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Information Theory

Outage minimisation in wireless relay networks with delay constraints and causal channel feedback

James C. F. Li and Subhrakanti Dey*

ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN), Department of Electrical and Electronic Engineering, University of Melbourne, Victoria 3010, Australia

SUMMARY

Motivated by delay-sensitive information transmission applications, we solve an optimal power allocation problem with a K-block delay constraint on data transmission using a cooperative relay network assuming a block fading channel model. Channel information is fed back to the transmitter only in a causal fashion, so that the optimal power allocation strategy is only based on the current and past channel gains. We consider the two simplest schemes for information transmission using a three node (a source, a relay and a destination) relay network, namely the amplify and forward (AF) and decode and forward (DF) protocols. We use a dynamic programming (DP) based methodology to solve a (K-block delay constrained) general expected cost optimisation problem with a short term (over K blocks) sum power (total transmission power of the source and the relay) constraint. By specialising the cost function appropriately, we solve the delay constrained outage minimisation problem in this paper. We also propose a simple but sub-optimal power allocation scheme based on a high signal to noise ratio (SNR) approximation, which is computationally much less demanding than the DP-based optimal method. Extensive numerical results are presented for Rayleigh and Rician fading channels, including results demonstrating the performance gain obtained by optimally allocating the (sum of source and relay) power to the different blocks as opposed to equally distributing the total power across all blocks. The accuracy of the high SNR approximation based power allocation scheme is also illustrated. Copyright © 2009 John Wiley & Sons, Ltd.

1. INTRODUCTION

Relay networks were first proposed in Reference [2], and capacity bounds for such networks were extensively studied by Cover *et al.* [3] in the 1970s. Fostered by the increasing importance of *ad hoc* and wireless sensor networks, of late a great amount of exciting research has gone into relay networks. Data transmission with relay(s) not only raises the achievable rate of information transmission but also provides alternative routes when direct transmission is resource-consuming or totally infeasible. For wireless networks, the outage performance, defined as the probability of the instantaneous mutual information falling below a basic rate threshold, has been shown to improve dramatically due to the diversity gain offered by relay networks. This type of diversity has been named as cooperative diversity [4], which, as the term implies, is provided by the cooperation among various communication units. The authors of Reference [4] suggested (amongst others) two simple relay schemes: amplify and forward (AF) and decode and forward (DF). Various other protocols such as compress-and-forward, estimate-and-forward etc. have been suggested in the literature as well. For a survey of these and other possible relaying protocols and their capacity results, see Reference [5]. There has been a significant number of studies on optimal power allocation in wireless relay networks over fading channels. Optimal power allocation for information theoretic achievable rate maximisation has

* Correspondence to: Subhrakanti Dey, ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN), Department of Electrical and Electronic Engineering, University of Melbourne, Victoria 3010, Australia. E-mail: sdey@ee.unimelb.edu.au

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been studied in detail in Reference [6] (see also references therein), whereas Reference [7] has studied the problem of optimal power allocation for outage probability minimisation in relay networks. There have been parallel studies on optimal power allocation for signal-to-noise ratio maximisation for single and multiple relay networks in Reference [8] amongst many others.

While optimal power allocation can maximise achievable rates or minimise outage probabilities specially with channel state information (CSI) at the transmitter (and receiver), it is also critical to consider other important quality-of-service (QoS) criteria such as delay in designing a reliable wireless system, specially for voice/video communication as well as delay sensitive data communication applications in wireless *ad hoc*/sensor networks. In these applications, using lengthy codewords to capture the ergodicity of the fading channel and achieve the maximum expected throughput is not a useful approach. This has led researchers to consider various different notions of capacity of fading channels other than ergodic capacity—such as outage capacity and delay-limited capacity etc. A comprehensive survey on various capacity notions for fading channels can be found in Reference [9]. Delay constrained capacity optimisation for wireless channels has also been a fruitful area of research. Following the seminal work of Reference [10] where the average queuing delay was minimised by optimal power and rate control for data transmission over wireless fading channels, there have been further advances on optimal power and rate control for minimal average delay in Reference [11]. Similarly, optimal power allocation for capacity maximisation with coding delay using causal channel information was investigated using dynamic programming (DP) techniques in Reference [12], and throughput maximisation with both coding and queuing delay constraints has been studied in Reference [13]. More recently, a stochastic power allocation method for expected capacity maximisation with a finite coding delay constraint has been investigated in Reference [14]. See also references within these papers.

While there is a rich literature on optimal power control and scheduling with delay or deadline constraints for traditional wireless networks, the same cannot be said for cooperative wireless networks. Delay constraints have been considered as embedded in the concept of 'effective capacity' for cooperative networks and optimal power allocation for such problems has been derived in Reference [15]. Some preliminary investigations into cooperative transmission with queuing delay constraints have been made in Reference [16]. However, there has been no in-depth study on power allocation for cooperative transmission with coding delay constraints where data transmission takes place over a finite number of blocks. This is indeed the focus of our current paper. Similar to Reference [12], we also impose the practical constraint that only causal channel information is available to the transmitting nodes. In a companion paper [1], we have studied optimal power allocation for delay constrained expected achievable sum rate maximisation for a three node relay network with causal channel information using DP-based methods. Motivated by the high computational demands of the DP-based methods, in Reference [1], we have also derived computationally simple but sub-optimal power allocation laws using high and low signal to noise ratio (SNR) approximations. In the current paper, we focus our attention to the corresponding outage minimisation problem. This is motivated by the fact that for delay limited applications, where the sum of instantaneous mutual information over K fading blocks can be treated as a random variable, outage probability (or the related notion of outage capacity or capacity *versus* outage) becomes an important metric, as opposed to the expected achievable rate. Note that the outage probability here is defined as the probability that the achievable sum rate over the number of coding blocks falls below a basic required rate. In particular, the main contribution of this paper is to address an optimal power allocation algorithm to minimise the outage probability for a three node relay network with a pre-specified delay constraint (i.e. number of coding blocks) and a short-term (also known as peak) constraint on the total power (sum of source and relay power) over all blocks where only causal CSI is available.

In summary, the main novel contributions of the paper are as following:

- We provide explicit expressions for the outage minimising optimal power allocation results for the special case of a single fading block ($K = 1$) for both AF and DF protocols with peak power constraints. In particular, we provide explicit expressions for the optimal power allocation law in the AF case using the exact (non-concave) expression for the mutual information rather than using the moderate-to-high-SNR approximation which is often used to make the optimisation problem convex. To the best of our knowledge, this complete characterisation of the outage minimising power allocation law has not been appeared before in the literature.
- In the multi-block case ($K > 1$), the optimal outage minimising power allocation law needs to be derived by using DP-based techniques. Note that DP is an important optimisation tool that has been used by many authors including those of Reference [12], where a delay constrained capacity optimisation problem was studied for

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the direct transmission case. The novelty of our contribution does not lie in the mere use of DP-based methods (in the same sense that the mere use of a convex optimisation tool for solving convex optimisation problems does not constitute anything novel), but the readers are reminded that the results obtained by employing the DP methods serve an important purpose. In particular, they provide a benchmark for the outage performances of the AF and DF schemes under delay and causality constraints, which are then compared with the performance of the standard direct transmission scheme under similar constraints. We illustrate *via* simulation results that cooperative transmission offers significant advantages over the direct transmission even with finite coding delay. The relative performances of the AF and DF schemes are also illustrated for varying available power levels and number of coding blocks. Furthermore, the computational complexity of implementing the DP algorithm in discrete time is particularly high due to the increased dimensionality of the channel space in the cooperative case. This fact also highlights the difficulty we faced in obtaining these DP-based simulation results, which itself posed a non-trivial challenge.

- We also illustrate the benefits from using the optimal power allocation technique using DP methodologies in comparison with a sub-optimal technique that uses equal (sum of source and relay) power for all blocks. It is seen that in the low power regime, these benefits are really substantial whereas in the high power regime, they are not so substantial. The effect of moving the relay from close to the source to close to the destination is also illustrated.
- Motivated by the high computational demands of the discrete time DP- based schemes (incurred due to discretisation of continuous variables in implementation of the DP algorithm), we provide simple but novel sub-optimal power allocation schemes for outage minimisation in the high SNR regime, which are seen to be reasonably accurate (as compared against the benchmark provided by the DP-based performance) *via* simulation studies.

The rest of the paper is organised as follows. Section 2 describes the general background and the network model and underlying assumptions used through the paper. In Section 3, we study a case where there is only one block available for the transmission and present a summary of the relevant results of optimal power allocation for the single block case, some of which exist in various forms in the literature. DP-based algorithms are then introduced to solve the multiple-block outage minimisation problem in Section 4. We propose a high SNR approximation based simple sub-optimal power allocation method for outage minimisation in Section 5. Extensive numerical results are provided in Section 6 to illustrate the performance of these various algorithms, followed by concluding remarks in Section 7.

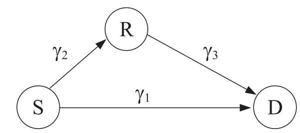

Figure 1. A relay network with random fading channels.

2. PROBLEM FORMULATION

We consider a three node cooperative relay network where the source sends data to the destination with the help of a relay node, as shown in Figure 1, and the relay does not produce its own data. It is assumed that the channel gains for all the three links—Source–Destination (S–D), Source–Relay (S–R), and, Relay–Destination (R–D)—follow a typical block fading model. In this model, time is divided into blocks where each block spans a codeword of a large number of transmitted symbols. Within each such block, the channels are constant but they change from one block to another in an independent and identically distributed fashion. We assume that the links are statistically mutually independent but not necessarily identically distributed.

In this paper, we consider the (coding) delay constrained case where data transmission takes place over a finite number (K) of such fading blocks. It is assumed that the delay constraint K is known *a priori*, depending on the specific nature of the application scenario. We consider the three simple transmission schemes: direct transmission (DT), AF and DF. In the DT scheme, obviously the source transmits only directly to the destination, while in the AF and DF schemes, the source also uses the relay for data transmission. We assume a half-duplex time division transmission scheme where every transmission block (or fading block) is divided into two halves. During the first half, the source transmits to the relay and the destination. During the second half, the relay transmits to the destination and the source does not transmit. As assumed in Reference [4], in the AF scheme, the relay only amplifies and forwards its received data to the destination. In the DF scheme, the relay first decodes and then forwards the decoded data to the destination. We also assume that there is no queue at the relay node.

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We use γ_1^k, γ_2^k and γ_3^k to denote the channel states of the S–D, S–R and R–D links respectively, during the kth block of data transmission. We use the notations $\gamma^k = (\gamma_1^k, \gamma_2^k, \gamma_3^k)$ and $\gamma^{(k)} = \{\gamma^1, \gamma^2, \ldots, \gamma^k\}$. Clearly, $\gamma^{(k)}$ represents the causal channel state vector including all the CSI until block k (inclusive). It is assumed that the destination node has exact knowledge of $\gamma^{(k)}$ and feeds this information back to the source and the relay nodes through error free feedback channels with negligible delay. Since the source and relay transmission powers in the kth block are allocated based on this causal CSI, we denote them by $P_s^k(\gamma^{(k)})$ and $P_r^k(\gamma^{(k)})$, respectively.

The objective of this paper is to minimise the expectation of a cost function $\mu(x)$ under a short term sum power (sum of source and relay) constraint over K blocks, where x denotes the sum of the achievable rates over these K blocks. Therefore, if the cost function is chosen to be $\mu(x) = -x$, then the optimisation problem boils down to an expected sum (over K blocks) achievable rate maximisation problem under a short term power constraint. On the other hand, if $\mu(x)$ is chosen to be an indicator function which takes value 1 if x falls below a predetermined threshold, the optimisation problem is transformed into an outage minimisation problem. To be precise, this general delay constrained cost minimisation problem with causal feedback is given as

$$\begin{aligned} \min\ & \mathbb{E}_\gamma \left[\mu \left(\sum_{k=1}^{K} C_* \left(P_s^k \left(\gamma^{(k)} \right), P_r^k \left(\gamma^{(k)} \right), \gamma^k \right) \right) \right] \\ \text{s.t.}\ & P_s^k \left(\gamma^{(k)} \right), P_r^k \left(\gamma^{(k)} \right) \geqslant 0,\ \forall k, \\ & \sum_{k=1}^{K} \left(P_s^k \left(\gamma^{(k)} \right) + P_r^k \left(\gamma^{(k)} \right) \right) \leqslant K P_0 \end{aligned} \tag{1}$$

where P_0 can be thought of as an average power constraint per block, and the function $C_*(P_s, P_r, \gamma)$ denotes either the instantaneous mutual information (for the DT case) or the achievable rate (for the AF or the DF case), which are given below:

$$\begin{aligned} C_{\mathrm{DT}} &= \log(1 + \gamma_1 p_s) \\ C_{\mathrm{AF}} &= \frac{1}{2} \log \left(1 + 2\gamma_1 p_s + \frac{4\gamma_2 p_s \gamma_3 p_r}{1 + 2\gamma_2 p_s + 2\gamma_3 p_r} \right) \\ C_{\mathrm{DF}} &= \frac{1}{2} \min\{\log(1 + 2\gamma_2 p_s), \log(1 + 2\gamma_1 p_s + 2\gamma_3 p_r)\} \end{aligned} \tag{2}$$

Note that these various mutual information/achievable rate expressions for the DT, AF and DF cases presented above Equation (2) assume that the relay is operating in the half-duplex mode and in the case of direct transmission, the source transmits during both halves of the transmission slot, as also assumed in Reference [4]. Here $\mathbb{E}_{\boldsymbol{\gamma}}$ denotes the expectation operator with respect to the joint probability density function of the random variables $\{\gamma^1, \gamma^2, \ldots, \gamma^K\}$. In Reference [1], we studied the delay constrained expected sum (over K blocks) achievable rate maximisation problem. Therefore, in what follows, we focus on the delay constrained information outage minimisation problem under a short term sum power constraint, where the above problem formulation (1) is specialised to the choice of the cost function $\mu(x) = 1_{x<KR_0}$, where $\mathbf{1}_{\mathrm{A}}$ is the indicator function taking value 1 if the event A is true, otherwise taking the value 0. Here R_0 can be thought of as the average (per block) basic rate requirement, which the relay network tries to meet in order to support data transmission in a delay constrained situation. A table containing a list of the major notations used in this paper has been provided on page 14 for the convenience of readers.

Remark 1. Note that although we focus on a peak power constraint (summed over the K blocks), one can easily extend the techniques of this paper to the case expected (or average) power constraints, where the peak power constraint in (refequ:CostMinFormulation) is replaced by an expected (over the channel fading process) sum power constraint. Note also that even with the expected sum power constraint, the number of codewords K remains finite, as opposed to the case where $K \to \infty$ and standard outage capacity with long term average power constraints based results can be applied. As a result, the computational complexity of the DP-based methods increases prohibitively (more so than the case of direct transmission studied in Reference [12] due to the increased dimensionality of the channel space) and it is for this reason that we do not study the expected sum power constraints in this paper. We believe that these constraints warrant the use of approximate reduced-complexity DP techniques [17] and will be investigated in future work.

Remark 2. We do not also consider optimising over the time durations of the two half slots in the half-duplex relay protocols. We simply assume that they are equal.

3. SINGLE-BLOCK TRANSMISSION

Before we attempt to solve the above outage minimisation problem mentioned above, it is instructive to look at a simple case when there is only one block available for transmission, that is, $K = 1$. Furthermore, the optimum power allocation results for the single-block case will be also useful in the later sections. Considering the specific choice of the cost function $\mu(x) = 1_{x<KR_0}$ as described in the previous section, the problem (for $K = 1$) translates to the following typical outage probability minimisation problem with a short term power constraint:

$$\begin{aligned} \min_{P_s, P_r} \quad & P(C^*(P_s(\gamma), P_r(\gamma), \gamma) < R_0) \\ \text{s.t.} \quad & P_s(\gamma) + P_r(\gamma) \leqslant P_0 \\ & P_s(\gamma), P_r(\gamma) \geqslant 0 \end{aligned} \tag{3}$$

where due to the fact that $K = 1$ and the fading variables are i.i.d, the dependence of the variables on k has been removed, and $C^*(P_s(\gamma), P_r(\gamma), \gamma)$ can denote any of the instantaneous mutual information/achievable rates for the DT, AF or DF schemes as stated in Equation (2).

It is obvious that the optimal direct transmission policy is to use the full power P_0 in the single block, and the corresponding outage probability is $F_{\gamma_1}(\frac{e^{R_0}-1}{P_0})$ where F_{γ_1} is the cumulative distribution function of γ_1.

For the AF and DF schemes, the basic idea behind solving the outage minimisation problem is discussed in References [7, 18] and is based on the outage minimisation for block fading channels as presented in Reference [19]. Adapting Proposition 3 of Reference [19], the outage minimising power allocation with a short term power constraint for the cooperative case is given by

$$\{P_s^*(\gamma), P_r^*(\gamma)\} = \begin{cases} \{P_s^{sh}(\gamma), P_r^{sh}(\gamma)\} & \gamma \notin \mathcal{U}(R_0, P_0) \\ \{g_s(\gamma), g_r(\gamma)\} & \gamma \in \mathcal{U}(R_0, P_0) \end{cases} \tag{4}$$

where $g_s(\gamma)$ and $g_r(\gamma)$ are arbitrary functions such that $g_s(\gamma) + g_r(\gamma) \leqslant P_0$ with probability 1, $\{P_s^{sh}(\gamma), P_r^{sh}(\gamma)\}$ are the solutions to the instantaneous mutual information maximisation with a peak power constraint problem

$$\begin{aligned} \max \quad & C^*(P_s(\gamma), P_r(\gamma), \gamma) \\ \text{s.t.} \quad & P_s(\gamma) + P_r(\gamma) \leqslant P_0 \\ & P_s(\gamma), P_r(\gamma) \geqslant 0, \end{aligned}$$

and $\mathcal{U}(R_0, P_0)$ is the *outage region* given by $\{\gamma : C^*(P_s(\gamma), P_r(\gamma), \gamma) < R_0\}$. Note that the solution may not be unique due to the arbitrary nature of $g_s(\gamma)$ and $g_r(\gamma)$. Although these optimal power allocation policies for outage minimisation with short term power constraints have been solved for in References [7, 18] for various relaying protocols, the explicit expressions for the power allocation policies are not provided. Therefore we provide them below for the DF and AF cases, for the benefit of the reader.

3.1. DF protocol

It is easy to derive an optimal policy (again, note that it can be non-unique due to the arbitrary nature of the policy within the outage set as mentioned above Equation (4)) for the DF protocol due to the fact that $C_{\text{DF}}(p_s, p_r, \gamma)$ is jointly concave in p_s, p_r, thus rendering the peak power constrained instantaneous mutual information maximisation problem a convex optimisation problem. The optimal peak power constrained instantaneous mutual information maximising allocation policy in this case is given by

$$(P_s^*, P_r^*)_{\text{DF}} = \begin{cases} (P_0, 0) & \gamma_1 > \gamma_2 \\ (P_0, 0) & \gamma_1 \leqslant \gamma_2 \text{ and } \gamma_1 > \gamma_3 \\ \left(\frac{\gamma_3}{\gamma_2+\gamma_3-\gamma_1} P_0, \frac{\gamma_2-\gamma_1}{\gamma_2+\gamma_3-\gamma_1} P_0\right) & \gamma_1 \leqslant \gamma_2 \text{ and } \gamma_1 \leqslant \gamma_3 \end{cases} \tag{5}$$

The maximum achievable rate over a single block under the short term power constraint is then given as

$$R_{\text{DF}} = \begin{cases} \frac{1}{2}\log(1 + 2\gamma_2 P_0) & \gamma_1 > \gamma_2 \\ \frac{1}{2}\log(1 + 2\gamma_1 P_0) & \gamma_1 \leqslant \gamma_2 \text{ and } \gamma_1 > \gamma_3 \\ \frac{1}{2}\log\left(1 + \frac{2\gamma_2\gamma_3}{\gamma_2+\gamma_3-\gamma_1} P_0\right) & \gamma_1 \leqslant \gamma_2 \text{ and } \gamma_1 \leqslant \gamma_3 \end{cases} \tag{6}$$

3.2. AF protocol

Due to the non-concavity of the AF mutual information expression $C_{\text{AF}}(p_s, p_r, \gamma)$, it is more complicated to compute the optimal policy in this case. We can reformulate the optimisation problem for the AF case as the following:

$$\begin{aligned} & \max \gamma_1 P_s + \frac{\gamma_2 P_s \gamma_3 P_r}{\frac{1}{2} + \gamma_2 P_s + \gamma_3 P_r} \\ & \text{s.t.} \quad P_s + P_r \leqslant P_0, P_s, P_r \geqslant 0 \end{aligned} \tag{7}$$

The corresponding Lagrangian function is

$$L(P_s, P_r, \lambda) = \gamma_1 P_s + \frac{\gamma_2 P_s \gamma_3 P_r}{\frac{1}{2} + \gamma_2 P_s + \gamma_3 P_r} - \lambda(P_s + P_r - P_0)$$

and

$$\frac{\partial L}{\partial P_s} = \gamma_1 + \frac{\gamma_2\gamma_3 P_r\left(\frac{1}{2}+\gamma_3 P_r\right)}{\left(\frac{1}{2}+\gamma_2 P_s+\gamma_3 P_r\right)^2} - \lambda \tag{8}$$

$$\frac{\partial L}{\partial P_r} = \frac{\gamma_2\gamma_3 P_s\left(\frac{1}{2}+\gamma_2 P_s\right)}{\left(\frac{1}{2}+\gamma_2 P_s+\gamma_3 P_r\right)^2} - \lambda \tag{9}$$

Note that we could categorise the optimal power allocation policy into two situations: direct transmission ($P^*_{s\,\mathrm{AF}} > 0$, $P^*_{r\,\mathrm{AF}} = 0$); and relay transmission ($P^*_{s\,\mathrm{AF}}$, $P^*_{r\,\mathrm{AF}} > 0$) which are described below:

- *Direct Transmission*:
  In this case, by using the Karush–Kuhn–Tucker (KKT) necessary conditions, we obtain

$$\lambda = \gamma_1 \geqslant \frac{\gamma_2\gamma_3 P^*_{s\,\mathrm{AF}}}{\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}} \Rightarrow (\gamma_3-\gamma_1)\gamma_2 P^*_{s\,\mathrm{AF}} \leqslant \frac{\gamma_1}{2}$$

  This condition holds only when either $\gamma_1 \geqslant \gamma_3$, or, $\gamma_1 < \gamma_3$ and $P_0 \leqslant \frac{\gamma_1}{2\gamma_2(\gamma_3-\gamma_1)}$. The corresponding capacity in this case is $\frac{1}{2}\log(1+2\gamma_1 P_0)$.
- *Relay Transmission*:
  Using the KKT conditions, it can be derived from Equations (8) and (9) that for this case,

$$\begin{aligned}\lambda &= \gamma_1 + \frac{\gamma_2\gamma_3 P^*_{r\,\mathrm{AF}}\left(\frac{1}{2}+\gamma_3 P^*_r \mathrm{AF}\right)}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2}\\ &= \frac{\gamma_2\gamma_3 P^*_{s\,\mathrm{AF}}\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}\right)}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2} \text{ also}\end{aligned} \tag{10}$$

  In order for the second equality to hold, γ_1 has to be less than γ_3, otherwise

$$\begin{aligned}\mathrm{LHS} &= \gamma_1 + \frac{\gamma_2\gamma_3 P^*_{r\,\mathrm{AF}}\left(\frac{1}{2}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2}\\ &> \gamma_1\\ &= \gamma_1 \frac{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2}\\ &> \gamma_1 \frac{\gamma_2 P^*_{s\,\mathrm{AF}}\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}\right)}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2}\\ &\geqslant \frac{\gamma_3\gamma_2 P^*_{s\,\mathrm{AF}}\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}\right)}{\left(\frac{1}{2}+\gamma_2 P^*_{s\,\mathrm{AF}}+\gamma_3 P^*_{r\,\mathrm{AF}}\right)^2} = \mathrm{RHS}\end{aligned}$$

  which is not possible when $P^*_{s\,\mathrm{AF}}$, $P^*_{r\,\mathrm{AF}} > 0$. Rearranging Equation (10), (since $P^*_{s\,\mathrm{AF}}$, $P^*_{r\,\mathrm{AF}} > 0$ in this case) $P^*_{s\,\mathrm{AF}}$ can be expressed as a function of $P^*_{r\,\mathrm{AF}}$:

$$\begin{aligned}P^*_{s\,\mathrm{AF}} &= f(P^*_{r\,\mathrm{AF}})\\ &= \frac{-b(P^*_{r\,\mathrm{AF}}) + \sqrt{b(P^*_{r\,\mathrm{AF}})^2 - 4ac(P^*_{r\,\mathrm{AF}})}}{2a}\end{aligned}$$

  where

$$\begin{aligned}a &= (\gamma_3-\gamma_1)\gamma_2^2 > 0\\ b(P_r) &= -\left(2\gamma_1\gamma_2\gamma_3 P_r + \gamma_1\gamma_2 - \frac{1}{2}\gamma_2\gamma_3\right)\\ c(P_r) &= -\left[(\gamma_1+\gamma_2)\gamma_3^2 P_r^2 + \left(\gamma_1\gamma_3 + \frac{1}{2}\gamma_2\gamma_3\right) P_r + \frac{\gamma_1}{4}\right] < 0\end{aligned}$$

  It can be shown that the function $f(P_r)$ is monotonically increasing in P_r. Therefore, the minimum sum power necessary is given by

$$\begin{aligned}\inf_{P_r}(P_s+P_r) &= \inf_{P_r}(f(P_r)+P_r)\\ &= f(0) + 0 = \frac{\gamma_1}{2(\gamma_3-\gamma_1)\gamma_2}\end{aligned}$$

  If the above minimum power necessary is greater than P_0, there will be no solution for this relay transmission case, and the relay node will be shut down. Otherwise, we can use a suitable nonlinear equation solver to find $P^*_{r\,\mathrm{AF}}$ as the unique solution of $f(P^*_{r\,\mathrm{AF}}) + P^*_{r\,\mathrm{AF}} = P_0$. The corresponding achievable rate in this case is

$$\tilde{C}_{\mathrm{AF}}(P^*_r, \boldsymbol{\gamma}) = \frac{1}{2}\log\left(1 + 2\gamma_1 f(P^*_{r\,\mathrm{AF}}) + \frac{4\gamma_2 f(P^*_{r\,\mathrm{AF}})\gamma_3 P^*_{r\,\mathrm{AF}}}{1+2\gamma_2 f(P^*_{r\,\mathrm{AF}}) + 2\gamma_3 P^*_{r\,\mathrm{AF}}}\right)$$

In summary, the corresponding maximum achievable rate is

$$R_{\mathrm{AF}} = \begin{cases} \tilde{C}_{\mathrm{AF}}(P^*_{r\,\mathrm{AF}}, \gamma) & \gamma_1 < \gamma_3 \ \& \ P_0 > \frac{\gamma_1}{2\gamma_2(\gamma_3-\gamma_1)} \\ \frac{1}{2}\log(1+2\gamma_1 P_0) & \text{otherwise} \end{cases} \tag{11}$$

Clearly, the above solution is quite complicated and an explicit expression for the optimal power allocation or the maximum achievable rate is not available. A popular approximation (valid in the moderate to high signal-to-noise ratio (SNR) regime) is given by

$$1 + 2\gamma_2 P_s + 2\gamma_3 P_r \approx 2\gamma_2 P_s + 2\gamma_3 P_r \tag{12}$$

This approximation is widely used due to the fact that the resulting achievable rate expression for the AF case becomes concave jointly in P_s and P_r (see for example Reference [15] amongst many others). It can be shown that the maximum achievable rate for the AF scheme with this approximation is

$$\tilde{R}_{\mathrm{AF}} = \begin{cases} \frac{1}{2}\log(1+2\gamma_1 P_0) & \gamma_1 > \gamma_3 \\ \frac{1}{2}\log\left(1+\frac{2uP_0}{1+u}(\gamma_1+\frac{\gamma_2\gamma_3}{\gamma_2 u+\gamma_3})\right) & \gamma_1 \leqslant \gamma_3 \end{cases} \tag{13}$$

where $u = \frac{\gamma_3(\gamma_1+\sqrt{\gamma_1\gamma_3-\gamma_1\gamma_2+\gamma_2\gamma_3})}{(\gamma_3-\gamma_1)\gamma_2}$.

Given the above optimal power allocation policies for an instantaneous channel realisation, one can calculate the minimum outage probability of a single block transmission system by numerical methods or *via* Monte Carlo simulations, by averaging over a large number of simulated channel realisations. While exact expressions of the outage probability are rare, there are high SNR approximations to the outage probability available in various papers such as References [4, 20] etc.

4. OPTIMAL POWER ALLOCATION FOR $K(>1)$ BLOCK DELAY CONSTRAINT

In this section, we solve the original dynamic power allocation problems as proposed in Equation (1) for the outage probability minimisation based on causal CSI, that is, based on past and current channel gains only. It should be obvious that for a given finite K, it is not possible to obtain the optimal power allocation for each block in closed form as in the case for $K = 1$. In fact, even if all the channel states (including the future ones) are available, it is still quite difficult to calculate the outage minimising power allocation for AF and DF cases (whereas the DT case can be solved by the well known channel inversion based power allocation algorithm [19]) using constrained optimisation techniques. This is simply because evaluating all possible combinations of conditions on channel triple γ^k, as in Equations (6), (11) and (13) and obtaining the corresponding optimal power allocation is computationally prohibitive even for moderate values of K. This motivates us to use the DP methodology as also used in Reference [12] for solving a delay constrained capacity optimisation problem for the DT case over fading channels. Using DP techniques, the dynamic power allocation problem can be solved in $K+1$ stages. Starting at stage $K+1$, a three-Dimensional (3D) array $\{S_k\}$ (which is a function of the transmission power as well as the achieved sum rate) needs to be derived in a backward fashion. An algorithmic description of this method is described below in Algorithm 1 in terms of the general cost function $\mu(x)$.

Algorithm 1 Power_co-op

Initialization $S_{K+1}(R, P) = \mu(R)$
for $i = K$ to 1 **do**
  $S_i(R, P) = \mathbb{E}_\gamma\big[\min_{0\leqslant p_s+p_r\leqslant P} S_{i+1}(R + C_*(p_s, p_r, \gamma), P - p_s - p_r)\big]$
end for
Optimal_Cost $= S_1(R = 0, P = KP_0)$.

As discussed before, one can substitute $\mu(x)$ by the indicator function $\mathbf{1}_{x\leqslant KR_0}$ in order to achieve the optimal outage probability.

Recall that in these algorithms, C_* can possibly symbolise C_{DT}, C_{AF}, or, C_{DF} as shown in Equation (2).

Adapting Theorem II.1 in Reference [12] to the cooperative case, the optimal solution to Problem (1) is given by $S_1(0, KP_0)$ for the outage probability minimisation case. It is straightforward to conclude this for the AF and DF cases when the achievable rates are modified accordingly. We can now compute all the values of $S_i(R, P)$ where $i = 1, \ldots, K+1$ and $0 \leqslant P \leqslant KP_0$ and $0 \leqslant R \leqslant KR_0$, using Algorithm 1. In a practical system, such computations can be carried out at the destination node (before the nodes start communicating data) which is assumed to have the knowledge of all the channel gains. This S-array can then be fed back to the source and relay nodes and stored at these nodes. When the nodes start transmitting, upon feedback of causal CSI data, the source and the relay can then use

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another Algorithm 2 (described below) to allocate dynamically the powers for their current transmission block where $R^{(i)}$ and $P^{(i)}$ denote the sum rate achieved before transmitting block i and the remaining power for blocks $i - K$, respectively.

Algorithm 2 Outage_co-op allocation

Initialization $R^{(1)} = 0$ and $P^{(1)} = KP_0$
for $i = 1$ to K **do**
  $(P_s^i, P_r^i) = \text{argmin}_{0 \leqslant p_s^i + p_r^i \leqslant P^{(i)}} \; S_{i+1}\Big(R^{(i)} + C_*(p_s^i, p_r^i, \gamma^i), P^{(i)} - p_s^i - p_r^i\Big)$
  $R^{(i+1)} = R^{(i)} + C_*(P_s^i, P_r^i, \gamma^i), \;\; P^{(i+1)} = P^{(i)} - P_s^i - P_r^i$
end for
if $R^{(K+1)} < KR_0$ **then**
  Outage
else
  Successful Transmission
end if

In order to implement the DP-based Algorithm 1 in the destination node, in principle, the 3D array S needs to be computed for all possible values of R and P within the appropriate range. This is clearly impossible since they are both continuous valued variables. Therefore one needs to carry out an appropriate discretisation of the rate R and the power P according to the computational capability of the destination node and the storage capability of the source and relay nodes. The 3D array S is then computed and stored at the destination and forwarded to and stored in the source and relay nodes. When the system is online, as long as the current CSI is available at the source and relay nodes (*via* feedback from the destination node), the system can allocate the transmit power to the source and the relay by using Algorithm 2 instead. Since the CSI is assumed to be i.i.d. over all blocks (for each individual links), the outage probability averaged over a large number of channel state instances is expected to be a 'good' estimate of $S_1(0, KP_0)$, as long as $S_1(0, KP_0)$ is also relatively accurate, which can be achieved by a sufficiently high number of discretisation levels for the rate, power and the channel gains. A few words on the computational complexity of the DP-based algorithm are now in order. The bulk of the computation arises due to the computation of the array S. While implementing the DP-based algorithm, we adopt the following discretisation strategy: the number of discretisation levels for the available power KP_0 and required sum rate KR_0 are taken to be proportional to the number of blocks K. We divide the intervals $[0, P_0]$ and $[0, R_0]$ into N_P and N_R discretised bins respectively, and each of the three channel gains is discretised into N_γ levels (by uniformly dividing the interval $(0, 10 * E[\gamma_i])$) where $E[\gamma_i]$ denotes the mean of channel $\gamma_i, i = 1, 2, 3$. Other types of discretisation schemes such as logarithmic [12] are also possible. It is then easy to show that the complexity of the procedure for calculating the S-array is $O(K^5 N_R N_P^3 N_\gamma^3)$. It is clearly seen that even for moderate values of the parameters K, N_R, N_P and N_γ the number of computations can quickly become prohibitively high. This is the main disadvantage of the DP-based algorithm from a computational point of view. In Section 6, we will provide further details on these parameters that will be used in the implementation of the DP-based algorithm.

5. APPROXIMATE POWER ALLOCATION SCHEMES FOR HIGH SNR

In the last section, we solved the original problem with a multi-block coding delay by a DP-based algorithm. However, this algorithm is computationally highly complex due to computation of the 3D array S. Therefore, in this section, a simplified power allocation scheme to compute the outage probability is introduced, which is based on a high SNR approximation of the instantaneous mutual information for AF and DF protocols.

Remark 3. Note that in Reference [1], we provided simple approximate power allocation schemes for maximising the delay constrained expected achievable rate for both the high and the low SNR regimes. For the outage minimisation problem however, we have noticed that in the low SNR regime, the outage probabilities are inherently high unless the basic rate requirement is made extremely small. Furthermore, it was shown in Reference [21] through an outage capacity analysis in the low SNR regime (with no constraints on the coding delay) that the optimal protocol is a *bursty amplify and forward* (BAF) rather than the usual AF, while the DF scheme is strictly sub-optimal. This analysis was also carried out under an assumption of a low outage probability. This warrants a detailed study of the (BAF) protocol in the low SNR regime in the context of coding delay constraints and causality restrictions of the current paper, rendering the pursuit of sub-optimal power allocation policies in the low SNR case beyond the scope of this paper.

To proceed, we adopt a reasonable assumption: when the CSI is only known for the current block and unknown for all the future blocks (which are i.i.d), after assigning the sum (source and relay) power to the current block, we equally distribute the remaining power to the rest of the blocks [14]. As a result, we may assume that the total available power (for use in blocks $i, i+1, \ldots, K$) $P^{(i)}$ defined in Algorithm 2 is divided into two parts: (1) the power allocated to the current block P^i which is the sum of transmit power on the source P_s^i and the relay P_r^i, and (2) the (sum of source and relay) powers equally distributed to the rest of the blocks which are equal to $P^k = \frac{P^{(i)}-P^i}{K-i}$ where $i = 1, \ldots, K-1$ and $k = i+1, \ldots, K$. This assumption essentially implies that the optimisation in Algorithm 2 can be substituted by the maximisation of the sum of the rate of the current block and the expected achievable sum rate of the future blocks. Therefore, the DP-based algorithm is now simplified by reaching a tradeoff between those two parts, the details of which are given below.

5.1. High SNR approximation for the AF protocol

With a high SNR approximation for the AF protocol, we can approximate the achievable rate for future blocks (assuming the current block is the ith block) as follows:

$$
\begin{aligned}
&\hat{C}_{\mathrm{AF}}^{i+1\sim K}(P^{(i)} - P^i) \qquad (14)\\
&\triangleq \mathbb{E}_{\boldsymbol{\gamma}}\left[\sum_{k=i+1}^{K} C_{\mathrm{AF}}\left(P_s^k, P_r^k, \gamma^k\right)\right]\\
&= (K-i)\mathbb{E}_{\boldsymbol{\gamma}}\left[\frac{1}{2}\log\left(1 + 2\gamma_1^k P_s^k + \frac{4\gamma_2^k P_s^k \gamma_3^k P_r^k}{1 + 2\gamma_2^k P_s^k + 2\gamma_3^k P_r^k}\right)\right]\\
&\approx (K-i)\mathbb{E}_{\gamma}\left[\frac{1}{2}\log\left(1 + 2\gamma_1^k P_s^k + \frac{2\gamma_2^k P_s^k \gamma_3^k P_r^k}{\gamma_2^k P_s^k + \gamma_3^k P_r^k}\right)\right]\\
&\leqslant \frac{(K-i)}{2}\log\left(1 + 2\mathbb{E}[\gamma_1^k]P_s^k + \frac{2\mathbb{E}[\gamma_2^k]P_s^k\mathbb{E}[\gamma_3^k]P_r^k}{\mathbb{E}[\gamma_2^k]P_s^k + \mathbb{E}[\gamma_3^k]P_r^k}\right)\\
&\leqslant \frac{(K-i)}{2}\log(1 + \bar{c}_{\mathrm{AF}} P^k)\\
&= \frac{(K-i)}{2}\log\left(1 + \bar{c}_{\mathrm{AF}}\frac{P^{(i)} - P^i}{K-i}\right) \qquad (15)
\end{aligned}
$$

The first inequality follows from Jensen's inequality and the constant $\bar{c}_{\mathrm{AF}}$ in the last two lines can be derived from Equation (13) by substituting $\gamma_1, \gamma_2, \gamma_3$ with their expected values—$\bar{\gamma}_1 = \mathbb{E}[\gamma_1]$, $\bar{\gamma}_2 = \mathbb{E}[\gamma_2]$ and $\bar{\gamma}_3 = \mathbb{E}[\gamma_3]$, respectively. Note that here we have suppressed the time index for the channel gains because of the (assumed) i.i.d. property of the block fading channel. We also make the reasonable assumption that the channel state of the relay-destination link is, with high probability, better than the one of the S–D link resulting in $\bar{\gamma}_3 > \bar{\gamma}_1$ (otherwise we may simply prefer direct transmission). This allows us to write $\bar{c}_{\mathrm{AF}} = \frac{2\bar{u}}{1+\bar{u}}(\bar{\gamma}_1 + \frac{\bar{\gamma}_2\bar{\gamma}_3}{\bar{\gamma}_2\bar{u}+\bar{\gamma}_3})$ where $\bar{u} = \frac{\bar{\gamma}_3(\bar{\gamma}_1+\sqrt{\bar{\gamma}_1\bar{\gamma}_3-\bar{\gamma}_1\bar{\gamma}_2+\bar{\gamma}_2\bar{\gamma}_3})}{(\bar{\gamma}_3-\bar{\gamma}_1)\bar{\gamma}_2}$. Therefore, the approximation of the achievable sum rate $\hat{C}_{i+1}^K(P^{(i)} - P^i)$ does not depend on any further CSI but only the three averages of channel gains, which means an approximate power allocation scheme can now be derived in a causal fashion. If we rewrite the optimisation problem in Algorithm 2 through this approximation, the new optimisation problem is given by

$$\max_{P^i} \; C_{\mathrm{AF}}^i(P^i, \gamma^i) + \hat{C}_{\mathrm{AF}}^{i+1\sim K}(P^{(i)} - P^i) \qquad (16)$$

$$\text{s.t.} \quad 0 \leqslant P^i \leqslant P^{(i)} \qquad (17)$$

where the objective function is the sum of the achievable rate in the current block and the approximate estimated sum rate for all the future blocks. Furthermore, $C_{\mathrm{AF}}^i(P^i, \gamma^i) = \frac{1}{2}\log(1 + c_{\mathrm{AF}}^i P^i)$, in which

$$
c_{\mathrm{AF}}^i = \begin{cases} \frac{2u^i}{1+u^i}\left(\gamma_1^i + \frac{\gamma_2^i\gamma_3^i}{\gamma_2^i u^i + \gamma_3^i}\right) & \gamma_1^i \leqslant \gamma_3^i \\ 2\gamma_1^i & \gamma_1^i > \gamma_3^i \end{cases}
$$

and $u^i = \frac{\gamma_3^i(\gamma_1^i+\sqrt{\gamma_1^i\gamma_3^i-\gamma_1^i\gamma_2^i+\gamma_2^i\gamma_3^i})}{(\gamma_3^i-\gamma_1^i)\gamma_2^i}$. This is a convex optimisation problem over the single variable P^i. Hence, using KKT conditions, it is easy to derive the optimal value

$$
{P^i}^*_{\mathrm{AF(H)}} = \min\left\{\left[\frac{c_{\mathrm{AF}}^i + \frac{c_{\mathrm{AF}}^i\bar{c}_{\mathrm{AF}}P^{(i)}}{K-i} - \bar{c}_{\mathrm{AF}}}{c_{\mathrm{AF}}^i\bar{c}_{\mathrm{AF}}\left(\frac{1}{K-i}+1\right)}\right]^+, P^{(i)}\right\} \qquad (18)
$$

Note that once the optimal power for the current (ith) block ${P^i}^*$ is computed, one can use the solution to the single block achievable rate optimisation problem (7) (with P_0 replaced by ${P^i}^*_{\mathrm{AF(H)}}$) to compute the optimal source and relay transmission powers for the current block.

5.2. High SNR approximation for the DF protocol

A high SNR approximate power allocation scheme can also be derived similarly for the DF protocol:

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$$\begin{aligned}
&\hat{C}_{\mathrm{DF}}^{i+1\sim K}(P^{(i)} - P^{i}) \\
&\triangleq \mathbb{E}_{\gamma}\left[\sum_{k=i+1}^{K} C_{\mathrm{DF}}\left(P_s^k, P_r^k, \gamma^k\right)\right] \\
&= (K-i)\mathbb{E}_{\gamma}\left[\frac{1}{2}\min\Big\{\log\left(1+2\gamma_2^k P_s^k\right),\right. \\
&\qquad \left.\log\left(1+2\gamma_1^k P_s^k + 2\gamma_3^k P_r^k\right)\Big\}\right] \\
&\leqslant \frac{K-i}{2}\min\Big\{\mathbb{E}\left[\log\left(1+2\gamma_2^k P_s^k\right)\right], \\
&\qquad \mathbb{E}\Big[\log\left(1+2\gamma_1^k P_s^k + 2\gamma_3^k P_r^k\right)\Big]\Big\} \\
&\leqslant \frac{K-i}{2}\min\Big\{\log\left(1+2\mathbb{E}[\gamma_2^k]P_s^k\right), \\
&\qquad \log\left(1+2\mathbb{E}[\gamma_1^k]P_s^k + 2\mathbb{E}[\gamma_3^k]P_r^k\right)\Big\} \\
&= \frac{(K-i)}{2}\log\left(1+\bar{c}_{\mathrm{DF}}\frac{P^{(i)}-P^{i}}{K-i}\right) \qquad (19)
\end{aligned}$$

where $\bar{c}_{\mathrm{DF}}$ can be derived from Equation (6) i.e.

$$\bar{c}_{\mathrm{DF}} = \begin{cases} 2\bar{\gamma}_2 & \bar{\gamma}_1 > \bar{\gamma}_2 \\ 2\bar{\gamma}_1 & \bar{\gamma}_1 \leqslant \bar{\gamma}_2 \text{ and } \bar{\gamma}_1 > \bar{\gamma}_3 \\ \frac{2\bar{\gamma}_2\bar{\gamma}_3}{\bar{\gamma}_2+\bar{\gamma}_3-\bar{\gamma}_1} & \bar{\gamma}_1 \leqslant \bar{\gamma}_2 \text{ and } \bar{\gamma}_1 \leqslant \bar{\gamma}_3 \end{cases} \qquad (20)$$

and $k = i+1, \ldots, K$. The first inequality in Equation (19) follows from Jensen's inequality and the second is simply due to a nice approximation in the high SNR regime [22]. Therefore, we can formulate a similar optimisation problem to Equation (17) for the DF case, which produces the sub-optimal power allocation scheme

$$P^{i*}_{\mathrm{DF(H)}} = \min\left\{\left[\frac{c_{\mathrm{DF}}^{i} + \frac{c_{\mathrm{DF}}^{i}\bar{c}_{\mathrm{DF}}P^{(i)}}{K-i} - \bar{c}_{\mathrm{DF}}}{c_{\mathrm{DF}}^{i}\bar{c}_{\mathrm{DF}}(\frac{1}{K-i}+1)}\right]^{+}, P^{(i)}\right\} \qquad (21)$$

where c_{DF}^{i} is computed based on the current CSIs (i.e, by replacing $\bar{\gamma}_1, \bar{\gamma}_2, \bar{\gamma}_3$ in Equation (20) by γ_1^i, γ_2^i and γ_3^i, respectively). The optimal source and relay transmission powers for the current block are then found by using Equation (5) (with a short term power constraint of $P_0 = P^{i*}_{\mathrm{DF(H)}}$), which is the solution to the single block achievable rate maximisation problem for the DF case. Once the power allocation for the ith block is decided, the system simply uses Algorithm 2 to record the achieved sum rate in $R^{(i+1)}$ and the remaining power. If this achieved sum rate is greater than the required rate threshold (KR_0 in our case), it means that an outage is avoided even if we do not allocate power to the rest of the blocks; otherwise, the system keeps solving the maximisation problem (17) (for the AF case) or an equivalent one for the DF case for each block. If all the power is used up before reaching the Kth block and the rate threshold is still not achieved or all the K blocks are exhausted without achieving the rate threshold, an outage happens. The resulting outage probability can be estimated *via* Monte Carlo simulations over a large number of channel realisations by averaging the indicator function $\mathbf{1}_{R^{(K+1)}<KR_0}$. Clearly, this scheme is less computationally demanding than the DP-based method simply because it does not have to rely on computing the S-array first in order to obtain the power allocation policy. We will illustrate the performance of these sub-optimal algorithms based on high SNR approximations and compare them with the performance of the DP-based algorithms in Section 6 next.

6. NUMERICAL RESULTS

In this section, we will provide a range of simulation results on the delay-constrained outage minimisation (see Reference [1] for results on the delay-constrained expected achievable rate maximisation) for the three node relay network using AF and DF schemes and compare their performances against the no cooperation (DT) scheme. We will present results in three subsections: A) in the first part, all links have no direct line of sight and undergo statistically independent Rayleigh fading (albeit with different means); B) in the second part, the S–D link has no direct line of sight (Rayleigh fading) but the S–R and the R–D links have direct lines of sight (modelled by independent Rician fading); C) finally, the results demonstrating the accuracy of the high SNR approximation based methods are illustrated. For simplicity, all the simulations are based on a network where the relay node is located on the straight line between the source and destination nodes. For Subsections 6.1 and 6.2, we use the discretisation parameters $N_R = 10$, $N_P = 10$, $N_\gamma = 100$.

6.1. *Rayleigh fading*

We assume that the channel triple $\gamma = (\gamma_1, \gamma_2, \gamma_3)$ in the Figure 1 are exponentially distributed with means given by

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inverse of the S–D, S–R and R–D distances with a path loss factor of 4. For computational simplicity, the S–D distance is normalised to 1 and, furthermore, the relay node is assumed to be at a distance of $d_{S--R} = 0.4$ unless otherwise stated. In the outage probability calculations for a K-block decoding delay, the required rate threshold is denoted by KR_0, where $R_0 = 0.1$ nats/Hz/Transmission. Obviously, this ensures a fair comparison between single and multi-block transmissions. Wherever applicable, the results are averaged over 100 000 channel realisations.

First of all, it should be obvious that in the case of cooperative transmission, the number of discretisation levels required for reasonably accurate calculations of the relevant S function is much higher (than the DT case) due to the increased dimensionality of the CSI space. Thus the inaccuracy of the S function inevitable due to limited computational resources is more apparent in the cooperative transmission case which is illustrated below in Figure 2 for the simple case of $K = 1$. The 'OM_Avg' lines represent the solution achieved by the outage minimisation approach in Section 3 followed by averaging over a large number of channel realisations and the 'Avg' lines represent the results obtained by averaging the power allocation obtained by using the S-array over a large number channel realisations. Figure 2 shows that in all the three cases—DT, AF, and, DF, averaging over a large number of channel realisations achieves similar results with DT having the best consistency (due to the single dimension of the CSI space) as expected. On the other hand, the inaccuracy of the S-array based calculations, while being negligible at low power (P_0)

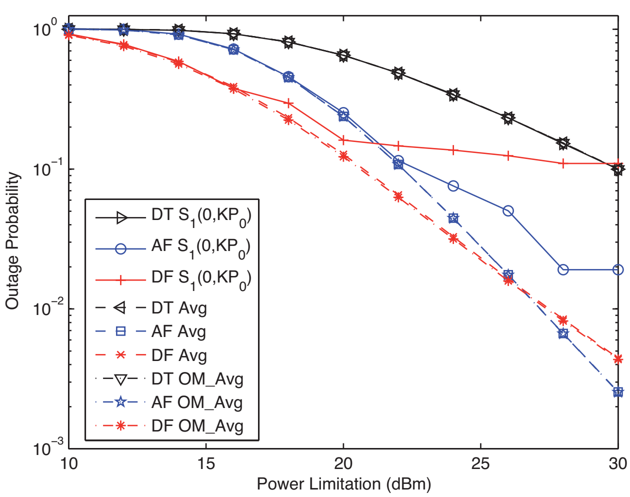

Figure 2. Optimal outage probabilities achieved by the S function, averaging of the S function , and averaging of the optimal single block outage minimisation.

levels, is exposed at high power levels. However, the outage probability results obtained by averaging provide relatively more accurate solutions. For this reason, unless otherwise mentioned, we will be using the method of averaging of the power allocation obtained by computing the S-array as the preferred method for computing outage probabilities for the multi-block ($K > 1$) case. It was also illustrated (graph excluded due to the space limitation) that the high SNR approximation for the AF scheme (12) resulted in outage probabilities that were close to the optimal solution at the high SNR range but were substantially lower in the low SNR (less than 17 dBm) range. Therefore while this approximation can be used as a lower bound on the optimal outage probability for the single block AF transmission scenario in general, it may not be a good approximation in the low SNR range. Consequently, wherever applicable, we use the exact optimal solution (see Equation (11)) to the AF single block transmission case.

Using the DP-based solution technique in Section 4, now we present some results for the multiple block case ($K > 1$) in Figures 3 and 4. Figure 3 plots the outage probability obtained by $S_1(0, KP_0)$ (solid line) and by averaging over the S function (dashed line) for $K = 1, 3, 5$ for varying S–R distances for the AF and the DF relaying schemes. We see that the results obtained by $S_1(0, KP_0)$ and by averaging over the S function are quite close to each other for all values of K for both transmission schemes.

We also compare the minimum outage probabilities achieved by the DT, AF and DF schemes for various power levels in Figure 4(a) and (b) for $K = 3$ and $K = 5$, respectively. All the dashed lines (labelled by 'ED' for 'equally distributed') symbolise the results achieved by equally distributing the total power among all blocks (i.e, by constraining the sum of source and relay powers for each block to P_0). The improvement obtained by using the DP algorithm is evident in these figures. It can also be illustrated that the cooperative transmission schemes (AF and DF) achieve better performances than the direct transmission scheme regardless of the position of the relay node (figures not included due to limited space).

6.2. *Rician fading*

Relay networks are often useful when the source and the destination nodes are far away from each other, or an unsurmountable obstacle prevents the two nodes having a line of sight (LOS) communication, whereas the Source-to-Relay and Relay-to-Destination links have LOS instead. In this case, it is more accurate to model the S–D channel as

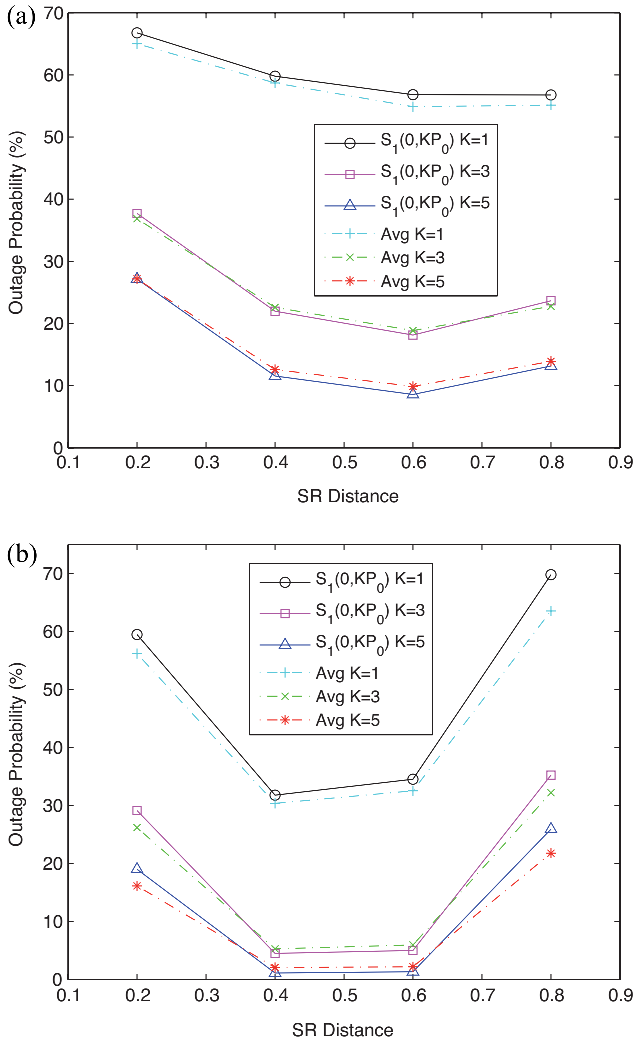

Figure 3. Optimal outage achieved by S function and averaging when $P_0 = 0.05$ W (17 dBm): (a) AF and (b) DF.

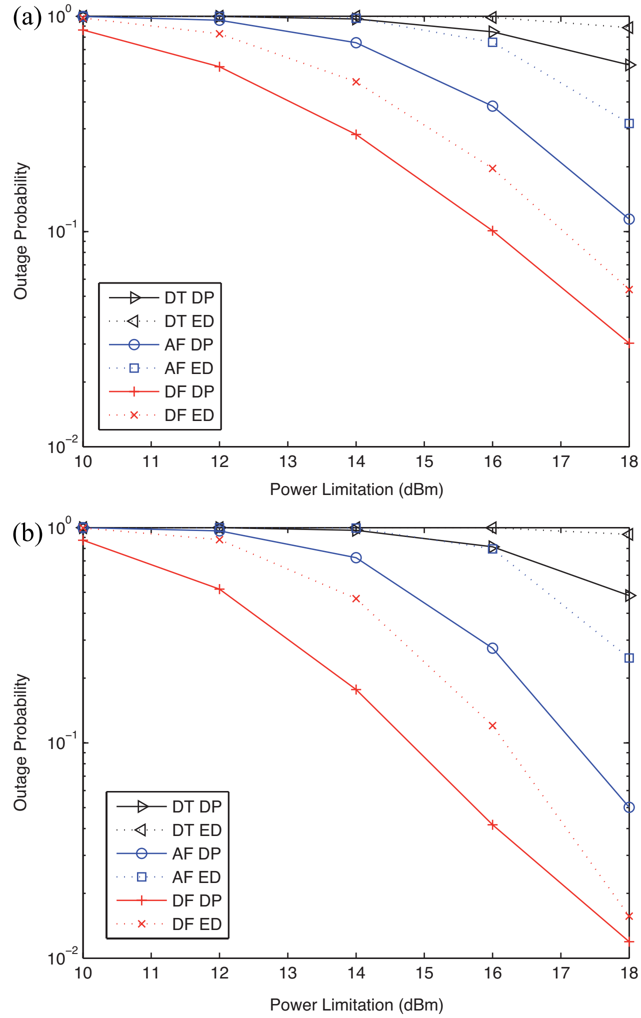

Figure 4. Minimum outage probabilities for DT, AF and DF *versus* those obtained by equally distributing powers to all blocks. Various power limitations for (a) $K = 3$ and (b) $K = 5$.

Rayleigh faded, and the S–R and the R–D channels as Rician faded channels [23].

In this section, we provide some numerical results for the above scenario where the average channel gains for the S–D and the R–D channels are still functions of the respective distances as in the earlier section, but the power attenuation is divided into two parts—due to LOS and due to multipath fading. The relationship between them is represented by the parameter $K_{\text{rice}} \triangleq \frac{A^2}{2\sigma^2}$, where A symbolises the amplitude of LOS component and σ^2 is the total power attenuation from multipath. It is well known that $K_{\text{rice}} = 0$ is equivalent to Rayleigh fading, and if $K_{\text{rice}} = +\infty$, there is no uncertainty and the channel is deterministic ($\gamma = \frac{A^2}{2}$ with probability one). The S–D channel is modelled as Rayleigh distributed as described in Section 6.1.

Figure 5 unveils the outage performance for different K_{rice} parameters. For both the AF and DF protocols, the outage probability with LOS communication on S–R and R–D links is lower than that obtained when all links undergo Rayleigh fading ($K_{\text{rice}} = 0$). The outage probability obtained in the Rician scenario for the cooperative schemes is clearly also lower than that obtained by direct transmission. In addition, for a fixed Rician fading parameter, we compare the performances of the AF and the DF schemes in Figure 6. It is seen that when the power level (P_0)

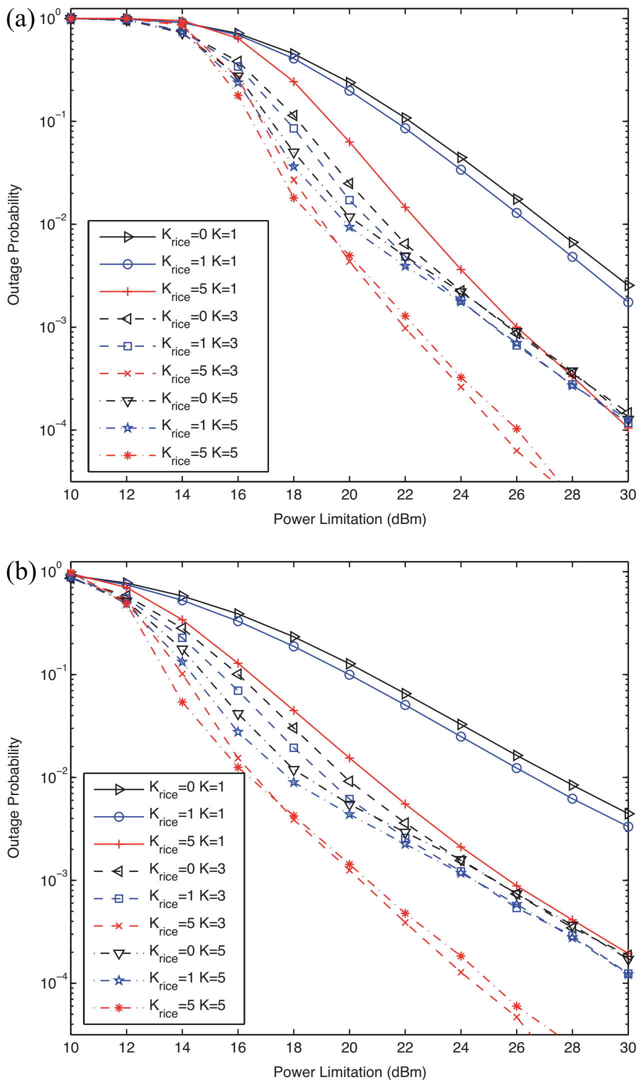

Figure 5. Outage minimisation for $K_{\text{Rice}} = 0,\ 1,\ 5$: (a) AF and (b) DF in rician fading.

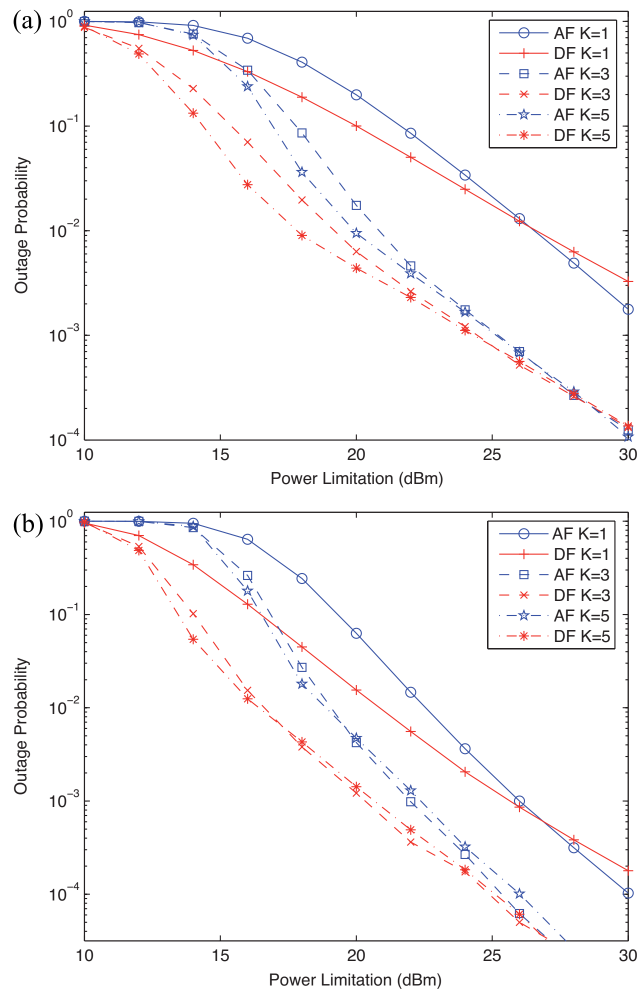

Figure 6. Outage minimisation for AF and DF in Rician fading: (a) $K_{\text{Rice}} = 1$ and (b) $K_{\text{Rice}} = 5$.

exceeds 22 dBm, the outage probabilities for $K = 3$ and 5 seem to be quite close as shown in Figure 6(a). Furthermore, Figure 6(b) illustrates that there is a possibility that a larger coding delay does not necessarily guarantee better outage performance when the power is greater than 20 dBm. This observation demonstrates that there is no strong correlation between the total power KP_0 and the outage threshold KR_0 when the system operates in the high power regime for the Rician fading case. This is possibly due to the fact that the improvement obtained due to the LOS links outperforms that obtained by time diversity using multiple blocks.

6.3. High SNR approximation based power allocation schemes

The results in this section are obtained for the same relay network configuration and Rayleigh faded channel statistics as described in Section 6.1. As opposed to the simulations in Sections 6.1 and 6.2, which are based on the computationally demanding DP algorithm, the results for this section are computed using the high SNR approximation of Section 5, whereby the complexity of the power allocation schemes are reduced dramatically. Figure 7 shows how the high SNR approximation power allocation scheme works. The normalised rate threshold R_0 is 0.5 for these simulations. It is clear from Figure 7 that these approximate methods provide

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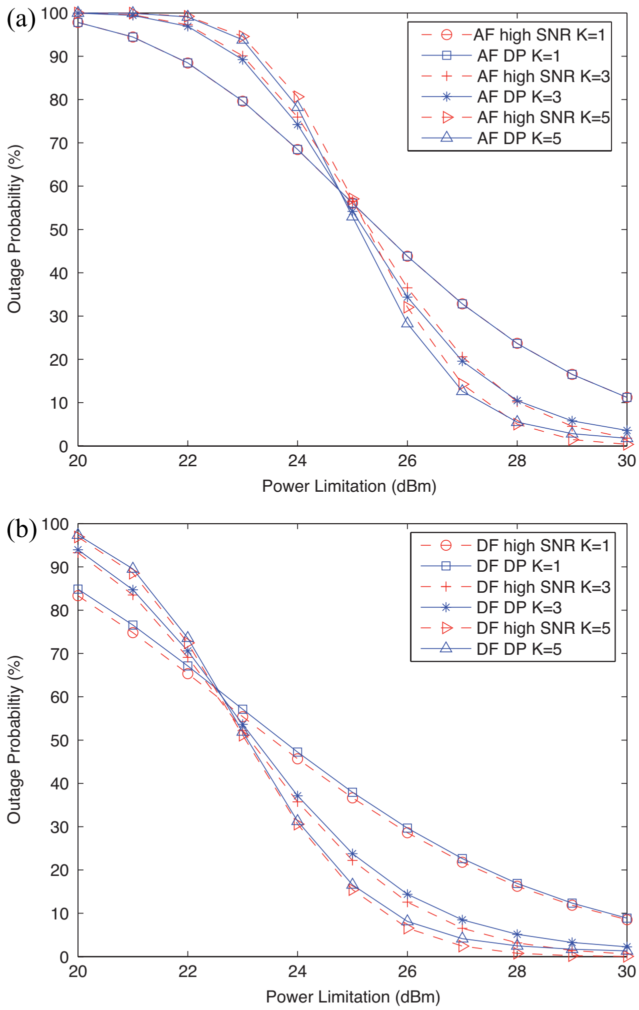

Figure 7. Sub-optimal outage probability in high SNR regime with the high SNR approximations: (a) AF and (b) DF.

Table 1. List of symbols.

S	Source node
R	Relay node
D	Destination sink node
k	Block index
K	Total number of blocks
γ_1^k	CSI of S–D link in kth block
γ_2^k	CSI of S–R link in kth block
γ_3^k	CSI of R–D link in kth block
γ^k	All 3 CSIs in kth block
$\gamma^{(k)}$	Causal CSIs before and in kth block
P_s^k	Tx power on source node in kth block
P_r^k	Tx Power on relay node in kth block
$\mu(x)$	Cost function
P_0	Power constraint for a single block
C_*	Achievable rate $* \in$ {AF, DF, DT}
$\mathbf{1}_A$	Indicator function
R_0	Rate requirement for a single block
F_γ	CDF function of γ
$\mathcal{U}(R_0, P_0)$	Outage region
u	Power control parameter $P_s = uP_r$
P_s^*	Optimal power for source node
P_r^*	Optimal power for relay node
λ	Lagrangian multiplier
R_*	Achievable rate $* \in$ {AF, DF, DT}
S	DP array (3D)
R	Rate variable
P	Power variable
$R^{(i)}$	Sum rate before Tx ith block
$P^{(i)}$	Remaining power for i–Kth block
N_R	No. of levels discretising rate
N_P	No. of levels discretising power
N_γ	No. of levels discretising CSI
$\bar{\gamma}_1$	Expectation of γ_1
$\bar{\gamma}_2$	Expectation of γ_2
$\bar{\gamma}_3$	Expectation of γ_3
K_{rice}	Rician K-factor
A	Amplitude of LOS attenuation
σ^2	Power from multipath

results quite close to the optimal solutions achieved by DP algorithm for both the AF and DF protocols.

7. CONCLUSIONS

In this paper, we have focused on an outage minimisation problem in a three node relay network with a finite coding delay and peak power constraints, where only causal channel information is available. AF and DF are the two cooperative transmission schemes that we have considered. In the single block case the optimal transmission power for the source and relay can be allocated explicitly based on the available CSI for all three links by using standard convex optimisation methods. In the multi-block transmission scenario (where the coding delay $K > 1$), a DP-based technique is used to find the optimal power allocation for the outage minimisation problem. Due to the high computational demands of the DP-based technique, we have also proposed a simple but sub-optimal power allocation scheme based on a high SNR approximation. Simulation results demonstrate the improvements obtained by using the DP algorithm over equally distributing total power to all blocks and the effectiveness of the high SNR approximation based sub-optimal outage minimising power allocation policies. Extensions of this work involving derivation of power allocation policies based on quantised or limited causal channel feedback are currently in progress.

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AUTHORS' BIOGRAPHIES

James C.F. Li (S'04) received his BE degree in Electrical Engineering from Beijing University of Technology, Beijing, China in 2002, and his PhD from the Department of Electrical and Electronic Engineering, The University of Melbourne, Melbourne, Australia, in 2008. He was a Post-doctoral Research Fellow with the same department until July 2009. His research interests include nonlinear optimization techniques and its applications, cooperative diversity and cross-layer design in wireless sensor networks. He worked with Hughes Network Systems and Hewlett-Packard before he began his PhD studies.

Subhrakanti Dey was born in Calcutta, India, in 1968. He received his BTech and MTech degrees from the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur, India, in 1991 and 1993, respectively, and the PhD from the Department of Systems Engineering, Research School of Information Sciences and Engineering, Australian National University, Canberra, Australia, in 1996.

He has been with the Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, Australia, since February 2000, where he is currently a Full Professor. From September 1995 to September 1997 and September 1998 to February 2000, he was a Post-doctoral Research Fellow with the Department of Systems Engineering, Australian National University. From September 1997 to September 1998, he was a Post-doctoral Research Associate with the Institute for Systems Research, University of Maryland, College Park. His current research interests include signal processing for telecommunications, wireless communications and networks, networked control systems, stochastic and adaptive estimation and control, and statistical and adaptive signal processing. Dr Dey currently serves on the Editorial Board of IEEE Transactions on Signal Processing and Elsevier Systems and Control Letters. He also served on the Editorial Board of the IEEE Transactions on Automatic Control during 2004–2007. Dr Dey is a Senior Member of IEEE.