_i + \varepsilon^*_i dx_i) = 0 \tag{44}$$

These can be solved for dq_i and dq^*_i to get

$$D^i dq_i = -(R^{*i}_{q^*q^*}\theta_i - \varepsilon^*_i\theta^*_i R^i_{qq^*})dx_i - (R^{*i}_{q^*q^*}\varepsilon_i\theta_i - \theta^*_i R^i_{qq^*})dx^*_i - R^{*i}_{q^*q^*}(ds_i + \theta_i\beta_i dx_j) \tag{45}$$

$$D^i dq^*_i = -(R^i_{qq}\theta^*_i - \varepsilon_i\theta_i R^{*i}_{q^*q})dx^*_i - (R^i_{qq}\varepsilon^*_i\theta^*_i - \theta_i R^{*i}_{q^*q})dx_i + R^{*i}_{q^*q}(ds_i + \theta_i\beta_i dx_j) \tag{46}$$

where

$$D^i = R^i_{qq}R^{*i}_{q^*q^*} - R^i_{qq^*}R^{*i}_{q^*q} = b^2_i(3 + r_i) > 0 \tag{47}$$

We assume D^i is positive to ensure stability of the output game. In the text we make frequent use of the following

$$R^i_{qq} = -b_i(2 + \alpha_i r_i) \qquad R^{*i}_{q^*q^*} = -b_i(2 + \alpha^*_i r_i)$$

$$R^i_{qq^*} = -b_i(1 + \alpha_i r_i) \qquad R^{*i}_{q^*q} = -b_i(1 + \alpha^*_i r_i) \tag{48}$$

R^i_{qq} and $R^{*i}_{q^*q^*}$ are negative from the firms' second-order conditions and the other two terms are negative provided home and foreign output are strategic substitutes.

A.2 Generalised reaction functions when spillovers are local

A.2.1 *Full commitment equilibrium*

To calculate the slopes of the generalised reaction functions under FCE, totally differentiate the two first-order conditions for a typical foreign firm (8) and (12). The total derivative of (8), the first-order condition for output, is given in (44) above and the total derivative of (12), the first-order condition for R&D, is

$$\pi^{*i}_{x^*x^*}dx^*_i + \pi^{*i}_{x^*q^*}dq^*_i = 0 \tag{49}$$

where $\pi^{*i}_{x^*x^*} = \theta^{*\prime}_i q^*_i - \Gamma^{*\prime\prime}_i < 0$ and $\pi^{*i}_{x^*q^*} = \theta^*_i > 0$. Solving eqs (44) and (49) for dx^*_i and dq^*_i and setting $\varepsilon^* = 0$ yields

$$\Delta^i dx^*_i = \pi^{*i}_{x^*q^*}R^{i*}_{q^*q}dq_i \tag{50}$$

$$\Delta^i dq^*_i = -\pi^{*i}_{x^*x^*}R^{i*}_{q^*q}dq_i \tag{51}$$

where

$$\Delta^i = \pi^{*i}_{x^*x^*}R^{i*}_{q^*q^*} - \pi^{*i}_{x^*q^*}\theta^*_i > 0 \tag{52}$$

Note that $\pi^{*i}_{x^*x^*}$ and $R^{i*}_{q^*q^*}$ are negative and Δ^i is positive from the typical foreign firm's second-order conditions. As mentioned in the text $R^{i*}_{q^*q^*}$ is negative if foreign output is a strategic substitute for home output. From (50) and (51) we can now see that: (i) x^*_i and q^*_i depend only on q_i; and (ii) the sign of the derivatives of the generalised reaction functions depend only on the sign of $R^{i*}_{q^*q^*}$.

A.2.2 Government-only commitment equilibrium

To determine the slopes of the generalised reaction functions under GCE we first need the total derivative of (19), the typical foreign firm's first-order condition for R&D under GCE

$$\pi^{*i}_{x^*x^*}dx^*_i + \pi^{*i}_{x^*q^*}dq^*_i + \pi^{*i}_{x^*q}dq_i = 0 \tag{53}$$

Compared to (49) there is now an additional term in dq_i. Let $E^{*i} = (1 + \alpha_i r_i)/(3 + r_i)$. Then under GCE: $\pi^{*i}_{x^*x^*} = \theta^{*\prime}(1 + E^{*i})q^*_i - \Gamma^{*\prime\prime}_i$, $\pi^{*i}_{x^*q^*} = \theta^*_i(1 + E^{*i}) + \theta^*_i q^* E^{*i}_{q^*}$ and $\pi^{*i}_{x^*q} = \theta^*_i q^* E^{*i}_q$. Stability considerations mean that we continue to assume that $\pi^{*i}_{x^*x^*}$ is negative and that Δ^i is positive. However, $\pi^{*i}_{x^*q^*}$ and a $\pi^{*i}_{x^*q}$ cannot be unambiguously signed. Nevertheless, $\pi^{*i}_{x^*q^*}$ will be positive provided the term in $E^{*i}_{q^*}$ is not too negative.

To obtain expressions for dx^*_i and dq^*_i, set $\varepsilon^* = 0$ and combine (44) with (53) to get

$$\Delta^i dx^*_i = \left\{\pi^{*i}_{x^*q^*}R^{*i}_{q^*q} - \pi^{*i}_{x^*q}R^{*i}_{q^*q^*}\right\}dq_i \tag{54}$$

$$\Delta^i dq^*_i = -\left\{\pi^{*i}_{x^*x^*}R^{*i}_{q^*q} - \pi^{*i}_{x^*q}\theta^*_i\right\}dq_i \tag{55}$$

From these equations we can see that x^*_i and q^*_i depend only on q_i and that strategic substitutes contribute to x^*_i and q^*_i falling in q_i.

A.3 Generalised reaction functions: International spillovers

A.3.1 Full commitment equilibrium

To compute the slopes of the generalised reaction functions in this case totally differentiate (12) to get

$$\pi^*_{x^*x^*}dx^* + \pi^*_{x^*q^*}dq^* + \pi^*_{x^*x}dx = 0 \tag{56}$$

(Note we have dropped the industry superscripts and subscripts as there are no inter-industry spillovers). Compared to (49), the presence of international spillovers implies that there is an additional term in dx. This is because in the international spillover case θ^* depends not just on x^* but also on x via the spillover effect.

The derivatives of the foreign marginal profit functions can be written more explicitly as: $\pi^*_{x^*x^*} = \theta^{*\prime}q^* - \Gamma^{*\prime\prime} < 0$, $\pi^*_{x^*q^*} = \theta^* > 0$, and $\pi^*_{x^*x} = \theta^*_x q^*$ which cannot be signed. To obtain expressions for dx^* and dq^* combine (44) with (56) to get

$$\Delta dx^* = \pi^*_{x^*q^*}R^*_{q^*q}dq + \left\{\pi^*_{x^*q^*}\varepsilon^*\theta^* - \pi^*_{x^*x}R^*_{q^*q^*}\right\}dx \tag{57}$$

$$\Delta dq^* = -\pi^*_{x^*x^*}R^*_{q^*q}dq - \left\{\pi^*_{x^*x^*}\varepsilon^* - \pi^*_{x^*x}\right\}\theta^*dx \tag{58}$$

From these we can see that: (i) x^* and q^* depend on q and x; (ii) the sign of the derivatives with respect to q of the generalised reaction functions is the same as the sign of $R^*_{q^*q}$ (and so is negative with strategic substitutability); and (iii) the derivatives with respect to x are positive provided $\pi^*_{x^*x}$ is not too negative. (This includes the linear case when $\pi^*_{x^*x}$ is zero.)

A.3.2 Government-only commitment equilibrium

To obtain the slopes of the generalised reaction function totally differentiate (36) to get

$$\pi^*_{x^*x^*}dx^* + \pi^*_{x^*q^*}dq^* + \pi^*_{x^*x}dx + \pi^*_{x^*q}dq = 0 \tag{59}$$

Compared to (56) there is an additional term in dq. This arises because the term in brackets in (36) depends on q.

The derivatives in (59) are somewhat harder to sign than in the FCE case. It will help in determining the signs of these derivatives to define: $T(x, x^*, q, q^*) = 1 + A^*(\bar{\varepsilon} - \varepsilon) > 0$. ($T$ is greater or less than unity as $\bar{\varepsilon}$ is greater or less than ε.) Then:

$$\pi^*_{x^*x^*} = T\theta^*_{x^*}q^* + \theta^*q^*T_{x^*} - \Gamma^{*\,\prime\prime} < 0, \pi^*_{x^*q^*} = \theta^*(T + q^*T_{q*}), \pi^*_{x^*x} = T\theta^*_x q^* + \theta^*q^*T_x,$$

and $\pi^*_{x^*q} = \theta^*q^*T_q$. To obtain expressions for dx^* and dq^*, combine (44) and (59) to get

$$\Delta dx^* = (\pi^*_{x^*q^*}R^*_{q^*q} - \pi^*_{x^*q}R^*_{q^*q^*})dq + (\pi^*_{x^*q^*}\varepsilon^*\theta^* - \pi^*_{x^*x}R^*_{q^*q^*})dx \tag{60}$$

$$\Delta dq^* = -(\pi^*_{x^*x^*}R^*_{q^*q} - \pi^*_{x^*q}\theta^*)dq - (\pi^*_{x^*x^*}\varepsilon^* - \pi^*_{x^*x})\theta^* dx \tag{61}$$

As in the FCE case, strategic substitutes contribute to foreign R&D and foreign output falling in q. However, as in the GCE case with local spillovers, they are not sufficient now due to the term in $\pi^*_{x^*q}$. The derivatives with respect to home R&D take the same form as those under FCE and can be interpreted in the same way.