

A Critical Comparison Between Parametric Approximation Methods for Radiation Forces in Wave Energy Systems

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ABSTRACT

Within the ocean engineering literature, and, particularly, within the wave energy research and hydrodynamic fields, different methods can be found that aim to identify a finite-order parametric model to represent the radiation force convolution term of the well-known Cummins' equation. Such an approximation process is required for several reasons: firstly, to obtain a mathematical representation that requires low computational effort and, secondly, to ease the model-based control/estimation design procedures. Recently, a new MATLAB toolbox has been released to obtain a parametric approximation of such a convolution term based on *moment-matching*. This paper aims to compare this new moment-matching-based application, with other well-established software. This comparison is based not only on the quality of the approximation, but also on the preservation of the intrinsic physical properties of radiation forces. Three different case studies are used as a basis for the comparison.

KEY WORDS: Radiation forces; Parametric form; Model order reduction; Frequency-domain identification; Time-domain identification

INTRODUCTION

Cummins' equation (Cummins, 1962) is commonly used to describe, in the time-domain, the dynamics of the motion of marine structures (floating bodies with zero mean velocity). However, such an equation includes a convolution operation to express the fluid memory effect associated to radiation forces, which can be a drawback for several applications relevant to several ocean engineering applications, such as, for example, applications belonging to the wave energy field. To start with, this convolution term represents a drawback from a simulation point of view, given that the numerical solution of such a mapping requires a high computational effort. In addition, it can be inconvenient from an estimation/control point of view, since state-of-the-art estimation/control techniques are usually based on the availability of a parametric state-space (SS) representation (at least in local co-

ordinates) of the system under analysis, i.e. a set of first order differential equations (Faedo et al., 2017). In order to overcome these drawbacks, researchers usually approximate the convolution term using a suitable parametric structure. To this end, several software utilities can be found in the literature, which attempt to compute an approximating parametric model for the convolution mapping introduced by Cummins.

The Centre for Ocean Energy Research (COER) has recently presented a moment-matching-based identification method for both the radiation convolution term and the complete force-to-motion (input-output) dynamics (Faedo et al., 2018). This strategy has its foundation in recent advances on model order reduction by moment-matching, developed over several studies such as (Astolfi, 2010) or (Scarciotti and Astolfi, 2016). This method allows for the computation of a model that *exactly* matches the frequency response of the original (target) system at a set of user-selected frequencies, depicting an efficient method to compute a state-space representation for the dynamics of Wave Energy Converters (WECs) or, more generally, floating bodies with zero forward speed. Motivated by both the solid theoretical and practical results obtained, the *Finite-Order Approximation by Moment-Matching* (FOAMM) MATLAB application has been developed within the COER (Peña-Sanchez et al., 2019), that systematically applies this method using, as input, only raw frequency-domain data provided by Boundary Element Method (BEM)-based hydrodynamic solvers, such as NEMOH (Babarit and Delhommeau, 2015) or WAMIT (Newman and Lee, 2002).

One of the best-established methods, for this particular approximation problem, is the method developed by Pérez and Fossen (2008) at the Norwegian University of Science and Technology. Such a strategy proposes a parametric identification of the radiation force impulse response function using only frequency-domain data from BEM solvers. One year after the theory of this strategy was presented, (Pérez and Fossen, 2009) developed a MATLAB toolbox that systematically applies their method, termed the *Frequency-Domain Identification* (FDI) toolbox.

To the best of the authors' knowledge, the only other available application is the associated with the WEC motion simulator *WEC-*

Sim (Yu et al., 2014). This software, termed *Boundary Element Method Input/Output* (BEMIO) (Tom et al., 2015), is specifically used within WEC-Sim to obtain a state-space approximation of the radiation force convolution term. BEMIO can directly read the outputs from BEM codes and save the relevant results in a standardised format, or calculate the impulse response function of the radiation force subsystem. Finally, and unlike FOAMM and FDI, which are only available for MATLAB, the developers of BEMIO offer a PYTHON-based version of the application.

We note that the time-domain approximation strategy based on Prony's method (De Prony, 1795) should be mentioned herein since, even though there is no specific software utility developed (for floating bodies), it is still a commonly used method within the ocean engineering literature. In brief, this strategy is based on identifying the radiation SS coefficients from the corresponding impulse response function, and it has been used in noteworthy studies, such as those involving the SEAREV WEC (Babarit and Clement, 2006) and the Spar-buoy WEC (Grilli et al., 2007).

That said, the objective of this paper is twofold. Firstly, this study aims to report the available techniques and applications for this particular approximation problem, and explain the different options presented by each software/method. Secondly, and as the main contribution of this paper, we aim to provide a critical comparison between different software applications (and Prony's method), both in the frequency- and time-domain. To fulfil this last objective, the methods are compared in terms of approximation quality, model order and preservation of physical characteristics of the radiation subsystem (including internal stability, passivity, among others). For the sake of completeness, the strategies are not only tested using geometrically complex WECs, but also with a vessel-shaped hull, as detailed in Section 4.

The remainder of this paper is organised as follows. Section 2 briefly discusses the equations to characterise the motion of a floating body. In Section 3 the different identification toolboxes considered in the study are introduced, while the test cases used for the numerical comparison are shown in Section 4. Finally, Section 5 illustrates, compares and discusses the results obtained for each application/method, while the main conclusions (and future work) are encompassed in Section 6.

EQUATIONS OF MOTION

This section provides a brief description of the linear hydrodynamic equations of motion used to describe the behaviour of a floating body, aiming to precisely describe what the different strategies are designed to approximate. Since the analysed applications, in general, only process single-input single-output (SISO) models¹, 1-DoF (Degree of Freedom) devices are considered within this study.

The equation of motion, for a 1-DoF floating body, can be expressed in the time-domain in terms of Newton's second law, obtaining the following formulation:

$$m\ddot{x}(t) = \mathcal{F}_r(t) + \mathcal{F}_h(t) + \mathcal{F}_e(t) + u(t), \quad (1)$$

where m is the mass of the body, $\ddot{x}(t)$ the acceleration, $\mathcal{F}_e(t)$ the wave excitation force, $\mathcal{F}_r(t)$ the radiation force, $\mathcal{F}_h(t)$ the

¹It should be noted that, even though a multiple-input multiple-output version of the moment-matching-based identification strategy has already been presented in (Faedo et al., 2019), the current FOAMM toolbox can only handle SISO models.

hydrostatic restoring force, and $u(t)$ represents any other external input force. The linearised hydrostatic force for a floating body can be written as $\mathcal{F}_h(t) = -s_h x(t)$, where s_h denotes the hydrostatic stiffness. The radiation force $\mathcal{F}_r(t)$ is modelled, from linear potential theory, using the well-known Cummins' equation (Cummins, 1962), as

$$\mathcal{F}_r(t) = -\mu_\infty \ddot{x}(t) - \int_0^{+\infty} k(\tau) \dot{x}(t - \tau) d\tau, \quad (2)$$

where $\mu_\infty = \lim_{\omega \rightarrow +\infty} A(\omega)$. $\mu_\infty > 0$ represents the added-mass at infinite frequency, $A(\omega)$ the radiation added mass and $k(t)$ the radiation impulse response, containing the memory effect of the fluid response. Regarding the external input force $u(t)$, for the WEC case, it is usually represented in terms of a Power Take-Off (PTO) force². Finally, the complete linearised equation of motion of the floating body can be written as

$$(m + \mu_\infty)\ddot{x}(t) + k(t) * \dot{x}(t) + s_h x(t) = \mathcal{F}_e(t) + u(t), \quad (3)$$

where the symbol $*$ represents the convolution integral operator. We note that such a convolution term is exactly what the applications analysed in this study aim to approximate using a approximated parametric model.

By applying the Fourier transform to Eq.(3), and considering velocity as the measurable output, the following representation

$$\hat{x}(j\omega) = \mathcal{F}_e(j\omega)H(j\omega), \quad (4)$$

where $H(j\omega)$ represents the force-to-velocity frequency response, holds. Specifically, $H(j\omega)$ is a function of a specific set of characteristic frequency-dependent parameters, namely

$$H(j\omega) = \frac{1}{B(\omega) + j\omega [A(\omega) + m] + \frac{s_h}{j\omega}}, \quad (5)$$

where $B(\omega)$ is the radiation damping of the device (Falnes, 2002). The hydrodynamic parameters $A(\omega)$ and $B(\omega)$ can be efficiently obtained using existing BEM solvers, such as those discussed in Section 1.

The relation between time-, Eq.(3), and frequency-domain, Eq.(4), models was established by Ogilvie (1964). Such a relationship is defined as a function of the hydrodynamic parameters $B(\omega)$ and $A(\omega)$, and the radiation kernel $k(t)$, using the definition of the Fourier transform as

$$\begin{aligned} B(\omega) &= \int_0^{+\infty} k(t) \cos(\omega t) dt, \\ A(\omega) &= \mu_\infty - \frac{1}{\omega} \int_0^{+\infty} k(t) \sin(\omega t) dt. \end{aligned} \quad (6)$$

Thus, the impulse response $k(t)$ can be written, involving the frequency-dependent hydrodynamic parameter $B(\omega)$, as

$$k(t) = \frac{2}{\pi} \int_0^{+\infty} B(\omega) \cos(\omega t) d\omega. \quad (7)$$

Eq.(7) allows a frequency-domain analysis of $k(t)$ using a direct application of the Fourier transform, i.e.

$$\hat{k}(j\omega) = B(\omega) + j\omega [A(\omega) - \mu_\infty]. \quad (8)$$

²The interested reader is referred to (Falnes, 2002), for a more comprehensive description of PTO systems.

The function $K(j\omega)$ represents the frequency response of the radiation force dynamics, and it is the *target* model to be approximated by frequency-domain methods. On the contrary, for the strategies that identify the radiation SS using time-domain data, the *target* system to be parameterised is the previously introduced impulse response function $k(t)$.

IDENTIFICATION APPLICATIONS

In this section, the different parameterisation applications considered in this study are introduced. For the sake of clarity (and economy of space), the underlying theory of each different method is not explicitly discussed herein and, therefore, the interested readers are referred to each corresponding theoretical study. It should be noted that the different applications/methods introduced in this section appear in order of release date, where the first application/method discussed refers to the oldest strategy.

FDI toolbox

This software runs in MATLAB, and it requires the user to have the commercially available *signal processing toolbox* in order to operate it. However, as explained in (Pérez and Fossen, 2009), alternative functions can be implemented by the user in order to avoid using such a toolbox. The application can be downloaded for free from their website www.marinecontrol.org.

The coefficients, and different options, of this application need to be defined in the main file composing this software suite, termed **FDIRadMod.m**. Then, **FDIRadMod.m** calls the different functions that comprise the toolbox, depending on the selected options. Thus, in order to work, this software requires the user to provide (as vectors) the frequency-domain hydrodynamic coefficients $A(\omega)$ and $B(\omega)$, along with a vector containing the corresponding (discrete) set of frequencies. Additionally, the infinite-frequency added mass μ_∞ and a structure (**FDIOpt**) containing information about the options of the software, can also be defined. Regarding the quantity μ_∞ , it can be either explicitly provided by the user (as a scalar) or, if **FDIOpt.AintFlag** is set to 0, the application automatically computes an estimate of its value based on Ogilvie’s relations (see Section 2).

Once **FDIRadMod.m** is called, the application asks the user to choose a desired frequency range over which to perform the parametric approximation. This frequency range is strongly linked to the frequency spectrum of the input of the system, as discussed in (Faedo et al., 2018). After selecting the frequency range, the software asks if there are any “wild” points in the frequency data to be removed before performing the approximation³.

Subsequently, the FDI toolbox automatically estimates the best order of the approximation, by linearly increasing the model order and computing the corresponding approximation error, as a function of both the approximated and the target radiation added mass and damping coefficients. Once this approximation error is less than a predefined value, the software stops increasing the order⁴, and the obtained model output is plotted along with the target data. At this point, the application prompts the

³The aim of this is to remove any possible information that the user considers unrealistic, as could happen for irregular frequencies (Penalba et al., 2017).

⁴It should be noted that, if the fitting condition is not fulfilled, the application will continue increasing the order until the maximum order specified in **FDIOp.OrdMax** is reached.

user to either manually introduce a new desired order for the approximating model, or to retain the last computed model and exit the function. If a new system order is specified, the model is re-computed and the output plotted against the target data, and the application consults the user one more time regarding the option to introduce a new model order, or exit the function. Such a loop is repeated until the user explicitly specifies that the obtained model is acceptable.

Regarding the identification method, three different options are given to solve the parameter optimisation problem (which can be chosen by changing **FDIOpt.Method**):

- linearise the optimisation problem and solve a linear Least-Square (LS) problem,
- solve an iterative LS problem (where the maximum number of iterations is given by **FDIOpt.Iterations**), or
- use the linear LS solution to initialise a nonlinear LS problem (solved using a Gauss-Newton method).

Once the system order and optimisation method are chosen, the function returns the numerator and denominator coefficients of the estimated transfer function for the radiation force subsystem, and the estimate of μ_∞ if **FDIOpt.AintFlag** is set to 0. Given that FDI offers, as outputs, the numerator and denominator of the input-output dynamics of the approximating model, in order to obtain the SS representation of the radiation impulse response, the user needs to compute a realisation of the obtained transfer function (for example, any canonical realisation can be used (Taghipour et al., 2008)).

In order to help the user to understand which parameters need to be pre-defined and how the application is run, different files with an example case are provided by the developers, along with the toolbox files.

BEMIO toolbox

In contrast to FDI, the BEMIO toolbox identifies the parametric model of the radiation convolution term using time-domain data, i.e. utilising $k(t)$. Though there exists a MATLAB version at the moment, the original platform of this application is PYTHON. However, since the conversion from PYTHON to MATLAB was specifically made to implement BEMIO in WEC-Sim (which only runs in MATLAB), the PYTHON-based BEMIO code is no longer supported by the developers. It is important to highlight that BEMIO does not require of any other additional toolbox to run (i.e. it runs in a plain MATLAB version), and it can be downloaded for free from <https://wec-sim.github.io/WEC-Sim/index.html>⁵.

The main file of this application is called **bemio.m**, which invokes a number of functions. The first function called explicitly depends on the BEM solver used to compute the hydrodynamic parameters. This software is capable of reading the output files from NEMOH, WAMIT or AQUA (ANSYS, 2013), and converting them into a standardised format. After pre-processing the hydrodynamic data (and storing it in a structure called **hydro**), **bemio.m** calls a function to compute $k(t)$ (which is also saved in the **hydro** structure). Finally, **bemio.m** invokes the **Radiation_IRF_SS** routine, where the SS model of the radiation impulse response is specifically computed.

⁵See <https://wec-sim.github.io/bemio/> for the Python version.

The only mode offered by BEMIO, is the automatic order selection mode, where the user selects the maximum order of the SS realisation and an accuracy threshold for the fit of the SS realisation (these are indeed the only options that the user can freely select). Therefore, to obtain a SS representation of a (fixed) user-defined order, one can set up the maximum order as the desired order, while setting the approximation threshold to 1^6 . Using this approach, the code increases the model order until the selected maximum order is reached. Though the computational time required by the toolbox is relatively low, this method is inherently sub-optimal with regard to its computational capabilities, given that the software always computes the totality of the parametric models going from order 2, up to the desired (or optimal) order.

After **Radiation_IRF_SS** is run, the function returns (inside the structure **hydro**) the matrices of the approximating SS model for the radiation force subsystem. Additionally, as for the FDI toolbox, the application is distributed with an example case, designed to help the user to understand the relevant working principles of the toolbox.

FOAMM toolbox

This is, to the best of the authors' knowledge, the most recently released toolbox for the approximation of radiation forces available in the literature. Similar to FDI, this toolbox identifies a parametric model based on frequency-domain data, i.e. from $K(j\omega)$. Even though the application uses a MATLAB toolbox to perform the underlying optimisation problem, since it is provided in an executable form, it effectively runs on a plain MATLAB version. However, it requires the user to have installed the correct matlab runtime version, for which the installer is provided, along with the FOAMM toolbox files. This application, along with the Matlab runtime installer and the toolbox manual, can be downloaded (for free) from <http://www.eeng.nuim.ie/coer/downloads/>.

The main function of this toolbox is the **main.m** file. **main.m** loads the corresponding hydrodynamic coefficients (which need to be provided as for the FDI toolbox), defines the different modes and options of the application (using the **Options** structure), and calls the **FOAMM** routine (file containing the identification algorithm).

As previously explained for the FDI toolbox, FOAMM also allows the user to choose the desired frequency range over which to perform the parametric approximation. Additionally, due to the fact that FOAMM is based on *moment-matching* methods, the resulting model *exactly* matches the target frequency response at a set of user-selected frequencies: the steady-state response of the obtained finite-order parametric model, and that of the target system, are exactly the same at the selected interpolation frequency points. This effectively helps to retain key underlying characteristics of the target system, such as, for example, the behaviour at the resonant frequency of the floating structure (i.e. the \mathcal{H}_∞ -norm of the system). Thus, a sensible choice of such interpolation frequencies can help to obtain an accurate low order parametric approximation of the target system.

Another noteworthy feature of FOAMM is that it not only considers an automatic order determination method, but three different identification procedures can be selected using **Op-**

⁶The approximation accuracy threshold for BEMIO is defined from 0 to 1, being 1 a 100% accurate approximation.

tions.Method. The first option is the *manual* identification method, where the user selects the desired interpolation frequencies, inherently determining the final order of the model (i.e. twice the number of interpolation frequencies selected). For the second identification method, the user selects the desired order, and the toolbox optimises the position of the interpolation frequencies. Finally, in the last method, FOAMM linearly increases the order of the model and optimises the position of the matching frequencies, until the accuracy thresholds (defined in **Options.ThresRel** and **Options.ThresAbs**) are fulfilled. For the last two methods, where the program optimises the location of the set of interpolation frequencies, FOAMM allows the user to pre-define a partial subset of these interpolation points. By way of example, if the user desires a model of order 4 (i.e. two interpolation frequencies), and explicitly defines one of the interpolation frequencies to be the resonant frequency of the device, FOAMM only optimises the location of the remaining free frequency.

The choice of both the frequency range and the interpolation frequencies can be carried out by explicitly pre-defining such values in the code itself, via a graphical interface (as in the FDI toolbox), or using MATLAB's command window. This can be changed using **Options.FreqRangeChoice** and **Options.FreqChoice**, as shown in (Peña-Sanchez, 2019). Additionally, if μ_∞ is not provided by the user, the application will automatically calculate its value using Ogilvie's relations (see Section 2).

Once **FOAMM** finishes the optimisation, the matrices of the SS representation of the selected target system are returned. As for the other two toolboxes, FOAMM is distributed along with an example case study in order to ease the understanding of the different options of the toolbox, and show how the hydrodynamic data should be pre-defined.

In contrast to the other two previously described toolboxes, FOAMM has an option to identify a parametric model of the complete force-to-motion dynamics of the device under analysis (and not only the radiation force subsystem). This feature can be easily activated using the **Options.Mode** flag. We note that, as shown in (Faedo et al., 2018), when identifying the force-to-motion dynamics, the dimension (order) of the final SS of the WEC is usually lower than if the radiation impulse response is identified separately, and later embedded into Cummins' equation (for the same approximation quality).

Prony's method

As mentioned in the introduction, even though there is no specific toolbox developed tackling Prony's method (in wave energy systems), such a strategy is included in this comparison since it is broadly used in the community. Regarding the algorithm, it can be coded by the user (as performed for the current comparison), or the MATLAB built-in function **prony.m** can be used. Finally, the accuracy of the obtained parametric model will depend (mostly) on the definition of $k(t)$.

TEST CASES

For the sake of completeness, three different geometries are considered for the comparison performed in this study, as depicted in Fig.1. The first geometry (**G1**), shown in Fig.1(a), is a CETO-like device (Fiévez and Sawyer, 2015), while a device similar to the float of the OPT WEC (Mekhiche and Edwards, 2014) is chosen for the second geometry (**G2**), as illustrated in Fig.1(b).

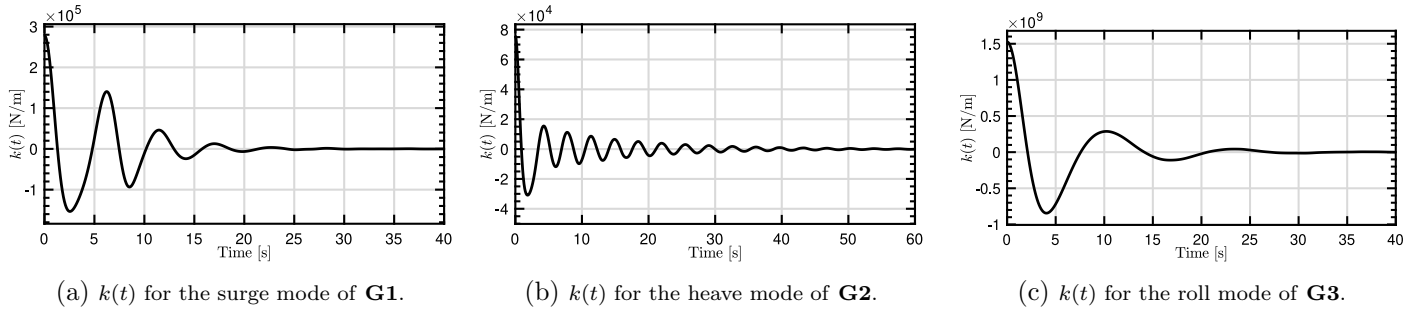


Fig. 3 Radiation impulse response kernel $k(t)$ for the different geometries considered in the comparison.

Finally, the last geometry considered for the comparison (**G3**) is chosen to be a vessel⁷, as shown in Fig.1(c).

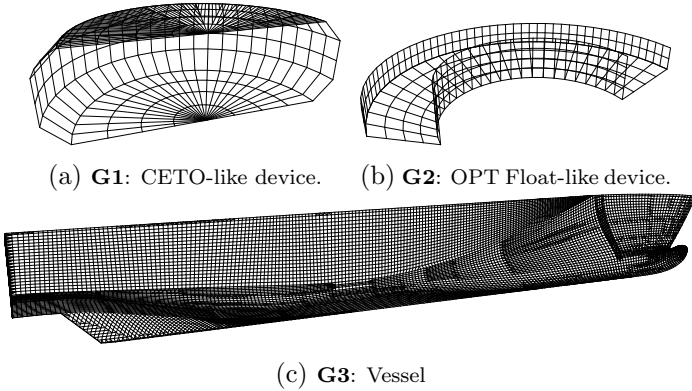


Fig. 1 The low-order meshes of the considered geometries (only half of the meshes is shown, since symmetry is assumed).

Table 1 contains information about the dimension of the geometries. It should be noted that, for **G1** and **G2**, the dimensions are the same as in (Penalba et al., 2017).

Table 1 Dimension of the geometries analysed.

G1	Diameter	17 m
	Height	6 m
G2	Outer diameter	9.5 m
	Inner diameter	6 m
	Draft	2.25 m
G3	Length	170 m
	Draft	20 m
	Width	30 m

The target frequency responses of the different geometries considered are shown in Fig.2. As depicted in Fig.2(a), the frequency response of **G1** is multi-modal, hence increasing its complexity when it comes to parametric approximation (Faedo et al., 2018). Regarding **G2**, the device has been chosen due to the underlying

⁷The mesh used to generate the hydrodynamic coefficients has been obtained from <https://www.friendship-systems.com/news/2017/online-demo-ropax-vessel-from-holiship/>.

complexity when identifying a device with a moon-pool, as depicted in Fig.2(b). Finally, even though the frequency response of **G3** appears to be “simpler” than those of the previous cases, and since the applications/methods under comparison are not exclusively used for WEC systems, evaluating their capabilities to approximate a vessel’s radiation force frequency response is also within the scope of this study.

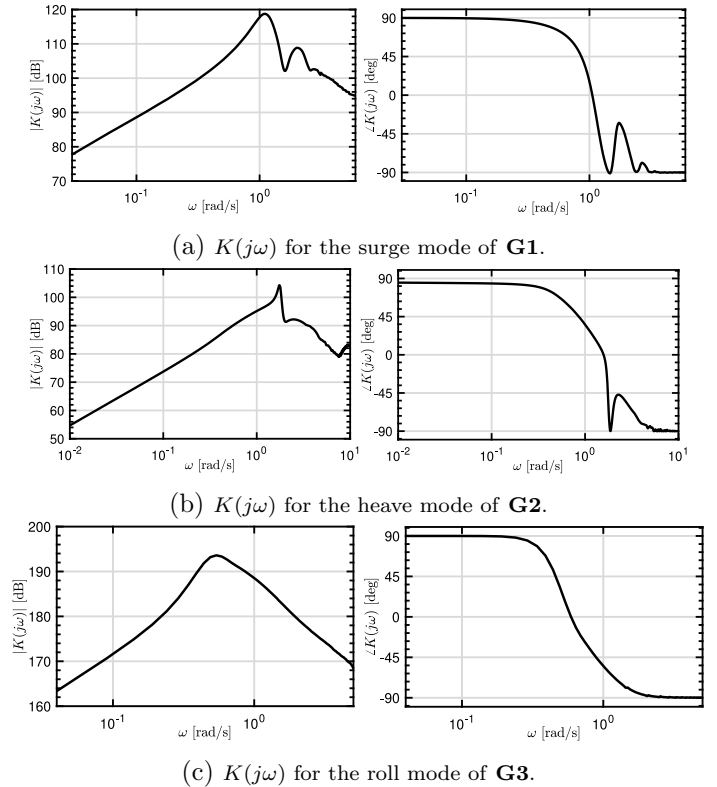


Fig. 2 Frequency response of the radiation kernels of the geometries considered for the study.

Finally, Fig.3 depicts the impulse response function $k(t)$ for each of the three different geometries. It can be seen how, for **G1** and **G3**, the impulse response function fades out to 0 in approximately 30 [s], while $k(t)$ for **G2** keeps oscillating up to around 50 [s]. Additionally, one can notice from the oscillations of each different $k(t)$ signals shown that **G3** has the slowest dynamics, followed by **G1** and **G2**.

COMPARISON RESULTS

In this section, the results obtained by the different identification toolboxes are discussed. First, an analysis is carried out on how the different options of the applications affect the results obtained. Finally, a summary of the obtained results, along with a comparison of the results obtained by the different toolboxes, is provided. It should be noted that the accuracy of the identified models is given in terms of the Normalised Root Mean Square Error (NRMSE), which is defined as follows:

$$\text{NRMSE}_f = \sqrt{\frac{\sum (f - \tilde{f})^2}{\sum f^2}}, \quad (9)$$

where \tilde{f} stands for the approximated f , and f represents the target frequency response $K(j\omega)$ or the impulse response $k(t)$, depending on the context.

FDI toolbox

The first step when using the FDI (and FOAMM) toolbox is choosing the frequency range where the approximation is computed. However, since such an option is not included in BEMIO, the frequency range is set to completely cover the vector of frequencies utilised to compute the hydrodynamic coefficients using BEM solvers, which varies for the different geometries (Fig.2).

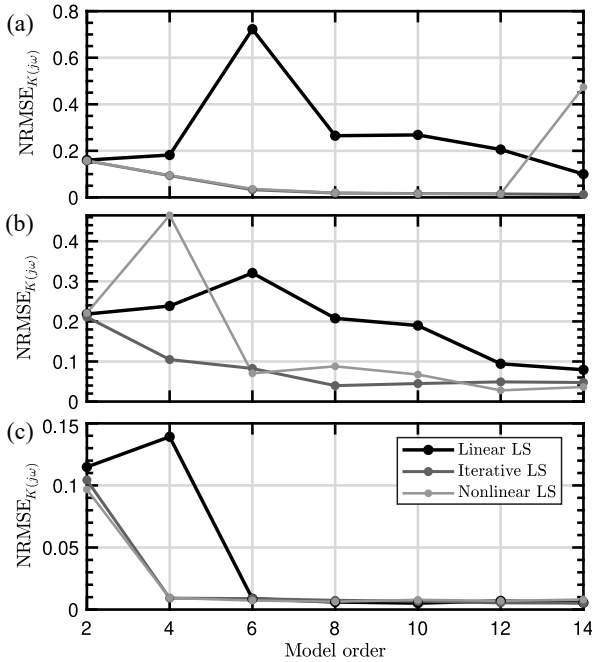


Fig. 4 NRMSE $_{K(j\omega)}$ for different model orders using the different identification methods of the FDI toolbox, when considering **G1** (a), **G2** (b) and **G3** (c).

Fig.4 depicts the accuracy of the obtained approximating model for the different identification (optimisation) methods contained in the FDI toolbox, and different model orders, for the three geometries under analysis. It can be appreciated that the linear LS methods gives, overall, the worst results. Additionally, even though for **G3**, represented in Fig4(c), both the iterative and non-linear LS methods give similar results, the iterative LS method obtains more reliable results than the nonlinear LS, as shown

for **G1** and **G2**. Therefore, the iterative LS is the identification method used (for FDI) in the reminder of this comparison⁸.

BEMIO toolbox

As mentioned in Section 3, since BEMIO has no other (user-defined) option than the fitting accuracy threshold and the maximum model order, the quality of the obtained model depends (mostly) on the definition of $k(t)$. Thus, as shown in Fig.5, the smaller the time-step (dt) used to define $k(t)$, the more accurate the obtained approximation. Since the accuracy improvement is not significant when reducing the time-step to less than 0.04 [s], this time-step is selected for the definition of $k(t)$ ⁹.

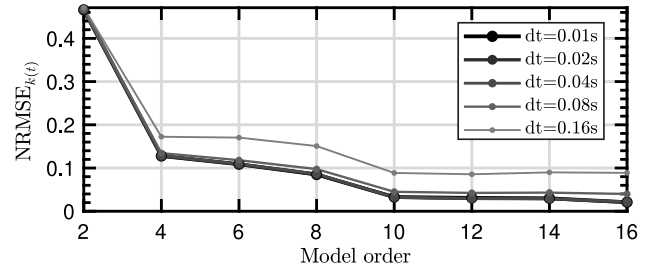


Fig. 5 NRMSE $_{k(t)}$ for different time-steps, when considering **G2**.

FOAMM toolbox

For the FOAMM toolbox, the manual method is used to identify the parametric models given that, as explained in (Peña-Sanchez et al., 2019), a wise selection of the interpolation frequencies improves the quality of the obtained models. The set of interpolation points for **G1** ($\{1\}$, $\{1, 2\}$ and $\{1, 2, 2.5\}$ [rad/s]), and **G2** ($\{1.75\}$, $\{1.75, 0.8\}$ and $\{1.75, 0.8, 2.6\}$ [rad/s]), are selected following the criteria developed in (Faedo et al., 2018) and (Peña-Sanchez et al., 2018), respectively. Regarding the frequencies selected for **G3**, since the frequency response is dynamically simpler than for the other two cases, a single interpolation point is chosen at $\omega = 0.55$ [rad/s]. Note that, for model orders higher than 6 (or 2 for **G3**, since a unique frequency is specified), FOAMM optimises the position of the remaining frequencies.

Comparison

As shown in Fig.6, the results obtained by FOAMM and FDI are similar for the totality of the analysed cases. Additionally, both produce a lower NRMSE $_{K(j\omega)}$ than BEMIO and Prony's method, which stems from the fact that both FDI and FOAMM parameterise radiation forces using a frequency-domain error minimisation criterion. We also note that the results obtained by applying Prony's method are the least accurate, obtaining (for a 10th order system for **G3**) a (Lyapunov) unstable model, which is not particularly useful as a parametric approximation.

It is noteworthy that the results obtained by Prony's method do not improve when considering the error on the obtained parametric model impulse response $k(t)$, as can be appreciated from Fig.7. This is not the case for the BEMIO toolbox, which obtains (overall, of among the different methods) the lowest

⁸Which is also recommended by Pérez and Fossen (2009)

⁹Note that this also applies to $k(t)$ used for the Prony's method.

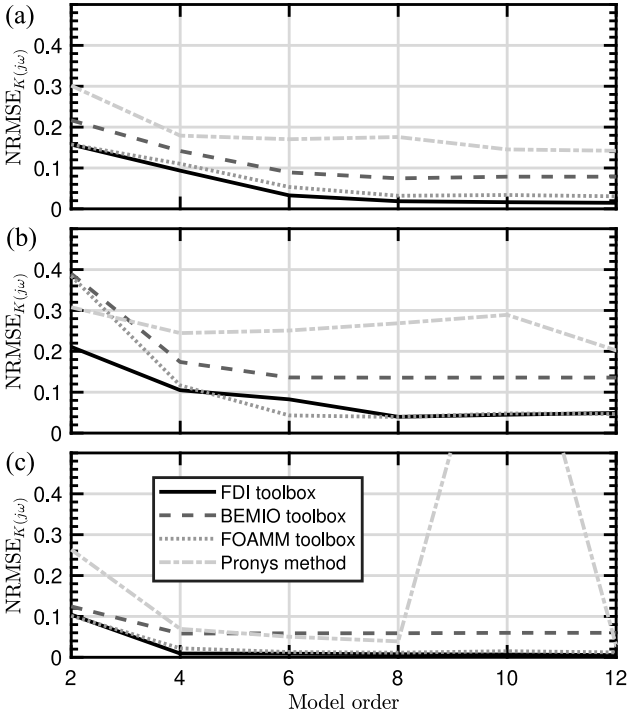


Fig. 6 $\text{NRMSE}_{K(j\omega)}$ for different model orders and toolboxes, when considering **G1** (a), **G2** (b) and **G3** (c).

$\text{NRMSE}_{k(t)}$. Regarding the FDI and FOAMM toolboxes, the obtained $\text{NRMSE}_{k(t)}$ are indeed similar to the $\text{NRMSE}_{K(j\omega)}$ shown in Fig.6, for the three geometries. However, one could notice that, while in Fig.6 all the parametric models obtained using the FDI toolbox give consistent results, there are some cases where the obtained $\text{NRMSE}_{k(t)}$ is peculiarly high. This has been already reported in (Pérez and Fossen, 2009), where the authors specify that if the order of the proposed model is too large, there will be over-fitting and therefore, the approximation error will increase. Additionally, it should be noted that, the results obtained by both FDI and FOAMM could be improved by selecting a different frequency range (Peña-Sánchez et al., 2018).

Radiation subsystem physical properties

The parametric models for the radiation force subsystem should respect the underlying physical properties of radiation forces. To be precise, the parametric model should be input-output stable, passive, strictly proper, and should have zeros at the origin (the interested reader is referred to (Pérez and Fossen, 2008) for comprehensive discussion of these properties). Regarding the stability of the parametric models, apart from the previously mentioned cases where, due to numerical inaccuracies, the obtained model is unstable (twice for FDI and once for Prony’s method), the identified approximating models are, in general, input-output stable.

As discussed in (Faedo et al., 2018b), passivity is an important property of radiation systems. It should be noted that even though none of the toolboxes explicitly guarantee passivity, BEMIO and FOAMM are the ones obtaining the higher number of passive models (around 75% of the cases). In fact, for the case of the FOAMM toolbox, a particular selection of the interpolation frequencies can be used to obtain a passive model, since there exist a relation between the interpolation frequencies, the spectral

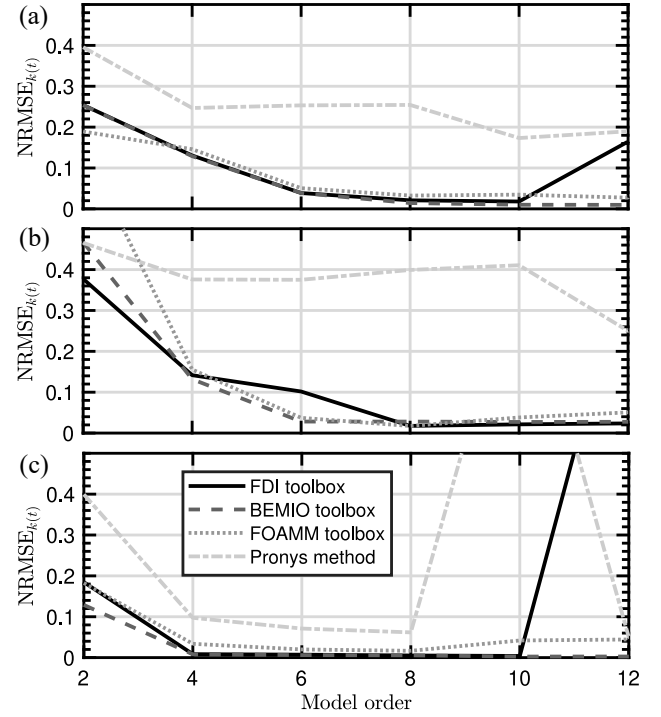


Fig. 7 $\text{NRMSE}_{k(t)}$ for different model orders and toolboxes, when considering **G1** (a), **G2** (b) and **G3** (c).

zeros of a system, and its passivity (Faedo et al., 2018b).

Finally, it should be noted that, in contrast to FDI, FOAMM or Prony’s method, the parametric models obtained using the BEMIO toolbox contain a non-zero feed-through matrix, which directly implies that the computed model is not strictly proper.

CONCLUSIONS AND FUTURE WORK

This paper presents a review of the available applications/methods to compute parametric models that approximate the radiation force convolution term of Cummins’ equation. Additionally, a critical comparison between the different available software (and Prony’s method) is provided, both in the time- and frequency-domain. The comparison is carried out using not only two geometrically complex wave energy converters, but also using a vessel-shaped hull, to further extend the analysis to more general marine applications.

The paper shows that the FDI and FOAMM toolboxes can obtain accurate results in both the frequency- and time-domain. On the contrary, the results obtained by BEMIO are accurate in the time-domain (which stems from the fact that BEMIO identifies the parametric model using time-domain data), while, in the frequency-domain, the approximating systems obtained are not as accurate as those computed by FDI or FOAMM.

Additionally, it is also shown that, while none of the toolboxes is able to enforce all the physical properties of the radiation force kernel, FOAMM is the only application able to consistently respect the full set of physical properties (apart from passivity, which is respected in more than 75% of the analysed cases). We note that, as introduced by Faedo et al. (2018b), passivity can be *enforced* using a moment-based approach. The developers of FOAMM plan to include this theoretical modification in an up-

coming version of the application.

Finally, future work will compare how the quality of the obtained radiation force parametric models impacts on the input-output (force-to-motion) behaviour of the floating bodies.

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REFERENCES

- Astolfi, A (2010). “Model reduction by moment matching for linear and nonlinear systems”, *IEEE Transactions on Automatic Control*, Vol. 55, No 10, pp. 2321–2336, 2010.
- AQUA, ANSYS (2013). “Version 15.0”, *ANSYS Inc., Canonsburg, PA, USA*, 2013.
- Babarit, A, Clément, A.H, Ruer, J, Tartivel, C (2006). “SEAREV: A fully integrated wave energy converter”, *In proceedings of the Offshore Wind and other marine renewable Energies in Mediterranean and European Seas, OWEMES, Rome, Italy*, 2006.
- Babarit, A and Delhommeau, G (2015). “Theoretical and numerical aspects of open source BEM solver NEMOH”, *In proceedings of the 11th European Wave and Tidal Energy Conference, EWTEC, Nantes, France*, 2015.
- Cummins, W E (1962). “The impulse response function and ship motions”, *Die Schiffstechnik, Heft*, Vol 47, pp. 101–109, 1962.
- De Prony, G (1795). “Essai expérimental et analytique: sur les lois de la dilatabilité de fluides élastique et sur celles de la force expansive de la vapeur de l’alkool, a différentes températures”, *Journal de l’école polytechnique*, Vol 1, No 22, pp. 24–76, 1795.
- Faedo, N, Olaya, S, Ringwood, J V (2017). “Optimal control, MPC and MPC-like algorithms for wave energy systems: An overview”, *IFAC Journal of Systems and Control*, Vol 1, pp. 37–56, 2017.
- Faedo, N, Peña-Sanchez, Y, Ringwood, J V (2018). “Finite-order hydrodynamic model determination for wave energy applications using moment-matching”, *Ocean Engineering*, Vol 163, pp. 251–263, 2018.
- Faedo, N, Peña-Sanchez, Y, Ringwood, J (2018b). “Passivity preserving moment-based finite-order hydrodynamic model identification for wave energy applications”, *In proceedings of the 3rd International Conference on Renewable Energies Offshore, RENEW, Lisbon, Portugal*, 2018.
- Faedo, N, Peña-Sanchez, Y, Ringwood, J (2019). “Parameterisation of Radiation Forces for Multiple Degree-of-Freedom Wave Energy Converter Using Moment-Matching”, *In proceedings of the 29th Ocean and Polar Engineering Conference, ISOPE, Honolulu, Hawaii*, 2019.
- Falnes, J (2002). “Ocean waves and oscillating systems: linear interactions including wave-energy extraction”, *Cambridge University Press*.
- Fiévez, J, Sawyer, T (2015). “Lessons learned from building and operating a grid connected wave energy plant”, *In proceedings of the 11th European Wave and Tidal Energy Conference, EWTEC, Nantes, France*, 2015.
- Grilli, A.R, Merril, J, Grilli, S.T, Spaulding, M.L (2007). “Experimental and numerical study of spar buoy-magnet/spring oscillators used as wave energy absorbers”, *In proceedings of the 17th International Offshore and Polar Engineering Conference, ISOPE, Lisbon, Portugal*, 2007.
- Kristiansen, E, Hjulstad, Å, Egeland, O (2005). “State-space representation of radiation forces in time-domain vessel models”, *Ocean Engineering*, Vol 32, No 17, pp. 2195–2216, 2005.
- Mekhiche, M, Edwards, K.A (2014). “Ocean power technologies powerbuoy®: System-level design, development and validation methodology”, *In proceedings of the 2nd Marine Energy Technology Symposium, METS, Seattle, US*, 2014.
- Newman, J N, Lee, C H (2002). “Boundary-element methods in offshore structure analysis”, *Journal of Offshore Mechanics and Arctic Engineering*, Vol 124, No 2, pp. 81–89, 2002.
- Ogilvie, T F (1964). “Recent progress toward the understanding and prediction of ship motions”, *In proceedings of the 5th Symposium on Naval Hydrodynamics, Bergen, Norway*, Vol 1, pp. 2–5, 1964.
- Penalba, M, Kelly, T, Ringwood, J V (2017). “Using NEMOH for modelling wave energy converters: A comparative study with WAMIT”, *In proceedings of the 12th European Wave and Tidal Energy Conference, EWTEC, Cork, Ireland*, 2017.
- Peña-Sanchez, Y, Faedo, N, Penalba, M, Giorgi, G, Mérigaud, A, Windt, C, García Violini, D, L, Wang, Ringwood, J (2019). “Finite-Order hydrodynamic Approximation by Moment-Matching (FOAMM) toolbox for wave energy applications”, *Submitted to the 13th European Wave and Tidal Energy Conference, EWTEC, Naples, Italy*, 2019.
- Peña-Sanchez, Y, Faedo, N, Ringwood, J (2018). “Hydrodynamic model fitting for wave energy applications using moment-matching: a case study”, *In proceedings of the 28th Ocean and Polar Engineering Conference, ISOPE, Sapporo, Japan*, 2018.
- Pérez, T, Fossen, T I (2008). “Time-vs. frequency-domain identification of parametric radiation force models for marine structures at zero speed”, *Modelling, Identification and Control*, Vol 29, No 1, pp. 1–19, 2008.
- Pérez, T, Fossen, T I (2009). “A matlab toolbox for parametric identification of radiation-force models of ships and offshore structures”, *Modelling, Identification and Control*, Vol 30, No 1, pp. 1–15, 2009.
- Scarciotti, G, Astolfi, A (2016). “Model reduction by matching the steady-state response of explicit signal generators”, *IEEE Transactions on Automatic Control*, Vol. 61, No 7, pp. 1995–2000, 2016.
- Taghipour, R, Pérez, T, Moan, T (2008). “Hybrid frequency-time domain models for dynamic response analysis of marine structures”, *Ocean Engineering*, 2017.
- Tom, N, Lawson, M, Yu, Y-H (2015). “Demonstration of the recent additions in modelling capabilities for the WEC-Sim wave energy converter design tool”, *In the proceedings of the 34th International Conference on Ocean, Offshore and Arctic Engineering, OMAE, Newfoundland, Canada*, 2015.
- Yu, Y-H, Lawson, M, Ruehl, K, Michelen, C (2014). “Development and demonstration of the WEC-Sim wave energy converter simulation tool”, *In the proceedings of the 2nd Marine Energy Technology Symposium, METS, Seattle, WA*, 2014.