

## The search for equilibrium relationships in international finance: the case of the monetary model

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This paper considers some of the main long-run equilibrium relationships in international finance. We supplement the Phillips–Perron test, which has a unit root under the null, with the new KPS test statistic which is based on a stationary null and apply them to the various exchange rate fundamentals. The application of the Johansen test for multiple cointegrating factors finds evidence of the existence of stable money demand functions in both the USA and UK with relatively short-lived perturbations. However, there appears to be insufficient information in the data to distinguish whether the real exchange rate has a unit root or is persistent and mean reverting. The consequent persistent deviations from purchasing power parity appears the only source of rejection of the equilibrium monetary model.

International finance has proved a particularly fertile area for the application of the relatively new econometric technique of testing for unit roots and cointegration. Since financial markets are volatile and our understanding of risk premium, the causes of excess volatility, and departures from equilibrium are as yet incomplete, the profession has understandably focused on long-run equilibrium relationships. While exchange rates appear to conform to the behavior of other asset prices and are apparently martingales and risk premia in forward markets are invariably stationary, little other reassuring evidence has accumulated on even the long-run properties of the major relationships we would expect. In particular the notion of purchasing power parity (PPP), even as an equilibrium relationship, appears dubious and no satisfactory model has been found to explain the determination of nominal exchange rates.

In this paper we consider cointegration and the implication of equilibrium relationships within a standard monetary model of exchange rate determination. The model we present in Section I is standard except that we hypothesize that the demand for money balances and the uncovered interest parity relationships are subject to stationary disequilibrium disturbances and that the real exchange rate is persistent and may or may not be mean reverting. This last assumption allows for long-lived but finite deviations from PPP. We also briefly discuss some extensions of the model and their implications for long-run behavior.

Section I of the paper discusses the evidence in favor of unit roots in exchange rates and the ‘fundamentals’ derived from the monetary model. We apply various

unit root tests to the US dollar–UK pound bilateral exchange rate relationship estimated from March 1973 through May 1990. We discuss some of the limitations of this methodology and we supplement the now familiar Phillips–Perron test with the new KPS test due to Kwiatkowski *et al.* (1990). The KPS test has a null hypothesis of stationarity and appears to provide helpful information in conjunction with the Phillips–Perron tests. There is strong evidence that the nominal exchange rate and nearly all the fundamentals, such as US and UK money supplies, real outputs, and interest rates have a unit root, or are  $I(1)$  processes. However, the inflation rates in both countries appear to be fractionally integrated and persistent, but mean reverting, while the price differential between the USA and UK appears to be  $I(1)$ . On using the Johansen test for multiple cointegrating factors, we find there is strong evidence in favor of a long-run money demand curve in both the USA and UK. Disequilibrium disturbances around these equilibrium relationships appear stationary,  $I(0)$ , and mean reverting. However, data limitations prevent us from formally distinguishing whether the real exchange rate has a unit root, or alternatively exhibits long-run persistence but eventual mean reversion consistent with a fractionally integrated process.

### I. An equilibrium monetary model

The monetary model of exchange rate determination provides a convenient vehicle for considering some of the major equilibrium relationships in international finance. There is considerable evidence that the static model, as originated by Frenkel (1976), can be empirically rejected. However, it seems reasonable to consider the monetary model as being an appropriate specification of long-run equilibrium relationships, where the nominal exchange rate can depart from equilibrium due to a variety of possible shocks or disequilibrium disturbances. We therefore consider a rather more general specification of the monetary model of exchange rate determination with various disequilibrium errors being included to give the following set up:

$$\begin{aligned} \langle 1 \rangle \quad & s_t = p_t - p_t^* + r_t, \\ \langle 2 \rangle \quad & m_t - p_t = \phi y_t - \lambda i_t + u_t, \\ \langle 3 \rangle \quad & m_t^* - p_t^* = \phi y_t^* - \lambda i_t^* + u_t^*, \\ \langle 4 \rangle \quad & i_t - i_t^* = E_t s_{t+1} - s_t + v_t. \end{aligned}$$

In this model  $s$ ,  $p$ ,  $m$ ,  $y$ , and  $i$  are the nominal exchange rate, domestic prices, money, real output, and interest rates, respectively, and are all measured in logarithmic form except for the rate of interest. Asterisks denote variables that are foreign rather than relevant to the domestic country.

Equation  $\langle 1 \rangle$  is an identity and merely specifies the real exchange rate,  $r_t$ . In accord with the available empirical evidence we assume that all the variables are nonstationary, except for  $u_t$ ,  $u_t^*$ ,  $v_t$  and possibly  $r_t$ . In particular  $u_t$  and  $u_t^*$  are disturbances around the money demand functions and are assumed to be  $I(0)$  or covariance stationary processes, so that equations  $\langle 2 \rangle$  and  $\langle 3 \rangle$  define long-run money demand functions which are also cointegrating relationships. Similarly,  $v_t$  is also assumed to be  $I(0)$  since it is the disturbance around uncovered interest rate parity and is a composite error since it includes a rational expectations error

and a possibly time dependent risk premium. As for the real exchange rate  $r_t$  we remain agnostic and note that it appears to exhibit persistent deviations around its equilibrium value of zero. Lothian (1987) provides a detailed description of the behavior of real exchange rates in the recent past.

Solving <1> through <4> gives a rational expectations equation of

$$\langle 5 \rangle \quad s_t = x_t + \left( \frac{\lambda}{1 + \lambda} \right) E_t s_{t+1},$$

where

$$\langle 6 \rangle \quad x_t = (1 + \lambda)^{-1} [(m_t - m_t^*) - \phi(y_t - y_t^*) + w_t],$$

and are the 'forcing variables' or 'fundamentals' supplemented with the composite disturbance term

$$\langle 7 \rangle \quad w_t = \lambda r_t - u_t + u_t^* - r_t.$$

The forward looking solution of <5> is

$$\langle 8 \rangle \quad s_t = \sum_{j=0}^{\infty} \left( \frac{\lambda}{1 + \lambda} \right)^j E_t x_{t+j}.$$

In practice  $s_t$  and the linear combination of the fundamentals  $x_t$  are likely to be I(1) processes and Baillie (1989a) discusses how models such as <8> can be expressed in terms of stationary and/or cointegrating relationships. Equation <8> is essentially similar to the type of model discussed by Campbell and Shiller (1987) and it follows that <8> can be transformed to

$$\langle 9 \rangle \quad \lambda E_t \Delta s_{t+1} = (s_t - (1 + \lambda)x_t).$$

Since  $s_t$  is I(1) and  $\Delta s_{t+1}$  is I(0) it follows that the linear combination of  $s_t$  and  $x_t$  on the right-hand side of <9> is also stationary and hence defines the cointegrating relationship between  $s_t$  and the fundamentals  $x_t$ .

If the real exchange rate is also stationary then  $w_t$  is I(0)

$$s_t - (m_t - m_t^*) + \phi(y_t - y_t^*) - w_t \sim I(0),$$

and hence

$$\langle 10 \rangle \quad s_t - (m_t - m_t^*) + \phi(y_t - y_t^*) \sim I(0),$$

so that the nominal exchange rate can be expected to be cointegrated with relative money supplies and relative real outputs. Rejection of <10> could be due to the nonstationarity of any of the components of  $w_t$ , such as the real exchange rate.

While the model given by equations <1> through <4> is very basic apart from the inclusion of the stochastic disturbances, there have been many attempts at increasing the realism of the model. In particular Dornbusch (1976) has considered a sticky price model where there are short-run deviations from PPP, so that  $r_t$  is essentially I(0). Woo (1985) assumes a Goldfeld type adjustment model in the real balances equations, which leads to a second-order rational expectations equation. Wickens (1984) shows that a similar second-order equation comes out of Dornbusch's (1976) overshooting model under rational expectations. However, both these extensions still imply the same form of equilibrium cointegrating relationship given by <6>; albeit with a different cointegrating vector.

As it stands, the monetary model given by equations <1> through <4> is essentially incomplete since the money supply and aggregate supply relationships are omitted from the model. To complete the model we can specify that both countries have aggregate supply equations, price adjustment equations and money supply rules of the form:

$$\langle 11 \rangle \quad y_t = -\alpha(i_t - E_t \Delta p_{t+1}) - \beta r_t + q_t,$$

$$\langle 12 \rangle \quad \Delta p_t = \gamma y_t + \eta_t,$$

and

$$\langle 13 \rangle \quad \Delta m_t = \mu + \kappa(i_t - \bar{i}) + \omega_t,$$

where  $\bar{i}$  is desired level of interest rates and  $q$ ,  $\eta$ , and  $\omega$  are disturbances. Lane (1991) has solved such a system under rational expectations. It transpires that if  $q_t$  is I(1) so that shocks to aggregate supply are persistent, then the real exchange rate will also be I(1) and the nominal exchange rate will not be cointegrated with money supplies. When the shocks  $q_t$ ,  $\eta_t$ , and  $\omega_t$  are all I(0), then the real exchange rate  $r_t$  will also be stationary. Modifications and extensions of the basic model give similar results; the most important assumptions always concern the properties of the shocks to aggregate supply.

## II. Testing for equilibrium relationships

As previously indicated, virtually all of the variables in the monetary model can be expected to be nonstationary and the detection of equilibrium (cointegrated) relationships critically depends upon the judicious use of the various testing procedures. The now standard augmented Dickey–Fuller test, due to Dickey and Fuller (1979, 1981), and the Phillips–Perron test, developed by Phillips (1987), Phillips and Perron (1988), and Perron (1988) are both based on a null hypothesis of a unit root existing in the autoregressive representation of a univariate time series and typically seek rejection against a stationary alternative. Recently, Schwert (1987) and DeJong *et al.* (1989) have questioned the power of these procedures in distinguishing between trend stationary and difference stationary processes. Schwert (1987) has also shown the Phillips–Perron test to have low power when the true data generating process has a substantial moving average component.

The implementation of classical statistical hypothesis testing ensures that the null hypothesis is accepted, unless there is very strong evidence against it. If an investigator wishes to test stationarity as a null, and has a strong prior in its favor, then it is not clear that the familiar Dickey–Fuller parameterization is very useful. An alternative approach is provided by the KPS statistic, introduced by Kwiatkowski *et al.* (1990). The KPS approach assumes the univariate series can be decomposed into the sum of a deterministic trend, random walk and stationary I(0) disturbance; and is based on a Lagrange Multiplier score testing principle. The KPS test is defined as

$$\langle 14 \rangle \quad \eta = T^{-2} \Sigma S_t^2 / s^2(k),$$

where

$$S_t = \sum_{i=1}^t e_i$$

is the partial sum of the residuals  $e_t$ , when the series has been regressed on an intercept and possibly also a time trend; and  $T$  is the sample size.  $s^2(k)$  is a consistent nonparametric estimate of the disturbance variance; it is computed in an identical manner to its equivalent in the Phillips–Perron test by using a Bartlett window adjustment based on the first  $k$  sample autocovariances as suggested by Newey and West (1987). When the residuals are computed from an equation with only an intercept, the test statistic is denoted by  $\hat{\eta}_u$ , and when a time trend is included in the initial regression, the test statistic is denoted by  $\hat{\eta}_\tau$ . Under the null hypothesis of the series being  $I(0)$ , KPS show that both  $\hat{\eta}_u$  and  $\hat{\eta}_\tau$  are asymptotically functions of a Brownian bridge and they produce tables of critical values. The critical values for  $\hat{\eta}_u$  and  $\hat{\eta}_\tau$  are 0.739 and 0.216 at the 0.01 level and 0.463 and 0.146 at the 0.05 level, respectively.

The combined use of the Phillips–Perron (PP) and KPS test statistics gives rise to four possible outcomes:

1. Rejection by the PP statistic and failure to reject by the KPS is viewed as strong evidence of covariance stationarity, *i.e.*, an  $I(0)$  process:
2. Failure to reject by the PP and rejection by the KPS statistic appears to be strongly indicative of a unit root, *i.e.*,  $I(1)$  process:
3. Failure to reject by both the PP and KPS statistics is probably due to the data being insufficiently informative on the long-run characteristics of the process.
4. Rejection by both the PP and KPS statistics presumably indicates evidence of some process that is neither well described by an  $I(1)$  or  $I(0)$  process.

Table 1 presents results on applying the PP and KPS tests to the fundamentals of the US dollar–UK pound bilateral exchange rate. The data are monthly, seasonally unadjusted and are from March 1973 through May 1990, a total of 207 observations. The nominal exchange rate  $s_t$  is the end-of-month closing bid price on the New York foreign exchange market,  $y$  and  $y^*$  are proxied by the Indexes of Industrial Production, and prices are the relevant CPI series. Interest rates are the Federal Funds rate for the US and the London Interbank Offer Rate (LIBOR), which is an appropriate equivalent for the UK. All these series were obtained from Citibase. Finally M1 was used for the US money supply and M0 for the UK money supply, and these series were obtained from the International Monetary Fund.<sup>1</sup>

Following the work of Meese and Singleton (1982) and Baillie and Bollerslev (1989) who applied the ADF and PP tests, respectively, we find very strong evidence that nominal exchange rates have a unit root and appear to be a martingale. This is of course consistent with the properties of other asset prices. However, as described by Schwert (1987) and DeJong *et al.* (1989) there is less consensus about the properties of aggregate macroeconomic time series data. From the combined use of the PP and KPS test statistics many of the series in Table 1 appear to be  $I(1)$  processes. Most uncertainty exists over the US and UK inflation rates and the real exchange rate  $r_t$ . Inspection of the autocorrelation coefficients for the two inflation series indicates a typical pattern of persistence with the first-order autocorrelation being between 0.4 and 0.5 and the subsequent ones very slowly declining with increasing lag so that the coefficient at lag 15 is around 0.3. As previously described by Baillie (1989b), such a pattern is typical of CPI inflation series for the G7 and is not really consistent with either an  $I(0)$

TABLE 1. Tests for orders of integration.

Variable	$H_0: I(1)$		$H_0: I(0)$		Conclusions
	$Z(t_{z^*})$	$Z(t_{\bar{z}})$	$\hat{\eta}_u$	$\hat{\eta}_t$	
$s_t$	-1.794	-1.635	1.279 <sup>a</sup>	0.175 <sup>b</sup>	I(1)
$\Delta s_t$	-6.947 <sup>a</sup>	-6.980 <sup>a</sup>	0.140	0.076	I(0)
$p_t$	-3.383 <sup>b</sup>	-0.598	2.343	0.546 <sup>a</sup>	I(1)
$p_t^*$	-3.846 <sup>a</sup>	-1.822	2.283 <sup>a</sup>	0.573 <sup>a</sup>	I(1)
$r_t = s_t - p_t + p_t^*$	-1.658	-1.684	0.200	0.202 <sup>b</sup>	Cannot distinguish between I(0) and I(1)
$\Delta p_t$	-4.293 <sup>a</sup>	-5.540 <sup>a</sup>	1.063 <sup>a</sup>	0.150 <sup>b</sup>	Reject both I(0) and I(1)
$\Delta p_t^*$	-6.806 <sup>a</sup>	-8.177 <sup>a</sup>	1.210	0.138	Reject both I(0) and I(1)
$p_t - p_t^*$	-2.882 <sup>b</sup>	-2.189	2.010 <sup>a</sup>	0.465 <sup>a</sup>	I(1)
$\Delta(p_t - p_t^*)$	-9.642 <sup>a</sup>	-10.023 <sup>a</sup>	0.620 <sup>b</sup>	0.154 <sup>b</sup>	Reject both I(0) and I(1)
$m_t$	-0.419	-1.951	2.400 <sup>a</sup>	0.247 <sup>a</sup>	I(1)
$m_t^*$	-2.801	-1.996	2.275 <sup>a</sup>	0.550 <sup>a</sup>	I(1)
$m_t - m_t^*$	-1.858	-2.262	0.603 <sup>b</sup>	0.522 <sup>a</sup>	I(1)
$y_t$	-0.232	-2.479	2.147 <sup>a</sup>	0.113	I(1)
$\Delta y_t$	-6.216 <sup>a</sup>	-6.213 <sup>a</sup>	0.081	0.049	I(0)
$y_t^*$	-0.416	-2.237	1.716 <sup>a</sup>	0.281 <sup>a</sup>	I(1)
$\Delta y_t^*$	-22.752 <sup>a</sup>	-22.724 <sup>a</sup>	0.137	0.041	I(0)
$y_t - y_t^*$	-1.616	-2.872	1.989 <sup>a</sup>	0.200 <sup>b</sup>	I(1)
$i_t$	-2.028	-2.017	0.275	0.274 <sup>a</sup>	I(1)
$i_t^*$	-2.455	-2.482	0.128	0.114	Cannot distinguish between I(0) and I(1)

Notes.  $Z(t_{z^*})$  and  $Z(t_{\bar{z}})$  are the Phillips-Perron adjusted 't-statistics' on the lagged dependent variable in regressions with an intercept only, and intercept and time trend, respectively. The 0.05 critical values for  $Z(t_{z^*})$  and  $Z(t_{\bar{z}})$  are -2.86 and -3.41, respectively, and 0.01 critical values are -3.43 and -3.96, respectively.

$\hat{\eta}_u$  and  $\hat{\eta}_t$  are the KPS statistics<sup>3</sup> described in the text and are based on residuals from regressions with an intercept, and intercept and time trend, respectively. The 0.05 critical values for  $\hat{\eta}_u$  and  $\hat{\eta}_t$  are 0.463 and 0.146 and the 0.01 critical values are 0.739 and 0.216, respectively.

The superscripts a and b denote calculated test statistics which are significant at the 0.01 and 0.05 levels respectively.

All test statistics in the above table are based on a Newey-West adjustment with eight lags.

or I(1) process. Many previous studies have estimated ARIMA (0, 1, 1) processes for inflation and found a large negative moving average coefficient, which almost cancels with the differencing operator. Baillie (1989b) and Schwert (1987) discuss this situation in more detail, and Schwert (1987) notes the poor performance of the PP test in this situation. As discussed by Pecchenino and Rasche (1991) the behavior of the post-war US CPI may be consistent with an I(1) process in different monetary policy regimes. However, the price differential,  $p_t - p_t^*$ , appears unambiguously I(1), so that some type of fractional cointegration as described by Granger (1980) may be occurring.

Given that the nominal exchange rate and many of the fundamentals in both countries appear to be well described by I(1) processes, it is then appropriate to consider whether some form of cointegration is present that is consistent with the equilibrium monetary model. If the vector  $Y_t$  contains  $g$  random variables

all of which are I(1) processes, then the VAR( $p$ ) model can be written as

$$\Delta Y_t = \sum_{j=1}^{p-1} \Phi_j \Delta Y_{t-j} + \Phi_p Y_{t-p} + \varepsilon_t,$$

where  $\varepsilon_t$  is a vector white noise process. If the rank of  $\Phi_p$  is  $r$ , where  $r \leq g - 1$ , then there exists  $r$  linear, cointegrating vectors. Johansen (1988) has developed a testing procedure for determining these cointegrating vectors. Ideally the Johansen test could be applied to a vector containing all nine variables in equations <1> through <3>. However, the substantial autocorrelation present in the differenced price and money series requires a large number of lags in the VAR. This substantially reduces the degrees of freedom and probable power of the test. Accordingly, Table 2 presents the Johansen trace statistics for testing for cointegration when  $Y_t$  contains different subsets of variables implied by the monetary model. Interestingly enough there is evidence of the existence of one cointegrating vector between the group of variables ( $m_t - p_t$ ),  $y_t$ , and  $i_t$ , for both the USA and UK. On normalizing the coefficient of real money balances to be unity, Table 3 reports estimates of the cointegrating vector and the parameter standard errors. For both countries it seems reasonable to interpret the vector as an equilibrium money demand function; the size and sign of the long-run elasticities appears quite reasonable.<sup>2</sup> The disequilibrium disturbances  $u_t$  and  $u_t^*$  around each equilibrium money demand function were also examined. Both series are quite well represented by AR(3) and AR(2) models, although the US money demand disturbance exhibited some residual autocorrelation at lags 6 and 7. Both estimated money demand disturbances appear to be clearly stationary as evidenced by the KPS statistics and roots of the estimated AR processes. However,  $\hat{u}_t$  and  $\hat{u}_t^*$  are quite strongly autocorrelated with the median lag response to a shock being in the order of seven months for the USA and ten months for the UK. Hence unanticipated innovations which cause disequilibria in the US and UK money markets tend to be important for some time, but are nevertheless stationary and mean reverting.

TABLE 2. Johansen trace test statistics.

Variables	$p$	$r = 0$ (31.53)	$r \leq 1$ (17.95)	$r \leq 2$ (8.18)
$(m_t - p_t), y_t, i_t$	4	31.68	3.77	0.11
$(s_t - m_t + m_t^*), y_t, y_t^*$	5	29.08	6.71	0.91
		$r = 0$ (17.95)	$r \leq 1$ (8.18)	
$(m_t^* - p_t^*), y_t^*, i_t^*$	5	28.07	2.08	

Notes. The test statistics are the Johansen trace test from unrestricted VARs. Critical values at the 0.05 level are given under the value of  $r$ ; the values being obtained from the recently updated tables provided by Osterwald-Lenum (1990).

In the case of the UK money demand function it was found the data could not reject a long-run real income elasticity of one. This restriction was imposed in estimation and consequently reduced the maximum number of cointegrating factors from two to one.

TABLE 3. Estimates of the equilibrium money demand functions.

USA:

$$(m_t - p_t) = 0.5644y_t - 0.0369i_t + \hat{u}_t$$

$$(0.0932) \quad (0.0055)$$

$$(1 - 1.1653L + 0.4577L^2 - 0.2203L^3)\hat{u}_t = -0.4572 + \hat{\varepsilon}_t \quad Q(8) = 21.78$$

$$(0.0696) \quad (0.0991) \quad (0.0541) \quad (0.0240)$$

Median lag on  $\hat{u}_t$  is 7.6 months and mean lag is 11.9 months.

UK:

$$(m_t^* - p_t^*) = 1.0000y_t^* - 0.1845i_t^* + \hat{u}_t^*$$

$$(0.0070)$$

$$(1 - 1.1761L + 0.2357L^2)\hat{u}_t^* = 2.5496 + \hat{\varepsilon}_t^* \quad Q(8) = 2.06$$

$$(0.0619) \quad (0.0579) \quad (0.1717)$$

Median lag on  $\hat{u}_t^*$  is 10.3 months and mean lag is 15.8 months.

*Notes.* The equilibrium money demand functions are the estimated one cointegrating vector in a VAR(4) for the USA and VAR(5) for the UK. Standard errors appear in parentheses below corresponding parameter estimates, and are derived from the Wald test statistics on cointegrating vectors developed by Jøhansen (1989). In both cases the real money balances coefficient was normalized at unity. For the UK the long-run real income elasticity was constrained to be unity.

The residuals  $\hat{u}_t$  and  $\hat{u}_t^*$  are the estimated disturbances around the equilibrium money demand functions and are represented as AR(3) and AR(2) processes, respectively. The KPS,  $\hat{\eta}_u$  statistic applied to  $\hat{u}_t$  and  $\hat{u}_t^*$  was 0.262 and 0.355, respectively, indicating the null hypothesis of stationarity could not be rejected.

The  $u_t$  and  $u_t^*$  disequilibrium disturbances are also the estimated error correction terms in a VAR with the differenced real balances as the dependent variables and lagged differences of real output and interest rates also appearing in the equations. We do not report such estimated equations since they are tangential to the interests of this study.

Table 2 also reports the Jøhansen trace statistic based on VARs including other variables in the  $Y_t$  vector. In general there does not appear to be any support for the nominal exchange rate being cointegrated with relative money supplies or real outputs, as hypothesized by equation <10>. One possible explanation for this lies in the fact that the estimated equilibrium money demand functions imply different long-run real income and interest rate elasticities for the USA and UK. Accordingly, money supplies and real incomes were examined separately with the exchange rate; but again no evidence of cointegration was found. Full details of these results are omitted from the paper for reasons of space but are available from the authors on request. Hence no evidence is available to support the long-run version of equation <10>.

### III. Persistence and the real exchange rate

The combined use of the PP and KPS testing approach in Table 1 reveals that most of the fundamentals appear to be well described as I(1) processes. Three



series stand out as not conforming with this general pattern: the real exchange rate and the inflation rates in both the USA and UK. As previously discussed the issue as to whether inflation is stationary has concerned many previous authors and inflation series generally have a characteristic pattern of persistence in their autocorrelation functions which is suggestive of an ARFIMA type of fractionally integrated processes. Following Granger (1980) and Granger and Joyeux (1980) a process  $y_t$  is said to be integrated of order  $d$  where  $0 \leq d \leq 1$  if

$$(1-L)^d y_t = \omega_t,$$

where  $\omega_t$  is  $I(0)$  and can be represented as a stationary and invertible ARMA  $(p, q)$  process. In the simplest case where  $\omega_t$  is i.i.d.  $(0, \sigma^2)$  white noise,  $y_t$  is said to be fractionally integrated white noise. One-sided representations of the form

$$\langle 15 \rangle \quad y_t = \sum_{j=0}^{\infty} \pi_j y_{t-j} + \omega_t$$

and

$$\langle 16 \rangle \quad y_t = \sum_{j=0}^{\infty} \psi_j \omega_{t-j}$$

exist from binomial expansions. The coefficients have the form

$$\pi_j = \frac{\Gamma(j-d)}{\Gamma(-d)\Gamma(j+1)}$$

and

$$\psi_j = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)},$$

where  $\Gamma(\cdot)$  is the gamma function. Clearly when  $\omega_t$  is i.i.d.  $(0, \sigma^2)$ ,  $\langle 15 \rangle$  and  $\langle 16 \rangle$  have the interpretation of being finite autoregressive and infinite moving average representations. For  $-0.5 < d < 0.5$  the process  $y_t$  is covariance stationary and the moving average coefficients decay at the relatively slow hyperbolic rate compared with the regular stationary and invertible ARMA process where the  $\psi$  coefficients decline exponentially with  $j$ . Granger (1980) and Geweke and Porter-Hudak (1983) give expressions for the autocorrelation function of  $y_t$ .

Diebold and Rudebusch (1991) have provided interesting simulation evidence to show that conventional Dickey-Fuller unit root tests fail to reject a unit root sufficiently often when the true data generating process is fractionally integrated.

We estimated ARFIMA  $(0, d, 1)$  models for the US and UK inflation series and have reported the results in Table 4. When  $d=1$  the familiar ARIMA  $(0, 1, 1)$  appears to provide a reasonable representation for both inflation series. Application of the Geweke and Porter-Hudak (1981) two step estimator provided an initial estimate of  $d$ , the fractional differencing parameter that was sensitive to the range of ordinates used in the spectral regression devised by Geweke and Porter-Hudak (1983). However, for both the US and UK inflation series an initial estimate of  $d = 0.40$  seemed reasonable. We then used this value of  $d$  to filter the inflation series from  $\langle 15 \rangle$  to obtain a filtered series  $\hat{\omega}_t$  and then fitted an ARMA model to this filtered series, having discarded the first 25 observations that were probably corrupted by the initialization. The results in Table 4 indicate

TABLE 4. Estimates of the model.

$$(1-L)^d \Delta p_t = b + (1+\theta L)\varepsilon_t$$

Country	$d$	$b$	$\theta$	Log likelihood	$Q(10)$
USA	1	-0.0009 (0.0090)	-0.5099 (0.0586)	-16.51	34.22
USA	0.4	0.0389 (0.0222)	0.1607 (0.0668)	-6.75	12.42
UK	1	-0.0063 (0.0117)	-0.8366 (0.0431)	-189.25	17.39
UK	0.4	-0.0571 (0.0631)	-0.1000 (0.0549)	-185.85	15.52

*Notes.* All models were estimated by approximate maximum likelihood and were conditioned on the value of  $d$ .  $Q(10)$  is the Box-Pierce portmanteau statistic based on the first ten autocorrelations of the residuals.

the ARFIMA (0, 0.4, 1) model to have a significantly higher maximized log likelihood value than for the model with  $d=1$ . Although these results are maximum likelihood estimates conditional on the value of  $d$ , they are highly suggestive that the mean reverting fractionally integrated process is at least as likely as a process containing a unit root.

Similar analysis for the real exchange rate series,  $r_t$ , was less striking, with most of the Geweke and Porter-Hudak (1983) spectral regressions estimating  $d$  as being close, but possibly slightly less than unity. It should be noted that most studies of the real exchange rate in the current float have concluded that it is nonstationary, *e.g.*, Adler and Lehmann (1983). While Diebold *et al.* (1991) provide evidence of the real exchange rate being mean reverting and persistent over a period of 100 years covering both floating and fixed exchange rate regimes. Analysis of recent history only covering the current float suggests it is hard to distinguish between a process that has a unit root, or is very persistent but mean reverting. Clearly either more data and/or more economic theory is required to explain the behavior of the US/UK real exchange rate.<sup>4</sup>

#### IV. Conclusion

After the initial optimism surrounding the monetary model of exchange rate determination, it has subsequently fallen out of favor as mounting empirical evidence has accumulated against it. In this study we have shown how tests of cointegration can be interpreted as providing evidence on the validity of components of an equilibrium monetary model. The results of multivariate tests of cointegration appear to provide surprisingly strong evidence for a stable equilibrium money demand function in both the USA and UK. Furthermore, disturbances around the equilibria are quite strongly autocorrelated but nevertheless definitely stationary and mean reverting.

However, just over 18 years of data may well be far too short to make strong inferences about other properties of the model. In particular it is clearly difficult on the basis of the available information to draw any definite conclusions as to whether the US and UK inflation rates and the real exchange rate have a unit

root or are fractionally integrated and mean reverting processes. On the strength of this evidence we may well have to admit that either hypothesis is consistent with the data, and it may well not be possible to devise a test to clearly differentiate between them.

Related work concerning the apparent instability of cointegrating relationships between different exchange rates has been provided by Sephton and Larsen (1991).

Apart from having to wait for more data, as suggested by Hakkio and Rush (1991), there is also a clear need to develop other theoretical models which can explain the price adjustments and deviations in the real exchange rate.

### Notes

1. We are grateful to Timothy D. Lane for making this data set available to us.
2. Hoffman and Rasche (1991) report estimates of long-run money elasticities for a longer period.
3. We are very grateful to Robert H. Rasche for providing us with software that he had written.
4. Lothian (1985) has provided some supportive evidence on long-run relative purchasing power parity and also the relationship between changes in exchange rates and money growth differentials.

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