The generation of output supply and input demand elasticities for a Johansen-type model of the Irish agricultural sector[*]

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Summary

This paper attempts the estimation of an integrated output supply and input demand system for the Irish agricultural sector. The objective of the exercise is the generation of a set of elasticities to be used in the construction of a Johansen type model of the sector. Drawing on separability principles, the estimation problem is divided into three levels: the estimation of an aggregate profit function; a revenue function for aggregate output; a revenue function for aggregate tillage output; and a cost function for aggregate fertiliser consumption. Elasticities are obtained for each level and the integrated production system.

Introduction

No comprehensive modelling framework has been developed for Irish agriculture. The demand for such a framework has been evident for some time given the policy upheaval which has occurred in the Common Agricultural Policy (CAP) in recent years. No adequate tool exists, for instance to analyse the effects of dairy quotas; quotas or set aside measures for cereals; taxation or quantitative restrictions on nitrogenous fertiliser use; restrictions on intervention in the beef sector. Moreover the decoupling of income support from price setting has generated the need to assess the impact of these and other policies (for example direct income transfers) at disaggregated levels of analysis e.g. farm size, geographical region, farming system (i.e. cattle, dairy, sheep, tillage etc.).

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This paper attempts the estimation of an integrated output supply and input demand system for the Irish agricultural sector. To date published quantitative analysis has mainly focused on single product or single input models.[1] Exceptions are Boyle (1981) who estimated a cost function for Irish agriculture based on time series data and Higgins (1986) who estimated a profit function based on cross-sectional farm accounts data. None of these studies, however, attempted to simulate impacts of policy developments. The only avowed policy study is by Sheehy (1982) who assessed the likely impact of quotas and price cuts for the dairy industry. His production parameters were, however, assumed rather than estimated.

This paper is a preliminary attempt to develop a methodology for the purpose of agricultural policy assessment at levels of disaggregation appropriate to the CAP reform measures now being mooted. As a first step we attempt to develop a means of implementing the Johansen modelling approach at the level of the so-called 'national farm'.[2] The Johansen methodology has been extensively used in the apparently successful ORANI project (Dixon et al., 1982) and has attracted some adherents among European economists in recent years (Munk, 1984, 1985; and Mahé and Munk, 1988).

A key ingredient of the Johansen methodology is the matrix of output supply and input demand elasticities for the farm system. Given the number of outputs produced and inputs consumed in the Irish farm sector, a single step approach to the estimation of an integrated output supply and input demand system would not be feasible. This paper drawing on separability principles breaks up the estimation problem into feasible components. While the use of separability results to generate a set of elasticities for the entire agricultural sector has been employed *inter alia* by Munk and Mahé, their approach relies on the assumption of values for certain key elasticities. The relatively novel feature of our procedure is that we derive all our elasticity values by econometric estimation of the production relationships at each stage of our assumed separability structure. We believe that this approach to the estimation of the complex set of inter-relationships within the farm economy has potential applicability to many other agricultural sectors within the EC.

The format of this paper is as follows. Section 1 outlines the Johansen idea and presents a method of generating elasticities for the 'national farm'. Section 2 discusses the estimated model and elasticity estimates obtained using an explicit separability structure. Section 3 outlines some implications for future work.

1.   Methodology — The Johansen procedure

We assume agricultural output and input prices are given. If producers are assumed to maximise short-run restricted profits we obtain by Hotelling's

Lemma a set of profit maximising output supply and input demand relations (Varian, 1978).

$$Y_i = f_i(P_i, W_j, Z) \, i = 1, ..., m \tag{1}$$
$$-X_j = f_j(P_i, W_j, Z) \, j = 1, ..., n$$

where,

> Y are outputs; X are inputs; P and W are the corresponding prices and Z is a vector of quasi-fixed factors (labour, capital etc).

The Johansen solution proposes that we solve for this system by linearising in terms of percentage changes and then given a specification of the endogenous and exogenous variables (the 'closure' assumption), the solution is obtained by matrix inversion

$$Y = -A_1^{-1} A_2 X \tag{2}$$

where,

> Y is a $(L \times 1)$ vector of endogenous variables
> X is a $(K \times 1)$ vector of exogenous variables
> and
> A_1 is a $L \times L$ and A_2 an $L \times K$ matrix of elasticities.

If the profit maximising levels of outputs and inputs were the endogenous variables than A_1 would consist of ones on the diagonals and zeros elsewhere while A_2 would be the matrix of price and quasi-fixed factor elasticities.

The main drawback of the approach in (2) is the linearity of the model which limits simulations to small changes in exogenous variables. However, there are advantages. First, alternative closure assumptions can be readily accommodated. For example, to analyse the impact of a quota for a particular product the quantity becomes the exogenous variable and the model solves for the shadow price. Second, the elasticities can be derived from flexible functions. Third, the solution is fast and cheap. For small changes the solution is adequate and for large changes an iterative procedure can apparently give reasonable estimates (Dixon et al., 1982).

The elasticity set

As with any modelling exercise the core is the elasticity set. In principle the estimation strategy is straightforward. Given a suitable flexible form (for example the translog) the entire set of elasticities could be estimated for the system in (1). The practical problems are numerous. Flexible forms do not

globally satisfy the necessary curvature restrictions for profit maximisation. Using the Cholesky decomposition it is relatively easy to impose definiteness of estimated parameter matrices (see Ball, 1988), but this is not sufficient to impose positive-definiteness of the hessian of the translog profit function. Imposition of the required curvature assumptions cannot circumvent the very real problems of data aggregation and multicollinearity.

Given the information published on the 'national farm' sector it is possible to obtain data on at least 11 outputs, 7 inputs and about 11 quasi-fixed factors. Each of the components are in turn aggregates with varying degrees of violence being done to the underlying production conditions. Not surprisingly preliminary attempts to estimate a profit function by only adhering to the separability structure implied in the published data proved fruitless. Minimum conditions of sign plausibility were violated. We were forced then to make certain explicit separability assumptions to obtain any kind of usable elasticities. This structure is shown in Figure 1.
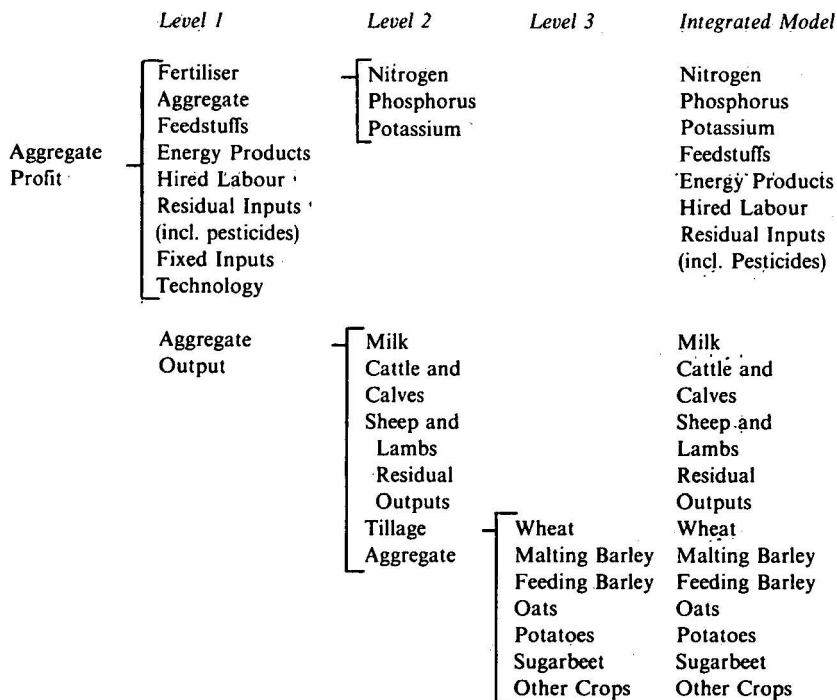

Level 1	Level 2	Level 3	Integrated Model
Fertiliser Aggregate	Nitrogen Phosphorus Potassium		Nitrogen Phosphorus Potassium
Feedstuffs			Feedstuffs
Aggregate Profit — Energy Products			Energy Products
Hired Labour			Hired Labour
Residual Inputs (incl. pesticides)			Residual Inputs (incl. Pesticides)
Fixed Inputs			
Technology			
Aggregate Output	Milk		Milk
	Cattle and Calves		Cattle and Calves
	Sheep and Lambs		Sheep and Lambs
	Residual Outputs		Residual Outputs
	Tillage Aggregate	Wheat	Wheat
		Malting Barley	Malting Barley
		Feeding Barley	Feeding Barley
		Oats	Oats
		Potatoes	Potatoes
		Sugarbeet	Sugarbeet
		Other Crops	Other Crops

Figure 1. *Explicitly assumed separability structure for the Irish farm economy*

At *level 1* we estimated the parameters of a translog profit function from the following system

$$\text{In } \pi = \alpha_0 + \sum_i \alpha_i \ln P_i + \tfrac{1}{2} \sum_i \sum_j \alpha_{ij}(\ln P_i)(\ln P_j) \tag{3}$$

$$+ \alpha_k \ln K + \tfrac{1}{2}\alpha_{kk}(\ln K)^2 + \sum_i \alpha_{ki}(\ln K)(\ln P_i)$$

$$+ \alpha_t T + \tfrac{1}{2}\alpha_{tt} T^2 + \sum_i \alpha_{ti}(T)\ln P_i$$

$$+ \alpha_{tk}(T)\ln K$$

$$S_i = \alpha_i + \sum_i \alpha_{ij} \ln P_j + \alpha_{ki} \ln K + \alpha_{ti} T$$

We adopt the netput convention. Outputs are positively signed. Profit (π) is defined as gross revenue less the value of variable inputs. The 11 quasi-fixed inputs are aggregated into a Divisia quantity index (K). This index comprises family labour and components of fixed and live capital. Technical change (T) is modelled as a simple linear trend. The aggregate output price index and the residual inputs price index are obtained using the Divisia formula which is exact for the linearly homogenous translog function.

For profit maximisation we require that the system satisfy the following conditions: $\sum_i \alpha_i = 1$ (the aggregate output share is omitted in the estimation);

$$\sum_j \alpha_{ij} = \sum_i \alpha_{ij} = \sum_i \alpha_{ki} = \sum_i \alpha_{ti} = 0; \; \alpha_{ij} = \alpha_{ji}.$$

Elasticities are obtained from (3) with the following expressions (see for example Binswanger, 1974; and Bannante and Sidhu, 1981).

Own price netput elasticity $\cap_{ii} = S_i - 1 + \dfrac{\alpha_{ii}}{S_i}$

Cross price netput elasticity $\cap_{ij} = S_j + \dfrac{\alpha_{ij}}{S_i}$

Netput quasi-fixed factor

elasticity $\cap_{ik} = \alpha_k + \alpha_{kk} \ln K + \sum_i \alpha_{ki} \ln P_i + \dfrac{\alpha_{ki}}{S_i} + \alpha_{tk} T$

Netput technical change

(time trend) response $\cap_{it} = \alpha_t + \alpha_{tt} T + \sum_i \alpha_{ti} \ln P_i + \alpha_{tk} \ln K + \dfrac{\alpha_{ti}}{S_i}$

At *level 2* we estimated the parameters of a linearly homogeneous translog revenue function for aggregate output and a linearly homogeneous translog cost function for the fertiliser aggregate. In the case of the revenue function, the aggregate crops price is a Divisia index. At *level 3* we estimated a linearly homogeneous translog revenue function for the defined crops aggregate. The price of other crops are aggregated using the Divisia formula.

The homogeneous restriction is a necessary and sufficient condition for the assumed separability structure (Fuss, 1977; Boyle, 1982). The assumption

of homogeneous separability implies that the individual output/fertiliser elasticity with respect to aggregate output/aggregate fertiliser is unity. The elasticities at *levels 2* and *3* are interpreted as compensated elasticities while those at *level 1* are conditional on the level of the quasi-fixed factor K and the time trend. This is, we believe, a reasonable postulate in the medium-run and thus adequate for medium-run simulations.

To obtain the complete set of Marshallian elasticities we must aggregate from *levels 3* to *1*. This is a relatively straightforward procedure for the assumed separability system and is set out in the annexe.

It should be emphasised that our estimation approach does not guarantee that the aggregated profit function will be convex in its price arguments. Indeed given the aggregative nature of our model it would be fortuitous if such were the case. For convexity we would require that the aggregate profit function be convex and increasing in the value of the revenue function and convex and decreasing in the value of the cost function. If the revenue function is convex and increasing in output prices then the profit function will be convex, increasing in output prices and decreasing in input prices.

2.   Empirical results

Details of the data set employed in the empirical analysis may be found in Boyle (1987). The data span the time period 1960–1982. The estimator employed for all regressions was maximum likelihood. For *level 1* we estimated the system as indicated in (3). For *levels 2* and *3* we only estimated the share relations. In all cases we allowed for contemporaneous error correlation. We also implemented a non-linear maximum likelihood estimator to capture a simple autoregressive structure. These estimates were very disappointing in two respects. First, they yielded very unstable coefficients. Second, the estimated AR(1) parameters implied unstable solutions. Accordingly we only report the regression results and the elasticity estimates obtained with the simpler estimation procedure.

Before considering out regression findings some comments on, key system equation diagnostics are warranted. The appropriate procedure in assessing parameter stability for a system of equations, no more than the computation of goodness of fit statistics, is open to considerable debate (see Judge et al., 1980 for a discussion). In this paper we report the R^2 obtained for each individual equation in the system.

We report two sets of regression results corresponding to the periods 1960–1982 and 1960–1972. These estimation periods were motivated by the changing regime facing Ireland's agricultural sector upon EC accession in 1973. Splitting the estimation period in this way allows us to test for parameter stability and by implication to ascertain how well the model predictions fit with reality over the period 1972–1982.

To test for parameter stability we compute the Chow predictive failure test for each equation of the system and we also derive a multi-equation version of this statistic. For each equation of the system the Chow test is given by the statistic (Chow, 1960)

$$C_i^x = \frac{(\varepsilon_i' \varepsilon_i - \varepsilon_{i1}' \varepsilon_{i1})/n_{i2}}{\varepsilon_{i1}' \varepsilon_{i1}/(n_{i1} - k_i)} \tag{5}$$

where,

C_i^x is the value of the Chow predictive failure test;

$\varepsilon_i' \varepsilon_i$ is the residual sum of squares for the ith equation estimated over the period 1960–1982;

$\varepsilon_{i1}' \varepsilon_{i1}$ is the residual sum of squares for the ith equation in the period 1960–1972;

n_{i2} is the number of observations for the ith equation in the period 1973–1982

and

$n_{i1} - k_i$ is the number of degrees of freedom for the ith equation for the period 1960–1972, where k_i is the number of parameters estimated.

C_i^x is asymptotically distributed as $F(n_{i2}, n_{i1} - k_i)$:

The apportionment of degrees of freedom to each equation of the system is problematic. Our approach was to simply allocate the degrees of freedom for the system as a whole, i.e. $\sum_i n_{i1} - \sum_i k_i$, in equal measure to each equation.

The multiequation version of the Chow statistic was obtained as

$$C^x = \frac{\left(\sum_i \varepsilon_i' \varepsilon i - \sum_i \varepsilon_{i1}'\right) \Big/ \sum_i n_{i2}}{\sum_i \varepsilon_{i1}' \varepsilon_{i1} \Big/ \left(\sum_i n_{i1} - \sum_i k_i\right)} \tag{6}$$

The results for the *level 1* regressions are given in Table 1. The coefficients are generally well determined with approximately two thirds and over 80% respectively of the estimated parameters for the periods 1960–1982 and 1960–1972 statistically significant. The most appealing feature of the findings is the relative degree of parameter stability as indicated by a comparison of the individual coefficients for the time periods.

Table 1.   *Translog coefficient estimates of the aggregate profit function (symnmetry and price homogeneity imposed)*

	1960–1982		1960–1972	
	Coefficient	Standard error	Coefficient	Standard error
α_0	5.08	0.05	5.68	0.04
α_{01}	-0.04	0.01	-0.02	0.01
α_{02}	-0.17	0.02	-0.15	0.02
α_{03}	-0.06	0.00	-0.05	0.00
α_{04}	-0.17	0.01	-0.16	0.01
α_{05}	-0.12	0.01	-0.13	0.00
α_{11}	-0.13	0.02	-0.20	0.05
α_{22}	-0.14	0.05	-0.10	0.11
α_{33}	-0.07	0.01	-0.04	0.00
α_{44}	-0.05	0.01	-0.06	0.01
α_{55}	-0.07	0.04	-0.07	0.02
α_{12}	-0.06	0.03	-0.11	0.05
α_{13}	0.04	0.01	-0.00	0.01
α_{14}	-0.02	0.01	0.01	0.01
α_{15}	-0.07	0.03	0.06	0.03
α_{23}	0.03	0.01	0.03	0.01
α_{24}	-0.02	0.02	-0.04	0.02
α_{25}	-0.08	0.03	-0.09	0.04
α_{34}	-0.00	0.00	-0.00	0.00
α_{35}	-0.05	0.01	-0.05	0.01
α_{45}	-0.02	0.01	-0.02	0.00
α_k	-0.04	0.47	-1.72	0.29
α_{kk}	1.88	2.54	9.28	0.19
α_{1k}	0.01	0.06	-0.05	0.02
α_{2k}	0.10	0.12	0.20	0.11
α_{3k}	0.01	0.02	0.01	0.00
α_{4k}	-0.00	0.03	0.04	0.04
α_{5k}	0.02	0.06	0.00	0.02
α_t	0.00	0.01	-0.18	0.00
α_{tt}	0.00	0.01	0.03	0.00
α_{1t}	-0.01	0.00	-0.01	0.00
α_{2t}	-0.01	0.00	-0.01	0.00
α_{3t}	0.00	0.00	0.00	0.00
α_{4t}	0.00	0.00	0.00	0.00
α_{5t}	-0.00	0.00	-0.00	0.00
α_{tk}	0.04	0.04	-0.03	0.00

Note: 1 = fertilisers; 2 = feedstuffs; 3 = energy products, etc.; 4 = hired labour; 5 = residual inputs; t = time trend; k = quasi-fixed factors. The numeraire price was aggregate output.

Regression diagnostics

Equation	Computed	Chow values Critical values		R^2	
		5%	1%	1960–1982	1960–1972
Profit	0.31	3.63	6.62	0.99	0.97
Fertiliser	34.00	3.63	6.62	0.88	0.97
Feedstuffs	0.91	3.63	6.62	0.76	0.67
Energy etc.	8.10	3.63	6.62	0.95	0.71
Hired labour	0.10	3.63	6.62	0.94	0.72
Residual inputs	5.00	3.63	6.62	0.89	0.69
System	0.49	1.63	1.98	–	–

The multi-equation Chow value confirms this stability but the individual equation values indicate instability for the fertiliser and energy equations particularly.

The derived elasticity estimates for the ultimate year of our data, 1982, are given in Table 2. The findings are of interest and appear reasonably plausible. Relatively high own price elasticities are obtained for all inputs and the aggregate output elasticity at about 0.7 is a plausible value. With the exception of the terms involving the price of aggregate output, the cross price effects are not large.

The quasi-fixed inputs yield elasticities with the expected signs and which are of a reasonable magnitude. The trend estimates appear quite strong but the negative sign on hired labour conforms well with our expectations.

For *level 2* we report two sets of elasticities. Table 3 gives the estimated translog parameters obtained for the aggregate output revenue function. With the exception of the cattle own price quadratic term, the remaining

Table 2. *Aggregate profit function price, quasi-fixed factor elasticities, and trend effect, 1982*[a]

	Price								
Quantity	W_1	W_2	W_3	W_4	W_5	P_q	Sum	K	T
X_1	−0.62	−0.13	−0.30	−0.02	−0.09	1.12	0.00	0.45	0.08
X_2	−0.07	−1.07	−0.19	−0.02	−0.08	1.43	0.00	0.24	0.07
X_3	−0.57	−0.68	−0.49	−0.07	0.17	1.64	0.00	0.41	0.05
X_4	0.07	−0.13	−0.11	−0.37	0.01	0.53	0.00	0.50	−0.04
X_5	−0.07	−0.12	0.07	0.00	−1.02	1.14	0.00	0.43	0.06
Q	−0.11	−0.28	−0.09	−0.02	−0.15	0.65	0.00	0.57	0.07

Note: X_1–X_5 refer to fertilisers, feedstuffs, energy products, hired labour and residual inputs and W_1–W_5 are the corresponding prices. Q and P_q refer to aggregate output and price respectively. K and T refer to the fixed factor and time trend respectively.

a: The translog parameter estimates are for 1960–1982 (Table 1).

Table 3.   *Translog coefficient estimates of the aggregate output revenue function (symmetry and price homogeneity imposed)*

	1960–1982		1960–1972	
	Coefficient	Standard error	Coefficient	Standard error
α_{01}	0.25	0.01	0.23	0.01
α_{11}	0.39	0.09	0.36	0.06
α_{12}	0.03	0.04	−0.06	0.03
α_{13}	−0.06	0.02	−0.05	0.03
α_{14}	−0.12	0.03	−0.10	0.04
α_{02}	0.37	0.01	0.37	0.01
α_{22}	0.14	0.03	0.21	0.02
α_{23}	−0.04	0.01	−0.05	0.02
α_{24}	−0.04	0.02	−0.03	0.03
α_{24}	−0.04	0.02	−0.03	0.03
α_{03}	0.05	0.00	0.05	0.00
α_{33}	0.05	0.01	0.06	0.02
α_{34}	0.00	0.01	−0.01	0.02
α_{04}	0.15	0.00	0.16	0.01
α_{44}	0.09	0.03	0.08	0.05

Note: 1 = milk output; 2 = cattle and calves; 3 = sheep and lambs; 4 = tillage crops. The numeraire price was residual outputs.

Regression diagnostics

Equation	Chow values			R^2	
	Computed	Critical values		1960–1982	1960–1972
		5%	1%		
Milk	8.10	3.10	5.10	0.66	0.77
Cattle	5.85	3.10	5.10	0.47	0.82
Sheep	1.35	3.10	5.10	0.70	0.56
Tillage	1.35	3.10	5.10	0.58	0.63
System	4.73	1.71	2.14	–	–

parameters exhibit apparent intertemporal stability. The Chow values, however, reject stability especially for the milk and cattle equations. An implication might be that EC entry caused a structural change.

The elasticities evaluated for the 1982 data point are supplied in Table 4. It is evident that implausible signs are obtained for the cattle and tillage own-price elasticities. We should note, firstly, that the estimates are relatively small in magnitude. As the elasticities are non-linear functions of the estimated parameters, the calculation of standard errors is difficult. If we employ a very simple formula for the standard errors such that, $SE = SE(\alpha_{ij}/S_i)$, which is an approximation for the formula suggested by Kulatilaka (1985), we can reject

Table 4. *Aggregate output revenue function price elasticities, 1982*[a]

Quantity	Price					
	P_1	P_2	P_3	P_4	P_5	Sum
Q_1	0.47	0.44	−0.14	−0.21	−0.56	0.00
Q_2	0.43	−0.25	−0.08	0.02	−0.12	0.00
Q_3	−1.38	−0.80	0.48	0.13	1.57	0.00
Q_4	−0.57	0.05	0.04	−0.18	0.66	0.00
Q_5	−1.49	−0.33	0.42	0.67	0.73	0.00

Note: Q_1-Q_5 refer to milk, cattle, sheep, tillage and residual outputs and P_1-P_5 are the corresponding prices.
a: The translog parameter estimates are for 1960–1982 (Table 3).

the tillage estimate as not being significantly different from zero but the cattle own price effect is significantly negative.

A number of possible reasons could explain these results. For instance, contrary to our specification, producers may not be in equilibrium in relation to the cattle or tillage variables. This appears a very plausible story as regards cattle output and possibly also for tillage output, which depends on specific pieces of capital, such as harvesting machines. Because of obvious physical constraints optimal adjustment of cattle output would be unlikely to occur within a one-year time period. Thus the finding of a significantly negative sign could be a plausible short-run response. In the long-run we would expect optimal adjustment. This possibility could be modelled by assuming cattle to be a quasi-fixed output using the procedures implemented by Kulatilaka (1985) and Squires (1987). Other explanations for the perverse signs may include risk and or income effects, neither of which are incorporated in our model.

The *level 2* fertiliser elasticities were not estimated for this study but are taken from Boyle (1982). These estimates are reproduced in Table 5. The interesting feature of these results is, firstly, that own price effects are seen to be important and secondly that substitution between nitrogen and other

Table 5. *Aggregate fertiliser cost function price elasticities, 1978/79*[a]

Quantity	Price			
	W_{11}	W_{12}	W_{13}	Sum
X_{11}	−0.76	0.54	0.21	−0.01
X_{12}	0.77	−0.71	−0.05	0.01
X_{13}	0.61	−0.11	−0.51	−0.01

Note: $X_{11}-X_{13}$ refer to nitrogen, phosphorus and potassium and $W_{11}-W_{13}$ are the corresponding prices.
a: The results are taken from Boyle (1982).

fertiliser products cannot be rejected. The latter result would apparently contradict experimental findings of complementarity.

The translog parameters for the tillage aggregate at *level 3* are given in Table 6. We can report reasonable intertemporal stability for these estimates. The system Chow values is marginally significant and the most unstable equation appears to be that for feeding barley but the Chow value is barely significant at the 10% level. The corresponding compensated elasticities are given in Table 7. The most notable feature of the findings is the high degree of substitutability obtained between the cereal products. The disappointing results in this table are the negative own price signs for sugar beet and other crops.

Using the aggregation procedure outlined in the Annexe we obtain the

Table 6.   *Translog coefficient estimates of the aggregate tillage revenue function (symmetry and price homogeneity imposed)*

	1960–1982		1960–1972	
	Coefficient	Standard error	Coefficient	Standard error
α_{01}	0.18	0.01	0.21	0.02
α_{11}	0.49	0.08	0.38	0.14
α_{12}	−0.10	0.03	−0.02	0.02
α_{13}	−0.43	0.08	−0.39	0.10
α_{14}	0.01	0.01	−0.02	0.02
α_{15}	−0.03	0.03	−0.04	0.05
α_{16}	0.04	0.03	0.07	0.06
α_{02}	0.08	0.00	0.08	0.00
α_{22}	0.22	0.05	−0.03	0.05
α_{23}	0.00	0.05	0.16	0.05
α_{24}	−0.03	0.02	−0.01	0.01
α_{25}	−0.04	0.02	−0.06	0.03
α_{26}	−0.02	0.01	−0.03	0.01
α_{03}	0.26	0.02	0.22	0.01
α_{33}	0.52	0.14	0.21	0.14
α_{34}	−0.02	0.02	−0.02	0.02
α_{35}	0.03	0.04	0.07	0.06
α_{36}	−0.04	0.06	−0.02	0.06
α_{04}	0.02	0.00	0.02	0.00
α_{44}	0.03	0.01	0.04	0.01
α_{45}	0.00	0.01	0.03	0.02
α_{46}	−0.01	0.01	−0.03	0.01
α_{05}	0.18	0.00	0.16	0.00
α_{55}	0.07	0.03	0.11	0.06
α_{56}	−0.02	0.02	−0.08	0.02
α_{06}	0.20	0.01	0.24	0.01
α_{66}	0.08	0.04	0.13	0.04

Note: 1 = wheat; 2 = malting barley; 3 = feeding barley; 4 = oats; 5 = sugarbeet; 6 = potatoes. The numeraire price was other crops.

Regression diagnostics

Equation	Chow values			R^2	
	Computed	Critical values		1960–1982	1960–1972
		5%	1%		
Wheat	0.31	3.23	5.54	0.56	0.14
Malting barley	3.84	3.23	5.54	0.56	0.28
Feeding barley	5.87	3.23	5.54	0.53	0.54
Oats	3.20	3.23	5.54	0.10	0.45
Sugar beet	5.16	3.23	5.54	0.32	0.66
Potatoes	3.05	3.23	5.54	0.44	0.55
System	2.38	1.58	1.90	–	–

Table 7. *Aggregate tillage revenue function price elasticities, 1982*[a]

	Price							
Quantity	P_{41}	P_{42}	P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	Sum
Q_{41}	2.49	−0.62	−2.56	0.08	0.35	0.03	0.23	0.00
Q_{42}	−1.39	2.57	0.38	−0.47	−0.22	−0.42	−0.45	0.00
Q_{43}	−0.99	0.06	0.76	−0.04	−0.01	0.30	−0.08	0.00
Q_{44}	1.02	−2.55	−1.37	1.63	−0.78	0.22	1.83	0.00
Q_{45}	0.56	−0.15	−0.04	−0.09	−0.07	0.01	−0.22	0.00
Q_{46}	0.07	−0.12	0.51	0.01	−0.07	−0.46	0.04	0.00
Q_{47}	0.41	−0.24	−0.35	0.24	−0.13	−0.03	0.10	0.00

Note: Q_{41}–Q_{47} refer to wheat, malting barley, feeding barley, oats, potatoes, sugarbeet and other crops and P_{41}–P_{47} are the corresponding prices.

a: The translog parameter estimates are for 1960–1982 (Table 6).

matrix of gross elasticities in Table 8. Because these elasticities are conditional on the levels of the quasi-fixed factor index and technical change we would not expect them to necessarily conform to the expectations of Sakai (1974), namely, that gross substitution and regressive relationships between inputs and outputs is ruled out. Nonetheless, it is encouraging that our results indicate no evidence of regressive relationships and in general we do not find much indication of gross substitution between outputs or inputs.

3. Concluding comments

This paper indicates that the employment of sensible separability assumptions can yield elasticity estimates which contain sufficient detail to be useful

Table 8.   Complete set of short-run gross elasticities for the Irish 'national farm', 1982

Quantity/Price	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	K	T
Milk (1)	0.70	0.68	-0.11	-0.02	-0.01	-0.05	-0.00	-0.01	-0.03	-0.01	-0.47	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Cattle (2)	0.66	-0.01	-0.05	0.02	0.01	0.04	0.00	0.01	0.02	0.01	-0.04	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Sheep (3)	-1.15	-0.56	0.51	0.03	0.01	0.08	0.00	0.02	0.05	0.02	1.66	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Wheat (4)	-0.34	0.29	0.07	2.48	-0.63	-2.60	0.08	0.34	0.01	0.22	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Barley:																				
Malting (5)	-0.34	0.29	0.07	-1.40	2.57	0.34	0.47	-0.23	-0.44	-0.46	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Barley: Feed (6)	-0.34	0.29	0.07	-1.00	0.06	0.72	-0.04	-0.02	0.28	-0.09	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Oats (7)	-0.34	0.29	0.07	-1.00	-2.56	-1.4	1.63	-0.79	0.20	1.83	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Potatoes (8)	-0.34	0.29	0.07	0.55	-0.16	-0.08	-0.09	-0.08	-0.01	-0.23	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Sugarbeet (9)	-0.34	0.29	0.07	0.07	-0.13	0.47	0.01	0.07	-0.48	0.04	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Other Crops (10)	-0.34	0.29	0.07	0.37	-0.29	-0.36	0.24	-0.14	-0.05	0.09	0.75	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Residual																				
Outputs (11)	-1.27.	-0.09	0.45	0.11	0.05	0.29	0.01	0.08	0.17	0.07	0.82	-0.06	-0.04	-0.02	-0.28	-0.09	-0.02	-0.15	0.57	0.05
Nitrogen (12)	0.39	0.40	0.03	0.02	0.01	0.06	0.00	0.02	0.03	0.01	0.15	-1.06	0.33	0.10	-0.13	-0.30	0.02	-0.09	0.45	0.08
Phosphorus (13)	0.39	0.40	0.02	0.02	0.01	0.06	0.00	0.02	0.03	0.01	0.15	0.47	-0.92	-0.15	-0.13	-0.30	0.02	-0.09	0.45	0.08
Potassium (14)	0.39	0.40	0.03	0.02	0.01	0.06	0.00	0.02	0.03	0.01	0.15	0.31	-0.31	-0.62	-0.13	-0.30	0.02	-0.09	0.45	0.08
Feedstuffs (15)	0.50	0.52	0.04	0.03	0.01	0.07	0.00	0.02	0.04	0.02	0.19	-0.03	-0.02	-0.01	-1.07	-0.19	-0.02	-0.08	0.24	0.07
Energy (16)	0.57	0.59	0.05	0.03	0.01	0.08	0.00	0.02	0.05	0.02	0.21	-0.28	-0.19	-0.10	-0.68	-0.50	-0.07	0.17	0.40	0.05
Labour (17)	0.18	0.19	0.02	0.01	0.00	0.03	0.00	0.01	0.02	0.01	0.07	0.03	0.02	0.01	-0.13	-0.11	-0.37	0.01	0.50	-0.04
Residual																				
Inputs (18)	0.40	0.41	0.03	0.02	0.01	0.06	0.00	0.02	0.03	0.01	0.15	-0.03	-0.02	-0.01	-0.12	0.07	0.00	-1.02	0.43	0.06

Note: These elasticities are derived from Tables (2), (4), (5) and (7) using the procedure set out in the Annex.

in a policy analysis context. Given the elasticity set, the Johansen solution can be invoked to yield a modelling device which has the not inconsiderable merit of being easy to use. We thus feel that the model could provide a reasonable basis for assessing the impact of policy changes on the production system. Given the restricted nature of the profit function the model can only be employed for short to medium-run simulations.

The failure of our model to satisfy even the necessary conditions for profit maximisation given the incorrect sign of the cattle elasticity raises important questions concerning the methodology of policy modelling. It would certainly be of great help in interpreting and explaining model outcomes if the sector as a whole behaved as if it were a single profit maximising entity. Since our estimated model has not turned out to be a profit function we have two options, short of abandoning the goal that model outcomes have an easy and clear interpretation. We could either force the function to become a profit function, or, we could change the specification (for example by treating cattle as a quasi-fixed output) to get the sector modelled in a way that facilitates the interpretation of model outcomes. The imposition of such curvature constraints must await future research but the latter strategy has been recently implemented with promising results (Boyle and Guyomard, 1989).

Several other possibilities for improvement exist. While the application of this methodology can give long-run estimates this may not be always of interest to the policy maker. The long-run may be too far distant to be of much relevance in a policy context. Moreover the path to long-run equilibrium may be of interest. While techniques exist within the duality framework to model the dynamic adjustment path they are very much *ad hoc* and are difficult to implement. An alternative procedure might be to adopt a two step model. In the first stage we could apply the duality methodology as we have done for the variable netputs. In the second stage we could model with traditional single-equation dynamic methodologies the behaviour of important quasi-fixed factors which are likely to be in disequilibrium, such as family labour and capital.

An important extension would also be the building of a livestock-feed production block because of its interaction between the cereal and livestock industries.

ANNEXE

Generating the set of elasticities for the integrated system (Table 8)

The totality of entries in Table 8 will not be explained in detail since this would be tedious. The aggregation procedure is straightforward but it is helpful to document the approach for typical equations. Four types of equation characterise the elasticities

in Table 8. Milk, cattle, sheep and residual outputs form one type with the crop and fertiliser components forming another two and the remaining inputs (feedstuffs, energy, labour and residual inputs) fall into the final category. These categories reflect the underlying separability structure employed in the estimation.

Taking milk output as the typical equation for the first category, its Johansen equation is

$$
\begin{aligned}
q_1 = {} & n_{11}^1 p_1 + n_{12}^1 p_2 + n_{13}^1 p_3 + n_{14}^1 (r_{41} p_{41} + r_{42} p_{42} + r_{43} p_{43} \\
& + r_{44} p_{44} + r_{45} p_{45} + r_{46} p_{46} + r_{47} p_{47} + n_{15}^1 p_5 + n_{qq}(r_1 p_1 + r_2 p_2 \\
& + r_3 p_3 + r_4 [r_{41} p_{41} + r_{42} p_{42} + r_{43} p_{43} + r_{44} p_{44} + r_{45} p_{45} + r_{46} p_{46} \\
& + r_{47} p_{47}] + r_5 p_5) + n_{q1}(s_{11} w_{11} + s_{12} w_{12} + s_{13} w_{13}) + n_{q2} w_2 \\
& + n_{q3} w_3 + n_{q4} w_4 + n_{q5} w_5 + n_{qk} k + n_{qt}
\end{aligned}
\tag{A.1}
$$

where,

quantities and prices are in percentage changes, q_1 is milk output; $p_1 - p_q$ are the prices of milk, cattle, sheep, tillage, residual outputs and aggregate output; $p_{41} - p_{47}$ are the prices of wheat, malting-barley, feeding-barley, oats, potatoes, sugar beet, and other crops; n^1 are the compensated price elasticities and n are the Marshallian-type elasticities; r denotes the revenue shares corresponding to the particular outputs; $w_1 - w_5$ refer to the price of aggregate fertilisers, feedstuffs, energy inputs hired, labour, residual inputs; $w_{11} - w_{13}$ are the prices of nitrogen, phosphorus and potassium; s indicates the cost shares corresponding to the production inputs; k is the proportionate change in fixed inputs and n_{qt} is the proportionate change due to trend.

The reasoning underlying equation (A.1) is intuitive. The milk output equation is estimated conditional on the level of aggregate output. In our procedure we impose the condition that the elasticity of milk output with respect to aggregate output is unity. The proportionate change in aggregate output in turn depends on the arguments in the profit function, i.e. the aggregate output price index, which is approximated here as the weighted sum of proportionate changes in component prices, with the weights given by the revenue shares; the price of aggregate fertilisers; feedstuffs; energy; labour; residual inputs; quasi-fixed inputs and a trend term.

Gathering terms we have

$$
\begin{aligned}
q_1 = {} & (n_{11}^1 + n_{qq} r_1) p_1 + (n_{12}^1 + n_{qq} r_q) p_2 + (n_{13}^1 + n_{qq} r_3) p_3 \\
& + (n_{14}^1 r_{41} + n_{qq} r_4 . r_{41}) p_{41} + (n_{14}^1 r_{42} + n_{qq} r_4 . r_{42}) p_{42} \\
& + (n_{14}^1 r_{43} + n_{qq} r_4 . r_{43}) p_{43} + (n_{14}^1 r_{44} + n_{qq} r_4 . r_{44}) p_{44} \\
& + (n_{15}^1 r_{45} + n_{qq} r_4 . r_{45}) p_{45} + (n_{16}^1 r_{46} + n_{qq} r_4 . r_{46}) p_{46} \\
& + (n_{17}^1 r_{47} + n_{qq} r_4 . r_{47}) p_{47} + (n_{15}^1 + n_{qq} r_5) p_5 + (n_{q1} s_{11}) w_{11} \\
& + (n_{q1} s_{12}) w_{12} + (n_{q1} s_{13}) w_{13} + (n_{q2}) w_2 + (n_{q3}) w_3 \\
& + (n_{q4}) w_4 + (n_{q5}) w_5 + n_{qk} k + n_{qt}
\end{aligned}
\tag{A.2}
$$

The terms in parentheses are the gross or Marshallian-type elasticities. A similar structure applies for the cattle, sheep and residual output equations.

The story is slightly more complicated for the individual crop equations since we must aggregate through three stages to obtain the Marshallian elasticities. To illustrate the aggregation procedure, we document the Johansen equation for wheat output. This becomes

$$
\begin{aligned}
q_{41} =\ & n^1_{4141}p_{41} + n^1_{4142}p_{42} + n^1_{4143}p_{43} + n^1_{4144}p_{44} + n^1_{4145}p_{45} \\
& + n^1_{4146}p_{46} + n^1_{4147}p_{47} + n^1_{44}(r_{41}p_{41} + r_{42}p_{42} + r_{43}p_{43} + r_{44}p_{44} \\
& + r_{45}p_{45} + r_{46}p_{46} + r_{47}p_{47}) + n^1_{41}p_1 + n^1_{42}p_2 + n^1_{43}p_{43} \\
& + n^1_{45}p_{45} + n_{qq}(r_1p_1 + r_2p_2 + r_3p_3 + r_4[r_{41}p_{41} + r_{42}p_{42} \\
& + r_{43}p_{43} + r_{44}p_{44} + r_{45}p_{45} + r_{46}p_{46} + r_{47}p_{47}] + r_5p_5) \\
& + n_{q1}(s_{11}w_{11} + s_{12}w_{12} + s_{13}w_{13}) + n_{q2}w_2 + n_{q3}w_3 \\
& + n_{q4}w_4 + n_{q5}w_5 + n_{qk}k + n_{qt} \tag{A.3}
\end{aligned}
$$

Gathering terms we have

$$
\begin{aligned}
q_{41} =\ & (n^1_{4141} + n^1_{44}r_{41} + n_{qq}r_4.r_{41})p_{41} \\
& + (n^1_{4142} + n^1_{44}r_{42} + n_{qq}r_4.r_{42})p_{42} \\
& + (n^1_{4143} + n^1_{44}r_{43} + n_{qq}r_4.r_{43})p_{43} \\
& + (n^1_{4144} + n^1_{44}r_{44} + n_{qq}r_4.r_{44})p_{44} \\
& + (n^1_{4145} + n^1_{44}r_{45} + n_{qq}r_4.r_{45})p_{45} \\
& + (n^1_{4146} + n^1_{44}r_{46} + n_{qq}r_4.4_{46})p_{46} \\
& + (n^1_{4147} + n^1_{44}r_{47} + n_{qq}r_4.r_{47})p_{47} \\
& + (n^1_{41} + n_{qq}r_{41})p_1 + (n^1_{42} + n_{qq}r_2)p_2 \\
& + (n^1_{43} + n_{qq}r_3)p_3 + (n^1_{45} + n_{qq}r_5)p_5 \\
& + (n_{q1}s_{11})w_{11} + (n_{q1}s_{12})w_{12} + (n_{q1}s_{13})w_{13} \\
& + (n_{q2})w_2 + (n_{q3})w_3 + (n_{q4})w_4 + (n_{q5})w_5 + n_{qk}k + n_{qt} \tag{A.4}
\end{aligned}
$$

The intuition supporting equations (A.3) and (A.4) is that each crop supply function is conditional on aggregate tillage which is in turn conditional on aggregate output which is in turn dependent on all the arguments in the aggregate profit function.

The nitrogen, phosphorus and potassium equations are similarly obtained. For instance, the equation for nitrogen becomes

$$
\begin{aligned}
x_{11} =\ & e^1_{1111}w_{11} + e^1_{1112}w_{12} + e^1_{1113}w_{13} + e_{11}(s_{11}w_{11} + s_{12}w_{12} + s_{13}w_{13}) \\
& + e_{12}w_2 + e_{13}w_3 + e_{14}w_4 + e_{15}w_5 + e_{1k}k \\
& + e_{1t} + e_{1q}(r_1p_1 + r_2p_2 + r_3p_3 + r_4[r_{41}p_{41} + r_{42}p_{42} + r_{43}p_{43} \\
& + r_{44}p_{44} + r_{45}p_{45} + r_{46}p_{46} + r_{47}p_{47}] + r_5p_5) \tag{A.5}
\end{aligned}
$$

where, e^1, e are the compensated and gross price elasticities respectively.

$$x_{11} = (e_{1q}r_1)p_1 + (e_{1q}r_2)p_2 + (e_{1q}r_3)p_3$$
$$+ (e_{1q}r_4r_{41})p_{41} + (e_{1q}r_4r_{42})p_{42} + (e_{1q}r_4r_{43})p_{43} + (e_{1q}r_4r_{44})p_{44}$$
$$+ (e_{1q}r_4r_{45})p_{45} + (e_{1q}r_4r_{46})p_{46} + (e_{1q}r_4r_{47})p_{47} + (e_{1q}r_5)p_5$$
$$+ (e^1_{1111} + e_{11}s_{11})w_{11} + (e^1_{1112} + e_{11}s_{12})w_{12}$$
$$+ (e^1_{1113} + e_{11}s_{13})w_{13} + e_{12}w_2 + e_{13}w_3 + e_{14}w_4 + e_{15}w_5$$
$$+ e_{1k}k + e_{1t} \tag{A.6}$$

The remaining equations (feedstuffs, energy, labour and residual inputs) have the following relatively simpler structure. For example the feedstuffs equation is

$$x_2 = e_{22}w_2 + e_{23}w_3 + e_{24}w_4 + e_{25}w_5$$
$$+ e_{2k}k + e_{2t} + e_{21}(s_{11}w_{11} + s_{12}w_{12} + s_{13}w_{13})$$
$$+ e_{2q}(r_1p_1 + r_2p_2 + r_3p_3 + r_4[r_{41}p_{41} + r_{42}p_{42} + r_{43}p_{43} + r_{44}p_{44}$$
$$+ r_{45}p_{45} + r_{46}p_{46} + r_{47}p_{47}] + r_5p5) \tag{A.7}$$

Gathering terms we have

$$x_2 = (e_{2q}r_1)p_1 + (e_{2q}r_2)p_2 + (e_{2q}r_3)p_3 + (e_{2q}r_4 . r_{41})p_{41} + (e_{2q}r_4 . r_{42})p_{42}$$
$$+ (e_{2q}r_4 . r_{43})p_{43} + (e_{2q}r_4 . r_{44})p_{44} + (e_{2q}r_4 . r_{45})p_{45} + (e_{2q}r_4 . r_{47})p_{47}$$
$$+ e_{2q}r_5p_5$$
$$+ (e_{21}s_{11})w_{11} + (e_{21}s_{12})w_{12} + (e_{21}s_{13})w_{13}$$
$$+ e_{22}w_2 + e_{23}w_3 + e_{24}w_4 + e_{25}w_5 + e_{2k}k + e_{2t} \tag{A.8}$$

NOTES

1. References can be supplied by the authors.
2. Strictly the Johansen model was a prototype of the CGE modelling whereas our paper is concerned exclusively with profit generation in the farm sector. We retain the terminology employed here following Munk (1985).

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