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Decision Points in Mathematics Lectures

We report on a study of decision-making in mathematics lectures. The data comes from accounts of classroom incidents written over two academic years by three university mathematics lecturers who set out to reflect on, and improve, their practice by collectively engaging in a professional development project using Mason's (2002) Discipline of Noticing. The accounts were analysed to locate decision points during lectures. We report on a classification of these decision points and give examples. We found that 72% of the decision points that occurred were student-focussed while the remainder focussed on the practice of teaching. The professional development implications of our study will be discussed.

Keywords: Discipline of Noticing, decision points, professional development, reflection, university mathematics.

Introduction

Of necessity we, as university lecturers, develop habits and routine ways of acting in the classroom. As Mason (2002) explains: "We cannot afford to think out a response to each emerging incident. Habits must be developed in order to free our attention to keep in mind our over-all goal" (p.8). Often we will be happy to continue acting in a well-practiced way unless we experience some form of disturbance or trigger that challenges us, or we engage in professional development that causes us to question our tried-and-tested approach. To grow professionally as educators we need to become aware of missed opportunities in the classroom – occasions where we could have made a better decision or acted in a more productive manner. Schoenfeld (2007) views teaching practice as involving

> a significant amount of routine activity punctuated by occasional and at times unplanned but critically important decision making – decision making that can determine the success or failure of the effort (p.33).

Hatton and Smith (1995) describe a professional practitioner as one who

> is able consciously to think about an action as it is taking place, making sense of what is happening and shaping successive practical steps using multiple viewpoints as appropriate (p. 46).

Lecturers need to sensitise themselves to recognise the opportunity to act differently and have alternative actions come to mind as the opportunity unfolds. Mason (2011) claims that "the mark of effective professional development is that participants can imagine themselves in the future acting responsively and freshly rather than habitually" (p.38).

The three authors of this paper are mathematics lecturers from different institutions in Ireland who embarked on a collaborative professional development project with the aim of reflecting on their individual teaching practice in an effort to both examine and improve practice. Along with two other colleagues who were part of the original group, we followed the Discipline of Noticing as outlined by Mason (2002). In a subsequent analysis of our reflective accounts, it was remarked that they often contained descriptions of opportunities to make decisions that arose during a mathematics lecture – we have labelled these moments 'decision points'. In our conception of 'decision point', we include any instance when a non-trivial choice between alternative courses of action could be taken; thus contingent moments and events noticeable to an outside observer in the sense of Rowland, Huckstep and Thwaites (2005) are decision points, but so too are opportunities for choices observable only by the lecturer themselves. For example, the latter type of decision point might arise when a lecturer has a realisation concerning some of their teaching goals.

The aim of this paper is to outline the most common types of decision points in the lectures that were reflected on by the lecturers.

Literature Review

There is general agreement that some level of reflection is necessary if we are to learn from experience. Schön (1983) developed the concept of reflective practice as a means of developing professional excellence, making a distinction between 'reflection-on-practice' and 'reflection-in-practice'. He explained

> "Through reflection, he [the practitioner] can surface and criticize the tacit understandings that have grown up around the repetitive experiences of a specialized practice, and make new sense of the situations of uncertainty or uniqueness which he may allow himself to experience. Practitioners do reflect *on* their knowing-in-practice…But they may also reflect on practice while they are in the midst of it. Here they reflect-in-action" (p.61-62)

Quinton and Smallbone (2010) asserted "the ability to reflect on and analyse material in order to form reasoned judgements is central to critical thinking and deeper learning" (p.126), and advised that such reflection needs to be captured and recorded in some way. They explained how reflection can facilitate the development of skills in self-assessment, and the transference of tacit self-knowledge into clear improvement plans. Moreover, the value of reflection specifically for teachers at all levels has been widely acknowledged (Ticha and Hospesova, 2006).

In terms of professional development, decisions made by a teacher in a classroom setting can provide the context for a discussion on what alternative decisions might have been made by the teacher. As early as 1978, Peterson and Clark identified four possible decision paths used by teachers during classes. They describe how teachers observe student behaviour and judge whether or not it is within desirable limits. This is followed by a decision to continue the teaching process as planned or to search memory for an alternative mode of teaching which might serve to bring student behaviour back within the limits of tolerance.

Ainley and Luntley (2007) proposed a novel theoretical framework to account for the ways in which teachers make choices about how to act in the moment: in addition to subject knowledge, and general and subject-specific pedagogical knowledge they believe that experienced teachers draw on 'attention-dependent knowledge'. The latter is a "highly contextualised propositional knowledge that is made available by attending to aspects of the classroom situation" (Ainley & Luntley, 2007, p.4). It cannot be written down but becomes available during the complexity of the progress of a lesson.

The Knowledge Quartet (Rowland et al. 2005) is a framework that was developed to understand how teachers draw on their knowledge of mathematics and pedagogy in the classroom. It provides a means of discussing observed teaching practice with pre-service teachers and their tutors, and consists of four dimensions: foundation, transformation, connection, and contingency. The latter is particularly relevant for this study since it refers to the ability to 'think on one's feet' and to deviate from a planned agenda in order to respond to a student's contribution. This is what Schön (1983) referred to as 'reflection-in-action' while Rowland et al. (2005, Table 1 p.266) described this category as 'knowledge in interaction', thus emphasising the role of both the teacher and the student in a contingent moment. Rowland, Thwaites and Jared (2011) found that contingent situations in school classrooms were triggered by three categories of events: responding to student ideas (including students' responses, either correct or incorrect, to a question and their spontaneous responses to an activity or discussion), teacher insight (as the teacher monitors his own actions and self-regulates) and responding to the (un)availability of tools and resources. They found that the great majority of triggers of contingency were in the first of these categories.

There have been relatively few studies which involve mathematicians at university level reflecting on their own teaching or describing in-the-moment decision making in their classrooms (Speer, Smith and Horvath, 2010). However, some work has been done on this

topic recently for example by McAlpine, Weston, Beauchamp, Wiseman and Beauchamp (1999); and Paterson, Thomas, and Taylor (2011).

McAlpine et al. (1999) reported on a reflection project at two Canadian universities involving three mathematics professors and three mathematics education professors. The professors were interviewed prior to some of their classes, and again while watching a recording of the lecture. The study found that the participants monitored their teaching actions in order to see if their pedagogical goals were achieved. When monitoring the lecturers evaluated a multitude of cues: over 70% of these were related to students and included individual student's responses, expressions or written work. The cues, if judged to be outside a corridor of tolerance, could lead to a decision on behalf of the lecturer to change plan. McAlpine et al. (1999) believed the level of attention paid to student cues was due to the professors' recognition of the fact they are the primary vehicle for assessing student learning. As they describe it, "reflection is a mechanism for turning experience into knowledge…a process of formative evaluation in which one collects and evaluates feedback to revise and improve instruction" (McAlpine et al., 1999, p.116).

Paterson et al. (2011) examined selected excerpts of video recordings of undergraduate mathematics lectures. They drew on Schoenfeld's (2011) Resources, Goals and Orientations (ROG) framework of teaching-in-context to try to explain why teachers make particular in-the-moment choices, either consciously or unconsciously, while teaching, and claimed that the decisions that are made are consistent with the goals a lecturer sets for him/herself. Their analysis suggested that the creation of a forum to discuss the decisions involved in lecturing situations in a supportive manner can have a positive impact on professional growth.

The studies on reflection in university mathematics teaching above employed different conceptualisations of reflection but have at least two features in common - firstly,

reflection is undertaken as a collaborative practice involving both mathematicians and mathematics educators; and secondly, decisions made by the lecturers in class and/or the cues or factors that prompted these decisions, provide a focus for some of the productive discussions between both groups.

Methodology

All three authors lecture Mathematics at university level in the Republic of Ireland, each working at a different institution. They embarked on a Reflection on Teaching project along with two other lecturers, in which each individual undertook to write an account of moments or incidents which occurred in a number of their classes per week, over the academic years 2010/2011 and 2011/2012. The written accounts were then circulated electronically to all members of the group. The group met regularly (every six to eight weeks) over this time to discuss the accounts circulated and any matters of interest which arose. We will report here on the accounts written by the three authors. The courses in question were two calculus courses, a business mathematics course, and an introductory analysis course.

Throughout the project we endeavoured to follow the philosophy and ideas of Mason (2002) as described in the book 'Researching your own Practice: The Discipline of Noticing'. In that regard, our aim was to write accounts which were 'brief-but-vivid', that is, both as short and as free from opinion and value judgments as possible. Indeed, Mason (2002) defines a brief-but-vivid account as

> one which readers readily find relates to their experience. Brevity is obtained by omitting details which divert attention away from the main issue. The aim is to locate a phenomenon, so the less particular the description, the easier this is, without becoming so general as to be of no value….Thus description is as factual as possible. (p.57)

While Mason acknowledges that events or situations which stay in our memory are usually

those to which we have considerable emotional or intellectual commitment, he claims that we cannot analyse such events unless we can first be clear on what they consist of, as impartially as possible. This leads Mason to distinguish between an '*account-of*' which describes an event as objectively as possible, minimising evaluation and judgement, and an '*account-for*' which offers interpretation, explanation, value-judgement or criticism. (Further details of our experience of writing brief-but-vivid accounts and how the process followed in this project may have contributed to the findings can be found in Breen, McCluskey, Meehan, O'Donovan and O'Shea (2014)).

A general inductive approach (Thomas, 2006) was first used to identify themes emerging in a random sample of the full set of accounts collected and the experiences they documented. During this phase, it was noticed that many of the accounts described or implied points or moments during a mathematics lecture which provided the lecturer with an opportunity to make a synchronous decision, whether the lecturer was aware of this opportunity or not. We chose to focus specifically on decision points in lectures, where a 'lecture' was defined as a class involving one teacher and a group of students in which the dominant direction of communication is from the lecturer to the students (following Pritchard (2010)). Thus, each member of the group was asked to filter out accounts they had written which did not specifically deal with lecturing. This resulted in a subset of 162 accounts.

Initially two of the authors analysed all of the accounts selected and independently coded and categorised decision points they observed. After some discussion the two agreed on a set of codes and labels for the decision points. Then all three authors sequentially worked through all of the accounts to confirm their agreement or highlight disagreements with the decision points identified and codes presented. Further discussion led to a final set of agreed decision points and codes. One of the authors then grouped the decision point codes

into categories and a second examined these for legitimacy and consistency.

In what follows a ‘class exercise’ refers to a mathematical task assigned to students during a lecture, to be completed, either individually or in small groups, before the end of the lecture.

Results

We found 170 individual decision points in these accounts and following coding grouped the decision points into 7 categories. The category that appeared most often (n=40, 23.5%) was *How to deal with students’ mathematical difficulties*? which was followed jointly by *How to respond to students’ questions, answers or comments?* (n=24, 14.1%) and *How to deal with disruption?* (n=24, 14.1%). Together with the two other student-focussed categories (*How to engage students?* and *How to ask questions or gather information?*), these five categories represent 72% of the decision points identified. The *Other* category contained a small number of decision points about levels of attendance or classroom conditions. The two remaining categories concern the practice of teaching. Full details can be found in Table 1.

Type of Decision Point	***Frequency***	***Percent***
How to deal with students' mathematical difficulties?	40	23.5
How to respond to students' questions, answers or comments?	24	14.1
How to deal with disruption?	24	14.1
How to engage students?	22	12.9
How to conduct class activity, discussion or exercise?	22	12.9
How to conduct a lecture?	22	12.9
How to ask questions or gather information?	12	7.1
Other	4	2.4
Total	*170*	*100*

Table 1: Categories of Decision Points

We will now describe each of the categories briefly and give examples from our accounts.

The category *How to deal with students' mathematical difficulties?* mostly contains decision

points which occurred when the lecturers noticed their students' difficulties or misconceptions relating to particular mathematical topics. Some of these accounts related to difficulties experienced by the whole group and some to problems for individual students. Often the realisation that many of the students are experiencing difficulty led to the lecturer changing her plan, usually by revising or doing extra examples. This could pose a dilemma for the lecturer however, since it raises timing issues.

> I started today's class by revising briefly what we had covered the previous week. However, it was clear from asking the class some questions that we needed to spend some more time on this material. I ended up spending half the class on old material. Afterwards I felt that I was now even further behind where I wanted to be. (Account 1)

There were also moments when lecturers realised that the difficulties arose from previous knowledge or lack of basic skills, and wondered how to rectify the situation.

The accounts grouped under *How to respond to students' questions, answers or comments?*, as might be expected, mostly dealt with how the lecturer should respond to students' mathematical questions. The category also included instances when the students gave an incorrect answer (as in the account shown below), asked about examinations, or where the lecturer was not able to hear or understand the students' question.

> The students were presented with a particular example of a function and were asked to determine, in a whole-class situation, whether or not it was surjective. Some were answering "yes" and others "no" (the latter answer being the correct one), and so I asked for a show of hands. Only 3 or 4 students raised their hands for "yes". I hesitated for a moment before reminding the students that the codomain of the function was infinite (as had been mentioned earlier), while the domain was finite and therefore, the function could not be surjective. I wondered myself what had been my intention in asking students, so early in the semester, to identify themselves 'in public' as having the wrong answer. (Account 2)

The decision points in the *How to deal with disruption?* category arose from lecturers' efforts to respond to situations where some of the students in the class were talking (and disrupting others), texting, or (in one case) even sleeping. In the next account, we see a series of decision points when the lecturer has to choose how to handle the students' behaviour culminating in a reflection-in-action moment when the lecturer is able to remark on her own practice.

> Shortly into the lecture I had to stop to tell three guys and a girl to stop talking. Only minutes later, I told two girls to stop talking. Some minutes later, I tell the first group again to stop talking or get out. I look at the audience – most of them are staring straight at me and not moving. I realise that I am nagging and stopping the lecture for the sake of a few chatterers. (Account 3)

The category *How to engage students?* includes accounts dealing with issues such as what to do when students look bored, what to do with an unresponsive class or group within the class, how to make the lecture more student-centered or interactive, and even where the lecturer should focus her gaze while talking to the whole group. The following account arose from a moment where the lecturer had a sudden realisation that she had opportunities to teach in a different way and to actively involve the students more.

> Today I was happy with my 'performance' from a teacher-centred perspective as the lecture evolved: I felt I was coherent, explained and connected ideas well, used multiple representations of concepts and built on students' prior knowledge, However, I realised more than halfway through the class that my lecture was just that – very teacher-centred! I tried to rectify this but was not happy that my attempts were successful. (Account 4)

As the title suggests, the category *How to conduct class activity, discussion or exercise?* contains decision points emanating from lecturers' efforts to include class discussions or activities in their lectures. Some of these decision points arose when the activity or discussion did not go to plan, as for example in the next account when the students

did not all follow the lecturer's instructions and she encountered a decision point on what to do about this.

> I try out a technique today. I want to know whether the class think we should evaluate the function at q=999 or 1,000 or 1,001. I want to do a poll so I decide to ask them to close their eyes. Under half do not. "Close your eyes" I laugh. Some are still looking at me. "Close them! What a lack of trust" I joke. (Account 5)

Decision points classified in the *How to conduct a lecture?* category mostly dealt with the lecturers' aims or plans for the class or module; for example there were accounts on dealing with time pressure, whether to emphasise certain general aims, or, as in the next account, whether to change plan in the middle of a class.

> There were very few students in class today (about one third of the class) because of the snow. I decided to go ahead and cover implicit differentiation. At the very end of the class it occurred to me that I could have given the students some problems to work on themselves and could have gone around and talked to them individually. But I am so used to having hundreds in the class, I wasn't quick enough to adapt to the much smaller group setting. (Account 6)

Finally, the category, *How to ask questions or gather information?*, consists of accounts where the lecturer wondered whether to ask a question or how to ask a question in order to gauge the understanding of students in the class. It also contains accounts where the lecturer considered whether to ask a particular type of mathematical question, for example to look for conjectures or justifications.

> I was proving that a certain set was countable. The main part of the proof was to construct a bijection. It was obvious (to me) that the map we were considering was actually a bijection and usually I would say 'clearly g is a bijection' and leave it at that. Today I decided to get the class to explain why g was a bijection. It turned out that it wasn't clear to them at all and it took some effort to come up with a justification. In future, I'll be slower to say 'clearly'. (Account 7)

Discussion

Our analysis shows that opportunities to make significant choices happen regularly in lecturing situations despite the possible stereotype of a pre-prepared static lecture. We have also seen that these kinds of moments are similar but not identical to those identified in the school classrooms considered by Rowland et al. (2011). They found that in the great majority of cases a need for contingency arose in response to ideas put forward by students. Similarly, we found that the majority of the decision points in our accounts are student focussed though not always student generated. The influence of students on the reflections-in-action of a lecturer is also seen in the work of McAlpine et al. (1999) who found the majority of cues monitored by the lecturers in their study were related to students.

We did not originally set out to write about decision points or even to code for them but the initial analysis of our lecturing accounts emphasised the importance of these moments in our reflections. Finding new ways to act in the face of old challenges was one of the aims of our project and is at the heart of the Discipline of Noticing. Mason (2002, p.72) advises that thinking about these situations (or in our case decision points) in advance, along with possible strategies to deal with them, can help to prepare lecturers to act in new ways and break old habits and ultimately improve teaching. We see from our accounts that sometimes decision points occur but either no alternative course of action is available or none is taken (as in Account 6 for example). From their study, Peterson and Clark (1978) remarked that the likelihood of an alternative strategy coming to mind for a teacher while teaching increased in line with a teacher's engagement in reflection on his teaching. We endorse this perspective on the importance of reflection and propose that identifying decision points, sharing them with a group and discussing possible future strategies can be a powerful form of professional development for lecturers.

There is some evidence in our accounts (see Accounts 1 and 7) that the practice of noticing may have prompted the lecturers in this study to think afresh about aspects of their teaching, and in Accounts 3 and 4 we see lecturers engaging in 'reflection-in-action' and noticing elements of their teaching in real-time. We suggest that engaging in methodical reflection, such as that in the project described here, would provide lecturers with the opportunity to develop noticing or attentional skills and give them the capacity to decide how to respond appropriately to their students. This is turn could lead to the lecturer's teaching becoming more student-centred as was suggested by McAlpine et al. (1999). Ainley and Luntley (2007) believe the development of attentional skills should be a key component of teacher education; in the context of teaching in higher education, their development may fall to continuing professional development. We suggest that the use of Mason's (2002) Discipline of Noticing is one way of achieving this.

For us, the collaborative nature of our project, in sharing accounts and identifying the decision points therein, played an important role in the examination of our individual teaching practices and what we might learn from it (Breen et al. 2014). To borrow an explanation from Mason (2002): "reflective practice is precisely that, but offers little in the way of confirmation or validation; professional development involves interaction with a community" (p.159). Indeed, Paterson et al. (2011) stated that in discussing with colleagues decisions that arise in teaching situations, participants could develop an "awareness of unarticulated, taken as given, orientations and their consequent impact on teaching" (p.994), resulting in effective incremental professional growth.

We must acknowledge the limitations of our data; unlike other studies such as Peterson and Clark (1978) or Patterson et al. (2011), we did not have recordings of the lectures to work with. We only had the accounts that the participants chose to write. This could be viewed in a negative light, and certainly we may have missed many important

decision points in our lecturing. On the other hand, the accounts represent the incidents that meant enough to the lecturers to firstly remember them after class, and then to write an account of them to share with the group; this resonates with Mason's (2002) assertion that the reporting of an incident itself indicates an inherent significance. He remarked that "an incident is said by some to be critical if it turns out to be a turning point in their awareness, that is, which initiated a rethink about some implicit belief, theory, or practice in teaching" (Mason, 2002, p.117). The decision points identified in our accounts were often such turning points in awareness; for instance, see Accounts 4 and 7 above. Tripp (2012) also discussed 'critical incidents' in teaching and suggested that while these incidents may appear routine or even trivial, the analysis of them can lead to 'understanding and control over professional judgments and thereby over practice' (p. 24). Our experience from this project supports this view.

Our study highlights the opportunities for making choices inherent in mathematics lectures. We are in the process of identifying and classifying the triggers for the decision points identified in our accounts and will report on these elsewhere. This would build on the work of Rowland, Thwaites and Jared (2011) on classifying triggers for contingent moments in teaching, but in the setting of a university lecture. We also aim to identify what actions (if any) were taken by the lecturer as a result of decision points that arose. In particular, this could complement the research of McAlpine et al. (1999). Finally, in this study we only analysed brief-but-vivid accounts written by the authors about a critical incident while lecturing. By analysing each lecturer's entire collection of accounts, we hope to describe the implications that engaging with Mason's (2002) Discipline of Noticing has on an individual lecturer's practice, and explore the use of this technique in the professional development of lecturers.

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