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Constraint handling in extremum-seeking control for wave energy systems: A case study

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***Abstract*—Wave energy converters (WECs) are a promising technology aimed at harvesting energy by converting the device motion induced by ocean and sea waves. The control strategy adopted to guide the power take-off (PTO) system is among the most crucial aspects in the energy conversion process. Indeed, the solution of such an *energy-maximizing optimal control problem* is fundamental for the economic viability of this type of emerging technology. State-of-the-art WEC control technology can be almost fully enclosed within the family of model-based control strategies: The optimal control law, which maximizes energy absorption, is computed based upon knowledge of a dynamical model of the device, able to predict the associated motion. Nonetheless, models constructed for WEC control purposes are inherently affected by several sources of uncertainty, especially in terms of the associated hydrodynamic effects. As a consequence, there is a significant appetite to adopt model-free control strategies to overcome this issue. In this study, we propose a perturbation-based *extremum-seeking control* (ESC) in which a class of soft-constraint handling has been introduced to deal with excessive motion values for a heaving point absorber WEC. We test such a strategy in different operating conditions to highlight the influence of the soft constraint mechanism on power absorption, applied control action, and WEC velocity.**

***Index Terms*—Wave energy converter, Model-free control, Extremum seeking control, Constrained optimal control**

I. Introduction

Among the present global challenges, among which the need for energy sources alternative to those from fossil fuels is rising, the scientific community is showing an increasing interest in renewable energy research, and, in particular, efficient extraction of ocean wave energy [1]. Devices aimed at wave energy conversion are called *wave energy converters* (WECs). Such devices are able to convert the energy from the motion caused by the waves in oceans and seas. Many solutions have been proposed, characterized by different methods to exploit the wave motion [2]. Nonetheless, due to the relatively immature stage of WEC technology development affected by different issues to be solved, none of the studied concepts have yet succeeded in achieving commercial success [3]. One of the main challenges to overcome in the process towards economic feasibility of these devices is the development of suitable energy-maximizing control systems, which are responsible for maximizing the energy extracted, while also minimizing the risk of component damage, and adapting to a constantly changing environment. It is worth noting that, since the process of computing such an energy-maximizing law can be formally posed in terms of a constrained optimization problem, WEC control solutions naturally fall under the umbrella of optimal control theory [4]. The solution of the associated optimal control problem (OCP) is virtually always performed by explicitly using a control-oriented model of the WEC. Some examples of model-based optimal control applications in the wave energy field are model predictive control (MPC) [4]–[6], and moment-based control [7]–[9]. The models adopted for such computations are, however, the result of a trade-off between accuracy and complexity. Indeed, to facilitate real-time computation of the associated control action, the WEC model complexity must be compatible with the computational capabilities of each specific controller. The resulting trade-off produces a model that approximates the WEC behaviour with a certain degree of inaccuracy and/or uncertainty [10]. To deal with this issue, a robust control solution can be developed [11], [12] or a model-free control approach can be followed. The former, which intrinsically requires a characterization of the corresponding uncertainty set, tends to be conservative by definition, since the control objective is that of minimizing the worst-case performance for the defined uncertainty set. The latter, on the contrary, avoids the definition of a model at all, and only operates in terms of accessible variables of the

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device (*i.e.* manipulated inputs and measurable outputs).
In this study, we propose a model-free control solution to the OCP for WECs. In particular, we extend the perturbation-based extremum-seeking control (ESC) strategy presented in [13] by incorporating a methodology to handle constraints on the motion of the considered device. Furthermore, we explicitly demonstrate the influence that this type of strategy has both on the constrained variable, and the performance measure, *i.e.* power absorption.
The remainder of the paper is organised as follows. In Section II, the model of the point absorber WEC, adopted as a case study, is presented, together with the definition of the associated energy-maximizing OCP. Section III introduces the adopted ESC framework, and describes the strategy proposed to include constraints on the ESC design. In Section IV, numerical analysis of the presented approach, in regular wave conditions, is provided. Finally, in Section V, a set of considerations are offered regarding the advantages of adopting this type of approach in the ESC-based solution of the OCP for WEC systems.

II. Problem definition

This section introduces the WEC control problem addressed in this study. In particular, the dynamics of a point absorber WEC is presented in Section II-A, together with the main equations describing its motion, while the energy-maximizing optimal control problem is formulated in Section II-B.

A. WEC modelling

Among WEC technologies, one of the most popular is the so-called point absorber. This type of device is commonly constituted by a hull moored to the seabed, able to extract energy from its (predominantly) heave motion by means of the *power take-off* (PTO) system. It is characterized by its small dimension with respect to the wavelength of the site in which it is deployed. Two different categories of point absorbers can be identified: floating, and submerged point absorbers [14]. In this study, we consider the submerged point absorber adopted in [13]. A graphical representation is presented in Fig. II-A. The equation of motion of this type of device[1] is given by

$$m\ddot{z}(t) = F_w(t) + F_r(t) + F_v(t) - F_{\text{PTO}}(t), \tag{1}$$

where m is the point absorber mass, $z(t)$ is the heave displacement, $F_w(t)$ is the force that the wave exerts on the hull (*i.e.* the so-called excitation force), $F_r(t)$ is the radiation force, $F_v(t)$ is the viscous drag force, and $F_{\text{PTO}}(t)$ is the force the PTO generates to extract energy (*i.e.* the control force). From now on, being this work a preliminary analysis, we assume the device is subject to regular waves, and hence the force $F_w(t)$ is formulated as:

$$F_w(t) = |f_{e_\omega}|\frac{H}{2}\sin\left(\omega t + \angle f_{e_\omega}\right), \tag{2}$$

[1]We consider a single degree-of-freedom (DoF) for simplicity of exposition. Note that similar arguments can be made for multi-DoF devices (see, for instance, [15]).

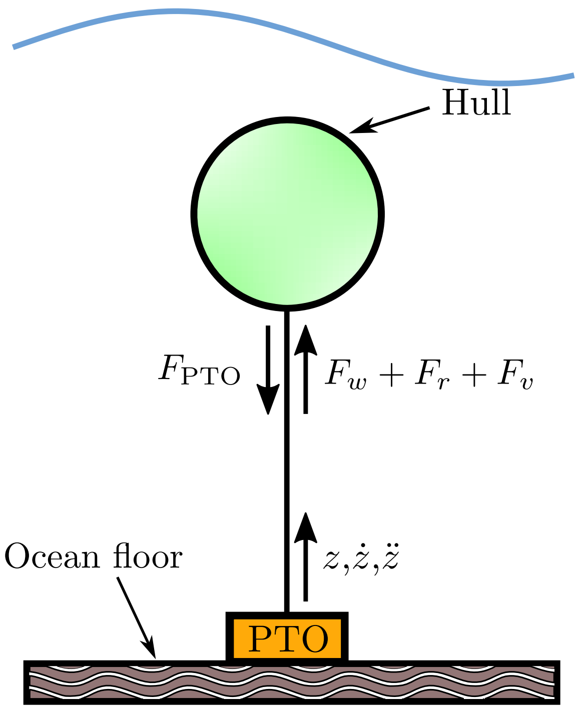

Fig. 1. Scheme of a submerged point absorber.

where f_{e_ω} is the geometry-dependent excitation force coefficient[2] at frequency $\omega = 2\pi/T$, with wave period T, and H is the corresponding wave height. Following Cummins' equation [17], the radiation force $F_r(t)$ can be expressed as:

$$F_r(t) = -A_\infty\ddot{z}(t) - \int_0^t K_r(t-\tau)\dot{z}(t)d\tau. \tag{3}$$

In (3), A_∞ is the added-mass at infinite frequency (see [16]), while the mapping $K_r \in L^2(\mathbb{R})$ represents the fluid-memory effect, modeled in terms of the so-called radiation impulse response function. Given the inherent representational and computational drawback associated with the convolution operator in (3), the radiation force term has been approximated by means of a state-space system, identified using the *Finite Order Approximation by Moment-Matching* (FOAMM) toolbox [18]. The viscous drag force $F_v(t)$ in (1) has been modeled following Morison's formulation of the drag force [19]. It must be highlighted that, in contrast to the floating point absorber case, the hydrostatic stiffness effect is not present in (1), since the considered point absorber is effectively submerged [13]. Because of that, the amount of submerged volume is the actual point absorber volume, and for this reason, it is always the same. As a consequence no recall force is generated by the heave oscillation.

B. Energy-maximizing optimal control problem

As introduced in Section I, the main objective of the control system in the WEC case is the maximization of the energy extracted over a certain time set $\mathcal{T} = [a,b] \subset \mathbb{R}^+$. Since the instantaneous absorbed power is the product of the PTO force

[2]See, for instance, [16].

$F_{\text{PTO}}(t)$ and the device velocity $\dot{z}(t)$, the control objective $\mathcal{J}$ can be formulated as[3]:

$$\mathcal{J}\left(F_{\text{PTO}}\right) = \frac{1}{T}\int_{\mathcal{T}} F_{\text{PTO}}(\tau)\dot{z}(\tau)d\tau, \tag{4}$$

where $T = b - a$. Aiming to maximize energy, while minimising risk of component damage, a set of limitations (*i.e.* constraints) is often introduced along with (4). In particular, constraints on maximum displacement z_{max}, maximum velocity $\dot{z}_{max}$, and maximum control action $F_{\text{PTO},max}$, can be incorporated as

$$\begin{cases} |z| \leq z_{max} \\ |\dot{z}| \leq \dot{z}_{max} \\ |F_{\text{PTO}}| \leq F_{\text{PTO},max} \end{cases} \tag{5}$$

with $\forall t \in \mathbb{R}$, and $\{z_{max}, \dot{z}_{max}, F_{\text{PTO},max}\} \subset \mathbb{R}^+$. Hence, in the general case, the optimal control problem can be fully written as

$$\begin{aligned} F_{\text{PTO}}^{\text{opt}} = \arg\max_{F_{\text{PTO}}} \ & \mathcal{J}\left(F_{\text{PTO}}\right) \\ \text{s.t.:} \ & \\ & \text{WEC dynamics (1),} \\ & \text{Motion and input constraints (5).} \end{aligned} \tag{6}$$

In order to solve (6), different strategies may be applied [20]. Most of them rely on the adoption of a model able to describe the system behaviour over the time interval $\mathcal{T}$. In contrast, as discussed in Section I, we adopt a model-free real-time ESC, following [13].

III. Extremum-seeking control

ESC is a *model-free* control strategy, since it does not rely upon access to a model for control action computation, and is purely based upon online measurements. As consequence of the latter characteristic, the control law is of a feedback type $f\left(x, \theta\right)$, where x denotes the state-vector of the associated system, parameterized in terms of a vector θ, and hence the adopted performance function $\mathcal{J}_{\text{ESC}}\left(x, \theta\right)$ depends on both x and θ. The controlled system behavior can be summarized as[4]

$$\dot{x} = g(x, u, d), \tag{7}$$

$$y = \mathcal{J}_{\text{ESC}}\left(x, \theta\right), \tag{8}$$

$$u = f\left(x, \theta\right), \tag{9}$$

where g is the function (not necessarily linear or known) describing the system dynamics, u is the control action (F_{PTO} in the WEC case), y is the evaluated performance, and d is an exogenous uncontrolled input (and hence not necessarily known) characterizing external disturbances (*e.g.* the wave force F_w in the WEC control application). It is important to note that the closed-loop behaviour of the system in (7) can be characterized by a set of equilibria directly depending on the control parameterization θ. To guarantee that the ESC formulation is stable for the general class of nonlinear systems described by (7)-(8)-(9), the following assumptions must be met [21]:

1) The equilibrium of system (7)-(8)-(9) is described by a smooth function l, $\theta \mapsto l(\theta)$, such that $g(x, u(x, \theta), 0) = 0$ if and only if $x = l\left(\theta\right)$.
2) For each θ, the equilibrium $x = l(\theta)$ is locally exponentially stable. As a consequence, the chosen control law (9) is stabilizing independently of the parameterisation of the law itself.
3) There exists an optimal parameter $\theta^\star$ such that the derivative of (8) with respect to θ is zero, *i.e.*
$$\frac{\partial \mathcal{J}_{\text{ESC}}\big(l(\theta^\star), \theta^\star\big)}{\partial \theta} = 0, \tag{10}$$
and the second derivative is strictly negative
$$\frac{\partial^2 \mathcal{J}_{\text{ESC}}\big(l(\theta^\star), \theta^\star\big)}{\partial \theta^2} < 0. \tag{11}$$
This assumption therefore implies that the output equilibrium map $y = \mathcal{J}_{\text{ESC}}(l(\theta), \theta)$ is concave in the parameter θ, and has a minimum at $\theta = \theta^\star$.
4) In steady-state conditions, the parameter θ, controlled by ESC, evolves much slowly than the system dynamics described by (7). As a consequence, the eventual time-dependence of the objective function $\mathcal{J}_{\text{ESC}}$ induced by the plant dynamics can be neglected. In such conditions, the time-derivative $\dot{\mathcal{J}}_{\text{ESC}}$ can be reasonably approximated as[5],

$$\dot{\mathcal{J}}_{\text{ESC}} \approx \frac{\partial \mathcal{J}_{\text{ESC}}}{\partial \theta}\frac{\text{d}\theta}{\text{d}t} = \frac{\partial \mathcal{J}_{\text{ESC}}}{\partial \theta}\dot{\theta}. \tag{12}$$

It is important to note that, in (12), the variation in time of θ is controlled by the ESC and, hence, measuring $\dot{\mathcal{J}}_{\text{ESC}}$ provides the algorithm with knowledge of $\frac{\partial \mathcal{J}_{\text{ESC}}}{\partial \theta}$. Since the ESC formulated as a gradient-based real-time optimization process, this latter information on $\dot{\mathcal{J}}_{\text{ESC}}$ is required to converge to the optimal parameter $\theta^\star$, and hence maximize the performance function. Moreover, an implicit assumption in (12) is that the evaluation of the objective function in ESC is considered to be time-invariant. As detailed in [13], by choosing an appropriate formulation for the performance map $\mathcal{J}_{\text{ESC}}$, such an assumption can still be reasonably adopted even in the wave energy case, where the optimal steady-state regime is effectively time-variant. In particular, [13] shows that the dynamics of transition between sea-states is sufficiently slow to allow the assumption of a slowly evolving performance mapping over the evaluation time required by the corresponding ESC.

In the following, we adopt the classical continuous-time perturbation-based ESC presented in [21], with the setting proposed in [13]. This type of ESC is composed of the combination of a high-pass filter (HPF) with cut-off frequency

[3] From now on, the dependence on t is dropped when clear from the context.

[4] In line with the single-DoF WEC system adopted in Section II-A, the ESC formulation is presented here for single-input single-output systems. Note that similar arguments can be adopted for multiple-input multiple-output systems (see [21]).

[5] The already mentioned dependence of $\dot{\mathcal{J}}_{\text{ESC}}\left(x(t), \theta(t)\right)$ from time-varying $x(t)$ and $\theta(t)$ is dropped here.

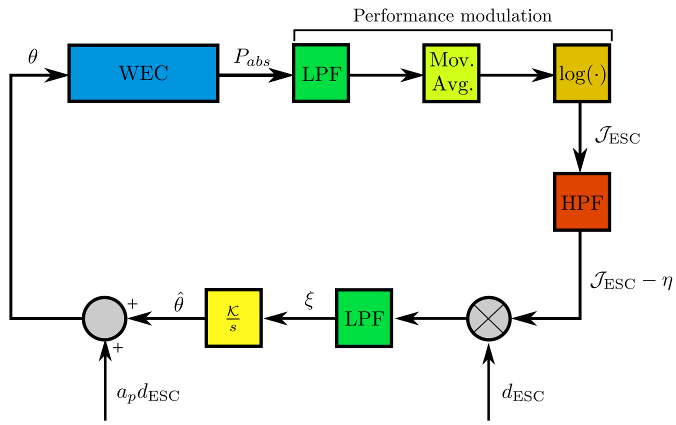

Fig. 2. Scheme of the developed perturbation-based ESC.

$\omega_H > 0$, a low-pass filter (LPF) with cut-off frequency $\omega_L > 0$, an integrator, and a sinusoidal dither signal with frequency $\omega_p > 0$, *i.e.* $d_{\text{ESC}}(t) = \sin(\omega_p t)$, introduced both in additive and multiplicative forms. The system of equations describing the behaviour of this type of ESC is:

$$\begin{cases} \dot{\eta} = \omega_H \left(\mathcal{J}_{\text{ESC}} - \eta\right), \\ \dot{\xi} = \omega_L \left(\left(\mathcal{J}_{\text{ESC}} - \eta\right) d_{\text{ESC}} - \xi\right), \\ \dot{\hat{\theta}} = \mathcal{K}\xi, \\ \theta = \hat{\theta} + a_p d_{\text{ESC}}, \end{cases} \tag{13}$$

where η is that part of the performance function $\mathcal{J}_{\text{ESC}}$ filtered by the HPF with cut-off frequency ω_H, while ξ is an estimate of $\frac{\partial \mathcal{J}_{\text{ESC}}}{\partial \theta}$, formulated as the output of the LPF placed after the first multiplicative perturbation d_{ESC}, as shown in Fig. III. Additionally, $\mathcal{K}$ is the integration gain, and $\hat{\theta}$ is an estimate of the optimal θ which, after the incorporation of the additive dither signal with amplitude a_p, composes the effective θ applied by the ESC. A graphical representation of the developed perturbation-based ESC is shown in Fig. III. In the design procedure, the gain $\mathcal{K}$ and amplitude a_p must be small enough to guarantee convergence, while the frequency ω_p must be lower than the frequencies that characterize the dynamics of the point absorber to guarantee equation (12).

A. On the inclusion of constraints in ESC design

In [13], the performance function $\mathcal{J}_{\text{ESC}}$ is purely based upon the measure of extracted power. Indeed, to compute an effective measure of the objective function, the absorbed power signal

$$J = P_{abs} = F_{\text{PTO}}\dot{z} \tag{14}$$

is used as input of the composite function consisting of (in order of interconnection):

1) A low pass filter with cut-off frequency $\omega_{L,\mathcal{J}} > \omega_H$.
2) A moving-average operator with a time-window defined as three times the period of the slowest operating wave, behaving similarly to another LPF but with different cut-off frequency.
3) A logarithmic function for data compression.

TABLE I
SCALED SPHERICAL POINT ABSORBER MAIN CHARACTERISTICS.

Symbol	*Quantity*	*Value*
D_s	Diameter	0.16 [m]
ρ_s	Homogeneous Mass Density	922.5 [kg/m^3]
d_s	Submergence Depth	0.25 [m]
d_w	Water Mean Depth	0.65 [m]

TABLE II
CHARACTERISTICS OF THE REGULAR WAVES ADOPTED.

Wave ID	T [s]	H [m]
1	0.625	0.0100
2	0.800	0.0200
3	1.000	0.0075

The output of this composite modulation function (highlighted in Fig. III) is the performance function $\mathcal{J}_{\text{ESC}}$. Since ESC is, by its nature, not capable of dealing directly with the constraints that may characterize a system (since the incorporation of state-dependent hard constraints inherently requires a model of the process), we include a term in J to penalize excessive violations of motion limits, *i.e.* a soft-constraint, in the spirit of equation (5). In particular, in the present case study, J is augmented, from (14) as

$$J = F_{\text{PTO}}\dot{z} - r\dot{z}^2, \tag{15}$$

where $r \in \mathbb{R}$ is the *weight* adopted for the penalization of excessive velocities. It is important to note that, with this soft-constraint formulation, constraint satisfaction cannot be guaranteed for all $t \in \mathbb{R}$, but rather in an average fashion, fully depending on the value selected for r. We discuss how to tune such a value in Section IV.

IV. NUMERICAL ANALYSIS

The presented approach to the ESC has been applied on a scaled-down point absorber with the same characteristics of the spherical one presented in [13]. The main dimensions of this scaled WEC are reported in Table I, while the set of regular waves, considered to evaluate the performance of the proposed strategy, are presented in Table II.

The ESC has been used in this application to optimally compute two parameters, described by a vector $\theta = [B, K]^{\text{T}} \in \mathbb{R}^2$, *i.e.* a classical PI (proportional–integral) control law is considered. Note that such a controller, which is often referred to as *reactive control* in the field of wave energy conversion [22]–[24], can be written as

$$u = F_{\text{PTO}} = B\dot{z} + Kz. \tag{16}$$

In (16), the control action results in a linear combination of a damping term proportional to the velocity $\dot{z}$, and of a stiffness contribution, proportional to the device displacement z. These two parameters are usually found in irregular wave conditions, by an extensive set of simulations, designing any constraint handling mechanism *a-posteriori*. In regular wave conditions, and for the unconstrained scenario, optimality is reached by

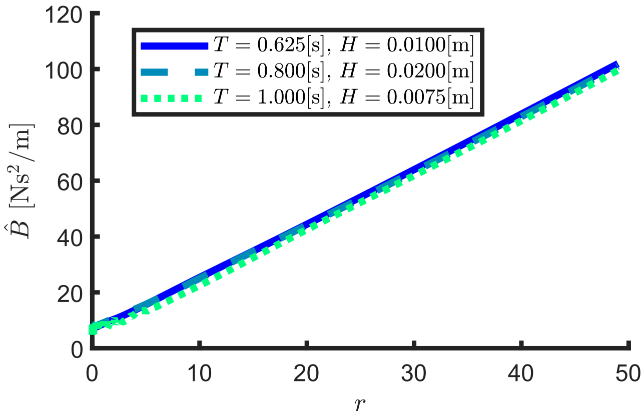

Fig. 3. Influence of r on optimal damping value $\hat{B}$ at convergence.

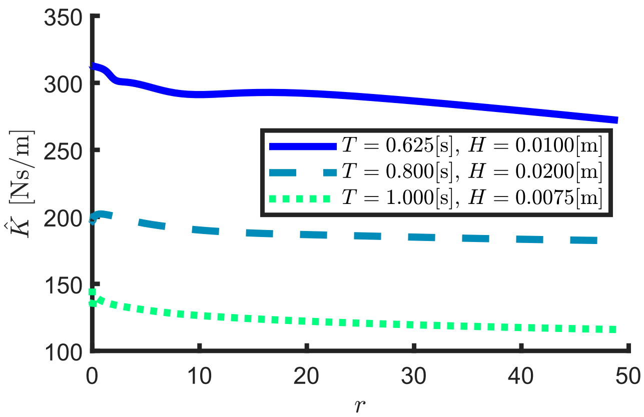

Fig. 4. Influence of r on optimal stiffness value $\hat{K}$ at convergence.

means of the solution of the so-called impedance-matching problem [23].

To evaluate the proposed strategy, 200 values of r logarithmically spaced between $r = 0$ (equivalent to an ESC without a soft-constraint), and a maximum weight value of $r = 50$, have been simulated for each of the 3 wave conditions in Table II. The length of the simulations has been set to secure convergence of the controlled parameters, in order to evaluate the corresponding performances at steady conditions. For each r value and wave condition, the value at convergence of the estimate of the optimal parameters $\hat{\theta} = [\hat{B}, \hat{K}]^{\mathrm{T}}$, mean absorbed power P_{abs}, mean-square of the velocity $\overline{\dot{z}^2}$, and mean-square of the control action $\overline{F_{\mathrm{PTO}}^2}$, have been considered in the analysis. Looking at Fig. IV, it is possible to notice that the optimal damping value $\hat{B}$ varies linearly in function of the weight r. Such a behaviour can be fully justified in terms of the formulation of J. In particular, for the PI control law described in (16), the contribution of the absorbed power $F_{\mathrm{PTO}}\dot{z}$ depends only upon the product of the damping coefficient B and the square of the velocity $\dot{z}^2$. In particular, note that the mapping (4) defines an inner-product operation in $L^2([0, T])$, where T corresponds with the period of the associated input wave. Under these conditions, the inner-product between steady-state displacement and velocity, arising from the definition of the PI control law in (16), is effectively zero, *i.e.* they are orthogonal

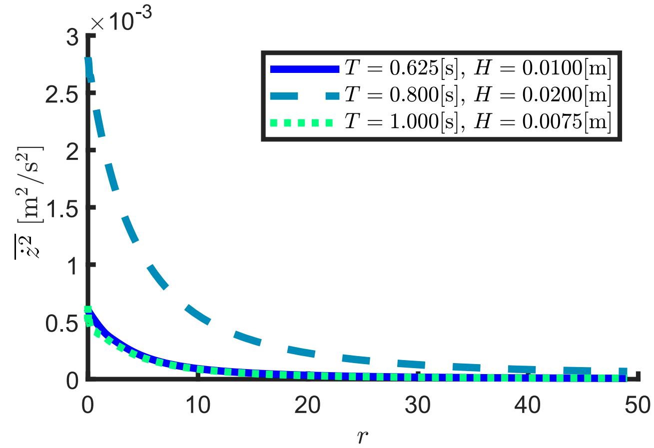

Fig. 5. Influence of r on the mean square of the heave velocity $\overline{\dot{z}^2}$ at steady-state conditions.

functions under the operator (4). As a consequence, the performance function in steady-state conditions is proportional only to the square of the device velocity, i.e.

$$\mathcal{J}_{\mathrm{ESC}} \mapsto (B - r)\dot{z}^2, \tag{17}$$

where the weight r acts as an additional damping term.

The stiffness parameter $\hat{K}$, on the other hand, is only slightly affected by variations in r, as shown in Fig. IV. The velocity behaviour, with respect to the weight r, is presented in Fig. IV. As it is possible to notice, the mean-square value of the velocity $\overline{\dot{z}^2}$ is strongly reduced by the addition of the penalizing term in J. It is important to note that, depending on the required level of constraint, this type of analysis is able to return a preliminary guidance on the design of the objective function. Moreover, since the reactive control action is also dependent on the velocity itself, the mean-square value of F_{PTO} is also directly affected and penalized by r. In this way, the soft-constraint on $\dot{z}$ also behaves as a soft-constraint on excessive PTO forces. However, the presence of such a term in the objective function has also an influence on the absorbed power, reducing the relative weight of the power absorption term in the optimality definition in the OCP. From the previously presented simulations, with $T = 0.625$[s] and $H = 0.01$[m], a graphical representation of the existing relation between r, the mean absorbed power P_{abs}, and the value of u_{ms} at convergence, is presented in Fig. IV. As can be directly appreciated, with a value $r < 6$, it is possible to significantly reduce the control effort, measured via u_{ms}, while only slightly affecting the obtained performance.

V. Conclusions

We propose, in this study, a solution for the inclusion of constraints in the formulation of an ESC strategy applied to a WEC. By means of a soft-constraint approach, we reduce excessive values of the heave velocity of a scaled-down point absorber. Because of the type of feedback control law adopted, the inclusion of the velocity-dependent penalizing term in the objective function also facilitates constraint handling of the control force F_{PTO} applied by the PTO. In addition, we

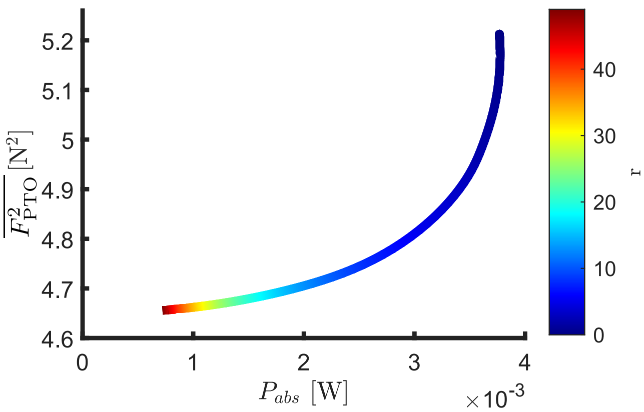

Fig. 6. Influence of r on the mean absorbed power P_{abs} and on the mean square of the control action $\overline{F^2_{\text{PTO}}}$ at steady-state conditions. Regular wave case, with $T = 0.625$[s], $H = 0.01$[m].

demonstrate that the relation between power absorbed and the penalizing weight, for low values of r, can significantly reduce motion and control values without generating a massive impact on the total absorbed power. The results presented in this study can help in the pathway towards an efficient model-free solution for WECs under realistic scenarios, directly contributing to effective commercialization of a wide variety of wave energy conversion systems.

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