A Visual Chaotic System Simulation in Arduino Platform Controlled by Android App

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Abstract—The application of computer and electronic technologies has grown extensively in recent years being a useful tool for teaching in various fields of knowledge. The use of these technologies aims at bridging the gap between theory and practice, improving learning outcomes. This work demonstrates the project of didactic prototype based on the Arduino platform to study and teach nonlinear dynamical systems. The teaching platform presented here aims at providing visual effects and better understanding of basic concepts of nonlinear dynamical systems, such as chaos. It is also demonstrated when the data loses the precision, also in a visual setup. Additionaly, input data may be configured by wireless communication, which makes it very attractive for young students.

Keywords— Dynamical Systems, Chaos, Didactic Prototype, Learning, Arduino, Non-linear Systems, Numerical Methods.

I. INTRODUCTION

Considering the great revolution of automation, information and communication technologies, the educational context has required great transformation in the school structure aiming at new postures that favor changes conducted to a new scenario of the contemporary world [1].

The use of these technologies aims at shortening the distance between theory and practice, improving the learning of specific knowledge [2]. There is a difficulty in presenting concepts related to non-linear dynamical systems, due to the difficulty in assimilating such knowledge.

Currently, many papers deal with nonlinear dynamics, studying and applying methods to better understand computational simulations [3] [4] [5]. It shows the importance of learning these concepts deeply, understanding better the loss of precision in simulations and developing new methods for research. However, in the educational environment, teaching such concepts is not easy, since it requires an extensive study and it is usually very difficult to demonstrate in a practical way.

Additionally, it is well known that learning must be acquired through intuitive processes, seeking different ways to represent the same concept [6]. In this way it is necessary to search for alternative solutions, such as the elaboration of didactic prototypes. Currently embedded platforms, such as Arduino, are used for the construction of educational kits. Several areas of knowledge have been applied Arduino widely for the purpose of teaching due financial cost and flexibility [7] [8].

The application of Arduino platform on the investigation of chaotic systems seems to be in a very early stage. A search on IEEE Xplore using the terms *chaos* and *Arduino* comes out with only two papers [9] and [10], but both of them without an explicit concern on educational purposes.

Applying technology in education is used to arise interest in the learning of students in the proposed area, likewise to serve as a link between software and hardware in the development of experiments in the laboratory and also to aid in research. This work presents the construction of a didactic prototype based on the Arduino platform, with the purpose of teaching about non-linear dynamic systems, more specifically, on chaos theory [11].

The didactic platform presented in this work aims at visual effects based on LEDs, to provide a better understanding of fundamental notions of non-linear dynamical systems, such as periodicity, chaos and bifurcation. It is also demonstrated a method for estimation of error propagation in simulations [12] [13], in which loss of precision is shown at each iteration. The data entry is made through an application that provides
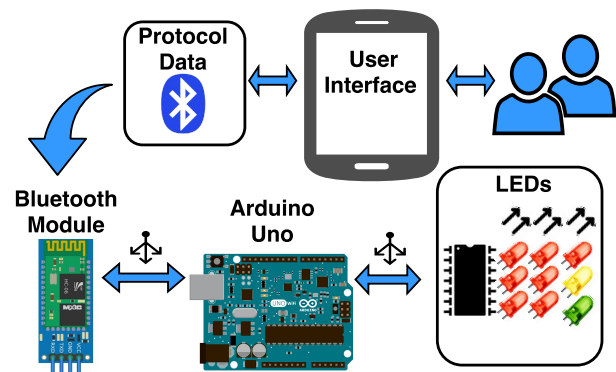

Fig. 1. Project procedures.

wireless communication in the bluetooth standard with the didactic prototype, being possible to change simulation parameters remotely. Figure 1 demonstrates a scheme of the project undertaken in this work.

This article is organized as follows: Section II describes general concepts for the understanding of nonlinear dynamics and accumulation of errors. Section III is dedicated to describe

the technologies and techniques implemented. Section IV describes the structure and operation of the remote user interface created. Section V presents the analysis of the results obtained through the didactic design tests. The key conclusions drawn from this research work are presented in Section VI.

II. PRELIMINARY CONCEPTS

The following section concepts that explain and base the subjects covered in the didactic platform, such as nonlinear dynamics and accumulated errors in numerical simulations.

A. Recursive Functions

For analysis and visualization of the concepts, recursive functions are simulated, which can be defined as follows: Let $\mathbb{I} \subseteq \mathbb{R}$ a metric space with $f : \mathbb{I} \longrightarrow \mathbb{R}$:

$$x_n = f(x_{n-1}). \tag{1}$$

Discrete time series can be generated by a simple iterative procedure of Equation (1). Depending on the choice of f, the series may exhibit behaviors of nonlinear dynamics such as bifurcation, periodic orbits and chaos. An important concept is the fixed point, as the point of $f(x_n)$ in which

$$x_n^* = f(x_n^*), \tag{2}$$

if the orbit of the map reaches the fixed point in a given iteration, the orbit there remains in the following iterations [14].

B. Logistic Map

Nonlinear maps present a rich and complex dynamic, in which one of the most studied is the logistic map, which is a recursive function that represents a population model [15]:

$$f(x_n) = x_{n+1} = rx_n(1 - x_n), \tag{3}$$

r is control parameter, which belongs to the interval $1 \leq r \leq 4$ and x_n at interval $0 \leq x_n \leq 1$.

A time sequence can be obtained from the logistic map equation by choosing a value for r and recursively iterating the map from an initial condition x_0. This sequence of map values obtained in response to each iteration is called an orbit. As the parameter r approaches 4, the path of the orbit becomes more unstable, causing the system to exhibit chaotic behavior.

C. Orbits and Pseudo-Orbits

Let f be a function e x is a real variable. Nepomuceno and Martins [12] present the following definition.

Definition 1: An orbit is the sequence of values of the function, represented by $\{x_n\} = [x_0, x_1, ..., x_n]$.
An orbit is clearly a pseudo-orbit, but a pseudo-orbit is a more comprehensive concept that encompasses, for example, the orbit of a dynamical system when viewed on a computer screen, due to the finite representation of real numbers in computer simulations, accumulated and propagated errors.

Definition 2: Let $i \in \mathbb{N}$ which represents a pseudo-orbit, which it is defined by an initial condition and extension of the function f, for some hardware, software, standard of numerical precision or mathematically equivalent expressions. The pseudo-orbit is an approximation of the orbit represented by

$$\{\hat{x}_{i,n}\} = [\hat{x}_{i,0}, \hat{x}_{i,1}, ..., \hat{x}_{i,n}], \tag{4}$$

such that

$$|x_n - \hat{x}_{i,n}| \leq \delta_{i,n}, \tag{5}$$

in which $\delta_{i,n} \in \mathbb{R}$ is the error.

D. Lower Bound Error

A method for estimation of error propagation in simulations was developed by Nepomuceno and Martins [12], represented by theorem:

Theorem 1: Be two pseudo-orbits $\{\hat{x}_{a,n}\}$ and $\{\hat{x}_{b,n}\}$ derived from two different extensions of the function f, lower bound error is defined as

$$\delta_{\alpha,n} = \frac{|\hat{x}_{a,n} - \hat{x}_{b,n}|}{2}. \tag{6}$$

Basically the lower bound error is a practical and simple implementation tool, to increase the reliability of the computational simulation. Calculated as half the distance of the two pseudo orbits each iteration, providing information that a minimum precision for the simulation is no longer satisfied.

E. Logistic Map Extensions

Extensions are mathematically equivalent representations of functions, which are differentiated by the modes that can be written, for example, when inserted in computer algorithm [16]. The didactic platform consists of the calculation of the recursive logistic map function by means of the Arduino, in the algorithm is determined two extensions of the logistic function based on Table 1.

Although mathematically equivalent they are not seen in this way by computers. The basic axioms of multiplication and division are not contemplated by computers using floating-point arithmetic [17] [18].

TABLE I
LOGISTICS FUNCTION EXTENSIONS.

$L1$	$x_{n+1} = (rx_n)(1 - x_n)$
$L2$	$x_{n+1} = r(x_n(1 - x_n))$
$L3$	$x_{n+1} = (r - rx_n)x_n$
$L4$	$x_{n+1} = r(x_n - x_nx_n)$
$L5$	$x_{n+1} = rx_n - (rx_n)x_n$
$L6$	$x_{n+1} = rx_n - r(x_nx_n)$

III. PROJECT DEVELOPMENT

This section presents an explanation of the technologies used in this work, demonstrating the electronic resources used, at the same time serving as a guide for new projects.

A. Arduino Uno Microcontroller Board

Arduino Uno is an Open-Source electronic prototyping platform that uses the ATMega328 microcontroller, the board supports the programming language based on C/C++. The Uno platform was chosen because it allows the creation of several projects for a low cost, and ease in the implementation of integrated circuits to the Arduino. Its main programming environment is the Arduino IDE, which has a simple and intuitive interface [19]. The arduino get the simulation parameters through the bluetooth user interface, processing the data and sending the results to the electronic circuit, which displays the results through visual effects.

B. Display Driver

The integrated circuit LM3914 is a linear voltage indicator, whose main function is to monitor the signals applied to its input. Internally it has 10 comparators each with a respective reference designated by the resistive network. This way your outputs switch from logic level high to low as your references are reached. The circuit produces a very interesting visual effect, which according to the voltage intensity applied to its input, the corresponding LEDs light up in the same proportion on a scale of 1 to 10.

C. Module Bluetooth

Bluetooth technology is already widely used. In Arduino is more a simple and inexpensive way to send and receive information remotely [20]. The Bluetooth module HC-06 is used for wireless communication between the Arduino and other devices. The information is received by the module and transferred to the Arduino via serial communication.

D. Bargraph Circuit

The circuit that displays the response through visual effects, is composed of an RC low-pass filter, connected to the LM3914 circuit. Due to the operation of the LM3914 circuit, it is necessary to have a continuous analog signal, in which the variable output of the Arduino provides a Pulse Width Modulation waveform. The RC filter is a simple circuit, used to convert a PWM signal into a CC signal. The continuous analog voltage varies by the duty cycle of the converted PWM wave, where it is proportional to the mean value. The resistors $R1$ and $R2$ regulate the reference voltage applied to the bar graph. Figure 2 shows the circuit.
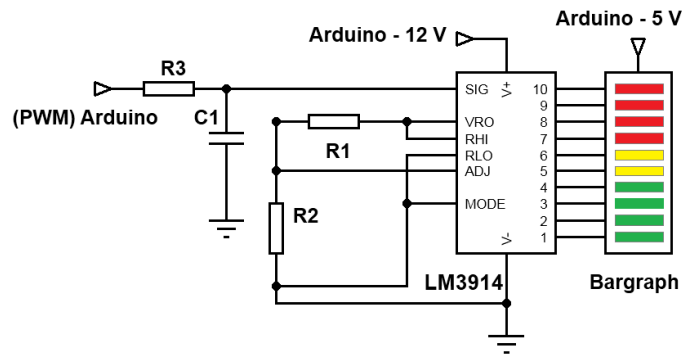
IV. REMOTE USER INTERFACE

An application has been designed to change the parameters of the simulation executed by the platform, in which it is possible to perform several experiments in a simple and fast way, excluding the need to change the program stored in the Arduino. The interface was created through App Inventor [21], in which is a software intended for the creation of Android applications. App Inventor has a simple, intuitive, block-based interface Built-in, a visual programming structure for education.
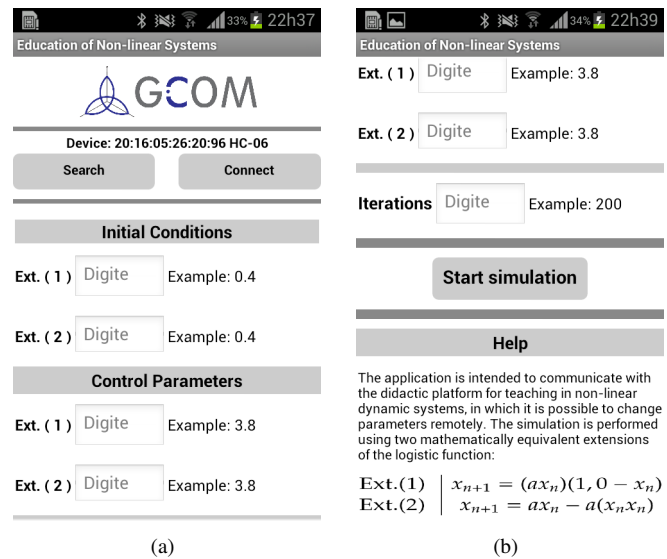

Fig. 2. Bargraph circuit.

Fig. 3. User interface screens.

Figure 3 shows the interface visual, in which are used Labels that show text on the screen, Button indicating actions of response and TextBox that stores and edits variables. Other hidden elements, we have the Notifier is an alert element on the screen, TinyDB which stores data in the application and BluetoothClient element responsible for the transmission of data via bluetooth.

The application and its elements create the connection of the Android device to the Arduino, as well as treat the signal and the variables involved in the simulation process in the Arduino. It creates alert elements and checks the limits of variables and graphical elements such as the bluetooth connection and initialization of the application.

For communication between Android and Arduino it was necessary to create a data protocol, because App Inventor provides the basis for sending characters. The protocol is done in packet format strings, namely a set of characters. After sending the String to the Arduino, a conversion of the variable to floating-point is done, for mathematical manipulations in the Arduino code.
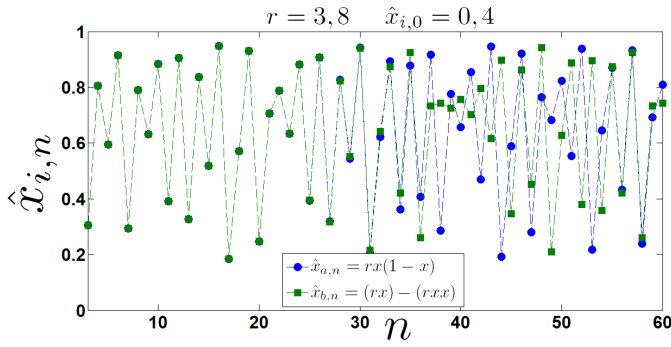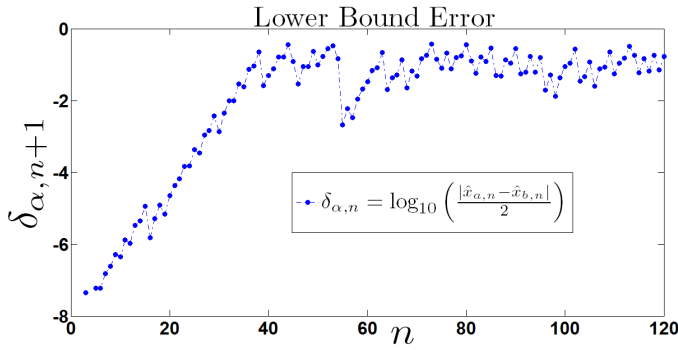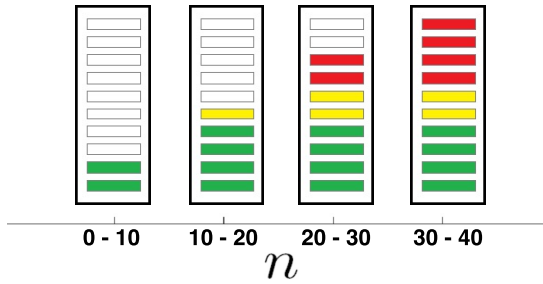
Fig. 4. Simulation of the Temporal Series.

(a)

(b)

Fig. 5. Simulation of the Lower bounder error at each iteration.

V. DEVELOPMENT AND ANALYSIS OF THE DIDACTIC PROJECT

The present work demonstrates a platform for visualization of phenomena involving the computational simulation of nonlinear dynamic systems. It is demonstrated visually the occurrence of errors in numerical computation, where we have three main sources of errors: rounding, imprecision and truncation. As also shown that the different behavior of the errors, considering different extensions that are mathematically equivalent.

Initially two extensions are chosen, which represent two pseudo-orbits during the simulation. The logistic function is calculated through each extension, in which two graphical bars represent the calculated points at each iteration. The graphs shown below represent the visible points on the graphical bars, illustrating the behaviors that are possible to observe in the

bargraph circuit. In the same way, the lower bounder error is calculated, defined by Equation 6.
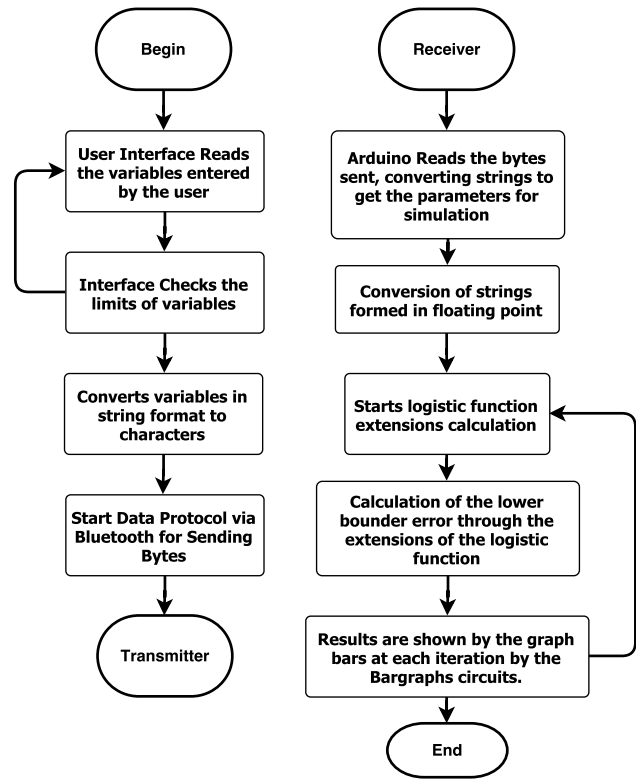

Fig. 6. Flowchart diagrams.

Is added the $\log$ on base 10, because the presentation of data in this scale covers a larger range of values, reducing to a scale for better visualization. Figure 4 shows the results obtained in the calculation of the logistic function using the Matlab software. It is possible to observe the distancing of the two time series that are mathematically equivalent.

It is presented the relevance of evaluating the possible extensions of the same equation, since the result from a number of iterations differs from one extension to another. In the electronic circuit two graphical bars represent the results obtained by the Arduino in the execution of the simulation of Figure 4, in which, even though they are the same parameters, the results can be differentiated because they are pseudo-orbits.

The calculation of the lower bounder error is presented by Figure 5, for the points obtained in Figure 4, where information is provided that a minimum precision for the simulation is no longer satisfied from the number of iterations. The first values between -8 and -7 represent the total number of decimal precision digits, considering that Arduino Uno uses the Single precision format, where floating-point numbers are stored as 32 Bits (4 bytes) of information, where in Arduino Uno is standardized from 6 to 7 digits of decimal precision.

The figures shows the loss of precision digits at each iteration in the simulation of recursive functions, which for a number of iterations the minimum precision is no longer satisfied, evidencing that there is no numerical reliability of

the values found. In the bar graph this concept is demonstrated by the colors of the LEDs in Figure 5, in which the colors green, yellow and red are used. Basically, the colors change according to the loss of accuracy of the results, with green color showing that the minimum precision is satisfied and we have confidence in the results, and the red color is opposite, emphasizing the loss of precision at each iteration, illustrating the sensitivity of the system to chaos.

The procedure of the platform is Illustrated in Figure 6, demonstrating the whole procedure of gathering and data processing. The process begins in collecting the data through the user interface that will be sent to the Arduino. In the microcontroller is made the calculation of the different extensions of the logistic map and the calculation of the lower bound error. Lastly the data is converted into visual effects on the bargraph circuit, the Figure 7 shows the circuit assembly in protoboard, which the bargraph is represented by the set of LEDs.

The machine settings used for the simulations are as follows: Processor Intel(R) Core(TM) i3-2348M, CPU @ 2.30 GHz, RAM 6,00 GB, 64-Bits, Windows 7 Ultimate.
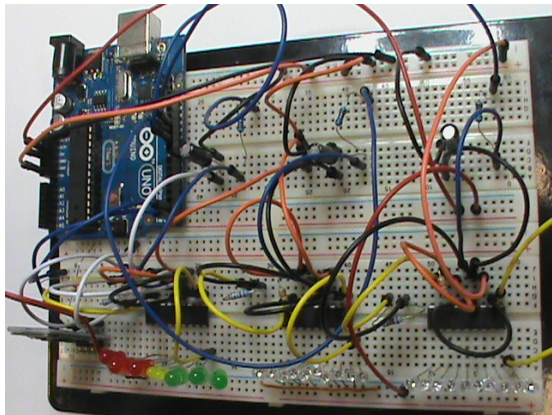

Fig. 7. Photo of didactic platform. Due to financial costs, the bargraph has been replaced by a set of leds.

VI. Conclusion

In this work, a didactic prototype was developed for the teaching of nonlinear dynamic systems, based on the Arduino embedded platform. It is composed of modules for signal processing which provide visual effects and a bluetooth communication system, coupled to an interface with allows the user to interact with the prototype. It is also an example that it is possible to combine research with practical projects, which illustrate and enrich the concepts addressed in the research.

The prototype was submitted to experimental tests allowing an analysis of the behavior and learning efficiency among the participants. It was observed that such dynamics allowed the reflection of the concepts discussed, revealing a stimulus and interest, resulting from the contact with the concepts presented, having the platform as mediator.

Teaching prototypes and graphical interfaces are forms of communication that transcend the limits of technology. Thus, they define scientific visualization as a tool for discovery and knowledge, since it allows to transform numeric data fields into images, thus creating more noticeable information, since the brain processes a picture better and faster than a set of numbers.

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