Extraction of small-signal model parameters of Si/SiGe heterojunction bipolar transistor using least squares support vector machines

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A novel straightforward methodology for extracting bias-dependent small-signal equivalent circuit model parameters (SSECMPs) of silicon/silicon–germanium heterojunction bipolar transistors is presented. The inverse mapping between SSECMPs and scattering (S) parameters is established and fitted using simulated data of the SSECM. Since the problem has large input space, S-parameters at many frequency points, the least squares support vector machines concept is used as regression technique. Physical SSECMPs values are obtained using the proposed methodology. Moreover, an excellent agreement is noted between the S-parameters measurements and their simulated counterpart using the extracted SSECMPs in the frequency range from 40 MHz to 40 GHz at different bias conditions.

Introduction: Heterojunction bipolar transistors (HBTs) are widely used for RF and high-speed applications. Silicon–germanium (SiGe) technology has a lot of advantages compared to its rival, III–V technology, such as low cost through the integration with Si CMOS, high thermal conductivity and lower operating voltage. An accurate small-signal equivalent circuit model (SSECM) of the device is very essential for evaluating process technology and optimising device structure. On the other side, they are indispensable to noise analysis and to design low-noise amplifiers and low-power high-speed optical receivers circuits [1]. The intrinsic characteristics of the on-wafer device are the main interest for IC designers. The problem of SSECM parameters (SSECMPs) extraction is mathematically described as follows: the aim of parameters extraction is finding the solution to system of ill-conditioned non-linear equations. These equations map SSECMPs, independent variables, into S-parameters, dependent ones, space. There are a lot of different techniques to extract SSECMPs. However, it can be categorised under two main conventional approaches, direct extraction, as an example [2], and optimisation-based extraction, as an example [3]. In the former approach, a series of transformations from Z-parameters to Y-parameters and vice versa of the SSECM nested layers in conjunction with some sort of frequency analysis are needed to extract SSECMPs. This procedure increases the uncertainty of the extracted SSECMPs values. Consequently, accurate S-parameters measurements are crucial and averaging over the frequency must be taken to obtain unique values. On the other hand, the latter approach utilises numerical algorithms to find the combination of SSECMPs that result the best fit of the SSECM calculated S-parameters to the corresponding measured ones. However, they suffer from local minima problems, and consequently, physical extracted values are not guaranteed. Moreover, long extraction time is needed for searching the best SSECMPs values.

In this Letter, to avoid the issues with utilising one of previously mentioned techniques, artificial intelligent (AI)-based inverse mapping technique is used to solve this problem. The philosophy of this technique is to build an inverse mapping from the S-parameters space to the SSECMPs space using data collected from simulations of the preassigned SSECM topology. The obtained data are composed of pairs from SSECMPs and the corresponding simulated S-parameters, respectively. Using the collected data, one of AI techniques is used to construct unique function for every SSECMP. As a consequence, once the mapping is established, every SSECMP could be extracted directly and independently. The problem has high dimensional input space, namely, all S-parameters multiplied by frequency points at which the measurement is done. The least squares support vector machines (LS-SVMs) is the technique which is qualified to perform this task. Furthermore, LS-SVMs technique does not have the local minima problem. Physical, reliable and frequency independent SSECMPs values are obtained, and consequently, all the disadvantages of the conventional techniques are overcome. To the authors' knowledge, it is the first time that SSECMPs of any RF active component could be extracted without using the traditional approaches.

Proposed extraction technique: The complete SSECM for a Si/SiGe HBT, as seen from the probe tips of vector network analyser (VNA), comprises of two main parts, namely, the intrinsic bias-dependent core of the device and extrinsic bias-independent parasitics. The contribution of the extrinsic part is removed from the S-parameters measurements using the pad, short and open dummy structures. On the other hand, the intrinsic SSECMPs, as depicted in Fig. 1, are the base resistance (R_b), the base–emitter junction capacitance (C_π), the dynamic base–emitter resistance (R_π), internal base–collector junction capacitance (C_u), external base–collector junction capacitance (C_f), DC transconductance (g_{mo}), transient time phase delay (τ) and collector–substrate capacitance (C_s). All the aforementioned parameters are extracted using the presented methodology. However, beforehand, the bias-independent emitter resistance (R_e) is extracted as in [2].

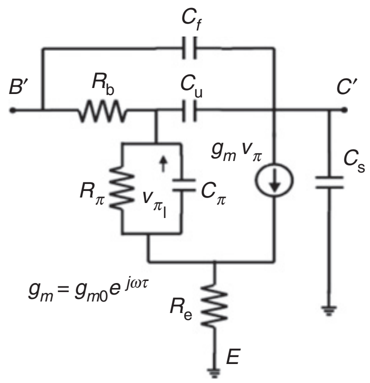

Fig. 1 *SSECM of intrinsic part of Si/SiGe HBT*

Let vector $\boldsymbol{S}_m$ represents real and imaginary parts of measured S-parameters at specific frequency point, m, and taking the following form:

$$\boldsymbol{S}_m = [\mathrm{Re}(S_{11})\ \mathrm{Im}(S_{11})\ \mathrm{Re}(S_{12}) \cdots \mathrm{Im}(S_{22})]^{\mathrm{T}} \in R^8 \tag{1}$$

Furthermore, the vector, $\boldsymbol{S}$, represents the S-parameters at all n frequency points. Algebraically, S can put in the following form:

$$\boldsymbol{S} = [S_1 \ldots S_m \ldots S_n]^{\mathrm{T}} \in R^{8n} \tag{2}$$

The extraction problem of SSECMPs vector ($\boldsymbol{p}$) is formulated as follows; assuming $\boldsymbol{p}$ is frequency independent, how to obtain values of $\boldsymbol{p}$ from the following equation:

$$\boldsymbol{S} = \varphi(\boldsymbol{p}) \tag{3}$$

This formulation causes problems in extracting $\boldsymbol{p}$ as it was explained in the introduction. Our solution is to inverse map (3) and find function ψ such that

$$\boldsymbol{p} = \psi(\boldsymbol{S}) \tag{4}$$

LS-SVM is used to construct ψ. In the remaining part of this section, a summary of the LS-SVMS theory [4] is offered. Consider N a given training data set $\{x_k, y_k\}_{k=1}^N$ with input data $x_k \in R^n$ and output $y_k \in R$, where n is the dimension of the input space. The LS-SVMs for function estimation has the following linear form in feature space:

$$y(x) = w^{\mathrm{T}} \varphi(x) + b \tag{5}$$

The non-linear mapping $\varphi(.)$: $R^n \rightarrow R^{n_F}$ maps the input space to a higher dimension feature space with dimension n_F. b is the threshold term; $\boldsymbol{w} \in R^{n_F}$ is the weight vector. The optimisation problem is formulated as

$$\min_{\boldsymbol{w},b,e} J(\boldsymbol{w}, e) = \frac{1}{2} w^{\mathrm{T}} \boldsymbol{w} + \gamma \frac{1}{2} \sum_{i=1}^{N} e_k^2 \tag{6}$$

subject to the equality constraints

$$y_i = w^{\mathrm{T}} \varphi(x_k) + b + \boldsymbol{e}_k \quad k = 1, \ldots, N \tag{7}$$

where $\boldsymbol{e}_K \in R$ is the error vector; γ is the regularisation parameter and used to control the trade-off between the smoothness of the function and the accuracy of the fitting. The optimisation problem (6) is considered to be a constrained optimisation problem and a Lagrange function is used to solve it. Instead of minimising the primary objective (6), a dual objective, the so-called Lagrangian, is formed of which the saddle point is the optimum. The Lagrangian for this problem is given as

$$\mathcal{L}(\boldsymbol{w}, b, e, \alpha) = J(\boldsymbol{w}, e) - \sum_{i=1}^{N} \alpha_k \left\{ w^{\mathrm{T}} \varphi(x_k) + b + \boldsymbol{e}_k - y_k \right\} \quad (8)$$

where α_k are Lagrangian multipliers.

This optimisation problem leads to a solution

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i K(x, x_i) + b \quad (9)$$

where $K(x, x_i) = \phi^{\mathrm{T}}(x)\phi(x_i)$ is the kernel.

The brilliancy of using the kernel function lies in the fact that one can deal with linear feature spaces of arbitrary dimensionality without having to compute the map $\varphi(.)$ explicitly. There are different kernel function types such as polynomial kernel: $K(x_i, x_j) = (x_i^{\mathrm{T}} x_j + t)^d$, with t is the intercept and d is the degree of the polynomial and radial basis function kernel: $K(x_i, x_j) = \mathrm{e}^{-(x_i - x_j{}^2)/2\sigma^2}$, with σ^2 the variance of the Gaussian kernel. In the following section, we will use (9) to build ψ defined in (4).

LS-SVMs extraction model: To obtain ψ using LS-SVMs model, training data are required. Therefore, the corners of the problem must be firstly determined. The proposed methodology is applied on the HBT device with geometry of 0.8 µm × 9.6 µm. The problem corners are the corners of the voltage region at which we want to extract SSECMPs. Base voltage (V_B) is chosen to vary from 0.8 to 0.9 V, while collector voltage (V_C) is varied between 1.0 and 2.0 V. The SSECMPs are extracted at these four corners using any traditional technique [2, 3].

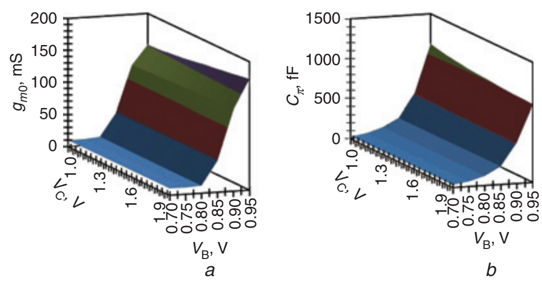

Fig. 2 *Bias dependency of some of extracted intrinsic SSECMPs*

a g_{m0}
b C_π

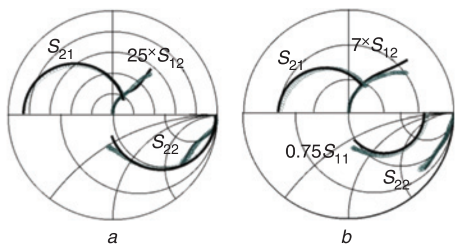

Fig. 3 *Comparison between measured (capital letter O line) and simulated (black rectangle line) S-parameters [40 MHz–40 GHz]*

a $V_B = 0.9$ V and $V_C = 1.5$ V
b $V_B = 0.8$ V and $V_C = 1$ V

In this Letter, the procedure followed in [2] is adopted. The lowest and highest extracted values of each parameter are listed and considered the approximated boundary of each SSECMP. The initial boundary is stretched by 20% from the upper and lower sides, respectively, to guarantee that the region of interest is well fitted inside the training region. The final boundary values are used in Monte Carlo simulation tool integrated in advanced design system (ADS) package to generate the needed data. One thousand five hundred data samples are generated for training and test phases' purpose. Every individual sample composes of the pair of SSECMPs values and the corresponding *S*-parameters resulting from their simulation. The value of n is 10, and therefore the dimension of input space, S, is 80. One LS-SVMs model per SSECMP is built. First, the generated training data are used to get the γ and σ values with the auto-tuning (cross validation) utility in the LS-SVMlab [5], integrated in MATLAB. Using the obtained optimised values for γ and σ and the same training data set, we train the LS-SVMs model with suitable kernel to obtain α_k and b values. All models use polynomial kernel except the one of g_{m0} parameter which utilises RBF kernel type. The criterion of kernel type selection is which one achieves minimum test error.

The bias dependence of the extracted SSECMPs follows the physical behaviour of the device. As an example, the bias dependence of g_{m0} and C_π reveals exponential-like dependency on V_B and quasi-independency on V_C, as shown in Figs. 2*a* and *b*, respectively. As consequence of this, an excellent agreement is noted between the *S*-parameters resulting from the simulation of the extracted SSECMPs with the corresponding measured quantities, as shown in Fig. 3 for two different biases. However, there is a small discrepancy starting from 20 GHz and onward for tiny S_{12} and it is attributed to standard 12-term VNA calibrations do not correct S_{12} background error caused by probe-to-probe coupling.

Conclusion: In this Letter, the inverse mapping function is established between Si/SiGe HBT SSECMPs and *S*-parameters by using LS-SVMs. The LS-SVMs is exploited to map high input *S*-parameters' space into lower output SSECMP space. Once LS-SVMs model is well trained, in fractions of second, physical, reliable and unique SSECMPs values are extracted. Therefore, all disadvantages of the traditional extraction techniques are overwhelmed. The offered distinguished extraction technique is very beneficial for circuit design, noise analysis and process technology evaluation purposes. On the other side, it could be applied to extract SSECMPs of any RF component.

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One or more of the Figures in this Letter are available in colour online.

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