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THE EFFECTS OF ANNUITIES, BEQUESTS, AND AGING IN AN OVERLAPPING GENERATIONS MODEL OF ENDOGENOUS GROWTH

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We examine the effects of introducing actuarially fair annuity markets into an overlapping generations model of endogenous growth. The complete annuitisation of agents' wealth is not, in general, dynamically optimal; the degree of annuitisation that is dynamically optimal depends nonmonotonically on the expected length of retirement and on the pay-as-you-go social security tax rate. The government has an incentive to restrict the availability of actuarially fair annuities contracts, and can often move the economy from a pay-as-you-go to a fully-funded social security system via voluntary contributions to a government sponsored, actuarially fair pension today accompanied by reductions in social security taxes tomorrow.

When agents face uncertain lifetimes, Yaari (1965) shows that the welfare of retirees is increased by the introduction of actuarially fair annuities. However, such annuities may be unavailable because of market failures caused by asymmetric information between the insurer and the insured. If this is the case, saving is, in general, suboptimally high in steady-state equilibrium (Eichenbaum and Peled, 1987) and the introduction of a mandatory, fully-funded social security scheme can often improve upon the steady-state market outcome (Sheshinski and Weiss, 1981; Abel, 1986; Eckstein *et al.* 1985*a*, *b*; Townley and Boadway, 1988). These results have led to the belief that increasing the availability of actuarially fair annuities will increase social welfare.

In this paper we examine the effects of introducing a government sponsored actuarially fair annuity market, *à la* Sheshinski and Weiss (1981), into an overlapping generations model[1] with external effects and growth. We examine two means of accessing the annuity markets. First, the government allows individuals to make voluntary contributions to an actuarially fair pension plan, but places a limit on contributions. Secondly, the government mandates contributions to an actuarially fair pension plan. We find that under both programmes, the complete annuitisation of agents' wealth, while individually optimal, is often not socially optimal, even in steady-state equilibrium, because agents may underaccumulate relative to the social optimum. Along an equilibrium growth path, full annuitisation may not be dynamically optimal because the 'excess' saving generated by unintentional bequests and incomplete annuitisation provides the fuel for the endogenously generated component of

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[1] The overlapping generations model was developed by Samuelson (1958) and Allais (1947), and extended by Diamond (1965) from an endowment to an economy in which goods are produced using labour and capital. This basic framework has been utilised extensively in recent studies of endogenous growth. See, for example, Jones and Manuelli (1992).

economic growth. Further, the degree of annuitisation (the percentage of wealth that is annuitised, voluntarily or by mandate) that is dynamically optimal depends nonmonotonically on the expected length of retirement under both pension plans, nonmonotonically on the pay-as-you-go social security tax rate under the voluntary pension plan, and on the weights the social planner places on current and future generations.

While incomplete annuitisation is usually dynamically optimal, Pareto improvements are often available via increases in annuitisation rates coupled with decreases in the future social security tax rate, and increases in the annuitisation rate coupled with the complete phasing out of the pay-as-you-go social security tax rate in the future. This suggests that the government could phase out the current social security system, replacing it with a government sponsored, actuarially fair pension scheme. Since full annuitisation of agents' wealth is not socially optimal, the government has an incentive to restrict the availability of actuarially fair annuity contracts.

I. THE ENVIRONMENT

Consider an infinite-horizon economy comprised of identical two-period lived agents, perfectly competitive firms, annuity markets, and a government. A new generation (called generation t) is born at each date $t = 1, 2, 3, \ldots$. Assume that there is no population growth, and that, at each date t, N agents are born. Without loss of generality assume N is unity.

Agents in this model, as in Eckstein *et al.* (1985*a*), are not altruistic: the old do not care for the young and the young do not care for the old. Agents in the first period of their lives, the young, are endowed with one unit of labour which they supply inelastically to firms. They divide their wages between their own current consumption, saving (held either as an annuity, as direct holdings of capital, or both) for consumption when old, and payment of social security taxes quoted as a proportion of their wages. Agents in the final period of their lives, the old, supply their savings inelastically to firms and consume their social security benefits and the return to their savings. An agent dies at the onset of old age with probability $(1-p)$ and lives throughout old age with probability p.[2] If an agent dies 'young', his unannuitised wealth is bequeathed to his children.[3]

Let the representative member of generation t's preferences be represented by

$$U = \ln c_t(t) + p \ln c_t(t+1),$$

where $c_t(t)$ is consumption by a member of generation t when young, $c_t(t+1)$ is consumption by a member of generation t when old.

[2] Agents in this model face uncertainty about the time of death but not about the maximum possible length of life. This implies that agents may die before they have exhausted their non-social security non-annuitised wealth, but not *vice versa*.

[3] This assumption of unintentional rather than altruistic bequests is consistent with empirical findings by numerous researchers: see Hurd (1990), Auerbach *et al.* (1992), and Börsch-Supan (1993), as well as the empirical finding of Altonji *et al.* (1992) that parents and their adult children are not altruistically linked. But, other research finds an operative bequest motive (Hamermesh and Menchik, 1987; Hurd, 1995), at least among the wealthy. Since the jury is still out, we will maintain the assumption of unintentional bequests.

The firms are perfectly competitive profit maximisers that produce output using the production function $Y(t) = A(t)\,K(t)^{\beta}\,N(t)^{1-\beta}$, $\beta \in [0, 1]$. $K(t)$ is the capital stock at t, $N(t)$ is employment at t, and $A(t) > 0$ is a productivity scalar. Capital depreciates fully in the production process.[4] The production function can be written in intensive form as $y(t) = A(t)\,k(t)^{\beta}$, where $k(t)$ is the capital-labour ratio. Assume, because of external effects of aggregate capital on productivity, as discussed in Romer (1986), $A(t) = a(t)\,K(t)^{\eta}$, $a(t) > 0$, $\eta > 0$, so that the aggregate capital stock, $K(t)$, enters the technology as a constant from the perspective of current producers.[5]

The government in this economy can impose social security taxes, $\tau(t)$, on the wages of the young at t. The government must fully fund all expenditures with tax receipts. Assume, following Sheshinski and Weiss (1981), that actuarially fair annuity contracts are unavailable on the private market. The government overcomes the market failure by establishing a market in actuarially fair annuity contracts. The government can control access to this market in one of two ways. It can either make the purchase of an annuity voluntary or mandatory. Under the voluntary plan, Plan V, each agent may place up to $\gamma^{v}(t)\,\%$ of his total saving in an annuity.[6] Under the mandatory plan, Plan M, each agent must place a fixed amount, s^{M}, in an annuity.

The representative agent at time t takes as given the wage, $w(t)$, bequests, $B^{t}_{t-1}(t)$, the return on saving when old, $\rho(t+1)$, the tax rate, $\tau(t)$, social security benefits, $T_{t}(t+1)$,[7] and the government pension plan. Under Plan V the agent takes as given the maximum percentage of saving that can be annuitised, $\gamma^{v}(t)$, and the excess return on annuities, $\alpha^{v}(t+1)$. Under Plan M he takes as given the mandatory contribution, s^{M}, and the return on these annuities, $\alpha^{M}(t+1)$. The agent chooses saving, $s(t)$, to maximise

$$\ln c_{t}(t) + p \ln c_{t}(t+1) \tag{1}$$

subject to, under Plan V,

$$c_{t}(t) = w(t)\,[1-\tau(t)] - s(t) + B(t), \tag{2}$$

$$c_{t}(t+1) = [1+\rho(t+1)+\alpha^{v}(t+1)]\,s(t) + T(t+1) \tag{3}$$

or subject to, under Plan M,

[4] The production process is over the course of a generation. Since empirically the depreciation rate is about 10% per year, capital is all but fully depreciated over the course of a 25 year generation. We assume, therefore, that capital is fully used up in the production process.

[5] We have chosen the functional forms for utility and production to guarantee a closed form solution and to facilitate the simulation exercises summarised in Section V, below.

[6] The voluntary scheme loosely resembles the current IRA legislation under which individuals can annually invest up to $2,000 in a tax deferred Individual Retirement Account. Here agents can save only their after tax income, but by restricting access to the annuity market, the government policy both affects the return on investment and bequests.

[7] We assume that the social security system is of the pay-as-you-go variety, and that one's benefits depend not on one's own contributions, but on the next generations'. In this system social security transfers are lump sum. This is consistent with the literature (see, for example, Cukierman and Meltzer (1989) or McCandless and Wallace (1991)), and can be interpreted as social security benefits being paid out at a constant rate over an agent's retirement. While most pay-as-you-go social security systems do link benefits to contributions, the redistributive element of these system and the legislated changes in the benefits formulae make the connection between one's contributions and one's benefits loose. To simplify the analysis we assume no link between benefits and contributions.

$$c_t = w(t)[1-\tau(t)] - s(t) - s^M + B(t), \qquad (2')$$

$$c_t(t+1) = [1+\rho(t+1)]\, s(t) + [1+\alpha^M(t+1)]\, s^M + T(t+1), \qquad (3')$$

where constraints (2) and (2′) encompass the assumption that bequests are allocated equally across all members of a generation, as in Hubbard and Judd (1987). This assumption ensures that bequests do not induce a non-trivial wealth distribution onto this economy comprised of representative agents. Further, it restricts uncertainty to the timing of death alone. Also, the return on saving in constraint (3) is stated as the sum of the return to direct holdings of capital, $1+\rho(t+1)$, and the excess return, prorated over all saving, of holding $\gamma^v(t)$ % of saving as an annuity, $\alpha^v(t+1)$. Since, as Yaari (1965) and Shenshinski and Weiss (1981) show, agents without a bequest motive would prefer to annuitise all their wealth, they will annuitise their wealth up to the legal restriction. Under Plan M, $1+\alpha^M(t+1)$ is the actuarially fair return on the government mandated pension.

Substituting constraints (2) and (3) ((2′) and (3′)) into the objective function (1) and maximising yields the first-order condition for Plan V

$$\frac{-1}{w(t)[1-\tau(t)] - s(t) + B(t)} + \frac{p[1+\rho(t+1)+\alpha^v(t+1)]}{[1+\rho(t+1)+\alpha^v(t+1)]\, s(t) + T(t+1)} = 0 \qquad (4)$$

and for Plan M

$$\frac{-1}{w(t)[1-\tau(t)] - s(t) - s^M + B(t)} + \frac{p[1+\rho(t+1)]}{[1+\rho(t+1)]\, s(t) + [1+\alpha^M(t+1)]\, s^M + T(t+1)} = 0. \qquad (4')$$

Agents equate the marginal rate of substitution between consumption and saving to the return to saving, the marginal rate of transformation.

The individual firm takes wages, rental rates, and the aggregate stock of capital as given. It hires labour and capital until their marginal products equal their factor prices

$$a(t)\, K(t)^\eta (1-\beta)\, k(t)^\beta = w(t), \qquad (5)$$

$$a(t)\, K(t)^\eta \beta k(t)^{\beta-1} = r(t). \qquad (6)$$

Because of the assumptions of constant returns production technology and inelastic labour supply, (5) and (6) also define factor market clearing.

The government must maintain a balanced budget. Thus, total outlays to the old who live throughout their last period of life must equal total revenues from social security taxes

$$pT(t+1) = w(t+1)\, \tau(t+1).$$

Therefore,

$$T(t+1) = \frac{w(t+1)\, \tau(t+1)}{p}. \qquad (7)$$

If an agent dies young, then the unannuitised portion of his wealth

$$B(t+1) = [1-\gamma^v(t)]\,(1-p)\,[1+\rho(t+1)]\,s(t), \tag{8}$$

under Plan V is distributed to his heirs, where $(1-\gamma^v(t))$ is the percentage of wealth not annuitised. The annuitised portion is distributed, *pro rata*, among the other holders of annuities

$$p\alpha^v(t+1)\,s(t) = \gamma^v(t)\,(1-p)\,[1+\rho(t+1)]\,s(t).$$

Thus, the excess return on saving from holding $\gamma^v(t)\,\%$ as an annuity is

$$\alpha^v(t+1) = \frac{\gamma^v(t)\,(1-p)\,[1+\rho(t+1)]}{p}. \tag{9}$$

If an agent dies young under plan M his wealth at death

$$B(t+1) = (1-p)\,[1+\rho(t+1)]\,s(t) \tag{8'}$$

is distributed to his heirs. The return on the mandated savings of the old who live throughout their old age is

$$[1+\alpha^M(t+1)] = \frac{[1+\rho(t+1)]}{p}. \tag{9'}$$

The goods market clears when demand for goods equals supply of goods. Under Plan V, goods market clearing is defined by

$$c_t(t)+pc_{t-1}(t)+s(t) = w(t)+r(t)\,k(t). \tag{10}$$

Substituting equations (2), (3) and (7)–(9) into (10) yields

$$s(t) = k(t+1), \tag{11}$$

where by arbitrage

$$r(t+1) = 1+\rho(t+1). \tag{12}$$

Under Plan M goods market clearing is defined by

$$c_t(t)+pc_{t-1}(t)+s(t)+s^M = w(t)+r(t)\,k(t). \tag{10'}$$

Substituting equations (2′), (3′) and (7), (8′) and (9′) into (10′) yields

$$s(t)+s^M = k(t+1) \tag{11'}$$

and (12). In equation (11′) let $s(t) = [1-\gamma^M(t)]\,k(t+1)$ and $s^M = \gamma^M(t)\,k(t+1)$. Saving at date t, under both plans, totally determines the capital stock at date $t+1$, and the return on capital equals the return on saving.

II. EQUILIBRIUM

A competitive equilibrium for a Plan V [Plan M] economy is a sequence of prices $\{w(t), r(t), \rho(t), \alpha^v(t)\ [\alpha^M(t)]\}_{t=1}^{\infty}$, a sequence of allocations $\{c_t(t), c_{t-1}(t)\}_{t=1}^{\infty}$ and a sequence of capital stocks, $\{k(t)\}_{t=1}^{\infty}$, $k(1) > 0$ given, such that given these prices and allocations, agents' utility is maximised, firms' profits are maximised, the government budget constraint is satisfied, and markets clear.

The Plan V [Plan M] equilibrium is fully characterised by equations (2)–(9), (11) and (12) [(2′)–(9′), (11′) and (12)]. Substituting (8)–(10), (11) and (12) into (4) yields for Plan V

$$k(t+1) = \left(\frac{a(t)\,p\{(1-\beta)\,[1-\tau(t)]+\beta(1-p)\,[1-\gamma^v(t-1)]\}}{1+p+\dfrac{p\tau(t+1)\,(1-\beta)}{\beta[p+\gamma^v(t)\,(1-p)]}} \right) k(t)^{\beta+\eta}. \quad (13)$$

Substituting (8′)–(10′), (11′) and (12) into (4′) yields for Plan M

$$k(t+1) = \left(\frac{a(t)\,p\{(1-\beta)\,[1-\tau(t)]+\beta(1-p)\,[1-\gamma^M(t-1)]\}}{1+p+\dfrac{\gamma^M(t)\,(1-p)}{p}+\dfrac{(1-\beta)\,\tau(t+1)}{\beta}} \right) k(t)^{\beta+\eta}. \quad (13')$$

These difference equations describe the dynamic paths of the Plan V and Plan M economies, respectively.

III. The Steady State

Sheshinski and Weiss (1981) find, in the context of an endowment economy with an exogenously fixed return on saving, that the socially optimal fully funded social security scheme for agents who have no bequest motive is one in which agents place all their savings in annuities. In their model individual optimality and social optimality coincide. In our models, because of the internal effect of capital accumulation on the interest rate and the external effect of the aggregate capital stock on productivity, this link is broken. Consequently, $\gamma^v = \gamma^M = 1$, or full annuitisation, while still individually optimal is not, in general, socially optimal in our models.

Proposition 1: *Let the social welfare function be defined as the steady-state utility of the representative agent. Further, assume that social security taxes, τ, and benefits, T, are uniquely equal to zero, and that $\gamma^v(t) = \gamma^v\,\forall t$, and $\gamma^M(t) = \gamma^M\,\forall t$. Then, social welfare is maximised at, for Plan V*

$$\gamma^v = \frac{p}{(1-p^2)\,\beta}\,[(1-\beta-\eta)-\beta(1+p)],$$

and, for Plan M, γ^M is implicitly defined by

$$\frac{(1+p)}{p+\gamma^M(1-p)} - \left[\frac{1+p(\beta+\eta)}{1-\beta-\eta}\right] \times \left[\frac{\beta}{(1-\beta)+\beta(1-p)\,(1-\gamma^M)} + \frac{1}{p(1+p)+\gamma^M(1-p)}\right] = 0.$$ [8]

In these models, because of the effects of capital accumulation on the interest rate and aggregate productivity, the income effect of bequests generated by an

[8] The proofs of the propositions are relegated to the Appendix.

annuitisation rate less than one may increase social welfare by generating a superior consumption allocation. This leads to a divergence of individual preferences from social preferences. When this is the case, the social planner will be compelled to restrict the proportion of an individual's wealth that can be invested in an annuity.

IV. COMPARATIVE DYNAMICS

The external effect of capital on aggregate productivity, while positive, has not been found empirically to be strong enough to generate growth. However, under conditions of exogenous technological progress, the external effect is growth enhancing. How growth is affected by full and/or partial annuitisation, the method of annuitisation, and of a longer period of retirement in various social security tax regimes is detailed below.

PROPOSITION 2: *Let the social security tax rate $\tau(t) = \tau \, \forall t$. Then, under both Plans V and M, economies with higher social security tax rates have slower rates of growth.*

An increase in the social security tax rate imposes a negative income effect on the young and a positive income effect on the old via higher social security benefits. Both effects reduce the incentive to save, which reduces capital accumulation and hence the growth rate. This result is common to many endogenous growth models.[9]

PROPOSITION 3: *Assume the annuitisation rates $\gamma^v(t) = \gamma^v \, \forall t$ and $\gamma^M(t) = \gamma^M \, \forall t$. Under both Plans V and M, in economies in which all wealth is annuitised, $\gamma^v = \gamma^M = 1$, growth is increasing in longevity*[10] *(the expected length of retirement, p). In economies in which wealth is not totally annuitised, $\gamma^v < 1\,[\gamma^M < 1]$, greater longevity leads to either higher or lower growth.*

If all wealth is annuitised, then the rate of growth is increasing in the length of life. This is because greater longevity heightens the incentive to save, thereby increasing capital accumulation. If wealth is only partially annuitised, an increase in life span reduces bequests while at the same time decreasing the marginal return to the annuity and decreasing social security benefits, since the tax revenues must be shared among the larger population of long-lived old. The first two effects reduce while the last effect increases the incentive to save; the net effect is ambiguous. Therefore, growth may either increase or decline as the population ages. If social security taxes are an increasing function of the expected length of retirement, a longer life may imply no change in annual benefits. In this case, greater longevity reduces capital accumulation and, thereby, growth.

PROPOSITION 4: *Assume first that the social security tax rate $\tau(t) = \tau = 0 \, \forall t$, and the annuitisation rates $\gamma^v(t) = \gamma^M(t) = \gamma > 0 \, \forall t$. Then, under Plans V and M economies*

[9] See, e.g. King and Rebelo (1990), Rebelo (1991) and Saint-Paul (1992).

[10] We model an increase in longevity as an increase in the length of retirement rather than an increase in the lengths of youth and old age. This is consistent with Hamermesh (1984) who finds that workers who live longer save more but do not work more than their shorter lived colleagues.

with higher annuitisation rates exhibit slower growth. Now assume that $\tau > 0$, *and* $\gamma^v(t) = \gamma^v > 0\,\forall t$ *and* $\gamma^M(t) = \gamma^M > 0\,\forall t$. *Then economies with higher annuitisation rates exhibit either slower or faster rates of growth under Plan V, and exhibit slower rates of growth under Plan M.*

If a greater percentage of wealth is annuitised, bequests fall, imposing a negative income effect on the young.[11] They respond to this by reducing saving and consumption today. However, an increase in γ^v increases the return to saving, which imposes a positive income effect on the young. At $\tau = 0$, under Plan V, only the negative bequest effect is realised, leading to a reduction in saving, and therefore growth. At $\tau > 0$ under Plan V, the bequest effect may be weaker than the annuitisation effect. If so, saving and growth will rise. If not, higher rates of annuitisation may lead to decreased saving and lower rates of growth: actuarially fair annuities may decrease dynamic social welfare. This is always the case under Plan M since changes in the rate of forced saving have no effect on agents' choices at the margin. Thus, policies that encourage the young to invest in annuities to supplement their social security retirement income may have the unintended side effect of benefiting one generation to the detriment of those that follow.

Proposition 5: *Under both Plans V and M, growth is maximised when the social security tax rate,* τ, *and the annuitisation rates,* γ^v *and* γ^M, *all equal zero.*

Growth in this model is linked to capital accumulation, so anything that reduces the incentive to save, reduces capital accumulation, and thus growth. Social security reduces the incentive to save by reducing disposable income during one's working years, and by guaranteeing a retirement income. Allowing the young to annuitise their saving, conditional on not paying social security, also reduces the incentive to save, because it reduces the risk of being unable to predict accurately the timing of death. However, conditional on there being a pay-as-you-go social security system, the optimal level of annuitisation is not necessarily zero. This is because partial annuitisation of one's wealth under Plan V increases the marginal return to saving and thus can increase capital accumulation enough to offset the loss in bequest income.

V. Dynamic Social Welfare

While policies to increase economic growth benefit future generations, they will not be Pareto improving if the current generation is made worse off. In this section we examine the effects of changes in the legal restrictions on annuities and of changes in the social security tax rate on the dynamic path of the economy via simulation exercises. All variations are of Plan V, since any increase in forced saving under Plan M is growth diminishing. Further, all variations hold the utility of the initial old constant.

[11] Weil (1993) examines the effects of uncertainty concerning the size and timing of bequests on the consumption/saving decisions of the adult children of the aged. By choosing to ignore uncertainty about bequests we recognise that our results concerning the effects of changes in the size of these bequests may be overstated. We thus look on our results as upper bounds.

We first created a set of baselines by calibrating the economy to achieve a 2% growth rate per year.[12] The parameters for the economy used in the baseline simulations, given in Table 1, are based on empirical estimates for the

Table 1
Parameter Values for the Baseline Simulation

Parameter	Value
τ	0·127
γ	0·165
p	0·20
β	0·30
η	0·10 to 0·70

US economy. The social security tax rate, τ, is derived from OECD data on social security contributions as a percentage of GDP, and adjusted for labour's share in output $(1-\beta)$.[13] The degree of annuitisation, γ, reflects the value of private pension funds as a percentage of US household net wealth.[14] The aged-dependency ratio, p, was set equal to the current ratio of the population aged 65 and over to the population between the ages of 20 and 64. The value for β reflects capital's share in output.[15] Since there is little empirical evidence on the value of the external effect of capital on productivity, we allowed η to vary between 0·1 and 0·7. Given the parameter values, the path of $a(t)$, total factor productivity, is chosen to produce the desired growth rate.

After calculating the baselines (one for each value of η), we changed various parameter values and re-simulated the economy, keeping $a(t)$ at its baseline path. Comparing the result of these simulations with the baselines allows us to make comparisons with respect to growth and dynamic social welfare.

Result 1: *Relative to the set of baseline simulations, simulations in which the annuitisation rate, γ, is raised up to 18·7% generate a growth path that dominates the baseline path. However, increases in γ holding the social security tax rate constant are not Pareto improving.*

From Proposition 4 we know that the growth rate increases or decreases with the annuitisation rate when the tax rate is positive. Result 1 shows that in comparison to the baseline, only minor increases in the annuitisation rate produce an increase in the return to saving adequate to generate an increase in saving. This increase in saving in turn raises the level of capital accumulation and thus economic growth. But, this increase does not translate into an

[12] In the simulations a period is 25 years, roughly equal to the time span of a generation.

[13] The social security data include employee and employer contributions to Medicare, and the old age disability and death portion of social security. Mandatory pension contributions covering federal employees are also included. For more details see OECD (1993).

[14] See Auerbach *et al.* (1992). The results presented in this section are invariant to small changes in the value of γ.

[15] Small changes in the value of β did not affect the results.

increase in the path of lifetime utility for all generations. The lifetime utility of the initial young rises but that of all future generations falls relative to the baseline.

Three factors affect the consumption/saving choice of future generations: the increased return to saving due to the higher annuitisation rate, the decline in bequests, and the increase in wage income resulting from the higher capital stock. The first factor raises saving and lowers consumption when young. The second factor lowers both saving and consumption. The third factor raises both saving and consumption. In sum, consumption when young falls and saving rises relative to the baseline. The increase in saving and the higher rate of return raises consumption when old. The net effect on lifetime utility is negative. Thus, lifetime utility for all future generations falls relative to the baseline.

RESULT 2: *Relative to the baseline simulation, simulations in which the annuitisation rate is raised above its baseline value (up to some threshold value) in period t and in which the social security tax rate is reduced in period* $t+1$, *generate a growth path that dominates the baseline path. Thus, increases in the annuitisation rate may be Pareto improving.*

Small increases in the annuitisation rate and the anticipated decrease in future tax rates lead agents to increase their saving. Above some threshold level, which is a positive function of the externality, η, and the size of the tax cut, the reduction in bequests leads to a decrease in saving and hence capital accumulation and growth. See Table 2.

Table 2

Maximum Value for γ *Generating a Growth Path Above the Baseline Path.*
(increasing γ *at t, decreasing* τ *at* $t+1$*)*

η	τ_{t+1}	γ_t
0·1 to 0·5	0·087	0·56
	0·047	0·77
	0	0·97
0·6	0·087	0·57
	0·047	0·78
	0	0·98
0·7	0·087	0·59
	0·047	0·79
	0	1

The initial young may be positively or negatively affected relative to the baseline. The anticipated decline in τ and the increase in the return to saving (resulting from the increase in γ) induce an increase in saving. The increase in the return to saving results in an increase in consumption when old. When the annuitisation rate is above the baseline level but below some threshold value, γ_L, which is a negative function of η and a positive function of the size of the tax rate cut, the benefit from the increased consumption when old is not enough to offset the disutility from the decline in consumption when young. Therefore the lifetime utility of the initial young declines. When $\gamma \geqslant \gamma_L$, the benefit from increased consumption when old offsets the disutility from the decline in consumption when young and lifetime utility of this generation rises. See Table 3.

Table 3

Minimum and Maximum Values for γ Generating a Lifetime Utility Path Above the Baseline (increasing γ at t, decreasing τ at $t+1$)

η	τ_{t+1}	γ_L	γ_H
0·1	0·087	0·33	0·41
	0·047	0·48	0·58
	0	0·65	0·76
0·2	0·087	0·32	0·45
	0·047	0·46	0·63
	0	0·62	0·82
0·3	0·087	0·31	0·48
	0·047	0·45	0·67
	0	0·60	0·86
0·4	0·087	0·30	0·51
	0·047	0·43	0·70
	0	0·58	0·90
0·5	0·087	0·30	0·53
	0·047	0·42	0·72
	0	0·57	0·93
0·6	0·087	0·29	0·55
	0·047	0·41	0·75
	0	0·55	0·95
0·7	0·087	0·28	0·58
	0·047	0·40	0·78
	0	0·53	0·98

The income of each successive generation is negatively affected by a decline in bequests resulting from an increase in γ, but positively affected by the increase in the capital stock, resulting from the increased saving of the initial young. If $\gamma_L \leqslant \gamma \leqslant \gamma_M$ then the capital stock effect dominates and consumption when young rises. Consumption when old always rises and thus lifetime utility rises. If $\gamma_M < \gamma \leqslant \gamma_H$ (see Table 3 for the values of γ_H) consumption when young falls. However, lifetime utility rises since the benefit from increased consumption when old offsets the disutility from the decline in consumption when young. Thus, if $\gamma_L \leqslant \gamma \leqslant \gamma_H$ a policy increasing γ at t and decreasing τ at $t+1$ is Pareto improving.

RESULT 3: *Relative to the baseline simulation, simulations in which the annuitisation rate is raised above its baseline value in period t and possibly in periods $t+1$ to $t+i$, $i>1$ as well, and in which the tax rate is reduced in period $t+1$ and possibly in periods $t+2$ to $t+i$, $i>2$ as well, generate a growth path that dominates the baseline path. Thus phasing out the pay-as-you-go social security system may be Pareto improving.*

Just how the government chooses to phase out the pay-as-you-go social security system depends on the relative weights it places on current and future

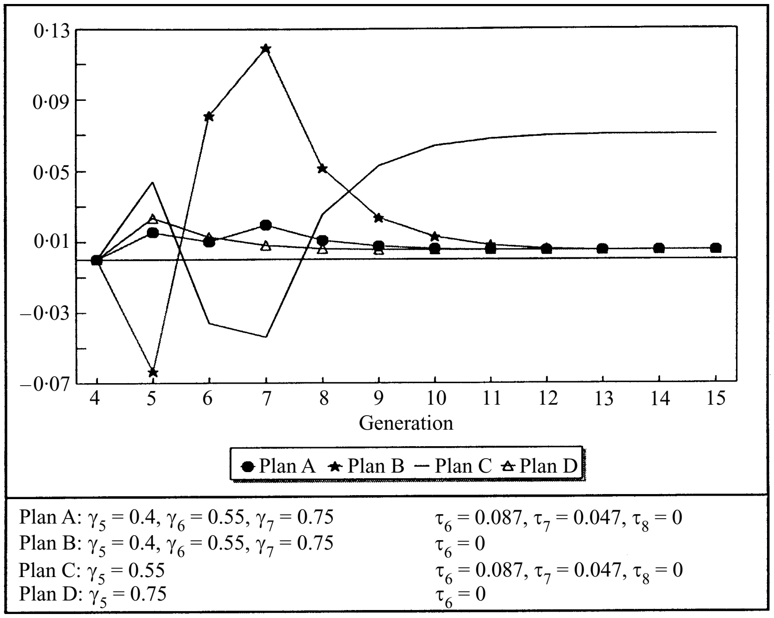


Fig. 1*a*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}1$.

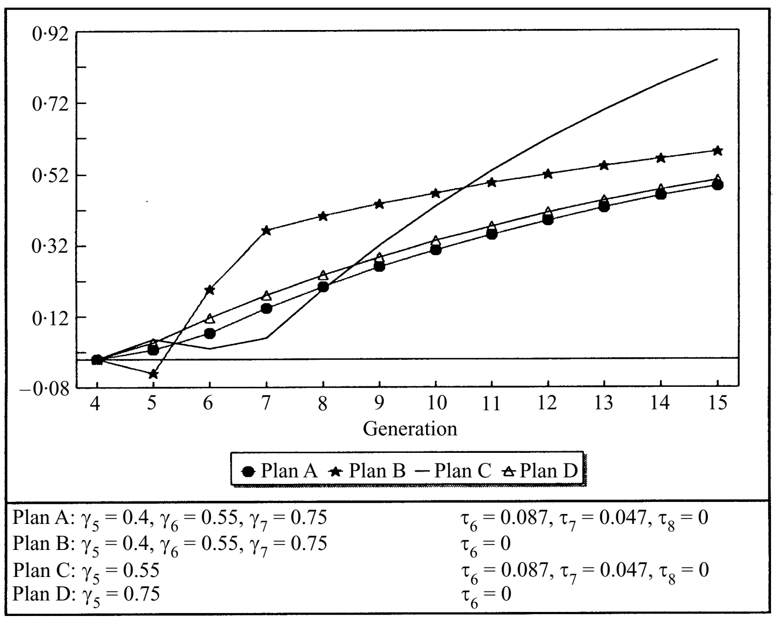


Fig. 1*b*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}6$.

generations, as well as the size of the externality, η. It is never optimal, however, to move to full annuitisation of wealth in conjunction with the elimination of the social security tax.

Fig. 1 presents four scenarios, Plans *A*–*D*, under which the social security tax is eliminated and the annuitisation rate is raised above the baseline rate. In all plans the economy starts with $\gamma = 0{\cdot}165$ and $\tau = 0{\cdot}127$. Beginning at $t = 5$ the annuitisation rate, γ, is increased and starting at $t = 6$ the social security tax rate, τ, is eliminated in one or more steps. The feasibility of each plan depends on the size of the externality, η. When η is small, only plan *A*, a gradual increase in the annuitisation rate coupled with a gradual elimination of the social security tax, and plan *D*, a one-step increase in the annuitisation rate followed by one-step elimination of the social security tax, Pareto dominate the baseline scenario, see Fig. 1 *a*. These two plans, however, are Pareto non-comparable. The plan favoured by the social planner depends on the relative weights placed on current versus future generations. If more weight is given to the generations born at $t = 5$ and $t = 6$, plan *D* would be chosen. Otherwise, plan *A* would be chosen.

When η is large all of the plans except plan *B* Pareto dominate the baseline scenario, see Fig. 1 *b*. In this case, plans *C* and *D* are Pareto non-comparable. While plan *D* Pareto dominates plan *A*, it does not follow that all variations on plan *A* are Pareto dominated by all variations on plan *D*, even assuming that the final annuitisation rate in the two plans is the same.[16]

The choice between plans *C* and *D* depends on the relative weights placed on the lifetime utility of each generation. The current young do best under plan *C* which combines a one-step increase in the annuitisation rate with a gradual elimination of the social security tax. If more weight is given to the near future generations (i.e. those born at $t = 6$, $t = 7$, and $t = 8$) then plan *D* would be chosen. Otherwise, plan *C* is preferred.

As discussed above, it is generally not possible to find one plan eliminating the social security tax and increasing the annuitisation rate that is optimal for all generations. Nor is there one rate of annuitisation that is optimal for all generations. As illustrated in Figs. 2 *a–d*, relatively high annuitisation rates generally favour the initial generation over future generations. For example, Fig. 2 *a* contrasts two versions of plan *A*. Both plans are based on a three step elimination of the social security tax. In plan *A*1 the annuitisation rate is increased in three steps to a maximum of 75 %. In plan *A*2 the annuitisation rate is increased in four steps. The first three steps mimic those of plan *A*1, with a final increase in the annuitisation rate to 85 % at $t = 8$. Lifetime utilities are the same under two plans for all generations up through those born at $t = 7$. For the generation born at $t = 8$ the increase in the annuitisation rate under plan *A*2 results in a higher lifetime utility. However, for all subsequent generations the higher rate of annuitisation under plan *A*2 reduces their lifetime utility below that of plan *A*1. The negative effect of lower bequests in plan *A*2 more than offsets the positive effect of the higher annuitisation rate. If η is small, as in the top panel of Fig. 2 *a*, it is possible that the lifetime utility of future generations will fall below the baseline if plan *A*2 is adopted.

Even when η is large, if the annuitisation rate is set too high, the lifetime utility of some generations will fall below the baseline, as shown in Fig. 2 *c*.

[16] For example, if plan *A* is: $\gamma_5 = 0{\cdot}35$, $\gamma_6 = 0{\cdot}5$, and $\gamma_7 = 0{\cdot}6$; $\tau_6 = 0{\cdot}087$, $\tau_7 = 0{\cdot}047$, and $\tau_8 = 0$, and plan *D* is: $\gamma_5 = 0{\cdot}6$ and $\tau_6 = 0$, then the two plans are Pareto non-comparable.

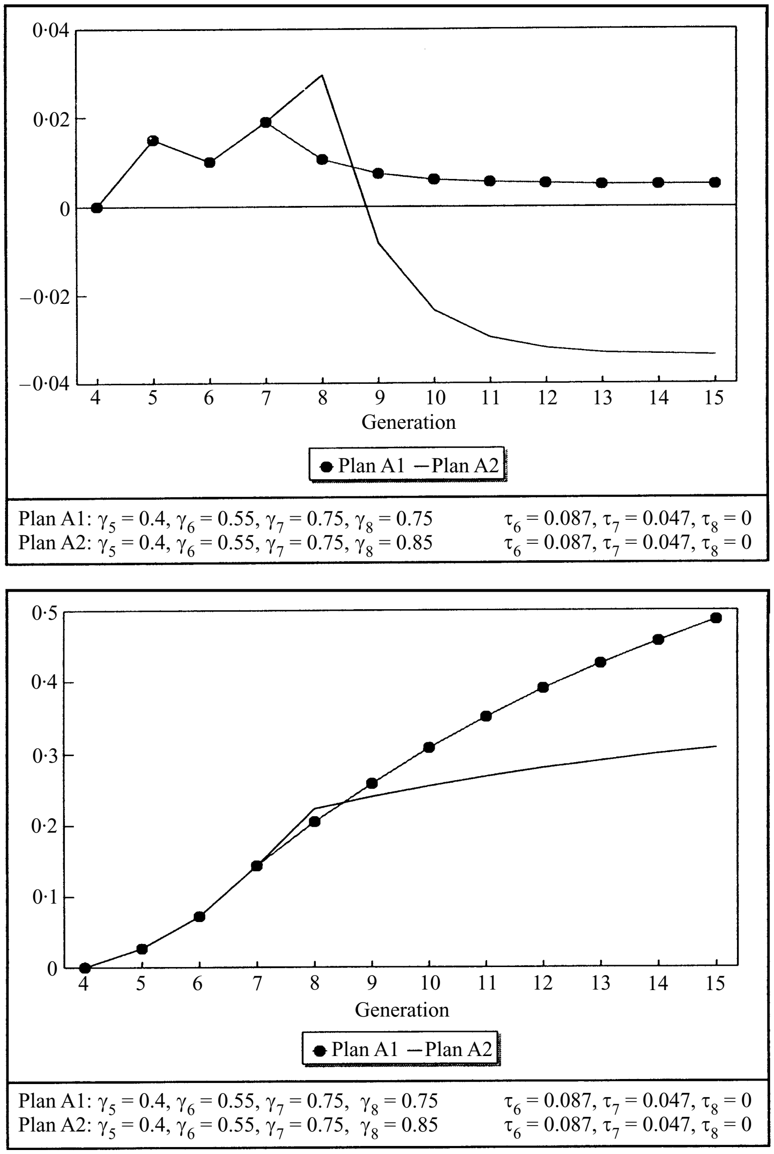


Fig. 2*a*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}1$ (top), $\eta = 0{\cdot}6$ (bottom).

This result does not imply that the lower annuitisation rate the better. As shown in Figs. 2*b* and 2*d* if the annuitisation rate is too low relative to the decline in the social security tax rate, the lifetime utility of one or more generations will fall below the baseline path.[17]

[17] To determine how sensitive our results are to the functional form of the utility function, we simulate the Plan *V* version of the model using the general CRRA utility function $c_t(t)^{1-\sigma}/(1-\sigma)+pc_t(t+1)^{1-\sigma}/(1-\sigma)$, for $\sigma = 0{\cdot}5$ and 2, all other parameters at their baseline values with

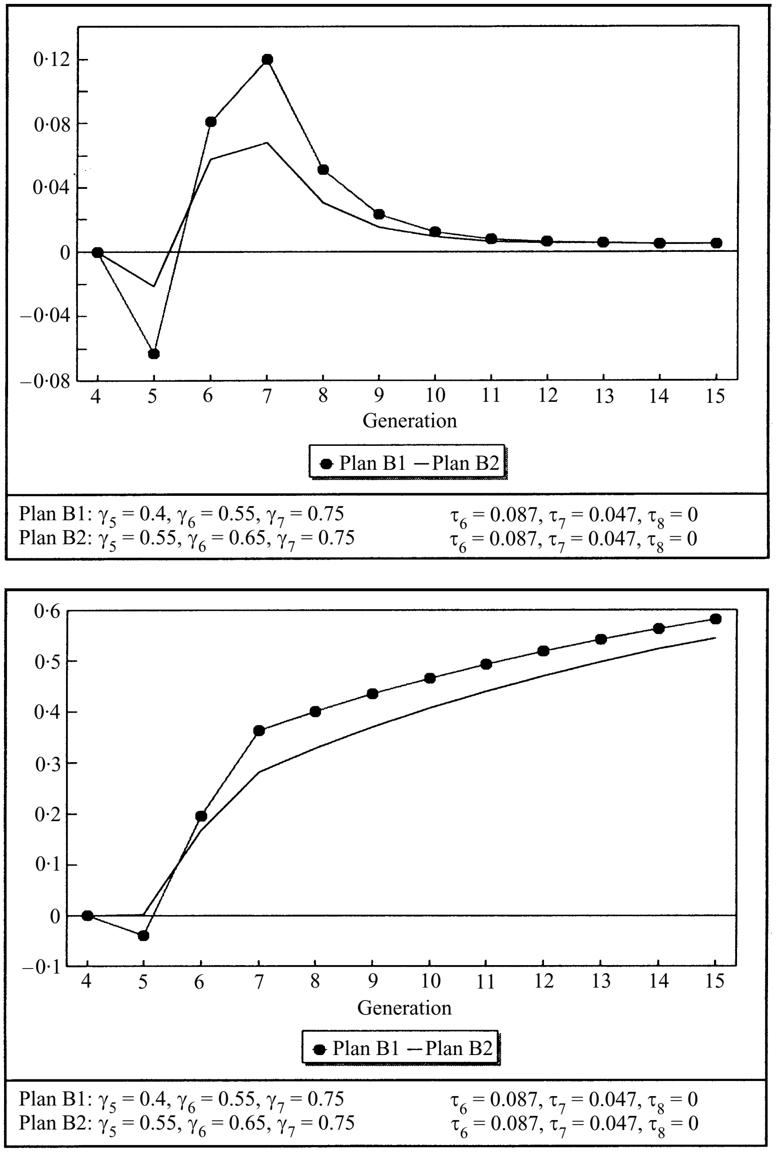


Fig. 2*b*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}1$ (top), $\eta = 0{\cdot}6$ (bottom).

$\eta = 0{\cdot}7$. For both values of σ we choose the root that yields positive consumption and well-behaved utility paths. We find that for $\sigma = 0{\cdot}5$ (2), γs below [above] the baseline value of 0·165 decrease (increase) [increase (decrease)] the growth rate of the economy and put the economy on a Pareto inferior (superior) [superior (inferior)] path. Further, when $\sigma = 0{\cdot}5$, increasing γ at t and decreasing τ at $t+1$ can be Pareto improving. However, unlike the log utility case ($\sigma = 1$), large increases in γ over the baseline are required. These simulation results lead us to believe that the log specification generates mid-range estimates of the effects of moving from a pay-as-you-go to a funded social security system. These simulation results are available from the authors on request.

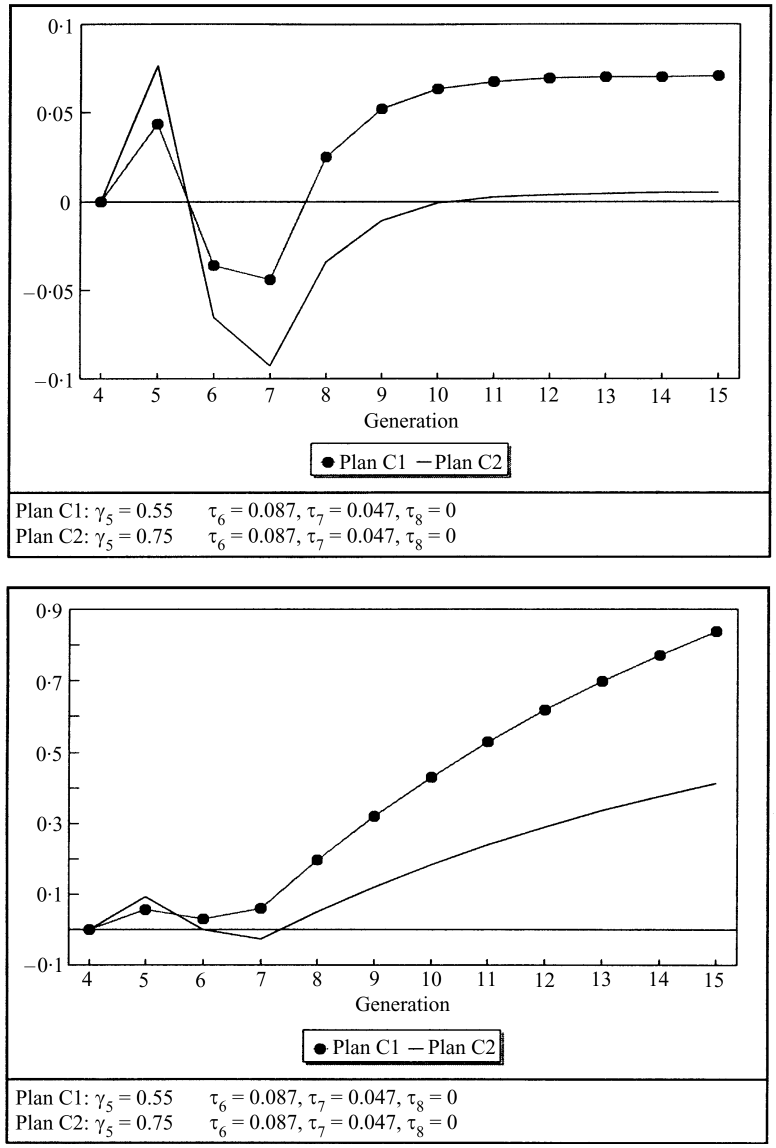
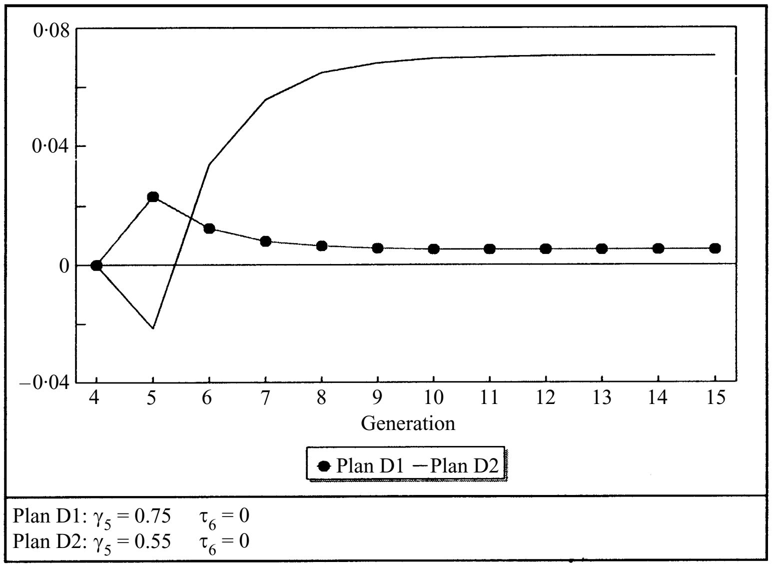


Fig. 2*c*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}1$ (top), $\eta = 0{\cdot}6$ (bottom).

VI. CONCLUSION

In this paper we examine the assertion that increased investment in actuarially fair annuities is Pareto improving. We derive conditions for this assertion to be true in steady-state equilibrium and along equilibrium balanced growth paths. Our results suggest that since annuities reduce the risk of being unable to predict accurately the timing of death, they can reduce saving. Further, by pooling the resources of a cohort, they reduce unintended bequests, which has the side effect of reducing savers' income and so saving. However, annuities,



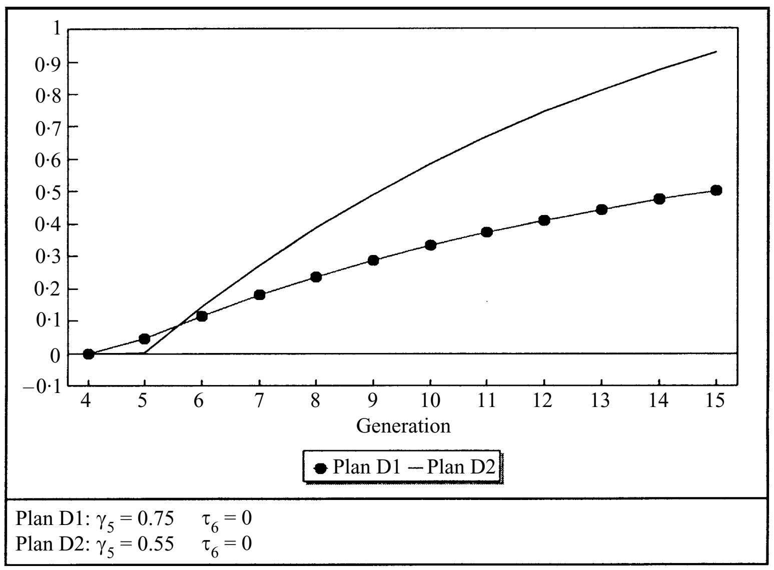


Fig. 2*d*. Lifetime utility eliminating the social security tax. Difference from baseline $\eta = 0{\cdot}1$ (top), $\eta = 0{\cdot}6$ (bottom).

when available by choice rather than mandate, also increase the return to saving, and thus the incentive to save. The interaction of these different effects determines whether an increase in the availability of actuarially fair annuities enhances the incentive to save, thereby increasing capital accumulation, growth and future social welfare.

An important result of this analysis is that in the model economy a pay-as-you-go social security scheme can be replaced by an actuarially fair pension system. Our results should be tempered by the acknowledgement that there are no information problems in this model, so everyone faces the same mortality risk which cannot be affected by individual actions. One suspects that if information problems were introduced, conditions similar to those derived in Townley and Boadway (1988) would be required for the government to be able to implement an optimal, fully-funded social security scheme.

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Appendix

Proof of Proposition 1

Under the assumptions on the utility and production functions an interior, stable, steady-state equilibrium in which all variables are constant for all t will exist and be unique if $(\eta+\beta) < 1$. In a steady state, assuming $\tau(t) = T(t) = 0\,\forall t$, for Plan V, $\gamma^v(t) = \gamma^v\,\forall t$ and $k^v(t) = k^v(t+1) = k^v\,\forall t$, for Plan M $\gamma^M(t) = \gamma^M\,\forall t$ and $k^M(t) = k^M(t+1) = k^M\,\forall t$.

Solving for the steady-state capital stock, consumption of the young, and consumption of the old, respectively, for Plan V yields

$$k^v = \left\{\frac{ap[(1-\beta)+\beta(1-p)(1-\gamma^v)]}{1+p}\right\}^{1/(1-\beta-\eta)},$$

$$c^{y,v} = \{a[(1-\beta)+\beta(1-p)(1-\gamma^v)]\}^{1/(1-\beta-\eta)}\left[\left(\frac{p}{1+p}\right)^{\frac{(\beta+\eta)}{(1-\beta-\eta)}}-\left(\frac{p}{1+p}\right)^{\frac{1}{(1-\beta-\eta)}}\right],$$

$$c^{0,v} = \frac{a\beta}{p}\left(\frac{pa}{1+p}\right)^{\frac{(\beta+\eta)}{(1-\beta-\eta)}}[p+\gamma^v(1-p)][(1-\beta)+\beta(1-p)(1-\gamma)]^{\frac{(\beta+\eta)}{(1-\beta-\eta)}}.$$

The social planner chooses γ^v to maximise

$$\ln c^{v,y} + p\ln c^{v,0}$$

subject to $\gamma^v > 0$. In an interior equilibrium the first order condition of the planner's problem can be rearranged to yield

$$\gamma^v = \frac{p}{(1-p^2)\,\beta}[(1-\beta-\eta)-\beta(1+p)].$$

Solving for the steady-state capital stock, consumption of the young, and consumption of the old, respectively, for Plan M yields

$$k^M = \left[\frac{ap[(1-\beta)+\beta(1-p)(1-\gamma^M)]}{1+p+\dfrac{\gamma^M(1-p)}{p}}\right]^{1/(1-\beta-\eta)},$$

$$c^{M,y} = \{a[(1-\beta)+\beta(1-p)(1-\gamma^M)]\}^{1/(1-\beta-\eta)}\left[\frac{p^2}{p+p^2+\gamma^M(1-p)}\right]^{1/(1-\beta-\eta)} \times\left[\frac{p+\gamma^M(1-p)}{p^2}\right],$$

$$c^{M,0} = a\left[\beta(1-\gamma^M)+\frac{\beta\gamma^M}{p}\right]\left[\frac{p^2a[(1-\beta)+\beta(1-p)(1-\gamma^M)]}{p+p^2+\gamma^M(1-p)}\right]^{\frac{(\beta+\eta)}{(1-\beta-\eta)}}.$$

The social planner chooses γ^M to maximise

$$\ln c^{M,y} + p\ln c^{M,0}$$

subject to $\gamma^M > 0$. In an interior equilibrium the first-order condition of the planner's problem is

$$\frac{(1+p)}{p+\gamma^M(1-p)} - \left[\frac{1+p(\beta+\eta)}{1-\beta-\eta}\right]\left[\frac{\beta}{(1-\beta)+\beta(1-p)(1-\gamma^M)} + \frac{1}{p(1+p)+\gamma^M(1-p)}\right] = 0,$$

which implicitly defines γ^M.

Proof of Proposition 2

Define for Plan V

$$g^v(t+1) = \left(\frac{a(t)\,p\{(1-\beta)[1-\tau(t)]+\beta(1-p)[1-\gamma^v(t-1)]\}}{1+p+\dfrac{p\tau(t+1)(1-\beta)}{\beta[p+\gamma^v(t)(1-p)]}}\right),$$

and for Plan M

$$g^M(t+1) = \left(\frac{a(t)\,p\{(1-\beta)[1-\tau(t)]+\beta(1-p)[1-\gamma^M(t-1)]\}}{1+p+\dfrac{\gamma^M(t)(1-p)}{p}+\dfrac{(1-\beta)\tau(t+1)}{\beta}}\right).$$

The growth rate of the capital stock under Plan V is

$$\frac{k^v(t+1)}{k^v(t)} - 1 = g^v(t+1)\,k^v(t)^{1/(1-\beta-\eta)} - 1,$$

which is positive if $k^v(t)^{1/(1-\beta-\eta)} \to 0$ more slowly than $g^v(t+1) \to \infty$. Assume this condition holds. The growth rate of the capital stock under Plan M is

$$\frac{k^M(t+1)}{k^M(t)} - 1 = g^M(t+1)\,k^M(t)^{1/(1-\beta-\eta)} - 1,$$

which is positive if $k^M(t)^{1/(1-\beta-\eta)} \to 0$ more slowly than $g^M(t+1) \to \infty$. Assume this condition holds. Then, anything that increases $g^v(t+1)$ $[g^M(t+1)]$ increases growth. Thus, holding the tax rate constant for all t, since, by inspection, both $g^v(t+1)$ and $g^M(t+1)$ are decreasing in the tax rate, the higher the tax rate the lower the rate of growth.

Proof of Proposition 3

$$\frac{dg^v}{dp} = \frac{a(t)}{\left[1+p+\dfrac{p\tau(t+1)(1-\beta)}{\beta[p+\gamma(1-p)]}\right]^2} \times \left(\left\{1+p+\frac{p\tau(t+1)(1-\beta)}{\beta[p+\gamma(1-p)]}\right\}\{[1-\tau(t)](1-\beta)+(1-\gamma)\right.$$
$$\times\,\beta(1-2p)\} - p\{[1-\tau(t)](1-\beta)+(1-\gamma)(1-p)\beta\}$$
$$\left.\times\left\{1+\frac{\gamma(1-\beta)\tau(t+1)}{\beta[p+\gamma(1-p)]^2}\right\}\right) \gtreqless 0 \quad \text{for} \quad \gamma \neq 1, \quad > 0 \quad \text{for} \quad \gamma = 1;$$

and

$$\frac{dg^M}{dp} = \frac{a(t)}{\left[1+p+\dfrac{\gamma(1-p)}{p}+\dfrac{(1-\beta)\tau(t+1)}{\beta}\right]^2} \times \left(\left[1+p+\frac{\gamma(1-p)}{p}+\frac{(1-\beta)\tau(t+1)}{\beta}\right]\right.$$
$$\times\{[1-\tau(t)](1-\beta)+(1-\gamma)\beta(1-2p)\} - \{[1-\tau(t)](1-\beta)+(1-\gamma)\beta(1-p)\}$$
$$\left.\times\,p\left(1-\frac{\gamma}{p^2}\right)\right) \gtreqless 0 \quad \text{for} \quad \gamma \neq 1, \quad > 0 \quad \text{for} \quad \gamma = 1.$$

Proof of Proposition 4

$$\frac{dg^v}{d\gamma} = \frac{a(t)}{\left[1 + p + \dfrac{p\tau(t+1)(1-\beta)}{\beta[p+\gamma(1-p)]}\right]^2}\left(\left\{1 + p + \frac{p\tau(t+1)(1-\beta)}{\beta[p+\gamma(1-p)]}\right\}[\beta(p-1)]\right.$$

$$\left. + p\{(1-\beta)[1-\tau(t)] + \beta(1-p)(1-\gamma)\}\left[\frac{p(1-p)(1-\beta)\tau(t+1)}{\beta[p+\gamma(1-p)]^2}\right]\right) < 0$$

if $\tau(t) = 0\,\forall t$, $\gtreqless 0$ otherwise;

and

$$\frac{dg^M}{d\gamma} = \frac{a(t)}{\left[1 + p + \dfrac{\gamma(1-p)}{p} + \dfrac{(1-\beta)\tau(t+1)}{\beta}\right]^2}\left(\left[1 + p + \frac{\gamma(1-p)}{p} + \frac{(1-\beta)\tau(t+1)}{\beta}\right]\right.$$

$$\left. \times [p\beta(p-1)] - \{[1-\tau(t)](1-\beta) + \beta(1-\gamma)(1-p)\}(1-p)/p\right) < 0$$

for all $\tau(t)$.

Proof of Proposition 5

This follows directly from Propositions 2 and 4.