WEIGHTED SUM RATE MAXIMIZATION FOR COGNITIVE MISO BROADCAST CHANNEL: LARGE SYSTEM ANALYSIS

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ABSTRACT

This paper considers the ergodic weighted sum rate (WSR) maximization problem for an underlay cognitive radio MISO broadcast channel, where a secondary network, consisting of a base-station with M transmit antennas and K single-antenna secondary users (SUs), is allowed to share the same spectrum with a primary user (PU), under an average transmit sum power (ATTP) constraint P_{av} and an average interference power (AIP) constraint on the PU. We show that the ATTP constraint is always active, and as $P_{av} \to \infty$, the ergodic WSR approaches infinity similar to the conventional non-CR network case. A low-complexity suboptimal beamforming scheme (called partially-projected regularized zero-forcing beamforming 'PP-RZFBF') with a closed-form beamformer is proposed. Due to the non-convexity of PP-RZFBF scheme, a large system analysis is conducted in the limit as M and K approach infinity with a fixed finite ratio $r = \frac{K}{M}$. We derive deterministic limiting approximations for the PP-RZFBF problem which enables us to determine asymptotically optimal beamformers for PP-RZFBF. Numerical simulations illustrate that the asymptotically optimal beamformers turn out to be quite effective even for small M, K.

***Index Terms*—** Broadcast channel, cognitive radio, beamforming, multiple-input single-output, large system analysis.

1. INTRODUCTION

Cognitive radio (CR), a promising approach to dramatically increase the spectrum utilization efficiency by allowing unlicensed/secondary users (SUs) to access the spectrum originally licensed to the primary users (PUs), has recently attracted a broad range of research interests. Effectively, three categories of CR network paradigms have been proposed: interweave, overlay, and underlay [1]. In the underlay systems (the focus of this paper), the SU can transmit even when the PU is active, as long as the resulting interference on PU does not excess an acceptable limit. Thus the transmit power of SUs should be controlled properly to achieve the best trade-off between maximizing the secondary throughput and minimizing the interference on PUs.

Under such a scenario, in order to enhance the secondary throughput, well studied MIMO technology can be employed to fully exploit the spectral efficiency of SUs. Throughput maximization of a CR point-to-point MIMO network with multiple PUs has been considered in [2], showing that beamforming is the optimal transmit strategy for the MISO case. In [3], the authors studied the weighted sum rate (WSR) maximization problem for CR Multi-SUs MIMO broadcast channel (BC) with a dirty paper coding (DPC) precoder (motivated by the fact that DPC is the optimal capacity achieving scheme for non-CR MIMO BC [4]). The corresponding non-convex problem can be transformed into an equivalent convex CR MIMO MAC problem by applying BC-MAC duality. In [5], the authors provided an state-of-art overview of CR MIMO (P-P/BS/MAC/ad hoc). Due to high computational complexity of DPC, suboptimal linear precoding techniques (such as beamforming) have become the focus of substantial research activities. The authors of [6] investigated a non-convex WSR maximization problem for a CR multi-user MISO interference channel with linear beamforming transmission scheme and obtained a locally optimum beamformer using an iterative algorithm.

In this paper, we consider the ergodic WSR maximization problem for an underlay CR multi-user MISO broadcast channel (CR-MISO-BC), subject to an average transmit sum power (ATTP) constraint at the secondary base-station (CR-BS) and an average interference power (AIP) constraint at the PU. The system setting is similar to [3], but unlike [3], we utilize a linear transmit beamforming strategy instead of dirty paper coding (DPC). Our main contributions are summarized as follows:

- In our CR-MISO-BC WSR maximization problem, we prove that the ATTP constraint is always active, and as $P_{av} \to \infty$, the ergodic WSR approaches infinity. This is different from prior research on the CR network with only a single antenna at the transmitters and receivers (such as [7][8]), where with fixed Q_{av}, as P_{av} increases, the capacity saturates since only the AIP constraint is satisfied with equality.
- Due to non-convexity, there is no explicit solution for our problem. Although we can numerically find a local optimum using an iterative algorithm similar to [6], it is computationally intensive and not amenable to further analysis. Motivated by this, we derive a low-complexity albeit suboptimal strategy (called 'PP-RZFBF') with a closed-form beamformer, by combining the regularized zero-forcing beamforming (RZFBF) [9] with the subspace projection idea proposed by [2][5][10].
- Designing the PP-RZFBF beamformer involves finding optimal values of the projection control parameter β and the regularization parameter α. Guided by a similar approach in non-CR networks, such as [11][12][13] (which only has parameter α), we derive deterministic large system approximations for the PP-RZFBF scheme in the limit as M and K tend to infinity with fixed ratio $r = \frac{K}{M}$. This allows us to derive the asymptotically optimal β, α for the PP-RZFBF scheme and also helps us characterize the asymptotic behavior of our CR-MISO-BC system. To the best of our knowledge, large system analysis on CR network has not been addressed in any existing literature.
- We also show that in the large system limit, as $P_{av} \to \infty$, the interference on PU caused by the secondary transmission goes to zero asymptotically.

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2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an underlay CR-MISO-BC, as illustrated in Fig.1, where a CR-BS equipped with M transmit antennas communicates with K single-antenna SUs, in the presence of a single-antenna PU. Regardless of the on/off status of the PU, the secondary network is allowed to share the same narrowband spectrum with the PU, as long as the degradation of the received signal quality of PU caused by the transmission of the secondary network does not exceed an acceptable level. All channels involved are i.i.d. and assumed to be flat-fading. For $i = 1, \ldots, K$, let $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$ and $\mathbf{h}_0 \in \mathbb{C}^{M\times 1}$ denote the channel vector from the CR-BS to SU$_i$ and PU, respectively, where the elements of each $\mathbf{h}_j, \forall j = 0, 1, \ldots, K$ are assumed to be i.i.d complex Gaussian random variables with zero mean and unit variance. Then, the received signal at each SU$_i$, indicated by y_i, is given as, for $i = 1, \ldots, K$,

$$y_i = \sqrt{p_i}\mathbf{h}_i^{\mathcal{H}}\mathbf{g}_i s_i + \sum_{j\neq i}^{K} \sqrt{p_j}\mathbf{h}_i^{\mathcal{H}}\mathbf{g}_j s_j + n_i, \quad (1)$$

where n_i is the additive white Gaussian noise [1] at the SU$_i$ and $n_i \sim \mathcal{CN}(0, 1)$; $p_i \geq 0$ is the transmit signal power for SU$_i$; s_i denotes the transmit symbol destined to SU$_i$ and is linearly precoded by the transmit beamforming vector $\mathbf{g}_i \in \mathbb{C}^{M\times 1}$. Subsequently, the ATTP constraint can be written as $E[\sum_{i=1}^{K} p_i|\mathbf{g}_i|^2] \leq P_{av}$, where P_{av} is the maximum average transmit power at CR-BS. Let Q_{av} denote the average interference power limit tolerated by the PU, then the AIP constraint can be written as, $E[\sum_{i=1}^{K} p_i|\mathbf{h}_0^{\mathcal{H}}\mathbf{g}_i|^2] \leq Q_{av}$.

Instead of applying dirty paper coding (DPC) at CR-BS (as [3]) or multiuser decoding at receivers to cancel the multiuser interference, each receiver SU$_i$ ($i = 1, \ldots, K$) decodes the transmitted symbol s_i by simply treating the multiuser interference as noise [14][15]. Then given full channel state information (CSI), i.e., $\mathbf{H} \triangleq \{\mathbf{h}_1, \ldots, \mathbf{h}_K\} \in \mathcal{C}^{M\times K}$ and $\mathbf{h}_0$, the signal to interference plus noise ratio (SINR) at each receiver SU$_i$ is given as, SINR$_i$ $= \frac{p_i|\mathbf{h}_i^{\mathcal{H}}\mathbf{g}_i|^2}{1+\sum_{j=1,j\neq i}^{K} p_j|\mathbf{h}_i^{\mathcal{H}}\mathbf{g}_j|^2}$. Therefore, with the assumption of perfect CSI, the WSR maximization problem for CR-MISO-BC, under both an ATTP constraint at CR-BS and an AIP constraint at the PU, can be formulated as,

$$\underset{\{\mathbf{g}_i\}_{i=1}^{K},\ \{p_i\geq 0\}_{i=1}^{K}}{\text{maximize}} \quad R_{\text{sum}} = \sum_{i=1}^{K} w_i E[\log(1 + \text{SINR}_i)]$$
$$s.t.\ E\left[\sum_{i=1}^{K} p_i|\mathbf{g}_i|^2\right] \leq P_{av},\ E\left[\sum_{i=1}^{K} p_i|\mathbf{h}_0^{\mathcal{H}}\mathbf{g}_i|^2\right] \leq Q_{av}. \quad (2)$$

where w_i is the weight for SU$_i$ and the expectation is taken over $\mathbf{H}, \mathbf{h}_0$.

Proposition 1 *In Problem (2), given any value of P_{av} and Q_{av}, the ATTP constraint is always active, and the ergodic WSR $R_{sum} \rightarrow \infty$, as $P_{av} \rightarrow \infty$. (Proof. Please See [16])* ■

It is easy to verify that Problem (2) is a non-convex optimization problem. In general, there is no explicit solution for Problem (2). Although we can numerically find a local optimum for Problem (2) by solving its Lagrange dual problem using an iterative algorithm similar to [6], it is computationally complex and it also hinders further analysis of the beamforming strategy. Next, we will derive a low-complexity albeit suboptimal strategy for Problem (2) with a simple closed-form beamforming solution.

[1]Here, similar to [2], the additive noise n_i at the receiver SU$_i$ is assumed to include any other interferences from outside of the secondary network, such as, the interference from the primary transmitters

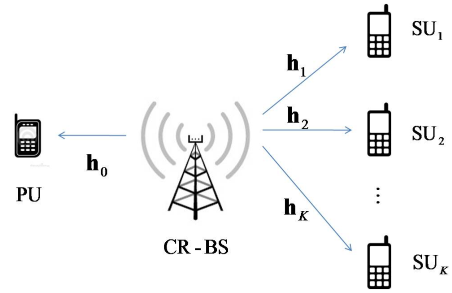

Fig. 1. CR-MISO-BC System Model

3. A SUBOPTIMAL BEAMFORMING SCHEME

Let $\mathbf{G} \triangleq \{\mathbf{g}_1, \ldots, \mathbf{g}_K\} \in \mathbb{C}^{M\times K}$ denote the beamforming matrix. Similar to [5], we first look at two extreme cases of Problem (2):

Case I: High Q_{av} case (large enough to make the AIP constraint inactive). In this scenario, Problem (2) reduces to the conventional non-cognitive MISO-BC WSR optimization problem subject to the ATTP constraint only. A popular, straightforward and effective suboptimal scheme for the conventional MISO-BC is known as zero-forcing beamforming (ZFBF)[9], where the beamforming vectors are designed to satisfy the ZF criterion (i.e., $\mathbf{h}_i^{\mathcal{H}}\mathbf{g}_j = 0, \forall j \neq i$), resulting in complete cancellation of the multiuser interference. One easy choice for the beamforming matrix $\mathbf{G}$ that meets the ZF criterion is the pseudo-inverse of $\mathbf{H}^{\mathcal{H}}$, i.e.,$\mathbf{G} = \mathbf{H}(\mathbf{H}^{\mathcal{H}}\mathbf{H})^{-1}$. However, as pointed out by [9], ZFBF has some shortcomings, such as, the inverse of $\mathbf{H}^{\mathcal{H}}\mathbf{H}$ may not exist (for example, when $K > M$, rank$(\mathbf{H}^{\mathcal{H}}\mathbf{H}) < K$) and when $K = M$, the sum-rate of ZFBF does not grow linearly with K (or M). These drawbacks can be improved by adding a regularization term to the ZFBF matrix, given as,

$$\mathbf{G} = \mathbf{H}(\mathbf{H}^{\mathcal{H}}\mathbf{H} + \alpha\mathbf{I})^{-1} \overset{(a)}{=} (\mathbf{H}\mathbf{H}^{\mathcal{H}} + \alpha\mathbf{I})^{-1}\mathbf{H}, \quad (3)$$

where $\alpha > 0$ is the regularization parameter and controls the amount of multiuser interference; (a) is given by [12]. This matrix is known as regularized zero-forcing beamforming (RZFBF) matrix. As shown by [9], RZFBF can achieve significantly better sum-rate performance than ZFBF, especially at low P_{av}, and with fixed K, as $P_{av} \rightarrow \infty$, the sum-rate of RZFBF approaches that of ZFBF.

Case II: $Q_{av} = 0$ case. In this case, the AIP constraint in Problem (2) becomes $\mathbf{h}_0^{\mathcal{H}}\mathbf{g}_i = 0, \forall i = 1, \ldots, K$, which implies any interference on the PU caused by the secondary network is completely removed. Similar to [2][5], we first project $\mathbf{H}$ into the null space of $\mathbf{h}_0$, given as

$$\mathbf{H}_{\perp} = (\mathbf{I} - \bar{\mathbf{h}}_0\bar{\mathbf{h}}_0^{\mathcal{H}})\mathbf{H}, \quad (4)$$

where $\bar{\mathbf{h}}_0 = \frac{\mathbf{h}_0}{|\mathbf{h}_0|}$, and then design the beamforming matrix G based on the projected channel matrix $\mathbf{H}_{\perp}$, so that the constraints can $\mathbf{h}_0^{\mathcal{H}}\mathbf{g}_i = 0, \forall i = 1, \ldots, K$ can be met. More specifically, from (4), we have $\mathbf{H} = \mathbf{H}_{\perp} + \bar{\mathbf{h}}_0\bar{\mathbf{h}}_0^{\mathcal{H}}\mathbf{H}$ and $\mathbf{h}_0^{\mathcal{H}}\mathbf{H}_{\perp} = \mathbf{0}$, which gives $\mathbf{H}^{\mathcal{H}}\mathbf{g}_i = \mathbf{H}_{\perp}^{\mathcal{H}}\mathbf{g}_i, \forall i = 1, \ldots, K$. Applying this to (2), Problem (2) again becomes a conventional non-cognitive MISO-BC with channel matrix $\mathbf{H}_{\perp}$, where according to **Case I**, the suboptimal RZFBF

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scheme can be applied to design the beamforming matrix, given as

$$\mathbf{G}=\mathbf{H}_{\perp}(\mathbf{H}_{\perp}^{\mathcal{H}}\mathbf{H}_{\perp}+\alpha\mathbf{I})^{-1}=(\mathbf{H}_{\perp}\mathbf{H}_{\perp}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\mathbf{H}_{\perp}. \quad (5)$$

Obviously, from (5), $\mathbf{h}_0^{\mathcal{H}}\mathbf{G}=0$, i.e., the constraint $\mathbf{h}_0^{\mathcal{H}}\mathbf{g}_i=0, \forall i=1,\dots,K$ is always satisfied.

In both of the above two special cases, the RZFBF scheme is employed to design the beamforming matrix based on a certain form of $\mathbf{H}$ (which preserves a certain orthogonality to $\mathbf{h}_0$). By comparing (3) and (5), it seems that as Q_{av} decreases, the amount of projection of $\mathbf{H}$ into the null space of $\mathbf{h}_0$ also increases. This motivates a general heuristic suboptimal method for Problem (2), called partially-projected-RZFBF (PP-RZFBF). In this method, similar to [2][5][10], first the secondary channel matrix $\mathbf{H}$ is projected into the null space of a certain subspace of $\mathbf{h}_0$, denoted as $\tilde{\mathbf{H}}$ and given as [10]

$$\tilde{\mathbf{H}}=(\mathbf{I}-\beta\bar{\mathbf{h}}_0\bar{\mathbf{h}}_0^{\mathcal{H}})\mathbf{H}, \quad (6)$$

where $0\le\beta\le 1$ is the projection control parameter. The RZFBF algorithm is then applied to $\tilde{\mathbf{H}}$ to obtain the beamforming matrix, namely,

$$\mathbf{G}=\tilde{\mathbf{H}}(\tilde{\mathbf{H}}^{\mathcal{H}}\tilde{\mathbf{H}}+\alpha\mathbf{I})^{-1}=(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\tilde{\mathbf{H}}, \quad (7)$$

which gives, $\mathbf{g}_i=(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\tilde{\mathbf{h}}_i,\ \forall i=1,\dots,K$ where the vector $\tilde{\mathbf{h}}_i$ is the i^{th} column of $\tilde{\mathbf{H}}$. Obviously, $\beta=0$ and $\beta=1$ are corresponding to **Case I** and **Case II**, respectively.

Therefore, Problem (2) with a suboptimal PP-RZFBF scheme can be simplified as,

$$\begin{aligned}\underset{\beta,\ \alpha,\ \{p_i\}_{i=1}^K}{\text{maximize}}\quad & R_{\text{PP-RZFBF}}=\sum_{i=1}^{K}E\left[w_i\log(1+\text{SINR}_i)\right]\\ s.t.\quad & E\left[\phi\right]\le P_{av},\\ & E\left[\psi\right]\le Q_{av},\\ & 0\le\beta\le 1,\ \ \alpha>0,\ p_i\ge 0,\forall i=1,\dots,K. \quad (8)\end{aligned}$$

where $\phi\triangleq\sum_{i=1}^{K}p_i\tilde{\mathbf{h}}_i^{\mathcal{H}}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-2}\tilde{\mathbf{h}}_i, \psi\triangleq\sum_{i=1}^{K}p_i|\mathbf{h}_0^{\mathcal{H}}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\tilde{\mathbf{h}}_i|^2$ and $\text{SINR}_i=\frac{p_i|\mathbf{h}_i^{\mathcal{H}}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\tilde{\mathbf{h}}_i|^2}{1+\mathbf{h}_i^{\mathcal{H}}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\tilde{\mathbf{H}}_{-i}\mathbf{P}_{-i}\tilde{\mathbf{H}}_{-i}^{\mathcal{H}}(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathcal{H}}+\alpha\mathbf{I})^{-1}\mathbf{h}_i}$, with $\tilde{\mathbf{H}}_{-i}\triangleq\{\tilde{\mathbf{h}}_1,\dots,\tilde{\mathbf{h}}_{i-1},\tilde{\mathbf{h}}_{i+1},\dots,\tilde{\mathbf{h}}_K\}\in\mathcal{C}^{M\times K-1}$ and $\mathbf{P}_{-i}\triangleq\text{diag}(p_1,\dots,p_{i-1},p_{i+1},\dots,p_K)$. Although the complexity is largely reduced compared to Problem (2), Problem (8) is still a non-convex optimization problem. However, for given β and α, a locally optimal power allocation scheme can be obtained by using the SCALE algorithm proposed by [17]. Then, one can utilize a two-dimensional exhaustive search method to get the optimal β and α, with which, a locally optimal PP-RZFBF beamformer can be obtained. Obviously, an exhaustive search over both β and α is rather difficult, especially when the searching range for α is from 0 to ∞. This motivates us to study the large system case, which enables us to derive simple but asymptotically optimal expressions for β^* and α^*.

4. LARGE SYSTEM ANALYSIS

In this section, we will investigate the large system approximations of the PP-RZFBF scheme in order to characterize the asymptotical behavior of our CR-MISO-BC system, and derive the asymptotically optimal solutions of Problem (8), in the limit as M and K grow jointly to infinity with a fixed ratio $r=\frac{K}{M}<\infty$ (called the large system limit). We assume that $\frac{1}{M}\mathbf{HH}$ has a uniformly bounded spectral norm for all M and $\max(p_1,\dots,p_K)=\mathcal{O}(\frac{1}{K})$. Then, guided by the approach in [11][12][13], we can show in the Theorem below that in the large system limit, SINR_i,ϕ,ψ converge almost surely to deterministic values, denoted by $\text{SINR}_i^{\infty},\phi^{\infty},\psi^{\infty}$, respectively. Later, this will enable us to determine the asymptotically optimal design parameters α^*,β^* for Problem (8).

Theorem 1 *As $M\to\infty$, $K\to\infty$ with a fixed and finite $r=\frac{K}{M}$,*

$$SINR_i\overset{a.s.}{\to}\frac{p_i z(r,\alpha_0)^2}{(1+z(r,\alpha_0))^2+\frac{rz(r,\alpha_0)\left(\frac{1}{K}\sum_{j=1}^{K}p_j-\frac{p_i}{K}\right)}{r+\alpha_0(1+z(r,\alpha_0))^2}}, \quad (9a)$$

$$\phi\overset{a.s.}{\to}\frac{rz(r,\alpha_0)}{r+\alpha_0(1+z(r,\alpha_0))^2}\left(\frac{1}{K}\sum_{i=1}^{K}p_i\right), \quad (9b)$$

$$\psi\overset{a.s.}{\to}\frac{(1-\beta)^2rz(r,\alpha_0)\left(\frac{1}{K}\sum_{i=1}^{K}p_i\right)}{\left(1+(\beta^2-2\beta)(1-\alpha_0z(r,\alpha_0))\right)^2(r+\alpha_0(1+z(r,\alpha_0))^2)}, \quad (9c)$$

where $z(r,\alpha_0)=\frac{1}{2}[\sqrt{\frac{(1-r)^2}{\alpha_0{}^2}+\frac{2(1+r)}{\alpha_0}+1}+\frac{1-r}{\alpha_0}-1]$ and $\alpha_0=\frac{\alpha}{M}$. (Proof. Please See [16]) ■

From Theorem 1, it is seen that $\text{SINR}_i^{\infty},\phi^{\infty},\psi^{\infty}$ are all deterministic quantities and do not depend on any CSIT ($\mathbf{H}$, h_0), which implies that in the large system limit, CSIT information is no longer required. Note that with assumption $\max(p_1,\dots,p_K)=\mathcal{O}(\frac{1}{K})$, the term $\frac{p_i}{K}$ in (9a) can be omitted since $\frac{p_i}{K}\ll\frac{1}{K}\sum_{j=1}^{K}p_j$ as $K\to\infty$ and without $\frac{p_i}{K}$ the convergence in (9a) still holds true. Let $\tau(r,\alpha_0)\triangleq\frac{rz(r,\alpha_0)}{r+\alpha_0(1+z(r,\alpha_0))^2}, \rho(r,\alpha_0,\beta)\triangleq\frac{(1-\beta)^2}{(1+(\beta^2-2\beta)(1-\alpha_0 z(r,\alpha_0)))^2}$ and $\upsilon_0\triangleq\frac{z(r,\alpha_0)^2}{(1+z(r,\alpha_0))^2+\tau(r,\alpha_0)\frac{1}{K}\sum_{j=1}^{K}p_j}$. Then, $\text{SINR}_i^{\infty},\phi^{\infty},\psi^{\infty}$ can be rewritten as $\text{SINR}_i^{\infty}=p_i\upsilon_0$, $\phi^{\infty}=\tau(r,\alpha_0)\frac{1}{K}\sum_{i=1}^{K}p_j$, and $\psi^{\infty}=\rho(r,\alpha_0,\beta)\tau(r,\alpha_0)\frac{1}{K}\sum_{i=1}^{K}p_j$, respectively . Therefore, in the large system limit, the Problem (8) becomes,

$$\begin{aligned}\underset{0\le\beta\le1,\ \alpha_0>0,\ p_i\ge0,\forall i=1,\dots,K}{\text{maximize}}\quad & R_{\text{PP-RZFBF}}^{\infty}=\sum_{i=1}^{K}w_i\log(1+p_i\upsilon_0)\\ s.t.\quad & \frac{1}{K}\sum_{i=1}^{K}p_i=\overline{P}. \quad (10)\end{aligned}$$

where

$$\overline{P}=\begin{cases}\min\{\frac{P_{av}}{\tau(r,\alpha_0)},\frac{Q_{av}}{\rho(r,\alpha_0,\beta)\tau(r,\alpha_0)}\}, & \text{when }\ \rho(r,\alpha_0,\beta)\ne 0;\\ \frac{P_{av}}{\tau(r,\alpha_0)}, & \text{when }\ \rho(r,\alpha_0,\beta)=0.\end{cases} \quad (11)$$

Applying the power constraint of Problem (10) into the definition of υ_0, we have $\upsilon_0=\frac{z(r,\alpha_0)^2}{(1+z(r,\alpha_0))^2+\tau(r,\alpha_0)\overline{P}}$, which is the same for all SU_i, $i=1,\dots,K$.

Next, we will find the asymptotically optimal design parameters by solving the optimization problem (10).

Theorem 2 *As $M,K\to\infty$ with a bounded ratio r, the asymptotically optimal regularization parameter α_0, asymptotically optimal projection control parameter β as well as the asymptotically optimal power allocation for Problem (8) are given, respectively, as:*

$$\alpha_0^*=\frac{r}{P_{av}}; \quad (12)$$

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- *When* $0 \le \sqrt{\frac{Q_{av}}{P_{av}}} < 1$, *we have* $\phi_1 \le \beta^* \le 1$*;*
- *When* $1 \le \sqrt{\frac{Q_{av}}{P_{av}}} \le \frac{1}{2}\frac{1}{\sqrt{\alpha_0^* z(r,\alpha_0^*)(1-\alpha_0^* z(r,\alpha_0^*))}}$, *we have,*

$$\begin{cases} \phi_1 \le \beta^* \le 1 \text{ or } 0 \le \beta^* \le \phi_2, \text{when } \alpha_0^* z(r,\alpha_0^*) < 0.5, \\ 0 \overset{(a)}{\le} \beta^* \le 1, \qquad\qquad \text{when } \alpha_0^* z(r,\alpha_0^*) \ge 0.5; \end{cases}$$

- *when* $\sqrt{\frac{Q_{av}}{P_{av}}} > \frac{1}{2}\frac{1}{\sqrt{\alpha_0^* z(r,\alpha_0^*)(1-\alpha_0^* z(r,\alpha_0^*))}}$, $0 \le \beta^* \le 1$.

where in (a), the equality could only happen when $\sqrt{\frac{Q_{av}}{P_{av}}} = 1$*;* $\phi_1 \triangleq J(\alpha_0^*, z(r,\alpha_0^*), t) - \frac{1}{2t(1-\alpha_0^* z(r,\alpha_0^*))} + 1$ *and* $\phi_2 \triangleq -J(\alpha_0^*, z(r,\alpha_0^*), t) - \frac{1}{2t(1-\alpha_0^* z(r,\alpha_0^*))} + 1$ *with* $J(\alpha_0^*, z(r,\alpha_0^*), t) \triangleq \sqrt{1 - \frac{1}{1-\alpha_0^* z(r,\alpha_0^*)} + \frac{1}{4t^2(1-\alpha_0^* z(r,\alpha_0^*))^2}}$, *and* $t = \sqrt{\frac{Q_{av}}{P_{av}}}$.

$\{p_i^*\}_{i=1}^K$ *is given by the conventional water-filling, i.e.,*

$$p_i^* = \left[\frac{w_i}{\lambda_\infty} - \frac{1}{\upsilon_0}\right]^+, \ \forall i = 1, \dots, K \tag{13}$$

where λ_∞ *is the Lagrange multiplier determined by* $\frac{1}{K}\sum_{i=1}^K p_i = \overline{P}$*. Without loss of generality, assuming* $w_1 \ge \dots \ge w_K$*, from (13), we obtain* $p_1 \ge \dots \ge p_K$*. Assuming there is* $\overline{K}$ *non-zero powers and applying (13) into the power constraint* $\frac{1}{K}\sum_{i=1}^K p_i = \overline{P}$*, we can obtain* $\lambda_\infty = \frac{\frac{1}{K}\sum_{i=1}^{\overline{K}} w_i}{\frac{\overline{K}}{K}\frac{1}{\upsilon_0} + \overline{P}}$*. (Proof. Please See [16])* ■

Based on Theorem 2, we have

$$R^{\infty}_{\text{PP-RZFBF}} = \left(\sum_{i=1}^{\overline{K}} w_i\right)\log\left(\frac{\overline{K}}{K} + \overline{P}\upsilon_0\right) + \sum_{i=1}^{\overline{K}} w_i \log w_i - \left(\sum_{i=1}^{\overline{K}} w_i\right)\log\left(\frac{1}{K}\sum_{i=1}^{\overline{K}} w_i\right) \tag{14}$$

and the optimal $\overline{P}$ is always $\overline{P} = \frac{P_{av}}{\tau(r,\alpha_0^*)}$. Note that in the case of $w_1 = \dots = w_K = w$, we have equal power allocation for each SUs, i.e., $p_1 = \dots = p_K = \overline{P}$, and the corresponding $R^{\infty}_{\text{PP-RZFBF}} = Kw\log(1 + \overline{P}\upsilon_0)$.

Proposition 2 *In the large system limit, for any given r, as* $P_{av} \to \infty$*, we have* $\alpha_0^* \to 0$ *and* $\beta^* \to 1$*. (Proof. Please See [16])* ■

Proposition 2 implies that in the large system limit, as $P_{av} \to \infty$, the secondary channel partial projection $\tilde{\mathbf{H}}$ is asymptotically orthogonal to the primary channel $\mathbf{h}_0$ and the interference on PU caused by the secondary transmission is asymptotically cancelled.

5. NUMERICAL RESULTS

In Fig.2, we compare the ergodic WSR between the optimal PP-RZFBF, obtained via an exhaustive search over all possible α and β, and the PP-RZFBF directly applied with the asymptotically optimal α^* and β^* obtained from large system analysis. We call it as 'PP-RZFBF-ABF', and since β^* is given by a range of values, a one dimensional exhaustive search over all possible β^* is also required but it is computationally less complex than the optimal PP-RZFBF case. These results are obtained for a finite system with $K = M = 3$ for different Q_{av}. Both cases have been averaged over 10000 independent channel realization. It can be observed from Fig.2 that, for each Q_{av}, these two performances are very close to each other. Thus even for small values of K, M, the asymptotically optimal α^* and β^* can provide pretty accurate approximations. Fig.3 depicts the difference between large system analysis based deterministic approximation of WSR of PP-RZFBF, i.e., $R^{\infty}_{\text{PP-RZFBF}}$ and the $R_{\text{PP-RZFBF}}$ of the optimal PP-RZFBF-ABF scheme for a fixed $P_{av} = 10$dB, $Q_{av} = 10$dB with $M = K$. From Fig.3, we can see that $R^{\infty}_{\text{PP-RZFBF}}$ becomes more accurate as M increases.

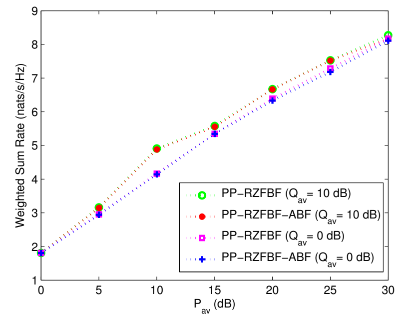

Fig. 2. Ergodic WSR performance comparison between PP-RZFBF and PP-RZFBF with the asymptotically optimal beamformer (PP-RZFBF-ABF) for $K = M = 3$.

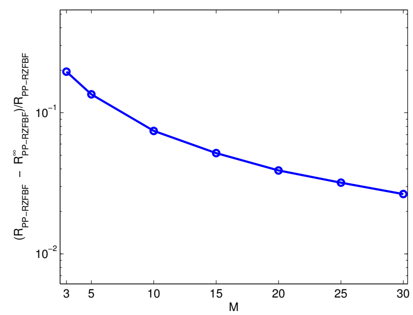

Fig. 3. The difference between the limiting approximation for WSR of PP-RZFBF and the corresponding optimal PP-RZFBF scheme for a fixed $P_{av} = 10$dB, $Q_{av} = 10$dB with $M = K$.

6. CONCLUSIONS

In this paper, we consider the optimal transmit beamforming scheme for an underlay CR-MISO-BC ergodic WSR maximization problem, subject to an ATTP constraint at the CR-BS and an AIP constraint on the PU. In order to further analyze the beamforming technique, we propose a low-complexity suboptimal beamforming scheme (called partially-projected regularized zero-forcing beamforming 'PP-RZFBF') with a closed-form beamformer. A large system analysis is then applied to derive deterministic approximations for PP-RZFBF scheme, based on which, asymptotically optimal values for the parameters β and α for PP-RZFBF can be obtained. Numerical simulations confirm the accuracy of asymptomatic optimal β and α for finite-sized systems and illustrate that the asymptotic expression of WSR approximates the PP-RZFBF behavior extremely well for large M with $r = 1$.

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