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Model Parameter And Time Delay Estimation In The Frequency Domain

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Abstract

This paper discusses the estimation of the parameters (including the time delay) of single input, single output (SISO) process models from an appropriate number of arbitrarily specified points on the process frequency response. The method involves combining an analytical approach with a least squares approach using a gradient algorithm, to provide accurate and robust estimates of the parameters.

1. INTRODUCTION

The estimation of the parameters (including the time delay) of a process model in the frequency domain may be considered to be divided into two stages: firstly, the estimation of the process frequency response over an appropriate frequency range and secondly, the estimation of the parameters of the model from the frequency response. The estimation of the process frequency response has been explored in detail in the published literature; estimation techniques range from the calculation of the frequency response in open loop by finding the output response of the system to a pulse input (Rajakumar and Krishnaswamy [1]) to the use of higher order spectral approaches (Nikias and Petropulu [2]). On-line measurement of the process frequency response using recursive Fourier transform calculations (Ringwood and O'Dwyer [3]) is another example of the many techniques that have been published. Model parameter estimation from the frequency response has also been considered in the literature; just two examples of the approaches that have been investigated are graphical methods based on Bode plots (as described by Seborg, Edgar and Mellichamp [4]) and least squares estimation of the parameters of a low order model plus time delay (Lilja [5]).

This paper focuses on the estimation of the parameters (including the time delay) of an appropriate process model from an appropriate number of arbitrarily specified points on the process frequency response. The frequency domain appears to be intuitively appropriate for the estimation of the time delay (in particular), as the process time delay affects the phase response of the process, but not its magnitude response. Dos Santos and De Carvalho [6] and Koganezawa [7] use this feature to separately estimate the non-delay parameters and the time delay. Lilja [5] estimates the parameters of a first order lag plus time delay (FOLPD) process model by estimating the non-delay parameters through the minimisation of an appropriate cost function; the time delay is estimated separately by calculating the global minimum of a non-unimodal cost function using a modified Newton-Raphson algorithm. These approaches have the disadvantage of separately estimating the non-delay parameters and the time delay; this leads to biased estimation of the time delay or difficulty in achieving reliable convergence of the time delay estimate to its optimum value.

These difficulties motivate an investigation of the possibility of estimating the non-delay and time delay parameters together. The process, of unknown model order, is assumed to be modelled adequately by either a FOLPD process model or a second order system plus time delay (SOSPD) process model (with no zero). Such an assumption is frequently made in process model identification, as a consensus exists that most industrial processes may be adequately modelled in such a manner. The use of the assumption means that, for a higher order process with time delay, the 'pure' time delay (for example) will not be identified; instead, a composite time delay, composed of the 'pure' time delay and contributions from higher order dynamics, will be identified.

Initial estimates of the relevant parameters of the model are calculated analytically; a least squares approach using a gradient algorithm is then employed to facilitate accurate and robust estimation of the parameters (a least squares approach to the problem was originally suggested by Palmor and Blau [8]). All of the parameters (including the time delay) are estimated together. The purpose of the analytical procedure is to facilitate convergence of the initial model parameter estimates to their optimum values, using the gradient method. This is achieved by ensuring that the cost function, equal to the sum of the squares of the sampled errors between the process and model frequency responses, is unimodal with respect to all of the parameter values, when the model in question is formed from the analytical estimates of the parameter values. This unimodality of the cost function is necessary as the gradient algorithm uses the first partial derivative of the cost function with respect to the appropriate parameter value, when updating the model parameter values.

The analytical formulae to estimate the parameters of a FOLPD model and a SOSPD model are developed in Sections 2.1 and 2.2, respectively. The least squares approaches to estimating the parameters of a FOLPD model and a SOSPD model, from the initial estimates of the parameters, are developed in Sections 3.1 and 3.2, respectively. Implementation issues and simulation results are discussed in Section 4. In Section 5, conclusions are drawn and future work is outlined.

2. ANALYTICAL ESTIMATION OF THE MODEL PARAMETERS

2.1 FOLPD Model Parameter Estimation

The transfer function of the model is defined as

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m} \tag{1}$$

with K_m = model gain, T_m = model time constant and τ_m = model time delay.
It may be shown that the parameters of the model may be analytically calculated from the following equations:

$$K_m = \frac{|G_p(j\omega_1)||G_p(j\omega_2)|\sqrt{\omega_2^2 - \omega_1^2}}{\sqrt{|G_p(j\omega_2)|^2 \omega_2^2 - |G_p(j\omega_1)|^2 \omega_1^2}} \tag{2}$$

$$T_m = \frac{1}{\omega}\sqrt{\frac{K_m^2}{|G_p(j\omega)|^2} - 1} \tag{3}$$

$$\tau_m = \frac{1}{\omega}\left[\phi_p(j\omega) - \tan^{-1}(\omega T_m)\right] \tag{4}$$

with $|G_p(j\omega)|$ = process magnitude and $\phi_p(j\omega)$ = process phase lag, at frequency ω.
The sensitivity (with respect to changes in the process data recorded) of equation (2) for calculating the gain, together with equations (3) and (4) for calculating the time constant and the time delay, has been explored in detail, both analytically and in simulation. The results show that, for a wide range of processes modelled by a FOLPD model, the sensitivity of the parameter estimates to changes in the process data recorded is lowered if
(a) K_m is calculated from magnitudes recorded at least a decade apart in frequency
(b) T_m is calculated when the magnitude values recorded are in a range of 0.25 to 0.75 times the value of K_m calculated
(c) τ_m is calculated using data corresponding to the magnitude of the response being less than 0.5 times the value of K_m calculated.

2.2 SOSPD Model Parameter Estimation

The transfer function of the model is defined as

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + a_{1m}s + a_{2m}s^2} \tag{5}$$

It may be shown that the parameters of the model may be analytically calculated from the following equations:

$$K_m = \sqrt{\frac{(\omega_2^2\omega_1^2 + \omega_3^4)(\omega_2^2 - \omega_1^2) - \omega_3^2(\omega_2^4 - \omega_1^4)}{\frac{\omega_2^2\omega_1^2(\omega_2^2 - \omega_1^2)}{|G_p(j\omega_3)|^2} - \frac{\omega_3^2\omega_2^2(\omega_2^2 - \omega_3^2)}{|G_p(j\omega_1)|^2} + \frac{\omega_1^2\omega_3^2(\omega_1^2 - \omega_3^2)}{|G_p(j\omega_2)|^2}}} \tag{6}$$

$$a_{2m} = \sqrt{\frac{K_m^2\left(\frac{\omega_2}{|G_p(j\omega_1)|}\right)^2 - K_m^2\left(\frac{\omega_1}{|G_p(j\omega_2)|}\right)^2 + (\omega_1^2 - \omega_2^2)}{\omega_1^2\omega_2^2(\omega_1^2 - \omega_2^2)}} \tag{7}$$

$$a_{1m} = \sqrt{\frac{\left(\frac{K_m}{\left|G_p(j\omega)\right|}\right)^2 - \left(1 - a_{2m}\omega^2\right)^2}{\omega^2}} \tag{8}$$

$$\tau_m = \frac{1}{\omega}\left[-\phi_p(j\omega) - \tan^{-1}\left(\frac{a_{1m}\omega}{1 - a_{2m}\omega^2}\right)\right] \tag{9}$$

The sensitivity of these estimates to changes in the process magnitude and phase values recorded has been explored in simulation. Detailed results have shown that, for a wide range of processes modelled by a SOSPD model, the sensitivity of the parameter estimates to changes in the process data recorded is lowered if

(a) K_m is calculated from three magnitude values that span at least a decade of frequency

(b) a_{2m} is calculated from magnitudes recorded at least a decade apart in frequency

(c) a_{1m} is calculated when the magnitude value recorded is in a range of 0.25 to 0.75 times the value of K_m calculated

(d) τ_m is calculated using data corresponding to the magnitude of the response being less than 0.5 times the value of K_m calculated.

3. LEAST SQUARES ESTIMATION OF THE MODEL PARAMETERS

3.1 FOLPD Model Parameter Estimation

The parameter vector to be estimated is $x = \left[K_m \quad T_m \quad \tau_m\right]^T$. (10)

A minimum of two data points on the frequency response is required to estimate the parameters. If just two data points are taken, the vector of frequency response values is

$$F = \left[\left|G_p(j\omega_1)\right| \quad \left|G_p(j\omega_2)\right| \quad \phi_p(j\omega_1) \quad \phi_p(j\omega_2)\right]^T \tag{11}$$

The error vector is formed as follows: $e = \left[e_1 \quad e_2 \quad e_3 \quad e_4\right]^T$ (12)

with

$$e_n = \frac{K_m}{\sqrt{1 + \omega_n{}^2 T_m{}^2}} - \left|G_p(j\omega_n)\right| \text{, n = 1,2} \tag{13}$$

and

$$e_n = -\tan^{-1}(\omega_n T_m) - \omega_n \tau_m - \phi_p(j\omega_n) \text{, n = 3,4} \tag{14}$$

The cost function, J, is formulated as $J = 0.5e^T pe$, with

$$p = \mathrm{diag}\left[\frac{1}{\left|G_p(j\omega_1)\right|} \quad \frac{1}{\left|G_p(j\omega_2)\right|} \quad \frac{1}{\omega_1} \quad \frac{1}{\omega_2}\right] \tag{15}$$

Such a filtering matrix is used to increase the range of parameters over which unimodality of the cost function exists.

3.2 SOSPD Model Parameter Estimation

The parameter vector to be estimated is $x = \left[K_m \quad a_{1m} \quad a_{2m} \quad \tau_m\right]^T$. (16)

A minimum of three data points on the frequency response is required to estimate the parameters. If just three data points are taken, the vector of frequency response values is

$$F = \left[\left|G_p(j\omega_1)\right| \quad \left|G_p(j\omega_2)\right| \quad \left|G_p(j\omega_3)\right| \quad \phi_p(j\omega_1) \quad \phi_p(j\omega_2) \quad \phi_p(j\omega_3)\right]^T \tag{17}$$

The error vector is formed as follows: $e = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{bmatrix}^T$ (18)

with

$$e_n = \frac{K_m}{\sqrt{(1 - a_{2m}\omega_n^2)^2 + \omega_n^2 a_{1m}^2}} - \left|G_p(j\omega_n)\right| \text{, n = 1,2,3} \tag{19}$$

and

$$e_n = -\tan^{-1}\frac{\omega_n a_{1m}}{1 - a_{2m}\omega_n^2} - \omega_n \tau_m - \phi_p(j\omega_n) \text{, n = 4,5,6} \tag{20}$$

The cost function, J, is formulated as $J = 0.5e^T pe$, with

$$p = \mathrm{diag}\begin{bmatrix} \dfrac{1}{\left|G_p(j\omega_1)\right|} & \dfrac{1}{\left|G_p(j\omega_2)\right|} & \dfrac{1}{\left|G_p(j\omega_3)\right|} & \dfrac{1}{\omega_1} & \dfrac{1}{\omega_2} & \dfrac{1}{\omega_3} \end{bmatrix} \tag{21}$$

As before, this filtering matrix is used to increase the range of parameters over which unimodality of the cost function exists.

For the estimation of the parameters of both models, the updated estimate of the parameter vector may be calculated from the following gradient algorithm

$$x(k+1) = x(k) + u\frac{\partial J}{\partial x} \tag{22}$$

with u = a constant.

It is clear from the formulation of the cost function that it is quadratic in the gain estimate (for all values of the other parameters) and is also quadratic in the time delay estimate (for all values of the other parameters). The cost function is not, however, quadratic in the time constant estimate (FOLPD model) or in the estimates of the denominator parameter values (SOSPD model). The cost function must be unimodal with respect to these quantities (allowing the other parameters to vary) if convergence of the model parameters to the process parameters is to be guaranteed using the gradient algorithm. An equivalent condition is that the first partial derivative of the cost function with respect to the time constant (FOLPD model) or with respect to each of the denominator parameter values (SOSPD model) may be equal to zero once only. Alternatively, the second partial derivative of the cost function with respect to the time constant (FOLPD model) or with respect to each of the denominator parameter values (SOSPD model) must always be greater than zero.

This latter condition acts as a convenient check on the unimodality of the cost function when the model is formed from the parameters calculated using the analytical approach.

4. IMPLEMENTATION ISSUES AND SIMULATION RESULTS

A number of simulations were performed to demonstrate the operation of the method. These simulations covered a reasonable range of time delayed processes, including a higher order process, a process with an underdamped step response and a non-minimum phase process. Simulation results from one SOSPD process, indicated below, are provided.

$$G_p(s) = \frac{2e^{-s}}{(1+1.5s)(1+3s)} \tag{23}$$

In all of the simulations, ten data points on the process frequency response are used in the calculations. For the purpose of the simulation, these data points are taken between phase lags of 0^0 and 270^0 (though the choice of data points on the process frequency response is arbitrary, in general). A simulated noise level of ±10% of the appropriate process frequency response is added to the data. The analytical estimates of the parameters are averaged, as appropriate, to improve the accuracy and robustness of the initial model parameter estimates.

4.1 FOLPD Model Parameter Estimation

The range of parameter values over which convergence of the model parameters to their optimum values is possible was first investigated. This was done by calculating the conditions under which the second partial derivative of the cost function with respect to the time constant was greater than zero. Unfortunately, it was not possible to calculate the required conditions analytically; however, the results of a large number of simulations revealed that the estimates of the parameter values found using the analytical approach should be as defined below (in either Case 1 or Case 2) to facilitate unimodality of the cost function with respect to the time constant variation.

Case 1:

$$K_m(\text{initial} - \text{LS}) = 1.5K_m(\text{analytical}) \tag{24}$$

$$0.25T_p \le T_m(\text{analytical}) \le 3.3T_p \tag{25}$$

$$\tau_m(\text{initial} - \text{LS}) = 0.5\tau_m(\text{analytical}) \tag{26}$$

Case 2:

$$0.83K_p \le K_m(\text{analytical}) \le 1.17K_p \tag{27}$$

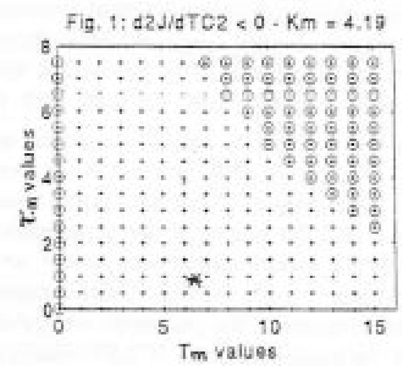
$$0.25T_p \le T_m(\text{analytical}) \le 1.25T_p \tag{28}$$

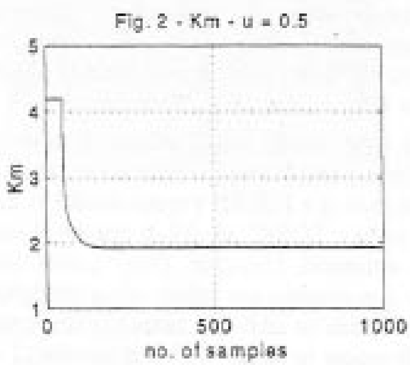
$$0 \le \tau_m(\text{analytical}) \le 1.1\tau_p \tag{29}$$

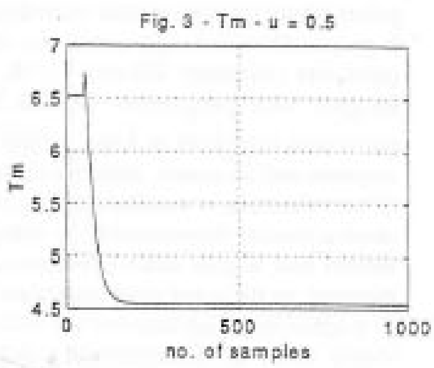
In both cases, K_p, T_p and τ_p are the optimum estimates of the gain, time constant and time delay, respectively.

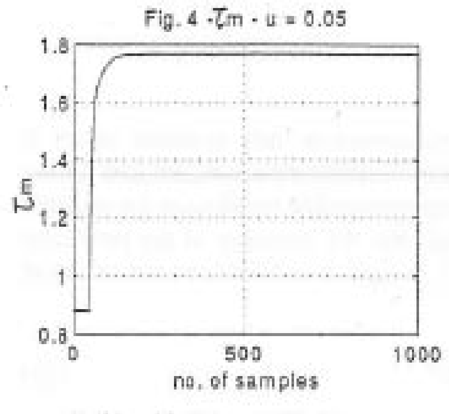
It is easier for the conditions in Case 1 to be fulfilled in practice, as there is a tendency for $T_m(\text{analytical})$ to be greater than $1.25T_p$ (at least in the simulations taken). The specifications in Case 1 and Case 2 are worst case specifications based on the simulations taken i.e. it may be shown that the parameter estimates converge to their optimum values when the parameter estimates calculated using the analytical approach are outside the parameter ranges given above.

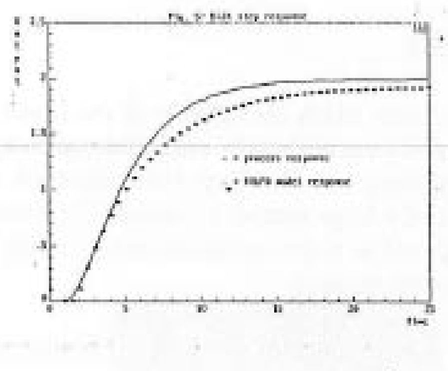
The analytical estimates of the parameters are first calculated; then the starting values of the gain and time delay for the least squares estimates of the parameters are put at 1.5 and 0.5 times the analytical gain and time delay estimates, respectively. This strategy increases the probability of convergence to the optimum values of the parameter estimates using the gradient method, though it does not guarantee such convergence. Figure 1 shows that, for the simulation taken, a wide range of initial parameter values is possible (In Figure 1, O = points where $\partial^2 J / \partial T_m{}^2 < 0$ and = points where $\partial^2 J / \partial T_m{}^2 > 0$). The model parameter values calculated analytically in this case were $K_m = 2.79$, $T_m = 6.53$ and $\tau_m = 1.76$. Therefore, the initial estimates for the parameters using the gradient algorithm are $K_m = 4.19$, $T_m = 6.53$ and $\tau_m = 0.88$; this estimated is marked as * on Figure 1. Figures 2, 3 and 4 show the convergence of these parameter values to the optimum values within 500 samples, using the gradient method. The optimum values of $K_p = 1.93$, $T_m = 4.55$ and $\tau_p = 1.77$ conform with the guidelines suggested in Case 1 above. Figures 5 and 6 show the step response and frequency response of the process and model together (using Program CC). The fitting of the process to the model in both domains is inaccurate (except at phase lags around 180^0), due primarily to an inaccurate estimate of the gain of the process. However, the apparent time delay of the process seems to be estimated well. Other simulation results show a similar deviation in the fitting between the process and the model, except when the process is itself of a FOLPD structure. It is possible, by restricting the range of phase values over which the process is identified, to yield a closer fitting between the process and the model in the frequency domain (over the corresponding frequency range) than that found in the simulation above. Of course, the acceptability of the fitting of the model to the process in any particular frequency range depends on the use to which the model is applied.

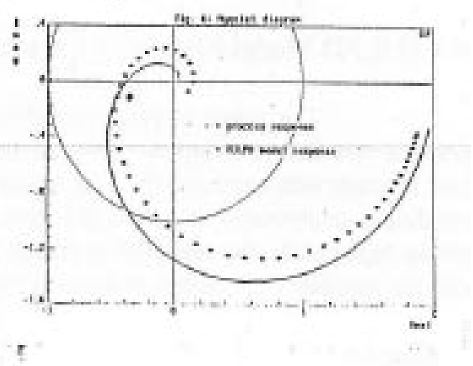












4.2 SOSPD Model Parameter Estimation

The range of parameter values over which convergence of the model parameters to their optimum values is possible was first investigated, as before. This was done by calculating the conditions under which the second partial derivative of the cost function with respect to each of the denominator parameters in turn was greater than zero. Unfortunately, it was not possible to calculate the required conditions analytically; however, the results of a large number of simulations revealed that the initial estimates of the parameter values found using the analytical approach should be as defined below (in either Case 1 or Case 2) to facilitate unimodality of the cost function with respect to the variation in each of the denominator parameter values in turn.

Case 1:

$$0.83K_p \le K_m(\text{analytical}) \le 1.17K_p \tag{30}$$

$$0.75a_{1p} \le a_{1m}(\text{analytical}) \le 1.5a_{1p} \tag{31}$$

$$0.5a_{2p} \le a_{2m}(\text{analytical}) \le 1.75a_{2p} \tag{32}$$

$$0.83\tau_p \le \tau_m(\text{analytical}) \le 1.17\tau_p \tag{33}$$

Case 2:

$$K_m = 1.5K_m(\text{analytical}) \tag{34}$$

$$0.75a_{1p} \le a_{1m}(\text{analytical}) \le 1.75a_{1p} \tag{35}$$

$$0.5a_{2p} \le a_{2m}(\text{analytical}) \le 1.75a_{2p} \tag{36}$$

$$\tau_m = 0.5\tau_m(\text{analytical}) \tag{37}$$

In both cases, a_{1p} and a_{2p} are the optimum estimates of the denominator parameter values.

For the estimation of the parameters of a SOSPD model, the specifications in Cases 1 and 2 are broadly similar; as before, both specifications are worst case conditions. The analytical estimates of the parameters are first calculated; these are used as the starting values for least squares estimates of the parameters. Convergence to the optimum values of the parameter estimates using the gradient method is of course not guaranteed (as in the case when the parameters of a FOLPD model are being estimated). The initial parameter values calculated in this case are $K_m = 2.24$, $a_{1m} = 5.36$, $a_{2m} = 4.98$ and $\tau_m = 1.04$. Figures 7 to 16 show that, for the simulation taken, a wide range of initial parameter values is possible (In Figures 7 to 16, O = points where the appropriate second partial derivative is less than 0,   = points where the appropriate second partial derivative is greater than zero, x = optimum denominator parameter estimates and [] = approximate allowed range of the analytical parameter estimates). Figures 17, 18, 19 and 20 show the convergence of these parameter values to final values within 4000 samples, with the gradient method. The final values ($K_p = 1.99$, $a_{1p} = 4.48$, $a_{2p} = 4.30$ and $\tau_p = 1.04$) conform with the guidelines suggested in Case 1 (and are close to the actual values of these parameters). Figures 21 and 22 show the step response and frequency response of the process and model together (using Program CC). The fitting of the process to the model in both domains is excellent and is better than if a FOLPD process model is estimated (as expected). Other simulation results show a similar improvement in fitting when a SOSPD model is estimated instead of a FOLPD model, if the parameters of second and higher order processes are estimated. However, better initial estimates of the model parameters appear to be required, as the worst case conditions for convergence are tighter when estimating the parameters of a SOSPD model compared to estimating the parameters of a FOLPD model; in addition, generally speaking, the convergence of the parameter estimates is slower when the parameters of a SOSPD model is being estimated compared to when the parameters of a FOLPD model are being estimated (though the speed of convergence of the parameter estimates may be altered by varying the value of u). As before, the acceptability of the fitting of the model to the process depends on the use to which the model is applied.

Fig. 7: d2J/da12 < 0, Km=1.67, τm=0.83

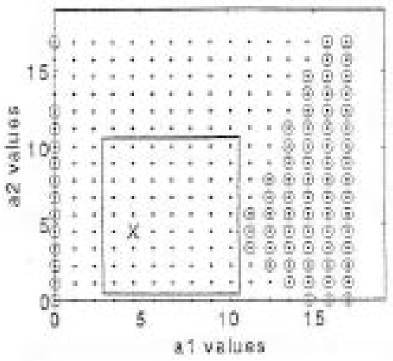


Fig.13: d2J/da22 < 0, Km=1.67, τm=0.83

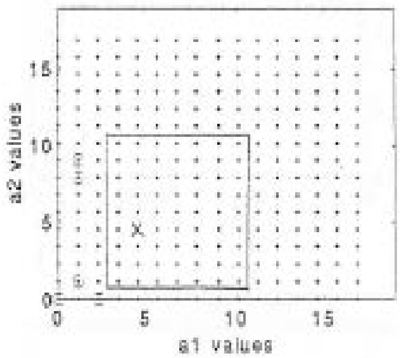


Fig.8: d2J/da12 < 0, Km=1.67, τm=1.17

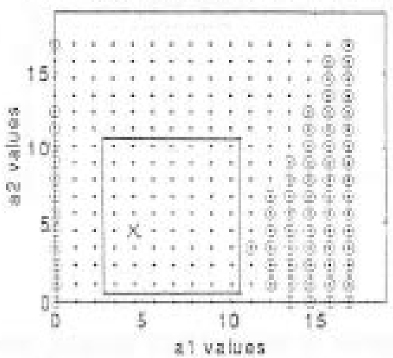


Fig. 11: d2J/da12 < 0, Km=2, τm=1

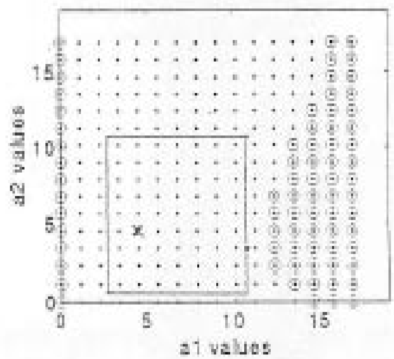


Fig.14: d2J/da22 < 0, Km=2.33, τm=0.83

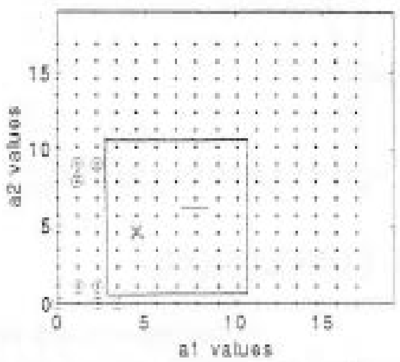


Fig. 9: d2J/da12 < 0, Km=2.33, τm=0.83

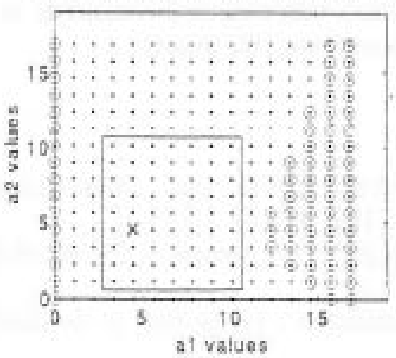


Fig.12: d2J/da22 < 0, Km=2, τm=1

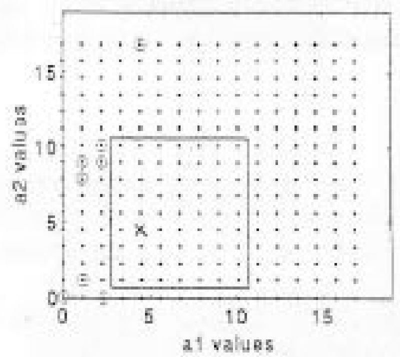


Fig.15: d2J/da22 < 0, Km=1.67, τm=1.17

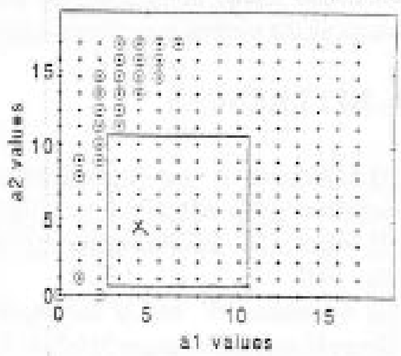


Fig.10: d2J/da12 < 0, Km=2.33, τm=1.17

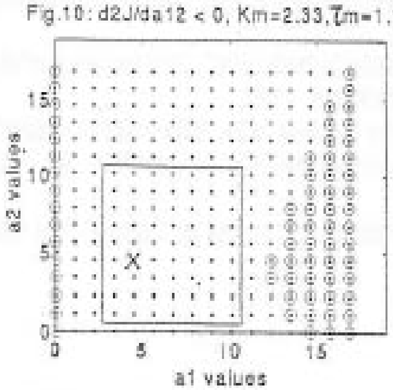
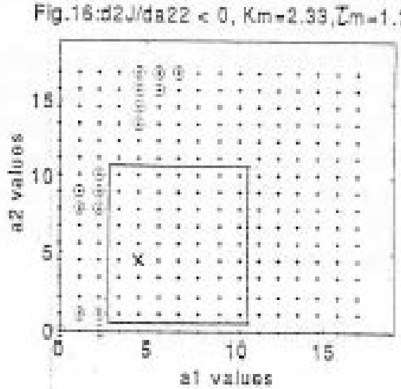
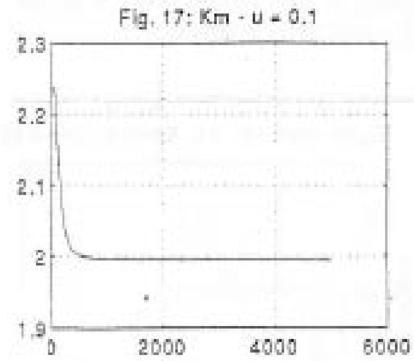
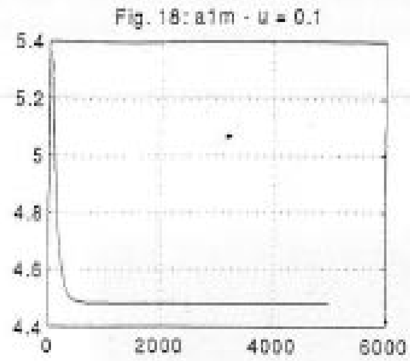
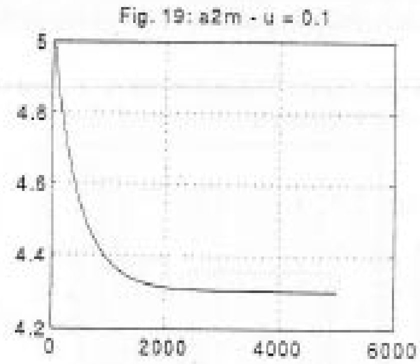


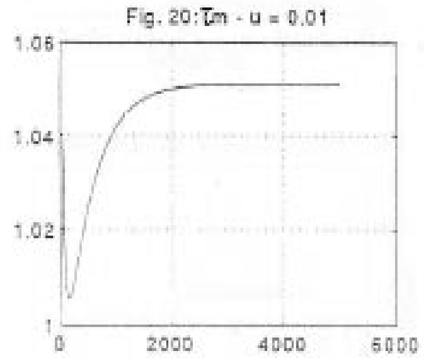
Fig.16: d2J/da22 < 0, Km=2.33, τm=1.17

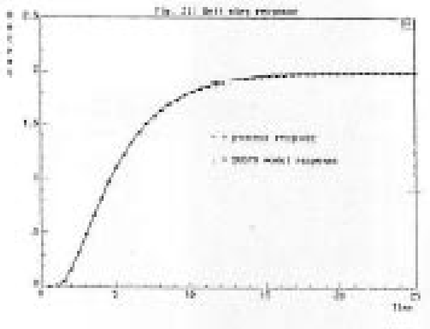


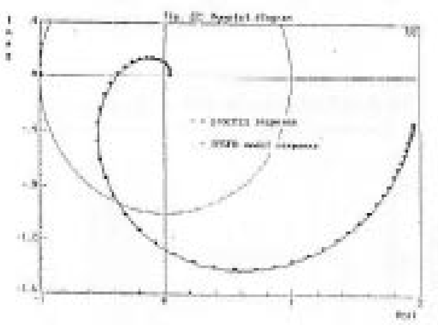












5. CONCLUSIONS

In conclusion, a new method for model parameter and time delay estimation in the frequency domain, that combines an analytical approach and a least squares approach using a gradient method has been defined. The method differs from other least squares approaches (such as that defined by Lilja [5]) as the model parameters and time delay are estimated together. In the paper, complete algorithms for the estimation of FOLPD and SOSPD model parameters have been developed. Simulation results using a sample process have shown the capability of the algorithms. Future work will concentrate on extending the method to facilitate the estimation of the parameters and the time delay of an arbitrary order process.

6. REFERENCES

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