An Action Research Investigation into Pedagogies of Enactment in Initial Teacher Education to Support Pre-Service Primary Teachers to teach Mathematics for Relational Understanding

Eddie Costello (BSc, BA, PGDE, MSc)

A thesis submitted to the Department of Education, Maynooth University in partial fulfilment of the requirements for the degree of Doctor of Education in the faculty of Social Sciences

Head of Department: Prof Aislinn O'Donnell

Deputy Head of Department: Dr Thomas Walsh

Supervisor: Dr Zerrin Doganca Kucuk

February 2022

Maynooth University
National University of Ireland Maynooth

Acknowledgments

There are so many people who have supported me in writing this thesis.

I would first like to thank my supervisor Zerrin who has been so giving of her time, knowledge, and expertise. She went above and beyond to help me finally cross the finish line.

I would like to thank Rose who offered me the opportunity to embark on this journey and fuelled my interest through thought provoking conversations along the way.

Thank you to my colleagues in the Froebel Department who offered support and expert advice. I would like to extend a special thanks to Rebecca who picked up the slack when I took a break from timetabling to write this dissertation.

I would like to sincerely thank my students who were central to this study. Without your time, patience, and feedback none of this would have been possible.

I would like to thank my parents who have instilled in me a drive to pursue difficult things! A special thanks to Finnuala and Pat, my parents-in-law, for all your help and support.

Finally, I would like to thank my wife, Orlagh, for her endless support, and our two babies Andrew and Ella who both arrived in the middle of this study.

Abstract

The problem of enactment describes a teacher's inability to translate effective theories of teaching into practice, and as a result they fail to produce effective classroom learning. It is common for pre-service teachers (PSTs) to experience the problem of enactment, and invariably they tend to enact instructional practices that are inconsistent with their beliefs (Kennedy, 1999).

My practice involves working with primary PSTs to develop their relational understanding of mathematics to support their classroom teaching. My concern, and motivation for this study, was my observation that PSTs did not enact their mathematical knowledge in the classroom, despite having proven competence in the area. Therefore, the objective of this study was to understand the reasons for this from the perspectives of PSTs enrolled in my content module (n=67) and make subsequent changes to my practice using Grossman and McDonald's (2008) *Pedagogies of Enactment* model of teacher learning.

The Action Research paradigmatic approach was chosen for this study because it involved making changes to my practice in a cyclic manner and evaluating if those changes had the desired effect. The study used two cycles of Action Research carried out over a period of two academic years. The specific aims of the study were to explore factors that contribute to the problem of enactment, and how an intervention could be developed to address these factors. It also aimed to examine PSTs' beliefs and the role they play in the enactment of mathematical knowledge.

Qualitative and quantitative data were gathered from PSTs using focus groups, classroom observations, and questionnaires, and analysed using Braun & Clarkes thematic analysis, the Mathematical Quality of Instruction (MQI) framework (Learning Mathematics for Teaching Project, 2011), and the ANOVA statistical test respectively.

The findings indicate that the Intervention can be effective in addressing the problem of enactment, but the model alone is not enough. Teacher educators need to work collaboratively to create an environment that is supportive of PSTs enacting and practicing their knowledge. Finally, this study recommends that ITE departments invest in professional development for teacher educators and reconsider the traditional model of ITE to work collaboratively with partner schools to improve PSTs' practical experiences.

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Acronyms

CT: Co-operating Teacher

ITE: Initial Teacher Education

LPP: Legitimate Peripheral Participation

NCCA: National Council for Curriculum and Assessment

MCK: Mathematical Content Knowledge

MKT: Mathematical Knowledge for Teaching

PST: Pre-service Teacher

SP: School Placement

SPT: School Placement Tutor

TE: Teacher Educator

Chapter 1: Introduction

1.1 Background

I am a teacher educator and researcher in a university Department of Education that offers undergraduate and postgraduate Initial Teacher Education (ITE) programmes for Primary School Teaching qualification. My department is the Froebel Department of Primary and Early Childhood Education and is named after nineteenth century German educator and philosopher Friedrich Froebel (1782-1852). Froebel was a pioneer for early childhood education and is credited with developing the first Kindergarten, a garden or nursery where young children could grow and develop at their own pace, under the watchful eye of supportive and knowledgeable adults (Tovey, 2013). From the outset, it is worth clarifying the Froebelian approach to education because it provides an important context for this study. The Froebelian approach puts the child at the centre of the teaching and learning process, and is built upon the following principles: respect for the child as a powerful learner; meaningful learning connected to childrens' experiences; play and active learning experiences to integrate learning; creativity as the essence of being human; freedom of choice and movements within an adult-guided framework; outdoor play in the natural world; a democratic learning community, connected to the wider community; positive, trustful and intellectually engaging relationships; and well informed qualitied teachers (Tovey, 2013).

This philosophy is the bedrock upon which both lecturers and pre-service teachers (PSTs) in the Froebel Department practice. Consequently, the Froebel Department website states:

> "*A Froebel graduate is recognisable for his/her ability to get the best from their students by nurturing their imaginations, creativity and critical faculties in an environment that respects the dignity and individuality of each child*".

(Froebel Department of Primary and Early Childhood Education, 2022)

In this context, my primary teaching role in the department is to improve pre-service teachers' (PSTs') Mathematical Content Knowledge (MCK) through a dedicated module called Maths Competency. Maths Competency is a compulsory component of the undergraduate Bachelor of Education (B.Ed.) programme, which is the most popular route to ITE qualification in Ireland. My teaching role initially appeared to be straightforward because it was primarily concerned with PSTs' acquisition of pure mathematical knowledge, rather than the pedagogical aspects of how PSTs learn to teach mathematics. That is, as a teacher educator (TE), my job was to ensure PSTs *understood* the relevant mathematics, and I did not need to factor into my teaching *how* PSTs enacted this mathematical understanding in the classroom. In this regard, maths competency pertains to promoting and developing PSTs' relational understanding of mathematics (Skemp, 1978) which is characterised by knowledge of an interconnecting web of relationships between mathematical ideas. It allows learners to develop mathematical knowledge on a schematic level thus understanding concepts and the relationships between them. This sort of mathematical knowledge essential for teachers to have for effective teaching (Wu, 2010), while also promoting democracy in education by conceptualising mathematics as "interacting constituent elements of the whole" (Freire, 1970, p.85) which necessitates an active process of inquiry by learners. See Section 2.3 for a more detailed discussion of relational understanding.

On the other hand, the pedagogical component of teaching mathematics is the responsibility of the maths methods module which focuses on teaching methodologies in the context of mathematics. Both modules run concurrently across the B.Ed. programme, but for all intents and purposes are separate modules.

My other role in the department is that of school placement tutor (SPT). As part of the B.Ed. programme, all PSTs in years 1-3 complete two School Placements (SP) each academic year for a duration of three weeks. In year 4, PSTs complete one extended 10-week placement. SPs are an opportunity for PSTs to enact their learning in a realistic practice-based situation. Consequently, I occupy a dual role in the Froebel Department as TE and SPT. On one hand I teach mathematics to PSTs, while on the other hand I observe and make judgments about how PSTs enact this knowledge in the classroom. Crucially, there is a clear distinction between these roles because as a TE I am a mathematics specialist, whereas as an SPT I am not. As an SPT I adopt a more generalised role, and invariably I am required to observe other subject areas other than mathematics.

As an SPT over the last number of years, I have observed that most PSTs do not enact, or even attempt to enact, the nature or essence of the mathematics taught in maths competency in their classroom practice. Instead, they tended to teach in an instrumental way (i.e., surface level, rote learned procedures), while ignoring more fundamental mathematical knowledge that leads to relational understanding. As a TE this was perplexing and frustrating. This observation challenged my basic assumptions about how PSTs learned, and learned to teach mathematics:

1. Engaging with the maths competency module would result in improved MCK for PSTs.
2. An improvement in PSTs' MCK would automatically result in improved mathematics teaching in the classroom.
3. PSTs would be able to use what they learned in maths methods, as well as their general pedagogical knowledge, to effectively enact their MCK in the classroom.

Additionally, as part of the B.Ed. programme, PSTs are required to attain a 70% pass threshold on end of year maths competency examinations. To date, albeit sometimes with additional supports, all PSTs have met this standard which suggests they have the necessary competency required to teach primary school mathematics, and therefore, my first assumption holds true. However, I assumed, perhaps naïvely, that teaching PSTs to understand the mathematics they were required to teach, but at a much deeper level, would automatically result in a better quality of mathematics teaching on SP. This has not been the case. This study is about understanding why PSTs can demonstrate competency in relevant mathematical knowledge and yet are unwilling or unable to enact this knowledge in the classroom on SP. It is also about addressing this problem in a practical way by making research informed changes to my practice.

I am considering this problem from the perspective of my practice as a teacher educator, and specifically how my practice influences PSTs and how they understand mathematics teaching and learning. Because it involves the interaction between my practice and PSTs' experiences of it, as well as the wider SP environment, this study is best suited to action research.

Another reason this study is suited to action research is because the research problem is about democracy. When PSTs enact a dominant technical-rational approach to teaching mathematics then both they, and more importantly, the pupils they teach, are engaging in a educational oppression because it denies them the opportunity to engage with mathematics in a critical and creative way. A more detailed justification for using action research is outlined in chapter 3 of this dissertation.

1.2 Conventions Used

Before describing the research area in more detail, it is necessary at this early stage to highlight some of the main style and linguistic conventions which will be used throughout this dissertation. Because action research relies heavily on reflection and elements of action and change, vignettes will be used to distinctly capture significant moments of practice to provide context for the reader, while also highlighting opportunities for reflexivity. Vignettes and reflections will be italicised to distinguish them from traditional academic writing.

In most cases, the acronym PST will be used to refer to the students on the B.Ed. programme rather than participants. This is because there is a blurred boundary between my research and practice, and as such, using participant and PST may cause unnecessary confusion for the reader. To protect confidentiality, pseudonyms are used when referring to individual PSTs. The word pupils, as opposed to children or students, is used to refer to the primary school children PSTs teach. However, the language of the theorists is preserved in Section 2.1 on my theoretical position, where Freire and Dewey use the terms student and children, respectively.

Maths competency and maths methods will refer to the mathematics modules relevant to this study. These informal naming conventions are used because they are indicative of their typical use within the Froebel Department by staff and PSTs. Because this study involves a change to my practice, there will be two different versions of maths competency, which shall be referred to, namely, as *original maths competency* and *the intervention*.

1.3 Reconnaissance Phase

Although the current study began in 2017, it was two years earlier, in 2015, that I recognised the problem of enactment. In October 2014, I was granted leave by my department from SP tuition to observe mathematics lessons for information gathering purposes to inform my practice. I observed several lessons from 3rd year PSTs on the B.Ed. programme, and recorded notes on each of them. These observations confirmed the preponderance of teaching was based on surface level, procedurally-based, instrumental mathematical understanding. However, there was one instance in a lesson that stood out above the rest because it captured the essence of the problem. This is described in the following vignette:

Orla was a 3rd year PST on the B.Ed. programme teaching 6th class pupils. The main learning objective for the day was rounding decimal numbers. For example, pupils were asked to round 5.23 to the nearest tenth, etc. Pupils were taught the rule for solving similar problems, and eventually asked to work independently to solve many of them. This was typical of the sort of mathematics lessons I had been observing on this information-gathering mission, i.e., procedurally-driven mathematics. As the lesson progressed, my attention was drawn to one pupil who I was sat beside near the back of the classroom. He was clearly disinterested in the lesson, which led to a loss of focus and some minor behavioural issues. He was checked by the PST on a few occasions because of his behaviour. At one point, however, something in the content grabbed his attention and he asked the PST why he was not allowed to remove the zero from 8.01 when he was allowed to remove the zero from 8.10. I was impressed by the depth of his question and eagerly awaited a proportionate response from the PST. She answered: "because there's another number after it". As a teacher educator, I was disappointed by this meaningless response which gave no insight into mathematical reasoning and left no opportunity for further discussion and inquiry. This made me question the effectiveness of my practice and how it may be contributing to this sort of PST classroom behaviour. On a human level, I was disappointed for the pupil who was courageous to ask a worthwhile and meaningful question but was met with an underwhelming and disappointing response. Sitting beside him, I could sense a palpable flatness to the mood of the lesson after he was given this response.

Since this lesson, and this general period of observation, I have been trying to understand why a PST who has proven content knowledge about fractions and decimals, as well as how they are related, would not use this knowledge to inform their practice. To continue with this process, in early 2017 I carried out some reconnaissance work in the form of an informal conversation with a small group of final year B.Ed. PSTs[1] about this difficulty of transferring mathematical knowledge into their classroom practice, and this offered me a glimpse into their realities. As we discussed their practice, we mutually acknowledged and agreed that they do not often use their ITE learned MCK[2] in their practice. I recorded their reasons for

[1] These PSTs were not involved in the current research project.
[2] For the purposes of this conversation this only refers to maths competency

not enacting their MCK in practice, and broadly categorised these responses into two groups based on the nature of the constraint: school-based constraints and beliefs-based constraints.

School-based constraints are those factors within the school setting that inhibit or prevent PSTs from enacting their MCK in the classroom, and the PSTs communicated these to me. The first constraint was PSTs' perception that the mathematics curriculum is overloaded, meaning there is only enough time to teach topics on a superficial level. This is exacerbated by the fact that only one week is usually dedicated to a topic. Furthermore, PSTs reported that schools encourage pedagogical practices that are is inconsistent with the underlying philosophy of maths competency lectures. All of this introduces a power differential whereby PSTs feel powerless to make any significant or meaningful difference in terms of how mathematics is taught. They believe change must come from those in leadership roles because enacting a pedagogy that is not based on traditional mathematics teaching and learning is too much responsibility for one teacher. Finally, PSTs reported feeling under pressure to teach to the test.

Constraints related to PSTs' beliefs are those deeply held assumptions about teaching and learning that act against a pedagogy for deep understanding. Firstly, PSTs reported that they find learning mathematics relationally difficult, and therefore it must also be too difficult for primary school pupils to learn. Secondly, they explained how there was a disconnect between the maths competency and maths methods modules, and agreed maths competency was ultimately for an examination, whereas maths methods was the module that was supposed to guide their practice. PSTs also explained that it was easier for them to teach mathematics the way they were taught in school, or from a textbook, rather than trying to include deeper knowledge learned from the maths competency module. Once again, this issue was exacerbated by pupils responding negatively to relational mathematics because it is, according to the PSTs, different than what they were used to. Finally, the PSTs explained that teaching instrumental mathematics, based on rote learned procedures, is easier because it allows pupils to get correct answers in a relatively straightforward way. These brief responses are enlightening and highlight the complex nature of the problem. That is to say, there are many barriers to enactment, emanating from different domains, which are not straightforward to address.

In addition to these responses, it has been quite common during post SP lesson conversations for PSTs to disclose to me that they revert to the way they were taught mathematics themselves in primary school, rather than making the appropriate pedagogical adaptations to the mathematical knowledge they learn in lectures. Additionally, in maths competency lectures, it is common for PSTs to question the necessity to possess deep mathematical knowledge, as well as its appropriateness for primary school pupils. For example, typical responses from PSTs regarding the content would be: “how would you teach this to a child?”, “will children be able to do this?”, and “why do we need to know this?”. Up to this point, my response has been to emphasise that pupils deserve to understand mathematics in this way, and PSTs must be able to make appropriate pedagogical adaptations to the content they are teaching. I encouraged PSTs to make connections between content learned in maths competency and the teaching methodologies they learn about in maths methods. Clearly, this approach is not effective. More fundamentally, PSTs do not seem to understand the purpose of maths competency and how it relates to their practice.

This anecdotal evidence is suggestive of Lortie’s (1975) apprenticeship of observation, particularly in cases where PSTs gravitate towards a mode of mathematics teaching reflective of the way they were taught themselves. More generally, the wider issue is indicative of problem of enactment, whereby novice teachers demonstrate the inability to translate theory into practice, and as a result fail to produce effective classroom learning (Kennedy, 1999; Grossman, 2008; Gardiner and Salmon, 2011). The problem of enactment in initial teacher mathematics education is internationally recognised in the educational literature (see for example Allen & Wright, 2013; Zimmerman, 2017; Lampert et al, 2013; Hlas and Hlas, 2012). Furthermore, specifically relevant to this study, Philipp (2008) recognised the inherent difficulty in motivating PSTs to learn mathematics relationally and subsequently use this knowledge in practice. These concepts introduced in this section, which will be interrogated further in the literature review, provide a suitable starting point for this study.

1.4 Initiating the Research

Considering the context provided in the previous sections, the research area will now be outlined with more clarity. From a high level, it seeks to explore the gap between PSTs’ mathematical knowledge on one hand, and their classroom practice on the other. It is hoped this exploration will lead to a comprehensive understanding of why PSTs do not enact mathematical knowledge in their practice, even though they have shown themselves to be competent in examinations. In this regard, it is a theory-practice problem. Using the reconnaissance work as a starting point, it will examine the main contributors to the problem of MCK enactment, and where they originate from, in the context of PSTs' mathematics teaching. It will seek to explore the nature of these contributors and how can they be best managed or eliminated, particularly in the context of my practice, so PSTs can begin to enact meaningful mathematical knowledge in the classroom, and ultimately a standard of mathematical instruction that is truly educational.

The data gathered from the reconnaissance phase provided a stimulus to examine my practice and conceptualise the problem within the wider educational context. To begin I talked with colleagues and engaged with literature on teacher education and teacher learning. Initial consultation with colleagues lacked criticality and usually resulted in placing the responsibility back on PSTs to enact content and theory learned in ITE. However, as these conversations progressed, and opened up to the wider educational community including my doctoral group, other perspectives were introduced. These conversations also introduced me to interesting and relevant literature including Powerful Teacher Education (Darling-Hammond, 2006) which deepened my knowledge of the problem of enactment. This led to the realisation that *how I teach* mathematics may be a contributor to the problem of enactment (Russell, 1997). Russell (1997) advised that PSTs are unlikely to enact practices based on the values of teacher educators unless the teacher educator's values are reflected in their actions. In other words, *how* teacher educators teach, as opposed to what they say, sends a message to PSTs about how they should teach. Consequently, in this study I will interrogate my practice as a teacher educator and challenge my actions in relation to my values (see Section 1.6) and the relevant literature. This will result in a values-based and evidence-based teaching intervention to address the problem of enactment (see Section 3.4), which will be carried out with PSTs over a period of two years. Using the action research approach, cycles of action and reflection will be used, with the input of PSTs, to evaluate and develop the intervention as necessary.

The research problem outlined in this section will be further developed in chapter 3 when situated within the context of relevant theory and literature critically reviewed in chapter 2. This will result in a refined set of research questions in Section 3.2 of the Methodology chapter. Before this, the two remaining parts of this section will further contextualise the study. The first of these (Section 1.5), presents an overview of my current practice (i.e., before any intervention was put in place to address the problem of enactment). This is important because this study is about my practice and how it impacts on others. Describing this practice provides a frame of reference from which to locate the problem of enactment, as well as a solid position from which a new practice will emerge. Section 1.6 presents my personal and professional values. Action research is a values-based paradigm (Cohen, Manion & Morrison, 2018), and it is from these values I can reimagine my practice, judge my practice, evaluate outcomes of this study, and ultimately transform my practice through these values (Brydon-Miller, Prudente & Aguja, 2016).

1.5 My Current Practice: The Original Maths Competency Module

The aim of the maths competency module is, as the name suggests, to improve PSTs' MCK. This aim was the same for the original modules and will remain the aim for any future versions of them. This section will describe how this aim was achieved in the original maths competency lectures prior to the commencement of this research study. The content of the module was based primarily on the *Number* strand of the primary school mathematics curriculum (NCCA, 1999b), with a focus on deep relational understanding of concepts. Where procedural knowledge was involved, the focus was about making sense of the concepts underpinning those procedures, including understanding how procedures are related to one another. Relational understanding of mathematics is discussed in more detail in the literature review, along with more general considerations for teachers MCK, in Section 2.3. to provide context, a high-level overview of the content for each of the four years of the maths competency modules are as follows:

> Year 1: Whole Numbers including number base, place value, and the arithmetic operations.
>
> Year 2: Simple fractions including the four operations.
>
> Year 3: Complex fractions including percentages, ratio, proportion, and rate.
>
> Year 4: The rational numbers and number theory

In this original format, direct instruction was the primary teaching methodology used in maths competency lectures, but typically with some group work included. This approach made sense to me, because the aim of the module was to teach mathematical content knowledge to PSTs. Furthermore, this approach was recommended by some of the education literature (Kirschener, Sweller and Clark, 2006), and is particularly suited to explicit teaching of mathematical facts, procedures and concepts (Hamachek, 1999). In my mind, this approach to teaching was justified because PSTs' grades in examinations indicated they developed an advanced knowledge of primary school mathematics. However, my initial observations and preliminary reconnaissance work suggests that even if PSTs were to develop an advanced knowledge of primary mathematics, they, for some reason, choose not to use it directly or indirectly in their practice. My difficulty in coming to terms with this apparent failure in *my* practice is captured in the following vignette:

I have started to read some of the literature on the problem of enactment and it suggests I need to adopt a practice-based approach to teaching mathematics, and that PST beliefs may also act as a barrier to enactment of meaningful mathematics. Intellectually, I understand this, yet find it difficult to reconcile this reality with my personal experiences of mathematics knowledge and ability to teach. As an adult I learned mathematics for various reasons including examinations, teaching, and personal enjoyment. I was, and am, motivated by mathematics as a subject in and of itself. It wasn't always like this for me. My fondness of mathematics was born from a deep frustration with how it was taught to me at school – primarily through rote memorisation of procedures. Because of this negative experience, as an adult, I took the time to explore some of the mathematics I learned in school and I became intrigued, sometimes fascinated, with the simplicity and the symmetry of it. As an undergraduate student of mathematics this attitude helped me a lot, even when the teaching approach at university was more like school mathematics than I would have liked. However, as a teacher of mathematics at various levels from the SEN school setting to adult education, my mathematical knowledge was my most valued asset. It was relatively deep and flexible which gave me the confidence to engage meaningfully with students and the mathematics. If there was something I didn't understand, I had the confidence and knowledge to address that gap. As a teacher, my mathematical knowledge allowed my pedagogical strengths to shine *through. This is the reason I found it difficult to understand why content knowledge alone did not automatically improve PSTs' teaching of mathematics in school.*

Further reflection led me to conclude there were other problems with my practice in addition to that of enactment. Firstly, it is difficult to create excitement about mathematics when the focus is on the fundamentals of numbers. For example, an explanation about why the long division algorithm works is tedious and intellectually demanding. It is hardly fascinating, particularly for PSTs who are not necessarily motivated by mathematics per se (Philipp, 2008). The second problem is more concerning from a knowledge acquisition perspective. Because the content of the modules was closely aligned to the PSC, every PST had familiarity with most of the concepts from when they were in school themselves. I realised that teaching old content in a new way is far more challenging than teaching previously unseen content because PSTs already had preconceived ideas about these concepts. I observed PST knowledge as overwhelmingly instrumental and when their notion of competence was challenged by a relational push, they often resisted. It appeared many failed to see the value of 'unpacking' concepts and breaking down explanations of procedures to create a more meaningful mathematical experience for both themselves and the pupils they teach.

While reflecting on these problems with my practice it occurred to me that they are possibly related in some way, i.e., if PSTs are uninterested in the mathematics and do not see the relevance of the mathematics, then it is reasonable to assume they are less likely to enact this mathematics in the classroom. These issues will be addressed as part of the teaching intervention, and explored during the data collection, analysis, and results.

1.6 My Values

My values as a teacher educator, and in particular a mathematics educator, originate from a range of personal and professional experiences. As alluded to in the previous section, I became frustrated with overwhelmingly instrumental nature of how I was taught mathematics in school, and this contributed to a lack of understanding of mathematical concepts and the subsequent disinterest in the subject. Despite an innate desire to understand relationally, I unenthusiastically persevered with school mathematics.

After graduating from secondary school, I studied computing at undergraduate and postgraduate level. My undergraduate degree had a strong focus on mathematics, particularly in the first three years. Following this and after several years of feeling somewhat unfulfilled working in industry, I made the decision to become a teacher of mathematics. My intention was to teach in a way that made sense, where concepts were explored and understood, so that learners' engagement with mathematics was meaningful, i.e., I wanted to teach mathematics for relational understanding.

After qualifying as a secondary school teacher, I completed a mathematics degree to complement the mathematics I learned in my undergraduate degree. Although there were examinations to pass, I felt I had the maturity, time, and autonomy to learn mathematics on a deeper level. As I learned mathematics this way, I experienced and became aware of certain characteristics that were absent from learning mathematics previously. Most notably, it was challenging, rewarding, and thought-provoking. It was also slow – to begin to understand mathematics relationally takes time. Over time, connections between different ideas became clearer and mathematics became more obviously logical than procedural. When solving a problem, rather than trying to remember the next step, it became about choosing the most logical and appropriate step from a wide bank of ideas to see if it worked. Crucially, mistakes were an important part of the learning process. These experiences helped to form my values about how I think about mathematics and my approach to *doing* mathematics. At this point, as an early career secondary school teacher, my values about mathematics were not informed by the literature or theory, rather by experiences.

In my early teaching career, I worked across a range of settings including secondary education, initial teacher education, and adult education. During this period my values about mathematics teaching merged with, or perhaps morphed into, my emerging values about education generally, which are outlined at the end of this section. As a teacher, particularly in secondary schools, my values were sometimes challenged and sometimes denied, and these experiences served to refine and strengthen those values. Some of these experiences mirror what Clandinin (1985) described as "watershed", which were the sorts of experiences that had a profound impact on my educational values. As an illustration, I will describe one of these watershed experiences, which involved teaching mathematics to a small group of Junior Certificate students. This was a team-teaching initiative including me (a novice teacher at the time) and a lead teacher who held a senior position in the school. Before the initiative began, the lead teacher described the five students involved to me as "borderline". Although I did not fully understand what was meant by "borderline", my assumption was that it meant they were at risk of failing the foundation level Junior Certificate[3] mathematics examination. This experience, and its significance, is captured in the following vignette:

Lessons were based solely on sample exam questions and structured so that there was direct teaching at the beginning, followed by one-to-one work on writing solutions to the questions. One lesson that stands out was the teaching of currency conversion, where I noticed that the other teacher and I had conflicting beliefs about how mathematics should be understood. Her explanations involved procedures to be rote learned, and the procedures varied depending on the type of conversion presented. Even I was initially confused about the procedures because there were several of them, and there was ambiguity about when to use them. Furthermore, there were no explanations about why any of these procedures worked or how they were derived in the first instance. The look of confusion on the students' faces remains a very strong visual image for me. Because of the power differential between me and the lead teacher, I felt I could not intervene. I felt frustrated because I could only look on and observe what I could consider miseducative experiences, while at the same time ashamed by my involvement with it. The same teacher advised me that they will forget what they have learned by the following day and so it was essential to keep revisiting the same sample questions with them. I couldn't understand why we were asking students with learning difficulties to rote learn increasingly more procedures and rules when this is the very thing they find difficult. Taught this way, mathematics had no meaning for them and was devoid of creativity, critical thinking, or enjoyment. This is just one of several watershed experiences that have impacted my educational values and my professional identity.

My educational values developed dynamically over time and became more apparent to me with experience. They emerged from my, more fundamental, ontological and epistemological assumptions. I will try to briefly describe the essence of these: From an ontological perspective I believe in the sort of respect for individuals which centres around equality and access to meaningful learning pursuits. From an epistemological perspective, I believe that

[3] Junior Certificate is an intermediate state mandated examination in Ireland.

even seemingly objective knowledge can be problematised and open to critique. Furthermore, how knowledge is interpreted, understood, and used varies between individuals. In essence, I believe meaning-making is an important part of accessing knowledge and this is, or should be, an individual pursuit which is best carried out in collaboration with others. In this regard, knowledge creation is a democratic process and not something that is necessarily expert-driven.

I consolidated my ontological and epistemological values and considered what they mean from an educational perspective. From this I derived a set of democratically orientated values that inform both my teaching and research. These are as follows:

- Education should be about understanding, and this understanding should be transparent and meaningful for individuals.
- Education should empower individuals to question and inquire. There should be a mutually reinforcing relationship between questioning and understanding.
- Education should benefit individuals and benefit society. It should be inclusive, interesting, thought-provoking, and lead to personal growth and fulfilment.
- Education should involve both individual and collective learning. All perspectives are important, and individual knowledge should be respected and considered legitimate. This necessarily involves a balanced dialogical relationship between teachers and students.

When I reflect on my practice with respect to the original maths competency modules, and question whether I am practising in accordance with my values, the answer is not straightforward. I do teach mathematics for deep understanding, and this is consistent with my values. However, I need to broaden my perspectives and the wider implications of my practice. If it does not benefit PSTs in the classroom, and by extension, the pupils they teach, then I cannot claim my practice benefits wider society. Nor does my practice include collective learning. I have struggled with this idea for a long time because, in my opinion, mathematics is objectively true, but teaching and learning mathematics are not. There are different ways of understanding mathematics, different ways of communicating it, different ways of appreciating it, etc. This has been largely ignored in my practice for a few reasons. Firstly, I assumed all PSTs would learn mathematics in the same way I did, and this learning would be easily transferred to the classroom. Secondly, this assumption precluded me from seeing alternatives and kept me in an intellectual echo chamber where my own ideas were mirrored in those I spoke to, and the research I read, which limited my exposure to new ideas.

A close examination of my practice highlights that I am not embedding my values and beliefs in my practice to the extent I would like to. This is suggestive of what McNiff (2005) refers to as a living contradiction whereby a practitioner does not act in accordance with their beliefs. This action research study is an opportunity for me to address this contradiction and realign my practice with my values to improve my practice for PSTs.

1.7 Chapter Conclusion

The goal of this study is to explore the issues in my practice that contribute to PSTs difficulties in enacting relational mathematics on SP, while being guided by my values along the way. Through observations, reflections, and the reconnaissance work, I established some starting points from which to problematise my practice. Some of these points, which appear to be related to the problem of enactment, are as follows:

- Addressing how PSTs' mathematical knowledge developed in ITE can be transferred to and successfully integrated into their practice on SP
- Challenging my own assumptions about how PSTs learn mathematics and how this impacts my practice.
- The efficacy of mathematics content examinations in ITE: PSTs have proven competence in maths competency examinations, yet this largely fails to impact on their classroom practice. It is necessary to consider other ways of developing and evaluating PST mathematical knowledge in a way that addresses identity development.
- The challenges PSTs experience when teaching mathematics including their own beliefs and school-based constraints to teaching in accordance with their beliefs
- A consideration of what PSTs consider to be relevant mathematical knowledge for primary school teachers, and the purpose of this knowledge.

This study is therefore, about researching my practice with PSTs and using their individual and collective knowledge to evaluate changes to my practice and help to make subsequent improvements. As such there is a blurred boundary between my research and my practice, and this must be carefully navigated from scholarly, practical, and ethical perspectives. Because PSTs will be involved in knowledge generation, it is important to acknowledge many will have differing values from mine, but which are equally legitimate to mine (Sullivan, Glenn, Roche & McDonagh, 2016). Although this may be a cause of tension, it can equally be an opportunity for learning. Researching with others in this way makes action research an appropriate approach for conducting the study because it is values-laden (Cohen et al., 2018), and is consistent with my values as a democratic and inclusive way of improving practice, leading to a participatory orientation to knowledge creation (Brydon-Miller at al., 2016).

1.8 Dissertation Overview

Chapter two presents my theoretical position on teacher education, and contemporary research which will provide necessary background to position the study in the broader theoretical and educational context. The literature will also inform the changes I make to my practice, and how I analyse data to evaluate those changes. Chapter 3 will present the overall methodological considerations for this study, including a detailed justification for the action research approach and other paradigmatic choices. This chapter will also present the proposed teaching intervention. The authentic voice of PSTs is a cornerstone to this study and so a primarily qualitative approach will be used for data collection. Instruments used for this are surveys, focus groups, reflections, and classroom observations. There will also be some quantitative data analysis to explore changes to PSTs' beliefs as a result of the intervention. Results will be presented in chapters 4, 5 and 6, followed by a discussion and implications for my practice and teacher education in general in chapter 7.

Chapter 2: Literature Review

2.1 Introduction

There is a dual purpose for this literature review, and the first is to fully understand the nuance in the research questions, and hence further develop them in a way that will result in meaningful learning. These are re-presented again for this purpose in Section 3.2. The second reason is to use this knowledge to inform the teaching intervention so that it is aligned with the pertinent research on teacher learning and the problem of enactment.

The chapter is divided into five distinct yet related parts. The first of these, Section 2.1, outlines my theoretical position on education which is used to provide an overarching structure to guide this study, enabling me to choose and interpret relevant literature, serving as a moral compass to guide my choice of teaching intervention, and ultimately interpreting results arising from this study. Section 2.2 is about teacher learning, competing knowledge domains in teacher education, and the problem of enactment. This section also presents a pedagogy of teacher education based on approximations of practice. Section 3.3 addresses mathematical knowledge, and the role of this knowledge for the preparation of PSTs and the pupils they teach. This section also critically examines my current practice as a teacher educator in the context of evidence presented in the literature. Section 2.4 addresses the issue of PSTs' beliefs about mathematics teaching and learning, and how these beliefs impact on the type of mathematics instruction they enact. The final part, Section 2.5, looks beyond initial teacher education to wider issues related to neoliberal influence on education and how this may influence how PSTs teach in the classroom.

2.1 Theoretical position

My theoretical position on education and educational research is underpinned by Paulo Freire's (1970) influential text *Pedagogy of the Oppressed* and multiple sources of John Dewey's work on education. Although separated by time, place, and context, Freire and Dewey form an appropriate nexus of educational theory which is grounded in the idea of democracy and the underlying belief in the transformative power of education. Both theorists have also fundamentally influenced my own educational and epistemological values, and as such are used to examine and scrutinise how I enact my values in practice. This democratically orientated theoretical position is consistent with the use of Action Research as a paradigmatic choice for this study because it is a collaborative and democratic approach enabling me to reflect on my practice and act against systems of oppression to promote a more democratically orientated approach to teaching mathematics (Brydon-Miller et al., 2016).

For Freire (1970), education is not neutral or value free because it can be used as an instrument for conformity, or for the practice of freedom. Freedom in this context refers to one's ability to deal critically and creatively with the world, which Freire calls critical consciousness, which can be submerged when education is used as a tool for conformity. He believed that if education is instead used as a tool for freedom, then people can begin to look more critically at the world and therefore become "more fully human" (Freire, 1970, p.56). Using education to develop this critical consciousness requires authentic reflection because this leads to action, and action leads to praxis. The central component of this is reflection and critical dialogue which must be carried out with the oppressed. In the context of this study, I am not suggesting that PSTs are oppressed in the way Freire described in *Pedagogy of the Oppressed*, nor do they need to be liberated from an oppressive regime. They are oppressed, however, as I described in Section 1.3 in that they are denied access to meaningful educational experiences, and they similarly become oppressors in their practice when they restrict pupils' mathematical understanding to that of instrumental. In this regard, I am interested in exploring Freire's epistemology and how it relates educational experiences to critical consciousness (Frankenstein, 1983).

Dewey, on the other hand takes a more utilitarian stance on education. He claims that students learn from books "only as they are related to experience" (Dewey, 1902/1990, p.17) and schools should prepare students to apply their learning to "everyday life" (p.75). Whereas Freire's theory is set in the context of adult education, Dewey was interested in the child. He advocated for the creation of a child-centred educational setting where children can be active agents of their own learning, and he claims this generally necessitates a certain disorder in the classroom.

For both Freire and Dewey, education and democracy are inextricably linked and despite their different backgrounds, democracy is where their theories meet. Dewey (1916) claimed the school ought to represent a miniature democratic society free from economic stress, whereby socialisation, communication, experimentation, and negotiation can be practiced through ongoing projects. Referring to the Deweyian approach, Chomsky (2004) argues that schools should not need to teach democracy, rather they should provide opportunities for children to experience democracy through practice. Such a conception of education, where students have an active role, encourages further educational activities and the acquisition of more advanced skills. This increases awareness for one's own individual talents and interests and increases capacity for future learning and growth (Dewey, 1916).

According to Freire (1970), education can stifle critical consciousness when it is conceptualised as, what he termed, a banking approach. This is an educational paradigm whereby students are conceived as empty vessels that are uncritically filled with information. Within this system, the more meekly the student uncritically and unconsciously allows themselves to be filled, the better the student is regarded. Chomsky (2004) refers to this form of pedagogy as a labyrinth of procedures and skills, and claims that the banking approach to education, consumed by rote memorisation and later regurgitation in state mandated standardised tests, deskills students, and prevents the development of independent and critical thought necessary to understand the reasons and linkages behind facts. He claims that this "mindless skills-based education" is gaining currency whereby the tests guide the teaching (Chomsky, 2004, p.27).

Dewey was also critical of this banking type approach to education, which he referred to as the "memorize and drill" pedagogy and "waste in education" where schoolwork is mass prescribed, impersonal and meaningless to individual pupils (Dewey, 1902/1990, p.75). He claimed education should not be about "telling" and "being told" but should be "an active and constructive process". He pointed out there is no educational value in "storing up, in accumulating, the maximum of information", and that the result of this is unhelpful competition between children, while at the same time sacrificing the opportunity to instil a sense of social cohesion and cooperation through active work, communicating and exchanging ideas (Dewey, 1902/ 1990, p.15). Dewey (1933) described these types of learning experiences as mis-educative and non-educative, where mis-educative refers to those learning experiences that impede further learning, and non-educative as learning experiences that fail to connect experiences and ideas with one another. Both experiences leave the learner either unchanged or incurious.

Freire (1970) described the banking approach to education as static, compartmentalised, and disconnected from the totality. It removes the need to think critically, and attempts to control thinking and action, to conform to someone else's reality (Freire, 1970, p.59). It works against democratic processes and independent thought, in favour of obedience and conformity (Chomsky, 2004, p.24). In *Pedagogy of the Oppressed*, Freire made just one reference to mathematics where he noted that students record, memorise and repeat "without really perceiving what four times four really means" (p. 52). This approach projects ignorance by negating education as a process of enquiry. In this way the banking method mirrors oppressive society, whereby students accept "the passive role imposed on them" (p.54). Dewey also rejected the notion that humans are inherently passive beings and believed that education should promote students as "active agents of their own learning" (Dewey, 1902/1990, p.17) to become individuals by providing opportunities to explore their own natural talents and interests. Mirroring this stance, Hogan (2011) asserts that schools should uncover those potentials that are most native to each person, and to cultivate these through practices of learning that realise the communal benefits of learning itself, not just individual benefits" (p.31).

The sense of a democratic community was clearly important to Dewey, and this sentiment is also echoed by Hogan (2011) who insists that learning environment then should be concerned with building and sustaining a community of inquiry. Freire (1970) also recommends a community of enquiry based on dialogue, through a problem-posing pedagogical approach where teacher and students can reflect simultaneously.

Freire's (1970) problem posing model encouraged simultaneous reflection between student and teacher without dichotomising their relationship and without dichotomising action and reflection. Respectful dialog is critical, and teachers should not be "offended by the contradictions of others" (p.71). Collective reflection allows for authentic dialogue and critical thinking leading to creativity, action, and enquiry. In this system, teacher and student generate themes to be explored. The role of the teacher here is to "re-present" ideas that students want to know more about, in a way that considers the world view of the students (Freire, 1970, p.76). In this model, knowledge is not to be conceived as fragmented unrelated parts but "interacting constituent elements of the whole" (p.85). With the problem posing approach, learners are not docile listeners, but critical co-investigators. Learners see challenges in relation to other challenges, much like Dewey's (1938) idea that growth leads to new growth. This, according to Freire (1970), leads to new understandings about reality, and education becomes a practice of freedom in which learners are not independent from, but considered in relation to the world (p.62). Dialogue must engage critical thinking to result in authentic forms of action and thought, and ultimately praxis. This necessitates a type of community of practice where students' expertise and experiences are valued within a climate of mutual trust. Such a conception of education cannot exist within the banking model.

For both Freire and Dewey, reflective practice is one of the core components of their respective pedagogies. Dewey (1933) asserted that reflective thought as an educational aim for novice teachers can foster freedom of mind and action. Reflecting on experiences creates growth enhancing habits, including emotional and intellectual dispositions, which are important for both learning and learning to teach (Dewey, 1938). It allows beginning teachers to engage in thoughtful examination of their educational experiences, which will allow them to reframe a difficult experience into a problem to be solved (Dewey, 1933).

2.2 Teacher Learning

Teaching and teacher education are inherently complex (Ball & Forzani, 2009), yet many PSTs and novice teachers enter the profession believing teaching is uncomplicated, simple, and transmissive (Loughran, 2006). This, combined with the competing knowledge demands within ITE (Loughran, 2006), present significant challenges for teacher education, and contribute to the problem of enactment. This is discussed in detail in the next section, followed by a synthesis of research-based principles that underpin good teacher education programmes (Korthagen, 2009). It concludes with Grossman, Hammerness, and McDonald's (2009) practice-based approach for teacher education, which forms the basis for the intervention used in this study.

2.2.1 Challenges with teacher education

One of the challenges with teacher education is that teaching is a complex practice that looks deceptively simple to PSTs and novice teachers (Grossman et al., 2009). Good teaching requires making specific instructional moves, coordinating learning outcomes, managing time, while simultaneously ensuring students are engaged and learning. It is intricate work comprising of multiple tasks and moves which are often invisible to the casual observer (Ball & Forzani, 2009). This complexity is equally not visible to novice teachers. This perceived oversimplification is largely a result of PSTs' apprenticeship of observation, which refers to the culmination of the thousands of hours spent observing their own teachers and building a mental model of how they think children should be taught (Lortie, 1975). The other factor that contributes to the oversimplification of teaching is the perceived naturalness of it. This perception results from the fact that people often teach each other informally in everyday life. While this sort of casual teaching occurs in a natural way, classroom teaching, when closely examined does not. There is a difference between informal everyday teaching and the types of complex activity inherent in quality classroom teaching (Ball & Forzani, 2009).

The misconception that teaching is a natural task emphasises the outdated notion put forward by economist Hanushek (1970) that good teachers are born not made. This notion, if taken seriously, has implications for teacher education because it denies that teaching is learnable, promotes a fixed mindset view of learning, reduces teaching to an oversimplified process while at the same time de-professionalising the practice. It is now accepted that developing a professional competence for teaching requires learning to do things related to teaching that are not common in everyday life, and that good teachers can be "made" with appropriate and deliberate teacher education (Ball & Forzani, 2009; Grossman at al., 2009).

While teaching is a complex task, this complexity is intensified in the context of teacher education because of the following idea: TEs teach about teaching and PSTs learn about teaching (Loughran, 2006). This dual role is specific to teacher education and does not exist in other systems of professional learning. For example, doctors do not treat their students; they treat patients and teach their students about medicine. On the other hand, teacher educators teach their students while simultaneously teaching them about teaching. This nuance needs to be recognised and carefully considered while designing a teacher education curriculum (Loughran, 2006).

Initial teacher education is further complicated by the competing cognitive and affective aspects of learning to teach and enact complex practices (Koster & Korthagen, 2001). Loughran (2006) argues that much of the work done in ITE focuses disproportionately on the traditional cognitive domain. This bias in favour of the traditional cognitive domain can have negative consequences on PSTs' ability to recognise and respond to their "emotions, feelings and reactions, all of which are so enmeshed in the experiences of learning and teaching about teaching" (p.3). Competing affective and cognitive demands in ITE are discussed in Section 6 of this literature review.

To call attention to the underlying complexity and the dual role of teacher education a specialised pedagogy of teacher education is necessary (Ball & Forzani, 2009). Teachers' moves, decisions and interactions all depend on specialised training – a special pedagogy for teacher education rooted in practice. Based on Shulmans (2005) idea of a signature pedagogy, the concept of pedagogy of enactment based around core practices of teacher education will be presented at the end of this section as an approach to PST learning (Grossman, 2009).

Next, epistemological approaches in ITE and how they influence PSTs' classroom behaviours will be discussed. This will include barriers to PST learning and how these can be addressed in ITE. Central to this conversation is the social constructivist nature of how teachers learn. This discussion is framed by Wenger's (1998) social learning theory and Korthagen's (2009) 3-level gestalt model of teacher learning. These theories set the context for Grossmans and McDonald's (2009) concept of 'pedagogies of enactment'. The section will conclude with implications for teacher education.

2.2.2 The Problem of Enactment

As described in the opening chapter, the problem of enactment is the central issue to this study. It is formally defined as situations whereby novice and pre-service teachers frequently demonstrate the inability to translate effective theories of teaching into practice, and as a result fail to produce effective classroom learning (Kennedy, 1999; Gardiner and Salmon, 2011; Darling-Hammond, 2006). On the contrary, PSTs tend to enact instructional practices that are inconsistent with their beliefs and the pedagogical commitments they profess (Kennedy, 1999) and this is one of the primary obstacles they must contend with in their practice (Darling-Hammond, 2006). Although it has been shown that PSTs enter the profession with optimism and ambition, they are soon faced with problematic scenarios in the classroom setting which they have not been sufficiently prepared to deal with (Korthagen and Wubbels, 2001). These scenarios are described in the remainder of this section.

Researchers have been aware of the problem of enactment in ITE for decades (Zeichner and Tabachnic, 1981; Muller-Fohrbrodt, Clohetta and Dann,1978; Bergqvist, 2000, Zimmerman, 2017). More than forty years ago, Zeichner and Tabachnick (1981) provided “overwhelming evidence” that the impact of ITE on PSTs’ learning was “washed out” by school experience (p.7). Adding to this, Korthagen (2005) argues PSTs are influenced more by existing practices in schools than they are by up-to-date literature on teaching and learning. Similar findings were put forward by Veenman (1984) who referred to the myriad of problems PSTs’ experience as they transition from ITE to in-service practice as the “reality shock” or “praxis-shock”. Veenman (1984) used these terms to describe the collapse of ideals formed by PSTs during teacher education by the “harsh and rude reality of everyday classroom life” (p.143).

Faced with the reality of the school classroom, it is quite common for PSTs to abandon their instructional ideals as they struggle to adjust to the challenges inherent in classroom instruction (Grossman & Thompson, 2008; Wood, Jilk, & Paine, 2012). This abandonment of their beliefs for less favourable behaviours can lead to frustration, early career burnout, and potentially leaving the profession (Hammerness, 2006; Veenman, 1984). It also leads to tensions between the TEs expectations of how PSTs *should* behave in practice, and the *actual* behaviour of PSTs. Tensions are further heightened when PSTs continuously fail to meet these expectations (Korthagan, 2005). As highlighted by Zimmerman (2017), the problem of enactment remains a major obstacle for teacher education.

There are many reasons put forward in the literature for the problem of enactment. In their seminal study, Zeichner and Tabachnic (1981) list the influence of co-operating teachers, the ecology of the classroom, the bureaucratic norms of the school, teacher colleagues, and even pupils as the contributing factors. There are also issues related to teacher identity as they make the transition from the role of student to teacher in a position of authority (Dugas, 2016), lacking the disposition or commitment to actualise their pedagogical ideals when the opportunity does arise (Diez, 2007). However, the most fundamental barrier to enactment is related to tensions that exist between the university context and the school context (Flores & Day, 2006; Valencia, Martin, Place, & Grossman, 2009). This is primarily related to learning in ITE lacking sufficient context, resulting in PSTs lacking the concrete tools and practices to put ideas they have learned into action in a complex setting (Darling-Hammond, 2006), even when these modules are running concurrently (Lampert, 2010: 24). Kennedy (1999) argues that the problem is compounded by PSTs’ apprenticeship of observation (Lortie, 1975; Borg, 2004) which often manifests in an inaccurate frame of reference which may be incompatible with what is taught on ITE (Darling-Hammond, 2006). This makes it likely that PSTs will teach the way they were taught in school, especially high stakes complex situations like SP, where it is natural to "trust what is most memorable" (Boyd et al., 2013, p.5).

Thus, there is a real challenge for PSTs to seamlessly integrate content knowledge, along with relevant theories of learning and pedagogical skill, into the already complex task of teaching. Lampert (2010, p.24) argues that putting this responsibility to "integrate it all" on PSTs is overwhelming. For example, in an extensive review of the literature on mathematics in ITE, Clift and Brady (2005) found that PSTs had great difficulty in translating university-based recommendations for teaching mathematics into classroom practice. At times, there was an outright conflict between teacher education programmes that promoted active and constructivist-based mathematics teaching, and the passive realities of classroom teaching. Clift and Brady (2005) also note time constraints and task demands on PSTs, inherent in university-based teacher education programmes, as barriers to the knowledge transfer.

2.2.3 Competing knowledge domains

A key consideration for addressing the problem of enactment is the model of learning used in initial teacher education. Korthagen (2010) argues the problem can be addressed by ITE departments moving away from a traditional cognitive model of learning to a situational model that accounts for the complexities inherent in teaching. Within the traditional model of initial teacher education, the university provides the knowledge, the school provides the placement setting and the PST provides the individual effort to assimilate and apply this knowledge in the practice setting (Wideen, Mayer-Smith & Moon, 1998, p.167). It is assumed that PSTs will be able to enact this theoretical knowledge in the classroom setting through engagement with specific strategies learned in methods courses. Grossman (2008) describes this basic assumption as a reductionist process-product conception of teacher education, which ultimately ignores the inherent complexity and theoretical underpinnings of teaching and teacher education.

There are further disconnects within the traditional teacher education model. A defining feature of the traditional model is the separation between foundation modules on one hand (e.g., the philosophy of education), and methods modules on the other, which tend to focus on practice (Grossman, 2009). The goals of each are very different. Foundation modules aim to impart conceptual tools to help PSTs make informed decisions in the classroom. This may include various theories and modes of learning, theories of motivation, etc. On the other hand, methods courses are concerned primarily with the practical aspects of classroom teaching, including strategies and tools for delivering lessons.

The challenge is to reconcile the specific nature of methods courses with the general nature of conceptual tools. Lampert (2010) argues that methods courses are often concerned with learning *about* instructional routines and approaches, and less about how to enact practice in an expert way. Failing to reconcile these competing knowledge domains (Loughran, 2006) results in a damaging separation of the technical from the intellectual (Chaiklin & Lave, 1996). This separation of the intellectual from the practical results in a fragmentation of teacher education, the most significant of which is the separation of theoretical knowledge from practical classroom work. It also relegates important aspects of teaching to individual modules, rather than a seamless integration into a PST's professional preparation and puts conceptual underpinnings at the core of teacher education at the expense of practice. To address this Chaiklin & Lave's (1996) view posits that practice should incorporate a seamless blend of technical and intellectual.

At the heart of this fragmentation is the fundamental assumption that the theoretical knowledge resides in the university setting and authentic practical knowledge, for the most part, can be reserved for school placements. Chaikin and Lave (1996) explain why this model is ineffective. Contrary to what teacher educators expect or desire, PST learning tends to exhibit characteristics of apprenticeship learning. This is at odds with the overly simplistic traditional model. Because of the disconnect between its various elements, the traditional cognitive model inevitably lends itself to transmission style teaching and the dominance of propositional knowledge (Korthagen, 2010). As well as the likelihood of universities reinforcing transmission style teaching, it is also likely PSTs will embrace the passive learning associated with it as it is what they were accustomed to by their own formal schooling (Loughran, 2006). Barone et al. (1996) argued that this system results in only tenuous links between theory and practice. This assertion is underpinned by the difference between nature of the knowledge that exists in the minds of PSTs that helps them to act effectively in the classroom, and the knowledge that is taught on teacher education programmes.

2.2.4 PST Knowledge is Socially Constructed

The first step in addressing the problem of enactment, and promoting long-lasting situational knowledge, is to understand the nature of how PSTs learn how to teach. In his seminal work, Wenger (1998) conceptualised PST learning as a process of social constructivism. According to Wenger, learning is determined by the extent to which an individual can learn to participate in the discourse of a particular community of practice. Wenger defines a community of practice as a group of individuals from a shared practice who share a common concern for what they do, and who strive to improve on what they do through interaction with each other. This conceptualisation explains the characteristics of apprenticeship learning observed in PSTs by Chaikin and Lave (1996), and why the model of teacher education based on the traditional cognitive model is largely unsuccessful. When learning is socially constructed, competence is not defined on an individual level, but socially negotiated by the community. It is through participating and contributing within these communities that individuals learn. In this regard, Lave and Wenger (1991) define learning as a special kind of social practice where the learner learns how to do and act within a particular context, as opposed to direct transfer of a discrete set of propositional knowledge from one context to another. Lampert (2010) mirrors this view in the context of teacher education and recommends that learning to enact should be a collaborative and contextualised process whereby PSTs and TEs work co-operatively together while actively using subject matter knowledge to guide practice. She maintains this approach will promote long-lasting situational knowledge for PSTs.

This notion is further supported by Eraut (2014) who posits that PSTs require a situational understanding to interpret knowledge requirements for practice successfully. Eraut (2010) claims that the only way to achieve this is through sustained experience, involving the availability and use of tacit knowledge for PSTs. The most important factor for novice practitioners to achieve this is confidence, that results in personal agency and motivation (Eraut, 2007).

In general, Lave and Wenger (1991) attribute professional learning to a process they identified as *legitimate peripheral participation* (LPP). LPP is a learning process whereby new members participate socially in a community of practice with more established members, and in doing so are given opportunities to observe more experienced practitioners' professional knowledge-in-action. In the context of this study, this will involve PSTs working with each other and with me, in a community of practice, guided by the philosophies of Freire and Dewey.

LPP allows novice professionals to engage in expert practice, albeit to a limited degree and with reduced responsibility than their experienced counterparts. Over time the novice professionals gradually develop expert knowledge and skills and take on increased responsibility to become fully fledged members of a community of practice. Constructive feedback to novice professionals on performance is a crucial element of this collaborative process (Philpott, 2014). In the context of this study, feedback will be informed by the literature and will be a shared responsibility between me and the PSTs in the study.

2.2.5 Gestalt Theory of Teacher Learning

Korthagen (2010) adapted Lave and Wenger's work on situated learning and legitimate peripheral participation to reconceptualise pedagogies of teacher education and reconcile the incompatibility between the traditional cognitive and situated models of learning. His aim was to develop a deeper understanding of teacher behaviour and learning, but more specifically to analyse the "friction between teacher behaviour in practice and the wish to ground teachers' practices in theory" (Korthagen, 2010, p.98). To do this, he developed a three-level "Gestalt" model of teacher learning which is described below.

Level 1 is the gestalt level: A gestalt refers to a cohesive whole of PSTs' past experiences. These experiences can refer to role models, needs, feelings, images, values, and routines – all of which are invoked, often unconsciously, during practical experiences (Korthagen, 2013). The gestalt has the effect of bringing together two different ways of seeing the same thing, i.e., traditional cognitive, and situational. This is done by taking into consideration the shift in the purpose of knowledge that takes place during a PST's development. During the gestalt level of learning, PSTs are largely unaware of their behaviour, so they are also unreflective about it. At this level, teaching is usually carried out in an instrumental way where pupils are treated as passive listeners. This behaviour is considered an automatic performance of actions. PST behaviour is determined by a wide range of factors, including cognitive, behavioural, and motivational. All these factors are intertwined to form the PST's gestalt which in turn determines their perception of the here and now situation. At this stage, the

PST is unaware of these factors and learning at this level is characterised by the onset of awareness of one's previously subconscious behaviours.

There are many examples in the literature of how PSTs' gestalts can interfere with, or even nullify, theory learned in university-based teacher education. Korthagen (2013) provides one illuminating example of this. In this example, a PST is faced with a pupil s/he sees as being unmotivated. Ideally, the PST would refer to and enact Wubbles and Levy's (1993) theory on interpersonal classroom behaviour s/he learned about in ITE but in reality, the situation conjures up a complex mesh of ideas, feelings, and images from the PST's past, along with the desire to change something in that pupil to affect some behavioural change. In this moment, that PST's gestalt can replace all the knowledge and theory learned during professional preparation. That PST may then, for example, confront the pupil in an oppositional rather than a co-operative way going against Wubbles and Levy's (1993) theory which says that in this type of situation a teacher would do better to opt for cooperative rather than oppositional behaviour.

By first recognising and then reflecting on their classroom behaviour, PSTs can progress to **Level 2: the schematic level.** Through reflection, PSTs can see generalisations in their learning and develop a schema of interrelated concepts. This happens when several similar situations occur, and the PST develops a generalised knowledge about the situation that occurs in practice (e.g., unmotivated pupils). Via this process, PSTs develop a conscious network of principles that help to describe practice.

This is not abstract as it is driven by a PST's desire to know how to act in specific situations. Then, this may lead to **level 3 – the theory level**. Driven by a desire to understand practice on a deeper level, PSTs may start examining relationships between schemata. Eventually, a theory or schema may be reduced to a single gestalt and used in a less conscious way, resulting in the emergence of new and improved PST behaviour. This allows the PST to recognise and reflect on other aspects of their teaching.

Teaching is a gestalt-driven activity, and gestalts are changed when they are recognised and reflected upon. They cannot be influenced by theory on its own. Because of this, ITE must present PSTs with meaningful practical experiences along with opportunities for reflection. Korthagen (2009) calls for the organisation of "sufficient, suitable and realistic experiences tailored to the needs and concerns" of PSTs, while preparing for schema development by offering opportunities for reflection on those experiences (p.104). Suitable experiences are those that are challenging enough to "offer opportunities for a confrontation with gestalts the educator would like to change" (p.104).

In the context of this study, I have no concern about PSTs' ability to reflect on their teaching. In fact, they are encouraged and supported to reflect on their learning experiences across modules, and there is a very intentional and deliberate focus on reflective practice on SP and other practical experiences. However, in the original maths competency modules there was limited opportunities for meaningful reflection about learning and its implications for practice. Therefore, a maths competency intervention to address the problem of enactment must include suitable practical experiences coupled with ongoing opportunities for meaningful reflection on those experiences.

2.2.6 Teacher Education Pedagogy

Professional learning, including ITE, is not for understanding alone, but to meet the demands of the profession. This involves bringing vast amounts of theory and bodies of knowledge into the practice setting. Shulman (2005) coined the term signature pedagogies to describe the process of how this is achieved. A signature pedagogy implicitly defines what counts as knowledge in a particular field and represents the deep structures and implicit values of the profession. It implicitly defines how knowledge is "analysed, criticised, accepted, or discarded" (Shulman, 2005, p.54).

A signature pedagogy of teacher education must simultaneously pay close attention to the content of what is being taught and how it is being taught (Loughran, 2006). For PSTs, this means they must learn what is being taught while simultaneously examining and questioning the nature of how this content is being taught. Loughran (2006) argues it is much easier for PSTs to pay attention only to what is being taught because this passive style of learning is encouraged by formal schooling experiences, as well as what traditional university teaching is more likely to reinforce. Furthermore, PSTs' conceptions of ITE are greatly influenced by their own school experiences, reinforcing the notion that learning is simple and transmissive. It requires a focused effort to change PSTs' perception of what meaningful or worthwhile learning is, and the belief that there is value in learning in a different way (Loughran, 2006).

Reflection and metacognition help address this issue by encouraging PSTs to be conscious of their own learning. This awareness promotes informed decision making as PSTs construct their personal pedagogies (Hoban, 1997, p.135), and facilitates progression to Korthagen's schema level. From here, PSTs can generate a conceptual schema and develop situational understanding they can apply to teaching in a general way (Loughran, 2006).

On the other hand, TEs need to think deeply about *how* content is taught and the underlying messages that this 'pedagogical turn' conveys to PSTs (Russell, 1997, p.44). TEs need to make their own pedagogical decisions explicit to PSTs and give them access to their thinking and reasoning that shapes their practice. Consequently, there is a similar competing agenda for TE to contend with – they must teach content while at the same time paying specific attention to how they teach that content. This is the challenge for teacher educators who need to recognise what aspects of teaching to unpack and when to do this. This should not be confused with modelling teaching practice because it involves unpacking teaching in a way that PSTs are given access to the "pedagogical reasoning, uncertainties and dilemmas of practice that are inherent in understanding teaching as being problematic" (Loughran, 2006, p.7). TEs must understand how teaching promotes meaningful learning, and how learning influences teaching such that there is a responsive relationship between teaching and learning. It is essential TEs give PSTs the opportunities, encouragement, and permission to make those mistakes that are part of learning to teach, followed by the necessary discussion and reflection to promote and enable growth.

2.2.7 Pedagogies of Enactment: A Model of Learning for Teacher Education

Underpinned by the idea of using knowledge in practice, Grossman and McDonald (2008) developed a TE pedagogy to prepare PSTs for the complexities of the classroom. Their approach provides PSTs with opportunities to "practice elements of interactive teaching in settings of reduced complexity", while simultaneously receiving feedback and reflecting on this feedback. It includes foundational elements of critical dialogue, public sharing of work, and engagement in community's learners (Parker et al, 2016), and addresses the problem of enactment by providing opportunities for "sustained inquiry about the clinical aspects of practice" (Grossman & McDonald, 2008, p.189). As a practice centred approach to teacher education that values the integrated nature of theory and practice, it is designed to close the theory-practice divide while at the same time addressing the complexity of teaching as a practice and the preparation of PSTs.

A core component of Grossman and McDonalds' (2008) framework is deliberate and collective reflection, including critical dialogue. This aspect of the framework gives PSTs opportunities to confront their gestalts through meaningful engagement in "sufficient, suitable and realistic experiences" tailored to their needs (Korthagan, 2009, p.104). Critical dialogue is the "best way to try to exert some control" over how PSTs think and act (Russell, 1997) and will expose PSTs to the complexities of classroom mathematics teaching and the uncertainty of knowledge (Parker et al., 2016). Furthermore, it will afford PSTs' opportunities to discuss their teaching experiences in a supportive environment where ideas, struggles, and concerns can be shared and learned from.

Lortie (1975) argued the lack of a "common technical language" in pedagogies of teacher education and this has epistemological consequences as it limits the ability of TEs and PSTs to communicate and access existing bodies of knowledge related to classroom teaching. Grossman and McDonald (2008) recognised this lack and included in their model a language for effective means of communicating ideas, practices, and research across the profession.

There are two key parts to their pedagogical framework. Firstly, is the idea of well-defined core practices of teaching which Grossman et al call high leverage teaching practices (HLTPs). These HLTPs are then analysed and enacted using representations, decompositions, and approximations of these practices to help teachers learn to use them in the classroom context. These are described in the following two sections.

2.2.8 High Leverage Teaching Practices

At the heart of this practice-based initiative is the idea that PSTs are given opportunities to practice teaching before entering the classroom. This is achieved through a set of established routines, known as high leverage teaching practices (HLTPs) that allow PSTs to integrate skills and knowledge and the necessary judgment required to put these skills to use in the classroom (Grossman, 2018). HLTPs are identifiable components fundamental to teaching that PSTs enact to support learning.

An extensive list of HLTPs have been developed and published on the TeachingWorks website (teachingworks.org). TeachingWorks is an initiative developed by teacher educators, teachers, and researchers to improving the professional preparation of teachers. Consistent with this study, the website section on HLTPs was informed by Ball and Forzani (2009), Grossman, Hammerness and McDonald (2009), and Grossman (2018). Their list of HLTPs include:

- Leading a discussion
- Explaining and modelling content
- Eliciting and interpreting
- Diagnosing patters of student thinking
- Implementing norms and routines for discourse
- Coordinating and adjusting instruction
- Establishing and maintaining community expectations
- Implementing organisational routines
- Setting up and managing small group work
- Building respectful relationships
- Communicating with families
- Learning about students
- Setting learning goals
- Designing lessons
- Checking student understanding
- Selecting and designing assessment
- Interpreting student work
- Providing feedback to students
- Analysing instruction

TeachingWorks refer to these HLTPs as a core set of fundamental capabilities for teachers. Furthermore, each of these HLTPs is further refined to reflect what each one looks like for specific content areas, including mathematics.

Grossman stresses that these HLTPs are *not* a set of competencies that can be checked off. They are strategies, routines, and moves that can be learned and unpacked by PSTs that are "deeply connected to the goals of disciplinary learning" (Grossman, 2018, p.4). HLTPs have the following features:

- Occur at high frequency in teaching
- Can be enacted across the curriculum
- Allow teachers to learn more about students and teaching
- Preserve the integrity and complexity of teaching
- Are research-based and improve pupils' achievement

A focus on HLTPs allow PSTs to experience teaching as a complex task, while at the same time intentionally addressing the key components of teaching, they need to be aware of. Examples of HLTPs include modelling mathematical content, eliciting and interpreting pupils' responses, implementing norms and routines, leading a mathematical discussion, and many others. As an example, Grossman and McDonald (2009) describe 'Leading a discussion' as a complex task of teaching that looks, simple, yet it may take dedicated teachers years to perfect. Like all HLTPs, it is composed of a range of learnable sub-practices as well as subject knowledge, knowledge of group dynamics, as well as issues of status/ equality and student development.

Focused attention to the detail of practices using HLTPs can allow PSTs to practice separating elements that make up that the corresponding practice, which can be integrated with experience over time. With any given practice, TEs must consider both the conceptual and practical aspects associated with it. Staying with the example of leading a discussion, TEs would point out the underlying theoretical rationale for using this practice. In this case, there is no point in trying to discuss a topic that is not discussable. In this way, PSTs learn not only how, but when it is appropriate to use the practice of discussion in the classroom. Through this process, "professional knowledge and identity are…interwoven around the practices of teaching" (Grossman, Hammerness & McDonald, 2009, p.278) and thus professional identity and learning is developed. Grossman and her colleagues noted that a recurrent challenge with leading a discussion, and other HLTPs, is concluding it to be responsive to the ideas that were raised during the discussion, linking it to future learning opportunities, as well as disrupting existing social hierarchies in the classroom.

Enacting HLTPs promotes a move away from instructor and concept-centred direct instruction where academic knowledge is delivered, to one where PSTs are given carefully planned opportunities to enact individual or collective elements of core practices in a controlled environment. It requires a significant increase in support for PSTs from TEs and this needs to be factored into planning. As PSTs enact HLTPs, they also learn about their underlying principles. This reduces the theory/practice dichotomy and the view that teaching can be learned as a finite set of techniques, thus maintaining the quality, integrity, and complexity of teaching as a practice. To facilitate this process, Grossman et al. (2009) developed the ideas of representations, decompositions, and approximations of practice. These are presented in the following sections.

2.2.8.1 Representations

Representations of practice invoke Lave and Wenger's (1991) concept of legitimate peripheral participation by making the work of expert teachers visible to PSTs (Grossman, 2018). This includes videos of practice, stories of practice, case studies, and narratives about practice. Representations of practice also include artefacts such as lesson plans, pupil work, and observations of actual practice. Both video recordings of classroom teaching and modelling of content are effective representations within university-based teacher education programmes. In essence, representations allow PSTs and TEs to jointly analyse aspects of practice and develop a deeper understanding of that practice. They are used to support PSTs in seeing holistically, as well as allowing for more in-depth analysis of specific components of practice (Grossman et al., 2009).

Representations are an important starting point for practice-centred teacher education. They allow complex practices to be unpacked by a process known as decomposition of practice. It gives PSTs' opportunities to identify and closely examine specific components of practice.

2.2.8.2 Decompositions

Because teaching is integrative work it requires decomposition of complex tasks into distinct teachable elements (Ball *et al.*, 2009). Decomposition is the process of breaking down complex practice into its constituent parts so that they can be made visible to PSTs or novice teachers (Grossman et al., 2009). It is used to isolate, identify, and practice different components of HLTPs. Because decomposition is designed to preserve the integrity of teaching, each of the component parts is ultimately recomposed to form the original complex practice. Effective decomposition is reliant on an established grammar of practice. Building a language to describe practice allows practitioners to name discrete components and establish relationships between them (Grossman, 2018).

2.2.8.3 Approximations

Approximations of practice refer to the enactment of complex teaching practices (HLTPs) in situations of reduced complexity and with varying levels of support (Grossman, 2018). They allow PSTs to engage in practice that is similar, but not identical to, the work of practicing professionals. They differ from professional practice because of lacking the additional complexity, such as classroom management. They offer PSTs' opportunities to enact parts of teaching in low-stake scenarios where they can receive support and targeted feedback from TEs and their peers (Grossman et al., 2009). This feedback and collective reflection allow PSTs to address the gap between content and theory learned in university and what they are likely to experience in the classroom setting (Grossman, 2009). Finally, PSTs should be afforded multiple opportunities to enact practices to develop fluidity.

Approximations of practice are becoming more popular in teacher education internationally (Grossman, 2018). In the Irish context they have been used to good effect by Twohill et al. (2022) to create opportunities for the development of PST efficacy beliefs, while Delaney (2013) used approximations of HLTPs to address the complexities inherent in learning mathematics for teaching.

Approximations lie on a spectrum whereby they can be more or less similar to the authenticity of the classroom. For example, PSTs teaching their peers in the university setting is less similar than the authenticity of the classroom setting, but it allows PSTs to get risk-free, immediate, and targeted feedback. Lampert et al. (2013, p.226) recommend the use of rehearsals as a form of approximation to embed "intellectually ambitious" teaching and learning in practice. The idea of intellectually ambitious refers to authentic pedagogy that aims to deepen pupils' understanding of content knowledge, encourage development critical thinking skills, and the capacity to construct and apply new knowledge to new situations (Smylie & Wenzel, 2006).

2.2.9 Conclusion

On entering initial teacher education, PSTs' beliefs about mathematics tend to be narrow, formal, and rigid and contribute to teacher-centred transmission style teaching. These beliefs act as powerful barriers to learning alternate ways of teaching mathematics (Larkin, 2012). To address these issues, it is the role of teacher educators to identify and design an appropriate signature pedagogy of mathematics teaching in ITE (Larkin et al 2012), and this design must be appropriate to facilitate the enactment of knowledge and theory in practice *(*Parker et al., 2016).

Grossman & McDonald's (2008) model of practice-centred teacher education address this shortfall by allowing for sustained inquiry into clinical aspects of practice. The framework described above allows practice to be parsed into its constituent parts so that teacher educators can provide professional education for PSTs to meaningfully engage in these practices. This approach to teacher education is consistent with Dewey's laboratory approach to "enliven and awaken teacher candidates to the meaning and vitality of educational principles" (Greenwalt, 2016, p.3) while preserving the relational nature of teaching.

2.3 Mathematical Understanding

There are wide ranging social and economic forces that impact on classroom practice on a national level, and these are discussed in Section 2.5 of this chapter. At the local level, perhaps the most important place to start is with teacher knowledge. To teach meaningful and intellectually ambitious mathematics (Lampert, 2010) that allows to pupils to experience productive struggle (Van de Wall, 2020), it is necessary for PSTs to have a corresponding level of mathematical competence (Reid & Reid, 2017). This competence should enable a better quality of mathematics teaching, and a more democratic approach to education, and the type of MCK taught in ITE should reflect this goal. In this regard, this section will outline what it means for pupils to understand mathematics, and what mathematical content knowledge is necessary for PSTs to have. To begin, a discussion about what it means for children, and indeed adults, to understand mathematics will be outlined. This will be framed by Skemp's (1978) ideas of relational and instrumental understanding. This will be followed a discussion about the depth of knowledge PSTs ought to have so that they can teach mathematics effectively. This will include a reflection and critique of this from the perspective of my practice as a teacher educator.

2.3.1 Mathematical understanding: relational and instrumental

Skemp (1978) categorised mathematical understanding into instrumental knowledge and relational knowledge. Relational knowledge, which is more commonly known a conceptual knowledge, can be described as “knowing what to do and why” (Skemp, 1978, p.2). Baroody (2003) also referred to this as meaningful knowledge, as it refers to an understanding of meaning; for example, knowing *that* $(n-2) \times 180$ is the formula to find the sum of the internal angles in an n-sided polygon is not the same thing as understanding *why* it is true. It includes knowledge that goes beyond a single concept to encompass contexts in which that concept is useful or applicable. It also includes an awareness of the coherency of mathematical concepts, including their relationships to one another. These relationships can exist between previously learned concepts, or previously learned and new concepts (Rittle-Johnson, Fyfe, & Loehr, 2016). Some researchers argue the rich links/ relationships between concepts are equally important as separate concepts (richly related) Rittle-Johnson & Schneider, 2015. Acknowledged this is disjointed in novices and can take time to develop/ become integrated. Richness of connections increases with practice

Relational understanding can be visualised as a connecting web of relationships (Hurrell, 2021) and allows learners to make connections to new and previously learned ideas by building up a conceptual schema (Skemp, 1978). This enables the ability to see relationships and navigate between seemingly unrelated mathematical topics. This schematic view is consistent with the work of Bruner (1966) on the development of concepts which, he argues, provide structure for a discipline, a framework from which individual components can be readily understood and retrieved, make transfer of learning possible, and provides a framework for lifelong learning.

Although it is the most difficult type of knowledge to develop because it must be built upon already existing knowledge (Willingham, 2009), it involves less memory work because mathematics is easier to remember as a connected whole (Skemp, 1978). This makes it inherently more inclusive. It encourages students to actively explore new areas and promotes “the kinds of fluency that enable the unearthing and the asking of searching questions” (Hogan, 2011, p.33). Learning becomes intrinsically more pleasurable, and PSTs are therefore more likely to voluntarily continue with the subject. Relational understanding is effective as a goal in itself and reduces the need for external rewards or punishments. By its very nature, learning relationally encourages a community of enquiry. It is consistent with Freire's problem posing pedagogy because it views mathematics not as fragmented, unrelated parts but as a set of "interacting constituent elements of the whole" (p.85).

Whereas relational understanding involves knowledge of more general principles, instrumental understanding involves the continuous acquisition of a multiplicity of rules that must be applied. It can be defined as one ability to use a series of steps or actions to achieve a task (Heibert & Lefevre, 1986; Rittle-Johnson, 2017). Other researchers characterise this mathematical knowledge as the capacity of follow predetermined steps in sequence in order to solve a mathematical problem (Canobi, 2009; Miller & Hudson, 2007; Rittle-Johnson & Schneider, 2015; Willingham, 2009). According to Skemp (1978), teaching for instrumental understanding only promotes the idea of "rules without reasons". In this regard, "understanding" refers to the ability of a student to perform calculations by uncritically following a procedure, without the need to understand why the procedure works or why the answer is correct. This definition of instrumental understanding characterises it as passive and uncritical, and mirrors Freire's (1970) banking model of education. It is fundamentally undemocratic because it is about instructing learners carry out actions without explanation as to why, and ultimately stifles growth and independent thought (Dewey, 1938; Freire, 1970; Chomsky, 2004). Some researchers disagree with this characterisation of instrumental understanding. For example, Baroody, Feil, and Johnson (2007) argue that there can be a relational nature to instrumental understanding because procedures are often interconnected or embedded within other procedures.

According to Skemp, the "teach to the test" pedagogical approach limits pupils' understanding to instrumental (Skemp, 1978). Importantly, this is a prevalent approach in the Irish school system (McKoy, Smith & Banks, 2012; Conway and Murphy, 2013; O'Leary et al, 2019) and this is discussed in more detail in Section 2.5. Other related issues which will be outlined in Section 2.5 include a culture of whole class didactic teaching and widespread use of textbooks as a pedagogical resource, both of which also impede the development of pupils' relational mathematical knowledge. These damaging practices are mutually reinforcing because teachers who take a procedural dominant approach are more likely to use textbook as a pedagogical resource, and, according to Boaler (1998) textbooks emphasise a computation and procedure approach to mathematics, while encouraging limited and inflexible mathematical learning. Furthermore, other research suggests that typical textbook "problems" are accompanied by suggested solutions which themselves are based on some algorithmic template which produces short-term learning but fails to enhance learners long-terms relational knowledge (Wirebring, et al., 2015). This is problematic because inhibiting development of relational understanding in learners critically inhibits their ability to generalise and transfer mathematical knowledge (Richland, Stigler and Holyoak (2012). Furthermore, the same researchers found that when learners knowledge base was instrumental in nature, they tend to demonstrate an inability to reason mathematically, and procedures tend to be incorrect or only partially correct. Ultimately, an instrumental approach to mathematics teaching interferes with the goals of mathematics education. it denies mathematics as an intellectual pursuit and promotes passive as opposed to critical thinking.

Despite the limitations of an instrumental only knowledge base, both types of mathematical knowledge are necessary (Richland, Stigler and Holyoak, 2012), and it is now accepted that an iterative bi-directional approach to teaching and learning mathematics is the most effective because conceptual knowledge and procedural knowledge are mutually reinforcing (Hurrell, 2021). From a cognitive science perspective, Willingham (2009) confirms that procedural knowledge does not imply conceptual knowledge, and conceptual knowledge does not guarantee procedural knowledge. Crucially, instructional practices that only emphasises only one component will result in limited mathematical understanding (Willingham, 2009; Reid and Reid, 2017). The relationship should be bi-directional but not necessarily equal in the sense that conceptual knowledge more often plays the lead role in supporting development of procedural knowledge rather than the other way around. Furthermore, there is a consensus in the literature that a relational first approach should be taken, followed by related instrumental mathematics. For example, Pesek & Kirshner (2000), found that learners who were taught concept first outperformed those who were taught by procedure first. This could be explained by Hiebert (1999) who found that learners who learned procedure first were less motivated to engage in conceptual reasons for those procedures. Teaching for relational understanding, including the meaning of concepts and the relationships between them, necessitates a corresponding level of teacher mathematical competence (Ma, 1999; Wu, 2010). This topic is discussed in the next section.

Before addressing teacher knowledge, it is necessary to examine the idea of procedural fluency. In the United States, the Common Core State Standards for Mathematics (CCSSM) document describes procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (CCSSI 2010, p. 6). Similarly, Baroody (2006) describes basic fact fluency as "the efficient, appropriate, and flexible application of single-digit calculation skills" (p.22). Furthermore, basis fact fluency must involve recall from memory of single-digit arithmetic operations from memory. Willingham (2009) adds that this retrieval from memory must be automatic (i.e., rapid and virtually attention free). The same author argues that the automatic retrieval of basic mathematics facts is important for solving more complex problems because complex problems have simpler problems embedded within them which require factual knowledge. According to Kling and Bay-Williams (2014), timed tests are regularly used to assess this mathematics knowledge in the classroom. However, timed tests offer little insight into pupils' levels of procedural fluency because they do not capture pupils' strategies, and how they select those strategies (Kling & Bay-Williams, 2014). Furthermore, timed tests have been shown to have a negative impact on pupils' development of procedural fluency compared with those who are not exposed to timed tests (Henry and Brown 2008). Boaler (2012) and Ramirez et al. (2013) have also demonstrated that timed tests can increase levels of mathematical anxiety in the pupils who take them, regardless of ability levels or how the pupil performs on the test. An important finding by Ramirez et al. (2013) was that those pupils who tended to use sophisticated mathematical strategies were the ones who suffered the most from mathematical anxiety. In other words, timed tests appear to have the most negative impact on the achievement of the best mathematical thinkers.

2.3.2 PSTs' Mathematical Content Knowledge

The 2007 McKinsey report concluded that "the quality of an education system can never exceed the quality of its teachers" (Barber & Mourshed, 2007, p.15). A fundamental requirement of a quality teacher is the possession of expert content knowledge (Shulman, 1986). However, there is an internationally recognised problem whereby, in general, PSTs and in-service teachers lack the necessary depth of conceptual understanding required for teaching primary mathematics (Wu, 2010; Ma, 1999; Philipp et al., 2007; Ball, 1990; van Es & Conroy, 2009). As a response, textbooks about mathematical knowledge have been specifically developed by mathematics teacher educators to address this issue, including Suggate, Davis and Goulding (2001), Haylock (2010), and Wu (2010). These authors agree on three things with respect to the preparation of primary teachers for mathematics:

1. PSTs' mathematical knowledge is predominantly instrumental in nature
2. Much of the mathematics they once knew is forgotten on entry to ITE
3. PSTs demonstrate a resistance to learning mathematics relationally
4. PSTs often carry with them psychological 'baggage' such as anxiety and unhelpful beliefs (see Section 2.4 for a discussion about beliefs and anxiety)

While studies looking at the MKT of in-service teachers in Ireland are limited (Delaney, 2010), multiple studies have found that PSTs have deficits in their mathematical knowledge. The following studies, which were analysed by Delaney (2010), point to deficits across the curriculum including the mean (Leavy & O'Loughlin, 2006), operations with decimals (Hourigan & O'Donoghue, 2007), and procedural and conceptual understanding (Corcoran, 2005). In a more recent study involving 381 PSTs by Hourigan & Leavy (2017) examining the geometric thinking levels of entry-level PSTs, the researchers found that half the participants demonstrated limited geometric thinking because of associated misconceptions. In my own department, my colleagues and I have investigated entry level PSTs' knowledge of fractions and found their basic knowledge to be severely lacking (Costello & Stafford, 2019); while similar deficits in postgraduate PSTs were found with proportional reasoning (Costello, Stafford, & Oldham, 2017).

While some researchers argue the MCK deficits of PSTs may be due to the lack of focus in ITE on such knowledge (Ball, 1990; Murphy et al, 2011), this does not explain why PST lack MCK on entry to ITE. Perhaps more accurately, Hourigan & Leavy (2017) question the extent to which pre-tertiary experiences develop appropriate foundations to facilitate a smooth transition into ITE mathematics programmes. This view is supported by Jeffs et al (2013) who found that although students engaging in Project Maths were familiar with procedure and regularly engaged in problem solving type questions, they found little evidence that students were engaging in reasoning, formulating proofs, communicating mathematically, or making connections between topics. Moreover, this lack of knowledge has resulted in an under-preparedness for third level courses containing mathematics elements (Kirkland et al, 2012). Given the importance of appropriate content knowledge for teaching, this is likely to mean that the pupils whom they teach will in turn inherit this limited understanding, and the problem will be perpetuated going forward.

According to Shulman (1986) content knowledge includes knowledge of concepts, theories, ideas, organizational frameworks, knowledge of evidence and proof. Knowledge and the nature of inquiry differ greatly between fields, and teachers should understand the deeper knowledge fundamentals of the disciplines in which they teach. In this regard, MCK is the fundamental mathematical knowledge required by teachers to be considered mathematically literate (Reid and Reid, 2017, p.853), and when this is developed in a deep and flexible way it will give PSTs the capacity to present ideas in powerful ways to make them accessible to children (Shulman, 1986; Darling-Hammond, 2006). On the contrary, PSTs without adequate MCK (i.e., relational understanding) will not be able to develop relational understanding in pupils, even if their general pedagogical knowledge and skills are good (Ma, 1999; NCCA, 2004). The rationale behind this is simple, and is summed up succinctly by Wu (2011, p.372) who concludes, “you cannot teach what you do not know”.

The nature of this mathematical understanding is captured by Ma (1999) as a profound understanding of fundamental mathematics (PUFM). In her seminal work on teacher mathematical knowledge, Ma (1999) conducted a study which compared teachers' understanding of fundamental mathematics in China and the United States. Ma (1999) concluded that US teachers lacked an understanding of fundamental mathematics, and their knowledge was overwhelmingly procedurally based.

Ma's conclusions about the US teachers' mathematical knowledge are broadly in line with other international research findings that the MCK of US and UK PSTs is limited, compartmentalised, and primarily procedural (Rowland, Huckstep & Thwaites, 2005; Conference Board of the Mathematical Sciences, 2001). In Ireland, despite entry to ITE requiring PSTs to be in the top 10-15 percent of their cohorts in the Leaving Certificate, research carried out by several researchers indicates the MCK of Irish PSTs is similar to that of their international counterparts (Corcoran, 2005; Leavy & O'Loughlin, 2006; Hourigan & O'Donoghue, 2015; Hourigan & Leavy, 2017). As previously alluded to, in the researchers own department, recent research found both undergraduate and postgraduate PSTs' to possess only a weak knowledge of procedural mathematics on entry to ITE (Costello & Stafford, 2019; Costello, Stafford & Oldham, 2017).

In Ma's (1999) research, the Chinese teachers what Ma referred to as a "profound understanding of fundamental mathematics" or PUFM, which essentially describes relational understanding. Ma provides a rich description of what this means: whereas the Chinese teachers' knowledge of mathematics was clearly coherent, the US teachers was fragmented; whereas the Chinese teachers were aware of interconnections between mathematical topics, the US teachers did not see connections. In other words, the US teachers lacked mathematical coherency which Wu (2010) agrees has concrete manifestations affecting every facet of mathematics. Ma (1999) points out that it is the Chinese teachers' mathematical substance of their knowledge that enables this coherency, which in turn enables them to teach the subject effectively. Furthermore, the Chinese teachers in this study expected their pupils to know mathematics in a similar type of way. One teacher even pointed to the fact that she required her pupils to write proofs for the invert-and-multiply rule for fraction division. Ma argues that this approach provides pupils with the fundamental knowledge of mathematics which serves as the foundations from which a more abstract and complex mathematical knowledge can be built on.

Going beyond the descriptive, Wu (2011) clearly defines what it means for PSTs to know mathematics. Consistent with the iterative bi-directional approach presented by Hurrell (2021), Wu defines what it means to know mathematical concepts and procedures. Knowing a concept includes knowing its precise definition, its intuitive content, why it is needed, and its role and context. On the other hand, knowing a procedure involves knowing its precise statement, when it is appropriate to apply it correctly in diverse situations, how to prove that it is correct, and the motivation for its creation (Wu, 2011, p.380). This knowledge of concepts and procedures will allow PSTs to make conjectures, justify claims and engage in mathematical argument, all of which is necessary for the primary school setting (Ball, 1990) and promoted in the 2011 Numeracy Strategy (DES, 2011).

In addition to this, there is a strong consensus in the literature that the level of mathematical knowledge required by teachers needs to go beyond that being taught to students (Ball et al, 2008; National Mathematics Advisory Panel, 2008; Wu 2011; Delaney, 2010). In fact, Wu (2011) asserts that teachers cannot claim to know the mathematics they teach without knowing a substantial amount of the mathematics of three or four grades before and after the grades they are teaching.

Wu (2011) also notes that the mathematical education of primary school teachers for mathematics must be consistent with the fundamental principles of mathematics. He claims that these principles form the basis for mathematical education and should be used as guiding principles for teacher educators. They are as follows:

1. Every concept should be precisely defined. Definitions form the basis for logical deductions.
2. Mathematical statements should be precise.
3. Every assertion can be backed up by logical reasoning.
4. Mathematics is coherent, and all concepts and skills are logically interwoven.
5. Mathematics is goal orientated and every concept and skill should have a purpose.

2.3.3 Reflection on epistemological approach

Guided by the research indicating that PSTs required a relational understanding of mathematics, when I first designed the mathematics competency modules, I adapted Wu's (2010) definition of what it means to know mathematics. I deliberately paid attention to the content knowledge requirements, but effectively disregarded Hill et al's (2008) contention that content knowledge alone was not sufficient (Hill et al, 2008). Darling-Hammond (2006) also agrees that while subject matter knowledge is essential, it is only one component of knowledge for teaching mathematics, and in fact must interact with pedagogical knowledge in the classroom to maximise teacher effectiveness.

Those other components of mathematical knowledge are captured in Ball, Themes and Phelps (2008) in their Mathematical Knowledge for Teaching (MKT) model. These researchers adopted Shulmans (1986) idea of PCK to develop the model, which they defined as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p.395).. They noticed that Shulman's description of content knowledge could be subdivided into four domains: common content knowledge (CCK), specialised content knowledge (SCK), knowledge of content and students (KCS) and knowledge of content and teaching (KCT). CCK and SCK are types of content knowledge, whereas KCS and KCT are types of pedagogical knowledge.

Common Content Knowledge (CCK) is the mathematical knowledge and skill used in settings other than teaching. It is about general competence with mathematics, and not aspects that are necessarily unique to teaching. Teachers use this knowledge to identify, for example, when a pupil gives an incorrect answer or when a definition in a textbook is incorrect. **Specialised Content Knowledge** (SCK) refers to the mathematical knowledge and skills unique to teaching. This involves the ability to unpack mathematics "that is not needed…in settings other than teaching" and might include looking for patterns in pupil errors or determining why a non-standard algorithm might work (Ball, Thames, & Phelps, 2008, p.400). **Knowledge of Content and Students** (KCS) refers to knowledge that combines knowing about students and knowing about mathematics. This includes knowing what students will find easy or difficult, boring or motivating, and so forth. Central to these types of tasks is knowledge about common pupil misconceptions. **Knowledge of Content and Teaching** (KCT) combines knowing about teaching and knowing about mathematics. This might include, for example, the most pedagogically appropriate way to introduce a sequence of topics, or choosing effective examples to teach a new concept.

While recognising the value and importance of the various domains of mathematical knowledge in the preparation of PSTs, I considered these outside of my remit and area of expertise. Furthermore, I believed my approach, which was outlined in chapter 1, was reasonable because my role was to teach mathematics to PSTs, not teach PSTs how to teach mathematics. I also believed that if PSTs could improve their general mathematical knowledge, then this would support the MKT domains. Consistent with these beliefs I adapted a blended instructivist – constructivist pedagogical approach, with more emphasis on the instructivist part. Thereafter it was the responsibility of PSTs to use independent learning opportunities to make sense of the mathematics for themselves, and this takes effort. There is significant evidence to support this approach. For example, constructivist researchers Spiro and DeSchryver (2009) agree that explicit instruction may be superior in well-structured domains such as mathematics. Research carried out by Alfieri, Brooks, Aldrich, and Tenenbaum (2011) supports this view. The authors carried out a two large-scale meta-analysis: the first to examine the effects of unassisted discovery learning versus explicit instruction, and the second to examine the effects of enhanced and/or assisted discovery versus other types of instruction (e.g., explicit, unassisted discovery). They found that outcomes were favourable for explicit instruction when compared with unassisted discovery under most conditions and concluded that unassisted discovery did not benefit learners. In contrast to this, enhanced discovery, including targeted feedback, worked examples, elicited explanations, and scaffolding, was the more efficacious approach when compared with other forms of instruction such as unassisted discovery and explicit direct teaching. Furthermore, Kirschner et al (2006) posit that explicit learning of knowledge and skills first is essential to be able to later use those knowledge and skills creatively or in a problem-solving scenario. These researchers argue that focusing on learning concepts and skills while at the same time problem solving is not consistent with what educational psychologists now know about cognitive architecture. In contrast with the well-structured nature of mathematical concepts and procedures, teaching is an Ill structured domain, which is better suited to more constructivist approach to learning (Spiro and DeSchryver, 2009).

All of this brings to light an apparent epistemological contradiction between my theoretical position on teacher education, and my position on how mathematics is learned. That is, learning to teach requires a constructivist or social constructivist approach, whereas learning mathematics requires an instructivist dominant approach. However, I did not intend for maths competency to be about "telling or being told"; it was supposed to be "an active and constructive process" (Dewey, 1902/ 1990, p.15).

This becomes clear when addressing the intent behind maths competency, which is about practicing a democratic pedagogy by enabling PSTs to access deeper relational understanding of mathematics which many PSTs may not have experienced in their secondary education (Burns et al, 2018; Shiel, Millar & Cunningham, 2020). Restricting access to this sort of mathematical experience is, itself, a form of educational oppression and mathematics competency aims to address this. Although the modules do not make explicit links to the everyday use of mathematics as described by Dewey (1902/1990), I take the view that doing mathematics per se is a worthwhile and educative and aesthetic use, just like reading a book, creating art, or appreciating music. Furthermore, maths competency was a deliberate move away from the memorise and drill pedagogy (Chomsky, 2004) towards an approach that empowers PSTs to make meaning from the mathematics they experience. On reflection however, the approach lacks the reflective and dialogical components that are integral to Freire's banking methodology. Importantly, when I look back on the opening chapter to this dissertation, I notice my use of transactional language such as “deliver” and “cover” which represents an undemocratic way of educating because it implies a pedagogy of “telling” and “being told” as opposed to “an active and constructive process” (Dewey, 1902/ 1990, p.15).

It is necessary to ask how these two opposing epistemological positions underpinning my practice can be reconciled to give PSTs’ opportunities to integrate it all into a single Gestalt. This further necessitates me to rethink what I do as a professional. It is worth considering the purpose of PSTs’ mathematical knowledge. For example, in Alfieri et al’s (2011) study, it was not stated the purpose of the mathematics that was being learned was for teaching. While mathematical knowledge in and of itself is useful, it doesn’t necessarily address the complexity of imparting this knowledge in the classroom. Although my explicit role is to help develop PSTs’ MCK, there is an implicit assumption that this knowledge will benefit PSTs’ professional practice and the pupils they teach. This is one of the main challenges in this study. Although the focus of mathematics competency is relational understanding, and is therefore democratically orientated, it lacks *deliberate* elements of reflection, dialogue, knowledge creation, criticality, and creativity. However, if I were learning mathematics as the PSTs are, I would be reflective about it, I would engage in dialogue about interesting or problematics aspects of the content, and I would explore it creatively. In fact, I did all of these things as both a student of mathematics, and when I was learning content to design the original mathematics competency modules. However, on deeper reflection I realised I have been implicitly contradicting my values outlined in chapter 1 about knowledge creation being an individual pursuit. PSTs are not the same as me, and my assumptions about them and how they learn are not necessarily true.

It is true that the B.Ed. programme contains all the components necessary to enable PSTs to develop into competent mathematics teachers. However, reflecting on these components with respect to the literature on teacher learning, it is also clear they are either incomplete (e.g., maths competency lacking a practice component) or not connected meaningfully. The research presented thus far will be used as a starting point to analyse and redesign the maths competency modules so that they become more democratic, while simultaneously addressing the problem of enactment. This intervention is presented in Section 3.4 of the methodology chapter. Before considering the methodology, there are two significant areas that need to be addressed to fully understand the nature of the research problem. The next section will critically examine the literature on affective issues and how they impact on how PSTs engage with mathematics. The final section of this chapter addresses the impact of neoliberal policies on mathematics teaching, with a focus on the Irish context. These will give a more complete picture of how PSTs learn mathematics for teaching, their willingness to enact mathematical knowledge, as well as providing essential background knowledge to fully engage with the research question.

2.4 Affective issues and beliefs

2.4.1 Beliefs about mathematics

Conceptions of teacher mathematical knowledge have been criticised for assuming a mainly traditional cognitive perspective, while ignoring affective issues such as teacher's beliefs (Petrou & Goulding, 2011). These include beliefs about mathematics as a discipline, beliefs about the teaching of mathematics, beliefs about the learning of mathematics, and beliefs about self within a mathematics community (Underhill, 1988; Kloosterman et al., 1996).

In the most general sense, beliefs can be defined as "psychologically held understandings and assumptions about phenomena or objects of the world that are felt to be true, have both implicit and explicit aspects, and influence peoples' interactions with the world" (Kunter et al, 2013, p.249). Therefore, knowledge cannot be considered distinct from beliefs. From a theoretical perspective, beliefs are units of cognition. They include not only what teachers consider factual knowledge, but also opinions and hypothesis (Wilson & Cooney, 2003). From a practical perspective, beliefs are essential considerations because they influence teacher behaviour directly, including planning and classroom practice (Lui & Bonner, 2016). Within ITE, Philipp (2008) argues that it is ineffective to teach courses designed to improve mathematical content knowledge if PSTs do not have the corresponding beliefs to meaningfully engage with such knowledge.

One of the fundamental problems in the preparation of PSTs is that they are not aware of the type of mathematics they need to know to teach effectively (Philipp, 2008). Philipp contends that many PSTs believe mathematics is primarily made up of rules and procedures, and pupils learn mathematics by being shown how to apply these rules and procedures in a step-by-step fashion, i.e., instrumentally. This closes PSTs off to approaching mathematics in a deeper, more conceptual way, than they themselves experienced in school. These beliefs are centred on two key arguments by PSTs:

1. If I, a university student, don't already know something, then a child should not be expected to know it.
2. If I already know something, then I should not have to learn it again in a different way.

(Philipp, 2008, p.7)

Phillips' contentions are supported by cognitive scientists Pesek and Krishner (2000) who describe cognitive interference and attitudinal interference as the mechanisms which explain this. Cognitive interference is when previous understandings of something are so powerful, they obtrude into subsequent learning. Attitudinal interference is where a PST's previously acquired opinions and attitudes block comprehensive engagement with a topic and therefore impede potential for learning (Pesek & Kirshner, 2000). Considered together, these phenomena explain the sorts of reactions many PSTs exhibit when confronted with alternative ways of thinking about mathematics teaching and learning.

These sorts of beliefs are problematic because they close PSTs off to the idea of teaching mathematics relationally, thus contributing to the problem of enactment, and as such they play an important role in classroom instruction. PST beliefs about mathematics can be categorised as follows: beliefs about the nature of mathematics and beliefs about the processes of mathematics teaching and learning (Ernest, 1989; Speer, 2005). These categorisations are discussed below.

2.4.2 The nature of mathematics and its impact on teaching and learning

Teachers' beliefs about the nature of mathematics refer to ideas about concepts, meaning, rules, mental images, and preferences about the discipline of mathematics (Yang et al, 2020). In this regard, PSTs' beliefs about mathematics are a sort of mathematical worldview or ideology that shapes practice (Speer, 2005) as well as influencing Gestalt formation. According to Speer, beliefs influence PSTs' decisions about all aspects of teaching mathematics including what knowledge is worth teaching, and what social and pedagogical norms and routines should be established in the classroom.

Beliefs about the nature of mathematics can be categorised into the opposing philosophical ideas of absolutism and fallibilism (Ernest, 1989; Lerman, 1990). The absolutist view sees mathematics as certain, absolute, value-free, and abstract. Mathematical knowledge is a result of the discovery by others of absolute truth. It is about establishing certainty and eliminating paradox. From an epistemological perspective, mathematics is an infallible "procedure-driven body of facts and formulas" which manifests itself as a definitive body of knowledge discovered by someone else which should be applied, but not necessarily understood, by students when instructed to do so (Yang et al, 2020, p.3). Within this philosophy of mathematics, the student's role is passive in the sense that they should not engage in discovery, inquiry or be creative. Teachers with absolute, or static, beliefs about the nature of mathematics typically enact transmission style teaching and learning in the classroom (Depaepe et al, 2020). This is mainly teacher-centred where pupils play a passive role in the classroom which is defined by following teachers' instruction and mathematical procedure.

The alternative is the fallibilist view that mathematics is socially constructed, notions of proof and truth are values driven and ultimately alternative "truths" exist, whereby mathematical knowledge is "subject to revolutionary change as much as other forms of knowledge" (Lerman, 1990, p.55). From a pedagogical point of view, such a philosophy is relevant at all stages of mathematical activity. It is not purely about application of prescribed algorithms and procedures invented by mathematicians who control the curriculum. It values problem solving, discovery, creativity, inquiry, and context. Teachers with fallible, or dynamic beliefs about the nature of mathematics typically enact constructivist style teaching and learning in the classroom (Depaepe et al, 2020). This is a student-centred view of mathematics pedagogy which is conceptualised as an active, inquiry-based process where students construct meaningful knowledge (Depaepe et al, 2020) through a process of sense making and pattern seeking (Felbrich, Kaiser & Schmotz, 2012).

Teachers' beliefs and their professional knowledge, both content and pedagogy, are related. Blömeke et al (2020) noted that stronger content and pedagogical knowledge of mathematics was associated with teachers with fallibilist beliefs. On the other hand, teachers with absolutist (static, transmission-orientated) beliefs had weaker knowledge of mathematics. This also impacted on the quality of classroom instruction. Because beliefs are assumed to be a filter through which one sees the world (Pajares, 1992), they guide teachers' actions and enactment of knowledge, and are therefore an essential consideration for classroom practice (Felbrich, 2012). Depaepe et al (2020) noted that beliefs act as a sort of buffer between teachers' professional knowledge and instructional behaviour and "seem to determine how teachers interpret mathematical classroom situations and how they act in these" (p. 182). Any meaningful change in instructional practices requires a corresponding change in individual teachers' beliefs about the nature of mathematics (Ernest, 1989, p.1).

Although Ernest (1989) and Lerman (1990) dichotomise the nature of mathematics between opposing absolutist and fallibilist perspectives, in reality, beliefs lie on a continuum ranging between the two views (Ross, McDougall & Hogaboam-Gray, 2002). In the United States, it was suggested that a range of instructional practices which correspond to this continuum (from student-centred to teacher-directed) should guide instructional practice (National Mathematics Advisory Panel, 2008). That is, instructional practice should not be entirely teacher-directed or student-centred. However, mathematics instruction is complex, and the best approaches are debated. For example, Van de Walle et al (2020) suggest an entirely constructivist approach, Kirschner et al (2007) suggest a direct teaching approach, while Hattie at al. (2016) and the Irish Mathematics Curriculum (NCCA, 1999) suggest a blend of both approaches.

2.4.3 Beliefs about mathematics achievement

There are another set of beliefs which are related to achievement in mathematics, or more precisely, beliefs about one's ability to achieve in mathematics. These beliefs are centred on the idea of a fixed or a growth mindset put forward by Dweck (2017). A growth mindset is characterised by the belief that ability is malleable, whereas a fixed mindset is characterised by the belief that ability cannot be altered with effort. Those people with characteristics of a growth mindset believe that effort, hard work, and collaboration can positively change one's ability. On the other hand, individuals who exhibit characteristics of a fixed mindset believe that one's ability cannot be changed, even with increased effort (Hamiovitz & Dweck, 2016).

When applied to mathematical ability, Boaler (2016) found that both teachers and students who had a growth mindset regarding mathematics outperformed their fixed mindset counterparts. This has implications for pupils because a growth mindset points to a strong self-efficacy and the confidence to generate and develop meaningful mathematical ideas (Beghetto and Baxter, 2012) while fully engaging in mathematical inquiry and problem solving. Furthermore, according to Boaler (2016) and Sun (2018), learners who demonstrate a growth mindset can make explicit connections to the multidimensional nature of mathematics, which are key components of relational understanding (Boaler, 2006; Skemp, 1978). This transfers directly to pupils' learning because it allows them to value connections between concepts and encourages multiple approaches to solving mathematics problem. This in turn allows pupils to experience success and develop a belief that everyone can improve their mathematical ability (Boaler & Staples, 2008). In this regard, mathematics instruction by teachers with growth mindsets promotes inclusivity by empowering pupils to access to complex and rich mathematical work (Boaler, 2016; Sun, 2018). The growth mindset about mathematical ability is commensurate with the fallibilist view of mathematics.

On the other hand, teachers who have a fixed mindset view of mathematics tend to enact classroom instruction that is one-dimensional in nature (Boaler, 2016; Sun, 2018) which puts instrumental understanding at the centre of learning (Kilpatrick, Swafford, & Findell, 2001). This type of instruction reinforces the notion that there is only one way to succeed in mathematics, and if pupils struggle with the prescribed way of doing mathematics they are likely to adopt a fixed mindset themselves and believe they are "not good" at mathematics (Sun, 2018). The fixed is commensurate with the absolutist view of mathematics. Importantly, mindset is not an individual construct. For example, a pupil's mindset in relation to mathematical ability is determined by peers, teachers, parents, and the culture of the school (Dockterman & Blackwell, 2014).

2.4.4 Mathematical Anxiety

Closely associated with mathematical beliefs is the problematic phenomenon of mathematical anxiety. Mathematical anxiety (MA) can be broadly defined an adverse emotional reaction to engaging with mathematics or even the prospect of engaging with mathematical tasks (Maloney and Beilock, 2012 p.404). Gresham (2018) describes it as an irrational fear which often manifests itself as feelings of uneasiness and uncertainty, of varying levels of intensity, when asked to engage with mathematics. It can range from mild feelings of anxiety to an irrational fear resulting in one's inability to think clearly resulting in an inability to meaningfully engage with or learn mathematics (Gresham, 2018, p.91). MA is relevant for this study primarily because if PSTs experience MA during maths competency, then they are less likely to engage meaningfully with the content and more likely to rely on instrumentally based mathematics instruction on SP. Secondly, if PSTs teach instrumentally on SP, then the pupils they teach are more likely to experience MA. These issues are discussed in more detail in the remainder of this section.

Although MA is indiscriminate and can impact on anybody, it often causes teachers to avoid mathematics and promotes the development of unhealthy attitudes towards it (Gresham, 2018). As with any form of anxiety, the most pervasive and unfortunate consequence of individuals with high levels of MA is avoidance (Ashcroft, 2002, p.181) and the generation of negative attitudes towards the subject (Zakaria et al 2012). Furthermore, MA is strongly associated with poor performance in mathematical related tasks. Despite being able to perform well across non-mathematical thinking tasks, individuals with MA underperform when numerical tasks are involved (Maloney and Beilock, 2012). This phenomenon was demonstrated by Fraust (1992) who found that when a group of people who displayed high MA were given increasing difficult mathematics tasks to complete, there was a corresponding set of associated physiological reactions such as increased heart rate. When the same participants were given verbal tasks of increasing difficulty, any change in physiological markers was insignificant.

MA is also strongly associated with poor performance and learning in mathematics (Gresham, 2018). Although some suggest that there is an association between low competency and MA (e.g., Hembree,1990), it has been shown that MA itself impedes normal cognitive function and the poor mathematical performance follows this (Maloney and Beilock, 2012). The mechanism by which this happens begins with anxious thoughts related to some upcoming mathematical tasks. These anxious thoughts compromise cognitive resources, particularly working memory, which is responsible for “the regulation and control of information relevant to the task at hand” (Maloney and Beilock, 2012, p.404). Maths anxious children as young as 6 years old who engaged in mathematical tasks showed increased neural activity in the right amygdala, which is responsible for regulating negative emotions. This increase in activity has a corresponding decrease in activity in parts of the brain that support working memory and mathematical processing (Young et al, 2012).

Although some individuals may be more cognitively and emotionally predisposed to developing MA, there is an important social contributor that originates in the classroom. Several studies including Haciomeroglu (2013) and Beilock (2010) found that mathematically anxious teachers passed on their anxiety, and other negative attitudes towards mathematics, to some of their pupils. What was particularly interesting about Beilock's study (2010) was the gender dimension whereby female teachers were more likely to transmit their anxiety and negative attuites to female pupils, while at the same time endorsing damaging gender stereotypes. This is of particular concern given women accounted for almost 90% of teachers at primary level in Ireland according to the CSO. Conversely, Hadley & Dorward (2011) found that female students who receive lower than average mathematics scores tended to also have high levels of MA. In addition to teachers' attitudes towards mathematics, the nature of the mathematical instruction teachers use plays a significant role in MA development. Mathematics teaching underpinned by the absolutist view which emphasises rote memorisation and drill of facts and procedures, followed by repeated textbook-based practice exercises, and insisting on only one way to solve problems all contribute to MA (Grisham, 2018).

It is important to address issues of MA in ITE because affective discomforts, particularly those relating to mathematical thinking, mathematical beliefs and other negative emotions PSTs have towards mathematics, tend to continue into the in-service level (Gresham, 2018). There have been many studies conducted on this topic (e.g., Aslan, 2013; Bekdemir, 2010; Bursal & Paznokas, 2006; Gresham, 2018; Haciomeroglu, 2013). These studies highlight that many preservice teachers experience high levels of MA with associated negative attitudes towards mathematics. This anxiety and negative attitude is likely to transfer to in-service teaching and even increase in severity over time (Gresham, 2018). This generally manifests in the type of mathematical instruction that itself causes MA in pupils, resulting in a self-generating negative cycle of mathematical anxiety (Perry, 2004). Rather than focusing on mathematical understanding and reasoning, making connections, and understanding concepts and procedures, PSTs with MA spend the majority of their instructional time focusing on computational procedures because they lack confidence in their ability to understand and teach more meaningful (i.e., relational) mathematics (Gresham, 2018). They use more traditional, whole class teaching instruction with a focus on basic numerical skills rather than deeper conceptual understanding (Finlayson, 2014), which will result in limited pupil understanding and associated increase in frustration and mathematical anxiety (Perry, 2004).

It is reasonable to assume there will be some levels of MA across the PSTs in this study given this largely homogenous cohort have been shown to possess only weak understanding of procedural primary level mathematics (Costello & Stafford, 2019) and this knowledge is challenged in maths competency modules. Indeed, MA can be addressed by improving the basic mathematical competencies of individuals with MA, including both pupils and PSTs. In younger pupils, this strategy can help reduce the likelihood of developing MA in the first place. PSTs, on the other hand, should be upskilled and empowered to enact this knowledge with non-traditional methods of mathematical instruction including the use of manipulatives to bridge the gap between concrete and abstract, the use of a problem-solving approaches, group, and individual instruction, as well as addressing attitudes to mathematics in the course of instruction (Lake & Kelly, 2014). One of the most effective instructional features that contribute to a reduction in MA is related to pace. Beilock & Willingham (2014) found when content was introduced very slowly without any assumptions about prior knowledge, and pupils afforded time to engage meaningfully with and discuss the material, MA was reduced. There is no reason to assume this is not the case with PSTs in the context of ITE.

However, Hadley & Dorward (2011) caution that many TEs teach mathematics in the traditional lecture style, promoting and rewarding PSTs' efforts to memorise mathematical procedure and algorithms at the expense of meaningful engagement with mathematics. This approach may unintentionally perpetuate MA across PSTs and the pupils they teach. Swars et al. (2009) and Hart et al. (2013) advise that mathematics courses in ITE should focus on building an in depth understanding of mathematics through inquiry and problem solving and should be informed by relevant theories relating to how mathematics is learned for teaching. Gresham (2018) cautions that PSTs who experience MA may be reluctant to embrace the use of alternative teaching approaches. However, when content is modelled accurately in for meaning and authentic experiences are created in which to enact this content, the anxiety levels tend to reduce. Allowing PSTs to work collaboratively and construct knowledge co-operatively are important aspects of this process.

2.4.5 Complexity of teachers' beliefs

It is important to note that teachers' beliefs are complex, and several researchers have found that teachers sometimes enact a pedagogy of mathematics that is inconsistent with the beliefs they espouse. Philipp (2007) conducted a critical literature review of research on inconsistencies between teachers' beliefs about the nature of mathematics and mathematics pedagogy on one hand, and their classroom practices on the other. The authors included in the review were Raymond (1997), Hoyles (1992), Skott (2001), and Sztajin (2003). Each of these researchers agreed that context plays an important role in teachers practice and this this context may appear to override their beliefs. Raymond (1997) put forward two factors that may lead to inconsistencies between beliefs and practice. First, teachers' beliefs about the nature of mathematics are not necessarily consistent with beliefs about mathematics pedagogy, and it is beliefs about pedagogy which takes precedence in the classroom, not beliefs about the nature of mathematics. Secondly, general educational issues such as time constraints, pupils' behaviour, and standardised tests can cause teachers to behave in a way that is inconsistent with their beliefs. Skott (1992) agreed that teachers' beliefs are often overshadowed by more general educational priorities, and what the teacher's goal is. Hoyles (1992) also argued that beliefs are situated, and how they are enacted depends on the context the teacher is in. On the other hand, Sztajin (2003) found that teachers practice in the classroom are determined not only by beliefs about mathematics, but also beliefs about society, pupils, and education more generally.

Building on this earlier research, Skott (2009) used a case study methodology involving one participant to develop a locally social approach to understanding the belief-practice relationship. He found that the "social perspective" which views classroom practices as something that emerges in, and through, social interactions. This means that inconsistencies between espoused beliefs and enacted beliefs, i.e., practices, "may be interpreted rather as one between espoused beliefs and the communal ways of acting that emerge in the locally social" (Skott, 2009, p.29). Simply put, what happens in mathematics classrooms may not be explained entirely by teachers' beliefs about mathematics. Skott questions the notion that many researchers view the development of teachers from a social perspective yet view the teacher practices in the mathematics classroom from a purely individual perspective. In short, he questions the premise that beliefs alone can explain teacher practice because this would leave little room for social interpretations of pedagogical practices. Skott's (2009) argues that to understand practice we need to contextualise the act of teaching in intersubjectively established and continually re-generated settings. He also suggests that "we acknowledge the simultaneous existence of multiple, possibly conflicting, actual and virtual communities of a teacher's practice" (Skott, 2009, p.45). Therefore, he argues that classrooms are social entities and therefore beliefs may not play a major role in teachers' practices.

In any case,when beliefs act as a barrier to learning and development they need to be challenged. However, beliefs are more difficult to change then emotions and attitudes (Phillipp, 2007) and this is largely dependent on whether beliefs are held evidently or non-evidently (Green, 1998). If beliefs are held non-evidently then they cannot be influenced by an evidence-based argument or reasoning. However, if beliefs are held evidently, they can be changed when the individual is presented with more compelling evidence or reasoning. For teachers in the latter category, reflection is a central component in the change process as it allows teachers to "learn new ways to make sense of what they observe" (Philipp, 2007, p. 281). It is not straightforward to determine if non-evidently held beliefs can be influenced, and to do so requires knowledge of individuals belief structures (Phillip, 2007). This is explained by Cooney (1999) who categorised how PSTs hold their beliefs into isolationist, naive idealist, naive connectionist, and reflective connectionist. PSTs who are characterised as isolationist may hold beliefs non-evidently and are problematic in relation to challenging existing beliefs through reflection. According to Cooney (1999, p.172) an isolationist:

> "…tends to have beliefs structured in such a way that beliefs remain separated or clustered away from others. Accommodation is not a theme that characterizes an isolationist. For whatever reason, the isolationist tends to reject the beliefs of others at least as they pertain to his/her own situation".

The other three characterisations are open to changing beliefs to different degrees, but it is the final one, the reflective connectionist, that is more likely to become a reflective practitioner as described by Schon (1983) because they are characterised by the ability to resolve conflict through reflective thinking (Cooney, 1999). The goal of Teacher Education should be to support the movement of PSTs from isolationist to connectionist, and Cooney (1999) contends that one possibility for achieving this is to introduce problematic and perplexing situations into instruction because making previously unproblematic scenarios problematic can greatly influence one's worldview.

2.4.6 Concluding remarks about beliefs

Philipp (2008) found that the cohort of PSTs in his study lacked the necessary content knowledge to teach for relational understanding, and the corresponding beliefs to enact such knowledge should they possess it. The sorts of beliefs that need to be considered in the preparation of teachers were presented in this section, including the related issue of mathematics anxiety. There are very clear parallels between various facets of PST beliefs about mathematics, and how these beliefs impact their practice, and hence the pupils they teach.

The absolutist philosophy of mathematics is consistent with PSTs who hold a fixed mindset view of mathematics and who generally conceive it as a subject based on rules and procedure, which in turn promotes teaching based on instrumental understanding. This absolutist view of mathematics may be suitable for some pupils, but in general leads to pupils often inaccurately believing that they are "bad at maths" just because their final answer to a task or problem is not correct. Ultimately this may lead to high levels of mathematical anxiety and all the problems associated with that such as avoidance and underperformance. This related set of absolutist orientated beliefs mirror a version of mathematics education that is anti-democratic in the sense that it is encourages conformity and restricts freedom to think creatively and critically (Freire, 1970), while leaving little space for authentic and meaningful socialisation, communication, experimentation, and negotiation (Dewey, 1916). Moreover, PSTs who have been accustomed to this form of education, and hence denied access to meaningful educational experiences, are likely to enact similar practices as teachers.

On the other hand, the fallibilist philosophy of mathematics is consistent with the growth mindset view of mathematics which is characterised by PSTs who are likely to embrace the multidimensional nature of mathematics. This view is typified by instruction based on a relational understanding of mathematics, which is more likely to lead to pupils who believe that mathematical ability can be improved. Importantly this will result in lower levels of mathematical anxiety and higher levels of engagement with the subject. Mathematics instruction underpinned by these beliefs encourage creative and critical thinking (Freire, 1970), while increasing capacity for future learning and growth (Dewey, 1916).

Although these concluding remarks present a simplified binary view of mathematical beliefs, it gives a general idea of how these beliefs are related to each other and how they relate to meaningful mathematics teaching. There are some key points to remember about beliefs. Firstly, it is important to remember that PST beliefs are strongly held and cannot be altered without resistance (Booker, 1996), and this must be part of any intervention to address the problem of enactment and enacting democratic mathematics education. As such it is worth recalling that the literature suggests enacting non-traditional methods of instruction so that PSTs can work cooperatively and construct meaning socially (Lake & Kelly, 2014). On the other hand, content should be modelled accurately for meaning and understanding, and this should be done at a slow pace (Beilock & Willingham, 2014). These recommendations provide more clarity around the instructivist-constructivist balance as problematised in Section 2.3.3.

Further contributions by Philipp (2008) suggest developing more nuanced beliefs in PSTs about mathematics teaching and learning, which he argued could be achieved by motivating them to look at the overall learning needs of pupils, rather than mathematics per se being the primary consideration. Hourigan and Leavy (2012) also suggest that it is possible to challenge PSTs' limiting beliefs about mathematics through the provision of opportunities to reflect on and critique upon their own experiences with mathematics. Finally, it is worth paying attention to the gender dimension because PSTs in this study, and in the wider context, are mainly female who are more likely to transmit anxiety and damaging beliefs to female pupils thus perpetuating damaging gender stereotypical views about mathematics.

2.5 The Political Landscape and Primary Mathematics

The next chapter will present a teaching intervention based on Grossmans (2008) pedagogies of enactment as described in Section 2.2.7. The intention for the intervention is to empower PSTs to enact meaningful and intellectually ambitious mathematics, underpinned by relational understanding, in the classroom. Furthermore, this approach will address problematic PST beliefs which, upon entering teacher education, tend to be absolutist in nature and contribute to transmission style mathematics teaching, reminiscent of Freire's banking method. However, even if these issues can be addressed so that PSTs become aware of, and reflective about, the nature of mathematics they teach, and are prepared to enact intellectually ambitious mathematics in the classroom, there is still a danger that this may be "washed out" by the reality of the classroom, as described by Zeichner (1981). The literature clearly suggests the neoliberal influence on educational systems, including schools, is an important consideration in terms of what influences teacher behaviour in the classroom (Mccoy, Smyth and Banks, 2012). This section will address this issue, with respect to mathematics instruction, because it is a significant consideration which is likely to impede, or at least curtail, enactment of intellectually ambitious mathematical instruction in schools.

Firstly, it is important frame this discussion in the context of the Irish primary school mathematics curriculum (NCCA, 1999a) and the key purposes of mathematics education (Ward-Penny, 2017). Both will now be discussed and later used to critique how mathematics is currently instructed in the Irish context.

The Irish mathematics curriculum (NCCA, 1999a) promotes mathematics instruction that is intellectually ambitious in nature (Lampert et al., 2013). The Irish National Teacher Organisation (INTO), who played a central role in the design of the mathematics curriculum (NCCA, 1999), interpret that it should not be designed primarily to meet commercial demands, but instead "focus on pupils' needs in preparation for life as adults capable of dealing with practical mathematics in real-life situations" (INTO, 2006, p.4). Beyond its economic utility, they claim that mathematics is an important dimension of general education, has an intrinsic social value, is a source of enjoyment and fascination, as well as being an intellectual pursuit in its own right. The curriculum conceptualises mathematics as a creative activity and "one of the most useful, fascinating and stimulating" areas of human knowledge (NCCA, 1999, p.2). It aims to give children the ability to solve the practical problems in everyday life, science, and industry. As well as teacher led instruction, the curriculum emphasises a constructivist approach to teaching based on discovery learning and social interaction (NCCA, 1999b). In this regard, collaboration and cooperation are key features of mathematical learning envisaged for pupils in Ireland.

2.5.1 The Purpose of Mathematics Education

Ward-Penny (2017) lists six key purposes of mathematics education. The first of these is **numeracy development for everyday life**. This is sometimes called functional mathematics, and it is so learners can apply basic mathematical techniques in everyday life situations. This might include, for example, measuring lengths, areas and volumes; telling the time; using money confidently, etc. The second purpose of mathematics education is to **prepare learners for work and vocational development**. For example, working with algebraic expressions is essential in many careers involving science, technology, and engineering, while probability and statistics are used widely across many professions. The third purpose of mathematics education is **enhancing one's ability to think critically** in such a way as to solve problems in general. Essentially, engaging in rich mathematical tasks teaches learners how to think: it develops the brain's ability to engage in logical thinking, enhances organisational routines, and decision-making processes, all of which develop cognitive 'muscles' such as cognitive reasoning, pattern spotting and visualisation. The fourth aim of mathematics education is to promote democracy by supporting the growth of **critical citizenship**. Mathematics can give learners a unique set of tools to understand and interrogate the social and political worlds in which they exist. For example, developing number sense can grasp the meaning of large sums of money, statistical understanding can allow learners to make sense of political claims, and an understanding of probability can help learners assess risk and prediction. Thinking algebraically, and indeed all mathematics, promotes a student's ability to follow logical argument and challenge empty rhetoric. The fifth aim of mathematics education is to facilitate the continuance of the subject by promoting it as an **intellectual pursuit**. Therefore, an educator's job is to share this idea to encourage public interest in the subject as an intellectual pursuit. The sixth and final aim of mathematics education is related to **mathematics as a gatekeeper**. In most educational contexts, mathematics is a high stakes subject, whose qualifications are used as gatekeepers to further and higher education, and subsequent employment opportunities.

However, there is growing evidence to suggest the aims of the mathematics curriculum (NCCA, 1999) are largely aspirational and also that, broadly speaking, the aims of mathematics education are not adequately being met both in Ireland and internationally (Conway and Murphy, 2013; Mccoy, Smyth and Banks, 2012). These researchers identified the neoliberal influence of the Global Education Reform Movement (GERM) for this, which appears to act as a significant barrier to high-quality intellectually ambitious mathematics instruction. This is discussed in the next section.

2.5.2 Global Educational Reform Movement

The Global Education Reform Movement (GERM), which is essentially a system of accountability, is the current internationally recognised discourse around contemporary education. It was inspired by three distinct educational phenomena outlined by Evers and Kneyber (2016). The first is the constructivist approach to teaching which began to take traction in the 1980's, and which removed the focus from teacher and put the child at the centre of teaching and learning. The second is the growing public demand for effective learning for all pupils, which resulted in the proliferation of nationally aligned standardised tests to ensure such learning is happening. The third phenomenon was the increase in decentralised governments which led to greater accountability for schools and teachers. This discourse around accountability was further strengthened by neoliberal principles which have influenced the design of accountability systems worldwide. Additionally, the regulation of the teaching profession became increasingly held to account by exogenous institutions such as the European Union (EU) and Organisation for Economic Cooperation and Development (OECD), who promoted the development of cross national regulatory frameworks (Conway and Murphy, 2013) and have a strong influence on Irish educational policy (Walsh, 2016). The OECD's Program for International Student Assessment (PISA) results are especially influential and enforce a system of accountability to comparative international standards.

Such a model of high stakes accountability naturally gravitates towards an emphasis on standardised testing (Conway & Murphy, 2013), which policy makers assume will improve the overall quality of teaching and learning (Evers and Kneyber, 2016). On the contrary, Evers and Kneyber (2016) explain that standardisation impedes on flexibility and freedom in the classroom resulting in reduced creativity and depersonalised learning. Furthermore, collaboration is stifled, and didactic teacher-led instruction becomes the dominant methodology. Furthermore, the interschool competition for academic attainment results in an inevitable narrow focus on literacy and numeracy to the detriment of other curricular areas (Conway and Murphy, 2013).

2.5.3 GERM in Irish Education

Mccoy, Smyth and Banks (2012) identified systematic evidence of neoliberal shift in the Irish primary education system. They attribute this shift to Ireland's accountability to the OECD in relation to the 2009 'PISA shock' following the publication of results which indicated Irish pupils performed "below average" on the influential standardised tests (Baird *et al.*, 2011). The Irish government's response to these perceived deficiencies was the roll out of the Literacy and Numeracy Strategy in 2011 (DES, 2011) whose high degree of specification, via Department of Education and Skills (DES) circulars in 2011 and 2012, pushed Irish education closer to the dominant GERM agenda by mandating a "systemic move towards attainment of results-type accountability" (Conway and Murphy, 2013, p.28). Eventually, the DES made standardised testing of mathematics and reading compulsory for primary school pupils at ages eight, ten, and twelve. DES Circular 0056/2011 mandated schools to report the aggregated results of these tests to boards of management and the DES at the end of the school year (DES, 2011b). Furthermore, under DES circular 0056/2011 schools are obliged to provide parents with the results of these standardised tests, as well as the pupils' chosen secondary school as part of their *education passport* (O'Leary et al, 2019). This approach has been supported by the INTO who, in a 2006 review of the 1999 curriculum, insisted there was a "pressing need for the most up-to-date…standardised tests…to be readily available for primary schools" (INTO, 2006, p.39). More recently, the stakes were increased further when it was decided by the DES to use the results of these tests as part of the criteria for the allocation of SEN resources in primary schools (DES, 2017).

In other jurisdictions, increases in high-stakes mandatory testing has been shown to have detrimental effects on the quality of education, of which the most notable is a culture of 'teaching to the test', resulting in spending excessive time coaching pupils to give correct answers to artificially raise test scores (Hoffman, Assaf, and Paris, 2001). It is also likely to inhibit pupil learning through narrowing the curriculum due to a reduction in content and activities not related to the testing, an increase in the use of teacher-centred teaching, and relegation of knowledge to that of factual (Jones et al., 1999).

2.5.4 Neoliberal Impact on Mathematics Education in Ireland

This neoliberal move towards standardisation has firmly established whole class didactic teaching as the dominant pedagogical approach in Irish schools, contrary to what was set out in the 1999 mathematics curriculum (Mccoy, Smyth and Banks, 2012). It has resulted in specific difficulties regarding teaching for differing abilities in mathematics and a strong reliance on textbooks despite the INTO predicting that once the curriculum was embedded in the school system this reliance would be reduced (INTO, 2006; Mccoy, Smyth and Banks, 2012). The narrow focus on basic literacy and numeracy has also had negative consequences for the remaining curricular areas and consequently resulted in achievement gaps in these areas (Mccoy, Smyth and Banks, 2012) while systematically reducing access to the broad and balanced curriculum as promised by the NCCA (1999a).

Further consequences were identified by O'Leary et al (2019) who conducted a large-scale study involving 1500 primary teachers in Ireland. This study aimed to gather data about the attitudes and practices of primary teachers in relation to standardised testing. While there were notable positive reactions to standardised tests (e.g., identifying pupils' strengths and weaknesses, selecting pupils for learning support, and broader whole school evaluation purposes), there were also some concerns that are relevant to this study. Firstly, these tests induce unnecessary levels of anxiety in pupils. 43% of the teachers in the study reported spending at least half of a day teaching strategies to cope with this anxiety. 10% of the teachers reported incidence of teaching to the test in their schools, while 7% were aware of pupils receiving grinds prior to standardised tests. At the same time, teachers felt pressure to improve test scores from parents, principals, inspectors, colleagues, and even pupils. Notably, about half the respondents felt that parents took the results of standardised tests too seriously. In a separate study looking at the primary-secondary transfer of mathematical knowledge involving 249 pupils, Ryan Fitzmaurice and O'Donoghue (2021) found a 7% drop in scores on standardised mathematics tests from the end of 6th class to the end of 1st year. This short-lived nature of pupils' mathematical knowledge raises questions about the efficacy of existing pedagogical approaches. Interestingly, in an in-depth study of Irish primary schools, Devine et al (2020) investigated the attitudes of almost 2000 second class pupils' attitudes towards mathematics and six other subjects. They were asked about how useful the subjects were, how interesting they were, and how good they were at each of the subjects.

While mathematics ranked 2nd out of the seven subjects for usefulness, it ranked sixth for interesting and fifth for their perception of how good they were. Furthermore, not only does teaching to the test cause a corresponding narrowing of the curriculum, but it also reduces the amount of time for quality instruction based on conceptual understanding, critical thinking, and problem solving, and, ultimately, does not result in an increase in student achievement (Welsh, Eastwood, and D'Agostino, 2014).

These results are not indicative of a primary school curriculum which proclaims itself to be both "broad and balanced" (NCCA, 1999a, p.10) and which also aims to promote mathematics as a "source of fascination" (NCCA, 1999b, p.3). When one also considers Ward-Penny's (2017) aims of mathematics education presented in Section 2.5.1, it is questionable whether pupils are being prepared to critically engage in the world or enjoy mathematics as an intellectual pursuit. When conceptual understanding, critical thinking, and problem solving are compromised, it is difficult to present mathematics as a subject that develops thinking skills, enhances citizenship, or promotes mathematics as an intellectual pursuit. Chomsky (2004, p.27) refers to this as "mindless skills-based education" reminiscent of Freire's banking concept of education whereby the test guides the teaching, and where pupils are uncritically filled with rote skills to be later regurgitated in state mandated tests.

2.5.5 Conclusion

This section highlighted a "systemic move towards attainment of results-type accountability" in Ireland (Conway and Murphy, 2013, p.28) which has put pressure on teachers to prepare students for standardised tests and has resulted in a pedagogy of memorising and executing basic mathematical skills (Mccoy *et al*., 2012). This stifles critical thinking, and also has negative consequences for children's mental and emotional health (O'Leary *et al.*, 2019). It is closely aligned to Freire's (1970) banking model and represents an undemocratic form of mathematics pedagogy because pupils are conceived as information takers as opposed to knowledge makers. It is based on rote memorisation of procedures and drills, which discourages independent and critical thinking. Furthermore, this approach is both miseducative and uneducative (Dewey, 1933) because it leaves pupils incurious and unchanged. This "mindless skills-based education" as described by Chomsky (2004, p.27) is reflected in much of the mathematics instruction I have observed on SP. If PSTs' experiences are part of this undemocratic educational environment, then this will potentially hamper efforts to enact meaningful and relationally understood mathematics instruction. This will be a key consideration in the design of the intervention presented in the next chapter.

Chapter 3: Methodology

3.1 Introduction

This chapter provides a technological and philosophical foundation for the research strategy and overall methodological design used in this study. As signposted in the opening chapter, Action Research was used as the paradigmatic framework, and its theoretical and epistemological underpinnings and its justification are presented in Section 3.2. However, this chapter will begin with a literature informed restatement of the research questions, including a detailed 'unpacking' of each of them. The data collection instruments used to help answer these questions, the justification for their use, and the practicalities around them are discussed in Section 3.6, followed by the analysis strategy in Section 3.7. The teaching intervention, which is the cornerstone of this study, is presented in Section 3.4. Ethical issues and trustworthiness are presented in Sections 3.5 and 3.8 respectively.

The methodological approach outlined in this chapter, including the action research paradigm and the methodological choices that followed, were underpinned by my theoretical position and my epistemological and ontological values. These connections are made explicit in this chapter.

3.2 Research questions revisited

Although the general research problem was clear from the outset, the specifics of the problems were not. The literature review has provided sufficient background knowledge to present the research questions in more detail in this opening section. I intend that this will give some insight into how the research problem evolved into a set of refined research questions. Experience and knowledge gained throughout the study have led to a set of questions that identified specific evidence-based issues that add another level of sophistication and nuance to the questions. This refinement of the questions allowed for more deliberate and informed data collection and analysis.

It is anticipated that the research context presented in chapter 1, combined with the theoretical background and the literature review from chapter 2, will help the reader to fully understand the rationale behind the questions. The research questions are:

1. What experiences contribute to the problem of MCK enactment for PSTs?
2. What is the optimal design for a pedagogy of enactment, in the context of mathematics education, to reduce the problem of enactment for my PSTs?
3. Can the intervention cause any change in PSTs' beliefs about mathematics and their levels of mathematical anxiety?
4. Can the intervention cause any change in PSTs' practice of teaching mathematics to become more democratic?

Each of these research questions are 'unpacked' below:

Question 1: What experiences contribute to the problem of MCK enactment for PSTs?

The literature review provided insight into why the decontextualised nature of the maths competency module is ineffective in the transfer of knowledge to the classroom. However, I am interested in exploring this idea on a deeper level and finding out about the nuanced aspects of PSTs' experiences and how these experiences shape how they act. Furthermore, I am interested in their holistic realities and would like to explore their experiences from the point of view of various contexts, including programmatic, modular, professional, and personal levels. Knowing this information can help to adapt Grossman's (2008) pedagogies of enactment to the needs of PSTs, while also considering issues related to SP tutors, other TEs and the ITE environment in general.

This question will also feed into question two by providing data to inform the development of a teacher education pedagogy in the specific context of mathematics education. Whereas the literature is concerned primarily with methods modules and educational theory, this question will address the ways in which content modules and methods modules can be tightly interwoven.

Question 2: What is the optimal design for a pedagogy of enactment, in the context of mathematics education, to reduce the problem of enactment for my PSTs?

The literature confirms that PSTs need to possess a significant level of mathematical understanding for effective teaching and achieving this level of competence takes a significant amount of time and effort. The literature also confirms that this knowledge needs to be contextualised within a model of practice, such as Grossman (2008), for it to be transferable to the classroom. This question is, therefore, about balancing the need to cover an adequate depth and breadth of mathematical content but doing so in a contextualised way. This introduces other sub-questions such as what content should be focused on, and what HLTPs should be prioritised, what will the content-practice relationship will be, and how can it be enacted. This question is heavily nuanced and requires PSTs' ongoing subjective responses to the intervention as well as their collective inputs. These should be combined with my reflections to result in a continuous process of refinement of the intervention.

Question 3: Can the intervention cause any positive change in PSTs' beliefs about mathematics and their levels of mathematical anxiety?

Beliefs "are one of the most significant forces affecting teaching" and determine behaviours such as "what knowledge is relevant, what teaching routines are appropriate, what goals should be accomplished" (Speer, 2005, p.364-365). The intervention aimed to fundamentally change PSTs experience of learning mathematics, with the intention that this would reorient existing beliefs in a direction consistent with teaching for relational understanding. I was therefore interested to find out if PSTs' beliefs were altered by the intervention, and if so, what was the nature of this change. Informed by the literature, PSTs' beliefs about the nature of mathematics, and beliefs about mathematics teaching and learning were explored. Mathematics anxiety was also included because of the demonstrably damaging effects it can have on PSTs' professional practice, including avoidance of mathematics and preference for instruction based on instrumental understanding. To determine what, if any, changes occurred because of the intervention, the different categories of beliefs were measured at different stages of the intervention. This is described in Section 3.6.1 of this chapter.

Question 4: Can the intervention cause any positive change in PSTs' practice so that it becomes more democratic?

Like all action research, this study is about making changes and evaluating the impact of those changes. In the first instance, I hoped for personal change resulting in what McNiff et al. (2005) describe as collective change, and that the decision to change my practice in accordance with my values would positively influence PSTs to improve their practice, with the aim of a higher and more democratically orientated standard of mathematics education for the pupils they teach. In this context a democratic orientation refers to understanding mathematics relationally in an intellectually ambitious way. I observed PSTs classroom behaviours to determine if the intervention was having its desired effect.

Data were collected from a variety of sources and analysed rigorously to answer these questions. The data collection instruments and the procedure for analysis of the data are described in Sections 3.6 and 3.7 respectively. Before that, the overall action research design, a description of the intervention, and ethical considerations, are presented in this chapter.

3.2 Research design: Action research

Action Research (AR) is a form of practitioner-based research that seeks to improve practice through some change or changes and to rigorously investigate the effectiveness of those changes (McNiff, 2017). There are various descriptions of the AR process in the literature. For example, it has been described as cycles of reflective action (Lewin, 1946) or a spiralling process (Kemmis & McTaggart, 1982). Also, it can be "a continuous process of acting, reflecting on that action, and then acting again in light of what you have found", essentially a "cycle of action-reflection", and when this is ongoing, it can be seen as a "cycle of cycles" (McNiff, 2005, p.58). It is basically a cyclical process of "planning, acting, observing and reflecting on what happens in order to be able to learn from it' (Ulvik, Riese & Roness, 2018, p.276), and from this seek practical solutions to issues of concern by bringing together collective action and reflection, and theory and practice (Bradbury, 2015). Common to all approaches is the cyclical and reflective process (Erbilgin, 2019, p.30) and the fact that AR has no well-defined ending because the end of one cycle leads into the beginning of the next (Cohen et al., 2018).

AR is a potentially transformative approach that focuses on rigorous data collection, reflection, action and finally, knowledge generation (Brydon-Miller *et al.*, 2016; McNiff & Whitehead, 2010). My knowledge claims are presented in the next chapter. As opposed to traditional forms of social research, AR is "an enquiry by the self into the self, though always in company with other people" (McNiff, 2017, p.4). It is also distinguishable from traditional forms of research in that it is grounded in the values of the practitioner (Sullivan et al., 2016).

As described in the introduction chapter, my decision to use the AR approach emanated from the reflective process and experiencing myself as a living contradiction, which Whitehead (1993) described as a *dissonance* between actions and values. My experience of myself as a living contradiction was not a watershed experience, but a slowly evolving process that over several years resulted in my decision to use AR to address issues of concern in my practice.

According to Noffke (2009) there are three major dimensions to AR: the political, the personal and the professional. It is professional in that it is practitioner-based and aimed at improving practice through learning about practice. In fact, it goes beyond good professional practice in that it also questions the motives for practice while at the same time taking other perspectives into account (McNiff et al., 2005). It is personal because it focuses on improving one's own practice and self-knowledge (Erbilgin, 2019). Although it is a form of personal enquiry, it is always done collaboratively, to achieve commonly agreed goals (McNiff et al., 2005, p.14). It is political because taking action that has consequences for other individuals or wider society is intentionally political. It is about questioning the status quo and practicing according to values, which is often contestable and uncomfortable (McNiff et al., 2005, p.15)

Noffke (2009) explains that all three dimensions are connected, and this is the case for the current study. It is personal because it is about improving my practice, and this is underpinned by my values which have been informed by my personal and professional experiences and validated by theory. However, it is also political because it brings into question historical and contemporary social contexts that are deeply contestable, especially around how and why we teach mathematics. It looks at the problem area from the point of view of my own practice, but this cannot be considered in isolation. My practice is located within the wider educational landscape of practice and policy, but also at a local departmental level. In this regard, there may be professional implications for colleagues which may result in the confrontation of opposing epistemological values.

3.2.1 Epistemology

My ontological, epistemological, and educational values presented in Chapter 1 are rooted in the idea of democracy. These values are consistent with the AR paradigm because it involves working with others in a naturally democratic and inclusive manner to improve practice, which leads to a participatory orientation to knowledge creation. This participatory orientation of this study, which is central to AR, is further explained in the next section, and developed throughout this chapter. Importantly, my choice of the AR paradigmatic approach is consistent with my theoretical position and my values that guide this study because AR is intentionally rooted in the ideas of Dewey and Freire (Brydon-Miller, Prudente & Aguja, 2016), both of whom translated the ideas of democracy in education and the transformative power of education into action using reflective practice, collaboration, and authentic problem solving as the basis of knowledge generation (Dewey, 1916, 1938b; Freire, 1970)

Within the AR paradigm, knowledge generation is firmly rooted in the interpretivist epistemological stance whereby the research relies on the accounts and observations of others to explain phenomena (Coe *et al.*, 2017). This is consistent with my stance that knowledge is "personal, subjective, and unique"(Cohen, *et al.*, 2018, p.5) and that, through critical dialogue, the research becomes a practice of freedom in which learners are not independent from, but considered in relation to the world (Freire, 1970).

Because AR is values laden, uses problem posing and is reliant on critical reflection for knowledge generation (Cohen *et al*., 2018), it is epistemologically compatible with Grossmans (2008) pedagogy of enactment described in Section 2.2.7. As such, the methodological foundations of this study and pedagogical design of the intervention sit in the same space, philosophically and practically speaking. Consequently, as participants and coresearchers, PSTs will be active agents in both learning and knowledge generation.

3.2.2 Critical Reflection

Critical reflection is a key component of AR for improving practice and professional understanding (Sullivan et al, 2016). Lindsey et al. (2015, p.13) describe it as a "conversation with ourselves that leads to even deeper understanding of our own values and beliefs" Reflection of this nature necessitates critical thinking, the skill of challenging thinking of others or the status quo (Sullivan et al, 2016). I began to engage in intentional critical reflection from the beginning of this research study, focusing on all areas of my practice. This was guided by Brookfield's (1995) four lenses for critical reflection which are: (i) the lens of my personal autobiography, (ii) the lens of PSTs' perspectives, (iii) the lens of colleagues' perceptions and (iv) the lens of educational literature to examine my practice. To support this process, I maintained a reflective journal to record observations and problematise my practice, while at the same time being aware of my values in relation to how I carry out my practice. It was also necessary to include PSTs in the reflective process, and this was achieved in several ways. Firstly, PSTs were encouraged to reflect on their learning after each session. This was largely successful across the interventions and is well documented in data collected in Cycle 2. Second, and perhaps more importantly, we engaged in ongoing collective critical dialogue and reflected publicly as a group. This type of collective critical reflection was a key part of this study's emancipatory style of AR and encouraged PSTs to challenge hegemonic systems and taken-for-granted value systems (Cohen et al, 2018), including how mathematics is taught.

Praxis happens when reflection leads to meaningful action (Freire, 1970), and this is the goal of AR, whereby a morally committed action is informed through ongoing and collective critical reflection (Cohen *et al*., 2018). This action must be informed by the lived experiences of others and committed to a set of values (McNiff et al., 2005). To facilitate praxis, I worked towards a situation of minimal power differentials through the reciprocal co-creation of knowledge (Buber, 1958) to encourage open and symmetrical communication between researcher and participants (see Section 3.5.2). This helped to create a dialogical and democratic relationship between me and the PSTs, key elements of Freire's (1970) methodology. This also ensured theoretical consistency between research methodology, teaching methodology, and the nature of mathematical content. All were interconnected by a single philosophy of democratic education.

Despite the pre-existing power differential inherent in the TE-PST relationship, PSTs very naturally engaged in shared dialogue and openly questioned my pedagogical decisions, possibly because they were encouraged to do so. However, PSTs were more reluctant to critique each other's work publicly. This is understandable given the somewhat vulnerable position PSTs are in when publicly problematising mathematics instruction. Efforts were made to reduce this reluctance and encourage more critical, yet respectful, dialogue between PSTs as the cycle progressed by critiquing groups rather than individuals and utilising existing frameworks (e.g., the MQI framework) as a stimulus for critique. Furthermore, efforts were made to create a community of inquiry with a corresponding atmosphere and organisation to encourage this. For example, instructional time became less formal, including less rigid seating arrangements for both PSTs and me as TE. This also included a fundamental change in relationship between PSTs and me to one where my primary role was that of critical facilitator rather than that of a typical lecturer.

Reflexivity was a core idea in this study. As an action researcher, it enabled me to navigate the dual role of participant and a practitioner in the research process, which necessarily integrated me into the social world I was studying (Cohen et al., 2018). Reflexivity has important epistemological consequences also, particularly for emancipatory AR, because it takes the view that data is authentic and represents the experiences of all PSTs. Such democratic relations mean my ideas, knowledge, and experiences, even if based on theory, are no more valid than the views of the PSTs.

3.2.3 Action Research Cycles

Embedded within and between AR cycles are the reciprocal and complementary processes of action and learning. Actions improve a situation and learning happens through actions. Learning in this case happens through critical reflection as described previously (Sullivan et al., 2016). To ensure the authenticity of this research, I consistently acknowledge the relationship between my actions and my learning via my reflective journal, and this is complemented by rigorous data collection and analysis (McNiff, 2017).

For this AR study, I used five steps recommended by Cohen *et al*. (2018): diagnosis, planning, action, assessment, and reflection on and communication of learning. Diagnosis is about defining what needs to be investigated and clarifying the overall purpose of the research. The starting point for this was the overall research problem that was outlined in the introduction chapter, i.e., the problem of enactment. Planning involved making a decision about what intervention to put in place, the nature of the data that should be collected to help inform decisions about the problem, and the type of data collection instruments which are best suited to collect this data. Action is about putting the planned intervention into practice, deciding what the contents of the intervention should be, how it should be organised, and who should be involved in it. Assessment involved analysing and interpreting the data to make a judgment about how well the intervention has addressed the research problem. Critical reflection, in the context of my values, allowed for a deep examination of the experience to determine what was learned from it. PSTs were involved in each of the steps throughout the duration of the study and provided formal and informal evaluations to guide the intervention and judge its success. The final step is to disseminate my learning for external validation by the academic community.

This AR project had two major cycles, each spanning an academic year. One academic year was chosen as a cycle length because this allowed an adequate amount of time to execute the research design including collecting initial baseline data, implement the intervention, collect, and analyse further data, and make relevant changes. It also had the advantage of researching with the same PSTs over an extended period, thus building trust and a professional rapport with them with the intention of maximising the effectiveness of the intervention for everybody and increasing the trustworthiness of the results of the study.

Within each cycle, there were multiple action/reflection iterations, usually resulting in some new diagnosis, or refinement to the original diagnosis. This was facilitated by PST feedback within each cycle that also resulted in useful refinements to the original problem.

3.3 Sampling

The participants in this study were PSTs enrolled on the 4-year B.Ed. (primary education) programme in the Froebel Department of Education, Maynooth University. In line with the national average, approximately 90% of the PSTs were female. To gain entry to undergraduate ITE programmes in Ireland, including the Froebel department, the participants were required to apply through the Central Applications Office (CAO), which is the entity with responsibility for allocating undergraduate places in Ireland. Prior to this, the participants had to complete the Leaving Certificate examination and achieve enough points from this to gain entry to an ITE programme via the CAO system. For each degree programme, the required points are usually determined by popularity and, because primary teaching is held in high esteem in Ireland (Hourigan & Leavey, 2017) the points required to gain entry are relatively high and difficult to attain. In 2017, the year the majority the participants in this study completed their Leaving Certificate examination, 485 points out of a possible 600 were required to gain entry to the programme in the Froebel Department. This places the participants in this study in the top 15% (approximately) nationally of the students who completed their Leaving Certificate in 2017.

Furthermore, each of the participants would have been required to take the recently developed Project Maths curriculum in their final two years of secondary school, which was also examined as part of the Leaving Certificate. Project Maths was developed in 2008 and rolled out nationally in 2010. Its aim was to distinguish itself from the previous (1994) syllabus, which was criticised for its overreliance on recall and application of procedures, by promoting understanding of both procedure and concepts, problem solving, representing mathematical ideas, making connections between topics, as well as the ability to reason and prove mathematical assertions. However, some shortcomings were found in relation to the new curriculum shortly after its rollout. In a study by Kirkland et al. (2012), it was found that students' lack of conceptual knowledge has resulted in an under-preparedness for third level university courses containing mathematics elements. Furthermore, despite what Project Maths purported to do, Jeffs et al. (2013) found little evidence that students were engaging in reasoning, formulating proofs, communicating mathematically, or making connections between topics. A more recent study by Tracey, Faulkner, and Prendergast (2016) looked at the basic mathematical skills of beginning undergraduate university students who have just completed the Project Maths syllabus. These researchers observed "significant declines" in the students' performances on a diagnostic test to measure basic mathematical skills.

In this two-year longitudinal study, the same cohort of PSTs were invited to participate in the research in both cycles. Cycle 1 was carried out in 2019/20 with PSTs in year 3 of the B.Ed. programme, and Cycle 2 was carried out with the same PSTs in 2020/21 in year 4 of the programme. Although there were ordinarily 67 PSTs in this cohort, only 58 PSTs were available in year 3 because 9 were on an Erasmus program. All PSTs were enrolled in the compulsory maths competency module.

At the beginning of Cycle 1, information letters and consent forms (see Appendix 1) were distributed to each PST explaining the purpose of the study and expectations of them should they wish to participate. The same PSTs, with the addition of the returning Erasmus students, were re-invited to take part in the study for the second cycle at the beginning of year 4 with a follow-on information letter and up-to-date consent form.

Participants were selected from year 3 of the B.Ed. program because they had already taken 2 full years of the maths competency module and were therefore familiar with its purpose, content, structure, and approach. This was important for them to be able to situate the intervention within the already established context and norms of the maths competency modules. Also, year 3 PSTs were scheduled to teach senior primary classes (4^{th} – 6^{th} class) on school placement (SP) making the maths competency content more suitable for their placement.

During Cycle 2, Froebel departmental regulations stipulated that PSTs were allowed to choose whether they taught junior classes (JI - 3^{rd} class) or senior classes (4^{th} – 6^{th} class) on SP. To maintain a consistent approach to data collection, only those PSTs scheduled to teach senior classes were included in the potential pool of PSTs for SP observation and focus groups. The details of this are discussed in the sampling strategy below.

For clarity about school placements, SP1 and SP2 will be used to refer to school placements during Cycle 1 and Cycle 2 respectively.

3.3.1 Sampling strategy: overall

At the beginning of each cycle, PSTs were asked to participate in the study and to consent to providing data for analysis. Whether or not PSTs agreed to participate, all of them would be taking part in the intervention as this is part of their normal teacher education. A requirement of this was to maintain a reflective journal which was also required to be submitted, regardless of whether they agreed to participate in the study or not.

For PSTs who agreed to participate in the study, there were two options representing distinct levels of involvement. The first level of participation involved only completing questionnaires (see Section 3.6) which contained both qualitative and quantitative parts and were used to gather data about PSTs' beliefs about mathematics and the effectiveness of the intervention. Non-probability convenience sampling was used (Creswell & Plano Clarke, 2007), and as such all available PSTs were invited to complete a questionnaire with a combination of Likert type quantitative questions, and open ended qualitative parts for PSTs to justify their responses. These were completed at 3 distinct points: before the intervention, in the middle of the intervention between AR Cycles, and after the intervention.

Pre-intervention questionnaire

This was administered at the beginning of year 3 in September 2019. This was the beginning of the academic year and before any changes were made to the maths competency module. the intervention began after this questionnaire was completed by participating PSTs.

Intermediate questionnaire

This questionnaire was administered between cycles in December 2019. It was completed by PSTs after the intervention and on return to college after SP1. Data collected from this questionnaire was used to guide changes to the intervention in Cycle 2.

Post-intervention questionnaire

This final questionnaire was administered in March 2021 after the entirety of the intervention and after SP2.

In Cycle 1, 57 out of a possible 58 PSTs consented to complete the pre-intervention and intermediate questionnaires. At the beginning of Cycle 2, 61 out of 67 PSTs consented to complete the post-intervention questionnaire. While all of these were used for qualitative data analysis, only those PSTs who completed the Likert sections of all three questionnaires ($n = 37$) were included for parametric data analysis. See Section 3.6.1 for details of the questionnaires.

3.3.2 Cycle specific sampling

Due to the restrictions introduced as part of the Covid-19 pandemic, there was a lack of uniformity between cycles in terms of data collection. Indeed, the questionnaires were the only data collection instrument that was the same in Cycle 1 and Cycle 2. Observations carried out in Cycle 1 could not be replicated in Cycle 2 because Covid-19 restrictions meant I could not make direct observations at placement schools. This resulted in collecting data in alternative ways. The sampling processes used for the collection of data across

both cycles are described below.

3.3.2.1 Cycle 1 sampling

In Cycle 1, PSTs were asked to provide consent to take part in a smaller subgroup for SP observations with post-observation discussion, and subsequent focus groups. Of the available

58, 45 PSTs agreed to be part of this subgroup. The size of this subgroup was initially planned to be approximately 15 based on the time available to visit various SP sites. Therefore, proximity to the university was a key criterion when compiling the final subgroup. Convenience sampling, based on SP location, was used to purposefully select this group.

To begin the selection process, the SP locations of all PSTs who consented were compiled and ordered by proximity to Maynooth University. From this, 14 PSTs who were located closest to the university were selected. Prior to observations, each of the 14 PSTs were contacted to confirm their willingness to participate and all 14 agreed. One visit was cancelled due to logistical reasons resulting in a total of 13 observations carried out in Cycle 1.

These 14 PSTs were subsequently invited to take part in one of two scheduled focus groups, and 12 agreed to participate. In addition, 2 PSTs who were not part of the original subgroup requested to participate. This resulted in two focus groups of 7 PSTs. See Sections 3.6.2 and 3.6.3 respectively for details of observations and focus groups.

3.3.2.2 Cycle 2 sampling

Due to Covid-19 restrictions, classroom observations could not be carried out in Cycle 2. In lieu of this, lesson planning documentation was collected and analysed to gain insights into what the PSTs *intended* to do in the classroom. Because teaching was done remotely, some of these PSTs also created instructional videos which were also a useful source of data. One focus group with PSTs was carried out online. Amended information letters were written for Cycle 2 to reflect the changing circumstances and sent to all PSTs. As explained in Section 3.3, 61 out of 67 PSTs consented to their survey responses being analysed.

A further 37 PSTs consented to be included in the subgroup for analysis of SP documentation and to take part in the online focus group. Like Cycle 1, only a small number of PSTs could be selected for inclusion in this subgroup due to time constraints and the qualitatively dense nature of the data. Again, 8 of these were purposefully selected because they were teaching senior classes and, for convenience and efficiency, I was also assigned as SP tutor to this group of PSTs. This also gave me a deeper insight into their SP experiences through regular conversations and contributed to the development of a mutual trust between me and the PSTs which was a significant advantage when engaging with them in the focus group. On the other hand, it raised a significant power differential which needed to be addressed. This is discussed in Section 3.5.2 on power relations below.

3.3.2.3 Overview of participants

Table 3.1 below outlines the PSTs who took part in focus group in Cycle 1 and Cycle 2, including SP observations in Cycle 1 and SP document analysis in Cycle 2.

Table 3.1: Overview of participants

Cycle 1				**Cycle 2**		
Name	**FG1a**	**FG1b**	**Observation**	**Name**	**FG2**	**SP Documentation**
Claire	✓		✓	Fiona	✓	✓
Gillian	✓			Lilly	✓	✓
Aoife	✓		✗	Robbie	✓	✓
Freda	✗	✗	✓	Maeve	✓	✓
Claire	✓		✓	Emily	✓	✓
Helen	✓		✓	Helen	✓	✓
Paul		✓	✓	Paul	✓	✓
Gemma		✓	✓	Gemma	✓	✓
Derek		✓	✓			
Tony		✓	✓			
Mary		✓	✗			
Jenny		✓	✓			
Vicky		✓	✓			
Megan	✗	✗	✓			

Three PSTs, Helen, Paul, and Gemma, participated in both cycles, and these PSTs are highlighted in Table 3.1. At this point there are some details about the participation of PSTs in this part of the study that should be highlighted. In Cycle 1 both Megan and Freda were observed on SP and invited to take part in FG1. However, both were absent on the days FG1 and FG2 took place (4th & 11th December 2019) and were therefore not included in either focus group. On the other hand, Aoife was not observed in Cycle 1 because she was not included as part of the original subgroup, but personally requested to take part in FG1a and this request was obliged.

While there would have been advantages relating to continuity to include entirely the same PSTs in Cycle 1 and Cycle 2, there were two reasons why this did not transpire. Firstly, to maintain consistency, it was important that the PSTs were teaching in the senior primary classes (preferably 5th and 6th class, see Section 3.3). However, of the 16 PSTs who took part in FG1 and FG2, only eight choose to teach senior classes on SP2. Of this eight, I was assigned

SPT to three of them, namely, Gemma, Helen, and Paul, who were all teaching either 5th or 6th class. The remaining five PSTs (Tony, Aoife, Mary, Vicky, and Gillian) were all teaching 4th class and were assigned a different SPT for SP2.

When it became apparent the same PSTs would not be taking part in FG2 as FG1a and FG1b, I made the decision to maintain the democratic nature of the study by including a wider range of PSTs in the process. All of the new PSTs who took part in FG2 were teaching either 5th or 6th classes. In this way, fewer voices were omitted from data collection and, it was hoped, this would result a more holistic critique of the intervention.

Despite mostly different participants taking part in FG2, an effort as made to maximise continuity between the themes generated from the focus groups in Cycle 1 and the data that was gathered from the focus group In Cycle 2 by presenting the PSTs who participated in FG2 with an overview of the study to date. This included a reminder of the original research problem, the design of the intervention and the changes made for cycle 2, and the themes generated in Cycle 1. Using this presentation as a stimulus, the PSTs were asked what further changes they would recommend for the intervention, and how the intervention has changed, or will change, their classroom practice.

3.4 The Intervention

The teaching intervention represented a “signature pedagogy” (Shulman, 2005) of teacher education to reimagine and reorganise my teaching so that the problem of enactment could be addressed. The intervention has its theoretical foundations in Korthagen’s (2010) Gestalt model of teacher learning, and practically based on Grossman and McDonald’s (2008) pedagogies of enactment, as discussed in Section 2.2.7. Grossman and McDonald’s model ensured PSTs were given opportunities to “practice elements of interactive teaching in settings of reduced complexity”, while simultaneously receiving feedback (Grossman and McDonald, 2008, p.190). Underpinning the entire intervention, including PSTs’ involvement in it and any data collection associated with it, was the concept of democratic education as outlined in my theoretical position.

3.4.1 Overview

The intervention was based on three pillars: high leverage teaching practices (HLTPs), Kersaint's (2015) '*100 questions to promote mathematical discourse'*, and the mathematical quality of instruction (MQI) framework which was designed as part of the Learning Mathematics for Teaching Project (2011). Each of these is described below.

High leverage teaching practices (HLTPs) formed the core of the intervention. Enactment of HLTPs allowed for meaningful and deliberate engagement in practice while preserving the relational nature of teaching (Grossman et al., 2008). The importance of critical reflection and dialogue was emphasised (Freire, 1978), both individually and collectively, to promote growth and enhanced understandings about their practice (Freire, 1978; Dewey, 1933). The intervention started at the beginning of Cycle 1 with two HLTPs: *leading a mathematical discussion* and *modelling content*. These were chosen specifically to promote critical dialogue and relational understanding in mathematics. Additional HLTPs were gradually introduced as PSTs became more familiar with and comfortable using the initial two. These included *norms for mathematical discourse* and *eliciting and interpreting pupils' thinking*. TeachingWorks (2022) provided an open collection of practical teacher education resources based on HLTPs. The contributors to this website identified "practices of teaching that are particularly *high-leverage* for children to flourish" (TeachingWorks). These resources included videos of approximations, descriptions of decompositions, and materials depicting representations of practice specific to mathematics. A description of these HLTPs and their decompositions can be seen in Appendix 14.

There were two other crucial elements of the intervention. The first of these is a publication called '*100 questions to promote mathematical discourse*' by Kersaint (2015) (see Appendix 11). The 100 questions are presented as an infographic and are grouped depending on the purpose of the question. These groups include questions that promote mathematical reasoning, collaborative work, group work, process evaluation, problem solving, etc. The nature of this resource complemented the HLTPs and encouraged PSTs to design and enact lessons to allow pupils to reflect on their mathematical thinking and make sense of mathematics in a collaborative and supportive learning environment. It's versatility, portability and ease of use allowed it to be employed in real-time during approximations of practice within the ITE environment and the SP setting.

The final component of the intervention was the MQI framework (Learning Mathematics for Teaching Project, 2011). This component was used as a guide for maintaining the quality of mathematics used in approximations of practice and retrospectively evaluate that quality. It was also a useful tool for the basis of discussion and reflection. The MQI framework was also used for data collection and analysis. These are described in Section 3.6.2 where a deeper description of the framework is available.

3.4.2 Organisation

The teaching intervention was not implemented continuously across two academic years. To phase it in gradually, it was implemented in two block periods during the first two months of both academic years/AR cycles. These intervention blocks were strategically chosen to precede an SP block. This ensured learning was fresh in the minds of PSTs, thus allowing them the opportunity to enact learning from the intervention in the SP block that followed it. The remaining maths competency lectures reverted to the type of direct teaching that was typical of maths competency pre-intervention, where the focus was purely on the development of mathematical content knowledge. In the first cycle, maths competency lectures took place only in semester 1. An overview of this is outlined in Table 3.2 below.

Table 3.2: Overview of Intervention

Key: I = intervention, SP= School Placement, DT = direct teaching.							
Intervention Cycle 1: 2019-2020							
Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr
I	I	SP	DT	/	/	/	/
Intervention Cycle 2: 2020-2021							
Sept	Oct	Nov	Dec	Jan	Feb	Mar	Apr
I	I	I	SP	SP	SP	DT	DT

The intervention involved PSTs learning about HLTPs and enacting these practices in the context of the primary school mathematics curriculum, while using the MQI framework to ensure a high quality of mathematics. Learning about HLTPs typically started with a representation of that practice. For example, when PSTs were learning about modelling mathematical content, they watched three short videos aimed at showing what modelling is, and importantly, what it is not. The next step was to show PSTs the decomposition of that practice. Decompositions were discussed and then examined in the context of representations of that practice. PSTs were then given opportunities to enact that HLTP through collaboratively planning various levels of approximations. This was an iterative and cyclical process done in the context of open dialogue and collective reflection.

In keeping with the collaborative and democratic nature of the intervention, PSTs were fully involved in teaching and learning processes. I tried to empower PSTs to teach and learn from one another using their knowledge of HLTPs, the MQI framework, and their professional experiences. Once PSTs learned about the frameworks, I was able to take a step back and assume, primarily, the role of facilitator. This involved planning and directing activities within each session, providing resources, asking critical and open-ended questions, and allowing PSTs time to discuss and deliberately reflect on their practice.

Role play was an integral part of this. During approximations of practice, PSTs were assigned distinct roles, including that of teachers, pupils, and a feedback role. Each feedback role usually judged approximations of practice on a specific area. For example, one PST might judge an approximation from a mathematical quality perspective, while another might judge it from a decomposition perspective. This feedback was used to generate meaningful dialogue between PSTs involved, which led to critical reflection and learning. See Tables 3.2 and 3.3 below for an overview of the intervention in Cycle 1 and Cycle 2, respectively. The changes to Cycle 2 resulted from my reflective process and data collected from Cycle 1, which is discussed in detail in Section 4.5 of the next chapter.

Table 3.3: Intervention Cycle 1

Date	Session number	Lecturer Activities	PST Activities
Friday 6/9/2019	Session 1	• Introduction to research study • Present challenges in teacher education explained, including theory-practice divide. • Presented motivation for intervention.	• Reflect on current practice • Research curricular areas and discuss in groups • Identify challenges teaching this topic

Monday 9/9/2019	Session 2	• Present a flawed representation of practice for analysis. • Introduce HLTP: Leading a mathematical discussion.	• Complete questionnaire • Reflect on and discuss representation of practice in groups and feedback to class • Discuss how HLTP can improve representation of practice.
16/9/19	Session 3	• Present decomposition of Leading a discussion. • Share instructional video of practice using discussion and moderate whole class reflection on this.	• Groups comment on elements of decomposition evident in the video and reflect on learning from this. • Identify important mathematical knowledge in video
23/9/19	Session 4	• Recap approximations of practice • Organise PSTs into groups • Explain task: pick topic, approximate practice using discussion • Engage with PSTs in reflective process • Along with PSTs, provide feedback	Approximating practice in small groups (5 groups) • Decide discussible topic and prepare in groups • Approximate topic for class • PSTs engage in role play (teacher, pupils, and feedback roles). • Decomposition and '100 questions' used to provide feedback.
1/10/19	Session 5	• Introduce modelling mathematical content: decomposition, representations and approximations • Show video outlining differences between modelling, explaining, and demonstrating. • Engage PSTs in reflection about developing a mini lesson to include both modelling and discussion.	• Identify and discuss the differences between modelling, explaining, and demonstrating content • Choose topic and approximated practice using modelling in small groups • Reflect on learning
7/10/19	Session 6	• Organise visiting pupils, PSTs and assigned teaching spaces. • Observed different groups and took notes.	• PSTs prepare mini lessons to include discussion and modelling. • Groups approximate lesson with small groups of visiting pupils.

14/10/19	Session 7	• Engage PSTs in collaborative reflection on learning.	• Consolidation and reflection on learning.

Table 3.4: Intervention Cycle 2

Date	Session	Lecturer Activities	PST Activities
24th September 2020	Session 1	• Open discussion and collective reflection on the previous year • Presentation: pedagogies of enactment and Gestalt theory of teacher learning, sharing of learning from data collected from Cycle 1 • Shared plan for the year ahead including changes to the intervention and my learning. • Emphasised the need for reflection and MCK, reviewed MQI framework, • Engaged with PSTs on planning introduction to division.	• Reflected on learning from previous year. • Engaged in discussion about teacher learning, and the plan for the year ahead. • In groups, reviewed meaning of division and various pedagogical approaches.
1st October 2020	Session 2	• Discussed MCK around meaning of division • Explained instructions: develop a lesson plan for introduction to division. • Provided PSTs with lecture notes and other resources for planning. • Worked collaboratively with different groups • Engaged in collaborative reflective process.	• Using notes and resources, PSTs worked in pods to develop division lesson plan. • Pods took turns presenting their initial plan to the class. • Remaining PSTs helped to assess specific areas: discussion, modelling, and MQI. • All PSTs gave verbal feedback and because all groups were doing the same topic this resulted in a process of group reflection and refinement.

			• PSTs noticed where they could make improvement through others teaching of the same topic. • Process: Plan – present – feedback – critique – refine.
7th October 2020	Session 3	• Provided handouts to enhance knowledge of division. • Reminder to focus on MQI and the following HLTP's: norms for mathematics discourse, discussion, modelling. • Review of *100 Questions to promote mathematical discourse*. • Ongoing critical feedback	• Refinement of lesson plans • Discuss new resources/ ideas and how these can be used in the refinement process. • Enact lesson plan for class • Use process: Plan – present – feedback – critique – refine.
13th October 2020		**Approximation of practice:** One pod (Gillian, Ella, and Samantha) volunteered to teach their *introduction to division* lesson plan to a BEd year 1 tutorial group. This approximation of practice facilitated by me and recorded with the permission of PSTs as a representation of practice for analysis in next session.	
14th October 2020	Session 4	• Facilitated analysis of Gillian, Ella, and Samantha's video representation of practice. •	• PSTs gave overview of their lesson, and content involved. • Watched video with intermittent pauses for reflection. • Other PSTs asked questions and recommended feedback.
28th October 2020	Session 5	Cancelled due to Covid-19.	
5th November 2020	Session 6 (online)	• Introduced new HLTP: Eliciting and Interpreting Pupils Thinking • Content: written methods for division (i.e., division algorithms)	• PSTs watched video representation and used observation too to note instances of it.

		• Explained new HLP and motivation for its use • Presented its decomposition	• PSTs worked in pods to create lesson plans to include new and existing HLTPs with the usual MQI focus. One pod enacted their lesson plan online.

3.4.3 The Flipped Classroom

Informed by the literature review about the importance of mathematical knowledge, and because the learning outcomes of the maths competency module did not change with the intervention, it was necessary to maintain the same quality and quantity of MCK from the original maths competency module. Another reason to maintain a high level of MCK is motivated by my theoretical position and captured by Freire's (1970) notion of literacy, which in this context refers to deep numerical literacy. This numerical literacy, enabled by relational understanding of mathematics, is required to empower PSTs to use and interpret mathematical knowledge critically as opposed to uncritically receiving it from me.

As time slots for the intervention were limited, it was a challenge to try to balance the need to seriously address PSTs' mathematical knowledge while at the same time implementing an intervention that is based primarily on enacting knowledge in practice. Shulman (2005) acknowledged such conflicts when he cautioned about tensions arising through the competing demands of the academy and the "contradictions inherent in the multiple roles and expectations for professional practitioners" (p.53). To address this conflict, the 'flipped classroom' model was integrated into the intervention. This is a pedagogical model whereby lecture time and "independent work" are reversed. This was done by developing concise, high-quality teaching videos containing the traditional mathematical content from the original maths competency module (Bergman & Sams, 2012). This allowed PSTs to watch short videos in their own time and later use this knowledge during teaching sessions. The use of innovative technologies for such purposes is becoming more popular in ITE and has been shown to improve learning for PSTs by giving more time to collaborative and student-initiated activities (Leming, 2018). This resulted in a more dynamic learning environment where PSTs were able to engage more creatively with the lecture content (Vaughan, 2014). It also contributes to democratic educational processes as it promotes individualised learning, allowing PSTs to learn at their own pace, and providing opportunities to catch up when they miss classes (Hickman, 2016). Within lectures, it has the potential to promote what Dewey (1990) envisages as a community of enquiry because it allows PSTs to problematise their mathematical knowledge in the context of practice.

3.5 Ethical Considerations

Ethical approval was granted from the Maynooth University Social Research Ethics Sub-Committee before both cycles (see Appendix 7). I was guided by the Maynooth University Research Ethics Policy throughout the study.

There are ethical principles that are unique to AR because the research is not simply researching with participants, rather the aim is to influence human participants (McNiff, 2005, p.49). This requires the provision of documentation, including ethics and permission letters, negotiating access with PSTs, schools, and the university, ensuring confidentiality of information, identity, and data.

In addition to this, it is necessary to conduct practice in a way that is professional and academically sound. Throughout the process, PSTs were aware of their right to withdraw at any time. Within the AR design, there are additional considerations such as the knock-on effects of the intervention, such as the increased workload put on the PSTs (Locke et al., 2013). At all times, beneficence was taken seriously in this study by striving to minimise any possible harm and maximising any possible good to PSTs (Sullivan et al., 2016).

Because of the nature of this AR study, which is also guided by my democratic values, I wanted to pay particular attention to informed consent and power relations. These are discussed below.

3.5.1 Informed consent

Owing to the democratic and participatory nature of this study, all PSTs were fully informed about it (Sullivan et al., 2016). This was done by distributing informed consent forms and information letters to PSTs at the beginning of Cycle 1 and the beginning of Cycle 2 (see Appendices 1 & 4). The purpose of the second information letter was to ask PSTs to continue with the research into a second cycle and to reflect the changing circumstances due to Covid-19 restrictions.

It was explained in the information letter that PSTs were under no obligation to participate, and if they did, they could withdraw at any point. It was also outlined in the information letter that there would be no penalty for not participating (Nolen & Vander Putten, 2007). Each PST was asked to select the appropriate box on the informed consent form indicating whether they would like to participate or not, and to what extent they would like to be involved.

In the first information letter, PSTs were asked to consent to two different levels of involvement. The first was to complete a questionnaire on two separate occasions. The second was to be observed on SP and subsequently participate in a focus group. The second information letter and consent form asked PSTs to explicitly give permission for their reflections and SP data to be analysed. This was because, in the first cycle, reflective data was not adequately collected. In addition to this, because I was not able to directly observe PSTs on SP due to Covid-19 restrictions, this data would play a more key role in answering the research questions.

Prior to observations, information letters were sent to the principals of all participating schools requesting formal permission and explaining the nature of the study and my role as an observer in the classroom. See Appendix 2 for a copy of this.

Confidentiality was ensured for all PSTs throughout the research process. Participants were asked to include student identification numbers on questionnaires rather than names, and the key was only available to my supervisor and me. Identification numbers were later mapped to PSTs' names, and these were maintained securely on a password-protected file. Names were only used to make links between data from questionnaires, and data from observations and focus groups. Otherwise, identification numbers were the primary identifier for PSTs in the study. This unique identifier was especially useful for quantitative data analysis, allowing for the mapping of everyone's responses across data sets. All PSTs involved in the study, including SP observations, focus groups, or any other personal encounters, are referred to by pseudonyms.

3.5.2 Power Relations

To fully make sense of my practice and the implications of my actions, it was necessary to access PSTs' subjective knowledge about their beliefs, experiences, and ideas related to my practice and any changes made to it. To achieve this, I tried to integrate myself fully into the research as a participant as well as the researcher. As suggested by McNiff (2017) and Cohen, *et al.* (2018), I tried to reduce the power dynamics and include PSTs as co-researchers. This was intended to promote honesty and allow for the legitimate negotiation of knowledge through critical dialogue and collective reflection (Sullivan *et al.*, 2016; Freire, 1978)

Power exists in the relationships between people (McNiff, 2005, p.152), and due to the interpersonal nature of the study, it was important to consider the impact of these relationships carefully. In the traditional lecturer-student relationship, there is a clear power differential. This is evident even in terms of who gets to speak, express an opinion, as well as the parameters within which discussion is permitted. Power imbalances can reduce critical thought and encourage "uniformity, complacency and acceptance" (McNiff, 2005, p.152). Such a scenario is the anthesis to the essence of this study. It would invalidate the results, reduce the effectiveness of the intervention, and contradict the central idea of democracy in education. In addition to this, voluntary participation may cause PSTs to agree to participate in research that they might otherwise reject just to appease me as the person in a position of power (Sullivan et al, 2016, p.99). This may also result in acquiescence bias whereby PSTs may provide answers that would disproportionately concur with my known position on educational issues.

This issue was carefully addressed. Firstly, as explained earlier, PSTs were assured in the information letter and verbally that there would be no consequences for not participating in the study. Secondly, I did not request PST names on consent forms. Instead, I asked for student ID numbers to protect confidentiality. Although I could still identify student names through a key, I assured PSTs I would not do this unless there was a legitimate reason, related to the integrity of the research, to do so. I hoped that this would give PSTs additional reassurance to make independent decisions. I also ensured that even if PSTs did not volunteer to be part of the research, they would still fully take part in the intervention and would therefore not be disadvantaged in any way. The only difference between those PSTs who volunteered and those who did not was that those who volunteered would provide data.

Finally, I had an open discussion with PSTs about the balance of power within our new relationship. This included a public disclosure that I was not the exclusive source of knowledge in the class. In most cases, I merely gave PSTs access to the relevant knowledge and took a step back to allow PSTs to construct meaning from this. I encouraged PSTs to think openly and critically in the context of their learning and experience on the B.Ed. program to date.

I also adjusted my behaviour to reflect this new relationship. This included how I communicated and addressed PSTs, the room layout, and where I located myself in it. I also openly encouraged PSTs to challenge my inputs as well as each other's. This was a culture I tried to develop and normalise over two cycles.

To fully embrace this culture, I asked PSTs to focus less on assessment and more on their professional learning and how this would positively impact their classroom practice. I wanted to deemphasise the importance of passing an examination and emphasise the importance of understanding mathematical content for effective teaching. I assured them that they would be provided with all the necessary supports to do well in their examination, and I was willing to work with them fully to achieve the pass threshold.

I also extended this relationship to post-lesson discussions and focus groups to maximise honesty and integrity and to maintain the democratic nature of our interactions. This was especially important in Cycle 2 when the 8 PSTs in the subgroup were also my SP tutees. Specific conversations were held with them to ensure their continued consent and explain to them that their participation would not advantage or disadvantage them in any way. This was strengthened by the fact that, due to the online nature of the placement, performance would not be graded by the SPT. Instead, performance would receive assigned pass/fail based on post-lesson discussions and this helped to further reduce power differentials. SP files were graded in the normal way.

3.6 Data Collection Instruments

To maximise the trustworthiness of the results, multiple instruments and data sources were used (Taylor, 2004; Patton,2002). This section will give an overview of these and the purpose for which they were used (Sullivan et al, 2018). The data were collected from questionnaires, focus groups, and observations, and I also maintained and regularly updated a reflective journal to document my learning throughout the study. PSTs' reflections and artifacts related to SP were also used, although to much greater extent in Cycle 2. There is a more detailed justification for this in Section 3.6.4 of this chapter.

Before outlining the details of the data collection instruments, I will clarify when each of them was used within the context of the two cycles of the study. This is necessary because there were multiple data collection points, and each part fed into subsequent parts of the study. For example, initial results from the questionnaire data in Cycle 1 were used as a guide and discussion stimulus for the focus groups that followed SP1. When Cycle 1 was completed, all of the data from various sources were collated and analysed. The results were considered in terms of the research questions and implications for the intervention in Cycle 2. Although the data collection appears sequential, there was a process of revisiting and reviewing previously collected data in light of some other data or previously unread literature. For example, in Cycle 1 it was possible to better understand observation data when it was considered in the light of focus group data. Central to both cycles was my continuous reflective process. This enabled me to make sense of data collected in terms of my practice and the overarching research problem while maintaining a crucial awareness of my values throughout the process.

It is also important to emphasise that, due to restrictions put in place by the Covid-19 pandemic, data collection was not identical across both cycles i.e., Cycle 2 did not replicate Cycle 1. A detailed account of the key methodological points in chronological are described in Figure 3.1 below to provide clarity around the lengthy action cycles and the multiple data collection points. Figure 3.1 below should be considered in the context of Tables 3.1, 3.2 and 3.3 as presented earlier in this chapter which outline the intervention timeline, and the details of the intervention in Cycle 1 and Cycle 2, respectively.

Consistent with my epistemological and theoretical positions, this study is primarily qualitative. However, for the purpose of triangulation and data integrity, quantitative data were collected from questionnaires and analysed statistically to support knowledge claims emanating from qualitative data (Brydon-Miller, Prudente and Aguja, 2017). See Section 3.8.3 of this chapter for a discussion on triangulation in this study. The data collection instruments and the reasons for using them are discussed in the remainder of this section.

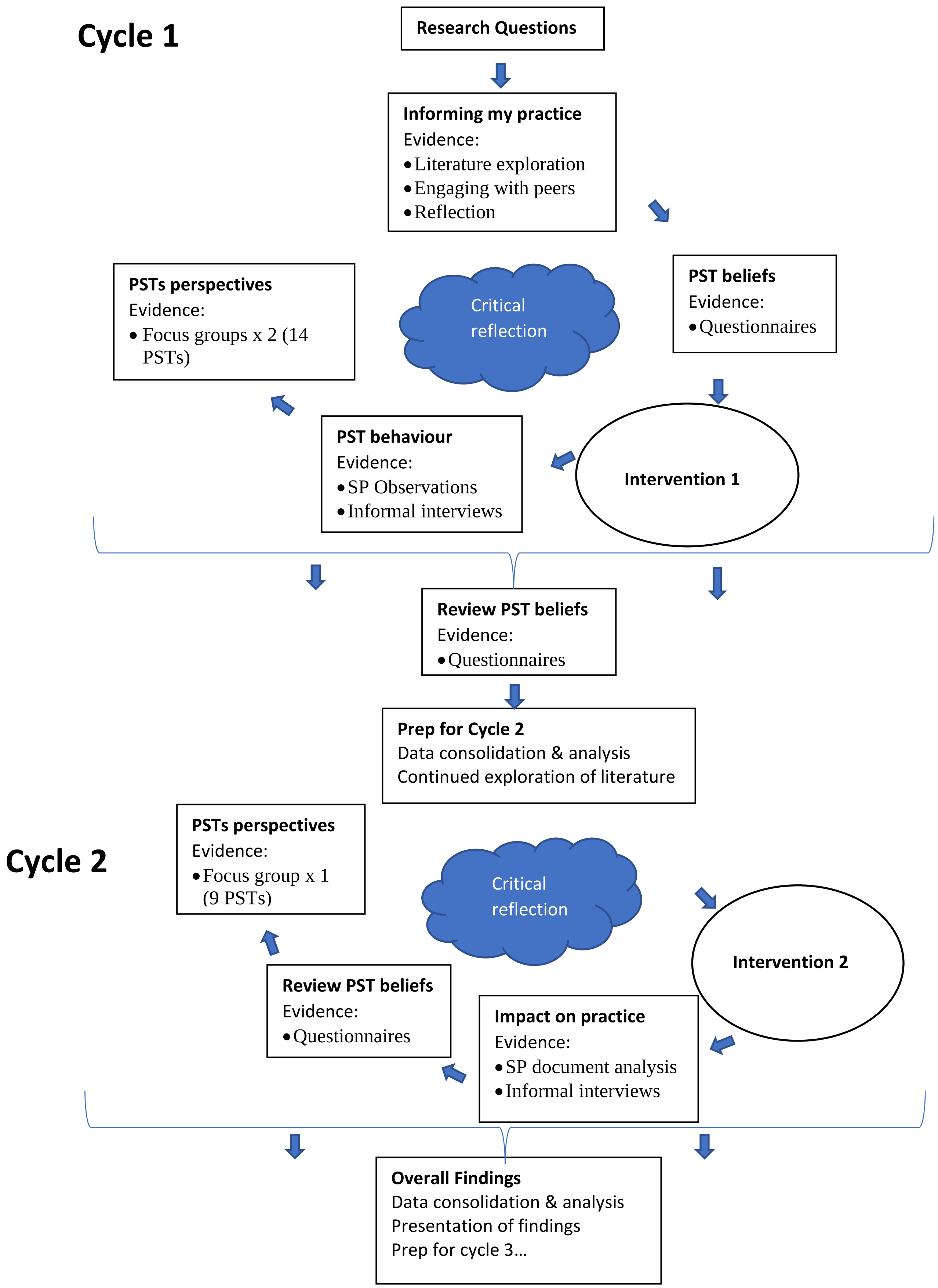


Figure 3.1: Research design flowchart

3.6.1 Questionnaires

Questionnaires were used to gather a blend of quantitative and qualitative data about PSTs' beliefs related to mathematics. Consistent with the literature, these beliefs fall into three broad categories: beliefs about mathematics, beliefs about the effectiveness of mathematics teaching and learning in the Froebel Department, and mathematical anxiety. For each of these categories, data were collected on a Likert scale followed by an open-ended qualitative section. For each category of belief, this process of data collection is described below. A copy of the questionnaire can be found in Appendix 3.

3.6.1.1 Beliefs about mathematics

This section of the questionnaire aimed to determine PSTs' beliefs about mathematics and make a judgment about how these beliefs might influence their behaviours and decision making in the classroom. The questionnaire designed for, and validated by, the Teacher Education and Development Study in Mathematics (TEDS-M; see e.g., Brese & Tatto, 2012) to investigate PSTs' beliefs about mathematics was adapted for this study. The questionnaire draws significantly on the opposing fallibilist and absolutist philosophies about the nature of mathematics which were discussed in Section 2.4 in the literature review, and this should be used to contextualise the remainder of this section.

The questionnaire used a 6-point Likert scale, ranging from 1 (strongly disagree) to 6 (strongly agree), to determine the extent to which PSTs agreed with a range of statements about mathematics. Statements related to beliefs about the nature of mathematics range from static (1) to dynamic (6); statements related to beliefs about learning mathematics range from a traditional pedagogy (1) to a constructivist pedagogy (6); and beliefs related to mathematics achievement range from a fixed mindset (1) to a growth mindset (6). Following each question, PSTs could complete an open-ended qualitative section to justify their choices and provide clarification where they felt necessary.

There were 12 items in the section of the questionnaire concerned with beliefs about the nature of mathematics. These were divided into the two opposing perspectives about the nature of mathematics: mathematics as a process of inquiry and mathematics as rules and procedures. The 12 items are split evenly between the two perspectives. The first perspective is the belief that mathematics is dynamic in nature, and this emanates from the fallibilistic philosophy of mathematics. It includes statements such as "mathematics involves creativity and new ideas" and "you can discover new things for yourself". The second perspective is the belief that mathematics is static in nature, and this emanates from the absolutist philosophy of mathematics. It includes statements such as "mathematics means learning, remembering and applying" and "mathematics is a collection of rules and procedures that prescribe how to solve a problem".

There were also two opposing perspectives for beliefs about learning mathematics. Emanating from the absolutist philosophy of mathematics is the traditional view of learning mathematics. This is primarily teacher-centred, and procedure driven. The traditional view includes statements such as "the best way to do well in mathematics is to memorise all the formulas" and "pupils learn mathematics best by focusing on the teacher's explanations". The second perspective is based on constructivist learning and emanates from the fallibilist philosophy of mathematics. This is primarily student-centred and involves students making sense of mathematics for themselves through inquiry and discovery. This perspective includes statements such as "pupils can figure out a way to solve mathematical problems without a teachers help" and "in addition to getting a right answer in mathematics, it is important to understand why an answer is correct".

The section on the nature of PSTs' beliefs about mathematical achievement contained eight statements, with each statement suggesting mathematical achievement is fixed. That is, agreement with these statements suggested a fixed mindset (Dweck, 2017). Teachers with a fixed view of mathematics are more likely to teach in a traditional fashion, whereas those who disagree (i.e., those with a growth mindset) are more likely to embrace constructivist teaching and learning, which necessitates more mathematically rigorous content standards (Bednarz & Proulx, 2009). Examples of statements include "to be good at mathematics you must have a kind of mathematical mind" and "some people are good at mathematics and others are not".

3.6.1.2 Beliefs about effectiveness of ITE

Because one of the research questions is about determining the effectiveness of the intervention, it was necessary to find out how well PSTs feel their teacher education program has prepared them to teach mathematics. There were two separate questions related to this. The first question is about preparedness to teach mathematics. This attempted to capture

PSTs' beliefs about the extent to which their ITE program had given them the capacity to carry out the main tasks of teaching to meet the demands of their classroom practice. Among other things, this question focused on PSTs' preparedness to engage pupils in effective learning and whether they feel they have become active members of a professional community (Tatto, 2003, p.56).

The second question is related to program effectiveness, and asked PSTs to give an overall indication of how well their ITE program has helped them learn to teach mathematics in practice. Questions focused on, among other things, the extent to which PSTs believed their lecturers "modelled good teaching practices and used and promoted research, evaluation, and reflection in their courses" (Tatto, 2003, p.56).

3.6.1.3 Mathematical Anxiety

To measure PSTs' mathematical anxiety, I used the Abbreviated Math Anxiety Scale (AMAS). The psychometric properties of the AMAS have been shown to be excellent as indicated by a high reliability and validity of the scale (Hopko et al., 2003). The instrument is comprised of 9 items which are responded to using a 5-point Likert-type scale, ranging from 1 (low anxiety) to 5 (high anxiety). For each PST, the total score is the sum of the nine items (Hopko et al, 2003). There are two subscales within the AMAS. These are learning math anxiety (LMA) and math evaluation anxiety (MEA). LMA refers to anxiety that one experiences while learning mathematics, for example "starting a new chapter in a math book". MEA refers to anxiety that one feels when their mathematical knowledge is being evaluated, for example, when taking a mathematics examination (Schillinger at al, 2018). I did not make a distinction between the two types of mathematical anxiety because this study is primarily about my practice and how it can be modified to improve the quality of learning for PSTs, and consequently the quality of instruction PSTs enact in the classroom. I was interested to discover if the democratic nature of the intervention would reduce negative feelings PSTs may have in relation to mathematics (whether that be learning anxiety or evaluation anxiety), as this would likely to increase the efficacy of the intervention for those PSTs. On the other hand, if overall levels of anxiety increased amongst the PSTs, then this would have implications for subsequent data collection and changes to the intervention.

3.6.2 Classroom Observations & Mathematical Quality of Instruction

The Mathematical Quality of Instruction (MQI) framework was briefly introduced in Section 3.4 as a pedagogical tool used during the intervention. To maintain a consistency of approach across the study, the MQI framework was used as a tool for collection and analysis of observational data. Some important background on this framework is presented in the following section.

3.6.2.1 Background

The MQI framework was developed as part of the Learning Mathematics for Teaching Project (2011). The framework is appropriate for this study because it is one of the few validated observational frameworks that look specifically at PSTs' MCK and how this is made available to pupils (Ingram et al., 2018). It also works on the premise that mathematical work in the classroom is distinct from generic aspects of teaching such as classroom management. Other frameworks, such as the Knowledge Quartet (KQ) and Watson Framework were considered, however, the MQI framework the MQI framework has the advantage of allowing for individual scores to be attributed to different dimensions of mathematical teaching. Having multiple dimensions also makes it flexible enough to capture the plethora of scenarios that can arise during mathematics classroom instruction (Ingram et al., 2018). The MQI framework has been rigorously examined from a validity and reliability perspective across multiple countries and cultural contexts (see Hill, Rowan and Ball, 2005) thus making it suitable for the current study.

The framework allows mathematical quality to be assessed across five different dimensions: richness of mathematics, errors and imprecision, working with students and mathematics; student participation in meaning making and reasoning; and connections between classroom work and mathematics.

Classroom observations were guided by the *richness of mathematics* element of the MQI framework. This element was chosen because it adequately captures the essence of relational understanding in the classroom. It also allows the observer to capture the extent to which relational understanding is included in mathematical instructing by easily categorizing MQI performance using a 4-point scoring system. Finally, focusing exclusively on *richness of mathematics* (from a scoring perspective) allowed for a high degree of accuracy as there were fewer elements to try to observe in a live and dynamic classroom environment. It also allowed more time to write rich descriptions of my observations which were consistent with the qualitative nature of this study. Where relevant, the remaining elements of the framework were used as part of the overall analytic process *after* the live observation.

Although it is recommended to use video recording with the MQI framework, this was not used in this study. It was more appropriate to use live observations and record rich descriptions using the MQI framework as a guide because I wanted to stay close to the reality of SP classroom observations, and I also wanted PSTs to work under conditions they had become accustomed to. Also, using video recording would limit observations to traditional classroom instruction (e.g., maths trails would be excluded) while potentially missing the nuance of meaningful interactions that require in person observation.

3.6.2.2 Post-observation Discussions

After each lesson observation, I facilitated an unstructured conversation-style interview with each PST. This discussion was PST-led and open-ended with the aim of promoting reflective dialogue and this added a qualitative depth to the observations (May, 2001). Leading questions and any lesson critique were intentionally avoided so that the PSTs could give their personal perspectives, even if these perspectives were contradictory with my observations. At times suggestions were made to PSTs to help improve the quality of future lessons, thus maximising learning from the interactions. Brief notes were taken during the discussions and, if necessary, expanded on later in the day.

3.6.3 Focus Group Interviews

Focus groups were used because they “work particularly well to determine the perceptions, feeling and thinking” of a group of people on a topic (Kreuger & Casey, 2009, p.8), and were used to help answer the research questions by providing rich data on PSTs’ thoughts and behaviours in relation to the intervention and its impact on their practice. In addition to this, because the data were collected in terms of PSTs’ “own words and contexts…there is a minimum of artificiality of response” (Stewart & Shamdasani, 1990, p.17). They were chosen over individual interviews because they encouraged a higher level of criticality amongst PSTs as they opened up in a comfortable space and responded meaningfully to each other’s views (Bryman, 2004).

Two face-to-face focus groups (FG1a and FG1b) were carried out in Cycle 1. Only one focus group was necessary in Cycle 2 (FG2) because the major themes had already been established and were, therefore, less open-ended than the focus groups in Cycle 1. FG2 was also conducted remotely because of Covid-19 restrictions. The purpose of FG2 was to refine existing themes in the context of changes to the intervention and another SP. To help with this process, and in the interests of transparency, PSTs were given a presentation before the FG2 began to remind them of the research questions, the findings from Cycle 1, and the resulting changes to the intervention. They were then asked specific questions about the changes to the intervention and the effectiveness of the intervention in terms of their most recent SP. See Appendix 14 for a copy of this presentation.

3.6.4 Lesson Planning Documentation

As a result of the Covid-19 pandemic, it was not permitted to observe PSTs in the classroom during cycle two and PSTs were required to teach remotely. Instead, SP documentation, including long-term planning, individual lesson planning and resources, were analysed with the permission of the eight PSTs who consented to be part of the subgroup, as explained in Section 3.3. Because PSTs were in their final year, they were not required to produce detailed lesson plans, and instead were asked to maintain more concise plans known as daily notes.

The pandemic resulted in the deployment of PSTs across a range of settings as determined by the school principal and cooperating teacher. As a result, some PSTs taught mathematics lessons each day, whereas others were scheduled less frequently or on an ad hoc basis. The setting also varied with some PSTs teaching in the SEN context as well as mainstream. Because teaching was done remotely, six out of a possible eight PSTs created instructional videos which were delivered asynchronously to their pupils. Where available, these were analysed in the context of their planning.

The purpose of analysing PSTs' plans, and videos was to look for evidence of the intervention in their teaching and evidence for teaching mathematics relationally as guided by the MQI framework. I was specifically looking for how their pedagogical approach to the delivery of mathematical content was influenced by HLPTs and the MQI framework. In the short-term schemes, these corresponded to learning objectives (content) and learning activities (practice), respectively. PSTs then used these schemes to develop individual lesson plans, which were analysed in the same way.

It would have been beneficial to supplement the planning documentation and instructional videos with additional data sources, such as individual interviews, as this would have provided rich descriptions of PSTs' content and pedagogical decisions, they made during SP2. While not doing so is an acknowledged limitation, there were several reasons why this was not possible, or it was felt, appropriate. Firstly, the Ed.D. programme which I am undertaking is limited in time (and dissertation word count) which puts very practical constraints on the amount of time, and hence scope, that collecting, analysing, and reporting additional data would entail.

Secondly, as outlined in Section 3.4, I was assuming the dual role of SPT and researcher. While SPTs met with PSTs remotely to discuss their teaching, this was scheduled to happen in the evening, three times over duration of the placement, and not directly after specific lessons. During these discussions I felt it would have been inappropriate to probe too deeply into their individual mathematical practices for two reasons. Firstly, planning documentation had not yet been analysed which would make discussions around this less beneficial. Secondly, SPTs and PSTs were made aware of specific guidelines about what to discuss at predetermined times during the placement. Deviating from this would have been unfair on PSTs in terms of their opportunities to reflect on their remote teaching experience in the same way PSTs not involved in this part of the study did.

While it would have been useful to know why PSTs made the decisions they did during SP2, it was not essential. Firstly, triangulation was used to make sense of their decisions, particularly with regard to the qualitative data from the surveys about their beliefs about the teaching and learning of mathematics. Secondly, the analysis that resulted from this triangulation procedure was analysed in light of the relevant literature on the topic of inconsistencies between PSTs' purported beliefs and their classroom practice, as outlined in Section 2.4.5 of the Literature Review.

3.6.5 Reflections

As noted in in the introduction to this section, it was intended that both PSTs and I maintain reflective journals to document learning and observations related to the study. Using Brookfield's Lenses, reflective journal was regularly updated, and was a rich source of data, particularly when reflecting on the intervention and deciding what changes needed to be made going into Cycle two.

PST reflections were not collected in Cycle 1 for several reasons. Firstly, I admittedly did not provide enough explicit direction to PSTs around the writing and retainment of these. For example, PSTs were not directed to type these, and they were not part of the formal

University module descriptor. Handwritten reflections were also difficult to collect when working remotely due to the Covid-19 pandemic. Although PSTs reflected on learning and experiences, unfortunately it did not result in the collection of meaningful data that could be analysed. This experience resulted in improvements in Cycle 2, resulting in the systematic online submission of PSTs' reflections, and reflections formally included in the module descriptor.

3.7 Data Analysis

The data were analysed qualitatively using Braun & Clarke's (2006) thematic analysis, and quantitatively using the Analysis of Variance (ANOVA) statistical test. The MQI framework was used to analyse PSTs' practice qualitatively and quantitatively. Each of these are explained in the sections that follow.

3.7.1 Thematic analysis

Braun & Clarke's (2006) thematic analysis was used to analyse the two focus groups in Cycle 1 (FG1a and FG1b) and one focus group in Cycle 2(FG2). This process involved looking for patterns of meaning to identify themes in the data. For FG1a and FG1b a blended inductive-deductive approach to coding was used. That is, my approach was informed by the theoretical perspectives and pertinent literature presented in the literature review while also being open to unique perspectives from PSTs that may not naturally fit into my preexisting conceptual frame. The deductive approach was chosen because there was a specific set of research questions, and it necessitated becoming familiar with a breadth and depth of relevant theory and literature. However, coding was done with an open mind, and seeking always to generating inductive codes. In essence, when looking at the data I asked myself "Does it capture something important in relation to the research question(s)?". If so, it was coded appropriately. At times, unexpected codes prompted further review of the literature in search of meaning which allowed for a more nuanced analysis of the data. Post-lesson discussions, PSTs' reflections, and my reflective journal were used for triangulation purposes and analysis of these was guided by the themes generated from the focus groups. Triangulation is discussed in more detail in Section 3.8.3 below.

All of the themes, except one, were generated from analysis at the semantic level (Braun & Clarke, 2006), where explicit meaning from PSTs' responses was searched for in the data. This is an important approach in terms of the epistemological assumptions which underpin this study. Consistent with my values, the voice of the PST was central to this study, and as such, I assumed a "unidirectional relationship" between "meaning and experience and language" underpinning the requirement for a sematic level of analysis (Braun & Clarke, 2006, p.14). Only one theme was generated from data analysed at the latent level in FG1a and FG1b to identify neoliberal influences on PST practices. I also aimed for a sophisticated level of analysis to tell a story, as truthfully as possible, from the PSTs' perspectives, which could be theoretically analysed to present an informed argument for reconceptualizing my practice.

Braun & Clarke (2006) suggest 6 phases that make up the thematic analysis process. These are: becoming familiar with the data; generating initial codes; searching for themes; reviewing themes; Defining and naming themes; Producing the report. In this study, themes were generated first for FG1a and FG1b and these themes were later used to code FG2 deductively. This was not a linear process because some codes were generated later in the process and others were deleted or merged with others. Because of the importance of the themes from FG1a and FG1b and their implications for the entire study, I have detailed the process in the next section.

3.7.1.1 Focus Group 1a and 1b theme generation

To begin phase one of the TA process, I transcribed the data myself. Although time-consuming, this process involved paying very close attention to the data. Following this I read over the transcripts several times, allowing me to spot basic patterns, and develop initial ideas about how the data related to the research questions.

As a natural progression from this familiarisation step, phase 2 of the TA I started to generate codes in a more formalised way by identifying "features of the data that appear[ed] interesting" in relation to the research questions that would later form the basis for themes (Braun & Clarke, 2006, p.18). Codes were generated from the FG1a first and then from FG1b. However, this was a non-linear, recursive process involving going back and forth between FG1a and FG1b. Codes initially generated from FG1a were recorded and presented in an organised way under general headings. When FG1b was being coded, the codes from FG1a were used to code FG1b and were assigned to segments of the transcript as appropriate. In several instances, additional (new) codes were generated in FG1b that were not identified in FG1a. In these instances, it was then necessary to review FG1a for that same code and code it if applicable. After this initial coding process involving both focus groups, there were a total of 55 codes.

Because of the large number of initial codes generated, I carried out a refinement process involving deleting meaningless codes, merging similar codes, and deleting duplicate codes (i.e., codes with different names but capture the same meaning). At the end of this refinement process, a set of 47 codes were established and clustered together to form themes. See Appendix 16 for details of code refinement and theme generation process. This resulted in thematic mapping, which is a visual process, whereby patterns within the data and relationships among themes were established. Braun and Clark (2006) recommend using a central organising concept to help keep the process focused, for which I used the problem of enactment as described in Section 2.2.2 of the Literature Review chapter. These themes were used as a starting point to address the research questions and provided a strong stimulus for the reflective process to consider evidence-based changes to the intervention going into Cycle 2.

3.7.1.3 MQI Analysis

As described in section 3.6.2 above, the *richness of mathematics* element of the MQI framework was used to analyse classroom observations in Cycle 1, and later PSTs' planning documentation in Cycle 2. Using the 4-point MQI scale, each PST received a numerical score relating to the quality of their mathematics teaching.

For the classroom observations, I used a limited version of the 4-point MQI *richness of mathematics* scale. This construct allowed me to objectively observe the following factors of mathematical quality: linking between representations; mathematical explanations; mathematical sense-making; multiple procedures or solution methods; patterns and generalisations; mathematical language; and the overall richness of the mathematics.

The 4-point scale ranges from 0 (not present) to 3 (high) for each of the above factors of mathematical quality (see Appendix 13). After each lesson, an MQI score was calculated for each PST and represented as a percentage. This allowed me to categorise PSTs into high, middle, and low/ absent based on their MQI scores. These were abbreviated as MQI-H, MQI-M and MQI-L/A, respectively. Participants were categorised in this way, not to internally rank them or compare with previous cohorts per se, but so that pertinent features of practice could be identified in a systematic way. As well as helping to answer the research questions, the data gathered this way was readily anonymised and can be used as representations of practice in future iterations of the intervention.

Descriptors of the relevant MQI codes were available to me as I observed each mathematics lesson to guide my notetaking. An effort was made to record as much qualitative detail as possible and immediately after each observation I completed the MQI assessment tool for each PST as well as some initial thoughts around analysis. Qualitative data (i.e., detailed notes) were retrospectively analysed using a broader application of the MQI framework to include the additional elements of task cognitive demand, remuneration of pupils' errors, and pupils' contributions, as well as the richness of mathematics. Reflections were written for each PSTs' observation to bring criticality to the analysis as well as to contextualise the data within the broader research area.

For each PST, lessons were analysed under headings relating to the structure of a lesson, i.e., introduction, development, and conclusion. The results are outlined in Section 4.3 of the next chapter.

3.7.2 Quantitative data analysis

The Likert scale data collected from the questionnaires for mathematical anxiety and beliefs were analysed quantitatively using the Analysis of Variance (ANOVA) statistical test. ANOVA is a difference test, a statistical technique to compare means within the population. The data from these surveys was used to answer research question 3, i.e., can the intervention cause any change in PSTs' beliefs about mathematics and their levels of mathematical anxiety? Therefore, the quantitative results play a significant role in answering the research questions and determining the overall efficacy of the intervention.

To measure the results in a consistent way, some of the Likert values for beliefs were reversed so that those that indicated a fixed mindset, traditional views about mathematics, or an absolutist orientated philosophy, were assigned the lower end of the scale. Similarly, those that indicated a growth mindset, constructivist views about mathematics, or an fallibilist orientated philosophy, were by default assigned the higher end of the scale. To maintain consistency with this, the scale for mathematical anxiety was also reversed so that lower numbers were associated with high MA and higher numbers were positively associated with low MA.

ANOVA was deemed suitable for this study because it allowed PSTs' beliefs and MA (dependant variables) to be measured three times over the course of the study: at the beginning, middle and end of the intervention (independent variable) to determine if there were changes in PSTs' beliefs, whether this change was positive or negative, and whether it is statistically significant or not.

The null hypothesis, $H0$, is that there will be no statistically significant difference between the means at three different points across the intervention. The alternative hypothesis, $H1$, is there will be some statistically significant difference between the means. This is a directional hypothesis because it assumes the intervention will lead to a positive change in PSTs' beliefs and mathematical anxiety. The level of significance, α, was set at the 0.05 level and the SPSS statistical software package was used to perform the calculations.

There were 67 PSTs involved in the study, but to be included in ANOVA parametric testing PSTs must have completed all three questionnaires. In this case there were 37 PSTs (*n*=37) who completed all three. This reduction was due to the loss of 9 PSTs who were on Erasmus for the first survey, and the remaining numbers due to other PSTs being absent for any one of the three surveys.

The dependant variables and abbreviations used to represent them were as follows:

- Anxiety (anxiety_1, anxiety_2, anxiety_3)
- Beliefs about the nature of mathematics (bnom_1, bnom_2, bnom_3)
- Beliefs about learning mathematics blm_1, blm_2, blm_3
- Beliefs about mathematical achievement bma_1, bma_2, bma_3
- Beliefs about preparedness for teaching mathematics (bptm_1, bptm_2, bptm_3)
- Beliefs about program effectiveness (pe_1, pe_2, pe_3)

The numbers appended to the end of the abbreviations indicate the stage at which the variable was measured where 1, 2 and 3 represent the three different stages the questionnaires were administered in chronological order.

Statistical significance means that the effect is unlikely due to chance and likely because of the intervention, but it does not indicate the size of the effect of the intervention on the dependant variables (Cohen et al., 2018). Therefore, in cases where the alternative hypothesis ($H1$) was supported then it was decided to "proceed to tests of magnitude of difference" (Cohen, 2018, p.776), and therefore effect size was used to measure the magnitude of any changes that exist (Cohen et al., 2018). In addition to this the Bonferroni post-hoc test was applied to determine exactly where, or at what point, the differences lie.

3.8 Trustworthiness

Bryman (2004) recommends using the idea of trustworthiness in qualitative research. Trustworthiness is determined by credibility, dependability, and confirmability. Each of these is discussed below in the context of this study. Credibility and dependability are discussed together. Confirmability, related to objectivity, is discussed in section on its own because of the complex role of objectivity in AR. Also addressed below is the related idea of making validity claims in AR. In this part, I will address the knowledge generation process and its transformation from embodied to explicit knowledge, and how my validity claims around this knowledge can be legitimized by the external world of researchers (McNiff at al., 2005).

3.8.1 Credibility and dependability

Credibility ensures congruence between concepts and observations (Bryman, 2004), and involved ensuring my interpretations of PSTs' responses and feedback was accurate. This has added importance in AR given the epistemological position of this study that the PST is regarded as a legitimate knower. Observations, for example, were followed up with unstructured conversations to clarify my interpretations with PSTs' voices. Clarification around meaning was always sought for during focus groups, and other interactions with PSTs. In a general sense, credibility was ensured by striving to be honest, open, and transparent with PSTs at all stages in the study.

Dependability, also known as reliability, sought to ensure that the methods capture what was supposed to be measured (Bryman, 2004). Sullivan et al. (2016) describe it as a test for accuracy of findings. To ensure this, complete records of all phases of the research were fully maintained and dated. These were presented to, and discussed with my supervisor, to confirm that I was following procedures appropriately. My supervisor scrutinised the accuracy and reliability of my research design and methods and provided comments and critical feedback throughout the process (Cohen et al., 2018).

3.8.2 Confirmability

Confirmability is concerned with remaining objective as a researcher and not allowing personal values or bias sway the findings of the research (Bryman, 2004). Objectivity is an important guiding concept in inquiry and is something that is traditionally achieved by distancing the researcher from the research process and from any interaction with the research object (Lindhult, 2019). It requires being impartial and free from bias to the extent that trustworthiness is not compromised. However, Lindhult (2019) notes that distancing oneself from the research, in particular participatory and action research, results in a limited view on the use of meaning and concept. It assumes "a spectator view of knowing and a positivistically influenced understanding of inquiry" (p.21). Similarly, Takacs (2003, p.27) notes that "few things are more difficult than to see outside the bounds of your own perspective". To take such a positivistic approach is inconsistent with the underlying philosophy of AR. This brings into question the subjective-objective debate where objectivity is typically associated with truth and trustworthiness, whereas subjectivity, on the other hand, is associated with unreliability, bias, and potentially error. Lindhult (2019) argues that fully objective knowledge is unattainable because we necessarily sense the world through basic categories of the mind. Paradoxically, actors and participants in AR can, through their subjective influence, enhance the objectivity of the study. However, Lindhult cautions that knowledge production is "based on relevant competencies and capacities to maintain norms of good inquiry by inquiring actors and their communities" (2019, p.24). At a minimum, clarification of ontological and epistemological assumptions, and values should be outlined so that they can enhance research.

These assumptions are pertinent for AR as they are by definition values laden and are concerned with living and working in line with these values. This has serious implications for issues of justification and validation (McNiff, 2005, p.16). My values and assumptions were outlined in the opening section to this dissertation, and throughout the process I have scrutinised my research and practice using my values as a means of educational and moral judgment. These values and the validation process play a significant role in my knowledge claims in the remaining chapters. These knowledge claims will be evaluated against the literature, but also in relation to a set of criteria that were informed by my professional values and assumptions. These criteria, referred to by McNiff et al (2005) as "new scholarship criteria", include my professional learning in relation to my practice, and the extent to which my values were lived out over the course of this study. I ensured the validity of my claims by being transparent about them at all stages throughout the study, particularly regarding assumptions and contradictions about them. In addition to this, I ensured validity of my claims for colleagues, critical friends, and other researchers through conference presentations, and presentations for professional learning. Consistent with my theoretical position, open dialogue and criticality were always encouraged and welcomed during these public claims to knowledge.

However, as recommended by Lindhult (2019) I was guided by Bacon's (1960) idols which help researchers to reduce the influence of factors that may undermine good practices of inquiry. Bacon (1960) called these four idols the idol of the tribe, the idol of the cave, the idol of the forum, and the idol of the theatre. The idol of the tribe refers to knowledge resistance whereby researchers believe to be true what they wish to be true, despite evidence that suggests otherwise. This is more commonly known as confirmation bias. The idol of the cave refers to the problem of working within an echo chamber where one-sided, and perhaps even false information, is reproduced and this is particularly relevant in an age of social media. The idol of the forum is, according to Bacon, the most problematic. This is the notion that narratives, words and meanings can be "twisted" by outside actors to suit their personal interests. The idol of the theatre refers to passed down wisdom, accepted uncritically, which can act as a barrier to intellectual advancement.

Bacon (1960) does not argue that these idols should be, or even can be, eradicated. Instead, he recommends that inquirers develop an acute and critical awareness of them so their impact can be minimised. This is called critical subjectivity whereby objectivity, based on developed forms of subjectivity, is used to maintain good forms of inquiry. There is a moral dimension to this form of objective inquiry as fair, impartial and unbiased. To address the necessary subjectivity in AR, more subjects (in this case, PSTs) should be included in the inquiry (Lindhult, 2019) because they are "experts on their own subjective experience" (Grover, 2004, p.91). Objectivity is therefore developed through a process of inter-subjectivity which evolves from the collective agreement between participants. This broad collaboration is one of the strengths of this AR study because it included the entire cohort of PSTs who were collectively encouraged to voice their ideas and critique the intervention and their ITE experiences in a supportive and open environment. Furthermore, when contributed to knowledge generation, this new knowledge was shared with, experienced by, and critiques by the other PSTs. This allowed for inclusive and meaningful collaboration of PSTs with me, together creating new knowledge for the advancement of teacher education, and by extension society and the common good.

3.8.3 Triangulation

Triangulation, the use of more than one method or data source, was a methodological cornerstone of this study. According to Taylor (2004), it gives a study "the ability to enhance the trustworthiness of an analysis by a fuller, more rounded account, reducing bias, and compensating for the weakness of one method through the strength of another" (p.43). From its inception, this study was about understanding PSTs' perspectives and it was therefore important to ensure their collective voices were heard. This collective voice was accounted for in several different ways using different data collection tools, and in the case of the questionnaire data, different paradigmatic modes of analysis. This allowed me to capture the complexity and richness of PSTs' behaviours by considering it from multiple perspectives within and across research strategies, as well as providing a more balanced picture of the research (Sullivan et al., 2016, p.82). The triangulation dynamic was particularly useful in reconciling apparent inconsistencies between PSTs' espoused beliefs and pedagogical aspirations which were captured by the qualitative and quantitative parts of the surveys, versus a sometimes-different observable classroom practices, which was captured by the classroom observations in Cycle 1. Although it was not possible to directly observe classroom practice in Cycle 2, a range of planning documentation and instructional videos were used to gain some insight into PSTs intentions for teaching mathematics. These data sources were analysed individually, and then considered in the context of survey data. Data collected from the focus groups in Cycle 1 and Cycle 2 were also used to additional criticality to the analysis of the other data sources.

In Cycle 1, data collected from post lesson discussions was used to make further sense of the observations that preceded them. These were used, for example, to explain PSTs' actions in the classroom, justify their used of approaches to teaching, and determine if they were aware of their approaches to teaching mathematics.

The study also considered my voice as researcher and teacher educator, with my own set of values, through ongoing reflection using Brookfield's Lenses as a guide. This process allowed me to recognise, and accept differences between my educational values and assumptions, and those of the PSTs in the study. In addition to this, critical friends were used, particularly my supervisor and some colleagues, to challenge my educational assumptions and give alternative perspectives on my practice. As well as enhancing overall trustworthiness, the triangulation process ensured a level of concurrent validity by using multiple sources and diverse kinds of evidence to address the research questions (Cohen et al., 2018, p.381).

3.9 Conclusion

This chapter presented a refined set of research questions, the motivations for which were presented in the opening chapter. The literature review was used to refine the questions while also providing direction for the research design and intervention presented in this chapter. The research was designed to best address the research questions in a way that aligned with my theoretical position, as was the intervention design. In both cases, PSTs are active participants in teaching, learning, knowledge generation, and other decision-making processes. PSTs' experiences in this context are rooted in democracy such that socialisation, communication, experimentation, and negotiation were practiced (Dewey, 1916). Freire (1970) argued this freedom to engage critically and creatively with their experiences using authentic action and reflection, will allow them to become more fully human. Action research was chosen because it is fully compatible with this theoretical position, while the opportunity to critically reflect on and improve my practice ensured I was researching and teaching in correspondence with my values.

The next two chapters, chapters 4 and 5, presents the qualitative results from Cycle 1 and Cycle 2 of this study respectively, while chapter 6 presents the results from the quantitative parts of the study.

Chapter 4: Cycle 1 Analysis

4.1 Introduction

This section will present the analysis of the data from Cycle 1 of the study, starting with focus groups FG1a and FG1b. This is followed by the results of classroom observations in Section 4.2 including post-lesson discussions with PSTs. As the Mathematical Quality of Instruction (MQI) framework (Learning Mathematics for Teaching, 2011) was used to guide these observations, a discussion piece is presented in Section 4.3.1 which attempts to characterise lessons where high levels of MQI were displayed versus those lessons where lower levels of MQI were noted. Finally, in Section 4.5, these results are consolidated through the lens of my reflective practice to develop reflexivity in my practice and devise an action plan for Cycle 2. I have also included several reflections, presented distinctly from the analysis, to add content for the reader and give insights into my thinking as a practitioner.

As a reminder for the reader, the research questions which were presented previously in chapter 3, are as follows:

1. What experiences contribute to the problem of MCK enactment for PSTs?
2. What is the optimal design for a pedagogy of enactment, in the context of mathematics education, to reduce the problem of enactment for my PSTs?
3. Can the intervention cause any change in PSTs' beliefs about mathematics and their levels of mathematical anxiety?
4. Can the intervention cause any change in PSTs' practice of teaching mathematics to become more democratic?

4.2 Focus Groups

Braun and Clarkes (2006) thematic analysis process as described in Section 3.7.1 was used to generate five broad themes from FG1. These are as follows:

- Participants experiences and motivations
- Froebel mathematics modules
- Barriers to enactment

- Child-centred teaching and Froebelian philosophy
- Neoliberal influence on mathematics pedagogy

Data from both focus groups carried out in Cycle 1, FG1a and FG1b, were analysed using Braun and Clark's (2006) thematic analysis to search for meaning in the data, which resulted in the generation of the themes listed above. In the following sections, each theme is further categorised into sub-themes and presented through the lens of the research questions. The first four themes were generated at the semantic level whereby meaning was interpreted explicitly. The final theme, that considers the neoliberal influence on mathematics education, was generated at a latent level by examination of the previously generated four themes through which an underlying neoliberal influence was identified. As noted in Section 2.5.4 of the Literature Review, this neoliberal influence has a demonstratable negative impact on democratic educational practices and is therefore a key consideration for the study.

There are some conventions used in the presentation of the themes that follow. Firstly, PSTs' responses will be referenced using the focus group they were part of and the line number from which their response is in the transcript of the focus group. For example, if a quotation was taken from Mary on line 52 of focus group b this would be referenced as (Mary, fg1b52). All PSTs' quotations are italicised. Secondly, some PSTs referred to maths methods as MSE (Mathematics and Scientific Enquiry) because this is the official name used for the module. MSE contains both science and mathematics elements and as such contains maths methods within it. To maintain the integrity of the transcripts and the voice of the PST, whatever name was used was kept in the transcript and used in quotes where necessary. Finally, where PSTs are quoted directly, these quotes will be italicised for clarity. It is important also to understand that contributions from PSTs, whether direct quotes or paraphrased by me,

represent how PSTs interpreted reality at the time. For example, on page 129 of this section Derek rephrased what he believed a school principal said, and Paul described his interpretation of what the principal meant by his words. This epistemological position is important because this study is about PSTs' interpretations of reality and how this influences their practice as novice teachers.

4.2.1 Theme 1: Participants experiences and motivations

4.2.1.1 Instrumental teaching and learning

PSTs' past experiences of school mathematics was, in some cases, described as a negative one which was underpinned by instrumental understanding, with little criticality involved in their learning of the subject. This was reported explicitly seven of the PSTs in FG1 and FG2: Claire, Paul, Vicky, Sharon, Gillian, Jenny, and Aoife.

For example, when discussing the need for relational understanding Claire claimed that this is not a feature of current classroom practice and that has not changed from when she was in school. She recalled: "*I remember in primary school it was the same, they just call out the answers to the homework. Where it's not, oh, did anyone like think about why it happened*" (fg1a34). The same PST had a similar experience in secondary school, particularly when preparing for Leaving Certificate[4] mathematics (fg1a78). These experiences created a mindset focused on rote learning of rules and procedures whereby "*you weren't told why but you knew you had to know it to pass your leaving cert*", and this "*need to pass*" attitude clearly transferred into ITE and the maths competency module. Both Paul (fg1a6), Vicky (fg1a8) and Mary (fg1a4) had similar experiences of school mathematics which clearly impacted on their relationship with mathematics and how they teach the subject. In relation to how the instrumental nature of school mathematics impacts on her current teaching, Mary reported "*it was just taught to us and we have it in our heads now but we don't know why we do it that way*" (fg1a4). Additionally, Vicky acknowledged that when she first started her teacher education programme, she would have taught mathematics instrumentally by default. More recently, the relational nature of the maths competency modules has made her question this approach:

> *"Because when I came in, I would have just, like the same as Mary, I would have been 'this is the way teacher said, that's the way we do it'. Well now I kind of have an idea of why we're doing it. So I can actually go out into my class and be like "this is why we are doing it*" (fg1b8).

Paul, who is self-professed "*good at maths*" is confident in his mathematical ability and enjoyed mathematics in school also agreed his experiences of school mathematics were

[4] The Leaving Certificate Examination, commonly referred to as the Leaving Cert, is the final exam of the Irish secondary school system and the university matriculation examination in Ireland.

primarily instrumental (fg1a14). However, he did not mind because his answers were usually correct, and this was his "*gratification*".

When asked about her experience of school mathematics, Claire talked about the nature of secondary school mathematics, and explained that this involved almost exclusively instrumental mathematics. This approach has compromised Claire's ability to learn mathematics for relational understanding in ITE. She explained:

> "I think it kind of comes as well from the leaving cert. Like it's the same thing that you were kind of just taught something. You weren't told why but you knew you had to know it to pass your leaving cert. Like, I think that's the same way I came in in first year, the same thing for maths competency. Oh, I just need to pass. The same thing as the leaving cert just learn it off. So I think that's the mentality, the attitude in secondary school".

The instrumental approach to mathematics teaching in secondary school reported by the PSTs has potentially compromised their disposition to learn mathematics relationally in ITE, and more importantly to enact this knowledge in the classroom.

4.2.1.2 Affective concerns

Sharon explained that she has negative feelings about teaching mathematics, and specifically talked about her current anxieties about teaching division. She also explained that this anxiety is rooted in her past experiences of learning mathematics in school:

> *"I was terrified to go near division. Absolutely terrified when the teacher gave me the topic. But I think it all goes back, not to necessarily what we did in college, but secondary school. So I just had a really bad teacher in secondary school who told us we were all going to fail our leaving certs and am would basically argue with me if I had a right answer telling me it was wrong and different things even though it as right. So I still questioning myself all the time. So I was like I don't know if I can teach division. I don't know if I know it"* (fg1a56).

As reported by Sharon, the root of her anxiety and lack of confidence stems from her secondary school experiences, and as a result she now questions her own mathematical ability and her ability to teach mathematics. This is captured in the following quote from Sharon: "*you know it but you feel like maybe you just don't know and I feel like it's all going back to* *secondary school where confidence…*". Sharon was attempting to articulate that her confidence was low in secondary school because one of her mathematics teachers was overly critical about her ability to do mathematical work.

Similarly, Gillian explained how she struggled with financial mathematics, ever since being introduced to it in primary school and highlighted that she "*wouldn't feel comfortable teaching it*" (fg1a62). Jenny, who likes mathematics, never really questioned the subject as a learner but as a teacher she admits that the instrumental approach to mathematics causes "*panic*" when a child asks for an explanation. Paul

Aoife explained that her experiences of how mathematics is assessed, and the accountability related to this, has contributed to her negative feelings about mathematics as a PST. She referenced standardised testing in primary school, entrance examinations for secondary school, followed by high stake state examinations in secondary school. She also pointed out that this continued into ITE with the 70% pass threshold for the mathematics competency examinations. In relation to this she said: "*I just feel like there are so many barriers, that just, it's hard to be like yeah I can do this it's ok*" (fg1a65).

It is clear from the data presented in this section that negative feelings about mathematics, and feelings of anxiety in relation to how they perceive and engage with mathematics generally (i.e., not necessarily in the classroom) are problematic for some PSTs. This is not entirely surprising as it was pointed out in Chapter 1 that PSTs often carry with them psychological 'baggage' such as anxiety and unhelpful beliefs. In was also pointed out in Section 2.4.4 of the Literature Review that these sorts of feelings can be problematic because they often lead to underperformance in, and avoidance of, mathematics related tasks (Ashcroft, 2002; Maloney and Beilock, 2012). This is of concern because it may result in PSTs disengaging from maths competency which will have corresponding negative consequences for their MCK and subsequent classroom performances. Furthermore, these feelings are also associated with a lack of confidence in relation to mathematical ability, which consequently often results in PSTs relying on instrumental mathematics in the classroom (Gresham, 2018), and this is clearly at odds with the goal of the maths competency module. It is hoped that the intervention as described in Section 3.4, will be implemented in a way that will reduce PSTs' negative feelings about mathematics and allow them to engage in the meaningful learning of content and pedagogy.

4.2.1.3 Pupils motivate PSTs to learn mathematics

Analysis of the focus groups in Cycle 1 revealed the key to PSTs' motivation to engage meaningfully with mathematics is the welfare of the pupils they teach. In other words, they are not intrinsically motivated by a desire to learn mathematics per se, rather they appear to be more motivated by improving the educational experiences of the pupils they teach. Throughout conversations in both focus groups there were several references, both explicit and implicit, to Froebelian Philosophies of Education, (as outlined in chapter 1) and child-centredness. Perhaps this is not surprising given that Froebel PSTs are encouraged to use a child centred Froebelian approach in their teaching. Furthermore, with reference to these occurrences, it is difficult to separate the Froebelian philosophy from child-centredness because the Froebelian philosophy is essentially a child-centred one. For example, PSTs in both focus groups discussed content being relevant to children's lives, catering for diverse types of learners, helping the weaker children, thinking like children (when modelling content), and the impact of their attitudes on children's engagement. Whereas all of these can be considered Froebelian, it is difficult to determine if PSTs were explicitly and deliberately enacting the Froebelian philosophy in their teaching.

Mary addressed the idea of child-centeredness and its impact on her learning of mathematics. She argued the original maths competency module was about passing an exam and "*for our own competency*", but with the introduction of the intervention it has become more about helping pupils learn mathematics. This finding supports the theory that PSTs' learning is socially constructed within a community of practice and confirms also that the traditional cognitive model (i.e., the original maths competency module) is a less successful mode of teacher learning (Wenger, 1998). It also supports Philipp's (2008) contention that PSTs MCK, and corresponding beliefs, is best improved when earning is framed around the overall learning needs of pupils, rather than mathematics per se being the primary consideration. In this regard the exam has become secondary consideration, with pupils learning being the primary motivator (fg1a132). Vicky shared this view. She noted that with the intervention it was "*really being reinforced to us that it's to help the children*" and "this year I am not really taking it as exam based, I am kind of taking it as more of a "*yeah I know I have an exam at the end but*..." (fg1a93). This focus evidently motivates PSTs to engage in maths competency in a more meaningful way.

4.2.2 Theme 2: Froebel Maths Modules

This theme is particularly important in addressing the aims of this study because it represents PSTs' critique of mathematics instruction in the Froebel Department, including both maths competency and maths methods, as well as the relationship between them. It also provides critical information to help shape the teaching intervention for Cycle 2 of this study. The main findings relating to this theme are outlined under the following sub-themes.

4.2.2.1 Role of the original maths competency module

There was a general agreement amongst PSTs that maths competency, and its focus on relational understanding, helped to improve their mathematical content knowledge and somewhat address the damaging effects of their instrumentally based school experiences.

> *"I suppose for breaking down, as you always say, you know, how multiplication or division of fractions - how that works. But we do not know why we dot it a certain way; it was just taught to us and we have it in our heads now, but we do not know why we do it that way. Like why do you invert and multiply, all that kind of thing. We did all that. You show us why do we do that. So, I suppose when we are teaching that to the children it makes it more obvious why you have to do it a certain way and break things down more for the children. You can't just go in and say oh we'll just invert and multiply to the children"* (Mary, fg1a4).

Derek, who claims to struggle with learning and teaching mathematics is now "*able to approach a problem with the class in a more meaningful way*" because it has increased his "*confidence to explore different avenues and different answers to get an answer rather than just saying that's the procedure that's it*" (fg1a10). For Vicky, "*it makes it clearer*" and gives the confidence to say why things are true in maths. She said she can:

> "*actually go out into my class and be like 'this is why we are doing it' instead of just saying, because I would have said 'oh ya that is just what you do', like I would have said that myself whereas now we've gone right through it I now know not to do that but it was never done to me so I would have just done it that way*" (Vicky, fg1b8).

Paul noted that the original maths competency module "*moves from when I was taught, of being like, you do that because the teacher says you do it and that's the reason to...the reason* *behind it is like*" (fg1a6). Aoife emphasised the importance of maths competency in being able to "*back up what you're saying to them*".

There was also some agreement that pupils would benefit from relational understanding. For example, Vicky agreed that "*some children do need the actual reasoning behind it...*" (fg1a12), while Mary responded with her belief that "a*ll children need the reasoning behind it. I think it is easier to understand when they know how...why it is a certain way, or how it works*" (fg1a13).

It is very clear the PSTs in the focus groups do at least acknowledge and appreciate the importance of relational understanding and, despite being taught instrumentally in secondary school, they at least expressed a *desire* to teach mathematics relationally.

However, despite the sort of confidence and competence PSTs report from engaging with the original maths competency module, there is little evidence, based on my professional knowledge, to suggest they effectively used their MCK to teach mathematics relationally pre-intervention. In fact, it was evidence to the contrary, as outlined in Section 1.3 on my reconnaissance phase, that provided the initial motivation for this study.

There are some possible reasons that might explain this apparent contradiction. Firstly, although these participants were asked about the original maths competency modules, it is possible their responses were influenced by the intervention itself in favour of more positive responses, because at the time of the focus group, they have already experienced the intervention in Cycle 1. Secondly, PSTs may have experienced a dissonance in terms of how they believe mathematics should be taught versus how they actually teach mathematics in practice. This could be explained by the research on apparent inconsistencies between PSTs' espoused beliefs and their actual classroom practice, which may be influences more by context than by the beliefs they hold about mathematics (Raymond, 1997; Hoyles, 1992; Skott, 2009; and Sztajin, 2003).It is also possible, particularly pre intervention, they did not know what relational mathematics looked like in practice due to a misinformed frame of reference (Lortie, 1976). As a result, PSTs would have lacked the teaching skills necessary to enact such mathematical instruction.

4.2.2.2 Maths competency lacked pedagogical knowledge

Despite reporting an appreciation for its focus on relational understanding, PSTs also recognised that the MCK focus of the original maths competency module lacked the pedagogical input required to support them in the enactment of that knowledge. For example, Mary said: "*actually going in teaching it is a different thing altogether*", and highlighted the need for knowledge around methodologies, phrasing, materials, and generally knowledge of a pedagogical nature (fg1a132). Adding to this, Paul commented that it lacked the collaborative aspect necessary for professional learning of mathematics by referring to the lack of meaningful discussion in lectures and how PSTs "*just sat there in rows*" and watched the lecturer "*do maths for an hour*" (fg1a110). Paul explained how he would like to see lecturer's model what is also expected of PSTs in the classroom and agreed that such a collaborative approach, involving meaningful multidirectional discussion between PSTs and lecturer, is necessary because "*that's how it should be in the classroom as well*" (fg1a113). He explained he would like to see lecturer's model what is also expected of them.

There are consequences of PSTs' perceptions that the original maths competency module is for their own personal competence, and not something that should be integrated into classroom practice. Gillian shared her perspective on what it feels like to learn mathematics for teaching in the original maths competency module:

> *"but when you're doing it kind of feels separate to teaching. It does not feel like you can really link it in. I do not know if that was just me. It felt very separate from, sort of, classes where you learn about actually teaching stuff to children. Sort of, you were learning the stuff. It was not to pass it on. And it just…you were competent yourself. So it felt quite separate to the actual teaching part"* (fg1a3).

In her teaching, Jessica reported difficulty establishing links between topics designated to them on SP and those taught in maths competency (fg1a9), while Brona agreed that maths competency, maths methods and their practice were notably distinct in years 1 and 2 (fg1a141).

These findings are indicative of the traditional cognitive model of teacher education whereby the university provides the knowledge, the school provides the placement setting, and the PST provides the individual effort to assimilate and apply this knowledge in the practice setting (Wideen, Mayer-Smith & Moon, 1998). They also indicate the problem of separation between foundation modules on one hand (in this case maths competency), and methods modules (i.e., maths methods), which results in a fragmentation of ITE. Grossman (2008) describes this model as a reductionist process-product conception of teacher education, which ultimately ignores the inherent complexity and theoretical underpinnings of teaching and teacher education.

4.2.2.3 Connections between competency and methods

One of the main challenges for PSTs to enact their MCK in practice was the lack of coherency and connectedness between the maths competency and maths methods modules. This is essentially a theory-practice divide within the ITE programme. For example, regarding maths methods, Paul said:

> "*we don't necessarily look at specific things that is in any way related to anything we're doing in competency, it's more generic maths things or games like. They can be effective at certain levels like, but they don't necessarily link to anything that you're doing*" (fg1a125).

This lack of consistency and connectedness within and between the two mathematics modules has resulted in barriers to PST learning. This resulted in PSTs feeling disempowered to bridge the subject specific gap between their mathematics content knowledge and their general pedagogical knowledge, particularly for teaching senior (5^{th} and 6^{th}) classes. For example, Tony explained that most PSTs in his cohort understand the relevant mathematical concepts necessary to teach the primary school mathematics curriculum but, because of the internal disconnect between modules, they lack the pedagogical content knowledge to convey these concepts to pupils. Regarding what is learned in maths competency, Tony does not agree that "*MSE is teaching…us the methodologies to put that across*" (fg1a59).

Paul explained how the dominant play-based approach of maths methods is inconsistent with the more abstract nature of maths competency, and expressed his frustration at being competent in mathematics yet struggles to transfer this competency to the classroom. Although he understands the mathematics on a conceptual level, he "*wouldn't necessarily know how to teach it or how to pitch it at a certain level or present it*" (fg1a81). Tony argued that the fundamentally different approaches to learning between both modules resulted in difficulties making connections, and this makes enactment difficult (fg1a61-64). Vicky (fg1a85) made the same argument referring to the overreliance on games without adequate conceptual underpinning and this causes her to lose interest in approaches to enacting her mathematical knowledge. Mary agreed that she knows fractions "*inside out*" but does not have the teaching methodologies to get this content across to pupils (fg1a132). In particular, she struggled with appropriate use of phrasing and wording for children. Gillian (fg1a185) agreed with Mary and explained that one of her difficulties was to use her mathematical knowledge in a way that was comprehensible for pupils. In essence, PSTs were looking for a way to translate their mathematical knowledge into the practice setting but are constrained and frustrated by the lack of meaningful connections between the two mathematics modules.

4.2.2.4 A focus on the 'Intervention'

It is clear from the PSTs' responses that the intervention, with a focus on pedagogies of enactment, has benefited the PSTs and addressed some of the issues outlined in the previous section. It has helped them in two ways specifically:

i. their preparation for SP

ii. enacting mathematical knowledge during SP

These subthemes are discussed below. `

Preparation for SP

PSTs' concerns about their lack of PCK were somewhat addressed by their engagement with the intervention. Previously, PSTs learned content from maths competency and learned about methodology from maths methods, but links between the two were weak and this compromised their development. It seems that the intervention, particularly the use of representations, decompositions, and approximations, succeeded in creating strong links between content and pedagogy. This approach to learning mathematics for teaching encouraged PSTs to think deeply about their teaching, and importantly gave them useful frameworks to structure their approaches (i.e., HLTPs, MQI and deliberate reflection) (see fg1a74). For example, Tony stated that his PCK improved mainly from approximating practice with pupils who came in as part of the intervention (fg1a70). He said:

> "*because we actually had to think about it. And we'd done all the things like discussion, we'd done modelling, we'd done questioning in here... It was all stuff we did in lectures here*" (fg1b74).

Tony articulated that the intervention allowed him to see mathematics content in the context of representations and approximations in lectures. When the pupils visited, he was then able to take what he had previously learned and enact it in a way that closer approximated his practice in an authentic situation.

Sharon also reported significant benefits from approximating her practice with visiting pupils because it allowed her and her group to "*see what their base level of understanding might be before you go into a class because you get no chance when you're on observation to go and teach a small bit just to see what a 6th class might be like*" (fg1a105).

Approximations of practice with pupils addressed the general PST concern about how to pitch a topic to them (fg1a185) while giving PSTs the opportunity to enact their learning in practical situations (fg1a103). That is, it enables PSTs to plan in a way that allows them to transfer their MCK to the classroom.

For example, Gillian explained:

> "*yeah, how to take that knowledge and phrase it for children. Because obviously we'd have a higher level of understanding than they would so how do you bring it back down to what they know? And how to explain it to them? Because obviously if you're explaining it to us it's going to be at a slightly different level, you're going to be pitching it differently than to a 5th class child. So I think it would be helpful to know how to pitch it to that age*" (fg1a185).

In the minds of PSTs, the original maths competency modules were about passing an exam, but the intervention changed this perception and reorientated their MCK to the needs of the pupils they teach. Mary stated:

> "*Even things like using the area models. I would never have known that kind of thing, you know to multiply fractions, to use the area model, things like that. As you said we've just been told "do it this [way] and that it there's a rule and that's it you follow*

> *the rule". But with the area model you can kind of see more clearly why you do that kind of thing*". (Fg1a9).

Underlying all of this was a safe and collaborative learning environment where mistakes were learned from, and risk was a welcomed as opportunity. Derek noted how the feedback roles given to PSTs resulted in deep and meaningful dialogue "*because we were using what we had learned and looking at each other's teaching, and what was good, what was bad*". This informed the participants teaching "*so much more than the last two years of the plain competency*" (fg1b99). Tony summarised the philosophy around making mistakes: "*you said don't worry about making mistakes, make them here and then you can fix them in the classroom so you don't make the same mistakes in the classroom you get to practice*" (fg1a106).

Working collaboratively with other PSTs allowed participants to explore mathematical and pedagogical ideas as a group which was effective for planning lessons and generating and exploring new ideas. For example, Mary referred to the idea that the specific representation and decomposition for modelling content (in this case the area model for fraction multiplication) helped simultaneously improve her mathematical knowledge and pedagogical skills:

> *"Even things like using the area models. I would have never have known that kind of thing, you know to multiply fractions, to use the area model, things like that. As you said we've just been told "do it this [way] and that it there's a rule and that's it you follow the rule". But with the area model you can kind of see more clearly why you do that kind of thing"* (fg1b9).

Derek also reported benefiting from the use of representations of practice during lectures. Regarding the use of videos to analyse practice he said the benefits have "*stayed with me since*" and positively influenced preparation for SP (fg1a99).

As a mathematics teacher educator and researcher, I found these results very interesting. Whereas the approach taken with then intervention seemed to enable PSTs to see the relevance of the mathematics they were learning, the original maths competency module did not. My reflection on this is captured in the following vignette:

Vignette

I find it interesting that PSTs reported an improvement in their MCK because of the intervention. This same cohort of PSTs were taught the content covered by these representations to address their content knowledge in years 1 and 2 of the maths competency modules, yet their reported learning is as a result of the representations and approximations enacted as part of the intervention. Evidently, the decontextualised learning from the pre intervention maths competency does not have the same relevance as that which happened as part of the intervention. Clearly, the context in which PSTs learn to teach maths makes a difference. This sort of contextualised mathematical learning, made possible by approximations of practice, combined with the opportunity to work with actual pupils allowed PSTs to get a sense of "what a 6th class is like" (fg1a109).

Enacting mathematical knowledge during SP

Following on from the lectures and workshops as part of the intervention, there was a consensus amongst PSTs that the intervention had a positive impact on their classroom practice, and they provided numerous examples to support this. Some participants took specific instances of the intervention and adapted them for enactment in the classroom, while others enacted broader, more general, ideas learned as part of the intervention.

An example of a specific instance of enactment is the area model for fraction multiplication. This was a representation of practice used as part of the intervention to teach PSTs about the HLTP of modelling mathematical content and it was hoped that by examining its decomposition that PSTs would learn to enact the more general practice of modelling content for pupils.

Helen enacted this specific representation during SP. She recalled:

> "*Even with the area model and things like that…I found showed some of the children who were struggling to do the multiplication*" (fg1a13).

While this was a very specific example of using modelling, Aoife used the same HLTP more generally:

> "*the whole way through placement. I think probably the way I was using it for was just the explanation and I think modelling really helped because when you're thinking out loud, you're kind of asking yourself the same questions children are asking. I found that really useful*" (fg1a93).

Similarly, Claire reported using the HLTP of 'leading a discussion' in her teaching. To enact this, she gave her pupils the stimulus mathematical statement that "*a square is a type of rectangle*" and used discussion to allow the pupils to debate the validity of this statement. Claire used this to try to "*get them to think a bit more*" (fg1a97). Mary also agreed this HLTP is useful for their practice, particularly for engaging children in problem solving activities. She described how she used this discussion one day per week for such activities (fg1a107). Claire also referred to how using video, as a means of representing practice, is generally useful for PSTs. She reported that "*if I'm ever going to teach that topic I can look back on that video and have an idea of, not how to teach it exactly, but you have an idea of like what good teaching looks like*" (fg1a180). The other participants in FG1a agreed with this.

Although not its intended use, most participants reported that the post lesson discussion (which was framed as supportive, collaborative, reflective and evaluative) was beneficial.

Tony struggled to teach the concept of fraction multiplication and this difficulty formed the basis of a discussion after a lesson observation. He described how this discussion about the use of the number line, not only later helped the children to understand the concept, but also how it helped clarify his own understanding of the concept of fraction multiplication. He described this as a change in mindset and admitted that "I would have definitely just followed procedures" in the past (fg1a18).

As part of the intervention PSTs were given access to and encouraged to use Kersaints' (2015) '100 questions to promote mathematical discourse' as part of their teaching. Only one participant, Vicky, appeared to use this resource intentionally as part of her SP teaching. She emphasised that it was a very powerful tool for teaching and learning because it allowed her to pose meaningful questions. She also noted that having access to this resource allowed her to engage dynamically with pupils which was reassuring and enhanced her sense of preparedness.

The flipped classroom

One of the key features of the intervention PSTs found useful were instructional videos for learning mathematical content. They were essentially separate from the intervention and represented a new mode of engaging PSTs with the same content contained in the original maths competency modules. All PSTs in the focus groups agreed these were very useful for learning new content, even more so than with face-to-face teaching and learning. For example, Aoife said:

> "*they were really helpful as well, because the things you don't get in class you have something to refer to at home so there's a backup there*" (fg1a89).

Paul also found the videos useful and described how being able to watch a video at home allows the student to focus just on the content and not have to worry about taking down notes (fg1a98). Jessica explained that she can often be confused after lectures but now this can be addressed by watching videos "*a few times*" (fg1a7).

Referring to the original maths competency module, Derek felt the only discussion afforded to PSTs was to comment on whether another PST got the correct answer or not, and perhaps explain why. As noted earlier, he explained that PSTs "*just watched you do maths for an hour whereas now we do some maths and we discuss it*" (fg1a110). The flipped classroom model addressed this problem because PSTs come into lectures with the necessary mathematical knowledge to engage in more meaningful and practice orientated discussion. Importantly, PSTs are also more likely to ask for clarification on concepts and procedures because they have enough base knowledge to recognise misunderstandings (fg2115). Similarly, Jenny explained that, with the original model, PSTs would focus primarily on taking notes, and were therefore less likely to ask questions if they did not understand some concept.

Overall PSTs felt the flipped classroom model afforded opportunities for more meaningful engagement with lecture content. Rather than engaging in the dual process of taking notes and listening to lecture content, PSTs can now be fully present in lectures, and this results in more meaningful learning. Importantly, the model also gives PSTs freedom of time and space because they can watch the content videos at a time, place, and pace that suits their individual needs. Misconceptions can be easily addressed by watching parts of videos multiple times or simply pausing videos to take personal notes.

Even though the flipped classroom model potentially adds to PSTs' workloads outside of lecture time, it appeared the net benefit was positive. Furthermore, there were no direct criticisms about the model from PSTs in relation to this. Perhaps this is because the videos were quite concise, each lasting less than ten minutes in duration. Furthermore, I encouraged them to watch the videos, perhaps on their phones, at times that were convenient to them such as during commutes to college.

4.2.2.5 Improvements to the intervention

The intervention evolved over the course of Cycle 1 through ongoing reflection and subsequent implementation of changes. These changes were a result of the reflective process and critically engaging in meaningful dialogue with PSTs, colleagues, and critical friends. Changes to the intervention were also a result of data collected during the focus groups where PSTs were directly asked how they think the intervention could be improved. The following section will focus on this aspect of the Cycle.

Realistic approximations of practice

The first recommendation by PSTs related to more consistent work with pupils as part of the approximations of practice. For example, Paul noted how it would have been useful for the PSTs to have opportunities to work with the same pupils on subsequent approximations of practice so that they could have progressed their learning through action and reflection:

> "*It would have been nice to get that same group back again and progress just to see if you can change it with the same group. It all well and good for us to think about what the children will say or what they'll ask but it's actually better to get the children in a see what they actually have to see or what they actually think. You know because we're all like 20 or 21 and it's very hard for us to like a 10 or 11 year old and try to think of what they think*" (Paul, fg1b116).

Gemma explained that her group pitched the lesson too low for the visiting pupils and would have benefited from an opportunity to reflect on and refine their lesson for the same group on another occasion (fg1b117). Tony suggested maths methods could play a role here by combining both modules:

> "*Even if that, that thing with teaching children, even if that could link to MSE and MSE could go through do this with the four children that are coming in to you*" (Tony, fg1b121).

Gillian explained that working more closely with pupils in this way will help them to figure out "*what phrasing to use*" and "*how to word stuff in a way that makes it easier to explain to children*" (fg1a183).

Content knowledge

In Cycle 1, PSTs worked in groups and each group was assigned or chose a different content area to focus on. Although groups reported an improvement in MCK by focusing on one content area, this knowledge was not effectively shared amongst the entire cohort as effectively as it should have been. To address this, Gillian suggested each distinct group share their new knowledge in the form of lesson plans on Moodle, as this would provide a useful resource when planning for SP (fg1a114). Additionally, Brona suggested the possibility of sharing voice recordings of the approximations involving visiting pupils, while acknowledging the presence of children may present ethical issues (fg1a116). Gillian also suggested posting feedback forms about what went well, and what phrasing helped the most, so that others could learn from this.

Links to classroom teaching

The mathematics content covered during the intervention appeared to be relevant for PSTs only if it directly related to the content they are expected to teach on SP. This was the case, even when the content was part of a planned approximation of practice. Whereas PSTs explained unambiguously that the content in the original maths competency module was not considered relevant because it felt separate to SP, and so they regarded it as something that would not influence their teaching (e.g. fg1a3), it is not as straightforward with the MCK from the intervention. Helen explained that because fractions were covered as part of year 2 (original) maths competency, and because fractions were also used as specific representations during the intervention, she was able to make links to college work and SP because "*the content was the same*" (fg1a15-17). This suggests that there needs to be focused alignment between MCK and pedagogies of enactment, which is quite a challenge from a logistical and resource perspective.

Helen's view about learning fractions contrasts with the topic of number base that is not an explicit component of the primary school mathematics curriculum (NCCA, 1999). As it is not a topic and PSTs will be unlikely to be teaching it directly, it is difficult for them to link to SP. Gillian explained:

> "*Even when we were doing number base and stuff, I was kind of like, is this a topic in schools? Like how am I going to use this in a school? Whereas like with fractions is very obvious because fractions is a topic. But even like I know number base is really helpful and stuff but even if it was just kind of linked to a topic, even. Like when we're doing a topic to link it back to something on the curriculum. Like, oh this is…you could use this when you're teaching this or…"* (fg1a21).

PSTs also found it difficult to make links between their mathematical knowledge and the younger classes. For example, Sharon explained:

> "*I had 2d shapes as well and 3d shapes and a bit of division and I think it's only in 6th class that I started using things we did in competency because otherwise I just felt there wasn't much of a link between the topics I was teaching. like we were doing like the story of 9 or length or 2d shapes and I was trying to see the connection from what you were doing in class (lectures) to what you are teaching in the class. It's kind of hard then*" (Sharon, fg1a30).

Essentially, PSTs shared an expectation for explicit links to be made for them between content from maths competency and their classroom practice. Gillian said "*but even if it was just kind of linked to a topic, even. Like when we're doing a topic to link it back to something on the curriculum. Like, oh this is…you could use this when you're teaching this…*" (fg1a21). In other cases where lecture content and SP topics were not aligned, PSTs had to make these links themselves. Aoife reported:

> "*you actually have to put in work beforehand to prepare for the questions that might be asked and to have an answer that's valid*" (fg1a85).

There were other similar examples. Gillian said, "*I would have had to almost teach it to myself first*" (fg1a62) and Jessica noted that she did not have to upskill on multiplication because we did that in first year but "*otherwise 2d shapes I kind of learnt it myself I suppose*" (fg1a29). Similarly, Helen explained "*when I was doing fractions with 6th class I used a lot of knowledge myself to figure out why I was teaching what I was teaching. Like I found it easier to explain it because I knew the background knowledge myself*" (fg1a11).

There is evidence that, even with the intervention in place, the quality of mathematics teaching and learning suffers when specific content/ pedagogy links are not made explicit for PSTs. This is best explained with an example about Brona teaching percentages on SP. Because Brona's knowledge about fractions was weak, she took a purely instrumental approach to teaching the topic by "*just presenting it to the kids without actually giving them a reason for it*" (fg1a23). This is interesting because it suggests that, in this particular case, significant preparation regarding the pedagogy of mathematics teaching does not improve the quality of teaching when there is insufficient content knowledge.

Interestingly, Brona retrospectively made the link between content and practice when she returned to college after SP and explicitly learned about percentages via the flipped classroom approach. She said: "*now that we've come back and done a bit on percentages in the last two weeks my understanding is more clear now than probably when I was on placement…if I had done that before my teaching would have been better*". She retrospectively reflected on the quality of her mathematics teaching: "*And even multiplying by 100 over 1, I never actually thought about why we do that*" (fg1a23). When asked how the content-practice divide could be addressed, Tony suggested maths competency and maths methods be meaningfully coordinated around the goals of the intervention, so that the most practical elements of the intervention be enacted during maths methods (fg1a123). Specifically, those elements that most accurately approximate practice, such as teaching visiting pupils, could be carried out during the maths methods time slot. This would mean additional time could be spent in maths competency for studying content focused approximations and decompositions of practice, which could later be approximated in maths methods. Importantly, it would introduce a coherency between modules while bridging the content and pedagogy divide.

Open discussion

Derek suggested the idea of regular open discussion between students and lecturers to "*chat for a half an hour or an hour and talk about what's going well in maths competency or MSE and this is not going well*". He argued this would be useful because lecturers are "*trying to* *teach us certain things and we have to learn things off*" and so it would be "*good for us and for you because you'd know what's good and what's working or not working or if we have ideas we could suggest something else…*"(fg1a129). Derek also suggests that it is important that it is communicated to PSTs that they are not expected to know everything, in terms of maths content and pedagogy when they go out on SP (fg1a135).

4.2.3 Theme 3: Barriers to Enactment

The theme of barriers to enactment of mathematical knowledge in the classroom is arguably the most important of the five themes because it relates directly to the problem of enactment. The intervention was developed to support PSTs in the enactment of meaningful mathematics by helping them to develop a situational knowledge of mathematics teaching. However, implementing a teacher education pedagogy that supports this, in isolation does not guarantee enactment of relational mathematics. Consistent with the literature presented in Section 2.2.2, there are many other factors which can contribute to the problem of enactment. Therefore, it was not surprising that PSTs reported barriers to enactment which are not directly related to intervention, and these are discussed in the remainder of this section.

4.2.3.1 Challenges to teaching relational mathematics

Although PSTs agreed that pupils would benefit from understanding mathematics relationally (e.g., see fg1a13), it would appear that many of the pupils they taught resisted it. In general, PSTs reported that pupils were not used to learning mathematics relationally, and generally reacted uncomfortably to it. For example, Claire noticed in her class there was a culture of calling out the answers to questions without justification or explanation, and when she did ask pupils to explain their answers she noticed *"most of them couldn't because they've never been asked before*" (fg1a34). Tony noted that teaching relationally did help some pupils but also agreed "*some of the other kids in the class were really annoyed that I was teaching them why it worked, and just wanted the quick way around*"(fg1a18). Gillian agreed with Sharon that "*the whole concept of being asked why did they do something they actually kind of seemed uncomfortable with it*" (fg1a38). This apparent discomfort for pupils was something PSTs had to contend with when trying to teach mathematics relationally. Pupils did not comprehend that getting the correct answer was different from understanding something (e.g., fg1a40). Furthermore, challenging pupils on their understanding makes them feel like the teacher is "*making it more difficult*" for them (fg1a44). For example, when asked to defend an answer, a pupil gave the following response to Derek: "*why should I have to defend my answer, it's right*" (fg1a21). This sort of reaction from pupils subsequently impacts on PSTs' behaviour because they do not want pupils to feel this way. For example, Gillian explained:

> "*you kind of feel like you're doing a bad job when you go into a lesson and they feel like they understand something and they leave the maths lesson feeling like they understand it less. You can kind of see they're like oh I thought I got long division but I clearly don't understand it and obviously like as a teacher that's the opposite of what you want to be doing*" (fg1a89).

Related to this issue is PSTs' inability to push pupils outside of their mathematical comfort zones. In the previous paragraphs Gillian explained that pupils felt uncomfortable when pushed to explain their reasoning, and this made her feel like she was doing a "*bad job*". This was a recurrent theme. For example, Gemma "*didn't question them too much about why we're doing this, why we're doing that*" (fg1a52) because they were having difficulty using protractors. Similarly, Helen did not want to confuse them by introducing vocabulary not included in the textbook. Jessica was reluctant to "*try new things*" because "*you don't want to confuse them*" if the content or methods are not similar to what the children were accustomed to (fg1a91). Sharon discontinued using Dienes blocks because "*it just confused them completely*" (fg1a158).

It was interesting that one of the most influential factors on PSTs enacting meaningful mathematics in the classroom were the pupils they were teaching. That is, pupils appeared to have a very powerful influence over PSTs' classroom behaviour. Ultimately, if PSTs are unable to motivate pupils to engage in a deeper level of mathematical understanding enactment of such knowledge will remain aspirational.

4.2.3.2 Neatness of pupils' work

Very much related to the previous subtheme, the pupils that PSTs were teaching tended to value the neatness of their work over mathematical creativity or explanation. This was evident across both focus groups. For example, Gillian described the difficulty with trying to engage pupils in relational understanding of the long division algorithm. She explained how pupils really just wanted to get their "*neat little…long division sum and they done it and they* *got the answer and they ticked it… out of the way. And after that they just didn't want to look at the answer again*" (fg1a48). Furthermore, Sharon highlighted need for pupils to keep their copies in "*pristine*" condition and admitted "*to get the to do rough work was hard*" because of this (fg1a50). Mary also struggled with this as "*there's no space for messy work*" (fg1a33). Additionally, this focus on neatness hindered PSTs ability to integrate number lines in their teaching. For example, pupils who have been taught to rule down the middle of the page do not have room for a number line, and despite being told not to by Tony, pupils automatically ruled the page this way (fg1b28). Similarly, Gemma noted that the pupils in her class did not want to be messy and were encouraged by their cooperating teacher not to draw number lines of physical representations (fg1a20). Gemma said: "*there was some that were really struggling with what we were doing but as a whole class they had been taught to just write the equation and the answer into their copybook and not to draw number lines or to draw physical representations*". She noted that the "*stronger*" pupils were able to write down the answers quickly, but the "*weaker*" pupils struggled because they could not visualise the problem. Despite this, the "*weaker*" children still wanted to maintain "*neatness*" in their copies. In general, PSTs agreed there was a disproportionate emphasis put on this neatness, and Paul feels this "*neatness and tidiness*" comes from teachers, not children (e.g. fg1a24). The other implication of this is that it is time consuming because it takes time to maintain a neat copy which consequently reduces the time spent learning mathematics. For example, Aoife noted that she does not "*know if it was just like that they didn't want to do it but they were just spending so long just ruling the copy like, instead of doing the work and I was like come on it's been 5 minutes there should at least be a line down*" (fg1a52). This attitude amongst pupils is problematic from an enactment perspective, as maths competency is very much reliant on detailed explanations, often using number lines and other visual representations, to represent concepts, numbers, and their operations.

4.2.3.3 Pupils' perception that mathematics should be fast

In addition to a preference for neatness of presentation, seven PSTs reported that pupils perceive mathematics as a speedy execution of procedures, and significant value was placed on this. This perception contrasts with understanding mathematics on a relational level, which is typically a slow process and involves a deeper level of thinking. Speaking about the attitudes of the pupils he was teaching, Derek noted:

> "*Faster is smarter like. It's almost a culture among children*" (fg1b36).

Tony's comments about teaching fraction multiplication using the number line highlight pupils' preference for doing mathematics quickly:

> "[some of the children] *were really annoyed that I was teaching them why it worked, and just wanted the quick way around. They kept asking me what is the quick way. It is like they knew there was going to be a quick way of doing it, and they weren't going to have to use the number line the whole time. And they kept asking me what is the quick way. And they were getting worked up that they had to spend so long doing the equation*" (fg1b18).

Tony also added the following, highlighting his struggle to strike a balance between teaching relational mathematics and satisfying childrens desire for speed. He recalled:

> "[pupils got] *worked up that they had to spend so long doing the equation. They just wanted the quick way, and it was like really annoying because some of the really needed it and I was trying to work through it until they had it perfect, they knew why it worked and they could use the number line and these other kids, they were getting really angry like, just give me a quick way of doing it*" (fg1a18).

Jenny agreed that "*they just want to get it done quick*" and Paul believes this is a culture amongst primary school pupils (fg1a34-38). To speed up their mathematical work, Sharon reported some pupils even refuse to write things down believing that doing calculations in their heads was somehow better (fg1a50). Vicky feels this culture may be related to how game playing is implemented in the classroom such that the winner is usually the pupil who can produce the answer the quickest, and this competitive aspect appeals to children. Therefore, this culture tends to permeate into other mathematical activities also (fg1a37). There was an example of this sort of competitive game playing in Paul's class where children were required to rote learn their times tables.

When the CT was assessing this factual knowledge (i.e., procedural fluency), the pupils were required to line up and compete against one another each morning before the main mathematics lesson. This resulted in the "king and queen" of the line which resulted in the perception that those pupils at the top of the line were good at mathematics, while those at the bottom of the line were bad at mathematics. This was problematic because assessing procedural knowledge in this way, under timed conditions, has been shown to be ineffective in gaining insights into pupils' thinking, while also increasing pupils' levels of mathematical anxiety (Kling & Bay-Williams, 2014; Boaler, 2012), especially those pupils who use sophisticated mathematical strategies (Ramirez et al., 2013).

4.2.3.4 Secondary school mathematics

Related to, and compounding, pupils' attitudes to learning mathematics was the prospect of secondary school mathematics and particularly early preparation for the Leaving Certificate. For example, there were discussions amongst pupils in Jenny, Sharon's, and Paul's classes about the honours Leaving Certificate mathematics examination, the 25 additional points for passing this, and whether showing one's work is worth any points in the examination. Jenny strongly felt that, in 5th and 6th class, this is one of the factors that influence childrens attitude to learning mathematics. In her classroom, part of the mathematics work was preparing for Leaving Certificate mathematics by completing foundation level exam papers (fg1a47-49). Paul also noted a divergence from the standard primary school curriculum focusing instead on preparation for a scholarship exam for secondary school. PSTs agreed this initiative was driven by parents and implemented by the school. Pupils' focus on secondary school is also consistent with their attitudes about mathematics being fast and neat. As noted earlier, when asked to show their work, the pupils in Sharon's class queried whether "rough work" contributed to points in secondary school (fg1a81). Sharon explained to the pupils: "*if you show your rough work the examiner can see how you're doing*" and she explained this motivated them to focus more on showing some of the detail involved in the process of mathematics.

4.2.3.5 Cooperating teachers

Whether or not the CT was supportive of teaching mathematics for relational understanding influenced PSTs' ability to each meaningful mathematics on SP. Helen was working with a CT who promoted teaching for relational understanding with critical thinking and creativity in mathematics. In this class, Helen reported the pupils "*weren't focusing on getting the answer right as much as they were trying to get the steps. Like when he'd ask them why they were* *doing it they had to think about it*". Helen attributes this to the CTs excellent content knowledge of mathematics and "*his confidence kind of helped that a lot*" (fg1a66).

On the other hand, Gillian recalled how her CT was not interested in mathematics as a subject and did not enjoy teaching it. She believed the pupils in her class could sense this, and this consequently resulted in a negative response from children to the subject (fg1a74).

PSTs also agreed it was primarily the CTs who influence pupils' focus on neatness and speed in mathematics (fg1a22-23) and this made it difficult to teach relationally. However, there were other constraints to teaching relationally which can be attributed to the co-operating teacher. Some examples from this study include:

- The CT not exposing pupils to this type of mathematics before (e.g., fg1a38, fg1a74-76)
- The CT focusing more on completing a list of items based on the curriculum and textbooks than exploring mathematics on a deeper level (e.g., fg1a97, fg1a124, fg1a126-128, fg1a130)
- PSTs feeling under pressure to complete many different types of textbooks including standard texts, workbooks, and mental maths books (e.g. fg1a134), and therefore lacked opportunities to explore mathematics relationally.

In some cases, CTs promoted mathematics as an individual, and silent subject. On one occasion, Derek was commended by the school principal for keeping the pupils "*nice and quiet like it should be*" (fg1a31). Vicky also experienced this type of guidance from her cooperating teacher who encouraged her to use the textbook because the pupils were "*too noisy*" (fg1a44). Derek agreed, and explained his CT often discouraged collaboration and discussion while Paul agreed that most of the co-operating teachers, he has worked with did not teach mathematics as a creative subject. Derek felt this attitude from CT emanates from the fact they are under pressure to cover so many different elements in mathematics lessons:

> "*I feel like teachers are under pressure to deliver the curriculum … because time isn't allowed, teachers are just more interested in getting things done and getting that chapter finished so there's no time for discussion or exploration*" (fg1a46).

Similarly, Paul agrees that CTs also feel under pressure from parents to prepare pupils for secondary school entrance examinations, and this results in instrumental mathematics teaching and learning (fg1a50).

4.2.3.6 Synopsis of barriers to enactment

Some of these challenges pointed out by PSTs in relation to barriers may be related to the neoliberal influence on mathematics education in Ireland, and the "systemic move towards attainment of results-type accountability" (Conway and Murphy, 2013, p.28). This could explain the CTs overreliance on textbooks and pupils' focus on correct answers. It also points to the problematic mismatch between some teachers and pupils' preference for instrumental mathematics versus the implicit requirement for PSTs to teach relationally (Skemp, 1968). All of this creates a very particular context for PSTs that is significantly different from that experienced during the intervention. Some PSTs' apparent inability to enact the mathematics that correspond to their beliefs could be explained by this change in context. Several researchers, including Raymond (1997), Hoyles (1992), Skott (2001), and Sztajin (2003) found that context plays an important role in teachers practice and this this context may appear to override their beliefs.

4.2.4 Theme 4: Child-centred Teaching and the Froebelian Philosophy

There is a perception amongst PSTs that SPTs expect them to enact a Froebelian pedagogy on SP, and in this regard, SPTs can significantly influence PSTs' practice. However, according to the PSTs in this study, there is a lack of consistency regarding SPSs expectations and what constitutes Froebelian teaching. Sharon noted that their perceived "*success criteria*" in a lesson varies depending on the SPT assigned, and what their area of interest is. She argued that one SPT might insist on having concrete materials in every mathematics lesson, whereas another she recalled insisted on song singing as a methodological approach (fg1a175). In other cases, the SPTs were happy with mathematics lessons as long as they were relevant to pupil's lives (fg1a167), while another promoted the use of direct teaching (fg1a178). Gillian claimed:

> "[the] *inspector wouldn't be impressed if you didn't have something visual or something physical or some sort of game or something like that*" (fg1a162).

Consequently, most PSTs expressed some confusion about what exactly Froebelian mathematics teaching is, and this resulted in concerns that SPTs may not accept the type of teaching approaches learned during the intervention as Froebelian. For example, Helen agreed that teaching fraction multiplication using an area model, and the high leverage practice of modelling, was highly effective but she did not know "*if it was Froebelian or not but the kids completely understood*" so she would "*do it again because it helped their understanding better maybe than an activity*" (fg1a164). However, she was not aware of whether this can be considered Froebelian teaching or not. She said even though the representation was child centred and started with the childrens knowledge "*I don't know if an inspector or supervisor would take that into account. I feel like unless they have something in their hands and see the kids doing things I don't know if they understand where we're coming from*" (fg1a166).

For PSTs in both focus groups, the idea of making mathematics hands on appeared to be a key component of a Froebelian pedagogy of mathematics and most reported feeling under pressure to achieve this (e.g., fg1a155, fg1a166). For example, Gillian noted "*you have to be Froebelian, it needs to be hands on*" (fg1a157). However, in general PSTs reported struggling to enact this, particularly in a way that includes appropriate activities for senior classes. For example. Brona reported the following:

> "*I struggled to make things hands on for 5th and 6th class. I think it's easier with the younger groups than for 5th and 6th. It would be nice to know how to make it more hands on, and different activities you can do and stuff*" (fg1a141).

Similarly, Helen struggled with her perceived incompatibility of some of the more abstract features of fraction operations and using hands activities. She said she was "*a bit worried when I was doing fractions, like, how do you make it Froebelian* (fg1a152)".

Gillian reported a similar issue when teaching the long division algorithm:

> "*it's that kind of where you have to be Froebelian, it needs to be hands on…I struggled a bit with long division because when you're doing actual long division sums it's hard to make that...we're been told we have to sort of have our visual learners and kinaesthetic learners but it's quite hard to do kinaesthetic with long division because they have to just do out the sum and stuff*" (fg1a157).

Relating content to childrens lives is also part of the Froebelian approach. To address this, Jessica ensured that all her mathematics lessons started with a real-life application to the concept she was teaching. She said:

> "*I think linking it to like everyday life... helped with the kids I was teaching ... Because at one point they were like oh maths doesn't really like feed in. They were looking at it as a subject. But every topic I did, I did money and percentages...so I like to start at least with the viewpoint of where you see these in everyday life*" (fg1a145).

The enactment of Froebelian teaching practices can also conflict with approaches used by cooperating teachers. In line with their Froebelian principles, PSTs often planned activity-based lessons which would sometimes have to be sacrificed at the expense of completing pages from a textbook work. For example, Jessica was working with a CT whose mandated textbook work "*cut into time for other activities or like hands on kind of things*" (fg1a134).

There were examples across both focus groups where CTs encouraged the use of textbooks over more interactive mathematical activities (e.g., fg1a74, fg1a124, fg1b44). This is at odds with the PSTs' perceptions of the type of Froebelian approaches SPTs are expecting to see. This scenario put PSTs in a difficult position where SPTs and CTs have different expectations, and both could conflict with what PSTs' learn about in their ITE mathematics modules. Froebelian principles are important and are consistent with standards set out by the primary school mathematics curriculum and with my theoretical position as set out in Section 2.1. However, when these are misinterpreted, misunderstood, or miscommunicated then the quality of teaching and learning suffers because it causes PSTs to act in a way that they think is expected of them, which may vary depending on who is observing them, as opposed to acting in a way they know is right for the pupils they are teaching.

4.2.5 Theme 5: Neoliberal influence on mathematics teaching

The primary school mathematics curriculum, which guides teachers and teaching says the following about mathematics:

> "It should be recognised that mathematics is an intellectual pursuit in its own right, a source of fascination, challenge, and enjoyment. The exploration of patterns and relationships, the satisfaction of solving problems, the appreciation of designs and

shapes and an awareness of the historical and cultural influences that have shaped modern mathematics can contribute to the child's enthusiasm for the subject" (NCCA, 1999, p.3).

This vision of mathematics instruction was not evident from what PSTs described in the focus groups. Instead, a GERM orientated neoliberal slant towards mathematics as a procedure driven activity, for which the main purpose was performance in high stakes testing and examinations, was evident. This neoliberal influence permeates across the previous four themes generated from the focus group data. This section will highlight evidence for this by reviewing PSTs' responses through a neoliberal lens as described in Section 2.5 of the literature review.

To begin, the PSTs in the study experienced this neoliberal push in their own education. For example, Aoife talked about the pressure of standardised testing in primary school, followed by entrance examinations for secondary school, and then two major State mandated examinations made worse by the allocation of 25 additional points for Leaving Certificate mathematics (Aoife, FG1a65). Aoife was also critical of the 70% pass threshold for the maths competency modules. Collectively she referred to these as "*barriers*" (fg1a65) in the education system. Claire said the following in relation to the 70% pass threshold:

> "*I know there was reason behind it but I know my main focus in first year was just to get the grade and pass*" (fg1a2).

Consequently, when PSTs were preparing for their maths competency examinations, the aim of which is to develop relational understanding, Claire argued they are focused instead on achieving a grade over understanding. She claimed this mentality comes from secondary school and, in particular, the Leaving Certificate mathematics examination where "*weren't told why but you knew you had to know it to pass*" (fg1a78).

There was evidence of how this results-driven agenda affects pupils whose PSTs are expected to teach. Of particular concern is how pupils, and perhaps teachers, perceive mathematics as a silent subject, carried out individually, at the fastest speed possible, with correct and neat answers, and as little evidence of understanding put on a page as possible. This is, at best, a mis-educative (Dewey, 1933) conception of mathematics and is at odds with the relational nature of mathematics promoted by the maths competency modules. Sharon, for example, reported how a group of 6th class pupils questioned the necessity for "*rough work*" because they were more interested in getting quick answers. Ultimately what motivated them to show their work was the prospect of picking up additional points in the leaving certificate for showing their work. Similarly, in FG2 Jenny claimed that pupils in 5th and 6th class are already looking to maximise the additional 25 points for passing the leaving certificate higher level mathematics examination (fg1b47). In fact, the children in Jenny's class were already practicing the Leaving Certificate foundation level examination papers, and she feels this is a consequence of pressure from parents.

This neoliberal view of mathematics education is often reflected also by CTs. According to Gillian, the attitude of her CT to "*kind of just like tick it off, get it done. It was not...he didn't want them to question stuff, because he wasn't that interested*". In this case Gillian reported that the CTs motivation was to "*get the box ticked to finish the curriculum. Get it done and that was it*" (fg1a76). Similarly in Bronas' class there was "*no value placed on understanding it. Because understanding it doesn't get you points, it doesn't get you marks, it doesn't get you full marks in the test..*" (fg1a79). Claire talked about the rigidity of mathematics teaching on SP and "*even if* [the pupils] *knew it you still had to do it*" (fg1a124) and any deviation from this was questioned, perhaps subtly, by the CT. This commitment to external demands can impact a PSTs' ability to teach mathematics for deeper relational understanding. For example, Claire decided to teach her pupils about Euler's formula to give them the opportunity to appreciate mathematics as a "source of fascination, challenge, and enjoyment" (NCCA, 1999, p.3) but felt uncomfortable doing this because "*I knew the teacher was like, not pressure, but there were certain things you had to get done*" (fg1a97).
It is evident that the pressure to meet external demands negatively impacts PSTs' practice. Sharon felt pressure from demands of her CT, the workbook demands, and the demands of the curriculum (fg1a30), while Jessica felt the pressures of having to demonstrate work from *"a mental maths book, tables champion book, busy at maths and then another maths book that I didn't go near so I kind of felt pressure to get three filled out every day for him*" (fg1a134). The nature of this work reduces opportunities for more focused and meaningful mathematics learning.

Accountability is inherent in a neoliberal educational system and as noted earlier, Paul felt that CTs were under pressure, and this pressure is reflected on PSTs. He agreed with Jenny that the pressure comes from parents and having to complete sections of the curriculum and textbooks, which does not leave any time for "*discussion or exploration*" (fg1b46). He disclosed that some pupils were receiving specialised tuition for "*some entrance exam for a scholarship for some private school…because the parents asked them to*" (fg1b50). Paul proposed that the value of learning deeper mathematics should be communicated to parents which may result in a parental attitude shift and subsequently alleviate some of this pressure on teachers.

Not all participants reported a negative response in relation to mathematics from their CT. For example, Brona noted her CT gave her a reprieve from the normal demands of SP by giving her the space to spend more time on particular topics until she has happy the pupils' level of understanding was good (fg1a30). Similarly, Helens CT gave her the freedom to teach relational mathematics and encouraged her to focus on the process without the pressure of external demands.

4.2.6 Section Summary

The focus group data from fg1a and fg1b were analysed using Braun and Clarks (2006) thematic analysis. This analytical process resulted in several themes, which interact together, to influence PSTs' ability to enact mathematical knowledge. One of the primary factors that inhibits PSTs from enacting relational mathematics is their own inhibitions. All PSTs have unique and complex mathematical identities (Leatham, & Hill, 2010), often shaped by a turbulent relationship with mathematics, which informs their beliefs about mathematics and their associated behaviours. They also appear to have had a disjointed experience of learning mathematics in ITE, and this is exacerbated by the many external factors that make teaching for relational understanding difficult. Underpinning all of this is a neoliberal foundation that does not encourage mathematics to be taught as a slow subject underpinned by meaning and understanding, rather one that is dependent on completing textbooks and passing examinations. These ideas are summarised in Figure 4.1 below:

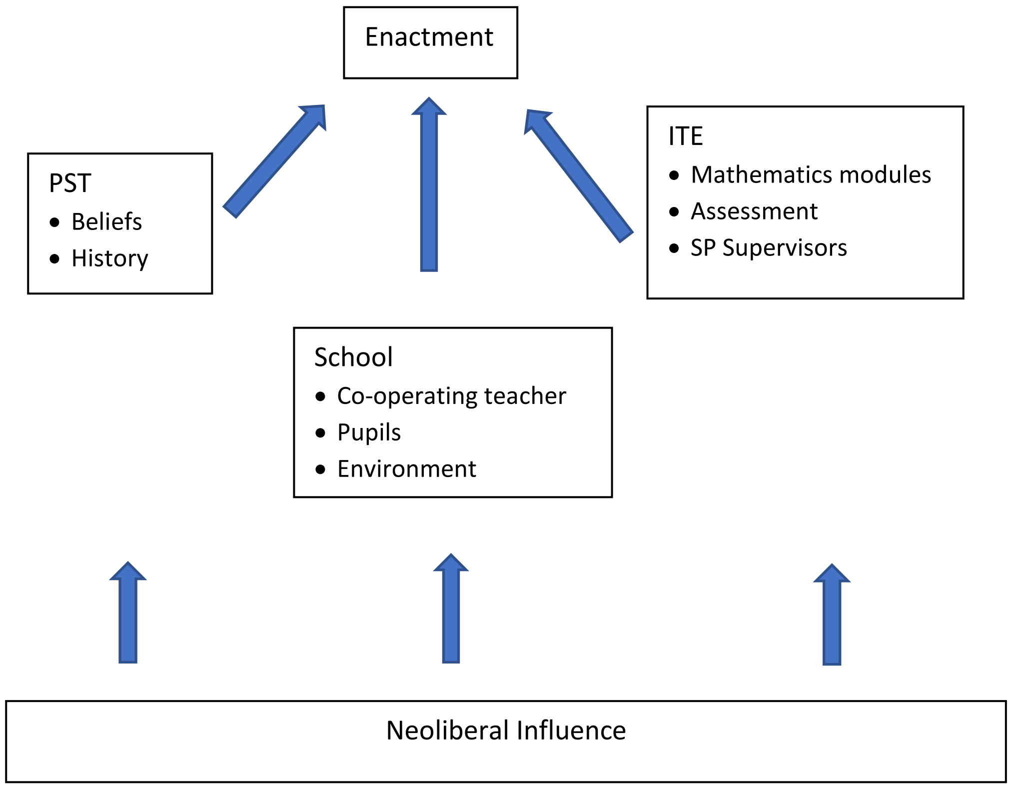


Figure 4.1: Factors that influence enactment

Significantly, those things that can act as barriers to enactment can also act as enablers, and this has important implications for Cycle 2 of this study. For example, CTs and SPTs can have a positive or negative influence on PSTs' experience. Admittedly, this is a simplified view of PSTs' experiences in a complex educational landscape, but it does represent the realities and authentic lived experiences of the PSTs in the study.

4.3 Classroom Observations

4.3.1 Analysing the Categories: MQI-H and MQI-L/A

An integral component of this research was to learn about the nuance of PSTs' mathematical understanding and how this is enacted in practice. To do this I focused on analysing the work of those PSTs who scored high on the 5-point MQI scale and those who scored low on the MQI scale. Of the 13 PSTs who were observed, this process resulted in a close examination of the lessons of 6 PSTs from an MQI perspective. These six PSTs were made up of the top 3 and the bottom 3 MQI scores from the cohort. Of the three lowest, two sored zero on the richness of mathematics scale, while one participant was categorised as low. The reason I included one

MQI-L was because it allows for the presentation of features of mathematical teaching that, due to the absence of any substantive mathematical teaching, were not present in the MQI-A example. It is important to note also that the analysis is a critique of the lesson content and pedagogy, and not an evaluation of the individual PST. Only six observations were chosen because the analysis is highly descriptive, paying particular attention to individual nuance which is characteristic of a complex teaching environment. Together, the 6 examples described below highlight aspects of mathematics teaching that should be celebrated and reinforced, and also those undesirable aspects of mathematics instruction that should be recognised and learned from in future representations of practice.

As convention, MQI-H is used to indicate those PSTs who scored highly on the MQI scale, while MQI-L/A is used to indicate those PSTs who scored low, or where features of the MQI framework were absent in a lesson indicating a score of zero. Each of the 6 lessons are discussed under the main headings of MQU-H and MQI-L/A. Within each of these subsections, each lesson is presented under the pseudonym of the PSTs involved, followed by a numerical MQI score, context, and a qualitative analysis of each lesson using the lens of the MQI framework. To ensure the voice of the PST is heard, each subsection also includes a critique of the lesson from the PSTs' perspectives. This overall section is concluded with a comparison between MQI-H lessons and MQI-L/A lessons.

To calculate an accurate MQI score, each PST was closely observed and detailed notes on this observation were written. Shortly after each lesson, these notes were analysed using the richness of mathematics dimension of the MQI 4-point version. This dimension captures the depth of mathematics offered to pupils.

The codes within this dimension are grouped into two categories: those that capture the extent to which instruction focuses on the meaning of facts and procedures (linking between representations, explanations, and mathematical sense-making), and those that capture the degree to which instruction focuses on key mathematical practices (multiple procedures or solution methods, patterns and generalizations, and mathematical language). The final code for Overall Richness of Mathematics was designed to capture the overall depth of mathematics offered to pupils (Learning Mathematics for Teaching, 2011).

Each code has associated detailed descriptors and based on these, PSTs mathematical instruction was assigned one of the following: not present, low, mid, or high. For each code within this dimension, the relevant aspect of instruction must be substantially correct to count as low, mid, or high. Richness elements that are not correct were ignored.

The MQI scoring document developed by Learning Mathematics for Teaching (2011) contains detailed descriptions of each code. See Figure 4.2 below for an example of this for the Linking Between Representations code:

Linking Between Representations

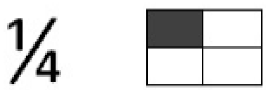
This code refers to teachers' and students' explicit linking and connections between different representations of a mathematical idea or procedure. To count, these links must occur across different representational "families" e.g., a linear graph and a table both capturing a linear relationship. So, two different representations that are both in the symbolic family (e.g., 1/4 and 0.25) are not candidates for being linked.

For Linking Between Representations to be scored above a Not Present:

- At least one representation must be visually present
- The explicit linking between the two representations must be communicated out loud

For Linking Between Representations to be scored Mid or High, two conditions must be satisfied:

- Both representations must be visually present
- The correspondence between the representations must be explicitly pointed out in a way that focuses on meaning (e.g., pointing to the numerator in 1/4, then commenting that you can see that one in the figure, pointing to the four in the denominator, pointing to the four partitions in the whole. "You can see the 1 in the 1/4 corresponds to the upper left-hand box, which is shaded, showing one piece out of four total pieces…")



For geometry, we do not count shapes as a representation that can be linked—we consider those to be the "thing itself." However, links can be scored in geometry if the manipulation of geometric objects is linked to a computation, e.g., showing that two 45-degree angles can be combined to get a 90 degree angle and linking that to the symbolic representation 45 + 45 =90.

Note: If links are made but underlying representation/idea is incorrect, do NOT count as linking between representations.

Figure 4.2: Overview of linking between representations

Each code description was followed by another set of descriptors which clearly outline what counts as not present, low, mid, or high. For example, for the above code the descriptors are outlined in Figure 4.3 (Learning Mathematics for Teaching Project, 2011) below.

Not Present	Low	Mid	High
No linking occurs. Representations may be present, but no connections are actively made.	Links are present in a pro forma way; For example, the teacher may show the above figure and state that one quarter is one part out of four. These links will not be very explicit or detailed; both representations need not be present.	Links and connections have the features noted under High, but they occur as an isolated instance in the segment.	Links and connections are present with extended, careful work characterized by one of the following features: • Explicitness about how two or more representations are *related* (e.g., pointing to specific areas of correspondence) OR • Detail and elaboration about the relationship between two mathematical representations (e.g., noting meta-features; providing information about under what conditions the relationship occurs; discussing implications of relationship) These links will be a characterizing feature of the segment, in that they may in fact be the focus of instruction. They need not take up the majority or even a significant portion of the segment; however, they will offer significant insight into the mathematical material.

Figure 4.3: Descriptors for linking between representations

Using these descriptors for each of the MQI elements, a score of 1 was assigned to codes that were not present, 2 for low, 3 for mid, and 4 for high. Because not all of the codes from the richness of mathemtimetics were required to be included in any one lesson, a percentage score was calculatred for each of the 13 PSTs to rank them from lowest to highest. From these, the 3 highest and the 3 lowest were chosen for detailed analysis. An overview of this is presented in table 4.1 below.

Table 4.1: MQI scores

Pseudonym	**MQI Score**	**MQI Category**
Sharon	67%	High
Claire	57%	High
Vicky	54%	High
Helen	30%	Mid
Megan	29%	Mid
Tony	25%	Mid
Gillian	25%	Mid

Freda	25%	Low
Jenny	18%	Low
Derek	18%	Low
Gemma	14%	Low
Jessica	0%	Absent
Brona	0%	Absent

Sharon, Claire, Vicky's lessons are used to demonstrate MQI-H lessons, while observations of Gemma, Jessica, and Brona's lessons are used to demonstrate lessons which were low in, or absent of, MQI.

4.3.2 MQI-H

Sharon

MQI score: 67%

Context

Sharon was teaching a multi-grade 5th and 6th class. As a lesson starter, she revised the concept of mirror image with her pupils. Mirror image was defined as a mathematical term and was presented using a child appropriate definition and represented using an accurate picture.

The Lesson

The main part of the lesson (development) was a 2-D shapes "maths trail" in the school yard, for which Sharon prepared an activity sheet to guide the pupils. Activities were related identification and characterisations of polygons and tessellations, regularity of 2-d shapes, and insightful and cognitively demanding true/false questions. For example, one of the statements was "a triangle can be both right angle and isosceles".

An interesting aspect of this lesson was that the mathematically rich content of worksheet led to mathematically rich interactions from the pupils. Children were afforded a lot of physical freedom to explore and hypothesise in the school yard which may have also positively contributed to these rich interactions. Furthermore, pupils were not intellectually constrained by workbook tasks or physically constrained by a desk. Importantly, they were intellectually prepared for the tasks due to the content knowledge the PST had delivered to them over the previous few lessons. For example, in the previous week children learned the definition of a tessellation as a “a tiled surface with no gaps”. A pupil I encountered used the example of a honeycomb as a naturally occurring tessellation with hexagons.

Another example that highlighted the quality of the lesson was when a pupil approached Sharon and asked her for a definition of a polygon. The first point of note here was that the pupil asked the question with genuine interest and had the necessary language to be able to do this. Secondly, the resulting interaction generated a meaningful discussion between Sharon and several pupils about the classification of 2d shapes. Finally, in terms of accessibility to language, the PST used the word poly pocket to explain its meaning as “many” to the pupil.

Sharon questioned another pupil about the regularity of a shape she had discovered in the yard. The child explained that the shape was irregular because “one side was longer than the other”. This was a perfectly valid explanation for a primary school pupil. Sharon recognised and acknowledged some pupils were confused between 2d polygons and 3d polyhedra, and effectively remediated them on a conceptual level using the example of a cube and a cuboid while using clear definitions.

From a mathematical quality perspective, there was a particular teaching moment that stood out above the rest. The pupils were tasked with finding lines of symmetry in the school yard. One pupil investigated the lines of symmetry in the circle located in the centre of a basketball court. The pupil pointed out 4 lines of symmetry by dividing the circle into 4 equal segments and Sharon praised the pupil for this but did not remediate in any way or offer additional input. Another pupil then suggested there are an infinite number of lines of symmetry in a circle (referring to diameters). Sharon recognised this as a learning opportunity and asked the pupil about the meaning of infinite. In response to this, the pupil replied, “more than 100, more than 1000”. According to Sharon, this child “struggled” with mathematics, but these contributions suggest otherwise. This scenario suggests when PSTs teach mathematics for relational understanding, and subsequently give pupils the freedom and opportunity to apply this knowledge, they can demonstrate creativity, criticality, and engage meaningfully with mathematics.

Post-lesson Discussion

In the post lesson conversation with Sharon, she expressed interest in further developing the concept of infinity with the children the following day. In general, she wanted to further develop the deep, meaningful sort of mathematics she was teaching so that it could become more relational. Sharon believes there is scope to be creative in her teaching and does not feel restricted by children's attitudes. Although the topics covered in this lesson were not yet addressed in maths competency, Sharon taught them well and demonstrated good MCK because she took the time to independently upskill in the relevant areas. In particular, Sharon intentionally dedicated a portion of her preparation time to learning about geometry, and specifically 2d shapes, important definitions and tessellations. Interestingly, in the previous academic year, Sharon scored higher on her maths competency examination than all of the other PSTs involved in these observations and second highest in her class overall.

Claire

MQI score: 57%

Context

Claire was teaching 6^{th} class pupils about 3d shapes and began by revising some of the shapes they were learning about in previous lessons. She asked several questions including: "what is another name for a triangular based pyramid?", to which a pupil answered tetrahedron correctly. She asked another pupil if a cylinder was a type of prism to which the pupil replied "no, because prisms have to have straight edges". Although a prism with a number of sides that approaches infinity may be considered cylinder, this was not discussed in the lesson.

Next, the teacher asked the pupils about the origin of the name of a pyramid. One pupil clarified that it was named after the shape of its base and gave the example of a square based pyramid. Another child identified and named the vertex at the top of the pyramid as an apex.

My only criticism of this introduction, based in the MQI framework, was the absence of pictures or physical shapes to accompany the definitions.

The lesson

For the main lesson Claire explained that they were going to prove a theorem (Euler's Theorem) and spent a few minutes eliciting children knowledge about what they understood a theorem to be. The pupils used words like theory and proof, and Claire eventually clarified its meaning in child appropriate language.

Claire then did some preliminary work with the pupils to clarify important background knowledge about the meaning of polyhedra and associated features: face, edge and vertex. It was collectively agreed that polyhedra have flat faces and straight edges. For a range of polyhedra, face, edge and vertex, were abbreviated with the letters f, e and v respectively. Perhaps a missed opportunity for learning, Sharon did not justify why these abbreviations were necessary or useful, nor did she highlight the usefulness of Algebra in solving mathematical problems.

For a range of regular polyhedra, children were asked to determine values for f, v and e and then, for each of these, calculate $f + v - e$.

The pupils were asked to answer the following questions:

- What do you notice? Is there a pattern?
- Can you think of a shape this doesn't work for?

Children collaborated in pairs on these tasks. They counted the edges, faces and vertices and performed the calculations.

This was a good attempt at rich mathematical learning, where meaningful communication between teacher and pupils was evident. Pupils also clearly communicated with each other using each other's ideas effectively. The lesson was well structured with a period of meaningful direct teaching followed by a process of mathematical inquiry.

Together the PST and pupils went through the findings of their investigations, which included the following:

Cube: $f = 3;\ v =\ 8;\ f + v = 14;\ e = 12;\ f + v - e = 14 - 12 = 2$

Tetrahedron: $f = 4;\ v = 4;\ f + v = 8;\ e = 6; f + v - e = 8 - 6 = 2$

This was displayed for several of the platonic solids, and it was collectively concluded by the pupils that $f + v - e$ is always equal to 2.

Conclusion

To finish the lesson, Claire consolidated and reinforced what the pupils had learned. They recapped some of the basic features of polyhedra, and determined that Eulers formula did not apply to the sphere because it did not share the same characteristics as regular polyhedra, i.e., flat faces and straight edges. Children also identified a rhombus and a cylinder as shapes that Eulers formula did not apply to because one is a 2d shape and the other has curved edges. When Claire looked for clarification about curved edges, a pupil replied that curved edges cannot have vertices. With Claires' guidance, the children concluded that Eulers formula only works for regular polyhedra. Finally, Claire reinforced for the pupils what a mathematical formula is.

Reflection

While this lesson was not perfect from an MQI perspective, there were some excellent aspects of it that gave it an overall high MQI score. The lesson as good in terms of generalisation, explanation, number sense, communication, collaboration, and language. Although, not a challenging lesson (for most pupils) in terms of the difficulty of the mathematics, there was a cognitive demand that is part of recognising pattern and establishing conjecture. There is scope for improvement, however, around giving pupils access to physical shapes as part of their investigation. The pupils were looking at 2d representations of 3d shapes which diminished their ability to fully explore and visualise the ideas in the lesson. There were also missed opportunities to historical context and mathematical implications of their work. For example, who was Leonard Euler and what else has he contributed to the field of mathematics? Why is Euler's formula useful? Claire could have used the term Platonic solids and discussed Plato. These historical and contextual aspects of mathematics can add to children's interest and motivation, as well as appealing to their sense of curiosity.

Post lesson

In the post lesson conversation with Claire, she described to me how there is an ongoing debate with the pupils about whether or not a square is a type of rectangle. Only one child was convinced a square is a special type of rectangle. The nature of this ongoing open discussion added a conceptual depth to the pupils' mathematical learning that captures the essence of fallibilistic mathematics. Furthermore, the pupils were not given the answer to this, and the pace at which this was done, over a period of several days promoted the idea of mathematics as a critical, creative, and open-ended pursuit, involving meaningful dialogue.

Vicky

MQI Score avg: 54%

Context

Vicky was teaching lines and angles to 6th class pupils. Although this lesson was a little one dimensional, it was categorised as MQI-H because the pupils were encouraged to think deeply about the lesson content while appropriately supported to justify the answers they provided.

Lesson starter

Vicky started the lesson by revising the previous day's work. Pupils discussed protractors, how to use them, different types of angles (right, reflex, etc.) and their definitions based on size in degrees. From this early part of the lesson, pupils were required and encouraged to justify their answers. For example, Vicky reminded the class that they looked at right angles the previous day and subsequently asked for more information to which one pupil responded that a right angle has 90 degrees. This line of discussion continued for a few minutes and the Vicky included pictures of various types of angles to complement this conversation while continuing to reinforce the definitions of angle types throughout the starter. The starter was effective because it provided a useful stimulus for the children that was very much related to the upcoming lesson.

Examining this early part of the lesson through an MQI lens there are some areas for improvement. For example, Vicky lacked linguistic precision when asking pupils about the size of various type of angles. For example, she asked "can anyone give me a degree for an acute angle”? While the children understood the meaning of this, it lacked clarity, simplicity, and mathematical convention. *Size* of angle is the key mathematical term missing from the Vickys mathematical vocabulary, and her inability to use this key mathematical term resulted in diminishing the quality throughout the remainder of the lesson where Vicky used “a degree” to refer to the size of an angle. For example, when Vicky was reminding the pupils of the previous days lesson she said: “*yesterday we had a degree and we had to draw that degree*”.

Vicky effectively used pictures to demonstrate the size of different angles, but the pupils would have benefited more had the relevant mathematical symbols been used, particularly those around equality and inequality. That is, Vicky used vocabulary to describe angle size such as less than, greater than, equal to, etc. without accompanying this with the associated mathematical symbols.

The lesson

The main lesson focused on estimating angles, and Vicky reminded the pupils of their prior knowledge about estimation, and what it means. She emphasised that, if an angle was, say, 50 degrees, and a pupil guessed 60 degrees, then this would not be wrong because it is an estimation. This generated high quality discussion about what various angles looked like, and pupils’ contributions were recognised and valued. Vicky provided explanations, accepted children’s explanations, contributed to children’s number sense, and used multiple methods for measuring angles. Almost every time a pupil gave an answer, they were encouraged to justify that answer. Even when an answer was correct, children were asked to share their thinking that led to correct answers.

When a pupil was asked to estimate the size of an obtuse angle he answered, “58 degrees”. Acknowledging this was inaccurate, she asked the pupil to justify his answer. This probing led the pupil to realise that the answer should have been 158 degrees, and the child omitted the 1. This may seem insignificant, but it is a good example of developing number sense because the child came to realise that the answer must have been 158 based on the reasonableness of his answer.

The main criticism of this section of the lesson was the lack of connectedness to other relevant areas of mathematics to develop pupils' relational understanding. While acknowledging errors in their estimations, they did not evaluate them on a deeper level. For example, a pupil guessed 163 degrees for an angle with an actual size of 145 degrees. Vicky could have used absolute error or relative error. Absolute error simply represents the positive difference between the pupil's estimation and the true value. In this case, $163 - 145 = 18$. As children have already studied percentages, this absolute error could be used to calculate the relative error. That is, $\left(\frac{18}{145} \times 100\right)$%. This would bring together several related areas of mathematics to encourage development of pupils' relational understanding.

Lesson conclusion

The final part of the lesson was used to assess pupils' ability to recognise different angles represented on a clock face. Before starting, Vicky reminded children of what a right angle and a straight angle look like by using arm gestures. She then wrote several questions on the board, which resulted in some significant teaching moments in this part of the lesson. Two of these questions are outlined below.

Question 1: How many degrees are in an hour?

It appeared the children did not already know there were $360°$ in a full rotation. One pupil "discovered" this by recognising that there were $90°$ from 12 to 3 and then added another $90°$ to this to get $180°$ from 12 to 6. He then doubled this to get $360°$ for a full rotation.

Question 2: How many degrees does the minute hand pass through in 30 minutes? In 15 minutes? In 5 minutes?

When it was established that the minute hand passes through $180°$ in 30 minutes, a pupil deduced it must pass through $90°$ in 15 minutes and justified her answer by saying "Half hour is double 15 so I halved last answer". Building on this, the same pupil concluded the minute hand passes through $30°$ in 5 minutes because $90 \div 3 = 30$.

Referring to each hour 1,2, … 12 on the clock as distinct division points, another child decided to find out how the size of each division point by dividing 90 by 3. Using this information, the child could then calculate other angles around the clock.

For each of these questions, and consistently throughout the lesson, Vicky asked pupils to justify their answers. Furthermore, she invited other pupils to agree or disagree with those answers and to give a reason for their agreement/ disagreement.

Reflection

This quality of mathematics teaching and learning in the final part of the lesson was high. It was relational in the sense that children were asked to justify their answers and evaluate the validity of other pupils' answers, while bringing together children's knowledge of arithmetic operations, circles, angles, and time. This contributed to rich and meaningful mathematics experiences for the pupils.

Post lesson

In the post lesson discussion Vicky explained how she intentionally used the '100 questions to promote mathematical discourse' as a core part of her teaching. She agreed that it helped children to discuss and share ideas with each other. She also explained how she believes that pupils learn more effectively when ideas are explained to them by their peers, and as such this was a key methodology used in this lesson. Vicky did not bring up the benefits of teacher revoicing as a tool for clarifying pupils' ideas. Revoicing is defined as the "reporting, repeating, expanding or reformulating a student's contribution so as to articulate presupposed information, emphasise particular aspects of the explanation, disambiguate terminology, align students with positions in an argument or attribute motivational states to students" (Forman & Larreamendy & Joerns, 1998, p. 106).

4.3.3 Low/ absent MQI

Gemma

MQI Score: 14%

Context

Gemma was teaching the strand of Money to 6th class pupils. More specifically, she was teaching the unitary method for calculating costs of items. She explained to me before the lesson that the pupils in her class were "very weak" and so she needed to go "back to basics" with them.

The lesson

Without revising any specific content, Gemma began the lesson with a worked example of finding the cost of one item. She did not explain the motivation for finding the cost of one unit or provide any background mathematics behind the procedures and strategies used to solve the problems in this lesson. This was the format of the remainder of the lesson.

What follows is a description of some of the problems from the lesson:

In the first example, Gemma presented the pupils with the following problem: if 8 bars of chocolate costs €7.52, how do we find the cost of 1 bar of chocolate.

This initial question showed promise because pupils were asked "how" as opposed to "what", which has the potential to lead to relational understanding.

Without opportunity for collaboration, one pupil was selected to provide a solution and explained to the class that you calculate $7.52 \div 8$. The pupil then went up to the board and completed the "short division" algorithm. Although Gemma reminded the pupil to put the decimal point in the correct position, there was no explanation as to why the decimal point goes in that position and no reference to how estimation could be used to determine the correct position of the decimal point. Furthermore, no explanation about why division was the correct procedure, or any suggestion that other operations could be used to solve the problem.

The next task was to determine the price of 4 bars. A pupil correctly suggested to multiply by 4 and Gemma suggested it was also possible to add the price of one bar to itself 4 times, but highlighted that multiplication is quicker. She did not refer to the relationship between multiplication and addition. The child executed the multiplication algorithm on the board, with a focus only on procedure.

There were other similar examples with some additional points of note. For example, given the price of one bun, a pupil was asked to find the price of 10 buns. Multiplying by powers of 10 is an important procedural skill with associated conceptual knowledge but this idea was not referred to in this example, with no attempt to generalise the idea. As part of the same example, the Gemma wanted to show that 6 buns costs €11.40 and to represent this she wrote $6 = 11.4$ which is a serious content error.

The next example involved finding the cost of 1 kilogram of a product. This example was very similar to the previous example, involving purely procedural use of the division and multiplication algorithms. It was the first example using kilogram but there was no explanation about what a kilogram is, or how it is related to grams.

At this point, the pupils who were able to continue with similar examples were instructed to work independently from their textbooks. At the same time, the pupils who were having difficulty were given another example to work on. Rather than giving a more basic example or taking the time to conceptually unpack some of the previous examples, then Gemma gave a more challenging problem. This example involved 800 grams of a product costing €2.40, and the task was to find the cost of 1800 grams. In this example, the children were instructed to divide €2.40 by 8 and multiply by 18. This was different from all previous examples because a unit value was not found, rather it was the price of 100 units but Gemma did not pay attention to this or explain why the new procedure was correct. Some of the pupils, unsurprisingly, did not grasp the idea of why they were required to multiply by 18. That is, she did not explain that $1800 = 18 \times 100$ or give a rationale for why this is useful.

In another example for the pupils who were struggling with the content, Gemma asked them: if 250mils of a liquid costs €160, how much does $1\frac{1}{2}$ litres cost? Gemma demonstrated (i.e., did not use the modelling HLTP used in the intervention) this using the same approach as the previous examples. The difference here is this problem involves fractions, so Gemma explained that 250mls is quarter of a litre but did not explain why this is true, or how she came to this conclusion. Similarly, she did not explain why there are two quarters in a half or make any attempt to explain why. On the board a child demonstrated that ½ litre must cost 3.20, and therefore a litre must cost 6.80. To do this calculation the child added 3.20 to itself using the standard addition algorithm, but Gemma did not pay any attention to why the decimal point was important, and when this was omitted by the pupil Gemma did not remediate this.

For the remainder of the lesson Gemma worked with individual pupils, while the majority of pupils worked on a task unrelated to the unitary method of calculation. A group of children were working on a problem next to me on a long division problem from the textbook, and I asked them to explain what they were doing. They had not "done" long division yet but were attempting to use the algorithm because the previous owner of the book had used it for the problem they were currently working on. I talked to the pupils about what division means, and within a short period they realized that they could solve the problem using multiplication. This incidental encounter highlights the danger of assuming pupils are "very weak" and they can engage in meaningful mathematics when they are encouraged and supported.

Lesson conclusion

The lesson concluded with children calling out answers to completed questions without explanation.

Post lesson

In our post lesson discussion, Gemma explained that she had concerns about pupils' levels mathematical anxiety, and consequently felt like she had to try to keep the learning simple so that her teaching did not exacerbate this anxiety.

Jessica

MQI score: absent

Context

Jessica was teaching 'value for money' to 5th class pupils as part of the Money strand of the mathematics curriculum.

Starter

For this activity, pairs of pupils were positioned standing back-to-back at the top of the class and were tasked with answering multiplication questions as quickly as possible. Jessica called out the two numbers and the pupils had to calculate the product and write it down on the board as quickly as possible. The pupil who wrote down the answer first was the winner.

The game was not related to the main learning objectives and provides little to no cognitive demand, mathematical thinking and involved only memorisation. The activity rewarded speed (both cognitive and gross motor) as well as a pupil's ability to work under pressure and therefore promotes these attributes as what it means to be a good mathematician.

Reflection on starter

There were other more inclusive and mathematically rich options that Jessica could have used here. For example, one of the examples used was $8 \times 8 = 64$. There is a myriad of things about this product that could have been explored with the pupils. In fact, the lesson starter need not have looked at anything apart of this product. For example, square numbers, rectangular numbers, compositive numbers, relationship to division, indices and square roots are just some options that could have been explored.

Main lesson

The main part of the lesson started with Jessica asking questions related to value for money. She started with the following question: Is it better value to purchase 1 item for €2.30 or 5 items for €10.50? There was no explanation regarding motivation for this question, or anything about the meaning of value, or the types of operations required to determine value. When asked, several pupils approached the board to demonstrate using division to solve the problem. Pupils were not encouraged or required to provide any sort of explanation or rationale for their work, while pupils sitting down watching this were not asked for input, nor was understanding checked for.

The next part of the lesson involved the PST setting up a “shop” with a list of items and their prices displayed on the board. The children were asked to find the total cost of all the items and then calculate the change they would get from €150. There were several more examples of this nature with no meaningful mathematics involved and requiring a low cognitive demand. Any remediation was purely procedural. For example, a pupil was stuck on a problem that involved the following addition: $5.99 + 5.99 + 3.5 + 3.5 + 6.21 + 24.95$. Jessica lined up the numbers as per the addition algorithm and completed the algorithm on the board. There were two things she focused the childrens attention to: insignificant zeros and decimal points.

She demonstrated by putting 0’s before single leading euro digit of the price. For example, 5.99 became 05.99, etc. However, she did not explain why this was necessary or useful, or why this operation does not change the value of the original number in any way. Jessica also lined up the decimal points, as per the algorithm, but did not pay any attention to the significance of this.

During this segment of the lesson, Jessica said some of the pupils’ answers “made sense” while others did not, without paying any attention to any of their reasoning. In this case, remediation of childrens misconceptions, as per the MQI framework, was classified as not present.

Lesson conclusion

For the lesson conclusion children were invited to the board to demonstrate their calculations. For this, Jessica asked that every pupil remains quiet and look up. The first child was reminded to “make sure your decimal point is in the right place”, again without explanation. The next child subtracted a 3-digit number from 1000 and explained that you can cross out the 1 and make all the other digits 9. While this is procedurally correct, the pupil was not asked to justify this in any way, not even superficially.

Jessica then explained to the pupils: "*ok we'll do one more and then we'll call out the answers*". For the remainder of the lesson, the pupils called out the answers: total cost and change. Some of them called out incorrect answers but there was no remediation or constructive feedback. These pupils were told their answers were incorrect and then more children were asked until the correct answer was called out.

During this part of the observation, I wrote the following in my reflective diary:

Reflection

This seems to be what pupils want….just the answer. This is consistent with idea that maths is about following instructions, without error, to get a single correct answer as fast as possible. …children are naturally curious and it's written in the curriculum that this should be encouraged. Yet, as educators, we seem to be complicit in systematically removing this sense of curiosity from young pupils.

Post-lesson Discussion

In our post lesson discussion Jessica did not have anything substantive to discuss. In her mind, the lesson went "well", and she explained that she did not want to challenge children too much because they are "too weak" at mathematics. She added that she was concerned about the remainder of the placement which involved teaching percentages because she questioned the pupils' ability to engage with the topic.

Brona

MQI Score: absent

Context

Brona was teaching the Money strand unit to 6th class pupils. Before the lesson Brona explained she would be doing money today because she did percentages last week and wanted to include percentages in the lessons on Money. Brona then explained to the children they would begin with mental maths as a starter, then as the main lesson they would be comparing the prices of items in two different supermarkets, and finally ending with a game working in pairs.

Starter

The starter involved Brona asking quickfire questions to the pupils relating to money, percentages, and the arithmetic operations. Some of these questions, and the interactions that followed, are worth considering. The first questions involved finding a percentage of a number. For example, Brona asked "what is 1% of 356"? and a child correctly answered 3.56 but no justification or discussion followed this. Brona then asked another similar question, "what is 1% of €230"? On this occasion, the pupil initially answered incorrectly with 23, and then attempted to revise her answer using her mini white board. However, Brona instructed her to stop writing because it was not permitted during mental maths. The questions then moved onto addition and subtraction, e.g. ($€3.25 + 75c$ and $€3.76 - 23c$). Again, pupils were not allowed to write down calculations, or share strategies for how calculations were executed. For the final part of the starter pupils were asked to solve the following division problem: if 3 bars cost €5, what is the cost of 1? A pupil suggested dividing 3 by 5, and Brona pointed out this was incorrect but did not explain why, or why division was the appropriate operation to use in the first place.

Main lesson

For the main lesson, pupils were given price lists from two popular supermarkets, and were asked to compare them. They were asked to examine the two lists and asked to identify which supermarket had the cheaper items and then find the difference between the expensive and cheaper items. This resulted in some confusion because Brona did not explain exactly what they were supposed to be doing, so pupils did not know whether they were required to calculate the sum of all the items on each list and compare this, or compare similar items on both lists.

When Brona asked the pupils how to compare the items, one suggested using subtraction. Brona told the pupil this was correct but did not indicate that ratio (i.e., division) could also be used to compare prices. This would have also been a good opportunity to use percentages, particularly since this was revised in the lesson starter. Finally, pupils were asked if four people split the bill, how much would each end up paying. While the pupils were doing this calculation in their copies, they were told they had “no excuse to talk…you’re working by yourselves”. The early finishers were told to choose 12 random items from the lists and calculate the sum of these items in both supermarkets and find difference between them.

Lesson Conclusion

Pupils played a Snakes and Ladders game which involved using the four arithmetic operators.

Post Lesson Discussion

Brona was initially happy with the lesson and talked about one pupil who usually requires additional help, but today he knew what he was doing and did it at a “quicker” pace. She also referred to one pupil who does things his own way and had his own strategies for comparing prices, so she “let” him do this “because its working for him”. Although sometimes she invites him to the board to share his strategies, she is concerned this may cause confusion for the other pupils.

Brona asked me how she should respond when pupils do not “get” something. Regarding percentages, she explained children are “doing it right” but not getting the right answer. They are doing, for example, multiplication…and getting the calculations wrong on this. She explained they were trying to do calculations in their head and getting them wrong as a result, which leads to the wrong answer. I suggested she explain operations clearly, with mathematical clarity and encourage pupils to use paper and pen.

She explained this would lead to resistance from pupils becaue they have one way to do things and “it is the way it is”. She also explained pupils do not like to explain the meaning behind mathematical procedures, even at the most basic level. For example, she explained that pupils will complete algorithms on board but will not speak to their basic actions. Interestingly, Brona described the pupils’ idea of mathematics as one of routine and habit.

Reflection

This was a very silent lesson, with no collaboration or discussion between pupils. The mathematics lacked any depth, cognitive demand was low and there was no productive struggle. Most solutions pupils provided were calculated by rote learned procedures. The one boy who did have alternative solutions was discouraged from sharing with the class because this may cause confusion, when in fact this is a significant learning opportunity. Furthermore, the lesson was not interesting for the pupils. I believe Brona, and some other PSTs, seem to think "real life situations" make up for mathematical understanding and explanation and games make up for inquiry and curiosity. Brona's approach seems to be motivated by what she perceives as her pupils' dislike of engaging in meaningful mathematics.

I also question the motivation for a version of mental maths that does not involve discussion of strategy, and alternative strategies. I also question the value of not allowing pupils to write things down, because this could promote mathematics understanding and is fundamentally more inclusive.

4.3.4 Comparison between high and absent/ low MQI

This section will use the data collected from observations, as presented above, to attempt to present a general idea of what mathematics lessons in both categories looks like in a way that can be used to inform future approximations of practice. This section uses a range of MQI scales including richness of mathematics, cognitive demand, remediation of errors, and pupils' contributions.

Richness of mathematics

Those participants in the high category delivered lessons steeped in rich mathematical teaching and learning. This included teacher and pupils' explanations of concepts and reasons, multiple representations of mathematical ideas to aid in pupils' understanding, and the use of generalisations. Mathematical language was well defined and used intentionally to help improve pupils' understanding and capacity to engage meaningfully in lessons.

For those in the low MQI category, overall richness of mathematics teaching and learning was generally low. Not only were aspects such as multiple procedure methods, generalisations, etc. not present, but participants seemed to have a lack of awareness about them. Explanations were procedural and were often no more than a recall of steps. Furthermore, PSTs appear to adapt this approach at least partly because they feel this is what pupils want, combined with a concern that more conceptually orientated mathematics will cause confusion for pupils.

Mathematical sense making was present and high in MQI-H lessons. This is a broad idea, which involves meaning, relationships, connections and whether or not things make sense. Claires' lesson on Eulers theorem is a good example of this. On the other hand, reminding a pupil to include a decimal point rather than reasoning where it should go is an example of the absence of mathematical sense making, and this is typical of a low-MQI lesson.

As I reflected on the quality of the lessons I observed, I noticed a feature common to all of the PSTs' lessons, which falls outside of the MQI framework. In my reflection I referred to this feature of mathematical flexibility, and the essence of this is captured in the following reflection:

Reflection

There is a related idea not explicitly included in the MQI framework which I refer to as 'mathematical flexibility'. This is a term I use to describe a PST's ability to broaden the conceptual scope of a lesson, even when the pupil, perhaps unknowingly, gives an opening for the PST to do so. I have observed a lack of this flexibility, even in high MQI lessons. For example, in Sharon's lesson when the PST realised there were infinite lines of symmetry running through the centre of a circle, there was a missed opportunity to broaden the discussion about the circle more generally. This is an important part of relational mathematics which is clearly Wu (2011) describes as a "coherent whole". I think this idea is important for PSTs and me to acknowledge in future approximations of practice, and address in a way that maximises potential learning of pupils.

Task Cognitive Demand

One of the main differences between those PSTs who demonstrated high MQI and those who demonstrated low MQI related to cognitive demand. This refers to pupils' engagement with tasks where they are required to reason and think deeply about mathematics. By and large, cognitive demand was present in MQI-H. Three examples from the MQI-H categories were children exploring the idea of infinity in Sharon's lesson, testing conjecture in Claire's lesson, and giving children the content knowledge and opportunity to justify strategies for estimating angle size in Vicky's lesson.

On the other hand, the cognitive demand of mathematical tasks for the lessons in the low MQI category was generally not present. Cognitive demand captures pupils' engagement with tasks in which they reason and think deeply about mathematics. It refers to enactment of the task, regardless of how the teacher initially designed it. However, these lessons centered around recall of well-established procedures and algorithms with reproduction of facts. Exploration of mathematical ideas was either absent or unsystematic and always surface level. These lessons also lacked the collaborative element which contributes to a low cognitive demand because it limited sharing and challenging ideas.

Cognitive demand, as described above, does not refer to the difficulty or challenge, which was also absent across participants performances. This may have something to do with participants not wanting to push pupils outside of their comfort zones and is also reflected in the fact that remediation is also generally low across participants. That is, when the difficulty of a task is low, there will be less necessity for any sort of remediation.

Remediation of Pupils Errors

Although remediation of pupils' errors was sparse for both low and high performing MQI, where it did exist, it was on a conceptual level in MQI lessons, and on a procedural level in low MQI lessons. Examples of conceptual remediation include Sharon correcting a pupil's confusion about the generalisation of shapes by asking her to think about its definition and make a judgment based on this. On the lower end, remediation is enacted in low MQI lessons by correcting procedure and not going beyond correcting pupils' answers.

Pupils' Contributions

A significant difference between MQI-H and MQI-L/MQI-A is the extent to which pupils were encouraged to contribute. Although there were obvious contributions from pupils across the observed lessons, it is the nature of these contributions that differed. On the lower end of the MQI scale, pupil's contributions could be described as "pro forma", with little influence on how the lesson developed as a result of that contribution. Those examples typically include calling out answers or steps in a procedure. PSTs' choice of language seemed to also reinforce this culture. For example, Jessica told the pupils: "I want everyone quiet and looking up…" as opposed to, for example, "listen carefully so you can compare this to your solution and perhaps come up with another way of solving this problem". A more extreme example was Bruna's request for silence when told her pupils: "…we have no excuse to talk…you're working by yourselves…if you have a question put up your hand".

Contributions from MQI-H lessons were fundamentally different. Pupils were actively encouraged to contribute to the lesson and communicate with the PST and other pupils. These contributions were valued and had an impact on the overall shape of the lesson. In this regard, learning was often incidental and occurred more naturally. Examples include pupils exploring geometry in the school yard and reporting back to Sharon and other pupils for discussion, or the pupils in Vicky's class justifying their answers which were then used to engage other pupils to develop the lesson further and maximise learning from individual incidents. Such an approach requires the PST to have confidence in their mathematical knowledge and flexibility around lesson planning and execution.

4.4 Concluding remarks on observations

The classroom observations highlighted three examples of high-quality mathematics teaching for the stage these PSTs are in their teacher education program. I have not observed lessons of this quality from B.Ed. year 3 PSTs in previous years which suggests the intervention is having some positive impacts on their practice. I also observed some poor-quality lessons which suggests there is more work to be done in terms of improving the intervention in Cycle 2. Incidentally, one of the things I noticed about the PSTs who enacted low MQI lessons was their lack of awareness of what high quality mathematics looks like. This suggests the intervention requires more explicit discussion about MQI coupled with more intentional reflection to make meaning from this.

Interestingly, PSTs who demonstrated a low MQI score does not necessarily mean they have deficiencies in MCK. For example, Gemma who scored low on her MQI was described to me by her peers as exceptionally able at mathematics, and her exam results confirm this. In post-lesson discussions, PSTs who scored low MQI offered deep mathematical explanations during post lesson conversations, which suggests they choose not to enact that knowledge for various reasons. At least in some cases, PSTs low MQI scores appear to relate to affective issues and their belief that pupils cannot engage in meaningful mathematical work. Reasons for this, reported by PSTs, include pupils lacking the ability, fear it will cause anxiety, or simply that pupils do not have a desire to lean mathematics in a relational way.

In fact, Gemma, Jessica, and Brona – who had the lowest MQI scores of the PSTs involved in this part of the study – appear to enact classroom practices that are inconsistent, not only with their competency, but also with their beliefs. All three PSTs enacted mathematics lessons that were purely instrumentally based, yet in the pre-intervention survey, when they were asked to articulate their beliefs about mathematics, each of them reported having beliefs that were more consistent with relational understanding. In relation to her beliefs about mathematics teaching and learning, Brona reported the following:

> "I think a hands-on approach featuring guided discovery and physical resources is the key to a child centred mathematics education. In doing so children can come to their own understanding of mathematical concepts at a pace that suits the individual child".

Similarly, Jessica explained:

> "I think it is important to put emphasis on understanding, not only on getting the correct answer. Hands on activities reinforce learning and are valuable [for pupils]".

Finally, Gemma reported the following:

> "Students may feel more engaged in maths if they are using hands on practical maths activities that make maths relevant to them or by actively exploring maths problems so that they make sense to the child".

These inconsistencies should be considered in light of the literature on the topic. As discussed in Section 2.4.2 of the Literature Review, teacher beliefs are complex, and several researchers have shown that teachers sometimes enact a pedagogy of mathematics that is inconsistent with the beliefs they espouse. Raymond (1997), Hoyles (1992), Skott (2001), and Sztajin (2003) agreed that context plays an important role in teachers practice and this this context may appear to override their beliefs. More specifically, Raymond (1997) found that general educational issues (e.g., upcoming standardised tests) can cause teachers to behave in a way that is inconsistent with their beliefs. Skott (1992) agreed that teachers' beliefs are often overshadowed by their current goals for the pupils. Hoyles (1992) argued that beliefs are situated, and how they are enacted depends on the context the teacher is in. On the other hand, Sztajin (2003) found that teachers practice in the classroom are determined not only by beliefs about mathematics, but also beliefs about society, pupils, and education more generally.

To determine the reasons for all the choices PSTs made on their SP is beyond the scope of this study, but as a teacher educator, it is important to understand that PSTs may, for various reasons, appear depart from their beliefs. Furthermore, it is important to share this knowledge with PSTs at the appropriate time and problematise the phenomenon with them via intentional reflection (Philipp, 2007) so that it can be addressed.

In any case, these PSTs tend to resort to mathematics based on instrumental understanding which, ironically, compromises pupils' mathematical understanding and increases the likelihood of developing mathematical anxiety (Beilock and Willingham, 2014; Lake and Kelly, 2014). It is also interesting to note that when participants demonstrated a low MQI score, the mathematics in their lessons was not necessarily incorrect or rife with errors. On the contrary, their use of mathematics was limited to instrumental understanding, there were not many opportunities for errors.

4.5 Implications for Cycle 2

This section presents some of the significant improvements that were made to the intervention based on the data collected from Cycle 1, and from entries from my reflective journal. These changes also add important context for the interpretation of results from Cycle 2.

Data from focus groups and observations suggest one of the main challenges for PSTs was dealing with pupils' attitudes towards learning mathematics, and an apparent aversion to understanding mathematics relationally. In many cases, these attitudes were at odds with the educational values and theories that underpin the intervention and my approach to teaching mathematics. These pupil attitudes put PSTs in a position of conflict because, on one hand they are encouraged to practice relational mathematics while on the other, pupils (and sometimes their class teachers) demonstrate a preference for instrumental understanding of mathematics. Coupled with this is the unpredictability of SPTs expectations regarding mathematics teaching. Although it is debatable why many pupils prefer instrumental mathematics, the overarching neoliberal influence on Irish education is likely part of it. From the point of view of the intervention, the question is how can this conflict be addressed?

I wanted to refine the intervention so that PSTs, and by extension their pupils, break free from the oppressive nature of instrumental mathematics. Reflecting on Freire (1970) and my theoretical position I decided to look for ways to include a level of criticality and authentic reflection in PSTs' practice. This necessarily involves a dialogical approach between PSTs and their pupils to give them access to educational freedom by moving away from the banking approach to education. Cycle 2 therefore introduced a new high-level teaching practice called 'norms for giving mathematical explanations' (See Appendix 14). This HLTP focuses specifically on mathematical explanations in the context of classroom dialogue, while promoting a relational understanding of mathematics (TeachingWorks, n.d.), and as such aligns with Dewey's (1902/ 1990) position that education should be an active and constructive process which reduces "unhelpful competition between pupils" (p.15), leading to a sense of social cohesion and cooperation through active work, communicating and exchanging ideas (p.15). Furthermore, these norms and routines are epistemologically aligned with the MQI framework and are complemented by Kersaint's (2015) '100 questions to promote mathematical discourse' (see Appendix 11).

The other major change to the intervention was in relation to the MQI framework. Observation data showed that PSTs enacted HLTPs reasonably well in their practice, and even explicitly acknowledged them in conversations I had with them. However, there was a lack of focused attention to the MQI framework, which was used to guide the enactment of PSTs' mathematical knowledge. There was little evidence of its impact on planning documentation or in PSTs' classroom practice. On reflection, its use during the intervention in Cycle 1 was perhaps too informal and ad hoc. Therefore, there is a clear requirement in Cycle 2 to place a more explicit emphasis on the framework, as a formative learning tool for approximations of practice, and link it also to PSTs' overall MCK. Along with relevant HLTPs it should be presented as central pillar of the intervention.

To maximise the effectiveness of the use of the MQI framework, I decided with agreement from PSTs, to focus on one content area at a time, i.e., each pod/ group focused on enacting the same content area. Although this resulted in less mathematical content included as part of the intervention, it allowed PSTs to share similar experiences, thereby opening opportunities for collaboration and critical dialogue. Of course, updated videos of content work were still available to PSTs. This change resulted in more meaningful opportunities for reflective practice, and this coincided with my decision to make PSTs' written reflections a formal part of the maths competency module in Cycle 2.

Finally, the intervention was carried out during Covid-19 restrictions in Cycle 2 between September and November 2020. Fortunately, most teaching remained face-to-face, but one session was cancelled while another was done online. Unfortunately, restrictions meant there was no scope for classroom visits to collect observation data. Instead, data was collected from planning documentation and post lesson interviews, and the focus group was conducted online.

Chapter 5: Cycle 2 Analysis

5.1 Introduction

This chapter presents the analysis from Cycle 2 of the study, beginning with the focus group interview, and followed by PSTs' lesson planning documentation, and online post lesson interviews. The previously mentioned Covid-19 restrictions provides important contextual information because it meant live observations were not possible. PSTs planned initially to teach face-to-face, but immediately prior to commencing SP, restrictions were introduced, which mandated that teaching should be carried out remotely.

5.2 Focus Group 2

As explained in the methodology chapter, just one online focus group was carried out in Cycle 2, and this took place shortly after PSTs finished their final 4th year SP. Braun and Clarkes (2006) thematic analysis was used to analyse the data from FG2 as it was for FG1a and FG1b. Directly before the focus group took place, the PSTs were given a short presentation to remind them of the overall aims of the study, the themes generated in Cycle 1, and the changes that were made to the intervention. Using this presentation as a stimulus, the PSTs were asked what further changes they would recommend for the intervention, and how the intervention has changed, or will change, their classroom practice. This was then used as a central organising concept (Braun & Clarke, 2006) for FG2 to generate a new set of refined themes to further improve the intervention and my practice as a Teacher Educator.

The focus group was carried out after an extended 10-week placement, five of which was in an SEN setting and the remaining five weeks in a mainstream setting. One of the limitations of the online platform was a reduction in group dynamics compared with the focus groups in Cycle 1, and this subsequently restricted the flow of conversation. Consequently, there was less balance of input across PSTs with the stronger personalities contributing disproportionately more. For example, Fiona provided quite a lot of input whereas Robbie's input was minimal. Although efforts were made to include all PSTs, as a moderator this was difficult to manage on an online environment. Apart from explicitly asking for input from more reserved PSTs, an effort was made to observe other forms of agreement or disagreement, including non-verbal gestures. This is an important contextual factor for the interpretation of the results presented in the remainder of this section.

5.2.1 The Intervention should begin earlier in the B.Ed. programme

Although vocalised by Fiona, many other PSTs agreed, either verbally or by gesturing agreement, that the intervention introduced at the beginning of year 3 was difficult to adjust to because it was so far removed from their previous experiences of learning mathematics. Although the adjustment period was not definitively communicated, PSTs agreed it was not until the second cycle of the intervention in year 4, that they began to get the most benefit from it. Fiona noted they "*got more benefit*" from the intervention in year 4 than in year 3 because of the radical change in pedagogical approach (fg2-5). For the first two years maths competency was described as "*passive*" whereas in in Year 3, participants were introduced to a range of new ideas, including approximations of practice, and they were expected to be active, analytical, reflective, and creative as part of the teaching and learning process. While participants agreed this was a welcome change, it was also overwhelming for them. Essentially, PSTs' experiences of learning mathematics in years 1 and 2 was a distinct compartmentalisation of content and pedagogy, and they then experienced a "*shock*" when they were asked to bring it all together in a new way (fg2-9). Fiona explained that because they were fully unfamiliar with the various elements of the intervention (HLTPs, MQI) that it was "*too late nearly for us to be hearing it*" (fg2-20).

Consistent with responses in Cycle 1, Fiona explained that in 3rd year it was difficult to let go of the 70% examination pass threshold, so much so that it was difficult to allow themselves to commit fully to the new approach. She explained: "*Because in 3rd year it was a small bit like… we didn't know and I think we were all far too focused on the exam and getting 70% then actually… allowing ourselves to commit to it in a way*" (fg2-4).

There was agreement amongst PSTs that the intervention needs to be implemented consistently across the four years of the B.Ed. programme to maximise its benefits. Referring to the content-pedagogy divide, Fiona questioned why enactment pedagogies had not been part of the module since first year because "*by the time we got to third year it was kind of hard then to… kind of go back and close that gap that was there*" (fg2-9). Eve characterised maths methods as a play-based approach versus "*the theoretical side of it*" in maths competency (fg2-16). In addition to this, Fiona was critical of the fact that "*the pedagogy side of it should have been explored more…in MSE*" and explained that it should not be the sole responsibility of maths competency to address this In any case, she explained there "*was* *something wrong*" in 1st and 2nd year and it was not until 3rd and 4th year that she felt prepared to teach mathematics, which she attributed to the intervention (fg2-15).

However, Fiona explained this gap, which was highlighted in Cycle 1 still exists, and this limited their learning:

> "*it would have been great if we had the chance to delve into it a little bit more…but I suppose like as well… we lost time as well due to covid and everything so maybe if we had a full term in 3rd year and 4th year at it then it would have been fine. But I just think that gap is kind of still there*" (Fiona, fg2-11).

Fiona's suggestion here is that content and pedagogy should be fully integrated across all 4 years of the B.Ed. programme and run for entirety of each academic year. That is, content and pedagogy should be fully integrated across the entire module.

5.2.2 Maths Competency is versatile

The benefits of the intervention were explored in Cycle 1, but these were established in the context of mainstream classroom teaching. FG2 offered additional perspectives from which to evaluate the intervention because it combined both SEN and mainstream components. In terms of SEN teaching, Helen explained that understanding the content herself allowed her to "bring it back to basics" (fg2-25). Using the example of division, she said if she did not fully understand the content, she felt she would not have been able to teach is as effectively to pupils with additional needs and learning difficulties. According to Helen, the children she was working with often asked more fundamental questions which required a deep rather than a superficial understanding to answer. She was asked, for example, about the meaning of division and how it related to grouping. She explained she was confidently able to answer such fundamental questions because it was similar to content explored during the intervention in Cycle 2. Specifically, Helen was able to use her content knowledge about different interpretations of division to explain the concept in a way that made sense to the pupils she was teaching. Regarding effective online teaching, Paul explained that the modelling content HLTP was useful when teaching asynchronously.

He explained:

"I was making videos of me doing questions and stuff and talking through it so I was kind of like doing the explanation and the modeling and steps…you do this, followed by this and this and this. So, I found that the modeling that we did…was very relevant… because that was the way that you had to teach on the online placement because obviously you weren't in the classroom and you couldn't really…give the children a problem and ask them to try that now and then discuss it then after" (Paul, fg2-96).

Because the content was carefully modelled and recorded to video, pupils could access the material at a time that suited them. Based on their feedback, Paul could then respond appropriately by modelling new content.

5.2.3 Practice based instruction in ITE improves MCK

There were several examples of PSTs talking about teaching division on SP (i.e., the shared topic from the intervention in Cycle 2). For example, in Helen's SEN example above, she explained how she was able to effectively teach division because it was explored during the intervention. This is significant because Helen was able to recall her MCK about division, which contrasts with descriptions about the problem of enactment discussed in the reconnaissance section. There were additional examples of the problem of enactment since this study began which are worth considering. During classroom visits in Cycle 1, Freda was teaching fractions to 6th class, and in our post-lesson discussion, she admitted not remembering *that* we studied fractions the previous year. At the time, this was very concerning because it suggested maths competency had minimal impact on practice. However, this is consistent with the literature because the original maths competency module lacked "sufficient, suitable and realistic experiences tailored to the needs and concerns" of PSTs (Korthagen, 2009, p.104). Looking at this, and similar occurrences, in the content of the literature brought a heightened awareness of the problem which made instances of it more recognisable. For example, in a post lesson discussion in Cycle 2, Lilly did not remember what content was covered in years one and two of the traditional maths competency module and was therefore unable to apply this knowledge in her practice. To examine this theory further, I asked year 4 PSTs to prove the distributive law for whole number multiplication over addition, and no one admitted to having knowledge of this, despite the fact it was a key component of year 1 maths competency and a distinct question on their examination. These examples confirm Korthagen's (2009) assertions that teaching content, mathematics or otherwise, without sufficient practical context, results in minimal impact on classroom teaching.

Brona agreed a practical context is essential for enactment and contributed additional nuance to this explanation. Because the focus in y1 and y2 was based on content that was ultimately going to be examined, she explained their primary motivation was to pass the exam. She described how this exam focused mindset began to change when approximations of practice were carried out with the visiting pupils in Cycle 1. At this point the priority switched from being about an exam and "*how can I get the best mark*" to "*how can I improve my practice, so that it benefits the children*" (fg2-33). She clarified that any modules based on an examination is experienced by PSTs in a "*vacuum*" and knowledge is temporary and "*gone out of your head…a week or two weeks later*". The practical elements of the intervention, and opportunities for reflective dialogue associated with this practice, helped PSTs frame the mathematics content in a way that made sense to them. In particular, the introduction of discussion had a positive influence on their learning. Brona explained that discussing content and practice with peers and lecturers "*is brilliant because you're going to remember a discussion you had, [but] you're not going to remember things that you learned off from a page*" (fg2-35).

Emily emphasised this point and explained that, during Cycle 2, it was the collaborative aspect of planning and enacting approximations of practice, and specifically teaching together "*was a really concrete way or showing how to bring it into the classroom. It was really effective*". She said:

> "*it made it really easy to remember how to teach effectively when you saw it being done in the lecture. Like, instead of just reading it out of a book or reading an article or something, it was just really helpful to just see it in action in the lecture room*" (fg2-59).

Brona was able to generalise this idea to ITE and explained that other competency subjects, assessed by examination, lack the necessary practice focus. The consequences of an examination being the primary motivator is that in a practice situation, learning becomes difficult to transfer to the classroom, and Brona provided another example of not remembering the specific content knowledge concerned with teaching apostrophes in an English lesson. Although, specific content knowledge around this had been covered in lectures in y1, she said "*I had to go back and teach it to myself*" because she could not remember back to first year (fg2-37). An interesting and significant aspect of this is that she did not look back over her designated notes to revise the topic, although she had the opportunity. Instead, she researched the topic herself to "*find the ways I can teach it in a child friendly language*" (fg2-39). Her motivation was to find ways to teach material in a way that pupils will understand. Brona appears to make little distinction between her learning of content and pedagogy, focusing instead on the needs of the pupils she teaches. This example further supports the requirement for the integration of content and pedagogy in mathematics teacher education.

5.2.4 Generalisability of Learning

During the intervention in Cycle 1, PSTs were offered a range of curriculum strand units (mathematics topics) to choose from to cover as much of the mathematics curriculum as possible. Because each group picked a different topic and were expected to share their learning, it was hoped this would result in all PSTs being prepared to teach content across the curriculum. This decision, in fact, resulted in less quality learning and less collaboration than was anticipated. To address this issue, one of the main changes to the intervention in Cycle 2 was to focus on just one topic. PSTs agreed this should be division because it is a topic which they find difficult on a conceptual and pedagogical level. It was hoped this focus would give them confidence with division by generating a shared understanding of it and refining this understanding through group reflections. Although this focus addressed a problem identified from Cycle 1, I had concerns about PSTs' preparedness to generalise their learning to other areas of mathematics. Generalisability in this case refers to:

- Teaching practices (discussion, modelling, etc.)
- Desired philosophy/ beliefs around teaching maths for relational understanding, as encompassed by the Mathematical Quality of Instruction (MQI) framework.

I asked PSTs about this in the focus group. Because we only looked at division (albeit through the lens of various practices) Brona explained that she feels prepared to teach division. She claims to "*fully understand division…and could teach [it] to 5th or 6th class in the morning*" (fg2-

45). She also expressed the desire for any future conception of the intervention to include more content areas if there was time. Adding to this, Gemma expressed the usefulness of discussion as part of the overall approach to the intervention for teaching mathematics and "*how that could be transferrable to different areas of maths where I might not have thought to use discussion so much*" (fg2-47). Gemma clarified she was referring to the 'Leading a Mathematical Discussion' HLTP, but also the more general methodological approach of sharing ideas, collaboration, group work and group reflections which were used as part of the intervention. Perhaps unsurprisingly, the HLTPs and other methodological approaches to teaching mathematics are generalisable across content areas. That is, once a practice is learned it seems likely to be enacted by PSTs in a range of different areas of mathematics teaching.

What is perhaps more significant is whether the relational approach to teaching mathematics can be transferable from one content area to another. In this context, a PSTs' approach to teaching mathematics is influenced by the enactment of HLTPs used in the intervention. For example, Gemma referred to how using discussion as a method for teaching mathematics can slow down the pace of the lesson and allow for deeper learning. Gemma was confident that it was possible to achieve this approach to deeper learning and referred to how the resource '*100 questions to promote mathematical discourse'* was a useful tool in supporting the enactment of *leading a mathematical discussion*. It was a means of demonstrating that it is possible to slow down and "*delve deeper*" into the content being taught, whatever that content may be (fg2-53). Of course, to "delve deeper" is a vague term and does not necessarily mean teaching for relational understanding. However, Gemma did also reference the content videos when she clarified they "*kind of helped too*" and complimented the HLTPs which suggests she is aware of the necessity to intentionally bridge the content/ practice gap. She also noted she found these videos "*really useful*" in terms of her own maths upskilling and "*helped with…how to teach it and how to actually implement it in a classroom, especially for the older years*" (fg2-55). She reiterated that MSE is good for the younger classes, but the videos helped to nurture a "*deeper learning*" with the older classes.

5.2.5 PSTs should be supported to enact relational mathematics

Results from Cycle 1 showed that enactment is not only dependant on PSTs, but also SPTs, CTs, and pupils, and changes were made to the intervention between cycles to address some of this. For example, education was provided to SPTs and the HLTP about mathematical norms was introduced. Nothing was put in place to specifically address the influence CTs have on enactment of mathematical knowledge. I made this choice because I did not want to interfere directly with SP, in a way that might be interpreted as coercive or mandatory. However, Fiona addressed this issue directly and made the point that despite high quality learning within ITE, when they enter the classroom, the external demands put on them on the CT "*might hold you back a small bit*" (fg2-4). She suggested that if PSTs had a structure based on various elements from the intervention (e.g., HLTPs and MQI) integrated into their planning, the CT "*just might allow it to happen more, instead of always focusing on the textbook*" (fg2-65). She argued this approach would give PSTs more confidence to approach the CT with ideas around enacting meaningful mathematics in the classroom "*instead of the teacher having to give them topics and pages of a book to do*" (fg2-65). Eve agreed with Fiona this would provide an evidence-based rationale for the approach they are taking. Rather than an ad hoc decision to include discussion as a core practice in mathematics lessons, she claims that its more legitimate to "*have the theory behind why you're doing it that way. Do you know like, you have an actual reason for it* (fg2-70).

Admittedly, what I thought might be undemocratic or coercive is a means of empowerment for PSTs within the SP environment where they could be seen as less powerful and where some teachers "*just won't accept*" PSTs' ideas (Eve, fg2-72). Providing PSTs with a framework for enactment could empower them while legitimising the work they are carrying out. Eve suggested that similar to their English schemes, if there was a structure to include headings and subheadings related to elements of the intervention then the structure would be there for them to enact their learning.

5.2.6 Mathematics Assessment

There were several previously unplanned changes to the research design and the intervention due to Covid-19 restrictions. Because large gatherings were prohibited the usual end of semester University examinations could not take place. As an alternative, maths competency examinations were converted to assignments and participants were given several weeks to complete them. The assignments were similar to normal examinations (i.e., a focus on content) with an additional section based on reflection and practice. This element allowed PSTs to reflect on the content of the examination (MCK) but in the context of classroom teaching and with specific focus on pedagogies of enactment. While completing the assignment, PSTs had open access to lecture notes and video content.

Since there are clear concerns about the effectiveness of traditional timed examinations in terms of knowledge transfer to the classroom, I wanted to find out the efficacy of this approach as a longer-term strategy as an alternative. When I asked about this, Paul offered his insight into why examination knowledge to classroom transfer problem exists. He explained that the examination "*doesn't really mean that much*" to them yet brings with it pressure and intimidation, both of which are exacerbated by the 70% pass threshold, which leads to "*rote learning*" and "*cramming to get the grade or get the marks as opposed to actually understanding necessarily what you're doing*" (fg2-79). Paul explained that with examinations of this nature "*you're just kind of learning something for that one- or two-hour period*". Maeve agreed "*when you're in an exam it's just learning off and you know, learn it off and forget within a few weeks*" (fg2-83).

In terms of learning mathematical content for use in the classroom, PSTs agreed the assignment is more effective because of the additional time engaging with the content in a less stressful situation. Paul emphasised that when this is done in a more relaxed way with the content videos as a guide the learning happens on a deeper more permanent level. Paul claimed that the videos were "*probably the biggest benefit of the whole intervention*" because it allows them to take the time to engage in the content independently and comprehensively (fg2-79). In agreement with this, Maeve said with an assignment "*you have to understand it so when you're doing an assignment, you're going to do so much reading and so much research*" (fg2-83).

5.2.7 Implications for practice

Because PSTs were due to graduate from the B.Ed. program in the next academic year, I asked them how the intervention, over the last two academic years, is likely to influence how they will teach mathematics.

There was general agreement that PSTs lacked autonomy on SP. For example, Eve described this pressure of "*trying to incorporate all these things like trying to make it fun and interactive and there's so much pressure on you to meet those standards*" (fg2-88). They reflected enthusiastically in anticipation of the autonomy that will come with having their own class. This, according to Fiona, will allow them to "*slow down and allow the time to delve into things instead of just rushing through the chapter of the book*" (fg2-87). She intends to use group work, discussion, and modelling to promote a deeper understanding of mathematics in an enjoyable way. Eve also expressed the desire the "*freedom*" to slow down, to "*go deeper*", and move away from the textbook, when necessary, to give children a chance to better understand mathematical concepts (fg2-88).

When I asked about what might obstruct of these freedoms Paul referred to wider school plans and cultural norms which might override individual teacher plans. However, Paul also believes that if teachers are knowledgeable enough and have a strong rational for "*why you're doing it a certain way I'm sure that any reasonable principle or team of staff will understand that, well this is why they're doing it and the children actually all understand this now*" (fg2-90). Paul strongly emphasised this need for children to understand mathematical content relationally and argued that if this is prioritised then time will be saved because you will not have to "*go back all over it again because [pupils]…don't know why they're doing it or what's the reason for it*" (fg2-90). Like the other participants, Paul believes that slowing down and taking the time to discuss concepts is how this level of understanding can be achieved. Paul also referred to the idea that having a deeper understanding of one concept could be beneficial for learning other concepts also, which is a tenet of relational understanding.

Several PSTs referred to teaching mathematics for conceptual understanding, and how it is important to slowly "delve" into concepts, primarily through discussion and modelling. Helen also mentioned this in the context of SEN. In response to this, I asked the group if there was a consensus on the importance of teaching for relational understanding, and there appeared to be general agreement. Emily responded that teaching procedure alone, such as formulae, does not have meaning for children, and results in boredom. She referred to how exploring the meaning of Pi in college deepened her understanding of the concept. She explained how this mathematical knowledge helped her explain Pi, and aspects of the circle, more meaningfully to pupils, which made it more interesting and memorable for them. She explained, "*It's like when you just explain why something is the way it is rather than just saying this is how it is, just remember that*" (fg2-92).

5.2.8 Summary and Conclusion

A positive finding from the data from FG2 was the change in the nature of the discourse around teaching mathematics, presumably because of more prolonged exposure to the intervention. Compared with FG1a and FG1b, PSTs were clearly knowledgeable about the HLTPs they learned about during the intervention and were able to contextualise these ideas in relation to their practice. Another significant development between the focus groups in Cycle 1 and the focus group in Cycle 2, was more clarity from PSTs around their approach to teaching mathematics. This was evident from their articulation of ideas and the intentional language they used. Although PSTs did not explicitly talk about relational understanding or MQI, they did pay attention to the essence of these concepts in their responses about providing pupils with access to deeper and more meaningful mathematics. They used informal language such as "go deeper", "delve in", slowing down which are all synonymous with relationally understanding mathematics. However, what is not entirely clear is participants motivation for this approach. For example, Paul talked about saving time later in the year, which could be more of an efficiency goal than an educational goal. For Eve it was a way of staying away from the textbook, and for Emily a deeper understanding can reduce boredom. It is important to find out if there is a common underlying motivation relating to educational values amongst participants, so that this can be harnessed as part of my practice.

However, it would be desirable for PSTs to explicitly name the big concepts that underpinned the intervention so that relational understanding and MQI become a cornerstone of their planning. This could be ensured by implementing Fiona's suggestion to systematically embed core ideas from the intervention in SP planning. As discussed, this small change could potentially result in a useful balancing of the power dynamic between PSTs and CTs and consequently promoting the enactment of relational mathematics. PSTs' responses regarding move away from examinations in maths competency towards a more creative and dialogical mode of assessment are consistent with my theoretical position and the underlying epistemology of the intervention.

Despite the changes after Cycle 1, the content-pedagogy "gap" in their teacher education programme is still an issue of contention for PSTs, and this needs to be addressed. A single consistent approach across both mathematics modules, informed by a common technical language, is a minimum requirement to address this problem.

5.3 Lesson Planning Documentation

As outlined in Section 3.6.2 of the methodology, and the introduction to this chapter, live classroom observations were not permitted during Cycle 2 due to Covid-19 restrictions. To compensate for this, PSTs' planning documentation and other SP related artifacts, including some instructional videos, were analysed to gain some insight into their intentions for teaching mathematics on SP2. The purpose of this section was to analyse these documents to determine if PSTs intentionally used ideas from the intervention, including HLTPs and the MQI framework, in their planning. It is important to note that using these concepts in their planning was not a requirement. To gain insights into the reality of PSTs' day-to-day planning and practice, I assumed the dual role of SPT and researcher. As explained in Section 3.5.2, power dynamics were explicitly addressed with PSTs so that they were fully aware this, and to reassure them their honesty could not diminish their performance in any way. Furthermore, this dual role was supported by the stipulation that teaching performance was not graded but instead awarded a pass or fail mark. This essentially transformed the nature of my SPT role to that of critical friend, which is commensurate with the role of co-researcher.

A great deal of PSTs' teaching was carried out on an ad hoc basis because of Covid-19 related restructuring within schools. One PST did not teach mathematics at all, some taught very little, and some taught mathematics on a more regular basis. Furthermore, in some cases, PSTs' originally planned schemes were used in their teaching, while in other cases, new schemes were required to be developed. Additional contextual factors are explained in each section below. It is acknowledged that planning documentation does not necessarily reflect the realities of how plans are enacted in practice, but they do provide important evidence about each PSTs' intention and, as such, I am assuming there is a strong correlation between PSTs' planning and implementation.

To maintain consistency across reporting, results are presented below similar to how they were in Section 4.2, with each heading names after the pseudonym of the PST involved. Subheadings used are learning objectives, learning activities, (both of which make up a scheme) and lesson plans.

PSTs were required to use the National Induction Planning toolkit to guide their planning for SP2 (National Induction Planning for Teachers, 2022). The primary toolkit for planning and preparation contains, amongst other things, planning templates, ideas for structuring plans, support for writing learning objectives, and differentiation and assessment strategies. PSTs are advised that learning objectives are not embedded in the curriculum. The NIPT toolkit states that "content objectives should be broken down into manageable learning objectives including skills as appropriate" based on the children's needs. In short, PSTs' planning should start with curriculum and adapt overarching outcomes from this (i.e., schemes), and develop learning objectives from this using Blooms Taxonomy.

To align the analytical process with the methodological tools and concepts used in this study, the intention was to equate lesson objectives with mathematical knowledge and MQI, learning activities with HLTPs, and lesson plans with enactment. Some PSTs also used instructional videos for teaching mathematics remotely, which were also analysed where they were available.

For this SP period, PSTs were not required to develop detailed lesson plan because they were in their final year of the B.Ed. programme. Instead, they were asked to include brief lesson notes in their planning, which made it more challenging to determine the exact nature of what PSTs planned to enact in their lessons. For convention, the term lesson plan will be used in this section.

5.3.1 Eve

Context

Eve was due to teach multiplication and division of whole numbers and decimals to 6^{th} class pupils. However, due to restructuring due to Covid-19 in Eve's school she did not teach mathematics on this placement, so the only available lesson plan was for her anticipated first lesson. Learning objectives and the associated learning activities were completed fully before placement was due to commence.

Learning objectives and activities

It is not clear from Eve's learning objectives (LOs) her intent regarding the depth of mathematics she intended to teach. There were no explicit or obvious statements that suggest her intent to teach mathematics for relational understanding or use the MQI framework as a guide.

For example, one of her LOs was the identification of patterns related to multiplication of powers of 10, as well as identification of "division facts between 0-100", the meaning of which is not entirely clear. She also planned for discussion of "different ways to solve practical problems" but in each of these cases she does not identify if there is a conceptual aspect to the learning. For example, patterns related to multiplication of powers of 10 can mean many things and can be instrumental or relational but this is not clear.

Another LO was to conceptualise multiplication as repeated addition, which progresses to "multiplication of two-, three- and four-digit sums and decimals" as well as "division of three- and two-digit sums". However, there was no link between these two outcomes nor was there any indication of how these calculations would be done. That is, there was no evidence of any formal plan for pupils to understand these operations on a conceptual level.

Eve's learning activities suggest a constructivist orientation to her practice and included talk and discussion, guided discovery, problem-solving based on real life problems, co-operative learning, use of ICT, and active learning. An example of an activity was "looking at patterns and similar traits when multiplying, finding out the best way to complete the tasks". However, her references to active learning was ambiguous, and she did not outline how these activities were to be used or for what purpose. Importantly, they lacked detail and any meaningful connections to the intervention or enactment of specific HLTPs and relational mathematics.

Lesson plans

There were three learning outcomes in Eve's only lesson plan:

- use repeated addition to look at patterns in multiplication

- look at the rules for multiplication
- work together through two-digit multiplication sums.

The ambiguity of these learning outcomes suggest it is unlikely there would have been a relational component to the lesson. There is no clear indication about what is means to "look at" some mathematical concept. The use of the word "sums" also suggests a lack of attention to mathematical language because the mathematics curriculum (NCCA, 1999) only uses this word in the context of addition, and its meaning has also been consistently emphasised in maths competency since Eve was in year 1 of the B.Ed. programme.

The main development component of the plan had a strong focus on procedural learning in relation to the standard multiplication algorithm. There is one part of the lesson where pupils are asked to "decide how to relate addition to multiplication" but it is unclear what is means or what the mathematics concepts involved in this are to this are.

5.3.2 Lilly

Context

Lilly was teaching units from the money and percentages strand units of the curriculum to 5th class pupils.

Learning objectives and activities

Lilly's LOs for both strands show little evidence of her intention to teach for relational understanding or be guided by the MQI framework. They focus more on applying mathematics than understanding mathematics. For example, one objective is to find percentages (or fractions) of amounts of money and "compare value for money using the unitary method" but there are no objectives around understanding why the unitary method works.

Her LOs for the percentages content are also about application rather than understanding of concepts. In addition to this, there are 8 objectives related to percentages for what is likely to cover a two-week period. These include comparing percentages, fractions and decimals; conversions between each; finding a fraction or percentage of a number; and finding a whole number given a fraction or percentage of it. Given that percentages is just one of the topics

Lilly needs to teach over a four-week period, it is highly unlikely her LOs could be taught in sufficient depth to ensure pupils understand them on a relational level.

Lilly planned separate learning activities for both topics. Under the money strand unit Lilly planned "activities, processes and language for the children, modelling actions and thinking and mathematical discussion" so children could "share and explain their thought processes through group/pair discussions". Although HLTPs from the intervention were not specifically mentioned, there is evidence to support that Lilly's planning for the Money strand unit was informed by the 'leading a mathematical discussion' and 'modelling mathematical content' HLTPs. . However, there were less activities planned under the percentages scheme and notably there was no discussion involved and it is not clear why there are inconsistencies between both strand units.

Lesson plans

Consistent with her LOs, there were several references to modelling content in Lilly's lesson plans. However, there was no reference to a decomposition of modelling or use of vocabulary associated with this HLTP. The brevity of daily notes means that it is difficult to determine what the intended detail of the modelling was, but there was certainly more focus on procedure (e.g., carrying out conversions) opposed to any indication that pupils were learning about the reasons behind the operations.

Instructional Videos

Lilly included instructional videos on fractions and percentages which evidenced a primarily instrumental approach to teaching mathematics. For example, in one of the videos Lilly uses "modelling" to teach pupils how to increase 14 by 50%. Here are the pertinent points from the video which demonstrate the instrumental nature of the mathematics

- Lilly started by saying "*some of you can probably even do this in your head*".
- She equated 50% with $\frac{50}{100}$ without any explanation why.
- Reduced $\frac{50}{100}$ to $\frac{5}{10}$ by crossing off zeros without explaining why this is allowed.
- Explained that $\frac{5}{10}$ "can be simplified to $\frac{1}{2}$" without explaining why.

- To find $\frac{1}{2}$ of 14 she explained to divide 14 by 2 but gave no explanation why.
- Use of "short division" algorithm for $14 \div 2$ but no attempt to explain this.
- Finally, Lilly added 7 to 14 without being explicit that 7 was 50% of 14 and to increase 14 by 50% we need to add this on.

The sentence "*some of you can probably even do this in your head*" actively discourages pupils to understand the mathematics relationally, and each instance of explanation further emphasise this approach, and this is indicative of a lesson that would be in the low MQI category. Furthermore, while the term modelling was used in Lilly's plans, her implementation is inconsistent with the definition of modelling that was used during the intervention which emphasises a meaningful mathematical point and making the teachers thinking visible for pupils. Finally, this video lasted less than one minute in duration leaving limited opportunities for deeper thinking.

5.3.3 Robbie

Robbie's topic were probability and length, and he was teaching 5th class for the duration of the placement.

Learning objectives and activities

Robbie's LOs included those related to estimating, using, comparing, and applying length. There were no outcomes related to probability, and no direct evidence in the outcomes of his intention to teach for relational understanding.

Similarly, there was no reference to HLTPs in Robbie's learning activities. In fact, there was some confusion around the meaning and purpose of learning outcomes and learning activities. For example, one of Robbie's learning activities was "measuring the perimeter of objects" which is a learning objective because it indicates what pupils are expected to be able to achieve by the end of the lesson, not the teaching methodology, or HLTP, that will be used to do this.

Lesson plans

Despite the lack of deliberate attention to relational understanding in Robbie's learning objectives, his lesson plans include elements of meaning, explanation, patterns, experimentation, understanding and linkages in mathematics teaching and learning. For example, in a probability lesson involving coin flipping he planned for pupils to investigate the differences between expected results and actual results. In a subsequent lesson he talked about proving that rolling a 6 has the same probability as rolling any of the other 5 values. He highlighted the commutive property of multiplication also in the context of a probability lesson. He also focused on the properties of 2d and 3d shapes and linked these to real life examples. There was an element of development of computational thinking in several of his lessons. This is all suggestive of relational understanding.

Although Robbie's schemes do not indicate a plan to focus on relational understanding, his lesson planning suggests desire to teach a depth of mathematics that, if executed to plan, will promote relational understanding. This raises a potential issue about the extent to which PSTs' planning determines enactment. The seminal research by Zeichner and Tabachnic (1981) list the influence of co-operating teachers, the ecology of the classroom, the bureaucratic norms of the school, teacher colleagues, and even pupils as some of the potential reasons why plans would not be enacted in the classroom. Furthermore, data gathered during Cycle 1 of this study further support the notion that CTs and pupils strongly influence PSTs' classroom practice.

In any case, PSTs need to understand the powerful role planning can play classroom performance, and the important relationships between the different components of planning. It should also be acknowledged that PSTs are under significant pressure on SP and consequently may neglect some aspects of planning. Finally, it is difficult to tell if Robbie's lesson plans were influenced by the intervention because he did not use language specific to the intervention.

5.3.4 Paul

Paul taught the strand units 'Length and time' and 'Fractions, decimals and percentages' to 6th class.

Learning objectives and activities

In both strand units, Paul's schemes are suggestive of favouring application over understanding of mathematics, and this is particularly evident under the fractions, decimals and percentages strand where the focus is on procedural understanding only. One of Paul's content objectives includes to "express improper fractions as mixed numbers and vice versa" which is a standard objective from the curriculum (1999). However, despite there being opportunities to do so, he does not state how this objective, or other, will be achieved or the sort of mathematical knowledge required. Therefore, it is not possible to determine Paul's intentions from these. For example, Paul documented the following breakdown for an upcoming lesson:

- Utilisation of instructional video to introduce the topic and recap on prior learning of fractions, decimals, and percentages.
- Children will complete questions from their Busy at Maths book and their Shadow book also.
- The children will also be assigned optional work from their shadow book too.

Finally, there was no explicit reference to HLTPs in Paul's learning activities.

Lesson plans

Paul's lessons plans were also based on instrumental understanding. For example, there were worksheets for early finishers which contained 9 addition and 9 subtraction problems. Each of these problems were similar, with no opportunities for pupils to develop a depth of understanding. In each case, the standard addition and subtraction algorithm was set up, leaving no option for ad hoc methods of calculation. A significant amount of the learning outcomes from an analysis of 19 different lessons included the phrase "identifying steps and methods to…" carry out various procedures This is consistent with Paul's instrumental approach to mathematical understanding.

It is worth taking a closer look at a lesson plan Paul developed to examine the relationships between distance, speed, and time. There was an excellent opportunity here for pupils to understand the definition of speed and its relationship to distance and time, as well as the relationships between fractions, multiplication and division in a way that transcends any one lesson. However, Paul chooses to use the "distance-speed-time triangle" which removed the necessity, and opportunity, for pupils to understand these relationships. Additionally, Paul created instructional videos to support pupils in completing these problems, and these were also entirely procedural.

This lesson, due to its dominant instrumental focus, represented Freire's banking model of education, which relegated pupils to passive spectators to be uncritically filled with information (Freire, 1970), leading to what Dewey called waste in education (Dewey, 1902/1990). However, Paul also claims to be "good at maths" and received a gratification from getting answers correct without necessarily understanding the mathematics from a relational perspective. This suggests Paul has not progressed from Korthagen's (2010) Gestalt level of teacher learning because he treats pupils as passive listeners and is seemingly unaware of this. Finally, Paul noted in his evaluation of the lesson that "maths was a struggle for several students" but was unreflective about why this might be and never questioned whether his approach to teaching mathematics could be a factor.

5.3.5 Maeve

Context

Maeve developed two schemes for her placement, one for the combined strand units of weight, operations, and fractions, and the other for shape and space.

Learning objectives and activities

Learning objectives for both strands were generally instrumental in nature and included identifying instruments to measure weights, estimating weights, renaming units, and "solving" (i.e., calculating) "sums". Although the word "sums" is used in the mathematics curriculum (NCCA, 1999), in Maeve's case it was used inappropriately and, similar to Eve, it signifies deficiencies in her knowledge of mathematical language. She used to word to describe "subtraction sums", "multiplication sums", and "division sums". She planned to use the long division algorithm to find equivalent fractions without reference to the fundamental fact of equivalent fractions (Wu, 2010). Objectives for shape and space were also mostly lower order and includes identifying equilateral triangles and identifying pairs of parallel lines in quadrilaterals. Overall, there was no reference to relational understanding in any of Maeve's learning objectives, nor was there any indication that relational understanding of any of the concepts was an objective.

Maeve listed a wide range of learning activities for both schemes. Although she lists modelling and talk and discussion as an activity there is no evidence, based on the language used, that this was inspired by the intervention. She also included both modelling and direct teaching as learning activities, but with no clear distinction between them which suggests Maeve has a misunderstanding about the meaning of modelling as it was described in the intervention.

In her fractions scheme Maeve included a list of "questions for eliciting discussion" which ware taken directly from the Kersants (2019) *100 questions to promote mathematical discourse*. Furthermore, she included suggestions about how these could be applied to the topics she was teaching.

Lessons plans

Despite its omission from her learning objectives and activities, Maeve's lesson plans indicated she would like to teach mathematics for relational understanding, and include meaningful and cognitively demanding tasks.

For example, in one of her early lessons on operations she pays particular attention to place value by differentiating between values of digits in different places. Across her plans she consistently refers to "breaking down all the mathematical information" through content modelling in her instructional videos (these videos are discussed below). For example, one of her lesson plans refers to a video to explain "the difference between 1 km and 1 m". Her demonstration of "dividing metres and kilometres" suggests a deliberate focus on relationship between m and km, although this relationship was not made explicit in her notes. She also intentionally used discussion alongside content modelling and other constructivist methodologies in her lessons.

Maeve regularly used mathematical vocabulary that promoted meaning. In one lesson plans she planned to explore the meaning of "parallel, acute, obtuse, angle, symmetry and congruent through a whole class discussion". In another lesson plan, pupils were tasked to "create their own definition of a quadrilateral by looking at images of a square, a rhombus and a trapezium and identifying what they have in common to be quadrilaterals". This task suggests an openness to cognitively demanding tasks with inherent creativity, and fundamentally provides opportunities for pupils to experience democracy through meaningful educative experiences (Dewey, 1916). Her final lesson was as planned Euler's theorem and pattern spotting. Although this lesson plan demonstrated a high MQI, it was included on my recommendation based on the success Sharon had with this lesson in Cycle 1, and I advised Maeve to collaborate with Sharon on this. This collaboration led to a successful lesson plan and is congruent with the reflective, collaborative, and dialogical process between teacher and students as conceptualised by Freire (1970).

As with most lesson notes, there was not enough transparency to make definitive judgements on the lessons in terms of HLTPs and MQI. In this regard, access to Maeve's instructional videos provide another useful layer for analysis.

Instructional Videos

In one of her lesson plans Maeve wrote: "Teacher models how to divide a decimal number (kg) by a whole number, using Microsoft whiteboard on a video for the children". However, in the instructional video where Maeve models this content, she does not explicitly model the multiplicative relationship between kg and g. In one example, she converted 4kg300g to g. she explained that 1kg is 1000g and then stated without any justification that 4kg = 4000g. She did not explicitly show the relationship between kg and g, i.e. kg = g x 1000 and did not refer to the role of multiplication in the solution. There were other videos where Maeve used modelling to perform calculations involving item weights, these were entirely instrumental and as such did not correspond to the definition of modelling used in the intervention.

Not all of Maeve's instructional videos were procedural in nature. She created one video about the classification of quadrilaterals which at least attempted to use appropriate mathematical language and precise definitions. However, there were inaccuracies within this also such as the definition of a rhombus as a quadrilateral with two acute angles and two obtuse angles.

5.3.6 Helen

Learning objectives and activities

Helen developed schemes for speed (including distance and time) and the circle. Her learning objectives for both schemes showed potential for the promotion of relational understanding in her teaching. For example, her objectives for speed included investigating the relationship between time, speed and distance using real life examples, developing formula for calculating time/speed/distance, and conducting experiments involving speed in the yard. These objectives are indicative of teaching that would score highly on the MQI richness of mathematics scale because they include relationships, patters, generalising, and investigative work.

Helen's scheme for the circle was designed for specifically for the SEN context. Objectives included knowing a definition for circle, identification of different parts of a circle, constructing a circle, the meaning of pi and some problem solving in relation to circles. Again, these objectives are indicative of teaching that would score highly on the MQI richness of mathematics scale.

There was no reference HLPTs from the intervention in activities for both schemes.

Lesson plans and Instructional videos

Helen's lesson plans are indicative of instrumental instruction and promote recall and procedural skills. However, considered in the context of the instructional videos Helen created for her teaching suggests a more relational approach.

There are three significant findings when Helen's instructional videos are considered in conjunction with her lesson plans. The first finding involves Helen's approach to teaching the standard multiplication algorithm. Whereas her lesson plans indicate a purely instrumental approach, her instructional videos attempt a hybrid approach involving elements of both relational and instrumental understanding. When outlining the steps for multiplication procedure she concurrently attempted to include a conceptual explanation for each step.

The main conceptual part involved Helen attempting to describe the distributive law and its role in the calculation but without expanding the numbers involved thus making it difficult to visualise how the distributive law works. This is likely to cause significant confusion for pupils, particular in relation to place value.

Although this is a notable attempt at including relational understanding, Helen's approach appears ineffective because it is not fully focused on either procedure or the underlying concepts. Choosing to explain either relationally or instrumentally would be more effective from a teaching and learning perspective. However, this is a good attempt at integrating relational understanding into her teaching, and it certainly serves as a good representation of practice going forward.

The second significant finding is that the SEN tasks on the circle involve rich mathematical learning. Helen presented two (i.e., multiple) methods for estimating the area of a circle. The first was counting unit squares contained within a circle drawn on a grid, where Helen drew pupils' attention to the concept of a unit square (in this case cm^2) and emphasised the result would be an approximation. The second method of calculating the area of a circle involved encapsulating the circle within a square so that the circumference of the circle touches the circumference of the circle at 4 distinct points. In this video Helen demonstrated an interesting relationship between the area of a circle and a square in which it is inscribed. That is, the area of a circle is approximately $\frac{3}{4}$ the area of the square. Although this is an interesting relationship, and it is beneficial to demonstrate multiple methods, it was not explored in a way that might allow pupils to grasp the concept at a deeper more generalised level. However, it was another notable attempt at teaching for relational understanding that contained significant features of the MQI framework including linking between representations, explanations, multiple solution methods, and mathematical language.

5.3.7 Fiona

Context

Fiona planned to teach the strand units Data and Measures, and the focus was on weight.

Learning objectives and activities

Fiona's learning objectives ranged from lower order (e.g., read and interpret bar charts and simple pie charts) to higher order (e.g., explain concept of area and discover the formula for area by identifying patterns between the dimensions and areas). This is very effective mathematics planning and lends itself to relational understanding and a potentially high MQI score when enacted in the classroom.

For both strand units Fiona's learning activities explicitly outlined HLTPs and described them in detail – both modelling content and group discussion. Furthermore, these HLTPs were linked to the specific work pupils would be carrying out, thus linking content and pedagogy into "interacting constituent elements of the whole" (Freire, 1970, p.85) carried out within a community of inquiry (Dewey, 1902/1990). Examples include, explaining and modelling content (how to draw a bar chart), eliciting and interpreting data (making conclusions based on results), and groupwork (creating surveys, collecting data).

Lesson plans

Although Fiona's lesson plans are concise, she did include explicit references to HLTPs, and her main focus was modelling content as she used this widely in her instructional videos. Two of her videos related to calculating perimeters of rectangles and irregular polygons. She accurately modelled content, her explanations were good, and she effectively focused on mathematical points about rectangles. From an MQI perspective, however, there were some criticisms.

Firstly, there were some mathematical inaccuracies. For example, her use of the terms regular and irregular are incorrect. In the context of describing the perimeter of a soccer pitch, Fiona used the term *regular* to refer to an oblong rectangle which is in fact an *irregular* polygon. Fiona seems to assign the term irregular to any a shape that is not a "typical" rectangle. In Fiona's example, this was a combination of several overlapping rectangles. She also uses the terms width, height, and length interchangeably without any justification for naming conventions.

Secondly, when calculating perimeters Fiona exclusively focused on the standard long multiplication algorithm without paying any attention to underlying concepts. For example, given that the soccer pitch was 105m x 68m there was an excellent opportunity to explore the distributive law and perhaps even link this to the long multiplication algorithm. Such decisions would have maximised pupils' learning and increased the overall mathematical quality of instruction in the lesson.

5.3.8 Emily

Context

Emily developed schemes for multiplication and division operations, which is part of the overall Number strand of the curriculum. However, when due to Covid-19 resurrections Emily was asked to teach remotely and change her topic to length. Emily developed schemes and one lesson plan for multiplication and division, which are discussed below. Her remote plans are discussed below under the section on lesson plans.

Objectives and activities

All of Emily's learning objectives were based on instrumental understanding. For example, one objective was "dividing a 3-digit number by a 2-digit number by solving long division equations". In some places there seemed to be some attempt to include some deeper learning, but this was misguided. This included asking pupils to "demonstrate their mathematical thought process clearly by recording each step carried out when solving an equation". This statement is contradictory because recording steps is a demonstration of a pupils' ability to recall from memory and does not necessarily involve any "mathematical thought".

As was the case with Robbie outlined previously, Emily demonstrated some confusion about the distinction between learning objectives and learning activities. In Emily's case, the content omitted from her learning objectives was included in her learning activities, which consequently resulted in an impressive blend of content and pedagogical considerations, even if they were organisationally incorrect. There were several instances of HLTPs integrated with content demonstrating a clear influence from the intervention. These include:

- leading a mathematical discussion to discuss commutative, associative and distributive laws of multiplication
- modelling to demonstrate multiplication and multiplication by powers of 10

- asking children to justify steps followed by teacher explanation of logic
- emphasis on pupils showing their work
- using estimations to support solutions
- using number lines to assist in generating solutions
- understanding of meaning of division
- different interpretations of division and its relationship to commutativity of multiplication
- explaining steps in long division algorithm

Unfortunately, this scheme was not used because of the move to online teaching. However, the fact that Emily a least planned such rich teaching and learning experiences involving relational understanding is promising and suggests a high MQI score. She also included 100 questions to promote mathematical discourse and her maths competency division notes in her resources, suggesting a deliberate attempt to link content and pedagogy.

Due to Covid-19 reorganisation, Emily was asked by her CT to develop a new scheme on the topic of length. Although the planned depth of learning was not as extensive as her original scheme, there were some instances of deeper learning. These included pupils looking at the difference between actual and estimated lengths of items in their homes. Although this was computationally elementary, it introduced pupils to the broader concept of mathematical error. Emily also introduced pupils to the Fibonacci sequence and golden ratio, which naturally require a high cognitive demand. However, it should be acknowledged these topics resulted from a post lesson discussion I had with Emily.

Lessons

In Emily's initial face-to-face scheme, and her first lesson plan, Emily planned the teaching of multiplication and division operations in a way that integrated HLTPs with high MQI. This included, for example, definition of multiplication as well as the basic laws of the operations.

Unfortunately, when teaching was moved online Emily's scheme was changed to measurements (length) and teaching and learning became more superficial and cursory in nature. For example, because pupils were working from home, measuring objects in the home became a core activity. Emily also included teaching on the relationship between mm and cm and used division as the operation for conversion. As alluded to in her scheme, there were some aspects of deeper learning such as finding the difference between estimated and true values. Unfortunately, this was a "bonus" task for pupils and not part of the core lesson. There was an opportunity for Emily to introduce the concept of relative error which would have been more appropriate and beneficial for 6th class children, resulting a high MQI score. In her final topic Emily introduced pupils to the Fibonacci sequence and the Golden Ratio. This was promising because it suggested a change in what PSTs are willing to try on SP

5.3.9 Gemma

Gemma planned to teach decimals and percentages, place value, and operations. Her teaching was more sporadic than most of the other PSTs and was divided almost equally between mainstream and SEN settings.

Learning Objectives

Gemma was the only PST involved in this part of the study to use pupils' understanding of mathematics as a learning objective. In one objective, pupils were expected to be able to "demonstrate understanding of divisor, dividend and quotient", and in another to "demonstrate their understanding of how division and multiplication are linked by using multiplication to check their answers", the latter ensuring pupils' awareness of the important inverse relationship between multiplication and division. There are several examples like this, which suggests Gemma values pupils' development of relational understanding. However, one of her objectives is for pupils to "demonstrate their understanding of long division by completing a series of equations without using calculators", which does not reflect relational understanding. This was inconsistent with her other objectives because the term understanding takes on an instrumental meaning here.

Although HLTPs were not explicitly mentioned in her learning activities, Gemma does include teacher questioning which including higher order questions to elicit pupils' thinking, teacher modelling including "thinking aloud and including the children in the process", and group discussion. The way these activities are described, and the language used to describe them suggests they were directly influenced by the HLTPs used as part of the intervention.

Lesson plans

Gemma's lesson plans were partly inconsistent with her schemes and her intention to teach for relational understanding was not always present. This was especially the case with her lesson plan for teaching long division for her mainstream pupils. As described in the schemes, understanding of vocabulary related to division was included, but the process of division itself was purely procedural. This was confirmed by the instructional video for the lesson where Gemma used arrows to "bring down" numbers to the next line etc., with no attempt to explain the reasoning for this.

On the other hand, Gemma created on instructional video for her SEN pupil explaining and demonstrating addition and subtraction of whole numbers, which was developed from a relational understanding perspective and demonstrated a higher MQI score. Gemma used definitions of both operations (adding on and taking away) in conjunction with a number line and manipulatives to explain the concepts.

It is unclear why Gemma would use instrumental understanding for mainstream pupils, and relational understanding for her SEN pupil. There are however, two likely explanations which could be attributed to this. Firstly, Gemma may have assumed her mainstream pupils had the capacity to learn "rules without reasons", while her SEN pupil required the additional explanation to grasp the concepts. The second possibility is related to Gemma's competence and the additional difficulty explaining the long division algorithm compared with the addition and subtraction algorithms.

5.3.10 Synopsis of the PSTs' experiences

The following table provides a useful summary of each of the PST's experiences, including evidence of the intervention in their enactment of mathematical knowledge. This will allow the reader to see at a glance, the sort of enactment that happened during this SP. The final column, evidence of the intervention in practice, refers to whether teaching was likely to be relational or instrumental, based on evidence available in PSTs' plans.

Table 5.1: Overview of PSTs' experiences on SP2

PST Name	**Topic**	**Setting**	**Explicit mention of HLTP/MQI in plans**	**Evidence of intervention in practice**
Eve	Whole number multiplication and division	6th class (Eve planned but did not teach mathematics)	No	n/a
Lilly	Money Percentages	5th class	Implicit evidence of HLTPs in Money scheme Modelling content in lesson plans but did not enact as per decomposition	Lesson plans indicate **instrumental understanding**.
Robbie	Chance Length	5th class	No	Lesson plans indicate **relational understanding**.
Paul	Length and time Fractions, decimals, and percentages	6th class	No	Schemes and lesson plans indicate **instrumental understanding.**
Maeve	Weight, operations, and fractions Shape and space	6th class	Modelling and discussion included in schemes but do not reflect HLTP 100 Questions to promote mathematical discourse explicitly used in fractions scheme.	Lesson plans indicate **relational understanding**. Instructional videos indicate **instrumental understanding**.
Helen	Speed The Circle	6th class	No	Lesson plans indicate **instrumental understanding.** Instructional videos indicate

				relational understanding
Fiona	Weight	5th class	Yes Modelling content and discussion in schemes and lesson plans.	Detail in lesson plans indicate a predilection for **instrumental understanding**.
Emily	Multiplication and division	5th class	Yes HLTPs integrated into scheme	Some **relational** focus in lesson plans.
Gemma	Decimals and percentages Place value Operations	6th class	No but HLTPs implicitly included in scheme.	Mainstream lesson plans suggest an **instrumental** focus. SEN lesson plans suggest a **relational** focus.

5.4 Conclusion

There were notable inconsistencies across PSTs regarding their enactment of HLTPs and elements of the MQI framework. Some refer to these in their lesson objectives and activities but do not in lesson plans, while others do not include them in their planning but do enact them to some extent in their lessons. It is possible, also, that there is some confusion amongst PSTs about the distinction between content objectives and learning objectives, where the latter is copied directly from the mathematics curriculum (NCCA, 1999). It is also possible that any confusion there was may have been exacerbated by the fact that this was the first time PSTs would have used the NIPT framework to guide their planning.

In any case, of the PSTs who effectively used ideas from the intervention in their lessons but did not name them using the formal language from the intervention, it was difficult to determine whether this enactment was a consequence of the intervention or not. Furthermore, some PSTs' plans were clearly influenced by the intervention, but evidence from instructional videos contradict this intention. There are some interesting individual cases also. Gemma teaches instrumentally with her mainstream group of pupils, but relationally with her SEN group. There is also the case of Paul who planned to teach instrumentally and followed through with this approach in his instructional videos, despite voicing a preference for relational understanding in Cycle 1. Of all the PSTs involved in this part of the study, only Helen attempted to enact mathematics lessons based on relational understanding.

Where these inconsistencies exist, pupils will be subjected to at least some elements of teaching and learning consistent with Freire's (1970) banking approach where education is delivered without opportunities for creativity or criticality. Mathematics education becomes about rote memorisation of procedures and skills when the MQI framework is not used effectively as a conduit for relational understanding, and this passive role in their learning results in a diminished capacity for future learning and growth as a human (Dewey, 1916; Freire, 1970). Ignoring or misrepresenting core teaching practices and HLPTs that provide the necessary environment to understand mathematics relationally, particularly those that allow for socialisation, communication, experimentation, and negotiation, deny pupils meaningful opportunities the development of independent and critical thought necessary to understand the reasons and linkages behind facts.

Therefore, these inconsistencies need to be addressed in future cycles of the intervention. The analysis suggests the inconsistencies are related to a gap in PSTs' knowledge of learning objectives and learning activities, how they relate to HTLPs and MQI, as well as how these can be integrated with the goal of teaching mathematics for relational understanding. This is an opportunity for teacher educators to work with PSTs to make these connections explicit. One of the recommendations that came out of FG2 was the idea of an information sheet for CTs which would provide a structure, and empowerment, for PSTs to plan deliberately rather than using ad hoc and perhaps ill-defined terminology in planning that have no meaningful impact on their practice. While this welcome recommendation would be a good start going forward, there are other considerations that will promote the enactment of meaningful practice.

Firstly, the Froebel Department requires PSTs to use the National Induction Plan for Teachers (NIPT, 2022) framework to plan for their SP. This framework contains a planning toolkit with its own set of vocabulary and terminology which could be interpreted as incompatible with HLPLs and MQI elements by a novice teacher, and thus sending contradictory messages to PSTs. If PSTs had a better understanding of the ideas in the NIPT framework, and how these map to concepts explored in the intervention then this would result in better planning and higher quality mathematics teaching. One way to achieve this is to work collaboratively with

PSTs and other teacher educators, using ideas from the intervention and the NIPT planning toolkit, to create research informed lesson plans which can then adapted to individual situations. This way, PSTs will see first-hand what expert planning looks like and consequently enact this in their lessons.

To integrate all these ideas, PSTs will need specific guidance. Firstly, there is a responsibility on me as teacher educator to ensure PSTs have a better understanding of relational understanding and the role it plays in democratising teaching. This will involve integrating this objective into the maths competency modules in a way that allows for meaningful links between theory and practice. The idea of relational understanding can then be integrated into the aforementioned lesson plans, perhaps using MQI as a frame, to encourage PSTs to see relational understanding of concepts as an outcome in and of itself.

A major limitation when interpreting the results from Cycle 2 was the exceptional nature of the circumstances of SP during Covid-19 restrictions, and as such there is a need to be cautious about the interpretation of PSTs' approaches to mathematics teaching and learning. Teaching online brings with it many challenges in relation to the type of communication needed to teach mathematics relationally, and the easier option, particularly in the face of external pressures from CTs and principles, may be to teach instrumentally. This can also be explained by the problem of enactment, which in general, occurs because ITE lacks sufficient context, and results in PSTs lacking the concrete tools and practices to put ideas they have learned into action in a complex setting (Darling-Hammond, 2006). The context in which PSTs were learning to enact relational mathematics, in Cycle 1 and Cycle 2, was very different from the remote nature of this placement. Arguably, this could explain why the PSTs on this placement did not enact elements of the intervention consistently across their planning. Furthermore, within this unfamiliar context, PSTs contended with extraordinary demands and responsibilities which may invoke their apprenticeship of observations causing them to teach in a more instrumental way and "trust what is most memorable" (Boyd et al., 2013, p.5).

Finally, the commendable performances by PSTs should also be acknowledged. For the first time since the maths competency modules were developed there is clear evidence, albeit with the above named inconsistencies, that the module is having some influence on PSTs' planning and practice. This is evident not only with the use of HLTPs and their associated language, but also in PSTs' approach to planning and teaching. Importantly, PSTs demonstrated an openness to change when provided with the correct guidance and support and demonstrate enthusiasm about these changes when they are in the best interests of pupils. There are clear indications from the above analysis for how the intervention can be shaped into the next academic year to provide the additional support the enactment of meaningful mathematics.

Chapter 6: Quantitative Analysis

6.1 Introduction

Surveys were distributed to PSTs at three points in time throughout the study: pre-intervention, mid-intervention, and post-intervention. As outlined in Section 3.6.1 of the methodology chapter, surveys were used to gather data about PSTs' beliefs related to mathematics anxiety (MA), mathematics teaching and learning, and ITE program effectiveness. The surveys used Likert Scale responses from PSTs to collect quantitative data from the PSTs. Likert Scales were chosen because they are the preferred method of measuring the mathematical beliefs of larger groups of teachers (Philipp, 2007). There was also an optional qualitative section for PSTs to add comments about their responses which are referred to in this section when they were able to provide extra context or nuance to the findings.

In Section 1.8, this study was described as mainly qualitative. While this may be true, the quantitative results are form an essential part of the overall study because they tell us about PSTs' beliefs about mathematics. Knowledge of PSTs beliefs is very important for several reasons. Firstly, the intervention is built upon the fallibilist philosophy of mathematics, and if PSTs' beliefs become closer orientated towards the fallibilist philosophy then this signifies a positive change in their beliefs. Secondly, if PSTs' beliefs tend towards a fallibilist philosophy then it is more likely their teaching will be informed by this philosophical orientation, although this is not always the case (Skott, 2009).

A one-way ANOVA was performed to compare the effect of the intervention on these beliefs, i.e., the dependent variables, at three different points during the intervention. The first survey was completed by PSTs in September 2019. This is the pre-intervention survey and was used to collect data on PSTs' beliefs before the intervention took place. The second survey was completed in December 2019 after Cycle 1. At this point PSTs engaged with the intervention in Cycle 1 from September to October 2019 and completed SP1, also in October 2019. The third and final survey was completed by PSTs in March 2021 after the entirety of the intervention and after SP2. ANOVA requires the participants to be the same in each group and this amounted to 37 participants included across the three surveys. The reason for the drop off in numbers is due to the usual absences, and the fact that only 57 out of a possible

68 were available for the first questionnaire because they were completing an international Erasmus programme.

For each belief being measured there are three variables to capture beliefs measures at the three different points in time across the study. Each of the three variables is differentiated by the number 1, 2 or 3 following the variable name. For example, Beliefs about the nature of mathematics is abbreviated to 'bnom' and the three variable names are bnom_1, bnom_2, and bnom_3. Table 6.1 below outlines each of the variables used in the test, the acronyms used, and a description of each of them.

Table 6.1: Variables used in the test

Name	**Variable acronym**	**Description**
Beliefs about the nature of mathematics	bnom (bnom_1, bnom_2, bnom_3)	bnom measures the extent to which PSTs believe mathematics is static or dynamic (i.e., absolutist or fallibilist philosophical orientations)
Beliefs about learning mathematics	blm (blm_1, blm_2, blm_3)	blm measures the extent to which PSTs believe mathematics is best learned through a traditional pedagogy (i.e., teacher centred, procedural) or a constructivist pedagogy (i.e., pupil centred, inquiry based).
Beliefs about mathematics achievement	bma (bma_1, bma_2, bma_3)	bma measures the extent to which PSTs believe mindset (fixed or growth) determines ones achievement in mathematics.
Beliefs about preparedness to teach mathematics	bptm (bptm_1, bptm_2, bptm_3)	bptm measures the extent to which PSTs believe their ITE programme has given them the capacity to carry out the main tasks of teaching to meet the demands of their classroom practice.
Program effectiveness	pe (pe_1, pe_2, pe_3)	pe measures the extent to which PSTs believe their ITE programme has helped them learn to teach mathematics for teaching.
Mathematics Anxiety	anxiety (anxiety_1, anxiety_2, anxiety_3)	This is a measurement of the mathematics anxiety PSTs experience at the three distinct points during the intervention.

The data were first analysed for normality and, where normally distributed, the results were analysed using the ANOVA statistical test. To provide a comprehensive picture, descriptive statistics were also included for the normally distributed variables. Where data were not normally distributed, ANOVA could not be applied and so these variables were analysed using only descriptive statistics.

Cohen et al. (2018) caution the use of null hypothesis statistical testing (NHST) alone and therefore, along with ANOVA, effect size was also used in this study. Additionally, efforts were made to increase statistical power, and this was done by using a high significance level ($\alpha = 0.05$), a homogeneous sample, and parametric tests where possible (Cohen et al., 2018). Cohen *d* values between 0.2-0.49, 0.5-0.79, and above 0.8 indicate small, medium, and large effect sizes, respectively (Huck, 2012).

6.2 Testing for Normality

ANOVA requires data to be normally distributed, i.e., scores related to variables must be normally distributed in the sample. If a variable is normally distributed in a population, then it should be normally distributed in some smaller sample also. Razali and Wah (2011) suggest applying the Shapiro-Wilk test for checking normality of the data. In this test, the null hypothesis, H0, states that a variable is normally distributed in the population, i.e., if p>0.05 we accept the null hypothesis and if p<0.05, we reject the null hypothesis. The results of the Shapiro-Wilk normality tests are displayed in Table 6.2 below.

It can be seen from Table 6.2, variables for beliefs about the nature of mathematics (bnom_2), program effectiveness (pe_1 and pe_3) as well as anxiety (anxiety_1, anxiety_2 and anxiety_3) are not normally distributed given their *p*-values are less than .05 (p<.05). Therefore, nongeneralisable and nonparametric tests were carried out on these variables. The remaining variables (blm, bma, bptm) were found to be normally distributed and ANOVA was applied to these. These are presented, along with their relevant descriptive statistics in the next section.

Table 6.2: Shapiro-Wilk Tests for normality

	Statistic	Df	Sig.
bnom_1	.970	36	.415
bnom_2	.941	37	.048
bnom_3	.963	37	.248
blm_1	.983	37	.836
blm_2	.983	37	.828
blm_3	.974	37	.536
bma_1	.962	37	.225
bma_2	.985	37	.892
bma_3	.972	37	.479
bptm_1	.984	37	.848
bptm_2	.971	36	.451
bptm_3	.972	37	.451
pe_1	.899	37	.003
pe_2	.942	37	.052
pe_3	.814	37	.000
anxiety_1	.929	37	.021
anxiety_2	.938	37	.039
anxiety_3	.927	37	.018

6.3 Inferential Analyses

Repeated Measures of ANOVA was applied to the blm, bma an bptm variables at three different time points: pre-intervention, min-intervention (intermediate test) and post-intervention. ANOVA findings for the normally distributed variables, including the F and sig. levels, are presented in the sections that follow. For each of these three normally distributed variables, assumption of sphericity was tested, and degrees of freedom were corrected using Greenhouse-Geisser estimates of sphericity. The Greenhouse-Geisser F-ratio was determined, as well as its associated significance level and effect size using partial eta squared.

Rather than presenting the non-parametric data for bpe and bnm in isolation, these were reported with related parametric data, to form a more robust argument. That is, blm is epistemologically connected to bnm, as both are fundamentally underpinned by the fallibilist philosophical orientation to mathematics. On the other hand, bptm is related to pe in a very practical way because both are measures of how well PSTs believe in their ability to perform quality mathematics teaching. In fact, open-ended responses for both items tended to be very similar, indicating PSTs did not see a clear distinction between them, and for this reason both are presented together. MA is reported alone using descriptive statistics.

6.3.1 Beliefs about Learning Mathematics

Descriptive statistics for blm including the mean and standard deviation are summarised in Table 6.3 below:

Table 6.3: Descriptive statistics for blm

	N	Minimum	Maximum	Mean	Std. Deviation
blm_1	37	14	84	64.05	5.652
blm_2	37	14	84	65.51	6.661
blm_3	37	14	84	68.78	6.844

The mean score for BLM pre-intervention (n=37) was 64.05 (SD=5.652). The mean increased to 65.51 during the intermediate period (SD=6.661) and further increased for the post intervention period with a mean of 68.78 (SD=6.844). Using the Greenhouse-Geisser values the difference in mean scores for blm were statistically significantly across the intervention. The p-value is less than 0.05 which is evidence of a significant main effect $(F(1.1618, 58.241) = 9.971, p < 0.0005)$. There is a small effect size given by 0.217. Because significant main effects were found, Bonferroni post hoc pairwise comparisons were conducted to identify where the significant differences could be found between pairs of means across the intervention (see Table 6.4). To interpret this, if the *p*-value is less than .05, then there is a statistically significant difference between the pairs of means.

Table 6.4: Pair-wise comparisons for blm

(I) testing	(J) testing	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
1	2	-1.459	.809	.239	-3.490	.571
	3	-4.730*	1.112	**.000**	-7.522	-1.937
2	1	1.459	.809	.239	-.571	3.490
	3	-3.270*	1.280	**.045**	-6.485	-.055
3	1	4.730*	1.112	.000	1.937	7.522
	2	3.270*	1.280	**.045**	.055	6.485

For the blm variable, there were significant differences between the intermediate survey and the post intervention survey, and the pre-test survey and the post-test survey. This is evidence of a gradual progress in participants' beliefs about learning mathematics throughout the entirety of the intervention, and this indicates PSTs' beliefs about learning mathematics became less traditional and transitioned to a more constructivist view of learning mathematics. These results imply that PSTs' tendency towards absolutist beliefs about nature of mathematics, which likely emanated primarily from their own experiences as pupils in school, appears to be challenged by exposure to the intervention.

The blm results are consistent with the parametric data for the beliefs about the nature of mathematics variable, as outlined in Table 6.5 below. The mean score for bnom pre-intervention (n=37) was 42.64 (SD=5.441). The mean increased to 46.36 during the intermediate period (SD=6.6029) and further increased for the post intervention period with a mean of 50.22 (SD=5.111). Although these scores are not generalisable, they do indicate that the beliefs of the PSTs in this study have moved along the absolutist-fallibilist philosophical continuum further towards a fallibilist orientation.

Table 6.5: descriptive statistics bnom

	N	Minimum	Maximum	Mean	Std. Deviation
bnom_1	36	12	72	42.64	5.441
bnom_2	36	12	72	46.36	6.029
bnom_3	36	12	72	50.22	5.111

6.3.2 Beliefs about mathematical achievement

Table 6.6 shows mean score for bma pre-intervention (n=37) was 35.03 (SD=6.543). The mean increased slightly to 35.46 during the intermediate period (SD=6.805) and further increased for the post intervention period with a mean of 37.51 (SD=6.167).

Table 6.6: Descriptive Statistics for bma

	N	Minimum	Maximum	Mean	Std. Deviation
bma_1	37	8	48	35.03	6.543
bma_2	37	8	48	35.46	6.805
bma_3	37	8	48	37.51	6.167

Furthermore, ANOVA indicated the difference in mean scores for beliefs about mathematics achievement (BMA) were statistically significantly across the intervention. The p-value is less than 0.05 which is evidence of a significant main effect $(F(1.605, 57.796) = 3.719, p < 0.0005)$. There was a negligible effect size given by .094.

Table 6.7: Paired comparison table of bma scores

	(J) testing	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
1	2	-.432	.709	1.000	-2.212	1.347
	3	-2.486	1.022	.060	-5.053	.080
2	1	.432	.709	1.000	-1.347	2.212
	3	-2.054	1.140	.240	-4.917	.809
3	1	2.486	1.022	.060	-.080	5.053
	2	2.054	1.140	.240	-.809	4.917

As indicated in Table 6.7, all p values for bma on the paired comparison table are greater than .050 and therefore, there no statistically significant difference between the pairs of means was identified by the test, despite the statistically significant main effect.

6.3.3 Beliefs about preparedness to teach mathematics

As indicated in Table 6.8, the mean score for bptm pre-intervention (n=37) was 31.28 (SD=7.021). The mean increased to 36.33 during the intermediate period (SD=7.441) and further increased for the post intervention period with a mean of 41.22 (SD=6.024). This tells us PSTs' beliefs about how prepared they are to teach mathematics increased throughout the intervention.

Table 6.8: Descriptive statistics for bptm

	N	Minimum	Maximum	Mean	Std. Deviation
bptm_1	36	13	52	31.28	7.021
bptm_2	36	13	52	36.33	7.441
bptm_3	36	13	52	41.22	6.024

ANOVA indicated the difference in mean scores for BPTM were statistically significantly across the intervention. The p-value is less than 0.05 which is evidence of a significant main effect $(F(1.966, 68.819) = 34.748, p < 0.0005)$. There is a small effect size given by 0.498.

Furthermore, as indicated by Table 6.9 below, the Bonferroni post hoc test identified significant differences between pairs of means across the intervention, indicating steady progress in PSTs' beliefs about their preparedness to teach mathematics throughout the intervention.

Table 6.9: Paired comparison table for bptm

(I) testing	(J) testing	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval for Difference	
					Lower Bound	Upper Bound
1	2	-5.056*	1.260	**.001**	-8.225	-1.886
	3	-9.944*	1.189	**.000**	-12.935	-6.954
2	1	5.056*	1.260	**.001**	1.886	8.225
	3	-4.889*	1.125	**.000**	-7.719	-2.059
3	1	9.944*	1.189	**.000**	6.954	12.935
	2	4.889*	1.125	**.000**	2.059	7.719

These results are consistent with the qualitative data gathered across both cycles of the study. It was clear from FG1 that PSTs engaged more meaningfully with mathematics content when they viewed it as beneficial for the pupils they teach, rather than something they need to know for an examination. Furthermore, the intervention gave PSTs a framework to link theory and practice and enact this in the classroom through realistic approximations in Cycle 1 of the study, and the sustained exposure to the intervention necessary to adjust to a new way of learning mathematics for teaching throughout Cycle 2.

I am making the assumption that beliefs about preparedness to teach are related to beliefs about program effectiveness (pe). This is a reasonable assumption given the similarities between the statements in the survey. One is about the programme (beliefs about preparedness to teach), whereas the other (programme effectiveness) is about the instructor. Based on this assumption, the above results are further supported by non-parametric data for PSTs' beliefs about program effectiveness (pe). For this reason, the above results are further supported by non-parametric data for PSTs' beliefs about program effectiveness. Table 6.10 below shows that the mean score for program effectiveness for teaching mathematics increased across the three surveys. This metric considers both maths competency and maths methods, and any other inputs from the B.Ed. program including SP that may contribute to PSTs' perception of its effectiveness.

Table 6.10: Descriptive Statistics for pe

	N	Minimum	Maximum	Mean	Std. Deviation
pe_1	37	6	36	24.35	5.682
pe_2	37	6	36	27.89	5.152
pe_3	37	6	36	30.73	4.273

These results, which indicate PSTs feel prepared to teach, are broadly supported by the qualitative parts of the surveys. These open-ended sections indicated that PSTs significantly valued the change to maths competency and agreed it helped them improve their practice. They particularly valued the practical nature of the intervention with the addition of HLTPs and the MQI framework, and six PSTs commended the use of reflections as an integral part of it with one commenting that it would have been useful if they had more opportunities to share these with each other. This really speaks to PSTs' reflective, dialogical, and collaborative experiences, and how the intervention supported them in experiencing a human element in their learning (Freire, 1970). On several occasions PSTs reiterated the gap between maths competency and maths methods as a significant barrier to learning, and this is something worth serious consideration in future cycles of the intervention.

6.3.4 Mathematical Anxiety

As described in Section 3.6.1.3, the abbreviated mathematics anxiety scale (AMAS) was used to measure PSTs' MA at the same three points across the intervention. As explained in the literature review, the Likert scoring for some items were reversed so that higher scores indicated lower MA consistently across the results. Therefore, in Table 6.11 below showing descriptive statistics for MA, a higher score equates to a reduction in MA.

Table 6.11: Descriptive Statistics for MA

	N	Minimum	Maximum	Mean	Std. Deviation
anxiety_1	37	9	35	19.89	5.435
anxiety_2	37	9	35	19.54	5.434
anxiety_3	37	9	35	35.35	4.855

The AMAS scores varied between 11 and 35 for the first survey, 10 and 37 for the second survey, and 25 and 42 for the final survey. The results presented above indicate a reduction of MA as measured in the third survey, with only a negligible difference between the first and second surveys. The reduction in MA, from the second to the third survey, is likely due to the inclusion of interactive and collaborative approaches to understanding mathematics for teaching, all of which is underpinned by relevant educational theory (Gresham, 2018). Interestingly, MA increased, albeit by a small amount, in the second survey compared with the first. A possible explanation for this is that this survey was carried out close to PSTs' assessment for maths competency which possibly resulted in higher levels of MA.

It is useful to compare the MA of the PSTs in this study to university students enrolled in other areas of study. Research carried out in Germany by Schillinger et al (2018) involved administering a German translated version of the AMAS (AMAS-G) to three hundred forty-one university students (221 females) between the ages of 18 and 35 years (M = 22.06, SD = 3.40). 50.1% of the students were enrolled in a psychology degree, 28.4% in science, 18.2% in humanities, and 3.3% in law or economics. The total score of the AMAS-G varied between 9 and 40 (M = 20.16, SD = 6.79). Female students in the study (M = 21.50, SD = 6.89) reported a significantly higher total score than male students (M = 17.68, SD = 5.87).

6.4 Conclusion

For the normally distributed variables, the difference between means has been shown to be statistically significant, meaning the intervention, as opposed to some external factors, did lead to a positive change in PSTs' beliefs. This is important in the overall context of this study because it means that "chance is an unlikely explanation" for the difference between the means (Cohen et al., 2018, p.739), i.e. the intervention is the most likely reason for the positive changes in PSTs' beliefs. Although not generalisable, non-parametric data further support these findings. However, the quantitative results should be interpreted with caution as bma was shown to have only a negligible effect size, while blm and bptm were shown to have a small effect size (Huck, 2012).

Considered in the context of the overall study, there are two important implications arising from these results. Firstly, despite the fact that beliefs are difficult to change (Booker, 1996; Philipp, 2007) all of the measured beliefs, including MA, did change and the statistically significant results indicate the changes were because of the intervention. As explained in the literature review, the intervention itself was evidence-based, and the concepts and theories underpinning this were shared openly with PSTs. This indicates that PSTs' beliefs about mathematics are likely to be held evidently and can therefore likely change as a result of further interventions (Green, 1998), if they are given sufficient opportunities to reflect on their learning (Hourigan & Leavy, 2012). Secondly, it is noteworthy that all the measured beliefs changed in a way that supports democracy in mathematics education. Beliefs about the nature of mathematics and beliefs about learning mathematics became further aligned to the fallibilist philosophical position, whereas beliefs about mathematics achievement became closer aligned to the growth mindset and mathematics anxiety was reduced. Common to all these changes in beliefs and anxiety is their manifestation of mathematics as a meaningful discipline which values relational understanding and is pedagogically active and inquiry -based involving critical thinking and creativity. In this regard, the intervention aligned to Freire's (1970) problem posing methodology and the experiential nature of Dewey's vision for education.

While beliefs are essential considerations and have been shown influence teacher behaviour directly, including planning and classroom practice (Lui & Bonner, 2016), other researchers have shown that context and the “social perspective” that form part of the PSTs teaching experience are likely to result in apparent inconsistencies between PSTs beliefs and their practices (Raymond, 1997; Hoyles, 1992; Skott, 2001 & 2009; Sztajin, 2003). As will be shown discussed in the next chapter (Section 7.2) such inconsistencies were apparent with some of the PSTs in the current study. Therefore, it is important to acknowledge the role of beliefs as part of the maths competency modules and try to develop PSTs’ beliefs concurrently with their content knowledge in future cycles of the intervention (Philipp, 2008).

Chapter 7: Discussion

7.1 Introduction

As discussed in chapter 2, there have been ongoing calls by prominent researchers in the field of teacher education for ITE programmes to develop signature pedagogies to allow PSTs to both pay attention to what is being taught, and how it is being taught (Russell, 1997; Loughran, 2006). This necessitates integrating elements of practice as a core component into ITE programmes in a way that PSTs are given opportunities to practice elements of teaching in situations of reduced complexity (Grossman & McDonald, 2008; Ball & Forzani, 2009). However, Grossman (2018) contends that meaningful elements of practice remain almost entirely to school based professional placements where the responsibility lies solely on the PSTs to seamlessly integrate knowledge and practice (Lampert, 2010). This approach contributes to the problem of enactment as PSTs tend to abandon their instructional ideals for less favourable practices (Korthagen, 2009; Grossman & Thompson, 2008; Wood, Jilk, & Paine, 2012). Consequently, Wideen's (1998) assertion that the traditional model of Initial Teacher Education (ITE) where the university provides the knowledge, the school provides the setting, and the pre-service teacher provides the individual effort to apply this knowledge remains relevant. We now know the task of integrating multiple domains of knowledge into the practice setting may be overwhelming for many PSTs (Lampert, 2010), and results in the abandonment of their instructional ideals. Furthermore, while School Placement (SP) is an essential aspect of ITE, it is one that teacher educators (TEs) have least control over, and this compromises PSTs' opportunities to engage in risk free meaningful practice with the freedom to confront their gestalts and construct their personal pedagogies (Loughran, 2006; Korthagen, 2010).

Using Brookfield's (1995) lenses to reflect on my practice, I realised I was enacting the traditional model of teacher education in the original maths competency modules, and this contributed to the problem of enactment that is at the heart of this study. My practice, therefore, promoted the idea that learning was a passive activity resulting in, for the most part, classroom teaching based on that idea that mathematical knowledge is transactional and instrumental. This was not my intention, and the oversight was due to a lack of knowledge about the inherent complexities of teacher education, and specifically the messages my pedagogical approach sent to PSTs (Russell, 1997). This lack of awareness promoted education as a tool for conformity and encouraged PSTs to enact mathematics teaching that discouraged pupils from engaging critically and creatively with the subject (Freire, 1970).

Grossman & McDonalds (2008) model of teacher education enabled me to design a teaching intervention to address the shortcomings in my practice by allowing PSTs to engage in sustained inquiry into clinical aspects of their practice. These pedagogies of enactment allowed PSTs to view the underlying structures of teaching, while giving them opportunities to engage in meaningful and educative practices in a risk-free environment (Grossman & McDonald, 2008). This approach to teacher education is consistent with Dewey's laboratory approach to "enliven and awaken teacher candidates to the meaning and vitality of educational principles" (Greenwalt, 2016, p.3), and it was evident, at times during the study, that it allowed PSTs and pupils to become "active agents of their own learning" (Dewey, 1902/1990, p.17) by affording them the freedom to explore their own natural talents and interests.

In this study, Grossman and McDonald's (2009) model was collectively adapted, with input from PSTs, to address their specific needs, the needs of the pupils they teach, and the mathematical knowledge requirements of the module. This process was guided by my educational values, particularly those about collectively problematising knowledge and accepted traditions, while always promoting a culture of inquiry. The intervention democratised teacher education by collectively developing a miniature democratic society free from neoliberal influences, whereby socialisation, communication, experimentation, and negotiation were practiced (Dewey, 1916).

The intervention is a signature pedagogy (Shulman, 2005) that was designed to give PSTs opportunities to "practice elements of interactive teaching in settings of reduced complexity", while simultaneously receiving feedback (Grossman and McDonald, 2008, p.190). It was based, primarily, on representations, decompositions, and approximations of practice (i.e., pedagogies of enactment) and used high leverage teaching practices to enact elements of practice. The MQI framework was used as a guide for maintaining the quality of mathematics used in approximations of practice and retrospectively evaluate that quality. The intervention was continuously improved by intentionally reflecting on it, and engaging meaningfully, openly, and professionally with PSTs over a two-year period. Data were collected informally by reflecting on everyday encounters and observations, and formally through data collection instruments. As well as finding out what changes needed to be made to the intervention, the research questions sought to determine the origins of the enactment problem for the PSTs in the study, and if the intervention resulted in changing their beliefs and behaviours in relation to mathematics teaching. To consolidate what was learned from the study, the research questions outlined in Section 3.2 are presented again below, followed by a discussion of the findings in the context of the relevant literature. Following this, a discussion about the implications for my practice and future cycles of the intervention is presented, and implications for programmatic design. In the final section, implications for teacher education are presented.

7.2 Revisiting the research questions

Each of the research questions is presented below, each with a discussion that draws on some of the more significant and nuanced findings from the study. Furthermore, the discussions around each individual question are not mutually exclusive and therefore, where possible, important connections across each individual discussion are made explicit for the reader.

Question 1: **What experiences contribute to the problem of enactment for PSTs?**

The literature provided a starting point to addressing this question. Over 40 years ago Zeichner and Tabachnic (1981) listed the influence of co-operating teachers, the ecology of the classroom, the bureaucratic norms of the school, teacher colleagues, and even pupils as the contributing factors to the problem of enactment. Essentially, the research carried out around this time identified the "harsh and rude reality of everyday classroom life" as the reason which caused PSTs to abandon their beliefs for less favourable pedagogical behaviours (Veenman, 1984, p.143).

More recent research acknowledges these issues were, and still are, exacerbated by models of teacher education that are underpinned by a primarily traditional cognitive model of learning, which ignore situational contexts that allows PSTs to link knowledge and practice (Darling-Hammond, 2006). These models of ITE promote the oversimplification of the practice of teaching (Ball & Forzani, 2010), while ignoring the complexity inherent in teacher education including the competing cognitive and affective domains of learning to teach (Loughran, 2006).

Broadly speaking, the fundamental reason for the problem of enactment in my practice was the decontextualised nature of the teaching, and disjointed connection between modules. Acknowledging these issues and informed the professional learning theories of Wenger (1998) and Korthagen (2010), I adopted Grossmans model to redesign the maths competency module.

However, the purpose of the intervention, and indeed any pedagogy of teacher education, was not about transferring Grossmans' model from one context to another. To effectively enact the intervention, it was necessary to understand the nuance underpinning PSTs' attitudes and behaviours and adopt Grossman and McDonald's (2008) model to these, and this necessitated a deeper exploration of their lived experiences.

From this perspective, there were a wide range of factors that contributed to the problem of enactment and knowing these helped to shape the intervention over the two-year period. In the first instance it is worth considering Philipp's (2008) contention that most PSTs start ITE with the belief that mathematics is a collection of rules and procedures, and this belief clashes with the conceptual approach taken by the maths competency module. While this appears to be true in this study from observing PSTs' behaviours, these behaviours are not necessarily consistent with their beliefs. While it is acknowledged the PSTs in this study have been in ITE for two years at the start of this study, there is evidence of a mismatch between their beliefs and their behaviours. As presented in Section 4.4, several PSTs reported beliefs about mathematics which suggest a preference for relational understanding (i.e., Gemma, Brona, and Jessica) yet enacted purely instrumental mathematics in practice. As discussed in section 2.4.5 of the Literature Review, there are several possible explanations for these apparent inconsistencies between PSTs' beliefs and their classroom practice. These include how PSTs feel they should behave in different contexts, enacting practice based on mathematics pedagogy as opposed to their beliefs about the nature of mathematics, or enacting practice based on beliefs about the pupil and society more generally (Raymond, 1997; Hoyles, 1992; Skott, 2001; Sztajin, 2003). While further research would be required to determine precisely why the apparent mismatch occurred for individual PSTs, based on the literature it is reasonable to assume it is related to the context and the "social perspective" in which they are teaching (Scott, 2009). As a teacher educator it is important to be aware of this phenomenon and, when appropriate, bring it to the attention of PSTs.

Another interesting point in the context of this study is that PSTs' classroom practice, in the first instance, appear to be influenced by their apprenticeships of observations, which developed from their personal experiences as pupils learning mathematics. As outlined in Section 4.2.1.1, seven of the 14 PSTs who took part in the focus groups in Cycle 1 reported their experiences of school mathematics, particularly in secondary school, included a significant amount of instrumental learning. In several cases, this experience of mathematics has resulted in current limited understanding of the subject which as implications for how they teach it on SP. If this default approach to teaching mathematics for some PSTs is left unchallenged in ITE it will very likely transfer to the classroom, thus inhibiting these PSTs from teaching mathematics for relational understanding.

This problem is often exacerbated by some pupils who tend to resist learning mathematics relationally, and who demonstrate a preference for shallow, product orientated mathematical learning. Importantly, in agreement with Philipp (2008), this study demonstrated that PSTs are reluctant to challenge pupils on their mathematical learning because of their beliefs that to do so is inconsistent with caring for them. Consequently, pupils have a very strong influence over PSTs' classroom behaviour and their approach to teaching mathematics. In some cases, the pupils' preference for instrumental mathematics appeared to be reinforced by the class teacher or seemed to stem from parental expectations. However, the data collected from this study suggests a fundamental issue, for pupils and PSTs, is the common belief that mathematics is a rule orientated discipline, whose use value is ultimately for examinations. When used primarily for this purpose, mathematics education becomes both mis-educative and noneducative (Dewey, 1933), and rather than teaching pupils to think creatively, critically, and analytically, it serves only to act as a gatekeeper to higher education and employment opportunities (Ward-Penny, 2017), and works against democratic processes in favour of obedience and conformity (Freire, 1970; Chomsky, 2004).

Unfortunately, PSTs often feel powerless to act against this, not only becaue of pupils' attitudes, but also because changes to their beliefs about how mathematics should be taught are often at odds with instructional direction from the class teacher (CT) or SP tutor (SPT). PSTs are therefore pulled in several different directions by mixed messaging by those in more powerful positions than they are.

Question 2: **What is the optimal design for a pedagogy of enactment, in the context of mathematics education, to reduce the problem of enactment for my PSTs?**

Adopting Grossman's (2008) pedagogies of enactment as a core part of the intervention allowed the PSTs in this study to contextualise mathematics knowledge in a way that acknowledged and embraced the complexity of teaching. Representations, decompositions, and approximations of practice provided opportunities for PSTs to closely examine their HLTPs to support their teaching of mathematics for relational understanding. The Mathematical Quality of Instruction (MQI) framework (Learning Mathematics for Teaching Project, 2011) was a necessary addition to this because it gave further clarity to what relational mathematics should look like in practice.

The PSTs in the study reported approximations involving visiting pupils introduced authentic situational learning which allowed learning to happen socially within a community of practice (Wenger, 1998). Unfortunately, due to Covid-19 restrictions such an authentic approximation was not possible in Cycle 2, so PSTs alternatively worked with a B.Ed. year 1 upskilling class to approximate their practice. Although the benefits of this were not as apparent for PSTs, it is important that some form of authenticity, whereby approximations closely resemble practice, is included within future cycles of the intervention. Therefore, approximations of practice should be progressive, range from low to high across the approximation scale, so that PSTs can engage in a full and graduated range of practical experiences.

A range of representations, including videos of practice, lesson plans, anecdotal evidence about practice, and examples of practice by previous PSTs, are also necessary components of the intervention. Video representations of practice are particularly useful because they are engaging and easily lend themselves to whole class reflection and subsequent discussion. During the intervention some of these representations were exemplary and effectively demonstrated the decomposition and enactment of HLTPs, while modelling expert content knowledge. However, representations of practice that show undesirable examples of practice were just as useful, and undoubtedly more thought provoking, invariably resulting in meaningful dialog and reflection which encouraged creativity, action, and enquiry (Freire, 1970). These representations included descriptions undesirable elements of practice which PSTs must then critique and develop alternative approaches.

The intervention developed in a way that moved beyond the scope of the maths competency module. The results clearly indicate a need to develop meaningful links between the currently compartmentalised nature of the maths competency and maths methods modules. While PSTs are left with the task to "integrate it all" (Lampert, 2010, p.24) they will continue to enact undesirable practices, and this is even more likely when the messages they are receiving are inconsistent. Therefore, all related modules must be underpinned by a common philosophical and practical approach and send a consistent message to PSTs. This is discussed in more detail in Section 7.3 below.

Further developing the discussion around research question 1, any teacher education pedagogy that is serious about addressing the problem of enactment must consider the relatively vulnerable position PSTs occupy on SP. The intervention has now entered a 3rd informal Cycle, and one of the primary concerns is to address this issue. In future SPs, PSTs will be empowered to enact meaningful mathematics by supplying them with official documentation which encourages them to adapt HLTPs used during the intervention, and which requires them to use the MQI framework to ensure the quality of the mathematics in their lessons. Furthermore, by addressing the separation between the maths competency and maths methods modules, PSTs will receive a single message from their university-based instruction. It is important also that this unified message is shared with SPTs so they too can support PSTs in the difficult task of enacting relational mathematics in the classroom.

Finally, enactment pedagogies need to be introduced from the start of ITE and should be the main approach, not something that is implemented six weeks before SP, or on an ad hoc basis. As a reminder, an enactment pedagogy refers to a practice centred approach to teacher education that values the integrated nature of theory and practice, it is designed to close the theory-practice divide while at the same time addressing the complexity of teaching as a practice and the preparation of PSTs (see section 2.2.2). Introducing this approach early is essential because it will take time for PSTs to become accustomed to a new way of learning, compared to the passive style they may have experienced in school, and within a larger university ecosystem that rewards transmission style learning (Loughran, 2006).

Question 3: **Can the intervention cause any change in PSTs' beliefs about mathematics and their levels of mathematical anxiety?**

PSTs' beliefs about mathematics, and their mathematical anxiety, changed throughout the intervention in a way that supports the enactment of relational understanding. As described in the previous chapter, the quantitative data indicates PSTs' beliefs moved closer to the fallibilist view of mathematics which values problem solving, discovery, creativity, inquiry, and context (Depaepe et al., 2020). Similarly, beliefs about mathematical achievement also changed in a way that likely supports the enactment of relational understanding. These beliefs promote the enactment of an active, inquiry-based approach to teaching mathematics where pupils are given opportunities construct meaningful knowledge (Depaepe et al., 2020) through a process of sense making and pattern seeking (Felbrich, 2012).

The results also indicated PSTs' beliefs about mathematics achievement became less fixed and more orientated towards a growth mindset as the intervention progressed. This includes the belief that mathematics ability is malleable, and that effort, hard work, and collaboration can positively change one's ability. This has important implications for teaching because PSTs with a growth mindset are more likely to teach mathematics for relational understanding and indicates increasing confidence to generate and develop meaningful mathematical ideas (Beghetto and Baxter, 2012). Beliefs of this nature increase the likelihood of PSTs empowering pupils to engage in meaningful mathematical work (Boaler, 2016; Sun, 2018).

There is also some evidence to indicate that PSTs' beliefs were held evidently and therefore,

beliefs can be further developed as the intervention evolves and improves over time (Green, 1998)..

Finally, the AMAS scores indicate that PSTs' mathematical anxiety reduced because of their engagement with the intervention. This reduction in anxiety, and associated negative feelings about mathematics, means PSTs will likely be less inhibited from focusing on mathematical understanding and reasoning, making connections, and understanding concepts and procedures in maths competency lectures (Grisham, 2018). The AMAS measured PSTs' anxiety in relation to learning mathematics and having their mathematical knowledge evaluated. In their open-ended responses, many PSTs specifically referred to these sources of anxiety in the context of learning mathematics in ITE. For example, in the final post-intervention survey Aoife reported the following:

> "*My levels of anxiety would depend on the topic being taught and my confidence with it. I would also be hesitant to ask questions, although I have gotten better at this, so if I didn't understand something on the board I would feel anxious. I would really need to revise for maths exams/tests in order to feel comfortable and confident although I feel with the problem-solving aspect of the numeracy model, this is improving. I enjoy teaching maths but not knowing something really throws me off, yet I would be anxious to research something too.*"

Aoife's response is indicative of many of those reported by PSTs. Importantly, she feels anxious when she does not understand something on the board yet does not like to ask questions to clarify her understanding. This scenario would be typical of what might happen in the original maths competency modules, which relied mainly on direct teaching. This scenario would be typical of what might happen in the original maths competency modules, which relied mainly on direct teaching. We know from the literature that PSTs who experiences these sorts of negative feelings are more likely to perform poorly in mathematical tasks or avoid engaging meaningfully with the subject altogether (Maloney and Beilock, 2012). In any case, these feelings will have negative consequences for their mathematical knowledge, and subsequent classroom performance. However, such a scenario is less likely to happen in the context of the intervention because of the collaborative, dialogical, and reflective nature of it. This is important in the context of the intervention because it means that PSTs are more likely to engage with meaningful learning and, it is assumed, this will have positive implications for their classroom practice. It is important to note that MA was measured only in relation to PSTs' responses to the intervention, and there is no evidence to suggest that PSTs' levels of anxiety will reduce in the context of teaching mathematics, relationally or otherwise, in their practice.

These changes in beliefs, from the point of view of the goals of this study, are very promising, and it is worth recalling Lui & Bonner's (2016) contention that beliefs directly influence teacher behaviour, including planning and classroom practice. However, this study suggests this is not always the case, and this suggests the forces that constrain enactment of relational mathematical knowledge are strong enough to cause PSTs to behave in ways that are inconsistent with their beliefs. In keeping with Kennedy (1999), these findings support the contention that PSTs may be enacting instructional practices that are inconsistent with their beliefs and the pedagogical commitments they profess, and that this may contribute to the problem of enactment. Once again, this brings to light the research of Raymond (1997), Skott (1992, 2001, 2009), Hoyles (1992), and Sztajin (2003) who agree that social context, as opposed to beliefs alone, plays a significant role in mathematics pedagogical practice. This social context and its role on PSTs' classroom practice should be acknowledged within ITE, and addressed by ensuring PSTs are provided with sufficient opportunities for reflection on how their beliefs impact their practice (Cooney et al., 2008). Perhaps also, it could be argued, if PSTs are supported to deliberately target and enact high MQI lessons with a focus on relational understanding in varying contexts, then their beliefs will be further strengthened so that their practices and beliefs are become closely aligned.

The discussion about PSTs' behaviour is continued in the fourth and final research question, but at this point it is clear the intervention has not fully prepared all PSTs to deal with problematic classroom scenarios (Korthagen and Wubbels, 2001).

Question 4: **Can the intervention cause any change in PSTs' practice of teaching mathematics to become more democratic?**

For this study, democratic mathematics instruction is that which is consistent with the work of Freire and Dewey as outlined in Section 2.1, and this necessarily involves teaching and learning mathematics for relational understanding. The MQI framework helped to achieve this by seeing past the complexity of classroom teaching to focus on those aspects of mathematics instruction that determine the quality of the mathematics instruction and identify opportunities for pupils to engage with mathematics in a meaningful way.

Using the MQI framework, data collected from classroom observations and other artifacts related to PSTs' classroom teaching suggests positive changes to their classroom instruction compared to that described during the reconnaissance phase, as described in Section 1.3. However, it should be acknowledged the data collected during the reconnaissance phase was anecdotal in nature and not formally collected, i.e., it lacks the same trustworthiness as the formally collected data. Good examples of mathematics teaching in Cycle 1 this was exemplified by Freda, Vicky, and Sharon who all enacted mathematics based on relational understanding, and in a way that required pupils to think creatively and critically about their work. Even though Covid-19 restrictions made it more difficult to evaluate, in Cycle 2, instructional videos by Gemma and Helen also drew heavily on relational understanding. Overall, it is clear, based on my professional knowledge and my experiences as a teacher educator working with these and similar PSTs, that the intervention did result in some noteworthily positive changes to how some PSTs teach mathematics. That is, it would have been unusual to see three examples of very good mathematics teaching within a small sample of year 3 PSTs in previous years.

Admittedly, this assertion is limited by the design if this research study, which did not use an experimental design, and as such there was not control group for direct comparison. However, my assertions are reliant on my professional opinion and honesty which were guided by Bacon's Idols, in particular the idol of the tribe. That is, I was critically aware of the impact of conformation bias, in the form of critical subjectivity, so that objectivity could be maintained when making judgments about the PSTs' classroom performances.

Although there was no control group, it is worth examining the reflections of the three PSTs who were involved in the subgroups in both Cycle 1 and Cycle 2: Paul, Helen, and Rachael, because doing so gives some insight into their thinking about the intervention and its impact on their practice. The PSTs were asked to reflect three times during Cycle 2 and submit these to me through their online learning environment. The following is an excerpt from Paul's first reflection:

> "*I feel that this lecture has really clarified for me the importance of reflection both in terms of my own education but also my practice*" (Paul, reflection 1).

We know that this reflective process is important for PSTs as it helps them to address the notion that learning is simple and transmissive this issue by encouraging them to be conscious of their own learning (Loughran, 2006). Crucially, this awareness promotes informed decision making as PSTs construct their personal pedagogies (Hoban, 1997, p.135), and facilitates progression to Korthagen's (2009) schema level.

In reflection 2, Paul wrote the following:

> *The lecture this week was extremely helpful for me personally, but I also feel it helped the whole class too. By preparing a lesson for the lecture and then comparing it to others and opening up a discourse about the lessons and why we had chosen to teach the lesson in a certain way we were able to understand each other's viewpoints and then structure our lessons based on the feedback from this*. (Paul, reflection 2)

Similarly, in reflection 2 Helen wrote the following:

> *"I found this lecture very beneficial. Prior to the lecture, I had written, with my pod, a lesson plan for the introduction of division to a third class. While we thought that we had prepared an adequate lesson plan, it was not until I heard from my other peers in the class that we had neglected to address a somewhat obvious question; what does division really mean?" (Helen, reflection 2)*

Both Helen's and Paul's reflections suggests the intervention was successful in addressing important elements of Grossman and McDonald's (2008) pedagogies of enactment by giving PSTs opportunities to "practice elements of interactive teaching in settings of reduced complexity", while simultaneously receiving feedback and reflecting on this feedback. This practice centred approach to teacher education values the integrated nature of theory and practice and is designed to close the theory-practice divide while at the same time addressing the complexity of teaching.

Paul concluded his second reflection with the following:

> "*I now have a better idea around what I would attempt to do in order to teach division to a class for the first time and I am more aware of what I will need to be focusing on for other maths lessons thanks to the sheet we received with the different focus' for teaching maths*"

There are some important points from the above quote. Firstly, Paul is confident that he can now teach division better because of the intervention. Secondly, because of how the MQI framework was use during this part of the intervention, he also feels he can teach other mathematics lessons better. Finally, he explicitly talks about an awareness of this which is further evidence that Paul is no longer in the Gestalt level of teacher learning, which is characterised by a PSTs lack of awareness of their behaviours and are therefore unreflective about them. Furthermore, at the Gestalt level PSTs' teaching tends to be automatic and instrumental in nature. Paul's apparent progression towards the schematic level will allow him to develop generalised principles about how to teach mathematics for relational understanding.

In reflection 2, Helen explicitly refers to the Korthagen's (2009) Gestalt model:

> *"I think that considering how a child may see division was extremely beneficial, and I could see more clearly what is meant by the Gestalt model. I think that it will result in a more thorough thought process prior to my teaching of new mathematic concepts in my school placement, and indeed in my first year of teaching" (Helen, reflection 1)*

Gemma also reported how her mathematical learning improved from engaging in meaningful elements of practice:

> *"We worked in our groups during the week to write up our lesson plans and while the devising of the lesson plan itself was not too difficult when it came time to "teach" it to our peers [in the place of school children] it highlighted the gaps in our knowledge when it came to the correct mathematical language and explaining the procedure in simple understandable terms" (Gemma, reflection 2).*

Once again, these reflections highlight the efficacy of the intervention and suggests these three PSTs developed an ability to recognise, reflect on, and be responsive to their learning needs as beginning teachers. They also support the assertion that PSTs' classroom practices have improved since the start of the intervention.

However, teaching, teachers, and teacher education are complex, and the things PSTs say about their practice does not necessarily lead to corresponding enactment of practice (Skott, 2009). As such it would be overly simplistic to look at the effectiveness of the intervention in a binary way. During Cycle 1 observations, even when MQI scores were low, there were legitimate reasons given by PSTs explaining why they did not enact relational mathematics. For example, Gillian tried to enact mathematics for relational understanding but the pupils in her class essentially rejected this, as did the class teacher, and this made it difficult for her to continue with this. Gemma and Jessica made the incorrect assumption that because they perceived their pupils to have low ability, they are better suited to instrumental mathematics. Interestingly, both Gemma and Jessica reported high levels of mathematics anxiety combined with a fixed mindset view of achievement in mathematics in the open-ended section of the first questionnaire. Gemma explained:

> "*I have always had a narrative that I am not good at maths, I found that this started during secondary school. When I hear mathematical terminology I feel anxious and overwhelmed*" (Gemma, Questionnaire 1).

In secondary school Gemma recalled how her teacher's suggestion to focus on easy questions because for her to attempt more difficult problems was a "waste of time" has led to her belief that she "wasn't great at maths". Jessica also reported feeling anxious when presented with mathematics she is unfamiliar with. Considering Gemma's and Jessica's beliefs, and corresponding levels of anxiety, their classroom behaviour is unsurprising because PSTs with fixed mindset are more likely to enact a one-dimensional pedagogy of mathematics based on instrumental understanding (Boaler, 2016; Sun, 2018). These beliefs also help to maintain self-generating negative cycles of mathematical anxiety, which also result in instrumentally based mathematics teaching (Gresham, 2018; Finlayson, 2014). It is therefore worth remembering that changing behaviours is a slow process which requires sustained exposure to "sufficient, suitable and realistic experiences tailored to the needs and concerns" of PSTs (Korthagen, 2009, p.104), which combined with deliberate reflections will lead to Gestalt formation and improved practice.

Finally, while Gestalt formation is an essential consideration for meaningful teaching, enactment is about more than the cognitive and the affective issues, which are essential but ultimately insufficient. A holistic approach, which removes barriers and creates structures that enable PSTs to behave in ways consistent with their beliefs, is required to support PSTs' development as teachers.

7.3 Implications for my practice

This study has transformed my practice from one that mirrored Freire's banking method where PSTs were mainly passive listeners, to one that is now built upon a community of inquiry that respects the democratic process by engaging PSTs and TEs in shared dialogue and critical reflection (Freire, 1970). The original maths competency module relied disproportionately on the traditional cognitive domain of learning, but the intervention introduced the affective aspects of learning which allowed PSTs to respond to their "emotions, feelings and reactions, all of which are so enmeshed in the experiences of learning and teaching about teaching" (Loughran, 2006, p.3).

A big part of the change to my practice, and indeed, how I perceive my practice is knowing what motivates PSTs to learn mathematics. PSTs in this study are, by and large, not motivated by the mathematics as a discipline per se, but more so by the way their knowledge of mathematics content and pedagogy can help develop the pupils they teach. This phenomenon is supported by the literature whereby Philipp (2008) found that PST competence and confidence can be improved by helping them develop more nuanced perspectives about mathematics and its role in education, which may then change their growth trajectory to one of relational understanding. Philipp (2008) suggests that this can be done by motivating the students, not to teach mathematics first, but to first look at the overall learning needs of the children they teach and making mathematics part of this. Similarly, in the Irish context, Hourigan & Leavy (2012), found that a more holistic approach to learning mathematics, including pupil responses and misconceptions, and the organisation of classroom structures to support learning, results in a more profound impact on PST learning compared with traditional methods of mathematical instruction.

Action research is an *ongoing process* of action and reflection and I intend to use it to continue to improve my practice over time. As such, there are two immediate issues that need to be addressed. The first of these is a significant reduction in the breadth of mathematical topics PSTs are required to engage with in the maths competency module. I noted early in my reflective diary about the danger of focusing on practice at the expense of content. I have learned it is unrealistic for one small module to adequately address both content and pedagogical requirements, which is an issue also addressed in the literature (Clift and Brady, 2005). The second immediate issue that needs to be addressed is the separation between maths competency and maths methods. Logistically, addressing this issue is more difficult than making changes to my own practice because it involves engaging with and negotiating with other TEs. However, the disconnect between modules is a problem PSTs in this study have identified from the beginning, and it is also identified by Grossman and McDonald (2009) as contributing to the problem of enactment because it reduces ITE to a simplistic process-product model, while ignoring its inherent complexity.

If the second issue is addressed in a comprehensive way, it will also fix the content issue. It is necessary, therefore, to work with other TEs to develop meaningful links between both modules. This will provide a range of benefits including additional time, resources, and expertise, while allowing PSTs to experience coherency in their mathematics education to bridge the theory-practice divide. Merging modules will also address the unintended consequence of content being marginalised for the sake of practice. I have a responsibility to ensure PSTs fully understand the mathematics they are required to teach, and this will necessarily require a deliberate but controlled swing back towards content. To ensure this content is enacted, the maths methods lecturer and I have arranged to work in partnership to develop a comprehensive experience for PSTs. This will require full transparency between modules, which will be led by the curriculum. Topics will be planned for each year of the B.Ed. program and enactment pedagogies will be implemented within the maths methods module. Maths competency will support this by addressing the content knowledge requirements for each topic, and PSTs will be required to demonstrate this knowledge as part of their practice in maths methods lectures. The MQI framework will remain a consistent feature of maths competency and will form a concrete guide for examining the nature of relational understanding in mathematics. The MQI framework will also be used in maths methods and as such will form an important part of bridging the gap between the two modules. Shared readings between modules will also be assigned to PSTs to explain theory and address gaps in knowledge, while forming the basis for reflective practice in both modules so PSTs can critically engage with and grow from their learning (Dewey, 1933). The intention is to create a seamless connection between modules, so PSTs experience them not as fragmented unrelated parts but “interacting constituent elements of the whole” (Freire, 1970, p.85).

Bringing the modules together in this way will allow PSTs opportunities to learn more mathematics, and at the same time be given more opportunities to enact their MCK in a contextualised way using research informed HLTPs “in situations of reduced complexity” (Grossman, 2009). This is an important component of the relationship between the modules because this study strongly indicates that to enact mathematics for relational understanding, PSTs need to have reflected on representations of practice or practiced approximations of practice, using that content. That is, if PSTs do not learn mathematics in a contextualised way within a community of practice (Wenger, 1998), not only are they likely to forget the MCK, but it is likely they will forget they had even learned it in the first place. This phenomenon is captured by a post-lesson discussion I had with a PST, Freda, after observing a lesson on fractions during Cycle 1. During the conversation she reported difficulty teaching fractions because she had not engaged with the topic since she was in school herself. When I asked her why she did not use what she had learned about fractions in maths competency the previous year she asked me “did we do fractions last year”? I had a similar post lesson discussion with Lilly in Cycle 2.

When reflecting on these experiences I realised they were not isolated and I recalled many others, and they reinforced the most basic requirement of meaningful practice-based learning. It has been shown in the literature that PSTs enter ITE with limited, instrumentally based mathematical understanding (Leavy & O’Loughlin, 2006; Hourigan & Leavy, 2017; Costello & Stafford, 2019), reinforcing the belief that mathematics is a static procedurally based discipline (Philipp, 2008). Although these issues were addressed by the intervention, I anticipate more substantive improvements will result from developing a holistic model of mathematics education that actively engages PSTs and focuses on explicit links between the curriculum, practice, and content. It is important that this model is implemented continuously from the start of the B.Ed. programme to address, early on, the inevitable consequences of the apprenticeship of observation (Lortie, 1975).

We know the most fundamental barrier to enactment is related to tensions that exist between the university context and the school context (Flores & Day, 2006; Valencia, Martin, Place, & Grossman, 2009), and this can be addressed partly by including approximations of practice in ITE. This study has also highlighted those external issues that exist independent of the mathematics modules and so one of the recommendations emanating from this study is to empower PSTs to teach in a way that is consistent their beliefs. Consequently, as part of the partnership between maths competency and maths methods, it was agreed to work collaboratively with PSTs to develop lesson plans that are consistent with the new model of instruction. This will involve intentionally collaboratively developing lesson plans using HLTPs and the MQI framework, and these will be used as representations of practice which can be safely approximated within the University setting and later enacted in the classroom.

Furthermore, there will be an expectation on PSTs to include this on SP as part of their assessment and CTs and SPTs will be fully informed about this.

A concerning finding from the study was the notion that PSTs teach in a way they think their SPT would like to see, rather than focusing fully on maximising learning for their pupils. This brings into question the efficacy of this current model of SP assessment, and the nature of the relationships between PSTs and SPTs. Interestingly, no PSTs in this study referred to SPTs as a tutor, choosing instead “supervisor” or “inspector”, neither of which speaks to the democratic nature of education or suggests building a community of enquiry where PSTs and tutors can reflect simultaneously to promote meaningful learning. For this reason, I recommend a teacher educator be assigned to each PST as a form of critical friend for mathematics support, who does not need to be concerned with assigning grades. This role would involve a meaningful reflective process so PSTs can identify and enact areas of improvement, resulting in Gestalt formation and authentic learning.

Finally, this study presented clear evidence that a focus on practice necessitates a move away from examination-based assessment and towards a model that values learning for teaching. In this regard, assessment should be aligned to practice, including reflection and enactment of mathematical knowledge. Currently, the traditional maths competency content examination has been removed for B.Ed. years 2, 3 and 4 in favour of an assessment that is connected to practice and values collaboration. While this decision will place the focus on learning to teach, it is also consistent with my educational values. This model of assessment will be developed in future to reflect the partnership with maths methods.

All these changes will bring their own challenges such as how course work is distributed between modules, what the focus will be on, and issues of assessment. The intervention will need continuous careful planning and added negotiation between TEs and PSTs in the best interests of pupils. Many of these decisions around the intervention move beyond the modular towards a programmatic level significant collaboration. This may result in professionally and politically contentious situations which will need to be carefully navigated and managed.

7.4 Implications for teacher education

More than 40 years ago Zeichner and Tabachnic (1981) listed the influence of co-operating teachers, the ecology of the classroom, the bureaucratic norms of the school, teacher colleagues, and even pupils as the contributing factors to the problem of enactment. It is startling how accurate these factors were reflected in the current study.

Yet, when I described the knowledge transfer problem in the opening chapter, I was not aware of the concept of the problem of enactment, and this resulted in frustration at the quality of mathematics I observed on SP. Crucially, for several years I was unaware that the way I was teaching contributed significantly to this problem. Although I was reflective about my practice, it was only when I was more intentionally reflective about the issues I was observing and adopted Brookfield's (1995) lenses to examine the problem from different perspectives that I was able to begin to fully appreciate the nature of the challenge. As I became increasing reflective and increased my awareness through engagement with the literature in the context of my practice, I started my personal journey of Gestalt formation.

As described earlier in this chapter this has resulted in substantive changes to my practice, but on a micro and personal level it has changed the very nature of how I think about my practice, how I think about teachers and teaching, and how I act. In a recent lecture I was able to link the ideas of relational understanding, mathematics anxiety, the purpose of mathematics education, and the purpose of education generally. This resulted in a more interesting, engaging, and contextualised discussion about mathematical knowledge, and its role in the meaningful education of pupils. If I had not undertaken this study, it is likely I would be practicing a transmission style of mathematics teaching, and it is also likely I would be unaware of this. This suggests to me that teacher educators need to learn, and perhaps and Korthagen's model of teacher learning is equally applicable to teacher educators.

In Section 2.3.2 it was noted that "the quality of an education system can never exceed the quality of its teachers" (Barber & Mourshed, 2007, p.15). But to what extent does that quality of teachers depend on the quality of teacher educators? Research by MacPhail, Ulvik, Guberman, Czerniawski, Oolbekkink-Marchand and Bain (2019) investigated the professional development of 61 University-based teacher educators from England, Ireland, Scotland, and the Netherlands. Their findings highlight that despite their complex roles, TEs receive minimal opportunities for preparation or professional development to fulfil these roles. Furthermore, the study revealed Universities have limited interest in the professional development of teacher educators, and this ultimately results in frustrations and tensions navigating these roles.

When I engaged with colleagues about the problem of enactment, there appeared to be a general acknowledgment about it, but there was little awareness about the specific causes or solutions. This is concerning given the extent of the problem in ITE, and the simple idea that there are structures in place that inhibit PSTs from enacting the quality teaching they learn about. University-based teacher educators should be given opportunities, and perhaps incentives, to learn about academic and practical issues in the field, including enactment pedagogies, to ensure the quality of graduate teachers.

All teacher education departments in Ireland must undergo a periodic accreditation process, known as Céim[5], by the Teaching Council of Ireland. The process sets out the requirements which all ITE programmes of qualification for teaching in Ireland must meet in order to gain accreditation from the Teaching Council (Teaching Council, 2020). The Céim guidelines focus mostly on the sorts of skills and knowledge PSTs should develop, and the experiences they should undergo to develop these skills. It recognises that:

> "*Appropriate staff development policies should be in place to ensure that staff continue to enhance their knowledge and expertise including that relating to reflective practice, research, curriculum development, inclusive education and professional development*".

(Teaching Council, 2020, p.15).

Teacher education departments should collectively acknowledge the need to take this seriously. Regardless of research interests and subject specialisms, teacher educators need to be aware of the fundamentals of how PSTs learn to teach, and the implications of this for their practice. Within university-based teacher education departments, this may lead to tensions between academic freedoms and the basic requirements of the profession. Regarding signature pedagogies and the preparation for the professions, Shulman (2005, p.53) noted:

[5] The Irish language translation for Degree

> "We have become increasingly cognizant of the many tensions that surround professional preparation, from the competing demands of academy and profession to the essential contradictions inherent in the multiple roles and expectations for professional practitioners themselves".

These competing demands, combined with minimal opportunities for the professional development of teacher educators (MacPhail et al., 2019), requires strong leadership at a departmental level to promote research informed professional development. Perhaps there is a role for teacher education departments to form internal committees for professional development or include professional development roles as part of suitably established committees.

The final recommendation from this study is that ITE departments move away from the traditional model of ITE where the university provides the knowledge, the school provides the placement setting, and the PST provides the individual effort to assimilate and apply this knowledge in the practice setting (Wideen et al., 1998, p.167). Zeichner (2010) argues this model, where academic knowledge is viewed as the authoritative source of knowledge, is outdated and should be replaced by one that instead by one that relies on non-hierarchical and non-dichotomous relationships between practitioner, academic, and partner schools. Zeichner (2010) conceptualised the idea of a *third space* where PSTs, TEs, and CTs can work collaboratively together in partnership, rather than just cooperating with each other. He argues this epistemology of ITE will create more authentic learning opportunities for PSTs that will better prepare them to be successful in enacting complex teaching practices. The model requires a hybrid space to close the cultural gap between schools and universities so that practical and academic knowledge can be drawn together and problematised collectively. It also requires an end to the separation of theoretical and subject-matter knowledge from practical classroom work to incorporate a seamless blend of technical and intellectual knowledge.

7.5 Limitations and future research

Several limitations, and opportunities for future research, were identified in this research study, and these are discussed in this section. To begin, a potential limitation is related to the dual role I played in Cycle 2, namely School Placement Tutor for PSTs and researcher with the same PSTs. This situation arose out of the necessity to continue with my research and at the same time carry out my role as a teacher educator in the challenging circumstances caused by restrictions introduced as a result of the Covid-19 pandemic. As SP tutor, I had responsibility for providing a pass/ fail grade for each PST performance and a percentage mark for their SP file. The pass/ fail grade for performance was based on three remote meetings I had with the PSTs in the study. SPTs were given specific topics to discuss with PSTs during these meetings, but ultimately the purpose was to encourage PSTs to reflect on their practice. This dual setup made it somewhat inappropriate to probe too deeply into their mathematics lessons as to do so would have blurred the blurred the lines between tutor on one hand, and researcher on the other.

The second limitation is related to the first. Because restrictions were put in place due to the Covid-19 pandemic, the second Cycle was treated somewhat differently to the first. In terms of data collection, the most significant of these was analysing PSTs' planning documentation in place of classroom observations which were carried out in Cycle 1. This, unfortunately, made it very difficult to make meaningful comparisons between Cycle 1 and Cycle 2 in terms of the sort of mathematics PSTs were enacting in their practice, or indeed determine with a great deal of certainty the extent to which PSTs taught mathematics for relational understanding. This difficulty was exacerbated by the fact that PSTs were also teaching remotely which limited their scope to enact the sort of mathematics pedagogy they might have in a face-to-face scenario. This was compounded further by the fact that the co-operating teachers, out of necessity, sometimes overrode PSTs' plans (i.e., topics to be taught) for teaching mathematics, or restricted the amount of mathematics they could teach at all.

Admittedly, it would have been beneficial to supplement PSTs' planning documentation and instructional videos with additional data sources, such as individual interviews, as this would have provided rich descriptions of the decisions PSTs made during SP2. While not doing so is an acknowledged limitation, there were several reasons why this was not possible, or it was felt, appropriate. The reasons for this were previously discussed in Section 3.6.4.

There is also a potential limitation in relation the questions on the survey related to beliefs about preparedness to teach mathematics and beliefs about program effectiveness. For each of these questions PSTs were asked to indicate a response based on the Likert scale for a number of statements. However, the statements did not distinguish between maths competency and maths methods, and this may be a source of conflict for some PSTs as indicated by some of their qualitative inputs. Perhaps this could be improved in future by adapting the survey so that there is a possibility to indicate the extent of their beliefs for just one or both modules separately, as this could provide more useful data.

Due to the limited scope of the EdD programme, the reflective data collected was not used to a great extent in this study. While my reflections were used implicitly throughout the study, particularly in relation to changes to the intervention, there is no dedicated section dedicated to the rigorous analysis of all of this data which amounts to approximately fifteen thousand words. Furthermore, reflective data was systematically collected from PSTs in Cycle 2, and while this was used in a limited way, there is scope for more rigorous analysis on this also. This presents an opportunity for a future study to analyse and present this yet largely untapped reflective data to seek further insights into the problem of enactment and the efficacy of the intervention in addressing this problem.

Finally, there are two areas that I became interested in and captured in my personal reflective diary which are worthy of including here for further exploration. These are PST agency and the gender dimension of PSTs' beliefs. Firstly, PST agency is something I reflected on early in the study in the context of Korthagen's (2009) 3-level gestalt model of teacher learning. The purpose of this model is essentially to bridge the gap between teacher knowledge on one hand, and practice on the other. Korthagen puts forward several reasons for the "transfer problem" including PSTs being socialised into existing patterns in schools, the inherent complexity of teaching, the apprenticeship of observation, and PSTs having "little time to think". Korthagen also suggests that there are affective issues which contribute to the problem. Although Korthagen doesn't specifically mention the phenomenon of teacher agency, it is in fact an umbrella term to describe the "transfer problem". This is especially true if we adopt Peristly, Biesta and Robinson's (2015) ecological conception of teacher agency which is based on the idea that agency is the interplay of one's capacity to act as well as the social conditions which impact how one acts. There are clear connections between this conception of teacher agency and the ideas put forward in this study relating to the problem of enactment. Perhaps this was a lost opportunity to reconcile both of these concepts into a single study given that there appears to be a sparsity of existing studies that have already done this. This certainly presents another area for future research.

The second area of interest which has arisen from this study is that of the gender dimension of PSTs' beliefs about achievement in mathematics. Just recently I was asked to provide a short presentation on this topic and as a result I delved a little deeper into the responses to just one of the eight statements on the survey item related to mathematics achievement. This statement read: "In general, boys tend to be naturally better at mathematics than girls". All of the PSTs in the study, of which approximately 90% are female, were asked to rate the extent to which they agreed or disagreed with this statement. The Likert scale ranges from 1 (strongly disagree) through 6 (strongly agree). The mean score for all PSTs in surveys 1, 2 and 3 were 2.3, 2.5, and 2.1. this means that the PSTs in this study, on average, don't fully disagree that boys are better than girls at maths. On average PSTs' beliefs lie somewhere between slightly disagree and disagree. Another interpretation of the results is that in survey 1 22% of PSTs agree boys are better than girls, in survey 2 29% of PSTs agree boys are better than girls, and in survey 3 20% of PSTs agree boys are better than girls at mathematics. These initial results are concerning because of the possible implications that may have for the classroom and are certainly worthy of further investigation in future research projects.

7.6 Conclusion

The motivation for this study resulted from a series of routine observations of PSTs' SP mathematics lessons in 2015. During these observations I noticed PSTs relied heavily on transmission style mathematics teaching, characterised by instrumental understanding, where pupils occupied a passive and uncritical role in their learning. I questioned why PSTs would teach this way because in maths competency modules I taught them to understand mathematics relationally, and I also assumed I was instilling an attitude that valued this sort of understanding. I later identified this as the problem of enactment, and this dissertation described my journey of investigating the problem from the perspective of my practice, and what I have learned along the way. Several reasons were put forward for the problem of enactment, but ultimately it is the decontextualised and compartmentalised nature of learning in ITE, underpinned by learning based on the traditional model of cognition. This is exacerbated by the neoliberal push within primary mathematics education in Ireland which tends to reward both teachers and pupils for embracing instrumental over relational understanding.

Along this research journey I was guided by the democratic principles outlined in my theoretical position, which provided a critical lens through which I could evaluate my practice, and PSTs' practice, as well as a tool to critique teacher education generally. From an epistemological perspective, the accounts and observations of PSTs formed the backbone of this study, and it was only through their generosity of time and openness to share, that led to the rich insights presented in this dissertation.

Freire's (1970) problem posing model of education guided me to decompose and rebuild my practice in a way that supported PSTs' development of relational understanding of mathematics in the context of authentic experiences involving creativity, critical dialogue, and inquiry. Knowledge in its many forms, including cognitive and situational, became "interacting constituent elements of the whole" (Freire, 1970, p.8). When I realised the disconnect also existed on a modular level, work was done to connect maths methods to maths competency, and as these two modules became increasingly connected, together it became clear they are unrelated from other parts of the B.Ed. programme and therefore static, compartmentalised and disconnected from the whole (Freire, 1970). As teacher educators we need to work together to address the problem of compartmentalisation in ITE so that PSTs can enjoy a coherent experience underpinned by a unified message and a set of implementable core principles.

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Appendices

Appendix 1: Information letter to PSTs Cycle 1

Maynooth University Froebel Department of Primary and Early Childhood Education

Roinn Froebel Don Bhun- agus Luath- Oideachas, Ollscoil Mhá Nuad

Dear third year Bachelor of Education students,

I (Eddie Costello) am currently completing a Doctor of Education degree and would like to ask each of you to become co-participants in my research. Participation in the research is entirely voluntary. This information sheet is to explain the research to you, as well as any risks and benefits that may arise from your participation, so that you can make an informed decision. If you have any questions about the information on this sheet, please ask me about it.

The research aims to find out if a different approach to teaching maths competency can bring about subsequent changes in the way maths is taught on school placement. As a result of this planned change in teaching, the structure of mathematics competency lectures will differ from previous years. This year all lecture content will be available for you to watch on video, and will be accompanied by lecture notes, as per previous years. This will allow you watch the videos at a pace that suits you, and at a time that suits you. You will also have the benefit of being able to re-watch the videos as often as you like. During lectures, this will give us the opportunity to discuss aspects of the content that you find difficult. This will be done in the first 15-20 minutes of maths competency lectures. The remainder of our lectures will be dedicated to practical work, discussions and reflections about the application of your mathematical knowledge in the primary school setting.

Lectures will take this format for 6 weeks up until your first placement, week commencing October 21st 2019. This is a total of 6 hours using the new pedagogical approach.

I would like to collect data from and with you. This data will be collected between September and November 2019. As a participant you will be asked to engage in the following:

- Two questionnaires relating to your beliefs about mathematics and mathematics education. The first questionnaire will be completed before mathematics competency lectures begin, and the same questionnaire will be repeated and completed after October school placement.

From those of you who do consent to participate in the research, a further 16 participants will be asked to participate further. This will involve being asked about your ideas and opinions about mathematics and mathematics education, which will add more depth to the research. This group will be selected based on survey responses. If you consent to be part of the subgroup you will be asked to engage in the following:

- Reflective writing: While all students will engage in reflective writing this year, as a participant you will be asked to submit your reflective journal for analysis. These will not be graded.

- Observation: you will be asked to volunteer to be observed by me for one of your maths lessons during SP. While I will take some notes about the lesson itself, this will not be graded. It will give us an opportunity to discuss how your maths lessons are going and offer you valuable feedback which will benefit you for future maths lessons. There will be no audio or visual recordings of the lesson or subsequent conversations.

- Focus groups: The purpose of this is to provide an opportunity for you to engage in conversations with your peers (in groups of 8), moderated by me, about maths education generally. This will be held after your October placement. It will be an opportunity for you to think about the new structure of the maths competency lectures and give your opinion about this new way of teaching and learning maths. These focus groups will be audio recorded and you will be asked to consent for this data to be used as part of the research project. Only the researcher will have access to these recordings.

As a participant and co-researcher, your honesty is crucial to the scientific impact of this research. It is very important that any data you provide represents your opinions, thoughts and ideas and not something you think the researcher would want to hear. There are no negative consequences for being critical or controversial about any of the topics we may discuss either individually or as a group. In fact, this is welcomed and necessary for the improvement of mathematics education in Initial Teacher Education.

Please note there are no risks to you as a participant. However, should you feel uncomfortable at any time during the research for any reason, you will be reminded of your right to withdraw from the process at any point without any judgment or penalty.

As a participant you will be provided with a unique participant number. There will be a text box provided on your questionnaire and your template for SP maths reflections in which you can write this number. The purpose of this is to ensure that your data is not identifiable to anybody but me, the researcher. The key which links the participant numbers to your names will be kept on a password protected file on an encrypted and password protected desktop computer in a locked office in Maynooth University. This key will be destroyed as soon as the research project ends. Similarly, all and any data that you submit will be kept in a secure filing cabinet in a locked office in Maynooth University. Your data may also be used for future publications and/ or conference presentations but your anonymity as participants will be guaranteed at all times. All data collected as part of the project will be destroyed after a period of 10 years.

It must be recognised that, in some circumstances, confidentiality of research data and records may be overridden by courts in the event of litigation or in the course of investigation by lawful authority. In such circumstances the University will take all reasonable steps within law to ensure that confidentiality is maintained to the greatest possible extent.

If you would like to participate in this research project, please sign and date the attached consent form. Please be aware that even if you do sign the consent form, you are under no obligation to partake in this research and can withdraw from the project at any time. If you do not sign the form, you will still attend lectures as usual but will not take part in any of the data collecting activities. Please also be aware that there are no negative consequences to non-participation in this research project. It will not negatively impact on your grades in maths competency or SP in any way. Additionally, non-participation will not have any impact on your good standing reputation as a student in the Froebel department.

Thank you all for taking the time to read this and considering becoming a research participant and co-researcher in this project. If you have any questions or concerns, please don't hesitate to contact me.

Kind regards,

Eddie Costello

Maynooth University Froebel Department of Primary and Early Childhood Education

T: +353 1 4747418 E: eddie.costello@mu.ie

If during your participation in this study you feel the information and guidelines that you were given have been neglected or disregarded in any way, or if you are unhappy about the process, please contact the Secretary of the Maynooth University Ethics Committee at research.ethics@nuim.ie or +353 (0)1 708 6019. Please be assured that your concerns will be dealt with in a sensitive manner.

Maynooth University
National University of Ireland Maynooth

Maynooth University Froebel Department of Primary and Early Childhood Education

Roinn Froebel Don Bhun- agus Luath- Oideachas, Ollscoil Mhá Nuad

By ticking this box, I confirm that I have read the attached information letter and am fully informed about the research project: ☐

By ticking this box, **I do consent** to be a participant in the main group. That is, **I do consent** to completing a questionnaire before School Placement and another questionnaire after School Placement. ☐

By ticking this box, **I do not consent** to be part of the main group. ☐

If you have volunteered to be a participant in the main group, you also have the opportunity to be considered to be a participant in the smaller group of 16 students who will take part in a School Placement observation and a focus group. Please indicate below whether or not you would like to be part of this smaller group.

By ticking this box, **I do consent** to be a participant in the smaller group of 16 students. That is, **I do consent** to taking part in a school placement observations and a focus group. ☐

By ticking this box, **I do not consent** to be part of the smaller group. ☐

Appendix 2: Letter to principals

October 31st, 2019

Dear Principal/ Chairperson of the Board of Management,

My name is Eddie Costello. I am a lecturer of mathematics in the Froebel Department of Primary and Early Childhood at Maynooth University. The modules I teach on are focused on improving the mathematical content knowledge (MCK) of pre-service primary school teachers (PSTs).

I am writing to ask for your co-operation in relation to research I am engaging in as part of my Doctoral studies. This research is an assessment of my own practice as a teacher educator, and part of this involves looking at the extent to which PSTs in the Froebel department use MCK while they are teaching mathematics lessons on their School Placement (SP). The proposal for this research has received ethical clearance from the Maynooth University Social Research Ethics Committee (Ref No. SRESC-2019-2374320).

The participants in the study are 3rd year Bachelor of Education students in the Froebel Department. One of these students, [insert name], is currently carrying out her/ his School Placement in your school. I would like your permission to observe one of [insert name] mathematics lessons over the course of his/ her placement. [insert name] has already kindly agreed to be a participant in this research project.

This visit is part of the existing Froebel Placement protocol and is designed to cause as little disruption as possible to the student and the school. The visit will be identical to a typical SP visit, with the exception that this visit will be only for mathematics and there will be no grade associated with it. As is typical with SP visits, I will have a brief conversation with the student post-observation. The purpose of this conversation is not for research purposes, but to guide the student in the reflective process and improve future practice.

Only data related to the *content* of the mathematics lesson will be collected, and I will look only at the PSTs actions in relation to this. No data will be collected from any of the pupils in your school, and they will not be negatively affected by the research in any way.

I plan to visit [insert name] over the next two weeks and will contact him/ her in order to give him/ her at least 24-hour's notice. I will also contact the school so that you are fully aware of my visit to [insert name].

If you have any queries regarding m y visit to [insert name] or the research project please do not hesitate to contact me by phone on 085 8140239 or by email at eddie.costello@mu.ie.

Yours sincerely,

Eddie Costello

Appendix 3: Questionnaire Cycle 1

Maynooth University Froebel Department of Primary and Early Childhood Education

Roinn Froebel Don Bhun- agus Luath- Oideachas, Ollscoil Mhá Nuad

Student ID:

Note: Do not write your name on this sheet.

Survey Information

This questionnaire is to find out your beliefs about mathematics (the subject), as well as about the teaching and learning of mathematics. You will also be asked about your opinions on the effectiveness of your teacher education programme for preparing you to teach mathematics. This data will be used as part of a research project to improve mathematics teaching in initial teacher education. Your opinions and ideas are a very important part of this. You should spend approximately 20 minutes completing this survey.

As a participant and co-researcher, your honesty is crucial to the scientific impact of this research. It is very important that any data you provide represent your opinions, thoughts and ideas and not something you think the researcher would like to hear.

You will be provided with a unique participant number. There is a space at the top of this questionnaire for this. The purpose of this is to ensure that your data is not identifiable to anybody but me, the researcher.

If you are unsure of an answer on any of the rating scales, please do not leave it blank. Rather, choose the answer you think best represents your opinion.

PERSONAL RESPONSE TO MATHEMATICS

Please rate your feelings for each of the 10 maths activities listed below on a scale from one (no negative feelings/ anxiety) to five (the worst feelings: the most tension, fear, worry, nervousness or anxiety)

A	Activity	Circle Closest Answer				
		No bad feelings or anxiety	Somewhat bad feelings	Fearful, tense, nervous, anxious	Very bad feelings	Worst feelings of anxiety/ fear
1	Having to use the *Formulae and Tables* booklet in a maths test.	**1**	**2**	**3**	**4**	**5**
2	Thinking about an upcoming maths test one day before it.	**1**	**2**	**3**	**4**	**5**
3	Watching a teacher or lecturer work an algebraic equation on the board.	**1**	**2**	**3**	**4**	**5**
4	Taking a maths exam.	**1**	**2**	**3**	**4**	**5**
5	Listening to a lecture in maths class.	**1**	**2**	**3**	**4**	**5**
6	Listening to another student explain a maths formula.	**1**	**2**	**3**	**4**	**5**
7	Having to take a surprise maths test in class.	**1**	**2**	**3**	**4**	**5**
8	Starting a new chapter in a maths book.	**1**	**2**	**3**	**4**	**5**
9	Learning to teach a maths concept you are unfamiliar with for senior classes on your upcoming SP	**1**	**2**	**3**	**4**	**5**

PERSONAL RESPONSE TO MATHEMATICS Continued.

If you wish to do so, please elaborate on any aspects of anxiety you feel about learning, taking exams in, or teaching mathematics on this blank page:

BELIEFS ABOUT THE NATURE OF MATHEMATICS

To what extent do you agree or disagree with the following beliefs about the nature of mathematics?

B		Strongly Disagree	Disagree	Slightly Disagree	Slightly Agree	Agree	Strongly Agree
1	Mathematics is a collection of rules and procedures that prescribe how to solve a problem.						
2	Mathematics involves the remembering and application of definitions, formulas, mathematical facts and procedures.						
3	Mathematics involves creativity and new ideas.						
4	In mathematics many things can be discovered and tried out by oneself.						
5	When solving mathematical tasks you need to know the correct procedure or there is a chance you'll get it wrong.						
6	If you engage in mathematical tasks, you can discover new things for yourself without your teacher telling them to you (e.g., connections between concepts, rules)						
7	Fundamental to mathematics is its logical rigor and preciseness.						
8	Mathematical problems can be solved correctly in many ways.						
9	Many aspects of mathematics have practical relevance.						
10	Mathematics helps solve everyday problems and tasks.						
11	To do mathematics requires much practice, correct application of routines, and problem solving strategies.						
12	Mathematics means learning, remembering and applying.						

BELIEFS ABOUT THE NATURE OF MATHEMATICS Continued.

Please elaborate on *any* aspects of your beliefs about the nature of mathematics on this blank page:

BELIEFS ABOUT LEARNING MATHEMATICS

From your perspective, to what extent would you agree or disagree with each of the following statements about pupils learning of mathematics?

c		Strongly Disagree	Disagree	Slightly Disagree	Slightly Agree	Agree	Strongly Agree
1	The best way to do well in mathematics is to memorise all the formulas.						
2	Pupils need to be taught exact procedures for solving mathematical problems.						
3	In a mathematics lecture or exam it doesn't really matter if you fully understand a mathematical problem, as long as you know how to get the right answer.						
4	To be good in mathematics you must be able to solve problems quickly.						
5	Pupils learn mathematics best by focusing on to the teacher's explanations.						
6	When working on mathematical problems, more emphasis should be put on getting the correct answer than on the process followed.						
7	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.						
8	Teachers should allow pupils to figure out their own ways to solve mathematical problems.						
9	Non-standard procedures should be discouraged because they can interfere with learning the correct procedure.						
10	Hands-on mathematics experiences are not worth the time and expense.						
11	Time used to investigate why a solution to a mathematical problem works is time well spent.						
12	Pupils can figure out a way to solve mathematical problems without a teacher's help.						
13	Teachers should encourage pupils to find their own solutions to mathematical problems even if they are inefficient.						
14	It is helpful for pupils to discuss different ways to solve particular problems.						

BELIEFS ABOUT LEARNING MATHEMATICS Continued.

Please elaborate on *any* aspects of your beliefs about learning mathematics on this blank page:

BELIEFS ABOUT MATHEMATICS ACHIEVEMENT

To what extent do you agree or disagree with each of the following statements about pupil achievement in primary mathematics?

D		Strongly Disagree	Disagree	Slightly Disagree	Slightly Agree	Agree	Strongly Agree
1	Since older pupils can reason abstractly, the use of hands-on models and other visual aids becomes less necessary.						
2	To be good at mathematics you need to have a kind of "mathematical mind".						
3	Mathematics is a subject in which natural ability matters a lot more than effort.						
4	Only the more able pupils can participate in multi-step problem solving activities.						
5	In general, boys tend to be naturally better at mathematics than girls.						
6	Mathematical ability is something that remains relatively fixed throughout a person's life.						
7	Some people are good at mathematics and some are not.						
8	Some ethnic groups are better at mathematics than others.						

BELIEFS ABOUT MATHEMATICS ACHIEVEMENT continued.

Please elaborate on *any* aspects of your beliefs about mathematics achievement on this blank page:

BELIEFS ABOUT PREPAREDNESS FOR TEACHING MATHEMATICS

Please indicate the extent to which you think your teacher education program (including Maths competency, LMSE, CMA, etc.) has thus far prepared you to do the following for teaching maths in senior classes on your upcoming SP.

E		Not at all	A minor extent	A moderate extent	A major extent
1	Communicate ideas and information about mathematics clearly to pupils.				
2	Establish appropriate learning goals in mathematics for pupils.				
3	Set up mathematics learning activities to help pupils achieve learning goals.				
4	Use questions to promote higher order thinking in mathematics.				
5	Use computers and ICT to aid in teaching mathematics.				
6	Challenge pupils to engage in critical thinking about mathematics.				
7	Establish a supportive environment for learning mathematics.				
8	Use assessment to give effective feedback to pupils about their mathematics learning.				
9	Provide parents with useful information about your pupils' progress in mathematics.				
10	Develop assessment tasks that promote learning in mathematics.				
11	Incorporate effective classroom management strategies into your teaching of mathematics.				
12	Have a positive influence on difficult or unmotivated pupils.				
13	Work collaboratively with other teachers.				

BELIEFS ABOUT PREPAREDNESS FOR TEACHING MATHEMATICS continued.

Please elaborate on *any* aspects of your beliefs about how well you feel you are prepared to teach mathematics to 5th and 6th class on this blank page:

BELIEFS ABOUT PROGRAM EFFECTIVENESS

To what extent do you agree or disagree with the following statements? The instructors who teaches mathematics related courses in your current teacher preparation program:

F		Strongly Disagree	Disagree	Slightly Disagree	Slightly Agree	Agree	Strongly Agree
1	Models good teaching practices in their teaching.						
2	Draws on and uses research relevant to the content of their modules.						
3	Models evaluation and reflection on their own teaching.						
4	Values the learning and experiences you had prior to starting the program						
5	Values the learning and experiences you had on your School Placement.						
6	Values the learning and experiences from other modules you have taken on your teacher preparation program.						

BELIEFS ABOUT PROGRAM EFFECTIVENESS continued.

Please elaborate on *any* aspects of your beliefs about <u>how well the maths related courses in Froebel prepare you to teach mathematics</u> on this blank page.

Overall, how effective do you believe your pre-service teacher education program (Froebel) is in preparing you to *teach* mathematics?

Check one box

A. Very ineffective ☐

B. Ineffective ☐

C. Effective ☐

D. Very effective ☐

Please elaborate on *any* aspects of your beliefs about how effective your teacher education program is in preparing you to teach mathematics in the space below:

Appendix 4: Information Letter to PSTs Cycle 2

Maynooth University Froebel Department of Primary and Early Childhood Education

Roinn Froebel Don Bhun- agus Luath- Oideachas, Ollscoil Mhá Nuad

Dear fourth year Bachelor of Education students,

As was communicated to you last year, I (Eddie Costello) am currently completing a Doctor of Education degree. I would like to invite each of you to continue to participate in this research. Participation in the research is entirely voluntary. This information sheet is to explain the research to you, as well as any risks and benefits that may arise from your participation, so that you can make an informed decision. If you have any questions about the information on this sheet, please ask me about it.

The research aims to find out if our new approach to teaching maths competency, based on pedagogies of enactment, can bring about subsequent changes in the way maths is taught on school placement. As you know this resulted in a change to the structure of our mathematics competency lectures last year, and this will continue in your final year. Similar to last year, all lecture content will be available for you to watch on video, and will be accompanied by lecture notes, as per previous years. This will allow you watch the videos at a pace that suits you, and at a time that suits you. You will also have the benefit of being able to re-watch the videos as often as you like. During lectures, this will give us the opportunity to discuss aspects of the content that you find difficult, while also dedicating time to practical work, discussions and reflections about the application of your mathematical knowledge in the primary school setting. Lectures will take this format for 9 weeks up until your extended school placement, week commencing 9th November 2020.

The data you provided last year was extremely valuable, and has helped to make additional changes to this years maths competency module. I hope we can continue to develop the module and for this I would also like to collect some additional data from you this year. As a student in maths competency lectures you will be required to submit weekly reflections after lectures. While these are a compulsory component of the module, they will not be graded. If you agree to participate in the research this year, you will be asked permission for these reflections to be analysed. As a participant you will also be asked to submit reflections after each of your maths lessons for analysis. The purpose of this is to get your views on how the maths competency module can be further improved.

On return to college after your school placement, each participant will be asked to complete a survey. This will be similar to the survey you completed last year and is related to your beliefs about mathematics teaching and learning.

A further 16 participants will be invited to participate in a focus groups to discuss mathematics pedagogy and content. This will be done on a voluntary basis. If more than 16 participants volunteer, I will do my best to accommodate everybody. Otherwise, names will be randomly selected. This will be held after your extended school placement, in February. The purpose of the focus groups is to provide an opportunity for you to engage in conversations with your peers (in groups of 8), moderated by me, about mathematics education generally. It will be an opportunity for you to think about and share your opinions on our maths competency lectures, and suggest how they can be improved. These focus groups will be audio recorded and you will be asked to consent for this data to be used as part of the research project. Only the researcher will have access to these recordings.

As a participant, your honesty is crucial to the scientific impact of this research. It is very important that any data you provide represents your opinions, thoughts and ideas and not something you think the researcher would want to hear. There are no negative consequences for being critical or controversial about any of the topics we may discuss either individually or as a group. In fact, this is encouraged and is necessary for the improvement of mathematics education in Initial Teacher Education.

Please note there are no risks to you as a participant. However, should you feel uncomfortable at any time during the research for any reason, you will be reminded of your right to withdraw from the process at any point without any judgment or penalty.

As a participant you will be provided with a unique participant number. There will be a text box provided on your questionnaire and your template for SP maths reflections in which you can write this number. The purpose of this is to ensure that your data is not identifiable to anybody but me, the researcher. The key which links the participant numbers to your names will be kept on a password protected file on an encrypted and password protected desktop computer in a locked office in Maynooth University. This key will be destroyed as soon as the research project ends. Similarly, all and any data that you submit will be kept in a secure filing cabinet in a locked office in Maynooth University. Your data may also be used for future publications and/ or conference presentations but your anonymity as participants will be guaranteed at all times. All data collected as part of the project will be destroyed after a period of 10 years.

It must be recognised that, in some circumstances, confidentiality of research data and records may be overridden by courts in the event of litigation or in the course of investigation by lawful authority. In such circumstances the University will take all reasonable steps within law to ensure that confidentiality is maintained to the greatest possible extent.

If you would like to participate in this research project, please sign and date the attached consent form. Please be aware that even if you do sign the consent form, you are under no obligation to partake in this research and can withdraw from the project at any time. If you do not sign the form, you will still attend lectures as usual but will not take part in any of the data collecting activities. Please also be aware that there are no negative consequences to non-participation in this research project. It will not negatively impact on your grades in maths competency or you school placement in any way. Additionally, non-participation will not have any impact on your good standing reputation as a student in the Froebel department.

Thank you all for taking the time to read this and considering becoming a research participant in this project. If you have any questions or concerns, please don’t hesitate to contact me.

Kind regards,

Eddie Costello

Maynooth University Froebel Department of Primary and Early Childhood Education

T: +353 1 4747418 E: eddie.costello@mu.ie

If during your participation in this study you feel the information and guidelines that you were given have been neglected or disregarded in any way, or if you are unhappy about the process, please contact the Secretary of the Maynooth University Ethics Committee at research.ethics@nuim.ie or +353 (0)1 708 6019. Please be assured that your concerns will be dealt with in a sensitive manner.

Consent form

ID:

Please read the following statements below and tick the appropriate box regarding your willingness to consent.

Please tick the relevant box below to indicate your consent for you reflections to be analysed:

I do consent for my reflections to be analysed ☐

I do not consent for my reflections to be analysed ☐

Please tick the relevant box below to indicate your consent to participate in a focus group:

I do consent to completing a questionnaire after School Placement and for the data from this to be analysed ☐

I do not consent to completing a questionnaire after School Placement ☐

Please tick the relevant box below to indicate your consent to participate in a focus group:

I do consent for to volunteer to participate in the focus group and for the data from this to be analysed ☐

I do not consent for to participate in the focus group ☐

Appendix 5: Letter to Research Committee for Cycle 2

7th September, 2020

Dear Members of Social Research Ethics Sub-Committee,

I am writing this letter to request that my previous ethical approval for my EdD be extended for the coming year. The reference number for this application is: SRESC-2019-2374320.

For this current application I have included a copy of last years completed application form with changes highlighted in yellow.

Supporting documentation is included as appendices as follows:

Appendix 1: My supervisor letter (no changes)

Appendix 2: Information letter to participants (this is a new letter)

Appendix 3: Questionnaire (no changes)

Appendix 4: references (no changes)

Appendix 5: The Intervention (changes highlighted in yellow)

Appendix 6: MQI framework (no changes)

Appendix 7: This cover letter (this is a new letter)

All of the documentation are included and attached as a single PDF file.

Kind regards,

Eddie Costello

Appendix 7: Ethical approval

MAYNOOTH UNIVERSITY,
MAYNOOTH, CO. KILDARE, IRELAND

Maynooth University
National University of Ireland Maynooth

Dr Carol Barrett
Secretary to Maynooth University Research Ethics Committee

16 September 2019

Eddie Costello
Froebel Department of Primary Education
Maynooth University

RE: Submission for ethical approval for a project entitled: Enacting mathematical content knowledge in the classroom: democracy through relational understanding

Dear Eddie,

The above project has been evaluated under Tier 2 process: expedited review and we would like to inform you that ethical approval has been granted.

Any deviations from the project details submitted to the ethics committee will require further evaluation. This ethical approval will expire on 31/08/2021.

Kind Regards,

Dr Carol Barrett
Secretary,
Maynooth University Research Ethics Committee

C.c. Dr Zerrin Doganca Kucuk, Education Department

Reference Number
SRESC-2019-2374320

Appendix 8: Focus Group 1a Transcript

Date: 4/12/19

Participants: Claire, Gemma, Gillian, Aoife, Sharon, Jessica, Brona, Helen.

Location: Education Department, Maynooth University.

Numbers correspond to paragraph numbers displayed in text references to this transcript.

1. EC: ok we'll start with maths competency pre-intervention. So we're talking really about 1st year and 2nd year. What I want to know is what did maths competency mean to you? What do you feel it's purpose was? And what was it useful for, if anything? Take your time.

2. Claire: To be honest I just learned off how to do each one. and that's what I did. I know there was reason behind it but I know my main focus in first year was just to get the grade and pass, and get into the next year. That's where I was coming from

3. Gillian: you can kind of see the thought behind it, that you understand it a bit more and I think it did help in that sense a little bit but when you're doing it kind of feels separate to teaching. it doesn't feel like you can really link it in. I don't know if that was just me. It felt very separate from, sort of, classes where you learn about actually teaching stuff to children. Sort of, you were learning the stuff. It wasn't to pass it on. And it just…you were competent yourself. So it felt quite separate to the actual teaching part.

4. EC: OK is that the general consensus?

5. [PST's nod in agreement]

6. EC: that's interesting. Why do you think you feel that way? I mean, do you think it's the content? Or is it the way the content is presented?

7. Jessica: I think this year, probably, it has become a bit easier having the videos so you can watch it a few times. Because after a lecture I might have went home very confused but now I have the videos to go back on.

8. EC: so you think having the videos makes it better, but would that encourage you to use it in your practice in any way, do you think?

9. Jessica: ya, you see the topics we are given on placement I feel are so separate but I'm sure there's a link as well.

10. EC: ok

11. Helen: I found that fractions, we did a lot of fractions like last year…when I was doing fractions with 6th class I used a lot of knowledge myself to figure out why I was

teaching what I was teaching. like I found it easier to explain it because I knew the background knowledge myself.

12. EC: ok so with the fractions you were teaching in Straffan you found that the fractions we did in 2nd year helped?

13. Helen: yeah, even with the area model and things like that I kind of just showed…I found showed some of the children who were struggling to do the multiplication things. like when I showed them the different ways we looked at it kind of helped a bit.

14. EC: That's interesting that in this particular case you did link back to some of the stuff from competency. But did you also feel that competency was just something to learn off?

15. Helen: Ya I think just because the topic I did on placement was…because we did so much fractions, like so, because we did so much work on fractions I think it was just because I was on that topic. Because it was more relevant I guess.

16. EC: So we did fractions in 2nd year and I know when we did the intervention there was some fractions there, and then you were teaching fractions. Is it the continuity?

17. Helen: So it's because there was a link, because of the topic I was doing on placement. Before this placement I probably wouldn't have…linked them as much.

18. EC: ok
19. Helen: but just because the topic was very much linked and the content was the same so…

20. EC: so what we do needs to be the same topic as what you're going to be teaching for it to be relevant is it?

21. Gillian: Ya I think it's just linked. Even when we were doing number base and stuff, I was kind of like, is this a topic in schools? Like how am I going to use this in a school? Whereas like with fractions is very obvious because fractions is a topic. But even like I know number base is really helpful and stuff but even if it was just kind of linked to a topic, even. Like when we're doing a topic to link it back to something on the curriculum. Like, oh this is…you could use this when you're teaching this or…

22. EC: ok very good. Anyone else have an opinion on this?

23. Brona: I taught percentages on placement, and I think when I was teaching them, now that we've come back and done a bit on percentages in the last two weeks my understanding is more clear now than probably when I was on placement…if I had done that before my teaching would have been better. And even multiplying by 100 over 1, I never actually thought about why we do that. And the kids, I was actually just presenting it to the kids without actually giving them a reason for it. But now I f I

was to go back and teach it now id be more confident in it because I have that understanding now myself. So I think that would help me.

24. EC: that's interesting. So retrospectively, now it makes more sense to you?

25. Brona: yeah

26. EC: but it would have been better if you studied percentages before you went on placement?

27. Brona: Yeah it was just the topic I was given. I think it's my understanding; I think it's how I would have delivered it to the children would have been a bit different because I would have had that understanding then.

28. EC: ok yeah, that's interesting. Ammm…Jessica, you mentioned the same thing. The topics that we get on placement don't always align with the topics we do in competency. Are you given topics to do?

29. Jessica: Ya I had 2d shapes and money for the 3 weeks so…I know we did long multiplication in first year I think it was so that was useful because they're using that in money obviously…but otherwise 2d shapes I kind of learnt it myself I suppose.

30. Sharon: ya I was in the same boat, I had 2d shapes as well and 3d shapes and a bit of division and I think it's only in 6th class that I started using things we did in competency because otherwise I just felt there wasn't much of a link between the topics I was teaching. like we were doing like the story of 9 or length or 2d shapes and I was trying to see the connection from what you were doing in class (lectures) to what you are teaching in the class. It's kind of hard then.

31. EC: ya I get that. So would you say that…in terms of relevance, is maths competency relevant? Or would it be more relevant then as you get towards the higher end and less relevant towards the junior classes?

32. PST's: agreed with this

33. EC: ok. Do you think, even if it wasn't directly relevant, does competency change the nature of your teaching? the way you approach maths in any way?
34. Claire: it's got me asking the children why, like why did you get that answer? Because even on placement, like, I asked the children why, or can they give a reason and most of them couldn't because they've never been asked before. Because even when I saw the teacher, I remember in primary school it was the same, they just call out the answers to the homework. Where it's not, oh, did anyone like think about why it happened.

35. EC: so it made them a little uncomfortable was it?

36. Claire: yeah.

37. EC: and Gillian, you had something similar to that if I recall?

38. Gillian: Ya I tried to do, because I had long division for the first week and they learnt it in 5th class and then the teacher had already done a little bit of work with them but when I was king of asking why do you do that they were like, "what do you mean?" and I said you know, why are you carrying that over, why are you doing this…and the whole concept of being asked why did they do something they actually kind of seemed uncomfortable with it. Like why are are asking that, does it matter? And they were like well I got the right answer so it doesn't matter. And I was trying to push them a little bit with like well do you understand what you're doing? And they were like well we can do it so it's fine. You know what I mean? It was kind of that attitude of why do I need to know why I'm doing t when I'm getting the right answer.

39. EC: so I know how to do it, that means I get it like?

40. Gillian: ya because I said well do you understand why and they said well I got the right answer. And that was the, sort of, attitude of the class. So it was quite hard. I didn't feel like I was really getting anywhere. Like if you had a class for a year or more you could probably do a good bit f work with them but in the space of 3 weeks it's quite hard to…when you're up against that attitude that they've been dealing with for 8 years.

41. EC: it kind of puts you off it?

42. Gillian: yeah, yeah

43. EC: Almost seem like an insurmountable goal to try to get them to come around to that way of thinking?

44. Gillian: yeah they just didn't like it. They didn't like been made felt like they didn't understand it. Because they were like we learnt this in 5th class, we know how to do it and we're getting the right answers. Why are you making it more difficult? That was the sort of attitude so they just really didn't like it at all, they were really uncomfortable kind a, when I was showing them different ways of doing it and seeing did anyone figure it out a different way or showing them like diagrams and stuff they were like why are we doing this? what's the point?

45. EC: so it it like we know a quick way of doing this and we can get the answers

46. Gillian: yeah

47. EC: so why overcomplicate things?

48. Gillian: Yeah so they had their neat little, sort of, long division sum and they done it and they got the answer and they ticked it and that's grand it's done, out of the way. and after that they just didn't want to look at the answer again. That was…if they got

it right that was it. If they got it wrong, they saw where they went wrong and they corrected it and moved on. So it was just that attitude in the class it was quite hard to push through it.

49. EC: yeah it's really interesting. Anyone else?

50. Sharon: yeah I was similar. I was doing long division as well. I had 5th so I was trying to teach 5th and 6th so some of the knew how to do long division some had no idea. And am, to get the just to show their rough work…they wanted to do it all in their heads. But because I know that when we're doing our exams and stuff we show all our workings so I was doing that on the board. But when it came to doing just integer (?) multiplication…before you…like how many times you can divide in and checking their answer, they were refusing to do it. They were like…we know it…but they were having problems with their multiplication so I had to show them well if you do it out on the side and you show your work, rough work, you can see where your problem is so some of them started to do that but 6th class did not want to do that at all because we know our multiplication, even though some of them didn't know their multiplication. So to get them to actually do rough work was hard. I think the pristine copy thing came up as well, they wanted to keep it so neat and tidy

51. EC: ya I remember that. Was there any others?

52. Aoife: there was a lot of time in my class spent just ruling out the copybook rather than actually like doing the sums. So I kind of say like right we're going to do q1 a b and c or something like that and to practice what I had taught…we were doing like lines and angles. So they were like drawing the triangles and stuff but I don't know if it was just like that they didn't want to do it but they were just spending so long just ruling the copy like, instead of doing the work and I was like come on it's been 5 minutes there should at least be a line down. Am I don't know maybe the fact that like I was in a gaelscoil made it a bit harder like, language wise and stuff but I found myself going back over a lot of things a lot of times looking for ways to try a reinforce it even though we had done it so many times before they were still kind of, a little bit unsure, and I think that's why I probably didn't question them too much about why we're doing this, why we're doing that? Because they couldn't even…like the protractor, they were finding that hard to just even use. Rather than like, why do we use a protractor…they're like hang on a second let me just use it first.

53. EC: so you didn't get a chance to go near the deeper why questions?

54. Aoife: no not really but I feel like myself I wouldn't even have the confidence to go near that like with my mathematical ability. I kind of just sway more towards playing a game for introduction and then do something on the board and then…I don't know if I have the confidence.

55. EC: that's a really interesting idea. I'm wondering actually, is your own confidence…what you believe your ability to be…because…is that something that would impact what you decide to teach in some way? like has it impacted you?

56. Sharon: yeah, I was terrified to go near division. Absolutely terrified when the teacher gave me the topic. But I think it all goes back, not to necessarily what we did in college, but secondary school. So I just had a really bad teacher in secondary school who told us we were all going to fail our leaving certs and am would basically argue with me if I had a right answer telling me it was wrong and different things even though it as right. So I still questioning myself all the time. So I was like I don't know if I can teach division. I don't know if I know it and then it went fine but…

57. EC: what did you get in your first year exam? Did you pass?

58. Sharon: oh ya

59. EC: of course you did! So you know, it's funny, you do know division. Am it's…do you want to talk more about that?

60. Sharon: no that's exactly…yeah like…you know it but you feel like maybe you just don't know and I feel like it's all going back to secondary school where confidence...I don't know maybe. I don't know if anyone else is the same?

61. Gillian: ya there are topics. There are certain topics and I'm like oh my God please don't ask me to do that.

62. Gillian: yeah I still struggle, even since primary school. You know the way they do a bit of financial maths? I don't know what it is like...fractions, anything...I'm fine, but financial maths. And the teacher was flipping through and was like maybe financial maths and I was like no please not that one like. I wouldn't have felt comfortable. I would have had to almost teach it to myself first because I think there's certain topics that even tough if I looked through the book I probably would have been fine, it's just like that title in itself like I think that would just be [inaudible]...and I just be like, oh my God I can't do that and so I wouldn't feel as comfortable teaching it. Like I feel there's certain topics where I'd just be like oh!...I've kind of decided in my head I'm not as comfortable teaching it even though it's only 5th and 6th class maths..I know I'm able to do it but...I think there's that sort of bit of hesitancy...

63. EC: avoidance?

64. Gillian: Yeah. Just avoidance really as well.

65. Aoife: I think there just like such an expectation with maths like from a young age. Like in primary school it's always like the Drumcondras and your entrance exams and all this, and in secondary it's higher level to get your 25 extra points and then coming here to get 70% to pass. I just feel like there are so many barriers, that just, it's hard to be like yeah I can do this it's ok. Do you know what I mean?

66. Helen: I think it's, the classroom I was in, the teacher himself, he had a good background in maths and like he was really good I think, like the way he taught

maths he kind of got them to question. Like they kind of had the why mindset a little bit more. Because, like, when I was teaching, say like adding fractions, like they weren't focusing on getting the answer right as much as they were trying to get the steps. Like when he'd ask them why they were doing it they had to think about it, like they wouldn't have any hesitancy. I think Like his confidence kind of helped that a lot.

67. EC: so the class teacher makes a difference?

68. PST: [general agreement]

69. EC: going back to the class teacher in general. Would you say that they can help or hinder?

70. PST: definitely. Yeah. [general agreement]

71. EC: in what ways might they hinder?
72. Jessica: probably in terms of any subject. Each teacher has something they're passionate about. Like you know it could be like Gaeilge, so they it would be like maths would slip.

73. EC: so you're inheriting something that's very difficult to work with and if you get something that difficult to work with then what happens?

74. Gillian: I think the children can kind of sense when you're comfortable, or when you enjoy teaching a subject. They can tell. So like I know, my teacher particularly, he said it to me from the start. He was like, oh...I'm not a fan of teaching maths. I just go by the book and I just teach it to them then they do the questions and that's kind of it. But I feel the kids that came across quite a lot because...it wasn't that obvious but I feel like the kids could pick up on it that he didn't enjoy teaching maths. It was kind of just like tick it off, get it done. It was not...he didn't want them to question stuff, because he wasn't that interested.

75. EC: passionate about it?

76. Gillian: he wasn't passionate about it so they...that wasn't passed on to them. there was no sort of exploration or anything like that. It was just kind of, he wanted to get the box ticked to finish the curriculum. Get it done and that was it. So even...I feel like every...he had like his plan done out and it was like...fractions has two weeks, long division has two weeks. And even of there was still questions, they didn't understand it just move on anyway because that was that time and if they didn't get it they didn't get it. And that's too bad it's ticked off anyway I've taught it. So I figured that kind of got passed on to the kids then. So that influenced their attitude towards maths as well.

77. EC: Claire are you thinking the same thing

78. Claire: yeah I'd say the same thing. I think it kind of comes as well from the leaving cert. Like it's the same thing that you were kind of just taught something. You weren't told why but you knew you had to know it to pass your leaving cert. Like I think that's the same way I came in in first year, the same thing for maths competency. Oh, I just need to pass. The same thing as the leaving cert just learn it off. So I think that's the mentality, the attitude in secondary school.

79. Brona: there's no value placed on understanding it. Because understanding it doesn't get you points, it doesn't get you marks, it doesn't get you full marks in the test...

80. EC: that's interesting. Has that attitude changed in any way?

81. Sharon: that came up actually in my class. They were asking already in 6th class when I was trying to get them to show their rough work, they were like oh but do your points in secondary school go towards your answer, or your rough work. And that was how I got some of them to do it. I was like well if you show your rough work the examiner can see how you're doing and that helped. But I just thought it was hilarious. Already in 6th class they were getting into that mindset.

82. EC: That's really interesting. That's difficult to work with. Can I ask you another thing? You're going in to teaching senior classes. How did you feel before, in September, how did you feel about going in to teach 5th and 6th class maths?

83. PST: nervous, scared

84. EC: why were you nervous
85. Aoife: I think it was because if they did...my own knowledge on the topics like it's not as simple as just go in. Like you actually have to put in work beforehand to prepare for the questions that might be asked and to have an answer that's valid, that you can give to the kids. Am... with the younger age groups I think you can kind of get away with, like you know the content and you're confident yourself. We need that extra bit of confidence going into the senior classes that if they do present questions that you're going to be able to back up what you're saying to them.

86. EC: ok back it up. And that's they why part isn't it? Very good. Anyone else feel the same way? Helen?

87. Helen: Ya I was going through the text books to get a bit more; to see what way they were having a look at things beforehand. Whereas for senior infants like for shapes it's the one approach. Because you know the way like you have different names of like renaming things and these different ways and I didn't want to confuse them more.

88. EC: did other people feel that way? Not wanting to confuse children by going deeper or showing multiple methods or anything like that?

89. Gillian: Yeah. Oh think I said this before. You kind of feel like you're doing a bad job when you go into a lesson and they feel like they understand something and they leave the maths lesson feeling like they understand it less. You can kind of see they're like oh I thought I get long division but I clearly don't understand it and obviously like as a teacher that's the opposite of what you want to be doing. You want to be going in and them know less and you teach them "abc" they come out more confident and more competent in that. But when you're getting them to question it, it feels like you're almost doing the opposite. You make them feel a bit more uncomfortable and it's just, it kind of goes against your unnatural teacher...

90. EC: I'm familiar with that. Would others agree with that. Jessica?

91. Jessica: Some teachers I feel, like you don't know how they taught it previously, in different years...so kind of what method they do things by. So you don't want to confuse them.

92. EC: ok very good thanks. Am...can we just talk about the intervention for a minute? I'll just throw it out there: what impact has the intervention had, if any impact at all, on your ability to teach maths, or the way you taught maths, or the way to think about teaching maths?

93. Aoife: I think the modelling, the strategies you gave us for modelling, I used that the whole way through placement. I think probably hey way I was using it for was just the explanation and I think modelling really helped because when you're thinking out loud you're kind of asking yourself the same questions children are asking. I found that really useful.

94. EC: so you found that aspect useful?

95. Aoife: yeah

96. EC: ok very good. Glad it was! was there anything else.

97. Claire: the discussion part we did about 2d shapes I asked them was a square a type of rectangle and it was like I had one boy and he was the only boy who said yes and the rest of them were like no a rectangle looks like this and square looks like that. We spent the whole week...everyday he'd come up with a new argument and the rest of them would just shoot home down. But even just that where we're challenging them. Like I did Euler's theory with them for 3d shapes and that was just...it was in the busy at maths book but I don't think I'd look at it if it hadn't been for like discussion and I knew that they knew that oh this is a vertex, this is a corner, face...they knew all that. It was to try to get them to think of something else. I thought that was a bit hard as well because I knew the teacher was like, not pressure, but there were certain things you had to get done like oh you have to know how to construct a triangle where, you could do that in like 20 minutes and they knew, where it was just trying to get them to think a bit more.

98. EC: So the discussion aspect gives you access to higher order thinking, is it?

99. Claire: yeah...or even leads on to different activities like. Like we had a discussion about 3d shapes and what we all knew about them and they literally knew, like from 5th class, they knew most of it because they were in a 5th and 6th class mixed so the teacher had done the 6th class content.

100. EC: so you needed a way to go a bit deeper on that content?

101. Claire: yeah

102. EC: ok very good that's excellent. I know we looked at two core practices. Was there anything else about the intervention, I know we mentioned the videos already, was it useful having the children in, and teaching each other and that sort of stuff?

103. PST: yeah that was good

104. EC: what made that good? What was it about?

105. Sharon: I suppose just to see what their base level of understanding might be before you go into a class because you get no chance when you're on observation to go and teach a small bit just to see what a 6th class might be like. You can plan because you have to have your four lesson plans done or whatever and your schemes so just to kind of know ok this is what they might know, this is the level they might have so I can try and build my schemes off that. Because you might only see one lesson on observation, if that, if they do maths that day. So to get a chance to see what a 6th class might be like.

106. EC: OK that's good. And the practice of teaching itself, was that of any use? You know, besides knowing the level they're at?

107. Gillian: no it was good working with other people as well. It was nice being in groups because you could kind of talk about different ideas and discuss different ways you might teach something. And even just seeing, I found it really interesting seeing how other people teach and just the way they go about things and...it was just really interesting seeing the way other people teach maths because you don't really get do a whole lot of observing of people teaching. So it was nice to see people at your own level and the way they go about it and the way they plan and other things they might pick up that you wouldn't think of. So I found that part really interesting.

108. EC: very good. And did it help, does anyone think, to link competency to what you were teaching? Did it help to take maths knowledge and apply it that you might not already have done so before?

109. Aoife: I think so. I think it kind of, like as Gillian says when you're working with others it allows you to explore different ideas in other ways rather than just like trying

to understand it in a lecture, do you know that kind of way like? You're talking about it and discussing it...having that open discussion made it kind of easier to plan and then to see like the children were actually like understanding it was kind of...you kind of have an idea of what 6th class is like. It's eye opening to be like, well, it does work.

110. EC: ok that's good. And was it useful to focus on one topic?

111. PST: I think so yeah

112. Helen: you properly got to know a topic instead of like, kind of bits from every topic.

113. EC: and did you get to see what other people did in terms of their topics?

114. Gillian: no not really. It would have been good if we could kind of share afterwards. Even if all the lesson plans were put onto Moodle or something. And we could have a look at them I think that would be helpful. Even if it was when you were going on placement you could see how other people were going to teach fractions or something. And you could go and see what way they did it in groups and just see from that.

115. EC: what would be the best way I wonder, to share that?

116. Brona: even if like you recorded, if everyone was happy to be recorded. But then there's the aspect of the children being there. Even voice recordings, do you know. Like even if you just hear this is how they explained this if you weren't sure how to use the language.

117. EC: yes I think that something that could possibly be done. Or even a video without the children in it.

118. Gillian: if you couldn't do a voice recording or a video even if you could do like a feedback form for every group with like what worked well, what phrasing do you think helped the most? Just if you couldn't do a video or anything.

119. EC: ok

120. Gillian: Like just feedback on what they think worked really well. I mean, maybe what didn't work as well. And then if you had, like, sort of feedback from all the different groups you could take what ideas worked well from the different groups.

121. EC: yup sounds good, anyone else have any other ideas around that?

122. PST's: No response

123. EC: that's good advice on that, thanks. Amm...ok, finally I suppose...we've already touched on some of these but...you know when we talk about rich mathematical knowledge, like when you engage in rich discussion or you give the reasons behind how you do long division, or you do proofs like Euler's formula, that sort of stuff. You know, when you go into detail. And that's, sort of, what maths competency aims to give you I suppose. And I'm wondering what are the constraints, and I know we've touched on some of these already, you know you talked about the class teacher you talked about your own knowledge, you talked about children's willingness to kind of engage in that sort of stuff. Are there any other constraints? Are there any other things that have prevented you, whether it be here in the college, or your own attitude, is there anything else that would prevent you from engaging in that type of knowledge?

124. Claire: I think time. I know on placement that it was just how you need to get a, b, c done. Like that's the main things and if you have extra time to do something else that fine. But you still...even if they knew it you still had to do it. I just felt like the class teacher was just kind of watching me to make sure that I was doing everything that was even in the book. Like I've done that, yeah grand. But if you did something that wasn't in the book it was kind of like oh ok why are you doing it?

125. EC: was that something that you felt? Or was it something that actually happened?

126. Claire: I know that I just kind of felt it because they had another teacher that was coming in for maths was well and everyday she'd ask me "oh why...what are you doing?" and it wasn't something from in the book she took the children out that she would deal with. So I was just kind of like, oh ok. So I don't know it was just a bit...but they liked whatever I was doing different, whatever I did but I just felt like it was a bit...

127. EC: prescribed?

128. Claire: yeah, yeah. Or even just that if you didn't do...so say if you didn't do constructing a circle on a certain day or in a certain week, ok, the you haven't got that covered.

129. EC: ok so it's very much prescribed. Do you feel the same way?

130. Brona: Ya I think there's so much to balance when you're on placement in terms of you have what the teacher wants you to do, then you have the workbook, then the curriculum...and you're trying to feed them all into one and get them all done in three weeks. Am...I was lucky in terms of my teacher. It was very, like, she didn't mind if I didn't get it completed. She was happy enough to continue on and that gave me the time I thin this time around to actually...if they didn't understand something...give it an extra day without any pressure. Oh I have to get this done to move on, do you know that kind of a way? But it was just because of the teacher. That I felt comfortable around her and she kept saying to me like, Brona it's

fine if you don't get that done or I'll make up for it here...I'd rather they understand it. Am...I think it comes down to the teacher and then trying to balance everything out to get it done in the three weeks as well.

131. EC: ok so you have a packed timetable and feeling like the teacher is putting you under pressure. You (Helen) didn't have an issue...because...in Straffan...?

132. Helen: yeah...and he's a Froebel teacher. Like he's very driven. He used to...they were sitting at differentiated groups so they were like if anything he just wanted to know what we were doing that day so he could help the weakest group. Like it had nothing to do with...[inaudible]...he just wanted to make sure they understood.

133. EC: Was there anyone else with teachers like that. Who let you off to let the curriculum be your guide and not, you know, off a workbook?

134. Jessica: yeah mine was pretty accommodating but they had a mental maths book, tables champion book, busy at maths and then another maths book that I didn't that I didn't go near so I kind of felt pressure to get three filled out every day for him. So that cut into time for other activities or like hands on kind of things or else you had a lot to catch up on and they had a lot of homework.

135. EC: so are teachers driven more by books than curriculum, do you think?

136. PST: general agreement

137. Gillian: I always thought the chapters of a book were like a checklist so you had to get all of them done. And if you didn't get all the sections in the book done that chapter wasn't finished. I think some of them just go by the book and they read the little section that explains it and they do an example and they do the questions then that part's done. And that's kind of it then.

138. EC: that's really interesting. Does anyone else have anything else they'd like to add? Are there things I have left out you'd like to talk about?

139. EC: if you had to go out to 5th or 6th class again next semester, do you feel like you'd be more prepared than if we hadn't done the intervention?

140. EC: just in terms of methodologies...going out to 5th and 6th class...do you feel like you need to have particular methodologies for particular topics? Or do you feel like you need to have maths knowledge? So what are you more worried about: your knowledge or a bank of methods that you can use to teach a topic? Is there a distinction or tension between methodologies and knowledge? Does that question even make sense to you?

141. Brona: You kind of need both...but I think it would be nice if we did a little bit more...like maths competency I feel is kind of...that's separate like that's your knowledge and that's you understanding it. Not as much now since the intervention

stuff but in 1st and 2nd year it was kind of separate but then I think it would have been nice going into 5th and 6th class to... I don't know I kind of struggled to make things hands on for 5th and 6th class. I think it's easier with the younger groups than for 5th and 6th. It would be nice to know how to make it more hands on, and different activities you can do and stuff.

142. EC: ok so the hands on aspect is something you were lacking is it?

143. PST:

144. EC: do you feel like there needs to be a hands on element to everything you do?

145. Brona: I think linking it to like everyday life...ammm...helped with the kids I was teaching anyway. Because at one point they were like oh maths doesn't really like feed in. They were looking at it as a subject. But every topic I did, I did money and percentages...so I like to start at least with the viewpoint of where you see these in everyday life, like situations, and like even by the end of the first lesson they were able to name out ten different situations where they see percentages. For example, when I was doing money I used, am, like shop comparisons, things that they would be interacting with. And they really then, I think that added like to their enthusiasm...in terms of it meant something to them.

146. EC: it made sense, it wasn't just doing addition or multiplication just for the sake of doing it? Ok that makes sense. Is anyone else in agreement?

147. Jessica: yeah I was the same as that. So, it was doing money and I actually brought in catalogues and then in pairs they did shopping lists so it was real to them.

148. EC: ok so being real...[inaudible]

149. PST: yeah definitely

150. EC: ok finally, the hands on thing. It seems to mean a lot to all of you. What do you mean by hands on. Give me a...can you give me just a little bit of insight into what hands on actually means?

151. Sharon: I was doing 2d shapes and 3d shapes so I brought them outside and there was a circle on the ground and they hand to come up with how many lines of symmetry there was and if I would be like imagine if there was a circle in your head and go and tell me how many lines of symmetry they wouldn't have got it. But then a kid got a ruler when were outside, went through it and then realised there was infinite lines of symmetry. So I feel like having it and being able to manipulate it with his body and then for like kinaesthetic learners to have something to manipulate. And then for 3d shapes as well they couldn't get the concepts in their heads so they so we made nets and they cut them out a stuck them so then they were able to manipulate and go

well here's the vertices here's the corner and just be able to hold it. I think you feel like it's stuck in their heads more.

152. Helen: I was a bit worried when I was doing fractions, like, how do you make it Froebelian and everything. And am the only think my inspector was saying was make it relevant to their lives so I did a lot of word problems with them, especially for the highest ability group because like they knew all the fractions straight away. So they used to write their own word problems and like make them as complicated as they wanted to. Like how they use fractions in everyday life so I think that made it a lot more relevant...that was the way I made it hands on. Because it was quite theoretical [means abstract] the things we were doing

153. EC: ok ya. I get it. So that's factions...it's not way to take the abstract and...was there any examples of abstract stuff that you couldn't for example use hands on...or make hands on in any way? that you really struggled with. Because I think things like geometry, you know it's obvious enough, you can get shapes and look at shapes. But fractions they are by nature abstract. So do you feel under pressure to always have an everyday life hands on experiences for the kids?

154. PST: general agreement

155. Claire: I think you know when your inspector is coming in that they're looking for that. So you know that you have to have it. They'll always be like oh where is that hands on resource or do you have something visual for them to use or something like that. Like I know it's really useful and it does help the children in the class but I think that's where some of the pressure might come from

156. EC: so your supervisor would also influence what you do?

157. Gillian: it's that kind of where you have to be Froebelian, it needs to be hands on. Like you said like...your supervisor coming in and saying oh is it hands on? I struggled a bit with long division because when you're doing actual long division sums it's hard to make that...like I was doing word questions and stuff to make t relevant to them but with regards to actual materials and stuff. Because you know you have to like, we're been told we have to sort of have our visual learners and kinaesthetic learners but it's quite hard to do kinaesthetic with long division because they have to just do out the sum and stuff. There's that little bit of pressure.

158. Sharon: I tried Dienes blocks for division and the tens sticks and the whole lot and it just confused them completely. But I had to leave them there because I'd be told by inspectors and we're told in maths class you have to have concrete materials...

159. EC: in maths class?

160. Sharon: in MSE...you told to have it right up through the years available for even senior classes, so they were there the whole time but none of them touched them. I offered them...would you like to use this? And no... they were like no it's totally confusing I don't get why they are stuck together. Why can't I pull them apart?

161. EC: it's interesting. I think there's something in maths maybe that...there is no concrete to it...sometimes maths is just abstract. And you can't always, we perhaps you could draw a picture, but there's not always a concrete idea around it. So you feel under pressure to be Froebelian. And to be Froebelian, well this is what I'm reading from you, means to have some sort of concrete physical object...be it concrete materials, Dienes blocks or games?

162. Gillian: I was thinking about when the inspector came in... taught it to them, did an example and did questions it wouldn't go well. Your inspector wouldn't be impressed if you didn't have something visual or something physical or some sort of game or something like that to take it from abstract to concrete. And sometimes it is quite hard, there are some concepts in maths where it's quite hard to do that but if you had an inspector then you would be expected to have something.

163. EC: do you remember the video we watched on fractions in the intervention. She taught the concepts of fraction multiplication. She didn't use anything expect a diagram on the board. Do you think that would be acceptable? Is that Froebelian?

164. Helen: I did that. I don't know if it was Froebelian or not but the kids completely understood. Even the ones...that were at 3rd and 4th class level but...every single one of them understood it after. That kind of method, I don't know if it's Froebelian or not but... I'd still do it again because it helped their understanding better maybe than an activity.

165. EC: so you achieved your objective and you don't know if it was Froebelian or not?

166. Sharon: I think it would be though because it was child centred and it was starting with their knowledge. But on the other hand I don't know if an inspector or supervisor would take that into account. I feel like unless they have something in their hands and see the kids doing things I don't know if they understand where we're coming from.

167. Helen: Some inspectors have different ideas about what that should be. Like my inspector...she was like just make it relevant to their lives and that's all you can do really. like she understood that I couldn't...Like other inspectors I've had were just like no that's...not practical...it depends just on peoples outlook, idea on it.

168. Claire: yeah I had on that was like oh you have to include song singing in your maths lessons, but I didn't think it added any maths value, but that's what they were

looking for. But it wasn't getting them thinking in maths or anything it just happened to be in the topic.

169. EC: that was this placement?

170. PST: no it was last year

171. EC: and it didn't necessarily help meet your [learning] objectives

172. Claire: no, it was 4th class so not really.

173. EC: so I'm sensing some inconsistency around what you feel is expected of you being dependant on who your SP tutor is?

174. PST generally agree

175. Sharon: Ya, and it changes every time then. I'm thinking of one...[inaudible]...oh yeah you definitely need to have concrete materials but then you could get the examiner that Claire has and then you'd have to sing so it changes every time...about what they think Froebelian means. And it depends too on if they are internal or external.

176. EC: and this is centred around what it means to be Froebelian? Is that where the inconsistencies lie?

177. PST's generally agree

178. Aoife: then one of mine was saying in the higher classes which we were teaching, like direct teaching, like more direct teaching. I was like do I need concrete materials...no! So that was a struggle too.

179. EC: ok, that's really interesting. Your insights have been fantastic. Is there anything else you'd like to add before we finish up. Is there anything else I could do better? I did two core practices, was that enough?

180. Claire: the videos that showed this is a good example of teaching. That was useful so I knew like, if I'm ever going to teach that topic I can look back on that video and have an idea of, not how to teach it exactly, but you have an idea of like what good teaching looks like.

181. EC: and you think the individual videos of you teaching the children would be good, or to document that in some way so we can share it out?

182. PST general agreement

183. Gillian: maybe just what phrasing to use and stuff. Like how to word stuff in a way that makes it easier to explain to children. So...just...I don't know...kind of what...like in the videos what ay did she phrase that and what way did she start, how did she start it off? And that kind of think.

184. EC: so that helped?

185. Gillian: yeah, how to take that knowledge and phrase it for children. Because obviously we'd have a higher level of understanding than they would so how do you bring it back down to what they know? And how to explain it to them? Because obviously if you're explaining it to us it's going to be at a slightly different level, you're going to be pitching it differently than to a 5th class child. So I think it would be helpful to know how to pitch it to that age.

186. EC: and you wrote reflections as we went along. Were they helpful?

187. Helen: they made you think about the class itself rather than just leaving it at the door, and just not going back to it until next week.

188. EC: ok I think that's it. Thank you very much for your time. If there's anything else you'd like to add you know where I am just pop up to my office and let me know.

Appendix 9: Focus Group 1b Transcript

Date: 11/12/19

Participants: Gemma, Vicky, Derek, Tony, Mary, Paul, Jenny.

Location: Education Department, Maynooth University.

Numbers correspond to paragraph numbers displayed in text references to this transcript.

3. EC: Brilliant, thanks very much. Ok we'll get started. I'm going to ask some questions. Take your time, think about it. So, the first thing I want to talk to you about is maths competency pre intervention. What I'm specifically interested in is, if you can remember back to September this year, when you first came in and you had the prospect of teaching 5th and 6th class. I want to know what did maths competency mean to you? What's it for? What's its purpose? What is it useful for? Or not useful for? What were your thoughts on it?
4. Mary: I suppose for breaking down, as you always say, you know, how multiplication or division of fractions - how that works. But we don't know why we dot it a certain way, it was just taught to us and we have it in our heads now but we don't know why we do it that way. Like why do you invert and multiply, all that kind of thing. We did all that. You show us why do we do that. So I suppose when we're teaching that to the children it makes it more obvious why you have to do it a certain ways and break things down more for the children. you can just go in and say oh we'll just invert and multiply to the children. Eventually they will think that way but initially you have to say why do you do that, or how you teach it so it's I'm a simple form for them.
5. EC: is that also what others think?
6. Paul: ya id agree with that. It moves from when I was taught, of being like, you do that because the teacher says you do it and that's the reason to...the reason behind it is like...i flower grows because it gets light and that kind of thing, it doesn't just grow because it just happens and that's it like, there's a reason. They we can actually understand why you do something rather than just do it because the teacher told you to do it and that's it.
7. EC: ok. Did you find it useful knowing why certain procedures worked the way they do?
8. Vicky: ya it makes it clearer. It makes doing maths easier for us as well to be able to go out an teach it. Because when I came in, I would have just, like the same as Mary, I would have been "this is the way teacher said, that's the way we do it". Well now I kind of have an idea of why we're doing it. So I can actually go out into my class and be link "this is why we are doing it" instead of just saying, because I would have said "oh ya that's just what you do", like I would have said that myself whereas now we've gone right through it I now know not to do that but I was never done to me so I would have just done it that way.
9. Mary: Even things like using the area models. I would have never have known that kind of thing, you know to multiply fractions, to use the area model, things like that. As you said we've just been told "do it this [way] and that it there's a rue and that's it you follow the rule". But with the area model you can kind of see more clearly why you do that kind of thing.

10. Derek: as someone who struggled with maths, it's never been my strongest subject. After maths competency I felt more able to approach a problem with the class in a more meaningful way. It's given me a better confidence to explore different avenues and different answers to get an answer rather than just saying that's the procedure that's it. Now I feel more confident actually saying why.
11. Paul: You have the confidence to teach maths and not be worried about a child asking you a question and you just being...you don't know what to say to them like. You're more confident to teach. Anything that we covered in maths, in competency, I'm more confident to teach it to a class now because I can explain, as Derek says, I can explain why you're doing something rather than just being doing maths and that's it like.
12. Vicky: I think as well a lot of children, they want...like my sister is like studying physics but she would even say like, when she was in school that she had to know why she was doing it because it didn't make sense to her if she didn't. And it would drive her teachers bananas because she'd be like why, why? And the teacher would just say learn it, just learn it. Whereas like, even now I can see why it make sense and if a child asked me now I can actually be like "this is why". So for children, say like my sister, it makes it easier for her. So some children do need the actual reasoning behind it...
13. Mary: I think all children need the reasoning behind it. I think it's easier to understand when they know how...why it is a certain way, or how it works.
14. Paul: Like for me, I would consider myself quite good at maths, it's one of my strong points and I would have been able to do maths without knowing why I was doing it. I would have just known like this is the procedure, this is how you get your answer, and this is how you check your answer. But I wouldn't have known why you were supposed to do that. And It didn't bother me because I always got the right answer, or I always go ten out of ten and that was good for me. That was my gratification of knowing that I was good at maths but I didn't know why I was doing maths, I just knew that I could do the sums but I didn't know why or what the sums were for...
15. Jenny: Ya I was similar like because I liked maths I never questioned it and an knew that if a child asked me in a class id panic because I didn't know why I...i just knew that's how you get the right answer. But now like if a child did ask you can break it down for them more easily.
16. EC: ok that's really interesting. I absolutely agree that if you're going to do something, I think you should know what it means. But there is definitely a difference between some students, children, who definitely require that knowledge in order for it to make sense, or else it's nonsensical to them. And then there's others, like Paul, who can make sense of it in a different way without knowing all the detail behind it.
17. EC: There are two things I'm wondering, and I'll just mention both of them now. One, somebody mentioned already, it's good that we covered a topic already because then I feel more comfortable teaching it. And I'm aware that's a very limited amount of material that we covered because it focuses mainly just on number. So we've only actually covered whole numbers and fractions. And the other thing I've noticed is that it's almost like a mindset change in terms of thinking "I would have been happy just teaching the procedures before". and it's like you're thinking now "maybe I should explain" or go into the detail behind the procedure. So I'm wondering two things at the same time, and you can decide which you want to talk about. Has your mindset changed, and is what you're learning only relevant to the content you will actually be teaching?

18. Tony: I think my mindset has definitely changed because I would have definitely just followed procedures as well. And then on placement I was teaching multiplying fractions by fractions. And you came in and said show them on the number line - why it does what it does, and use the definition of division. And even for my understanding that was so helpful [EC1] and there were so many children in the class who were struggling with it and found it way easier with the number line because they could see why the equation actually made sense. But then I did have other children who went against my mindset chance where I want to teach them why and it really helped some of the kinds, but some of the other kids in the class were really annoyed that I was teaching them why it worked, and just wanted the quick way around. They kept asking me what's the quick way. It's like they knew there was going to be a quick way of doing it, and they weren't going to have to use the number line the whole time. And they kept asking me what's the quick way. And they were getting worked up that they had to spend so long doing the equation. They just wanted the quick way, and it was like really annoying because some of the really needed it and I was trying to work through it until they had it perfect, they knew why it worked and they could use the number line and these other kids, they were getting really angry like, just give me a quick way of doing it. These number lines are so annoying like. I don't know, it was kind of tricky balancing both of them.
19. EC: ok that's very interesting. Did anyone else have that problem?

20. Gemma: In the class I was in there was some really strong students, and there was some that were really struggling with what we were doing but as a whole class they had been taught to just write the equation and the answer into their copybook and not to draw number lines or to draw physical representations. So while we were trying to do it on the board, and go through things when they went to do it in their copy they just went straight to doing the equation and get the answer. And they didn't want to have a messy copy. And no matter how much I tried to say it to them oh like maybe draw four bottles and write down a quarter of a liter of each of them and then you can count them, they just wanted everything to be neat in their copy and I think that mindset hindered a lot of them in the class, especially the weaker ones. Like the stronger ones were able to do it quite quickly but the weaker ones who wanted to keep the neatness in their copy at the same time weren't able to visualize what they were doing so with each, like id get them to do question 1a and then id get them to do it on their whiteboard and id ask them to do it in their copy but then they go straight to 1b and they go straight to writing the equation and they wouldn't know how to kind of go forward with it.
21. Derek: I feel like it's something that needs a whole school approach. It kind of reminds me of like template art almost. Like if you get children from a young age and start using that intervention from junior infants so they get used to it. Like, I was doing simple time problems with 5th class. And when I was asking them and using the language from what we were doing here, asking them to defend their answers they're looking at me saying like "why should I have to defend my answer, it's right". It's almost like it's difficult for them to shift their mindset when they're used to that so I feel like it's something that would have to be like a whole school approach almost. Am

especially for us going in for three weeks as student teachers it's very difficult to change the grain, you know?

22. EC: where do you think that mindset comes from?
23. PST: teachers, it has to be
24. Paul: There's a lot of emphasis in schools now in neatness and tidiness. Like a teacher doesn't want to see all like, stuff all over the copy. They want to like rule the top, rule the side, rule the middle...
25. EC: have you seen that?
26. [PSTs emphatically agree]
27. Paul: across, down, through the middle and you have to keep it in those boxes. [EC2]
28. Tony: I had big issues with number lines because they kept ruling down the middle because that's the way they were taught to rule them and I was like, every class I was like, don't rule down the middle because we're going to use number lines but automatically it was like oh no we just did it and then there was no space for number lines the like, do you know?
29. Paul: And it all has to be in that box, all perfectly on the lines and there's no such thing as doing sketches or drawing diagrams...
30. [PSTs all agree]
31. Derek: And it's even, this was on a previous placement, a principal walked through my class one day and the children had just been put to a quiet individual class and he's like oh great class Mr. Dalton nice and quite like it should be. and it's like that whole mind frame, even discussion, you know through maths...you know, a lot of teachers view is that maths is individual, it's on your own, it's not really collaborative, it's not exploratory, it's more... [inaudible]...it's like cheating really.
32. Paul: ...if I'm working with Derek, or if we're working together on a sum like I'm doing it and Derek is copying me or if Derek is doing it and I'm copying him they don't see it as collaborative. It's not every teacher, but some teachers feel that maths should be on your own otherwise you're kind of cheating.
33. Mary: and there's no space for messy work. You know the way when we did maths years ago there was all this space on the side for messy work. Now they might rub it out and stuff...or else they might just write the answer and you're like where did this come from? They'd have no procedure written down and it's all very neat and tidy.
34. Jenny: they just want to get it done quick.
35. Paul: the faster persons wins.
36. Derek: faster is smarter like. It's almost a culture among children, that doesn't come from the teachers but
37. Vicky: ya and my class played a lot where the teacher would give two people the sum and they would go head-to-head to see who said the thing fastest and then the person [who looses] would just sit down. And that was all on speed. I knew, and you could see some of the children like, some children would walk to a child they knew wouldn't get the answer. Like they knew themselves....knowing that they'd get that one. and I was like, no I wont play the game and they were like oh teacher come on come on we haven't played it in like four days and id just go oh yeah we'll play tomorrow. But like that's all they wanted to do and then when I found when they were doing the sums with the equations in their copies they were doing the same thing, they were just writing like the answer. Because in the book it would say 5 times 4, and they'd know because they were just saying it off, the answer was 20.

38. Paul: I had the same with mine where they had to learn off all their times tables. They wanted to do it every morning they had this big line and you had to go against the person beside you and if you got it right you got to move up a place and the two people at the top were the king and queen of the line. They just assumed they were good at maths if they were at the top of the line and if they were at the bottom of the line they assumed they were bad. And even the kind that were at the bottom of the line in that game, I tried not to play like but they just they wouldn't start the day unless you played it like. The teacher actually said to me just play the game. And the kids at the bottom of the line just wouldn't cooperate in maths lessons because they just assumed they were bad at maths like.
39. EC: ive seen that on many placements. I just want to bring it back to you for a second. There's a couple of things. First a very quick question: those things you described about maths in the classroom. You know the neatness, the answers, the speed, the discipline, the quietness, the exactness...does that apply to other subjects as well. Like if they had a very different subject, maybe art or music, would that apply to that? So, would the same rules be applied?
40. Paul: it depends on the teacher, some teachers would be for art like. They'd just be link coloring lines, and then everyone has the same piece of art and then it's all tidy, depends on the teacher.
41. EC: so if it's that teacher it would be kind of uniform.
42. Paul: most teachers would be like that for maths. A lot of teacher would allow for a bit of fun during art and they wouldn't during maths like.
43. EC: ok
44. Vicky: ya my teacher just kept saying to me that I had to ask them [INAUDIABLE] problems because they were too loud but when my inspector came in she asked me one thing: what could you improve on? Because my teacher kept saying it to me I was like, oh my classroom management and she was like not at all, she was like they're perfect. They're not learning when they're not talking. But my teacher kept saying to me "they're too noisy, they're too noisy" because he'd just use book, write down the answer, book. So it always depends, I found for me it was quite hard to do things because all he kept saying to me was they're too noisy.
45. EC: so would you say that in terms of what you do in the classroom, the way you teach maths, would you say that the classroom teacher has some influence over that?
46. Derek: I feel like they're probably under pressure as well. You know, like parents maybe, because maths is such an important subject and there is more things such as dyslexia/ dyscalculia...they are more recent phenomenon's or labels, I feel like teachers are under pressure to deliver the curriculum well and for students who struggle with maths, I feel like maybe, because time isn't allowed, teachers are just more interested in getting things done and getting that chapter finished so there's no time for discussion or exploration. So maybe it's just the pressure that teachers are under too. Maybe if parents were more informed of the value of the reason behind things and you know, they worked more together with teachers it might be clearer as to why we're doing things that way we're doing them. And even this intervention, why it's important for children's development in maths.
47. Jenny: I think with 5th and 6th class they're looking forward to secondary school as well and honors maths and then the 25 points extra for the leaving cert. They're kind of like looking at that...

48. EC: do they know about that already?
49. Jenny: ya they were talking about honours maths in my class anyway and even the teacher had exam papers and they'd be doing some of the foundation maths exam papers. They were a very strong class but they would be talking about like maths and that kind of thing. So I think the teacher are then under pressure if the children and parents are talking about it to try to get stuff covered.
50. Paul: Ya my teacher was under pressure with maths for some of hers because some of the children were going for some entrance exam for a scholarship for some private school or something so like she took them out during my placement and did some extra maths with them because the parents asked them to...often stuff we haven't done yet.
51. Vicky: in then in my class they'd just say, sure why are I doing this, I going to do ordinary level maths. I would say I had about 12 in my class who would say that to me. Ya my class knew about it.
52. Jenny: it probably comes from older brothers and sisters who are in secondary school as well so it's not from the primary school it's more from the home I'd say.
53. EC: so it's a societal thing?
54. Gemma: I know from the teacher in the class I was just in. We were doing a few fun based activities and I was worried that they were being too loud and noisy because she would usually be more for book work and calling out the answer. And I said it to her and she was saying no they are on task if you listen to them, they are on task. But I was kind of more worried like, oh if an inspector came in they would think this is all gone a bit haywire, but she was encouraging me say they're on task and they're fine. But there as was a oy in the class and he was struggling really badly with maths and he wasn't getting the basics, you know like the first parts we were doing on a Monday or Tuesday no matter how much time I was spending with him. And we'd move on and he's just be instantly stuck. And I said it to her. What do you do with him? how do you progress if we're moving on to the next topic and she said you just have to leave him which means that every topic he was getting to the end and being like oh well forget it because we're moving on now. This happened every time. And I suppose that's instilling this thing that maths is hard...
55. Mary: I was the same because they were split into three groups between mine and Holly was in next door, based on their ability. And the weaker group, I had them on week, and they were a lot weaker. There were three objectives set every week for the whole lot, but they weren't meeting, they we're even getting one objective. and I knew they weren't getting it at the end of the week and the teacher said no we'll move on. So they were really left behind like. so that kind od streaming type thing, I don't know if that worked really well.
56. EC: I spoke to the principal also and she wasn't sure if it was working.
57. Mary: ya I don't know I mean they only did three days of maths and then a day of problem solving and a day of something else. And a lot of them weren't getting one objective. Id say the stronger group were maybe getting three, but they were just moving on and I don't think that's good to leave a big group behind, not getting the basics you know?
58. EC: thanks for that. Can I move us on a little bit? Can I just talk about this distinction between, in general, your preparedness to teach maths...because one of the things that came out was I'm getting some methodology, I'm getting some competency but

I'm not sure how to marry the tow so that I feel like I'm prepared to teach maths. This is pre intervention again: do you see something around that?

59. Tony: we know the theory behind everything we are doing, or like number anyway. We know how to explain it to like anyone that's in our class say. That's serves it's purpose like, but I don't think MSE is teaching us how to teach that. I don't think they're giving us the methodologies to put that across. Like even for maths in 6th class, like I knew that stuff and I knew how I should explain it and I went back to MSE PowerPoints just to see if there was like of how to teach it, like a methodology or something and there was nothing. Do you know like I don't think MSE is actually very helpful at all. The maths part.
60. Vicky: I'd agree.
61. EC: for senior classes?
62. Tony: for all classes. Even the junior stuff like, you'd get stuff for maybe two lessons.
63. Paul: game based
64. Tony: yeah, and it's grand like for games but it's not...i wouldn't find it helpful at all to be honest.
65. Derek: I feel like there's a great sense of coherency with competency. We know what we're doing, we know whats left to do, we know what we've covered and how it connects to the curriculum. But there's no sense of coherency with MSE I feel like we go into those lectures and one day its based on a game and the next day it would be based on something completely different and we don't know what's going to come next. And it's very difficult...
66. Tony:...you could go from counting to long division...
67. Derek: exactly. So I feel like if they were more in harmony it would be so much more beneficial for us.
68. EC: when you say in harmony do you mean competency and methodologies?
69. Derek: maybe like if they just complemented each other a bit more. So like say if we're covering long division here and everyone is comfortable with it. How now would you transfer that over in maybe a game-based methodology a bit further in the same week or even the following week. That way our notes would be in alignment and our thinking would be easier, it would be easier to make these connections.
70. Tony: like MSE is about teaching us methodology but I got more from the question thing and the kids coming in here on how to teach maths then I have from MSE at all this year.
71. Derek: ya totally
72. Tony: and that was two lectures.
73. EC: with the kids coming in?
74. Tony: because we actually had to think about it. And we'd done all the things like discussion, we'd done modelling, we'd done questioning in here. We haven't done any of that in MSE like. Like I didn't use any MSE stuff when the kids came in here. It was all stuff we did in lectures here.
75. EC: so the discussion and the questioning, I know Vicky you used the questions, you found that good did you?
76. Vicky: yeah, yeah
77. EC: so that sort of idea, Tony, where you learned more from teaching the kids...ultimately you're teachers, you're going to be teaching children...
78. Gillian: there's only so many games you can do like as well. You have to be able to...

79. Mary: Ya there has to be discrete teaching as well. They have to be able to work on the problems you know in their copies as well, and not just games, games, games all the time you know.
80. EC: ok if we think about methodologies ourselves now and just what you said there, do you like you have to have games? Like, do you always have to have a game?
81. Paul: games are good, but they're better for reinforcing the learning than. From my understanding of MSE to be totally honest it just comes across that games are how you teach maths. And they expect us to have all these resources which schools just don't have. Like all the stuff we do in MSE it has all these resources and like you go out to schools and that doesn't exist. So like, what we learn here is like we learn the maths, we learn the procedures, we know the steps, I could explain to Derek every little thing that we do in something, or anyone here, but then if I was actually to go an teach it I wouldn't necessarily know how to teach it or how to pitch it at a certain level or present it because it's all like related to games and like inspectors come in and one day I and an inspection and I was doing maths and it was very game based and the inspector told me it was too much games[EC9] , there's not enough like teaching. So I think that games do have a part and it makes the maths that you do fun and that makes maths more enjoyable but I think it has to be the conclusion to a lesson to be like doing games because that reinforces the learning of what you taught instead of like...like you cant...it like playing sport...you can just give someone a ball and say go play a match.   and then when they come back another day like like ok now I'm going to teach you how to do a solo, and this is the rules of the match, of playing football and this is what you can do, this is how the scoring works and all these...you cant just say go play a game.
82. EC: so the games don't teach the maths?
83. Paul: The games may reinforce the maths for some children but the majority of the children wont understand the maths based on the game. You need to like, as Jacqueline says you need to do the discrete teaching of maths first and then games can help reinforce it or like make the learning more fun.
84. Derek: I totally agree with that. At the moment we have like a fantastic Drama lecturer and it's very clear what's expected of us in each lesson. Drama is a very kind of, it's a game subject in itself, sociodramatic, there's a lot of fun involved but we know like there are a number of games that are only used as a reflective exercise at the end and it's very clear that in intro and development there are things that are expected and that's for teaching drama. I feel like in the core subject of maths it should be very clear to us what's expected when we're teaching a procedure such as long division. Like in the introduction, conclusion and then appropriate materials and games that would sum it up. And maybe, ya, as Paul said help children who need the games but not set as the main introduction.
85. Vicky: and it sounds so bad as well but if I'm being totally honest like I switch off in MSE. I do, like, I go in and it's been three years now and I haven't used on thing from it. Like, do you know what I mean, so I just...and we have it two hours on a Monday. It goes to one hour in the second semester but like I think if I'm honest like the majority of the people do because you sit there and...you'd be listening but it's like a PowerPoint with words on it, a game like duck, duck, goose and say if you didn't know duck duck goose or like "round something" like there's no explanation of the game either it's just the game. And then that's it.

86. Mary: A lot of these are for early years, maybe early years mathematics. For using concrete materials and all that sort of thing, you need to move on.
87. Derek: It's just needs to be a bit more clear to use...what's appropriate and what's not appropriate for older classes, especially when we are being inspected as Paul says and being told that's games we are learning from MSE aren't appropriate in some of our lessons, and we're going into inspections and using these and taking these games at face value from these lectures, am, you know and they are great ideas it just the way we're applying them, it has to be a bit more clear and coherent.
88. EC: in terms of the module the I run, the stuff, the intervention this year, did that go some way in alleviating some of the frustration you may have been feeling around not being prepared.
89. Gemma: the videos were really helpful, if you're talking about the videos you put up online, they were really helpful as well, because the things you don't get in class you have something to refer to at home so there's a backup there as well.
90. EC: so you can watch them and re-watch them?
91. R: yeah
92. EC: that's good that's the intention, but as a teacher will that help you? Because I think there's competency...like lets say I was to ask you a question: competency, what does it mean to you? Does it mean an exam or does it mean I could be a good teacher?
93. Vicky: it would have previously meant to me exam. Like in first and second year I just learn all this stuff off and work it out then after the exam...whereas this year I think because the questions, because what we're doing, and looking back on it now it's been the way going through why you're doing it, but this year it's kind of really being reinforced to us that it's to help the children. The way like we're learning it so in order to be able to help the children. So this year I'm not really taking it as exam based I'm kind of taking it as more of a "yeah I know I have an exam at the end but..."
94. EC: what was it that made you think about the children this year?
95. Vicky: I think it was the questions. Knowing that like, I don't know what it was, just knowing I had the questions when we were going through it...
96. EC: the list of 100 questions?
97. Vicky: yeah and then the questions were kind of brought into it and then when the children came in that even though we were doing the same thing when I look back on it, we were doing the exact same thing for first and second year in maths like at the moment but, previous like, I don't know what it was, we just...
98. Paul: I think one of the things, sorry to interrupt, but I think one of the things this year is because you have all the maths questions up, like all the stuff we do in class is up on like video you can sit in class and focus on the maths that you're doing. So like, lets say like in first year and second year we never had any of these notes up online so in class you had to focus your time on taking down those notes so you knew how to do something. Whereas now if I want, like I still do take notes in class and write things down, but this year if I miss something or whatever for a second I can just sit there and watch you do it and take in everything you're doing and explaining and then I can go back later and watch the video so I can actually....like I don't know for other people but I find sometimes if I'm just writing stuff down, if you're on the third line and I'm on the first line second line third line then I'm not aware of the maths that you're doing or like the procedures or the steps I'm just...whereas now you have the opportunity to sit there and take in the maths that you're doing and the procedures

and the steps and all the reasons why and then if you still aren't sure or you want to look over something later on you can go back and look at the video. I think that then helps you as a teacher to teach the maths because I've see you teach me so then I can model some of those things to teach it on to the children whereas last year it was more focused on, as you said, these questions for the exam because if I miss this right now and I don’t know how to do this question then I've already thrown away 4% or whatever for the exam.

99. Derek: it was really valuable. There was a lecture where we looked a video of a woman teaching children, that was a lecture that stayed with me since. I always thought about it and I did during placement because it was so effective. I learned so much in that lecture. And there was another time where you had four students standing up, from our year group, and I was part of the adjudicator panel at the back, the discussions that we had in that lecture were really insightful and really informed me so much more than the last two years of plain competency. It was because we were using what we had learned and looking at each other teaching it, and what was good, what was bad.
100. Mary: yeah, it was discussion based
101. Derek: having those conversations are invaluable. Like that
102. EC: that was the lecture on discussion, was it?
103. Vicky: and putting it into practice. Using what we learned and putting it into practice in the class.
104. EC: so more of that?
105. Vicky: yeah, I think that worked
106. Tony: you said don’t worry about making mistakes, make them here and then you can fix them in the classroom so you don’t make the same mistakes in the classroom you get to practice.
107. Mary: well, the focus is on, a lot of maths with children nowadays, is problem solving and the best way to do that it to get into a group and discuss it and you know rather than just one to one teaching. Some of that’s very important too but I mean this year we did one day a week of problem solving and I put them into groups to do it and they had this peer teaching and peer learning, it's good for them.
108. EC: like in the video
109. Mary: yeah
110. Paul: last year the only discussion we had in maths competency was if you did a sum on the board and then you wrote another sum up and they you’d say like Jenny, can you come up and show the class how you’d do that? And that was the only discussion that every happened. And literally the only discussion was Jenny that’s right, or Jenny you did that wrong you need to fix this....you’d say if they did it right or if they did it wrong and if they did it wrong you said this is what you actually should have done, you missed this step or you forgot to do this bit and that was the only discussion and we just kind of sat there in rows and just watched you do maths for an hour whereas now we do some maths and we discuss it. I also think maths competency is becoming more effective because we’re as a group becoming more comfortable with each other, so people are less like afraid, like saying stuff or whatever like. Do you know like, no one cares about making mistakes or whatever but I still do think the fact that we have discussions in class definitely helps with the learning and helps to understand things, like. Because if you didn't understand something before you mightn't say anything and then you’d have to ask someone. Like

I found, this is going to sound weird, but like coming up to maths competency exams in first and second year I have had a lot of messages from people who were asking me how to do questions for the exam. I used to be doing questions and having to send them to people because they weren't sure, but now people ask questions in the class and how you explain them or someone else explains them in the class and people know how to do them.

111. Jenny: I think you're less afraid be like oh I don't get that. Whereas before when you were just taking notes down, you'd be so far behind like the notes would be going I'll just leave that...I'll catch up...but now you're just like oh I don't get that can you go through that again?

112. Derek: yeah the [inaudible] has been very positive like.

113. Mary: and that's how it should be in a classroom as well. Children should be able to have those discussions and kind of work things out between them.

114. EC: that's the idea of democracy, isn't it? What would you change? I had this discussion with the group last week and they were talking about, actually I won't tell you what they said for a minute, is there anything about the work you did with the children? Did you find that useful or not? So on the last day each of you had some children come in and you had a topic you had to teach them. That's the culmination of it all, you know? Where you tried to apply what you learned in the specific area you were learning about. What would you change about that, of anything? Did you find it useful?

115. Gemma: ya I found it really useful because you were able to see like how you would explain something to the children. I found that helpful but maybe like more of that, more opportunities to practice your teaching.

116. Paul: I would have been nice to get that same group back again and progress just to see if you can change it with the same group. It all well and good for us to think about what the children will say or what they'll ask but it's actually better to get the children in a see what they actually have to see or what they actually think. You know because we're all like 20 or 21 and it's very hard for us to like a 10 or 11 year old and try to think of what they think

117. Gemma: I think when we went in we pitched it too low for them and then you know so next time we could have been like so we know how this group is so next time we can pitch it higher or we can differentiate more between them

118. EC: so you're not kind of as blind then going in to your placement.

119. PST agreement

120. EC: the other group also said that there was so many different topics being taught that it might be nice, and I know I said I'd do this but I didn't do a very good job of it, is to share your ideas around teaching the different topics. Would you have any opinion on that? So for example before your placement you could look at the problems that other groups encountered with particular topics, the differentiation that they did...

121. Tony: Even if that, that thing with teaching children, even if that could link to MSE and MSE could go through do this with the four children that are coming into you.

122. Derek: I feel like there's only so much one course can do. You know I feel like you're covering a lot...

123. Tony: just to make it more [inaudible] because like one year say you're doing...like I know we're all doing different things and that would be difficult like but

then you go into MSE and you're doing something completely different...even if it links like that...like these kids come in you're going to be teaching them maths, that's what MSE should be. I'd find that more helpful.

124. EC: ok

125. Paul: so obviously every second week we have, like one week we have P and the next week we have B so we do science or geography one week and then the other week it's maths and then in the maths we don't necessarily look at specific things that is in any way related to anything we're doing in competency, it's more generic maths things or games like. They can be effective at certain levels like but they don't necessarily link to anything that you're doing.

126. EC: so maybe in 1st and 2nd year for MSE you do your normal MSE stuff and then in 3rd year, in know we're so limited with time I know we only had 6 hours or something like that, a possibility could be that the content would be on video and then we learn about the different practices like discussion like questioning like modelling, and then use MSE to go and actually enact that so you can teach each other, bring children in and teach them? Is that the idea.

127. General PST consensus on this.

128. EC: ok I think that's something that could work

129. Paul: I think that would be a good idea. I know what you're saying about time like but these groups, but this is actually really useful like. Just coming here and talking about all the things we find good and we find not good or whatever like. looking at some things that would work and be more effective, if there was able to be...obviously all the lecturers have lectures and we have lectures and different things and there's not a lot of time but if we could just come and chat for a half an hour or an hour and talk about what's going well in maths competency or MSE and this is not going well, this is useful. I think that would be a good idea...everyone wouldn't obviously want to come and obviously you and other lecturers have things to do and you have lectures to go to but if every now and again that we could touch base or feed in because you're obviously trying to teach us certain things and we have to learn things off you so if you could tell you wants good or what we'd like to improve on and you can ask us what's good and what's not working then that would be good for us and for you because you'd know what's good and what's working or not working or if we have ideas we could suggest something else like.

130. EC: I agree. I'm not sure about the feasibility of that but it's a good idea. Always know that lecturers are open to your ideas and what it is you have to say. I think feedback is really good, you know, a constructive discussion. And I think at the beginning of a lecture is always a really good time to do that. And always know, in competency anyway, it's welcome. So, in first year you might have been afraid to talk about your mistakes and that you didn't know something and that kind of stuff but as you go on it's really important that you get more comfortable. I know now it's difficult to get first years to just talk to each other. I ask them to work together and they're still whispering while I'm saying please talk out loud, it's ok. And so, communication is a really big issue I think. And I think it's really important for democracy and shared learning, it's really important.

131. Ok I think we're getting towards the end. Is there anything else you'd like to mention just before we finish up? Anything about teaching maths? Anything about

learning maths? Anything that you'd like to see included? Anything you'd like to change?

132. Mary: Maths competency is probably for our own competency as teachers but what we all find then when we're given a subject to go out on placement is actually going in teaching it is a different thing altogether. You might know fractions inside out but when you go to teach it it's a totally different thing so it's really those methodologies. And really the actual phrasing and the wording, I know you use concrete materials which is great, but how to phrase something so that children will understand it, that's really hard to get I think. You know equivalent fractions, you know what they are, but how do you phrase that to the children? It is really difficult to do that when you go out.

133. EC: trying to communicate it to a child?

134. Mary: They think differently and it's hard to get that across, I think.

135. Derek: I feel like going out to placement we're always expected to know all the answers. And I feel like maths, more than any other subject, when you stand up to teach maths you're standing up to teach any strand of the curriculum because any strand is going to come up through questioning or whatever. I feel like for students coming along in 1st and 2nd year it's very important for them to know going out to placement, because I know I faced that, that it's ok not to have all the answers. And I feel like even going forward when we are NQT's or qualified you're never going to know all the answers, and you're never going to know the perfect way to phrase things. I feel like that has to be really clear.

Appendix 10: Focus Group 2 Transcript

Date: 25/3/21

Participants: Eve, Fiona, Lilly, Robbie, Paul, Maeve, Helen, Emily, Gemma.

Location: Online

Numbers correspond to paragraph numbers displayed in text references to this transcript.

1. EC: …The plan is to let you know what the research questions are, give you the findings from last year as this is an action research project. So, I'll show you the findings from the first cycle, I'll tell you about the changes for cycle 2. The findings from the first cycle come from third year and the changes are those things that happened in fourth year so hopefully you noticed some changes, but I will try to point them out to you anyway and then I'd like to get your thoughts and ideas on those changes. I'd like to spend 1/2 an hour or so talking about your ideas.
2. [PowerPoint Presentation: research questions and problem of enactment explained to participants; findings explained to participants]
3. So, what do you think about the above changes? ...how did year three compare with year 4? And I'm wondering specifically what did you think about the first semester this year? Is there anything on your mind about the type of teaching that we did in the first semester?
4. Fiona: Eddie I think…probably in 3rd year when you changed our approach to it, I think it took us a while to get used to it a small bit…just because of the way it was in first and second year and then when we came in to 4th year we had a bit more of a focus and we knew what to expect a bit more so I think we would have probably maybe got more benefit from it this year I think. Because in 3rd year it was a small bit like… we didn't know and I think we were all far too focused on the exam and getting 70% then actually… allowing ourselves to commit to it in a way.
5. EC: Alright, that's brilliant Eve, thanks. So… it was a bit of a shock …?
6. [temporary connection loss]
7. PSTs: Sorry EC we lost you there
8. EC: I think that's a really good point. So, was it all a bit much in 3rd year, this new way of doing maths…a new way of learning?
9. Fiona: yeah I think personally, for me anyway, I think like just reflecting back on it and especially like we already pointed…[connection less]… the gap between MSE and competency like so then it was a bigger shock I suppose as well and we felt maybe in 3rd year, I'm speaking more for myself now probably, I kind of felt in 3rd year that why haven't we been doing this since first year because by the time we got to third year it was kind of hard then to… kind of go back and close that gap that was there.
10. EC: OK, fantastic! So do you think it would be better to start this type of teaching then before 3rd year?

11. Fiona: yeah definitely I think because [connection loss] …it would have been great if we had the chance to delve into it a little bit more…but I suppose like as well… we lost time as well due to covid and everything so maybe if we had a full term in 3rd year and 4th year at it then it would have been fine. But I just think that gap is kind of still there.
12. EC: OK because in 3rd year we only did it for a portion of the year up to SP and so it was half of a semester I suppose. And it was the same in fourth year. So, do you think it would be better if it was kind of consistently throughout the year before SP and after SP?
13. Fiona: Yeah definitely I think yeah, there would be no harm in it anyway.
14. EC: What do you think about the….balance between pedagogy and the content. I am obliged to teach that content that we do…like right now…I'm creating videos and putting them online. It feels a little bit separate…you know?
15. Fiona: I suppose I just think that the pedagogy side of it should have been explored more maybe in MSE…[connection error]… Maybe it was not the fault of maths competency maybe it was the MSE module I kind of felt…it wasn't until 3rd and 4th year I felt like I actually felt a bit more prepared to teach maths and that was probably down to maths competency and what we covered there, rather than MSE so maybe it's more …that there was something wrong there maybe in 1st and 2nd year.
16. Eve: I feel like there was a big divide between like in MSE it was all games and stuff, and then maths competency was very much like the theoretical side of it so there was kind of a gap between the games and that kind of side of it and then while you're teaching it kind of a thing. Like there was just…there was a gap in the middle if you get me?
17. E: Yeah I do. And that was one of the things, you know, the integration of the links between maths competency and MSE. Do you feel like that… they should work together to bring all the…the content and pedagogy together, the two of them it a kind of coherent way?
18. Participants: yeah definitely yeah
19. E: almost a seamless boundary between the two?
20. Fiona: Yeah I just think in 3rd year when … we started the high leverage practices and everything, like we had never, I don't think we ever heard of that before. I just felt like it was too late nearly for us to be hearing it. Like I was like… we should have just been hearing this from the start.
21. EC: That's very good. I get it because they are, there are basic things you know, model content, discuss maths, you know? I agree that they are something that are so basic but yet difficult, I suppose, to enact in the classroom. I do agree that they should be introduced a little bit earlier.
22. Do you want to say more about that is there anybody else who would like to add to it?
23. [looking at PowerPoint presentation again]

24. EC: what does maths competency do? What's the purpose of it? Right now, after you're finished with maths competency, and you've done third year and 4th year what do you think it's for?

25. Helen: I thought it was really [good]…just for my own knowledge, for teaching SEN Maths. I think I really had to know why I was doing the really basic steps in things because…explaining it to the kids in a mainstream class it didn't really work. So … I had to bring it back to basics. I think if I didn't understand why, say for long division…if I didn't understand why they were doing it, then I don't think I would have been able to teach it as effectively.

26. EC: OK fantastic. Was that on your SPN placement this year?

27. Helen: Yeah...SEN for the whole 10 weeks.

28. EC: OK that's really interesting. So, you had to find a way to explain the reason why behind what you were doing?

29. Helen: yeah and I don't think I would have been able to do that without, especially the division this year because they were asking me questions that, if I wasn't in some of the maths competency this year I don't think I would have been able to answer some of the questions.

30. EC: OK and do you know that knowledge you had? So we're talking about content knowledge now; where was the source of that knowledge? You know specifically within maths competency… was it something you learned in 3rd and 4th year or was it something you learned in 1st year?

31. Helen: There was parts of it from 1st and 2nd year when I was doing just multiplication with one group. Just explaining 3 + 2 is equal to 2 + 3 and that kind of thing. But then another time I was doing long division and it was from the lectures this year when we were kind of looking at… because they asked me the same questions that we spent a couple of lectures on…what does division mean? Is it three groups of this? Or is it how many groups of …do you know that kind of way?

32. EC: Ok yeah…That's an interesting point because, one of the things I wanted to ask actually…is… what you talked about there Helen…there are two different interpretations of division and it is because multiplication is commutative. And you said it was the lectures in fourth year that helped you. But I'm wondering, we actually did do that first year as well but very much for an exam. I'm wondering did anyone make that connection when we were doing it in fourth year? Or was it brand new to you? What I'm wondering is, when we do purely content knowledge does that have an impact? Does that stick with you for teaching? So, let's say there was no intervention…that content that we do that's not put in the context of teaching, does it stick?

33. Maeve: I think like, EC, the stuff that's taught in first year, and it's aimed towards the exam we are in student frame of mind. We're thinking exam, exam, exam. But when you brought in the kids for us to teach we got the chance to actually discuss it with one another. We were in, like, the teacher frame of mind where we were thinking "OK, how can I improve my practice, so that it benefits the children"? Not this idea

that “well how can I get the best mark, on the exam, will this fulfill the answer that you’re looking for”?

34. EC: ok, that's really significant isn't it, because if that's the case then, you know, if let's say maths for example if it's just for an exam what you learn doesn't last? Is that what you're saying?

35. Maeve: Yeah it's in a vacuum. It's if we take any kind of exam it's not just maths, you learn it for the exam. Then not even a week or two weeks later, it's in your head for that exam, then when the exam is over it's gone out of your head I think. But when you're actually putting it in practice, and I think like discussing it with one another is brilliant because you learn from one another as well. And even just discussing it with the lecturers is brilliant because you're going to remember a discussion you had you're not going to remember things that you learned off from a page.

36. EC: Are there other modules in Froebel that are purely exam focused and not practical enough?

37. Maeve: English would be a real exam focused one. Even… I had to teach apostrophes on placement in English and I couldn't remember, I had to go back and teach it to myself. And we had done it in first year but I couldn't remember back to first year to teach it on placement.

38. EC: OK and did you go back then to your English competency notes?

39. Maeve: No. I didn't even think of going back to the English competency notes. I just thought I'll research it myself and see what are the ways I can teach it in a child friendly language. And that's one of the main things with maths that's difficult as well, putting it in a way that children understand.

40. EC: I see where you’re coming from, its really interesting and its something that's been on my mind….how do I teach you a depth of maths to you…to give you that knowledge and also be able to give you the capabilities to use that knowledge to make sense of how you can teach it to children. Does that make sense?

41. Maeve: yeah

42. EC: Do you think this has been addressed in any way?

43. Maeve: I think if you are comparing third year to 1st year like that was such a big improvement because we got the chance to put it into practice… it wasn't just you know, something we had on a page. Even something as simple as a discussion does bring light to these kind of ideas and problems and how can I address this. You're getting like…different… other students points of view and you're getting the lecturer's point of view as well. Which is brilliant.

44. EC: ok very good….in fourth year …we actually only looked at one maths topic, we only looked at division. And when you think about all the topics we've covered since first year, there are a lot. I picked that topic because it was perceived as being difficult, I wanted to address something that was difficult. Do you think that's useful generally for maths, or is it only good for one topic?

45. Maeve: it was good for division, I think it was really good for division. You probably could do it for more topics, if there was time. Because it was so useful towards division because you would feel confident leaving the lecture that… "OK I fully understand Division now and I could teach division to 5th or 6th class in the morning". So, if it was possible for more topics to be explored that would be great.

46. EC: Excellent. I think somehow joining maths competency and MSE might be the way forward for that. Would anyone like to add to that?

47. Gemma: Just to add on what Maeve was saying I found the division the way we used group discussion about it so much I found that highlighted… how that could be transferrable to different areas of maths where I might not have thought to use discussion so much. I thought the group discussion and the group work and seeing the other groups teaching and getting ideas from that, could be transferable two other areas of maths. I haven't used it myself but, it's good to see how much discussion can be used rather than just going through the formula of how to do the questions and things like that.

48. EC: So, you can mirror that methodology in your own classroom?

49. Gemma: Yeah I thought it was interesting and it kind of showed that there can be so much more discussion and group work with maths rather than…in the classroom it seems to be quite just fast paced and getting through it and having it all very neat on a page. Whereas I feel there can be a more deeper learning, if there is more discussion around it.

50. EC: you talked about the fast pace, do you see that as being a challenge then? You are going out to teach in September, and it sounds like something you'd like to do… to slow down the pace and engage in… deeper learning… deeper intellectual work . Do you think that is something you will be able to achieve?

51. Gemma: yeah, even the list of questions that you put up. I think it was the 100 maths questions…?

52. EC: to promote mathematical discourse, yes.

53. Gemma: yeah. I thought that that was really interesting because it shows that you can wait a minute and then delve deeper into what a child is saying. And then the maths videos kind of helped too, I suppose…whats the word…, like it complemented it so we had the theory in the maths videos to help but…all of the practical side and the group discussion was more focused on… classroom based.

54. EC: OK, so would you actually …go back and watch those videos if you needed to upskill in a certain area? Are they useful to have?

55. Gemma: Yeah, I think they are definitely useful to have for your own knowledge on the Maths. And then what we were doing in the class I thought helped with like how to teach it and how to actually implement it in a classroom especially for the older years. As it was mentioned MSE is fine for the younger years but just to create I suppose a more deeper learning with the older class groups.

56. EC: that's great, thanks. Does anyone else have any ideas or opinions about the use of discussion and dialogue around maths?

57. Emily: I thought it was really good the way the group taught the lesson plan. Do you know the way we did the group lesson plans together? I thought it was really good the way one group taught it to the rest of us because it was a really concrete way or showing how to bring it into the classroom. It was really effective.
58. EC: so you found the dialogue between all the different groups useful for refining your Lesson plan?
59. Emily: yeah and it made it really easy to remember how to teach effectively when you saw it being done in the lecture. Like, instead of just reading it out of a book or reading an article or something, it was just really helpful to just see it in action in the lecture room. I thought that was helpful.
60. EC: OK. So would you recommend doing more of that sort of stuff for different topics?
61. P: Yeah definitely. I thought it was really good.
62. EC: OK brilliant. By the way please be as critical as you like, if there was anything that you didn't like about it please let me know. So, it looks like more of this sort of stuff, and start doing it earlier, probably from first year, certainly from 2nd year . And to get more topics in. And to bridge that gap between MSE and maths competency. That's kind of what I'm hearing so far.
63. [referring to PowerPoint]
64. EC: is there anything else that you would like to see, in Froebel as a whole? You know, for teaching and learning maths.
65. Fiona: Eddie, I think, it was just when Gemma was talking about the classroom and when you actually go out into the classroom, I think it would be probably good if, because I know I feel like sometimes you can get caught up with the class teacher and what they want might hold you back a small bit. So then when you actually go into the classroom, it's great learning and all of this in college… [loss of connection]…student teachers so they can say to the class teacher "Froebel want us to take this approach to teaching maths". So then maybe the class teacher…just might allow it to happen more instead of always focusing on the textbook. Because I know with literacy we take a reading workshop and the writing workshop approach to all of our English lessons so it would be great if student teachers could actually go in and say "I'm taking the high leverage practice approach to teaching maths…" [loss of connection]…and then they'd probably feel more confident going in with a plan to the teacher instead of the teacher having to give them topics and pages of a book to do.
66. EC: ok, a little bit like Gaeilge tri gaeilge as well?
67. Fiona: yeah. Just having that structure to it as well would make it more relevant for learners in college.
68. EC: OK , I was thinking about doing something like that but I didn't want to dictate what you did in the classroom. You know, I wanted to leave it open for you to decide. What you think it would be good to have that sort of a structure?
69. Fiona: well I think it would be great to be able to go into the class teacher and say, when he says or,…[loss of connection]…to do in maths well you just say "I don't mind

what topics but this will be the approach I'm taking", instead of them just asking you to do the textbook and trying to come up with active lessons based on the textbook.

70. Eve: It just gives you something to fall back on. Like instead of saying I want to have a discussion about maths, so you have the theory behind why you're doing it that way. Do you know like, you have an actual reason for it.

71. EC: ok, and then would it be good to have something in your teaching file then that I could create that that you could have, say a 1 pager…this is the approach, we will be using these practices and the maths needs to be based on the quality of instruction?

72. Eve: its just that some teachers like…most teachers would be fine if you said it to them but there will be teachers who just won't accept that. So if you had like, if there was a sheet or something there saying this is actually what we do and this is what's best for the students do you know like, then they, the teachers well then have to accept it [36:15]

73. Fiona: yeah it's kind of like when you're writing up a math scheme like I know in English most of our schemes would fall under subheadings in the three columns. It was always reading writing and oral language. It would be great for maths if you had high level practice like discussion modeling; like if you had these subheadings you had to use in your schemes. It would probably provide a lot of structure.

74. EC: great advice thanks for that. I think that's a really good idea. Is there anyone who would disagree with that?

75. Fiona: That's just an idea it might not actually work, I don't know.

76. EC: Ciaran, what do you think of that?

77. Robbie: yeah I thought it was really good, it's a good idea yeah. And you can transfer all the skills you learn in two different subjects as well which is another thing so…. Again, that structure that you use in division you could apply it to addition or whatever you're doing.

78. EC: Brilliant, thanks very much. I just want to go back and mention something that somebody mentioned before about the exam focus book versus the teaching focus. This year and last year because of covid your exams have become assignments and essentially you're armed with all the knowledge that you need to be able to complete those assignments without too much stress . I'm just wondering what you think about that approach? Not in terms of ease or difficulty or anything like that, but in terms of learning. The maths itself, the content knowledge. As in is it better to have an assignment than an exam?

79. Paul: I feel like the exam, it's like the spelling test in school. It doesn't really mean that much, the spelling test. But then kids make it into something bigger because it's a test. So, I feel like it's the same with us, in the sense that we have an exam and everyone feels like they're under a lot of pressure. Even though we know all the stuff we pretty much know all of it. It's just going into the exam setting is kind of like, I don't know not necessarily for everyone, but I know some people find the exam intimidating. Like the whole 70% thing as well it makes it more pressure. Whereas I feel like the assignment, you know just seems easier because it's less stressful and also allows you more time

to actually know what you're writing about. So, if you're going to do it for an assignment, because you have the time to do it for an assignment, you're going to spend time going over it and making sure that you have it right. Whereas, in an exam you're just kind of learning something for that one or two hour period. So that you know what you need to write down. I just feel like, the assignments make you learn the maths more in a sense because you spend more time going over it and watching the videos that you put up . I think that's probably the biggest benefit of the whole intervention is the videos, the fact that you can go back and look at something. I just remember in first and second year if you didn't take the right notes in the lecture that was it. You have to go to someone else and ask them for their notes or ask him about something because you don't know if you took it down right or if you understood something properly whereas with the videos you can go back and watch it 1000 times if you need to because they're just there online and you can go over them and if there's a certain thing that you're not sure of you can go back and look at it and go through it; whereas I find with the lectures sometimes say if you just doze off for a second looking out the window or something and you miss something then it's kind of gone forever. Whereas, especially with maths if you don't understand you don't understand , it's not like you can wing it. you either have the right ideas the right methods are you don't or you don't know the steps. I just think it's very important that…I find the videos very useful and very helpful and I think that's been the biggest benefit I think on the new practice that you done.

80. EC: OK great, thanks. So you're also saying that with the assignment you learn the maths better then you would in an exam?

81. Paul: I personally feel like you do yeah because you spend more time on it. Like, If you're doing an assignment you want to get the assignment perfect are you want to get as close to perfect as you can so you spend more time going over what you've written are going through it to understand it. Whereas in an exam it's kind of like rote learning nearly, and it depends how much pressure you're under as well. If you have loads of time before an exam it's easy to learn things but if you're feeling under pressure you're kind of cramming to get the grade or get the marks as opposed to actually understanding necessarily what you're doing. It's fine if you have a good understanding of maths. I feel like I have a decent understanding of maths and would be quite confident at maths so I don't find maths necessarily too hard. But then other people, some of my friends would find maths a more challenging subject so then I feel like, from talking to them as well, that those videos and all that you can just go back and look at and the assignment that gives you more time is more beneficial to actually understand and learn the maths.

82. EC: Great. Interestingly, you were just talking about English competency earlier on. Ruth , you mentioned that. Would is similar approach work better for English? I'm just trying to generalize the idea, you know like, you know if you had more time to just focus on the vocabulary and verbs etc in an assignment would it work better?

83. Maeve: Yeah I think so EC because I think when you're doing an assignment you're engaging with, to talk about something in an assignment you have to understand it so when you're doing an assignment you're going to do so much reading and so much

research into something. When you're in an exam it's just learning off and you know, learn it off and forget within a few weeks. When you're doing an assignment you go into the nitty gritty and you’re research something. Like even say we had an assignment for numeracy with Rose and she gave us all these different programs. And she taught us about the programs in the lectures, but if I'm going to be honest now, myself and a few of my friends hadn’t a clue of the content of the programs until we went and actually had to research it and had to discuss it in the actual assignment and then had to compare it to the curriculum…and actually think about it. So, I suppose if we were to think about English as well I'd say it's kind of the same. If you had to do an assignment and actually have to think about how am I going to teach this or what is the relevance of any of this then you'd actually get a chance to think about it.

84. EC: Brilliant, OK. Very useful. Thanks very much Maeve.

85. [referring to PowerPoint]

86. Ok so just very quickly I suppose, and they may have touched on this already: you are going to be in your own classrooms in September. Is what we've done in 3rd year and 4th year going to impact how you start out in September? Let's say we didn't do the intervention, or if you had to start teaching straight away after second year. Has the intervention, what we've done in maths competency, has that given you…given you ideas about how you want to start teaching maths in September?

87. Fiona: Yeah…what I would take from it anyway is just…[connection loss] … to like slow down and allow the time to delve into things instead of just rushing through the chapter of the book. Like I feel like you could sometimes do like … like I know when you're subbing and things like that and it's not your class you kind of don't mind as much, and you’re like I'll just get the work done. But when it's your own class it'll just be nice to be able to delve into topics like through group work and discussion and modeling and not be pressured I suppose to just be teaching content, but instead teaching the process of how to do stuff and just enjoying the experience I suppose more than feeling like it's a stress. Because I feel like on placement I'd always stress about maths but it is nice to know there is a different approach we can take to it.

88. Eve: I think with placement as well you were trying to incorporate all these things like trying to make it fun and interactive and there's so much pressure on you to meet those standards whereas in September we're going to have our own classes so we kind of can take, not a more relaxed approach, but actually go deeper into it. Like to kind of talk about it rather than…just get through it. Do you know like, if you want to just have a discussion, not necessarily do anything out of the textbook for a day and just discuss the [inaudible] about it. Because on placement it's very much do your warmup, do your games, do the content, that kind of way so ... In September we're going to have more freedom to be able to go deeper about them things.

89. EC: is there anything that you think might get in the way of that freedom?

90. Paul: probably restrictions of the school, like if the school has a plan maybe or something… that you need to get XY and Z done in September and whatever else done in November and stuff, that could be a constraint. But then if you can propose an argument for why you're doing it a certain way I'm sure that any reasonable principle or team of staff will understand that, well this is why they're doing it and the children

actually all understand this now. And I feel like if you invest time into something at the start then you can reap the rewards of it later because you won't have to spend time going over it again and…. if you spend the time going into something deep, on a deep level the first time, for people to understand it then in two months you're not going to have to go back all over it again because you find out that 15 people out of 28 don't actually know what to do, they just were able to do it on that day but they don't know why they're doing it or what's the reason for it. Whereas, you know like Eve said if you can just spend time talking about it for a day and just decide, in maths class today we're not actually going to do any problems in the book… we're actually going to sit down and discuss, like, in detail why we need to do these steps and what this means and like… you know even like, if I think back to when I went to school, you put the one down and you carry the one and all that, it wasn't until I came to college that I understood what that meant. I just knew that you did that but I didn't know that it meant like, this represents a 10 or a unit or whatever. I just knew that this is what you do, this is the step. If this comes up this is what I do, whereas if you just spend time in school explaining that that then the children will understand what it means and I feel like that carries on to other things that you'll be teaching them later on, and that would be more beneficial. You invest the time at the start and you get that time back at a later stage because you don't have to go over things or go into things in as much detail because children already understand it.

91. EC: OK very good… Eve was just talking about that depth of understanding, Paul you mentioned it and Helen mentioned it for SEN. Is there consensus around the need or the desire to teach for a depth of understanding in maths?

92. Emily: Yeah I think there is because when you just teach a formula or a way to do something it doesn't mean anything to the kids. And maths just gets so boring for them then…and they don't want to do it. But if you actually show them, like the thing we did with Pi and the string, like that's really interesting for them. It's like when you just explain why something is the way it is rather than just saying this is how it is, just remember that.

93. EC: So, it's got more meaning?

94. Emily: yeah

95. EC: before we go is there anything that anyone would like to mention about their online placement? If any of the stuff we did was relevant in any way?

96. Paul: I think modeling was the most relevant thing for online because, you're basically modeling all the questions yourself because…well I wasn't doing the live lectures… you're not doing live teaching except for one where I was going over work that she was finding difficult. But then when I was teaching the rest of the class, I was making videos of me doing questions and stuff and talking through it so I was kind of like doing the explanation and the modeling and steps…you do this, followed by this and this and this. So I found that the modeling that we did in…like was very relevant… because that was the way that you had to teach on the online placement because obviously you weren't in the classroom and you couldn't really…like you couldn't give the children a problem and ask them to try that now and then discuss it then after, and then model it because obviously you know some children do their work at 10 o'clock and others

do it at 2 o'clock so it wasn't really conducive to the online learning whereas modeling was just very straightforward and all the children could access the content at any stage and then obviously then as a teacher I was available then to answer questions or problems… and then you could delve into it deeper based on the feedback you got from the students.

97. E: OK, thanks Paul. Does anyone else have input there?

Appendix 11: 100 Questions to promote mathematical discourse

(Kersaint, 2015)

Think about the questions that you ask in your math classroom. Can they be answered with a simple "yes" or "no," or do they open a door for students to really share their knowledge in a way that highlights their true understanding and uncovers their misunderstandings? Asking better questions can open new doors for students, helping to promote mathematical thinking and encouraging classroom discourse. Such questions help students:

- Work together to make sense of mathematics.
- Rely more on themselves to determine whether something is mathematically correct.
- Learn to reason mathematically.
- Evaluate their own processes and engage in productive peer interaction.
- Discover and seek help with problems in their comprehension.
- Learn to conjecture, invent and solve problems.
- Learn to connect mathematics, its ideas and its applications.
- Focus on the mathematical skills embedded within activities.

Below are 100 questions from mathematics expert Dr. Gladis Kersaint to help you address these core areas and promote mathematical thinking and discourse in the classroom. Want these questions visible in your classroom? Curriculum Associates has released an infographic that you can print and have at your desk or in your class for quick access!

Help students work together to make sense of mathematics

1. What strategy did you use?
2. Do you agree?
3. Do you disagree?
4. Would you ask the rest of the class that question?
5. Could you share your method with the class?
6. What part of what he said do you understand?
7. Would someone like to share ___?
8. Can you convince the rest of us that that makes sense?
9. What do others think about what [student] said?
10. Can someone retell or restate [student]'s explanation?
11. Did you work together? In what way?
12. Would anyone like to add to this?
13. Have you discussed this with your group? With others?
14. Did anyone get a different answer?
15. Where would you go for help?

16. Did everybody get a fair chance to talk, to use the manipulatives, or to be recorded?
17. How could you help another student without telling the answer?
18. How would you explain ___ to someone who missed class today?

Refer questions raised by students back to the class.

Help students rely more on themselves to determine whether something is mathematically correct

19. Is this a reasonable answer?
20. Does that make sense?
21. Why do you think that? Why is that true?
22. Can you draw a picture or make a model to show that?
23. How did you reach that conclusion?
24. Does anyone want to revise his or her answer?
25. How were you sure your answer was right?

Help students learn to reason mathematically

26. How did you begin to think about this problem?
27. What is another way you could solve this problem?
28. How could you prove that?
29. Can you explain how your answer is different from or the same as [student]'s?
30. Let's see if we can break it down. What would the parts be?
31. Can you explain this part more specifically?
32. Does that always work?
33. Is that true for all cases?
34. How did you organize your information? Your thinking?

Help students evaluate their own processes and engage in productive peer interaction

35. What do you need to do next?
36. What have you accomplished?
37. What are your strengths and weaknesses?
38. Was your group participation appropriate and helpful?

Help students with problem comprehension

39. What is this problem about? What can you tell me about it?
40. Do you need to define or set limits for the problem?
41. How would you interpret that?
42. Would you please reword that in simpler terms?

43. Is there something that can be eliminated or that is missing?
44. Would you please explain that in your own words?
45. What assumptions do you have to make?
46. What do you know about this part?
47. Which words were most important? Why?

Help students learn to conjecture, invent and solve problems

48. What would happen if ___? What if not?
49. Do you see a pattern?
50. What are some possibilities here?
51. Where could you find the information you need?
52. How would you check your steps or your answer?
53. What did not work?
54. How is your solution method the same as or different from [student]'s?
55. Other than retracing your steps, how can you determine if your answers are appropriate?
56. What decision do you think he or she should make?
57. How did you organize the information? Do you have a record?
58. How could you solve this using (tables, trees, lists, diagrams, etc.)?
59. What have you tried? What steps did you take?
60. How would it look if you used these materials?
61. How would you draw a diagram or make a sketch to solve the problem?
62. Is there another possible answer? If so, explain.
63. How would you research that?
64. Is there anything you've overlooked?
65. How did you think about the problem?
66. What was your estimate or prediction?
67. How confident are you in your answer?
68. What else would you like to know?
69. What do you think comes next?
70. Is the solution reasonable, considering the context?
71. Did you have a system? Explain it.
72. Did you have a strategy? Explain it.
73. Did you have a design? Explain it.

Help students learn to connect mathematics, its ideas and its application

74. What is the relationship of this to that?
75. Have we ever solved a problem like this before?

76. What uses of mathematics did you find in the newspaper last night?
77. What is the same?
78. What is different?
79. Did you use skills or build on concepts that were not necessarily mathematical?
80. Which skills or concepts did you use?
81. What ideas have we explored before that were useful in solving this problem?
82. Is there a pattern?
83. Where else would this strategy be useful?
84. How does this relate to ___?
85. Is there a general rule?
86. Is there a real-life situation where this could be used?
87. How would your method work with other problems?
88. What other problem does this seem to lead to?

Help students persevere

89. Have you tried making a guess?
90. What else have you tried?
91. Would another recording method work as well or better?
92. Is there another way to (draw, explain, say) that?
93. Give me another related problem. Is there an easier problem?
94. How would you explain what you know right now?

Help students focus on the mathematics from activities

95. What was one thing you learned (or two, or more)?
96. Where would this problem fit on our mathematics chart?
97. How many kinds of mathematics were used in this investigation?
98. What were the mathematical ideas in this problem?
99. What is the mathematically different about these two situations?
100. What are the variables in this problem? What stays constant?

Appendix 12: Letter to Research Committee Cycle 1

26th August, 2019

Dear Members of Social Research Ethics Sub-Committee,

I am writing this letter to request that I stage out my application for ethical approval. There are several data collection points, the first of which is next Monday 2nd September 2019. This is a questionnaire which uses Likert scale type questions, and the opportunity for further explanation. I am requesting ethical clearance only for this initial phase of data collection at this point. Each of the data collection phases is outlined in my application form.

In the application form it is highlighted that I would like to request ethical approval initially for the questionnaire on 2nd September (the first phase). The remaining data collection phases include:

- Observations on School placement in October
- Focus groups in November
- Collection of reflective journals in November

The reason I am requesting this phased approach is due to the tight timeline resulting from the inflexible teaching and school placement schedule. I hope that requesting clearance only for the initial questionnaire phase at this point will be quicker than requesting clearance for the entire project, and thus allowing me to begin the project on time. I will subsequently seek ethical approval for the remainder of the research project on approval of this.

I would be very grateful if you could grand me this request.

Kind regards,

Eddie Costello

Appendix 13: Mathematical Quality of Instruction Framework

The MQI framework is explained in appendix 6. The MQI observational framework is designed to assign score to individual teachers based on discrete dimensions of mathematics teaching. these dimensions are: richness of the mathematics; errors and imprecision; working with students and mathematics; student participation in meaning making and reasoning; and connections between classroom work and mathematics. Each area contains sub-dimensions, and each of these will be scored by the researcher on a 4-point scale. For example, the sub-dimension 'mathematical sense making' is part of the 'richness of the mathematics' dimension. This sub-dimension aims to capture the extent to which the participant attends to the following: the meaning of numbers, relationships between numbers, connections between ideas and representations, and whether the modelling of solutions makes sense. This sub-dimension will be graded from 'not present' (a score of 0) to 'high' (a score of 3) and the descriptors for this is as follows:

Not present: Not present or incorrect (0)

Low: Participant focuses on briefly on meaning. (1)

Mid: Participant focuses more than briefly on meaning, but this work is not sustained or substantial. (2)

High: Participant focuses on meaning in a sustained way. (3)

(National Centre for Teacher Effectiveness, 2011, p. 7)

Richness of mathematics:

Linking Between Representations

i.e., explicit linking and connections between different representations of a mathematical idea or procedure.

[both representations must be visually present. Correspondence must be pointed out.]

Explanations

i.e., why a procedure works, why something is true, or why a method/ solution is appropriate…why an answer is true. (high: explanations major feature of teacher-student work)

Mathematical Sense-Making/ number sense

Meaning of numbers, relationships between numbers, connections between ideas, giving meaning to procedures, etc

e.g., the value of quantities, meaning of quantities, reasonableness of explanations, meaning of procedures/ expressions/ equations, using estimation.

Multiple Procedures or Solution Methods

i.e., different strategies and/ or approaches to solving mathematical problems.

Patterns and Generalisations

i.e., develop or work on a mathematical generalization; to notice, extend or generalize a mathematical pattern; to derive a mathematical property; or to build and test definitions.

e.g., whether a procedure works in all cases or building up a definition or deriving a mathematical property.

Mathematical Language

i.e., how fluently the teacher (and students) use mathematical language and whether the teacher supports students' use of mathematical language. Should be explicit.

Overall Richness of the Mathematics

i.e., the depth of the mathematics offered to pupils

Appendix 14: High Leverage Teaching Practices

Two important high leverage practices are described below. High-leverage practices are the fundamentals of teaching. These practices are used constantly and are critical to helping students learn important content. Two high leverage practice within mathematics teaching are *leading a mathematical discussion* and *modelling mathematical content*.

Remember we examined these in practice earlier in the year. As you are reading these, try to put them into some practical context. For example, you might recall what we did in lectures during the year and subsequently how this impacted on your school placement. Alternatively, you might try to imagine how you will use these practices in your future teaching.

Below is the decomposition of these two important practices into simplified teachable elements. You can find more information on http://www.teachingworks.org/.

Decomposition of leading a mathematical discussion

Discussion enabling (before launching task to be discussed)

- Select a task: Choose content/ topic that is “discussable”
- Topic must be worthy of discussing
- Teacher must identify the mathematical point of the discussion
- Teacher must anticipate student thinking and possible misconceptions

Discussion launch:

- Explain the goal of the discussion
- Determine and activate prior knowledge
- Pose open ended question related to the mathematical point
- Review norms/procedures for discussion

Discussion leading:

- Probe children’s thinking to clarify ideas
- Elicit multiple ideas/ views from children
- Orient children to connect to and build off one another’s mathematical ideas
- Ensure children are listening and responding to each other’s ideas
- Support children in connecting ideas
- Contribute strategically to the discussion
- Record and represent content of the discussion publicly, in order to support all students in having access to the ideas that are being shared.
- Maintain a focus on the mathematical point

Discussion Conclusion:

- Support children to remember and make sense of content
- Note progress made in discussion
- Acknowledge children’s competence

Decomposition of modelling mathematical content

Planning to model

- Decide if modelling is appropriate
- Select content to be modelled
- Choose appropriate representations

Framing

- Connect ideas to previous learning
- Explain the purpose of the content

Doing the content area work and Highlighting core ideas

- Work through the content for children to see
- Avoid highlighting aspects that are confusing or lead to misconceptions
- Use explicit verbal markers to draw children's attention to important aspects of the content; e.g., "Watch as…" "First … then…" and elaborate and emphasise the part of the explanation that is most complex or confusing.
- Highlight important ideas by naming key elements while progressing in a logical fashion and being careful not to skip any elements.

Making thinking visible by emphasising thinking and key elements

- Thinking aloud to make thinking visible.
- Using markers (verbal, tone, or visual) to indicate when thinking is being made visible.
- clearly articulating what you are doing and why you are doing it.

Using language and representations carefully

- Making explicit correspondences between the problem, the written explanation, and any representations.
- Use content-specific terms and representations clearly and consistently throughout the modelling
- Use language and representations that are developmentally appropriate and accessible to learners.
- Translate verbal ideas into a visual form in the public space in ways that are likely to support student understanding, are clearly labelled, and are accurate.

Norms for Giving Mathematical Explanations

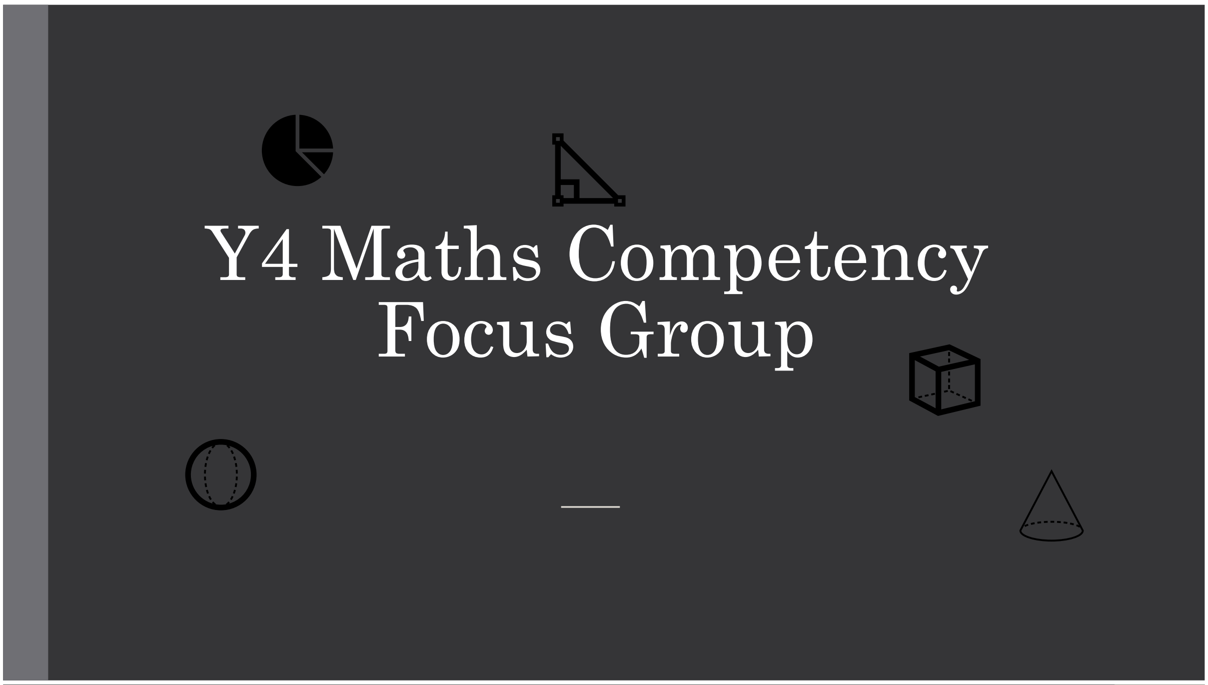
Source: TeachingWorks.com

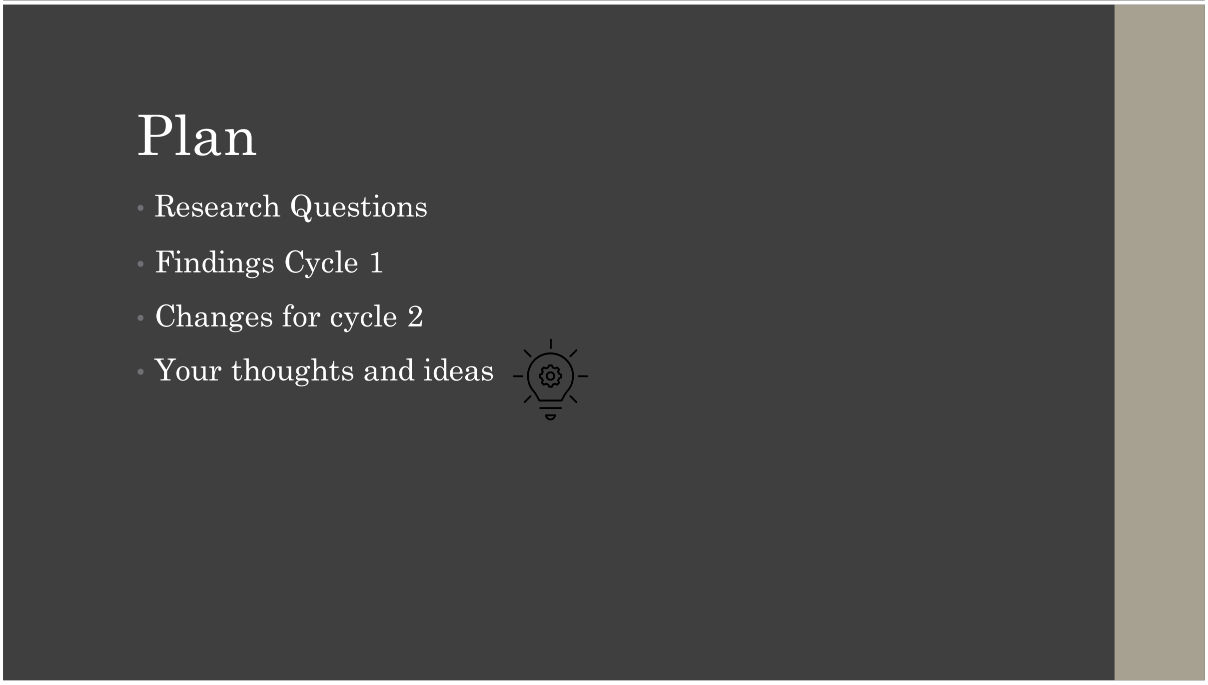
Mathematical explanations are a central part of mathematical discourse, in which the speaker communicates their mathematical ideas. The purpose may be to elaborate a mathematical concept, describe a set of calculations or a solution strategy, or convince others of the validity of a mathematical argument. Good mathematical explanations are precise, parsimonious, and tuned to the audience. Teaching students how to give good mathematical explanations involves making norms related to mathematical explanations explicit to students, providing frequent opportunities for them to explain their ideas, and giving constructive feedback on their explanations.

Below is a partial list of norms that can be used to support students at giving mathematical explanations that reflect the nature of discourse in the discipline. Explicitly excluded from this list are general norms about public speaking or other discipline-neutral norms. While acknowledging that the language might not be exactly what one would use with children, the intent of the list is to operationalize disciplinary discourse practices in ways that are understandable and practicable by children. As such, the norms in this list are written to go beyond general statements about what student should do. Rather, we describe how students should do those things.

- Don’t just describe your procedure or steps, but explain your reasoning behind the steps.
- To explain your reasoning, make connections between your strategy and/or solution and elements of the problem.
- To give a complete mathematical explanation, make sure you address all of the conditions of problem.
- Use appropriate mathematical representations, symbols, diagrams, etc. to support your explanation.
- Use appropriate/understandable mathematical language.
- Don’t assume that others have the same background/math knowledge that you do: make your assumptions and definitions explicit.
- Use mathematical reasons (and not other kinds of reasons, like appeals to authority or emotional arguments) to support your claims.
- Use precise mathematical language.

Appendix 15: Focus Group presentation Cycle 2





Research Questions

1. What causes of the problem of enactment?
2. How can maths competency be redesigned around pedagogies of enactment?
3. Will this approach change your beliefs about maths?
4. Can the 'intervention' change your teaching?

Findings

- Old maths competency: good for maths but little impact on practice
- MSE and maths competency not linked or coordinated
- MSE games focused, not great for senior classes
- The 'intervention'
  - Helped link content and pedagogy
  - Collaboration and working with children useful
  - HLP and reflection worked well, thinking deeper about teaching maths
  - Content online was better (flipped classroom model)

Findings

- Improvements still required:
  - Link competency and MSE
  - Improve sharing of ideas
  - Same pupils multiple occasions for refinement
  - More explicit links to SP topics

Findings

- Difficulties for you in the classroom:
  - Pupils
  - SP Supervisors
  - Cooperating teachers
  - You ☺
  - …and the conflict between all of these things!

Changes:

- More deliberate collaboration (full group)
- One topic
- Same HLP's
- More focus on MQI
- More deliberate reflection
- Education for supervisors

What I'd like to know more about:

- Maths Modules:
  - What do you think about above changes?
  - Does maths competency 'work'?
  - What advice would you give/ what recommendations do you have for future teaching of maths?

What I'd like to know more about:

Maths Modules:

- What do you think about above changes? (i.e. how did y4 compare with y3?)
- Does maths competency 'work'?
- What advice would you give/ what recommendations do you have for future teaching of maths in Froebel?

What I'd like to know more about:

Your Teaching:

- Did the intervention help with your online placement? If it were not online, would it have helped?
- Has this changed how you will teach maths from September? What might prevent this?
- Has it changed the way you think about maths?

Appendix 16: Codes and themes

Original 55 Codes:

1. mathematical understanding
2. maths comp relevance
3. reference to pedagogies of enactment
4. cooperating teacher
5. pupils' attitude
6. PST frustrations
7. PST Agency/ enactment (motivation)
8. preparedness for SP
9. topic/ age
10. Rote/ procedural/ instrumental
11. PST Confidence/ competence
12. maths expectations pressure
13. SP Tutor
14. PST needs/ suggestions
15. PST attitude/ mindset
16. textbook
17. PST as pupil
18. neatness/ messy
19. PST attempt at RU
20. Educational Value
21. maths pedagogy
22. critique maths methods
23. leaving cert
24. confusing/ offending/ discomfort pupils
25. child at centre of PST learning
26. hands on maths
27. teacher education dialogue
28. maths as speed
29. Froebelian maths
30. busy curriculum
31. maths comp Exam
32. maths as exam
33. gauging pupils prior knowledge
34. sums
35. disconnected from classroom
36. PST struggle with maths
37. concrete materials
38. methods and competency disconnected
39. pupil struggling maths
40. PST direct teaching on SP
41. maths as games
42. Flipped Classroom
43. PST calling for practice-based pedagogy
44. maths as cooperative
45. parents
46. maths as fun
47. SP too short
48. maths as quiet/ individual
49. maths panic/ anxiety
50. importance of communication
51. maths as competition
52. everyday life maths
53. fear of division
54. maths as a difficult/ hard subject
55. collaboration as cheating

The table below outlines the refined 47 codes and the themes they were assigned:

Theme	Code
Effectiveness of Froebel Maths Modules	maths comp relevance
	reference to pedagogies of enactment
	preparedness for SP
	disconnected from classroom
	PST calling for practice-based pedagogy
	methods and competency disconnected
	Flipped Classroom
	teacher education dialogue
	gauging pupils' prior knowledge
	critique maths methods
	PST needs/ suggestions
Froebelian Maths Instruction	hands on maths
	Froebelian maths
	concrete materials
	PST direct teaching on SP
	maths as games
	maths as fun
	everyday life maths
PST beliefs, thoughts and actions	PST Confidence/ competence
	PST attitude/ mindset
	child at centre of PST learning
	PST as pupil
	sums
	maths as cooperative
	fear of division
Influencers of PST agency/ enactment	cooperating teacher
	pupils attitude

	PST frustrations SP Tutor textbook neatness/ messy leaving cert maths as speed busy curriculum parents SP too short maths as quiet/ individual maths as competition collaboration as cheating
mathematical understanding	mathematical understanding
	Rote/ procedural/ insturmental
	PST attempt at RU
Effectiveness of Froebel Maths Modules	maths comp relevance
	reference to pedagogies of enactment
	preparedness for SP
	disconnected from classroom
	PST calling for practice-based pedagogy
	methods and competency disconnected
	Flipped Classroom
	teacher education dialogue
	gauging pupils' prior knowledge
	critique maths methods
	PST needs/ suggestions
neoliberal/ GERM	maths expectations pressure
	maths as exam
	Educational Value