PERGAMON

Computers, Environment and Urban Systems
25 (2001) 119–130

Computers, Environment and Urban Systems

www.elsevier.com/locate/compenvurbsys

Using moment invariants for classifying shapes on large-scale maps

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Abstract

Automated feature extraction and object recognition are large research areas in the field of image processing and computer vision. Recognition is largely based on the matching of descriptions of shapes. Numerous shape description techniques have been developed, such as scalar features (dimension, area, number of corners etc.), Fourier descriptors and moment invariants. These techniques numerically describe shapes independent of translation, scale and rotation and can be easily applied to topographical data. The applicability of the moment invariants technique to classify objects on large-scale maps is described. From the test data used, moments are fairly reliable at distinguishing certain classes of topographic object. However, their effectiveness will increase when fused with the results of other techniques.

Keywords: Moment invariants; Fourier descriptors; Large-scale maps; Cartographic data

1. Introduction

Automatic structuring (feature coding and object recognition) of topographic data, such as that derived from air survey or raster scanning large-scale paper maps, requires the classification of objects such as buildings, roads, rivers, fields and railways. Shape and context are the main attributes used by humans for this task. Our project combines shape recognition techniques developed for computer vision and contextual models derived from statistical language theory to recognise objects. This paper describes a measurement of shape to characterise features that will then be used as input into contextual models based on structure and statistics.

The technology to capture paper-based cartographic data through scanning is well founded and the production of raster data relatively easy. The vectorisation of raster

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PII: S0198-9715(00)00041-7

data, although not perfect, is also widespread in mapping organisations although it usually requires user intervention to ensure the quality of data. Vectorisation produces vector graphical data but most applications require the data to be structured so it models not only the geometry and topology but also logical contents, often stored as a set of attributes attached to the geometry. These attributes are usually captured manually by a human operator but this process of classifying and entering attributes can be a severe bottleneck in the production flow. This can result in both a scarcity of suitable searchable data and/or sparseness in its accuracy and detail. Automation of the recognition of objects is the obvious solution, but this is a complex problem.

Feature extraction and object recognition are large research areas in the field of image processing and computer vision. Recognition is largely based on the matching of descriptions of shapes. Numerous shape description techniques have been developed, such as the analysis of scalar features (dimensions, area, number of corners and so on), Fourier descriptors, moment invariants and boundary chain coding. These techniques are well understood when applied to images and have been developed to describe shapes irrespective of position, orientation and scale. They can be easily applied to vector graphical shapes (Winstanley, 1998). Experiments carried out to date include the application of scalar and Fourier descriptors as features of shape description and recognition. It is envisaged that these methods can be combined with moment invariants and other techniques of object recognition to produce an optimal result for the problem of shape description of general cartographic shapes on maps. This paper describes experiments that apply moment invariants to the problem.

2. Moment invariants

2.1. Background

Calculating and comparing the moment invariants of the shape of a feature is a well-established technique in image processing for recognition and classification. The invariant values model numerically the characteristics of an object that uniquely represent its shape (Winstanley, 1997). Invariant shape recognition is performed by classification in the multidimensional moment invariant feature space. Several techniques have been developed that derive invariant features from moments for object recognition and representation (Belkasim et al., 1991). These techniques are distinguished by their moment definition, the type of data exploited and the method for deriving invariant values from the image moments. It was Hu (Hu, 1962), that first set out the mathematical foundation for two-dimensional moment invariants and demonstrated their applications to shape recognition. They were first applied to aircraft shapes and were shown to be quick and reliable (Dudani, Breeding & McGhee, 1977). The values calculated are invariant with respect to translation, scale and rotation of the shape.

The moment of an object measures the distribution of its mass relative to the origin of some co-ordinate system. This origin is conventionally normalised to be the centre of mass of the object. In the physical world, the moment gives a measure of the object's propensity to rotate around an axis through this centre of mass. We can calculate different orders of moments, the second order moments measuring the spread of mass in an object. Zero and first order moments specify the total mass and centre of gravity of the object. Higher-order moments do not have simple interpretations although they do model aspects of the object's physical shape. The theory of moments can be applied in image processing by substituting the distribution of pixels depicting the object for mass.

Hu devised moment invariants — a set of seven values calculated from an object's moments to describe its shape. They are computed from central moments through order three and are independent to object translation, scale and orientation. Translation invariance is achieved by computing moments that are normalised with respect to the centre of gravity so that the centre of mass of the distribution is at the origin (central moments). Size invariant moments are derived from algebraic invariants, but these can be shown to be the result of a simple size normalisation. From the second and third order values of the normalised central moments, a set of seven invariant moments can be computed which are independent of rotation.

2.2. Theory

Traditionally, moment invariants are computed based on the information provided by both the shape boundary and its interior region (Hu, 1962, Prokop & Reeves, 1992). The moments used to construct the invariants are defined for the continuous domain but for practical implementation they are computed in the discrete form. Given a function *f(x,y)*, these regular moments are defined by:

$$M_{pq} = \int\int x^p y^q f(x,y)dxdy \tag{1}$$

M_{pq} is the two-dimensional moment of the function *f(x,y)*. The order of the moment is $(p+q)$ where p and q are both natural numbers. For implementations in discrete form this becomes:

$$M_{pq} = \sum_x \sum_y x^p y^q f(x,y) \tag{2}$$

To normalise for translation in the image plane, the image centroids are used to define the central moments. The co-ordinates of the centre of gravity of the image are calculated using Eq. (2) and are given by:

$$\overline{x} = \frac{M_{10}}{M_{00}} \quad \overline{y} = \frac{M_{01}}{M_{00}} \tag{3}$$

The central moments can then be defined in their discrete representation as:

$$\mu_{pq} = \sum_x \sum_y (x - \overline{x})^p (y - \overline{y})^q \tag{4}$$

The moments are further normalised for the effects of change of scale using the following formula:

$$\eta_{pq} = \mu_{pq}/\mu_{00}^{y} \tag{5}$$

where the normalisation factor: $\gamma = (p + q/2) + 1$. From the normalised central moments, a set of seven invariant values can be calculated and are defined by:

$$\begin{aligned}
\phi_1 &= \eta_{20} + \eta_{02}\\
\phi_2 &= (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2\\
\phi_3 &= (\eta_{30} - 3\eta_{12})^2 + (\eta_{03} + \eta_{21})^2\\
\phi_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{03} + \eta_{21})^2\\
\phi_5 &= (3\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right] + (3\eta_{21} - \eta_{03})\\
&\quad \times \left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]\\
\phi_6 &= (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})\\
\phi_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right]\\
&\quad + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03}) \times \left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{30})^2\right]
\end{aligned} \tag{6}$$

These seven invariant moments, ϕ_I, $1 \leqslant \mathrm{I} \leqslant 7$, set out by Hu, were additionally shown to be independent of rotation. The drawback for vector representations of shape is that they are computed over the shape boundary *and* its interior region.

2.3. New moments

When dealing with shape recognition of objects on maps we are dealing with objects in isolation, where we only know information about the outline of the shape. For this purpose, the moment invariants used in this paper are computed using the shape boundary only (Chaur-Chin Chen, 1993). In this case, using the same notation as above, the moment definition in Eq. (1) can be expressed as:

$$M_{pq} = \int_C x^p y^q \, ds \tag{7}$$

for p, $q = 0, 1, 2, 3$, where $\int_C$ is the line integral along the curve C and $ds = \sqrt{((dx)^2 + (dy)^2)}$. The central moments can be similarly defined as:

$$\mu_{pq} = \int_C (x - \overline{x})^p (y - \overline{y})^q \, ds \tag{8}$$

Given that the centroids are as in the regular method:

$$\overline{X} = \frac{M_{10}}{M_{00}} \quad \overline{y} = \frac{M_{01}}{M_{00}} \tag{9}$$

For a digital image, Eq. (8) becomes

$$\mu_{pq} = \sum_{(x,y)\in C} (x - \overline{x})^p (y - \overline{y})^q \tag{10}$$

Thus the central moments are invariant to translation. These new central moments can also be normalised such that they are invariant to changes of scale.

$$\eta_{pq}\mu_{pq}/\mu_{00}^{y} \tag{11}$$

where the normalisation factor is: $\gamma = p + q + 1$. The seven moment invariant values can then be calculated as before using the results obtained from the computation of Eqs. (7–11) above.

3. Moment invariants applied to topographic data

The recognition and description of objects plays a central role in automatic shape analysis for computer vision and it is one of the most familiar and fundamental problems in pattern recognition. Common examples are the reading of alphabetic characters in text (Dehghan & Faez, 1997) and the automatic identification of aircraft (Dudani, Breeding & McGhee, 1997). Most applications using moment invariants for shape recognition deal with the classification of such definite shapes. To identify topographic objects each of the techniques needs to be extended to deal with general categories of shape, for example those depicting houses, parcels and roads.

The data used for the experiments described in the following sections was extracted from vector data sets (NTF level 2) representing large-scale (1:1250) plans of southern England and the Isle of Man (Kelly & Hilder, 1998), an example of which can be seen in Fig. 1. A pre-processing operation was required to extract closed polygons from lines with the same feature codes. After extracting the required polygonal data from the maps, an interpolation method was applied to sample the shape boundary at a finite number (n) of equi-distant points. These points represent the x and y co-ordinates of the polygonal shape, are then stored, and then processed by a moment transformation on the outline of the shape. This produces seven moment invariant values that are normalised with respect to change of size (scale), change of position (translation) and change of orientation (rotation) and can be used to discriminate between shapes.

Given two sets of moment invariant values, how do we measure their degree of similarity? An appropriate classification procedure is necessary if unknown shapes are to be compared to a library of known shapes. The moment invariant implementation

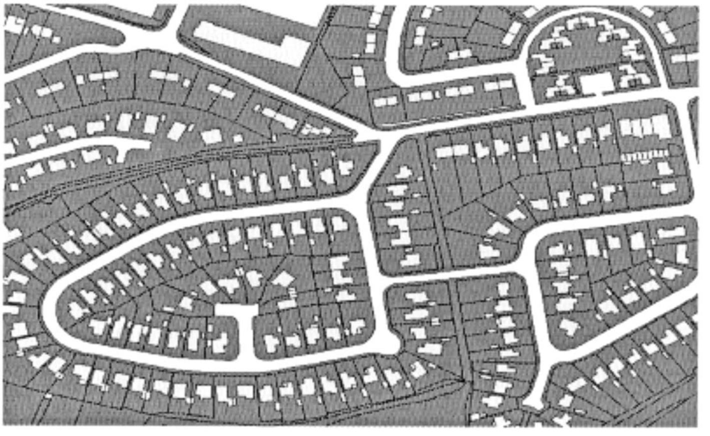

Fig. 1. Section of a 1:1250 plan, shading representing object classes.

produced sets of real values. If two shapes, A and B produce a set of values represented by $a(i)$ and $b(i)$ then the distance between them can be given as $c(i) = a(i)-b(i)$. If $a(i)$ and $b(i)$ are identical then $c(i)$ will be zero. If they are different then the magnitudes of the coefficients in $c(i)$ will give a reasonable measure of difference enabling discrimination between shapes. It proves more convenient to have one value to represent this rather than a set of values that make up $c(i)$. The easiest way is to treat $c(i)$ as a vector in a multi-dimensional space, in which case its length, which represents the distance between the planes, is given by the square root of the sum of the squares of the elements of $c(i)$.

4. Results

Below, a sample of the results produced by the application of the moment invariants technique is evaluated for shape discrimination between general classes of cartographic feature. Fig. 2 plots the average values obtained for five categories of objects from the sample maps. In order to classify shapes with any degree of certainty, the variation within classes must be less than that between classes.

To evaluate moments as a shape recognition technique, several shapes from the map (buildings, parcels and roads) were used as test images. As an example, Figs. 3 and 4 show, respectively, building and parcels on a portion of one map. The moment invariants are computed from the equally spaced (x, y) points (512 sample points) along the boundary of each test shape using the formulae derived earlier. The following table is an example of a set of seven invariant moments (IM) obtained for a house and parcel shape (starting at index IM(0) Table 1).

The moment invariants were calculated for three types of feature, namely buildings, parcels and roads in six different sub-categories used in Ordnance Survey large-scale data-sets:

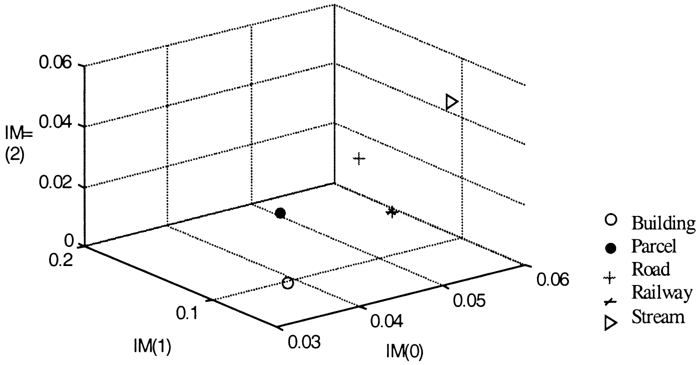

Fig. 2. Average moment invariants (IM) of five sample shapes.

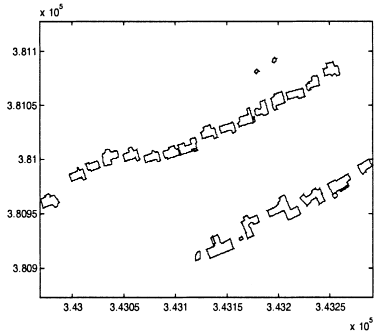

Fig. 3. Sample data representing house shapes.

1. buildings;
2. defined natural land-cover;
3. multiple surface land;
4. general unmade-land;
5. made-road; and
6. roadside.

Fig. 5 shows a plot of the mean values for each of the above named categories in three-dimensional space. The results obtained for each full data set were plotted using the invariants (IM(0), IM(1), IM(2)) to observe how well they formed separate groups. Fig. 6 shows the degree to which three of these data sets (unmade-land, surface land and buildings) cluster in (IM(0), IM(1), IM(2)) space.

Fig. 7 (left) shows the degree to which the data sets, building and defined land cover cluster and also in Fig. 7 (right), a cluster plot of the data sets, defined land cover and unmade-land. In Fig. 8 it can be seen how the features buildings and roads separate when plotted.

To analyse the results obtained, the repeatability function and mean value measurements were computed for each set or the sample shapes. The results can be seen

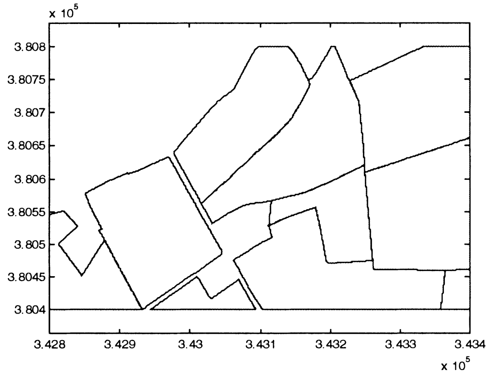

Fig. 4. Sample parcel shapes.

Table 1
Moment invariants calculated for a typical building, road and parcel

	Buildings	Roads	Parcels
IM(0)	0.00021913563	0.0191903068	0.19419031
IM(1)	1.4175713e-08	0.0028776518	0.0093515524
IM(2)	3.3163274e-12	0.0000022101	0.00055687797
IM(3)	7.332081e-14	0.0000002565	1.0685037e-05
IM(4)	2.4223892e-14	0.0000001930	5.696268e-05
IM(5)	−7.51903311e-18	−3.7718e-08	−6.2343667e-07
IM(6)	2.12921403e-26	−1.5393e-14	3.212549e-11

in Table 2. Only the first moment invariants measure, MI(0) is used here for clarity; it is the most significant invariant (Wang, Dayong, & Xie, 1996).

The values show that there is a significant separation between most of classes. Although overlap does exist (also seen through visual inspection of the plots), good classification occurs. In Table 2, the repeatability function for each class is represented by three times the standard deviation and can be seen in the shaded diagonal column of the table. All other values in the table represent the mean measurement between classes. On examining Table 2 more closely, it can be seen that the repeatability for the buildings is smaller than the distance between the mean values for all categories except for the surface-land data set, though these values are close. This is also true for the repeatability measure for the surface-land class; the distance between the mean values is larger except for buildings. Comparing the figures obtained for the other data sets it can be seen that repeatability is large than, but still close to, the mean distance in most cases.

In previous work, similar experiments were conducted using scalar and Fourier Descriptors (Keyes & Winstanley, 1999). Table 3 shows the values obtained for a sample of buildings and land parcels using the Fourier Descriptors. Here the repeatability of the measurements of the class is sizeably larger then the distance

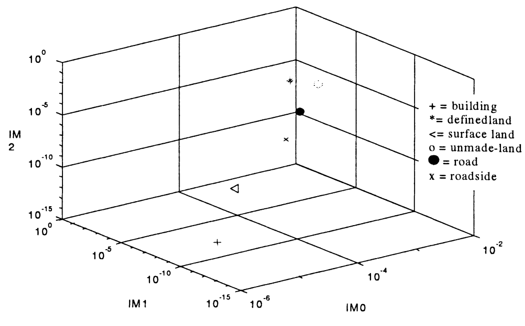

Fig. 5. Average moment invariants (IM) of six shape categories.

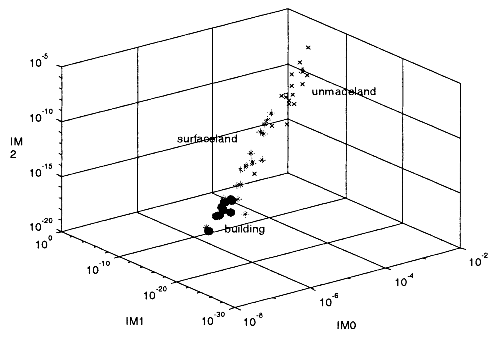

Fig. 6. Clustering of values for different feature classes of the invariants IM(0), IM(1) and IM(2).

between the mean values for the two classes indicating that Fourier descriptors are not very good for use in shape description where the data sets are of shapes in general classes. Similarly, in Table 4, the repeatability function is calculated for the data sets using the scalar descriptors. These results show that the distance between the means for the buildings is considerably larger than the repeatability of that class but smaller for the parcel class. This technique also shows considerable improvement over the Fourier descriptors but follows closely to the results obtained for moment invariants.

5. Conclusion

As a shape descriptor technique, moment invariants have been shown to be very good discriminators when dealing with specific shapes such as aircraft or alphanumeric characters (Hu, 1962). The aim of this paper was to investigate their usefulness for the identification of general topographic shape classes, e.g. houses, roads and parcels. When tested for these more generalised shapes, moment invariants seem to work. There is good distinction between classes although some overlap occurs. This indicates that moment invariants alone are not sufficient.

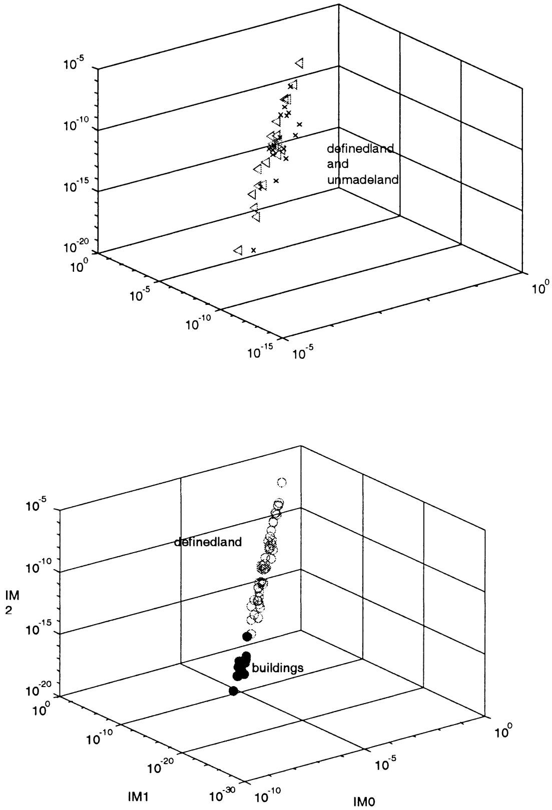

Fig. 7. Clustering of invariant values for different feature classes, (left) *buildings* and *defined land cover*, (right) *defined land cover* and *unmade-land* using IM(0), IM(1) and IM(2).

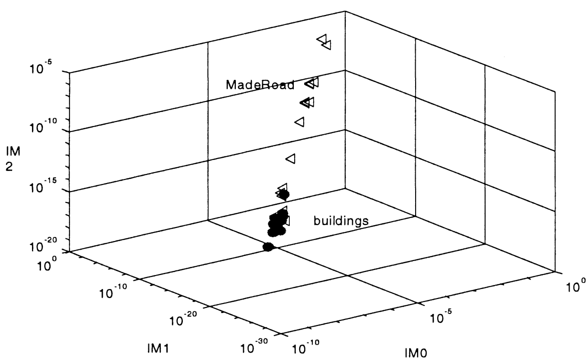

Fig. 8. Clustering of the polygon shapes, *buildings* and *made roads*, in three-dimensional space of the features IM(0), IM(1) and IM(2).

Table 2
Comparison of repeatability within feature classes (italic figures) and distance between classes

	Buildings	Defined land	Surface land	Unmade-land	Made Road	Roadside
No. polygons	*7976*	*6332*	*2889*	*2701*	*487*	*431*
Buildings	*5.2005e-005*	8.8572e-004	1.5488e-005	0.0034	0.0014	4.8116e-004
Defined land	8.8572e-004	*0.0138*	8.7023e-004	0.0025	5.5596e-004	4.0456e-004
Surface land	1.5488e-005	8.7023e-004	*3.9330e-004*	0.0033	0.0014	4.6567e-004
Unmade land	0.0034	0.0025	0.0033	*0.0231*	0.0019	0.0029
Made Road	0.0014	5.5596e-004	0.0014	0.0019	*0.0188*	9.6051e-004
Roadside	4.8116e-004	4.0456e-004	4.6567e-004	0.0029	9.6051e-004	*0.0048*

Table 3
Comparison of repeatability within feature classes and distance between classes for Fourier descriptors

	Buildings	Land Parcels
Repeatability (3σ)	FD(2) = 0.2562	FD(2) = 0.2814 FD (3) = 0.1644 FD(4) = 0.1200
	FD(3) = 0.2457 FD(4) = 0.2100	
Distance between means for buildings and parcels	FD(2) = 0.0067 FD(3) = 0.0123 FD(4) = 0.0137	

Table 4
Comparison of repeatability within feature classes and distance between classes for scalar descriptors

	Buildings	Land parcels
Repeatability (3σ)	Area = 906.8734 Perim = 121.2972 Points = 11.7001	Area = 159780.0 Perim = 1915.6 Points = 95.7411
Distance between means for buildings and parcels	Area = 38231.0 Perim = 587.4117 Points = 37.8071	

For optimal results, it is envisaged that moment invariants will be combined with other techniques currently being investigated. These include Fourier descriptors, scalar descriptors and boundary chain coding. All these techniques are looking at object shapes in isolation. Context is therefore an obvious next step to consider. The context of an object can be modelled by:

1. direct association between shapes;
2. statistical graphical language models built from a large corpus; and
3. analogical reasoning about context.

Future work will be to combine some or all the methods mentioned using data fusion techniques to produce a more reliable topographic object recognition system.

Acknowledgements

The work described in this paper is supported by Enterprise Ireland's Strategic Research Grants scheme, proposal number ST/98/021. The Isle of Man government and Ordnance Survey in Southampton loaned the topographic data.

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