

Abstract

Adaptive arrays can significantly increase cell capacity, improve signal quality, and reduce transmitter power requirements. In this article we establish a relationship between the information theoretic capacity of a mobile radio system and the beam pattern of a multisensor array.

We investigate the capacity improvement potentially achieved via an optimized design strategy for an unequally spaced array; that is, the positions and the weighting coefficients of the array elements are selected to improve the average system capacity subject to various constraints like minimizing the maximum sidelobe level or keeping the beamwidth of the main lobe to a minimum. Next, we investigate the effect of fading correlation on the performance of an unequally spaced adaptive array. Results are presented for optimum combining with flat fading as well as for frequency-selective fading using a two-path delay spread model. Computer simulations show that it is possible to achieve a gain of 1.5 dB for moderate to high signal-to-noise ratios when compared to the equally spaced array. Finally, it is shown that a base station with wide antenna element spacing has improved bit error rate performance over one with narrow element spacing under co-channel interference and multipath fading. In particular, we improve the performance for both uplink and downlink transmissions in a slow fading channel with cochannel interference. Promising results are presented for a 4 channel carrier time division duplexing system.

3G Wireless Capacity Optimization for Widely Spaced Antenna Arrays

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Future wireless communication systems have to support not only speech but also Internet and multimedia communications. This implies a tremendous increase in system capacity demand. One way of achieving this increase in capacity is to introduce smart antenna systems [1]. These are systems in which the base station antennas do not have a fixed pattern, but adapt to current radio conditions. There are three different smart antenna concepts: switched lobe array, phased array, and adaptive antenna array.

In this article we establish a relationship between the information theoretic capacity and the beam pattern of a multisensor array. Specifically, we investigate the potential capacity enhancement obtainable via an optimized design for an *unequally spaced* antenna array. We optimize the positions and weighting coefficients of the array elements to simultaneously improve the capacity subject to a constraint on the height of the maximum sidelobe and/or minimizing the width (beamwidth) of the main lobe. We illustrate with examples realizable gains over the equally spaced conventional design. A note on the design trade-offs is also included to help appreciate the sensitivities of the various degrees of freedom available.

To capitalize on the spatial dimension, multiple antennas are used adaptively to cancel interference produced by users who are occupying the same frequency band and time slots. We investigate the performance analysis of an unequally spaced array in a multipath fading environment. Optimum combining and signal processing with multiple antennas is not a new idea [2, 3]. In this article we use similar techniques to quantify the reduction in the average bit error rate (BER) for a system with N users in a flat Rayleigh fading environment. It is seen that an effective improvement of 1.5 dB is obtained for moderate to high signal-to-noise ratios (SNRs). To gain insight into the behavior of average BER vs. delay spread for a frequency selective environment, we use Monte Carlo simulation to derive 100,000 channel realizations and numerically calculate the average BER for each channel. Once again, the results show that substantial gains can be obtained for an

unequally spaced array design over the conventional equally spaced array.

Finally, a simulation study is presented for a widely spaced adaptive antenna array. There are three basic ways to provide low correlation (diversity gain): spatial, polarization, and angle diversity. For spatial diversity, the antenna elements are separated far enough for low fading correlation. Antenna element spacing is generally assumed to be half a wavelength ($\lambda/2$). The required separation depends on the angular spread, which is the angle over which the signal arrives at the receive antennas. However, the correlation between received signals for this configuration is high, and as a result reduces the ability of an antenna array to combat against fading (i.e., the lack of space diversity) [4]. For outdoor systems with high base station antennas located above the clutter, the angular spread may be only a few degrees, and a horizontal separation of 10–20 wavelengths is required [1]. This motivates us to investigate the performance of an adaptive antenna array with widely spaced elements to achieve greater space diversity. In particular, we improve the performance for both uplink and downlink transmissions in a slow fading channel with co-channel interference. Promising results are presented for a four-channel carrier time-division multiple access (TDMA) system.

This article is organized as follows. We introduce the concept of beamforming. We then present a mathematical model to compute the average system capacity of a mobile radio system. The performance analysis of such a system is discussed under fading channel conditions. Experimental support and graphical illustrations of analytical results in previous sections are also shown. We will investigate the performance of an adaptive array, followed by conclusions.

Beam Pattern Formulation

In the most general case, for a linear array made up of M omnidirectional elements and placed along the x -axis (Fig. 1), the beam amplitude $p(u)$ can be expressed as

$$p(u) = \left| \sum_{i=0}^{M-1} w_i e^{j \frac{2\pi}{\lambda} x_i u_i} \right|$$

where x_i is the position of the i th element, w_i is the related weight coefficient, $u = \sin(\theta) - \sin(\theta_0)$, θ and θ_0 being the angle of incidence of the plane wave and steering angle (Fig. 1), respectively, and $\lambda = 2\pi c/\omega$. The variable u can assume only real values included between -2 and 2 for any combination of θ and θ_0 . All signals arrive at the base station within $\pm\Delta$ at angle θ_0 . The normalized beam power (also known as the beam pattern) can also be written as $P(u) = (p(u)/Q)^2$, where Q is the sum of all w_i .

Each sensor has a complex weight and a carrier phase associated with it. The position of the main beam can be steered by varying the amplitudes and phases of these sensor weights. The simplest choice of sensor weights is uniform (i.e., identical amplitudes and phases). Thus, for an equally spaced array of aperture 6λ the positions of the elements are at $[0 \ 2\lambda \ 4\lambda \ 6\lambda]$. For the latter array the angular frequency response has a sinc-like behavior. A spatial aliasing effect is seen where the angular frequency response is replicated at intervals of $\lambda/2$.

Multi-User Information Capacity

The channel capacity, C (in bits per second), for an additive white Gaussian noise (AWGN) channel with bandwidth B , signal power P , and noise power σ^2 is given by the well-known Shannon formula.

In the case of one interfering user with a received signal power identical to that of the desired user, the capacity can be written as

$$C(u) = B \log_2 \left(1 + \frac{P(u_{\max})}{P(u) + \sigma^2} \right)$$

where $P(u_{\max})$ is the maximum power in the direction of the main beam, u_{\max} coincident with the direction of the desired user's signal, and $P(u)$ denotes the power of the interfering user's signal arriving from a direction u .

The maximum capacity, C_{\max} , for a system with one interfering user occurs when the interfering user's signal is expressed totally by nulls (i.e., $P(u) = 0$) in the beam pattern.

Subsequently, the minimum capacity, C_{\min} , is obtained when the interfering signal power is also $P(u_{\max})$ (i.e., the interferer is present inside the main beam). This is analogous to the situation when there is only one sensor (i.e., no array).

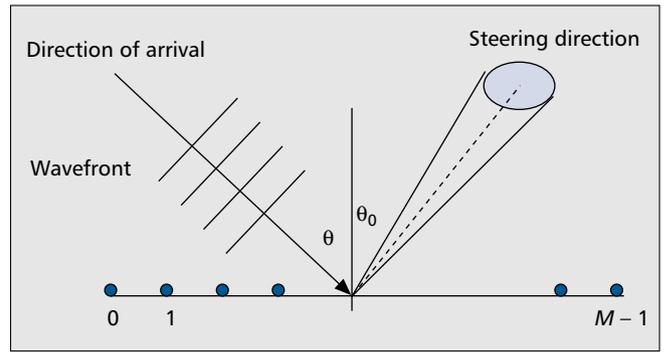
A more detailed discussion on the implications of these results can be found in [5]. In this article we maximize the expected system capacity, $E\{C(u)\}$. This is a more meaningful parameter since the maximum and minimum bounds are only special cases of a more general result,

$$E\{C(u)\} = \int_{-2}^2 C(u) f_u(u) du$$

where $f_u(u)$ is the probability density function of u where u is the angle of arrival (AOA) of the interfering signal at the base station antenna array.

Performance Analysis

In this section we develop a mathematical model for a multipath environment applicable in wireless digital communications. Consider a wireless system with N users, each with an



■ Figure 1. Geometry and notations for a linear array.

antenna, communicating with a base station with M antennas. The channel transmission characteristics matrix can be expressed as

$$C(\omega) = [C_1(\omega), C_2(\omega), \dots, C_N(\omega)]$$

where ω is the frequency in radians per second, and the column vector $C_k(\omega)$ represents the transmission characteristics from user k to all the antenna elements. Since each user is characterized by its own surroundings, and if the users are not on top of one another to within wavelengths, it is reasonable to assume that the columns in $C(\omega)$ are statistically independent. The correlation of fading between two antennas spaced d apart can be found in [6]. To compute the probability of error for flat fading and frequency selective fading environments, the $c_{ij}(\omega)$ s are modeled as complex Gaussian random variables at each frequency ω . The variation of $c_{ij}(\omega)$ depends on the delay spread model of the channel. For flat fading, $c_{ij}(\omega) = c_{ij}$ for all ω . Under this condition the "zero forcing" optimum combiner solution reduces to

$$MSE(C) = (C^*C)_{11}^{-1} N_0$$

In general, the probability of error is given by [2].

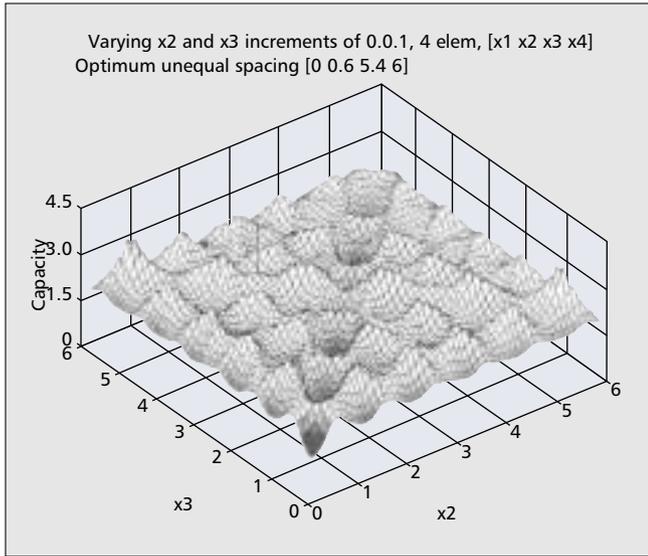
Results and Discussion

In this section we illustrate by simulations the potential improvements in capacity achievable for an optimized unequally spaced antenna over its equally spaced counterpart. The equally spaced array has four elements spaced 2λ apart. The positions of the four elements were at $X = [x_1 \ x_2 \ x_3 \ x_4]$. During the optimization process for the unequally spaced array, the spatial aperture and number of elements are fixed. The end elements were fixed at 0 and 6λ . The remaining elements, x_2 and x_3 , could assume any position within an interval of 0.1λ . The SNR was set to 100. The weighting functions were all identical.

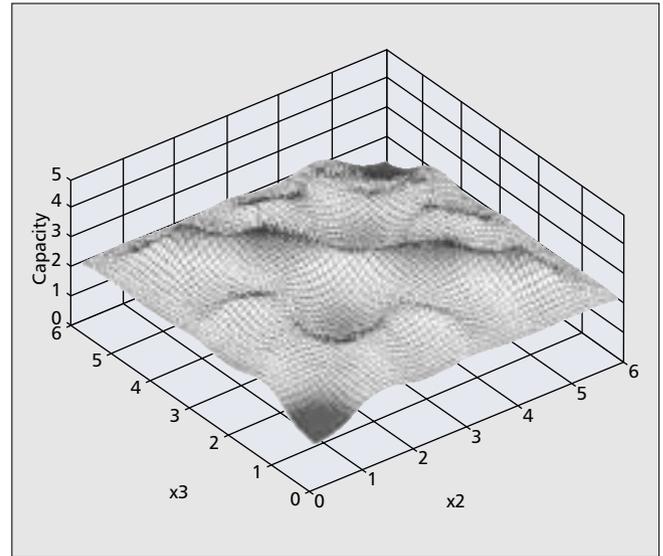
Figure 2 plots the expected capacity for varying position increments of x_2 and x_3 . It is seen that the maximum capacity is obtained when $x_2 = 0.6$ and $x_3 = 5.4$, (i.e., $X = [0 \ 0.6 \ 5.4 \ 6]$). The capacity for the unequally spaced array is 4.22, an increase of 32 percent over the equally spaced array (capacity = 3.2).

Similarly, Fig. 3 plots the capacity surface for the previous example — however, for a nonuniform probability density function (pdf). Specifically, we use a reuse factor of 1. Clearly the pdf of the AOA of the interfering users shapes the capacity surface differently. The maximum capacity is obtained for $X = [0 \ 1.2 \ 4.8 \ 6]$ ($C_{\max} = 3.76$).

It is important to note that the average system capacity is not the only criterion that determines the efficiency of the system. The height of the maximum sidelobe and the width of the main beam also determine the effective interference rejection capability. It is seen in [7] that a compromise in system capacity generates a much reduced sidelobe level.



■ **Figure 2.** A plot of capacity surface vs. varying antenna positions, uniform pdf of $f_u(u)$.



■ **Figure 3.** A plot of capacity surface vs. varying antenna positions, nonuniform pdf of $f_u(u)$, reuse factor = 1.

Performance with Fading and Interference

It is important to investigate the performance of an unequally spaced adaptive array under the effect of channel fading. Figure 4 considers the effect of correlation with flat fading for equally spaced arrays. It is seen that the BER improves as the spacing between the elements increases. This suggests that it may be possible to improve the error rate with unequally spaced arrays aimed to increase the average d/λ . Increasing the antenna spacing by a factor of 10 decreases the tolerable Δ by a factor of 10 as well. However, for very small beamwidth, the BER worsens rapidly. Figure 4 also shows the degradation in performance as the interference increases (i.e., $M = N = 3$, $X = [0 \ 0.382 \ 0.764]$).

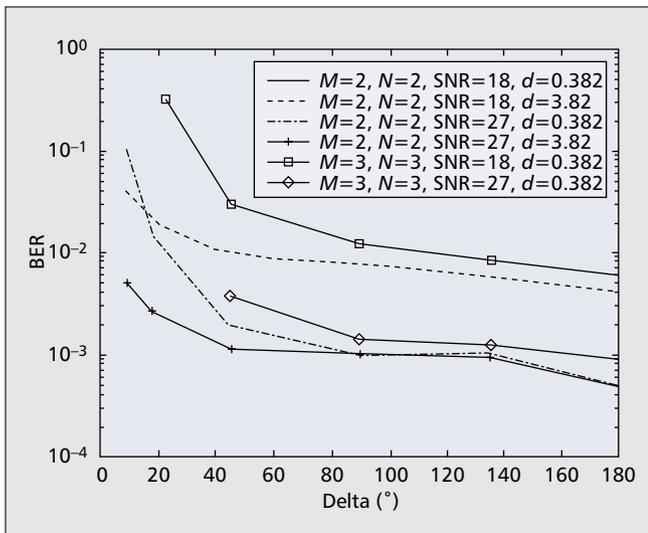
A Simulation Case Study

The array antenna is circular with four elements distance d apart arranged along the x and y axes. The incident waves come from a range of angles ($\Delta = 12^\circ$). Each wave impinging on the antenna has a constant amplitude, a random phase between 0 and 2π , and a Doppler frequency uniformly dis-

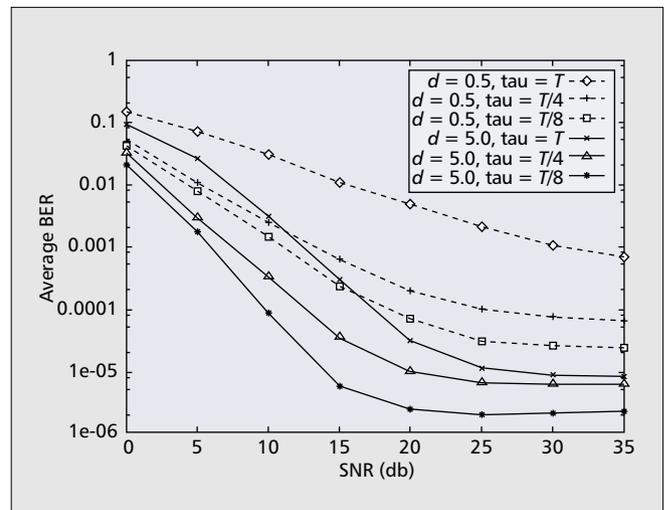
tributed between $-f_D$ and f_D (f_D being the maximum Doppler frequency). The simulation uses quadrature phase shift keying (QPSK) with coherent detection for modulation and demodulation, respectively. A root raised cosine filtering with a rolloff factor of 0.5 is used for pulse shaping. We assume a four-channel-per-carrier TDMA system. The length of a frame is 5 ms with a slot length of 120 symbols. The available bit rate was 384 kb/s. The adaptive weights are controlled by a least mean square (LMS) algorithm with a step increment of 0.01 [8]. We next evaluate the performance of the system outlined above for various channel models.

Uplink and Flat Fading

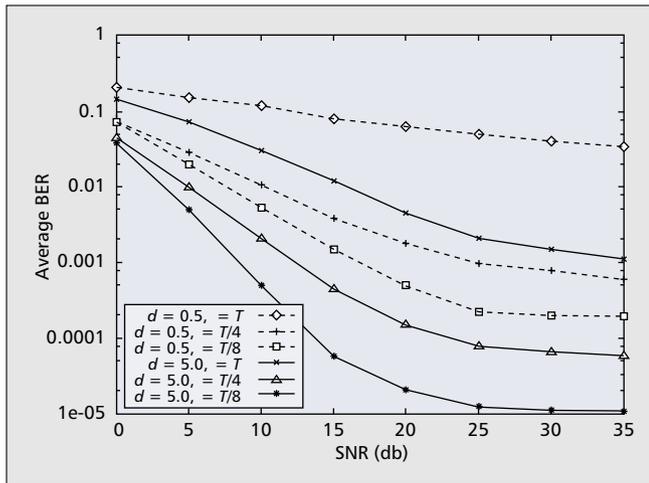
In this section a flat Rayleigh fading channel is used. The average BER performance is plotted for two array geometries, namely $d = 0.5\lambda$ and $d = 5\lambda$. Both the desired and undesired signals are incident at random. It is clearly seen that as the elements are spaced further apart there is a significant improvement in the BER. This can be explained by the reduced level of correlations in the received signals for larger element spacing. This translates to added diversity in which the antenna is now able to exploit and thus effectively cancel interference.



■ **Figure 4.** Average error rate vs. Δ with flat fading.



■ **Figure 5.** BER vs. SNR for uplink transmissions under frequency selective fading without co-channel interference.



■ **Figure 6.** BER vs. SNR for uplink transmissions under frequency selective fading with cochannel interference.

Uplink and Frequency Selective Fading

A standard two-path frequency selective fading model is assumed in this section. Fast fading can be modeled as a Rayleigh distributed random process. Independent fast fading is assumed for each resolvable path p with a specific time delay τ_p and AOA θ_p . The Rayleigh fading coefficients are generated from a complex Gaussian random process which is filtered using an IIR filter with the typical Jake's spectrum model.

Figure 5 plots the system performance in terms of average BER vs. SNR for a two-path frequency selective fading channel without cochannel interference. The Doppler frequency was 10 Hz. It is seen that as the delay in τ_p increases, the performance degrades. However, the antenna array with a larger element spacing consistently outperforms the narrowly spaced array ($d = \lambda/2$).

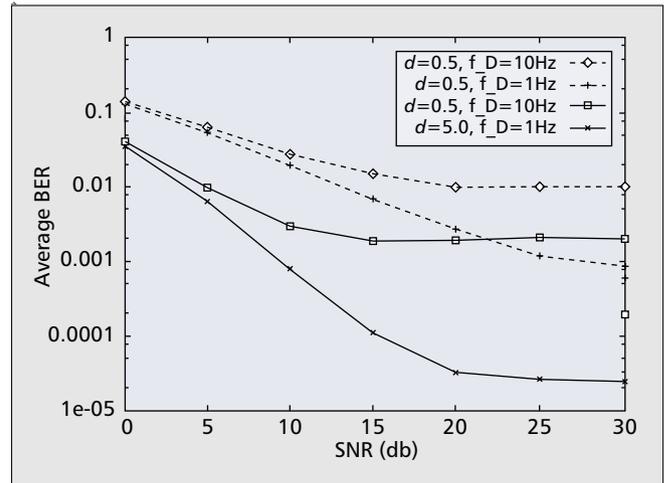
Figure 6 plots the system performance for identical conditions, but with the addition of co-channel interference. It is interesting to see that the BER degrades with the introduction of interference. However, the array with wider element spacing shows better performance once again. The BER floor in both plots can be attributed to the intersymbol interference caused by signal distortion.

Downlink and Frequency Flat Fading

In this section we investigate the performance at the mobile station. It is assumed that co-channel interference comes from the neighboring base station. Using adaptive antenna arrays (forming beams that track the mobile), the average interference power is significantly reduced. Figure 7 illustrates once again an improvement in BER due to the additional space diversity obtained from wider element spacing. It is seen that as the Doppler is increased, the BER degrades, since the adaptive weights lose track of the rapid fluctuations of the time-varying channel.

Conclusions

In this article we investigate the possibility of using unequally spaced antenna arrays in base stations and quantify the capacity gains that can be achieved under various constraints. The average system capacity was optimized subject to minimizing the maximum sidelobe and/or the beamwidth of the main lobe. In general an unequally spaced array outperformed its equally spaced counterpart by as much as 30 percent. The effect of correlation of the signal fading at the antennas is considered. For flat fading channels, the unequally spaced array outperformed the equally spaced array by 1.5–2 dB. Finally, it is shown that an adaptive array antenna with large



■ **Figure 7.** BER vs. SNR for downlink transmissions under frequency flat fading with cochannel interference.

antenna spacing at the base station can significantly improve the performance for both uplink and downlink transmissions in a slow fading channel with co-channel interference.

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