Examining the Perceptions of Able Mathematicians to Mathematics Teaching and Learning in Post-Primary Schools in Ireland.

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Declaration

I hereby declare that this thesis, which I now submit for assessment on the programme of study leading to the award of Doctor of Philosophy is entirely my own work, that I have exercised reasonable care to ensure that the work is original, and does not to the best of my knowledge breach any law of copyright, and has not been taken from the work of others save and to the extent that such work has been cited and acknowledged within the text of my work.

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Abstract

This thesis examines the perceptions of able mathematicians to their experience of mathematics teaching and learning in post-primary schools in Ireland. Able mathematicians are defined in this study as students who have done well in assessments but also students who have displayed an interest in the subject.

International research has shown that the classroom experience is fundamental in the development of students' mathematical understanding, motivation, and identity. Despite popular myths, able mathematicians have specific needs that require differentiated instruction in the classroom. It has been suggested that giving students opportunities to experience creativity is a means of providing the challenges they need. This research aimed to investigate how able mathematicians in Ireland feel about their mathematics classroom experience, whether they feel challenged in class, and how they would respond to being given opportunities to explore creative tasks.

The research design was based on the framework of Sriraman (2005), which advocated five principles to maximise creativity for students in the classroom. The first of these was engagement in the four-stage creativity process of the Gestalt psychology principle of initiation-incubation-illumination-verification. The remaining four principles recommend that students are given opportunities to experience uncertainty, the aesthetical beauty of mathematics, the freedom to think and take risks, and the scholarly aspect of mathematics.

Data was gathered from 92 Transition Year students in five second level schools in Ireland. The students participated in a series of two specially designed workshops in which they were presented with unseen tasks with multiple solutions. The study design included surveys to gather data on the participants' perceptions of their mathematics classroom experience, and to their workshop experiences. Sriraman's Principles were used for the design of a template to analyse interviews with the students and audio recordings of the workshops.

The analysis revealed that there is strong evidence to suggest that the opportunities to be creative are not being maximised in classrooms. There appears to be considerable rote learning and memorisation of algorithms, with little opportunity to have the freedom to think and explore mathematical tasks. In contrast, the workshops were shown to have given the particpants opportunities to experience Sriraman's five principles for creativity.

Table of Contents

List of Figures

List of Tables

List of abbreviations

CMR	Creative Mathematically founded Reasoning
CR	Creative Reasoning
IR	Imitative Reasoning
MST	Multi-Solution Tasks
NCCA	National Council for Curriculum and Assessment
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for Economic Co-Operation and Development
PISA	Programme for International Student Assessment
SREC	Social Research Ethics Sub-Committee
TDS	Brousseau's Theory of Didactical Situations in Mathematics
TIMSS	*Trends in International Mathematics and Science Study*
TY	Transition Year
ZPD	Zone of Proximal Development

Chapter 1 Introduction

1.1 Introduction

This study is concerned with second-level students who have displayed a high level of mathematical ability or motivation. It aims to examine whether these students feel challenged in post-primary schools in Ireland and what can be done, within a diverse classroom, to maximise their opportunities for challenge and enjoyment. The term diverse is used, rather than mixed ability, because it takes into account the varied ability there often is even within classes that have been divided according to ability. The term 'able mathematicians' was chosen to define the participants in this study, where 'able' refers not only to students who have done well in assessments but also to students who have displayed an interest in the subject.

This decision was in support of research which argues for the need to broaden the parameters for classifying "exceptionally able and talented" mathematicians (Reiss & Renzulli, 2010; Tomlinson et al., 2003; Sheffield, 2003). An important area of the research will be investigating what it is that motivates able mathematicians, and how to provide these students with opportunities to maximise their potential. This area of mathematics education in Ireland is, to date, under-researched, which makes it an important topic to investigate. There has been recent research on attitudes towards gifted mathematicians (Cross et al., 2018), mathematical assessment tasks given to higher-level students (O'Connor et al., 2019) and improving the problem-solving potential of high-ability students (Fitzsimons, 2021). However, there has been little research on the perceptions of the students themselves to their mathematical experience, and whether they feel there is anything they would change that would enhance their learning environment. In addition to those who have been identified as high-ability students, the intention was to give students, who may not achieve highly in traditional assessments, an opportunity to voice their opinion on their mathematical experiences and preferences.

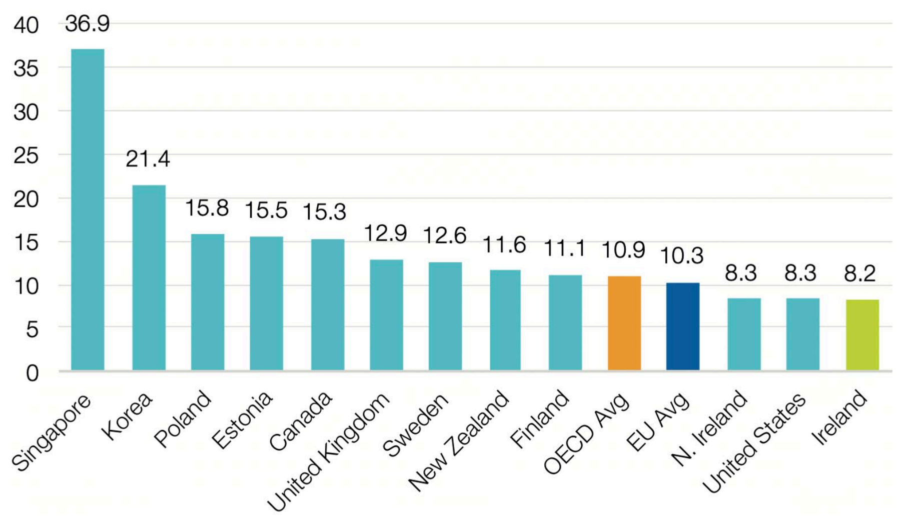
1.2 The Irish situation

In 1993, the Report of the Special Education Review Committee in Ireland (SERC, 1993) outlined best practice for students who were ‘exceptionally able and talented’ in a number of areas including ‘specific academic aptitude’ and ‘creative productive thinking’. Following this, the Education Act of 1998 recommended that the needs of all students, including those with special educational needs, must be identified and provided for (Education Act, 1998). It specifically defined ‘special educational needs’ to include the exceptionally able students. However, acts that followed, such as the Education for Persons with Special Educational Needs Act, (EPSEN, 2004), did not recognise the specific needs of highly able students and no additional supports were thus provided in schools.

In recent years there has been much international concern about the level of conceptual understanding of mathematics in students of all ages (De Corte et al., 1996; NCTM, 1980). Such reports prompted further research by Conway & Sloane (2006) into international trends in mathematics education which included a drive to find ways to enhance the Irish curriculum to give students a richer learning experience (NCCA, 2005). In 2010, the Project Maths curriculum reform was introduced into Irish schools at both Junior and Senior Cycle levels. The new syllabus placed much emphasis on teaching for conceptual understanding, reasoning, justification, and problem solving (NCCA, 2015) which is in line with the recommendations laid out in the aforementioned SERC report. The syllabus recommends differentiated teaching, but the level to which this is integrated into the classroom is dependent upon individual schools and teachers. Teachers have often cited the lack of guidance and resources as a reason for why they do not use differentiated instruction for high-ability students (Scager et al., 2014). To date, there has been limited research focused on how to specifically challenge able mathematicians, and this current study aims to contribute towards highlighting the diverse needs of these students and thus bridging the gap in research.

International assessments have shown that there are areas for improvement for mathematics students in post-primary schools in Ireland. It was seen in the 2015 TIMSS report that the students in Ireland ranked 27th internationally for experiencing 'very engaging teaching' and had the 8th highest ranking for less than engaging teaching (Mullis et, al., 2015). The PISA 2015 statistics revealed that, despite being ranked well above the OECD average overall, only 9.8% of Irish students performed at the highest proficiency levels five and six, which was below the OECD average of 10.7% (OECD, 2016). PISA grades their tasks from level one to level six, according to the level of difficulty, level six being the most difficult items on the assessment. The proficiencies in level five and six include reasoning, working strategically, communicating, applying insight and understanding, moving flexibly between representations, generalising and conceptualising. The results of PISA 2018 (see Figure 1 showed similar percentages with 8.2 % of Irish students performing at levels five and six compared with an OECD average of 10.9% (OECD, 2019). Similarly, the TIMSS statistics, of both 1995 and 2015, imply that Ireland is below par in the cognitive domains of applying and reasoning (Mullis et, al., 2019).

Percentage of students performing at or above Proficiency Level 5 in mathemaatics in Ireland, in selected comparision countries, and on average across OECD and EU countries



Note: Reproduced from PISA 2018, Learning for the future

Figure 1 Student Performance on Mathematical Literacy, PISA 2018

1.3 Motivation for research

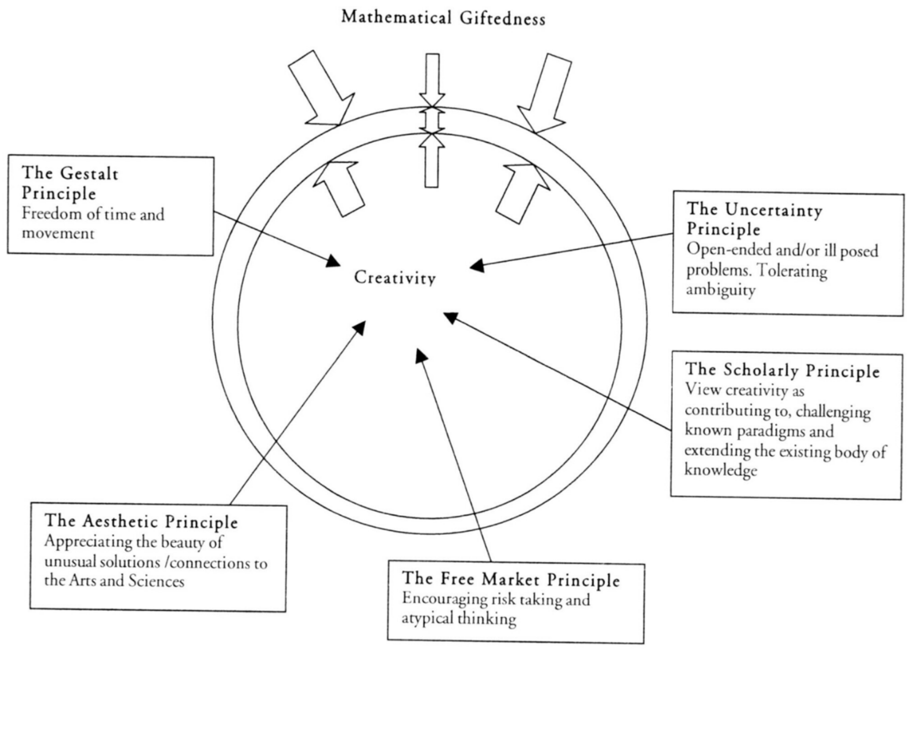
The mathematics classroom experience has been shown to be crucial in the development of students' mathematical understanding, motivation, and identity. International research has shown that students base their mathematical beliefs on their exposure to mathematics in the classroom (Brousseau, 1997; Nosrati & Andrews, 2022; Lampert, 1990) and this can have many limitations on their interest, self-perception, and success in the subject (Schoenfeld, 2016; Boaler, 2009; Kanevsky & Keighly, 2003). By examining the perceptions of able mathematicians towards their exposure to mathematics, it is hoped to gain an insight into what these students may need in order to feel stimulated. Given the lack of research in this area second-level mathematics education in Ireland could benefit from exploring this topic.

1.3.1 Research literature on creativity

The motivation for this research design came from a number of sources which all advocate the importance of creativity for challenging able mathematicians. Most influential were the works of Vygotsky (1978), Sheffield (2003, 2017), Sriraman (2005) and Leikin (2009, 2013, 2017), each of which will be outlined below and explore in more detail in Chapter 2. Vygotsky's work emphasised the need to provide opportunities for students to experience their zone of proximal development (ZPD) in order to reach their potential. His theories propose that exploring mathematical tasks that push students into their ZPD, provide that essential link between constructing new knowledge from previously known knowledge that is fundamental in the learning process. The findings of Sheffield were specifically focused on how to challenge 'promising mathematicians' and how these students are often neglected in schools. Her article titled 'Dangerous myths about "gifted" mathematics students' (Sheffield, 2017) raised some very important issues that the researcher believed to be highly significant for Irish post-primary education and therefore wished to explore. Of particular interest was the belief that promising students can teach themselves and so do not need special attention in class. Sriraman (2005) recommended ways in which teachers could adapt their methodology in order to foster creativity in the classroom. Sriraman's five principles to maximise creativity were used as a framework for the research and these will be outlined in paragraph 1.3.2 below. Leikin's (2009, 2013, 2017) work on multi-solution tasks and their role in acting as a tool for exploring and assessing creativity, provided a means by which the researcher could examine how the able mathematicians in the study might be challenged. The key message from all of the above research is that creativity is an essential part of mathematics teaching and learning for all students, and particularly instrumental for challenging able mathematicians.

1.3.2 Sriraman's five principles to maximize creativity

Sriraman's (2005) five principles were designed to show that creativity should not only be seen as a characteristic associated with professional mathematicians but could also be relevant for second-level mathematics. He recommended five principles that should be incorporated into teaching to maximise the students' creativity (See Figure 2). He labelled the principles the Gestalt Principle, the Uncertainty Principle, the Aesthetic Principle, the Free-Market Principle, and the Scholarly Principle. Each of these principles will be explained in more detail in the literature review in Chapter 2. The key features will be outlined here as a background for the rationale behind the research questions given in section 1.4 below.



Note. This is reproduced from Sriraman (2005).

Figure 2 Harmonizing creativity and giftedness at upper second level

The first of the five principles requires students to be given the opportunity to engage in the four-stage creativity process of the Gestalt psychology principle: initiation-incubation-illumination-verification (Kaffka, 1924). Of these four stages, the most important stage with regard to creativity is the incubation stage when the mind subconsciously reaches a solution or solution strategy. In the second principle, Sriraman emphasised the importance of exposing students to uncertainty in the classroom and encouraging them to explore methods without instruction. The Aesthetic Principle was concerned with appreciating the beauty of mathematics and of unusual solutions. The classroom atmosphere was essential for all of the above to happen and was incorporated into the Free Market Principle, which centred upon creating an environment that encourages student discussion. Finally, the Scholarly Principle was associated with the importance of learning and making a contribution to mathematics. These principles mirrored the ZPD environment suggested by Vygotsky and also identified specific classroom features that the researcher has used to form the structure upon which the data could be analysed.

In Chapter 3, it will be explained in detail how the five principles were used as a framework for designing the survey questionnaires, for coding the interviews and for analysing the audio data collected from the workshops. In his more recent work, Sriraman (2021) further explored the importance of uncertainty for providing creative opportunities and support for this will be seen to be embedded in each aspect of the methodology.

1.4 Research Questions

The results from international assessments such as PISA and TIMSS suggest that Ireland could do more to develop the talents of our able mathematicians at second level. In the study, the researcher wanted to gather data on the classroom experience of these students and their perceptions of mathematics and mathematics education. The aim was to find out the extent to which able mathematics students in Ireland encountered challenge as well as opportunities for developing creativity, given the importance of these ideas in the research literature. In addition, the researcher wanted to gather data on students' reactions to working on multi-solution tasks in specially designed workshops. Of particular interest was investigating if these types of tasks could be used to provide opportunities for enhancing creativity in second-level classrooms.

The research questions addressed in this study are:

1 What is the perception of able students in post-primary schools in Ireland towards their mathematics classes?
2 To what degree do these students feel challenged by the experiences they encounter in the mathematics classroom?
3 What comparison can be made between the opportunities these students had, in school and in the workshops, to experience each of Sriraman's five principles to maximise creativity.

The first research question seeks to examine how the students perceive their mathematics class, and what it was they enjoyed, or disliked, about school mathematics. The methodology was accordingly designed to probe a variety of common classroom features that could then be compared with the experience these students had in the workshops. The second research question explores whether we could be doing more to challenge highly able students in school, to help them reach their potential. In this aspect the researcher was motivated by reading the results of international assessments such as PISA (OECD, 2016, 2019) and TIMSS (Mullis et, al., 2015, 2019), which inspired an examination of classroom practises that may hinder or support students' mathematical proficiency. Further details on the influence of these assessments are outlined in the paragraph below. The third research question aimed to find out what comparisons the students made between their experience in school and in the workshops, which were specifically designed to provide the opportunities for creativity advocated by Sriraman.

1.5 Preliminary International Research

Given that the focus was on enhancing the experience of able mathematicians, a considerable part of the background research was spent investigating what was being done in other countries to challenge such students. In particular, the researcher was interested in countries where classroom diversity was most predominant, such as is a traditional feature of the Nordic model of education (Lundahl, 2016). The rationale for this was that useful teaching methodologies could be transferred into Irish schools where special needs facilities for highly able students were not the norm. A key influential factor was the research done by NRICH in the United Kingdom (www.maths.org) and by Mellroth (2008) in Sweden on the benefits of multi-solution tasks (MST) in diverse classrooms. The intended purpose of MST was to provide differentiation within a classroom where all students could work on the same task, but each could advance the level of difficulty to suit their individual ability. The important role MST play in the education of able mathematicians will be examined in more detail in Chapter 2.

An additional interesting feature of mathematics education in these countries was that both Sweden and the United Kingdom had a higher percentage of students performing above the OECD average in proficiency levels five and six in PISA 2018. Before deciding on the workshop design the researcher travelled to visit two schools in Sweden to discuss policies they had for challenging high-ability students. Following this, a meeting was organised with the NRICH team in Cambridge to discuss multi-solution tasks they had designed to provide differentiation in diverse classrooms. The third international trip undertaken was to Trondheim in Norway to experience the work of the Norwegian Centre for Mathematics Education. The centre is part of the Norwegian University of Science and Technology (NTNU) and specialises in methods and learning strategies for mathematics education at all levels. Of particular interest was their website (www.mattelist.no) which was designed to provide teachers with resources and students with opportunities for independent learning. Designing and collating tasks that would include specific content from the syllabus in Ireland was part of the long term plan, if the findings of the research showed that able mathematicians were motivated by independent learning. Hence, these visits were intended to inspire the researcher for the task selection phase of the workshop planning which will be detailed in Chapter 3.

1.6 Theoretical Framework

The research is guided by Vygotsky's Sociocultural Theory of Cognitive Development and Sriraman's Five Principles to Maximize Creativity (Figure 2). These frameworks form the structure upon which the study was designed. A central theme in both of these frameworks was giving students opportunities to think and to collaborate with their peers. By incorporating these into the theoretical framework it was hoped to create a learning environment conducive to answering the research questions. The aim was to investigate how challenged able mathematicians feel in Irish classrooms and what impact creating an environment based on Sriraman's five principles can have on their motivation, enjoyment and perseverance when solving unfamiliar tasks.

1.6.1 Vygotsky's Sociocultural Theory of Cognitive Development

Much of Sriraman's theories parallel a fundamental construct of Vygotsky's Sociocultural Theory which were also integrated into my framework. Vygotsky believed that creativity is the key to cognitive development because it enables students to construct their own knowledge through collaboration with others. He suggested that "development processes do not coincide with learning processes. Rather, the development process lags behind the learning process" (Vygotsky, 1978, p.90). For this reason, student learning should be in what he called 'zones of proximal development' (ZPD) where students work on more advanced tasks than those they currently feel comfortable with. Collaboration and exposure to tasks in the ZPD are considered critical in enabling students to reach their mathematical potential. This concept of the benefits of the ZPD for students was an instrumental factor in the design of the workshops. It will be shown in section 3.5 how creating an environment that was unfamiliar for the students was built into both the workshop structure and the tasks presented to the students.

1.7 Overview of research design

In order to address the research questions above, the researcher designed: a survey to gather data on students' perceptions of their mathematics classroom experience; a series of workshops to expose students to working on multi-solution tasks; a survey to gather information on students' workshop experiences; an interview protocol to investigate students' opinions on both their classrooms and the workshops. Work was carried out with groups of students from five different schools.

A series of workshops were designed through which the researcher could examine the perceptions of able mathematicians towards creative problem solving. The tasks selected can be found in Appendix B and will be described further in chapter 3. The workshops were designed to push the students into areas of uncertainty and encourage them to question, explore and discuss mathematics with each other. The intention was that the workshops would provide them with an experience that differed to that of their mathematics classes in school so that their perceptions could be compared. The research was carried out with a group of 92 post-primary Transition Year students, previously identified as highly able, or highly motivated, by their teachers. Details of the selection of students and the ethical approval required will be given in the research design in Chapter 3. Data on the student perceptions of school and the workshops was collected via two Likert scale surveys, a series of interviews, audio recordings of their collaboration in the workshops and worksheets of their solution methods to the tasks presented. An outline of the data collection and analysis carried out will be given below then each will be examined in more detail in chapters 5 to 7.

The surveys and interview questions were designed by the researcher to probe student perceptions towards their experiences. A combination of the researcher's own experience as a classroom teacher for 25 years, research literature (Cohen et al., 2018; Robson, 2011) and questionnaires designed by other researchers (Kloosterman & Stage, 1992; Lim & Chapman, 2013) were used to design the surveys. The students were given a pre-workshop survey based on questions that explored features of their experiences in school, such as: enjoyment of mathematics, their confidence in their own ability, the level of challenge they experience in class, and the types of mathematical problems that motivate them most. After the workshops they were given a second survey to explore their experiences in the workshops. The questions were grouped into scales according to the above features of their mathematical experiences and analysed the responses to each scale using Rasch analysis (Bond & Fox, 2007). A copy of the questions and scales for each survey can be found in Appendices D to G, along with a note explaining which questions were taken from other researchers' questionnaires, which are in Appendix C.

After the workshops 11 group interviews with 30 students in total were held. Each of the questions for the interviews were designed based on recommendations in the literature on how to conduct interviews (Brinkmann & Kvale, 2015; Cohen et al., 2018; King et al., 2019; Robson, 2011) and the researchers own experiences of what features may be important for students of this age (see Appendix H). The purpose of the interviews was to give the students an opportunity to express themselves more openly than what is feasible when answering a Likert scale survey. It was decided to use Template Analysis to analyse the interviews and Sriraman's five principles to form the templates for the coding. Chapter 5 will detail the rationale for using Template Analysis and how the final template evolved.

Finally, the data from the audio recordings of the workshops and the students answer books provided information on how the students responded to being presented with tasks without being given any guidance by the teacher. Being able to compare the students' experience in school and the workshops was a fundamental part of the research and the audio recordings were a good way to capture the atmosphere of the workshops when the students were engaged in discussion. There has been much research on comparing classroom practices that either promote rote learning or foster a deeper understanding of the mathematical concepts (Lithner, 2008; Schoenfeld, 2016; Skemp, 1978). By removing teacher instruction, the audio recordings also enabled the researcher to examine the students' strategies and their response to being given the freedom to think for themselves.

The research found evidence that the students in this study experience a very different learning environment to what is recommended by research for challenging high-ability students. Many differences between school and the workshops were identified by the students. Their insights into the extent to which they feel challenged, the opportunities they are given to think for themselves and what it is they would like to see in mathematics class are the most striking features of the analysis that will be reported.

1.8 Outline of dissertation

What follows is an outline of each chapter of the thesis:

Chapter Two presents a summary of the literature review informing the research. The literature examines what highly able students need, in terms of instruction, to be challenged. There is a particular focus on the effect of traditional classroom mathematics instruction on high-ability students and the importance of providing them with opportunities for creativity. Particular attention will be paid to the work of Sriraman which was instrumental in the framework for my research. Finally, the chapter will discuss the current situation with regards to mathematics teaching in Ireland and why this study is important in the field of post-primary mathematics education.

Chapter Three justifies and explains the research design for investigating the research questions. It will outline the three sources of data: the surveys, audio recordings from the workshops, and the interviews. The chapter will also include details on how the researcher selected the schools and students, the rationale behind the survey questions and details on the design of the workshops and interview questions. Details of the analysis of each section of data, including why the particular analysis methods were chosen and how they were carried out, will be included in the individual analysis chapters.

Chapter Four presents the findings from the survey data. It will describe how both the pre-workshop survey and the post-workshop survey were analysed using SPSS, a statistics software suite, and Rasch analysis software. In this chapter the analysis of the survey data will be presented and the key findings on the student perceptions of their experience of mathematics in their classrooms, in comparison to that of the workshops, will be described.

Chapter Five describes the rationale for the qualitative analysis of the post-workshop interviews. The choice of Template Analysis will be explained, why it was suitable for this study and how the templates from which the data was coded were developed. In doing this the researcher will illustrate how Sriraman's five principles to maximise creativity were embedded into the templates.

Chapter Six presents findings from the interview analysis. This will include how the data was coded to adhere to the templates representing Sriraman's five principles. The analysis will explain the rationale for each code and how they aligned to each individual principle. The student perceptions of their classroom experience, as discussed in the interviews, will be presented via the templates designed.

Chapter Seven presents findings from the audio recordings of the workshops. The chapter will give details on the most noteworthy discussions that took place within the student working groups. It will relate the student discussions back to the template based on Sriraman's five principles and give evidence of the opportunities the students had in the workshops to maximise their creativity.

Chapter Eight will present an overall discussion of the findings from the surveys, interviews, and workshop audio recordings. It will examine the findings in light of what the research literature recommends for challenging able mathematicians. It will also highlight which findings were felt to be more influential in expressing the perceptions and preferred learning environment of the students in the study.

Chapter Nine will present the conclusion from the research. It will outline the significance of the primary findings and the implications they might have for the future of mathematics education in Ireland. It will make suggestions for what could be implemented into the teaching of highly able mathematicians to enhance their experience. Finally, the chapter will outline any limitations to the study and what further research might advance the work presented in the thesis.

Chapter 2 Literature Review

Introduction

This chapter will outline the work that has been done previously on topics related to this thesis. An able mathematician will be defined in terms of previous work on highly able mathematicians. The chapter will then examine why able mathematicians need to be challenged and how researchers feel they can be challenged, within the classroom. The dangers, perceived by researchers, if such students do not receive adequate challenges will also be examined. It will be seen that, according to previous research, an important aspect of this is providing able mathematicians with opportunities for creativity. This will include looking at features of traditional classroom teaching that have been seen to hinder creative thinking. Finally, the current situation in Ireland will be examined in relation to where the teaching of able mathematicians can be positioned within the research literature outlined.

2.1 Defining an able mathematician

An able mathematician is defined for the purposes of this study as, primarily, a student who has the potential to achieve highly in this subject both in school and thereafter. However, in light of research which has shown that a lack of high achievement in school mathematics assessments does not prevent mathematical accomplishment (Pehkonen, 1997), the term 'able mathematician' in this study also refers to those who have a great interest in mathematics yet may not be high achievers in examinations.

Given the above definition, the literature reviewed here is concerned with what has been said about a wider section of the student population than the research solely on 'gifted' students. Even within those classified as 'gifted', by an IQ test, there is much diversity as is outlined below. In their research on the 'Three-ring conception of giftedness', Reiss and Renzulli, (2018) emphasise the multidimensional aspect of giftedness and that they see giftedness as a trait rather than as a type of person. The ‘Three-ring conception of giftedness’ comprise above average ability, creativity and task commitment. They also give support to the idea that students who are recognised as highly able are a diverse group who may or may not achieve highly in school, some even experiencing significant underachievement.

International research has acknowledged that the identification of ‘exceptionally able’ pupils is arbitrary (Sheffield, 2003; Mann, 2009; Mellroth, 2014) and the 1993 Report of the Special Education Review Committee in Ireland (SERC, 1993) also suggested that ‘adopting a precise cut-off point can have little practical application’. This 1993 Report cautioned that the proportion of the population classified as ‘gifted’ in the literature can be seen to range from 1% to 15%. It also advised of the need to put less emphasis on identifying ‘gifted students’ by IQ scores and more on ensuring that all students are getting the education they deserve. Research literature has recommended that standard assessments of students’ ability should not be the only determining factor in the identification of able mathematicians (Milgram & Hong, 2009; Renzulli, 2005; Sousa, 2009; Sternberg, 2017). Intelligence, personality, and perseverance need consideration as well, given the evidence that there are many high achievers who do not excel in traditional assessments (Mellroth, 2018; Nolte & Pamerien, 2017; Sheffield, 2003). These three characteristics play a key role in in a rapidly changing technological world where mathematics is becoming even more instrumental. ICT has been developed to perform algorithms, but able mathematicians have the potential to play a very important role in the creative changes required in science, business, finance and education in a changing world (Newton et al., 2022; Moholo, 2017; Schoenfeld, 2016).

2.2 Why able mathematicians need challenge

Research literature has revealed that high ability students need to be specifically challenged (Barbeau, 2009; Mann, 2006; Mellroth 2018; Nolte & Pamerien, 2017; Scager et al., (2014); Sheffield, 2003, 2017; Stillman et al., 2009). They possess valuable skills that modern society is in real need of to overcome challenges it faces such as deadly viruses, climate change, and environment and international terrorism (NCTM, 1980; Barbeau & Taylor, 2009). However, defining exactly what we mean by challenge is complex. The Cambridge online dictionary describes challenge as being faced with 'something that needs great mental or physical effort in order to be done successfully and therefore tests a person's ability'. By such a definition all students deserve to be challenged, but the difficulty comes with knowing how to challenge a class which contains students with a range of abilities and how to balance this with other priorities as a teacher.

2.2.1 Characteristics and needs of able mathematicians

Able mathematicians have been seen to possess different characteristics than average ability students and therefore need specific strategies to teach them effectively (Nolte & Pamperien, 2017; Sheffield, 2003; Sousa, 2009). Nolte & Pamerien, in their research on challenging students within the classroom, combine findings from researchers on the characteristics of high ability students. The consensus from this is that these students acquire new information rapidly, can learn from experience, comprehend complex ideas, work more quickly and have an 'enhanced capacity of working memory' (Nolte & Pamerien , p.122). It was also noted that they can recognise patterns, switch between different representations and generalise their findings. Sheffield (2003) refers to 'promising mathematicians' as those who show a mathematical frame of mind. This implies that they can think logically, can generalise mathematical problems, show mathematical creativity, curiosity and perseverance.

In terms of strategies best suited to challenging students with the characteristics noted in the research above, researchers agree on the type of tasks and method of presentation. It has been suggested that high ability students are more suited to inquiry-based learning, with less scaffolding and more real-life contexts (Gruber & Mandl, 2000; Milgram & Hong, 2009; Mc Clure & Piggott, 2007; Sousa, 2009). Scager et al. (2014) found that the three key factors students themselves had described as an important attribute of challenge for them were 'complexity, lack of teacher direction, and high expectations of both teachers and students' (Scager et al., 2014, p.659). The research from Nolte & Pamperien (2017) also advocates that a challenging learning environment is essential for the development of cognitive and emotional skills in high ability students. This environment should allow students to think like practicing mathematicians where they are given the freedom to work independently on tasks. Thus, it should include complex time-consuming problems that provide the possibility of struggle and students should be encouraged to explore further connections once they have solved the problem. The research is summarised by a quote from Leikin et al. (2016):

> "The special characteristics and needs of outstanding and gifted students require a unique learning environment and distinctive study tracks, with respect to pedagogic methods, and appropriate teachers and curricula" (Leikin et al., 2016, p. 119, in Nolte & Pamerien, 2017, p.123).

Sheffield (2003) suggests that challenge is also instrumental in the development of the brain and that mathematics is the ideal discipline in which to address this. She emphasises that promising mathematicians should be asked questions that make them think, not those which make them guess what the teacher is thinking. They should be given rich, time-consuming tasks with multiple solutions to be explored. Given that creativity is identified as one of the key characteristics of mathematically able students (Leikin & Lev, 2013, Sheffield, 2003) it should be an essential element of classroom instruction to provide challenges.

2.2.2 Myths about teaching 'gifted' students

Sheffield (2017) wrote about 'dangerous myths' about teaching 'gifted' students some of which she maintained could "discourage students' mathematical development of creativity, expertise and enjoyment" (Sheffield, 2017, p.13). The first of these myths was that mathematical ability is genetically determined. Sheffield gives her support for Dweck's work, on the 'growth mindset', that proposes mathematical ability is not a fixed entity and that students, and teachers must believe that they can improve with hard work (Dweck, 1986; Mangels et al., 2006). This belief will be discussed further in section 2.3.6.

Another myth stated is that mathematics is not creative, that students see it as a dead subject, driven by rules and procedures (Boaler, 2016). When discussing this myth Sheffield quotes Mann (2006) who warns of the dangers of not providing students with opportunities for creativity in mathematics. Mann suggests that this denies students opportunities to see the beauty of mathematics and to fully develop their talents. Recommendations given to overcome this myth are encouraging students to slow down and take risks with solution strategies and to make mistakes. Rather than worrying that students will remember incorrect answers, teachers need to realise that it has been shown in research that making mistakes deepens the students' enjoyment and understanding of mathematical concepts.

One of the most significant myths with regards to teaching high ability students stated that gifted students can develop on their own. In their article on talent loss, Milgram & Hong (2009) attributed the loss to the "popular prejudice that children will realise their potential talents without educational intervention" (Milgram & Hong, 2009, p.149). When refuting this myth Sheffield quotes the National Council of Teachers of Mathematics (NCTM) who claimed that "the student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world" (NCTM, 1980, p.18 in Sheffield, 2017, p.19). The need to identify, support and develop these students is seen as a priority for education, and, in particular, it is recommended that the focus should be on making programmes available to a larger number of students than is currently the practice. To challenge these promising mathematicians Sheffield suggests complex multi-faceted problem solving instead of accelerating the students through more memorised rules and algorithms.

2.3 Traditional classrooms

In the literature traditional mathematics teaching is characterised by a heavy reliance on example-led instruction, teacher guidance and memorisation which removes the agency of the student and the ability to apply mathematics to unseen situations. (Cooney, 2001; Leikin & Levav-Waynberg, 2007; Schoenfeld, 2016). Another key feature of traditional classrooms that hinders student agency is the practice of teachers relying on the textbook. Schoenfeld (2016) argued that teaching methods that are structured around the style of standard textbook questions encourage the memorisation of procedural knowledge that is non-transferable to applications in real life. This has been reiterated by Boaler (2006) where she showed that students who learn through textbooks rather than through open questioning and discussion find it difficult to apply the knowledge and make connections in other areas of mathematics. Classroom practices that neglect to prioritise the importance of understanding the concepts being taught have become common in examination-focused societies because students can often get the correct answer with limited conceptual understanding (Schoenfeld, 2016). The end result and speed are thus prioritised over understanding which results in teachers' instructions becoming 'a recipe for completing the task rather than a way of learning the material' (Schoenfeld, 1988, p.151). When students are predominantly taught in a traditional style where they rarely engage in activities that encourage metacognition the learning process is limited. Sousa, (2009) highlights the importance of giving students greater independence to tackle complex problems combined with specific strategies to help students develop self-awareness. This interest in metacognition has been shown to be an important feature of a challenging classroom for able mathematicians and is in sharp contrast with traditional teacher-led classrooms. There has been much research into the perils of classroom instruction based on teacher examples and the memorization of methods and rules which inhibits creativity (Hiebert, 2003; Lampert, 1990; Lithner, 2008, 2017; Schoenfeld, 1985, 2016).

2.3.1 Student views on mathematics

Students' experiences in the classroom have a huge effect on how they perceive the purpose of mathematics in school (Nosrati & Andrews, 2022). Without opportunities to be creative, students "see mathematics as a dead subject - procedures to memorize that they will never use and answers to questions they didn't ask" (Boaler, 2016, p.31). In order to improve motivation and confidence when problem-solving, students need to take more risks and consider their own view on what the nature of mathematics is. Their fixation with certainty and learning 'the' method and getting 'the' right answer can be a limiting factor in their mathematical development (Haylock, 1997; Diezmann & Watters, 2002; Mangels et al., 2006). This assumption that mathematics is all about certainty has a compelling effect on student beliefs:

> These cultural assumptions are shaped by school experience, in which *doing* mathematics means following the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth *is determined* when the answer is ratified by the teacher. (Lampert, 1990, p.32).

A challenge for future research is to identify and develop teaching strategies that will encourage all students to engage with creative solutions instead of relying on rigid procedural mathematical techniques. Providing students with unfamiliar tasks for which they do not already have a method will force them to experiment and conjecture and can have benefits for all students. This type of pedagogical approach has been endorsed by researchers who suggest that giving students opportunities to experience mathematical modelling in the classroom will have a positive effect on their motivation and their view of what mathematics is (Goldin, 2009; Chamberlin & Mann, 2021; Goos et al., 2023). The uncertainty embedded in modelling forces students to think mathematically, to persevere and be creative rather than focusing on getting a 'single correct solution' (Goos et al., 2023).

In her research project exploring traditional versus reform teaching environments, Boaler (2003) found that students in traditional classrooms believed that "their main role was to 'receive' the information teachers presented to them, remembering each demonstrated step". (Boaler, 2003, p.5). Whereas the students in the reform classes, who were encouraged to conjecture and ask each other questions, saw themselves as 'active knowers' rather than receivers. This research was centred upon student beliefs and how they positioned themselves as learners from the teaching they experienced. The key findings from the research emphasised the importance of the interaction between the teacher and student and how a teacher's actions can be so instrumental for creating learning opportunities for the students. Moving away from the teacher and textbook as being the authority for students and encouraging them to understand the mathematical concepts for themselves are fundamental.

2.3.2 Dangers of teacher-led classrooms

As Lampert (1990) and Boaler (2003) suggested above, teacher-led classrooms encourage students to see the teacher as the source of authority. Brousseau (1997) gives support to the negative effect of this style of teaching which has been seen to be evident in mathematics classrooms (Hourigan & O'Donoghue, 2007). Brousseau suggests that teachers often use 'task-solution' templates to reduce the demands of the task but the end result is that the students do not take responsibility for their learning (Brousseau, 1997). In his theory of didactical situations in mathematics (TDS) he states this as the reason for such teaching. He goes on to explain why memorising algorithms is ineffective because the knowledge cannot be transferred. The theory then emphasises the important role of the teacher in presenting tasks to the students where they are compelled to take ownership. The key message in TDS is that by removing the uncertainty embedded in a task the students do not experience the thinking and perseverance that is required to develop mathematically. This can be seen to be a danger for all students, but it has been suggested that countries, which have a high-stakes examination system, are more vulnerable to prioritising the procedural mathematics, over the conceptual understanding of the topics (Scager et al., 2014). The dangers of learning procedures laid down by the teacher, or textbook, from a conceptual viewpoint, have been examined in detail by many researchers (Hourigan & O'Donoghue, 2007; Lithner, 2008, 2017; Skemp, 1976; Kirkpatrick et al., 2001) and will be outlined below.

2.3.3 Imitative versus creative reasoning

Traditional classrooms described above have been acknowledged as such because of their focus on the rote learning of procedures. When experiencing this type of learning students are at risk of not grasping the concepts in a way that they can use them to solve more difficult tasks. Lithner's mathematical reasoning framework (2008) describes mathematical reasoning as either imitative reasoning or creative reasoning. Here reasoning is defined as "the line of thought adopted to produce assertions and reach conclusions in task-solving" (Lithner, 2008, p.257). Creative reasoning requires students to solve novel tasks and choose strategies to solve the task. In tasks involving imitative reasoning the solution method is already laid out for students. Lithner categorises imitative reasoning as either 'memorised reasoning' where students recall a complete solution, such as, learning off a proof, or 'algorithmic reasoning' which is learning off a sequence of well-practiced procedures. In both of these cases the students are not required to be creative and devise a strategy for themselves. Regardless of how the uncertainty is removed by the teacher, students will not get creative experiences when solving imitative reasoning tasks. He maintains that students having the opportunity to conjecture and create their own knowledge, such as Brousseau advocates in his TDS, is not common in teaching, textbooks, or assessments (Lithner, 2017). Hiebert (2003) concurs that the danger of students being consistently taught to memorise algorithms like 'robots with poor memories' is that the students do not learn how to discover the methods themselves (Hiebert, 2003, p.12). Both

Heibert and Lithner acknowledge that procedural facts must be taught, however they should not be taught at the expense of creative reasoning.

2.3.4 Developing mathematical competencies

Creative reasoning forms part of what Kilpatrick et al. (2001) describe as the cognitive changes they suggest are needed in students, for them to learn mathematics successfully. These ‘five strands of mathematical proficiency’ are comprised of: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Of these five, imitative reasoning would only satisfy the ‘procedural fluency’ strand. The other four proficiencies serve to help students grasp the higher-order concepts required to solve complex tasks and engage in mathematical discourse.

Similarly, learning mathematics as a set of procedures and rules is described by Skemp (1976) as ‘instrumental understanding’ where the students memorise off techniques without understanding the concepts. He quotes several widely known phrases used by teachers as instruction tools, such as: ‘turn it upside down and multiply’ or ‘move it to the other side and change the sign’, to illustrate this type of understanding common in schools. As discussed earlier in this section 2.3, Skemp’s work on ‘instrumental understanding’ continues to be a concern in mathematics educational research. In contrast, ‘understanding’ involves understanding ‘why’ a concept is applied in a particular situation and how it can thus be transferred. Skemp also emphasises the problems that exist when there is a ‘mismatch’ between teacher and student favouring one type of understanding over the other, and that this can occur in either direction. For able mathematicians this can be problematic: an able mathematician may want to learn relationally, but the teacher is teaching for instrumental understanding. Skemp suggests that this can cause much frustration for the student and even lead them to give up trying.

2.3.5 Problem-solving

Problem-solving has had a central position in mathematics education since the 1980’s because of its ability to provide opportunities to develop general cognitive skills, foster creativity, motivate pupils to learn mathematics and because it is such an integral part of the mathematical application process (Schoenfeld, 1985, 1988; Sheffield, 2013, 2017). Pólya (1990) first produced his book on 'How to solve it?', in 1945, where he set out a four-step heuristic outlining how to teach students to solve mathematical problems. This work has been credited as being responsible for bringing problem-solving to the forefront of mathematics education research (Schoenfeld, 2016). Defining a 'problem' is key to understanding its significance in mathematics education and for teaching able mathematicians in particular:

> We will use the concept "*a problem*" for a task situation where the individual is compelled to connect the known information in a way that is new (for him) in order to do the task. If he immediately recognises the actions needed to do the task, then it will be a routine task for him. Thus, the concept "problem" is bound to time and person (Kantowski, 1980).

The distinction attached by Kantowski to a 'problem' and a routine task is fundamental when considering how to challenge able mathematicians given that, as discussed in section 2.2.1, able mathematicians require more complex tasks to challenge them than average students. Other researchers have distinguished mathematical problems similarly by emphasising the lack of an algorithm to solve the task (Callejo & Vila, 2009). Both descriptions involve making mathematical connections and more complex thinking than would be required to solve tasks based on 'procedural fluency', as outlined by Kilpatrick et al. (2001).

The connection between problem-solving and mathematical creativity has only become prominent in recent decades, possibly because of the difficulty in defining creativity, which will be discussed in section 2.4 below. The first problem-solving heuristic to incorporate the creative side of problem-solving was designed by Jensen in 1976, (Sheffield, 2003). It was symbolized by a pentagon with each vertex denoting a problem-solving heuristic. The aim of the design was to give the students the freedom to start at any of the five points: Relate, Investigate, Communicate, Evaluate and Create (Sheffield, 2013). The benefits for the student were the lack of a prescribed ordered heuristic which was intended to promote creativity. An important feature of Sheffield's model was that students should not stop working when they have found a solution to the task but are instead encouraged to explore the task more deeply. Her intention was to offer a technique for challenging highly able mathematicians that did not involve giving them extension or acceleration tasks and could be incorporated into the classroom. The skills required to solve creative tasks are very different to those required for procedural tasks and need to be specifically cultivated by teachers. Chamberlin and Mann (2021), in their research on the Five Legs Theory of Creativity', suggest that student affect has a key role to play in developing a creative disposition. These 'five legs' include iconoclasm, impartiality, investment, intuition, and inquisitiveness which are also valuable characteristics for successful problem-solving. Chamberlin and Mann suggest that by carefully selecting open-ended tasks, that promote divergent thinking, teachers can build on these 'five legs' and have a positive impact on both mathematical affect and creativity.

2.4 Creativity

The research on creativity reviewed in this section, is concerned with mathematical creativity rather than creativity in general. There has been much interest in the connection between giftedness and creativity among researchers over the last 20 years. Many attempts have been made to provide workable definitions and to extend measures of assessment of a student's creativity in order to help foster it in the classroom (Leikin, 2009; Leikin & Lev, 2013). This section will provide a background to research on mathematical creativity and why it is seen as important for challenging high ability students. It should be noted that an important distinction has been made between the 'absolute creativity' of professional mathematicians and the 'relative creativity' of school students (Leikin & Lev, 2013). The definition that best guides this study is that creativity at school level is "the process that results in unusual (novel) and/or insightful solution(s) to a given problem" that are new, relative to the student's mathematical experiences and those of his peers (Liljedahl & Sriraman, 2006, p.19).

2.4.1 Background to research on creativity

Defining the term creativity has proved a complex task and there is a lack of a widely accepted definition amongst researchers. Up until recent decades it was felt that this "lack of an accepted definition for mathematical creativity hindered research efforts" (Mann, 2006). Torrance, the acclaimed "Father of Creativity" because of the development of his Test for Creative Thinking, offered the following definition of creativity:

> Creativity is a process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficult; searching for solutions, making guesses, or formulating hypotheses about the deficiencies; testing and re-testing these hypotheses and possibly modifying and re-testing them; and finally communicating the results. (Torrance, 1966, p. 8, in Mann, 2006).

Torrance based his assessment of creativity on a measure of the evidence of fluency, flexibility, novelty, and elaboration in a mathematician's work. These features have become well integrated into research on designing multi-solution tasks to challenge able mathematicians and measure their levels of creativity which will be discussed in section 2.5.3.

Much of the recent research on creativity is based on the work of Torrance and later Haylock (1997), who explored factors that inhibited creativity in school students. Haylock's research defines creativity in school mathematics as an ability to overcome fixations with content and algorithms. He quotes the work of Balkan (1974) who listed among his criteria for mathematical creativity "the ability to break from established mind sets to obtain solutions in a mathematical situation". The two types of fixation that he found were especially significant in school mathematics were self-restriction (Content-universe fixation) and adherence to stereotype approaches (Algorithmic fixation). Content-universe fixation is where students limit their thinking to 'an insufficient or inappropriate range of elements' and algorithmic fixation is when students persist with routine procedures even when they become inappropriate or unproductive (Haylock, 1997, p.68). He emphasises the importance of creativity for students to be able to progress from the incubation to the illumination stage when problem solving. In his framework, laid out in Haylock (1987), where he addresses the recognition of creativity within school mathematics, he focuses on identifying the kinds of thinking in mathematical tasks that qualify as "creative". These include the fluency, flexibility, originality and appropriateness evident in student responses to open ended tasks with multiple possible solutions.

2.4.2 The importance of creativity

Vygotsky spoke about creativity in terms of the 'imagination' of children as they develop (Vygotsky, 1930, 1984). He, and other constructivist educationalists (Piaget, 1957; Glasersfeld, 1995, 2001) emphasised the importance of constructing new information by making connections with previously known concepts. They may have disagreed on the process by which it happened but the student involvement in the creation of knowledge was instrumental. The significance of Vygotsky's theories on collaboration for able mathematicians will be outlined in more detail in section 2.6.1.

Research into the role creativity plays in mathematics has been ongoing since the early 1900's. In 1908, Henri Poincaré gave a presentation on "Mathematical Creativity" to the French Psychological Society in Paris (Liljedahl, 2009). Through this he laid the foundation for many others interested in mathematical inventiveness, including Wallas in The Art of Thought (1926) and Jacques Hadamard, whose lectures were published in 1945, in The Psychology of Mathematical Invention in the Mathematical Field (Hadamard, 1945). The most important contribution of this research was the further development and popularisation of the Gestalt (Koffka, 1935) four-stage process to describe mathematical inventiveness: initiation, incubation, illumination, and verification. Initiation involves reading around the problem to familiarise yourself with it and the methods previously encountered that may help to solve it. Incubation is the time given to allow the mind to work on the problem subconsciously. Illumination, often described as the 'AHA!' experience (Liljedahl, 2009), is the stage when a solution method appears which is then verified in stage four. Of these four stages, the most important stage with regard to creativity is the incubation stage when the mind subconsciously reaches a solution. There has been much debate in psychology over how this is reached but it has become generally accepted by mathematicians (Sriraman, 2005, 2021; Leikin, 2009, 2013; Lithner, 2008; Polya, 1963, 1990; Schoenfeld, 2002) that this stage is instrumental in solving unfamiliar problems.

2.4.3 Creativity as a tool for providing challenge

By focusing on ways of recognising creativity in students, and methods of fostering creativity in a diverse classroom, able mathematicians can be facilitated to reach their full potential (Goldin, 2009; Mellroth, 2018; Sheffield, 2003, 2009). Teachers may still need to be convinced of this because there is a tradition in diverse classrooms that, rather than being provided with challenges, high ability students are sometimes used by teachers to help the less able students (Freeman, 1996). Mann, (2006) stated there was a danger of neglecting the needs of these students and that it ‘may drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether’ (p.239). Despite the evidence to suggest the importance of creativity in problem solving (Schoenfeld, 1985; Diezmann & Watters, 2002; Sheffield, 2003), teachers often neglect it or are not aware of its essential role in mathematics. There has, however, been a shift in this mentality, and more recently it has been acknowledged that mathematical creativity can and must be developed in all students (Silver 1997; Sheffield 2003, 2009; Diezmann & Watters, 2002; Taylor, 2009; Mellroth, 2018). Goldin (2009) suggests students need to experience the positive aspects of struggle by exploring challenging tasks in an emotionally safe environment. In this way encouraging bewilderment, inventiveness and creativity will build mathematical perseverance and affect in all students, not just the able mathematicians.

2.4.4 Creativity in mathematics education research

Despite being recognised as an essential component of problem-solving involving flexible thinking and intuition, the US government’s focus on “back to basics” in the 1980’s and 90’s resulted in research on creativity taking a back seat (Sheffield, 2013). The last two decades have seen an increased interest in mathematical research on giftedness and creativity and, in particular, in the crucial role creativity plays. This heightened interest in creativity can be seen by the introduction of working thematic groups on creativity and giftedness in international conferences such as the International Congress on Mathematics Education (ICME) and The International Group for Mathematical Creativity and Giftedness (MCG). Research papers for these conferences have focused on the importance of creativity and on specific areas such as defining, recognising, and fostering creativity.

Pehkonen, who was instrumental in driving the introduction of the topic group on ‘fostering creativity’, in the International Congress on Mathematics Education (ICME-8) in 1996, emphasised the need to find a balance between logical and creative tasks given to students (Pehkonen, 1997). His research on the theory of functional asymmetry in the human brain has highlighted the danger of placing too much focus on routine algorithms which exercise the left hemisphere at the expense of the creativity and spatial awareness which exercise the right (Pehkonen, 1997). Pehkonen believes that modern society and culture reward student achievement that stems from actions of the left hemisphere more than that of the right. By not being given opportunities to explore visual solution methods, Pehkonen maintains that the common misconception that students have either 'logical' or 'creative' brains could deter students from exploring creative approaches (Pehkonen, 1997).

2.4.5 Challenging student mindsets

By providing students with unfamiliar, open-ended tasks, there is the potential to change their mindsets about what sort of learner they are (Spangler, 1992). Dweck's theories of intelligence suggest that an open mindset can be a significant factor affecting student motivation and achievement (Dweck, 1986). By this she referred to the belief that intelligence was not fixed, and that performance could be improved with effort and belief. Her research studied waveforms of the brain when students were given a general knowledge test followed by corrections to answers. 'Entity Theorists', who have a fixed mindset and emphasise performance goals, were found to be vulnerable to viewing feedback negatively, attributing their poor performance to their lack of ability. 'Incremental Theorists' who believe intelligence is malleable and emphasise 'learning goals' responded proactively to mistakes seeing them as a challenge to improve (Mangels et al., 2006). The above theories advocate that the predominant teaching and assessment methodologies employed in schools may need to be re-evaluated in order to encourage students and teachers that mathematical potential is not a fixed entity.

2.5 Fostering creativity in a diverse classroom

The opportunities and experiences provided to students have been shown to be a more significant factor in predicting low motivation and achievement than the actual ability of the students (Silver & Stein, 1996). With this research in mind, it can be seen that the creation of the right classroom environment is key for enhancing student motivation and learning. Factors that support high level cognition and mathematical competence include a focus on meaning, appropriate scaffolding, discussion and investment of time to think and explore mathematics (Goldin, 2009; De Corte et al., 2008; Mason & Watson, 2005; Sheffield, 2009). In contrast, inhibiting factors include lack of appropriate challenge and time and too much emphasis on getting the correct answer (Diezmann & Watters, 2002).

There is evidence that rather than providing special education for the students with high mathematical ability, rich learning tasks that are carefully chosen and implemented can provide learning opportunities within diverse classrooms (Nolte & Pamperien, 2017; Mellroth, 2018). Convincing teachers of the merits of investing time in such tasks is fundamental to their success. Hence, there is a need to provide teachers with specific support to enable them to create a classroom environment that promotes such tasks that will challenge all students (Cross et al., 2018; Silver & Stein, 1996). The advice from the research literature is that when selecting tasks to foster relative creativity in the classroom, open-ended, multi-solution tasks have proved to be a suitable mechanism with which to explore the mathematical potential of students (Mason & Wilder, 2007; Leikin & Lev, 2013). Multi-solution tasks will be further discussed in section 2.5.3.

Developing creativity in students requires teachers to encourage students to enquire and explore by providing them with an appropriate learning environment that is not based on memorising procedures (Haylock, 1997; Sheffield, 2009; Sriraman, 2005). Sheffield (2009) argues that students need to learn to ask questions, to suggest interesting problems to work on, to be allowed time to 'sleep on' difficult problems, to experiment with new ways to solve old problems, and to engage in collaboration and social construction of knowledge. By providing students with these opportunities they will be motivated to explore mathematics in a meaningful way.

2.5.1 Sriraman's five principles to maximise creativity

Sriraman (2005) suggested five principles to maximise creativity in the classroom. The first of these was to provide opportunities for experiencing the Gestalt psychology four-stage process of mathematical discovery, which incorporated initiation, incubation, illumination and verification. In a classroom situation this entailed giving students time-consuming questions that required perseverance for the satisfaction, or AHA! moment, to take place. The other four principles were providing opportunities for the students to experience uncertainty, the aesthetical beauty of mathematics, the freedom to think and take risks, and the scholarly aspect of mathematics. For second level students, Sriraman believed these principles manifest themselves in allowing students to discover mathematical methods themselves, to be unsure how to solve something, rather than be presented with routine algorithms. He also recommended providing opportunities for the appreciation of unusual solutions and collaboration with peers where they could take risks. He claimed that through collaboration and the exploration of solutions the students will learn that it is OK to be wrong. By learning that there is not always only one correct solution to a problem he suggests that students will reduce their mathematical fixation. The goal of Sriraman's five principles is to ensure that the mathematically creative students have opportunities to progress to become professional mathematicians and that they do not get overlooked when giftedness is assessed. In a society where giftedness is measured by IQ scores this is an important factor when researching high ability students.

Similar recommendations, for providing tools to foster creativity, can be found in other literature. These include suggestions to provide opportunities for uncertainty through open ended, unfamiliar tasks and the use of multi-solution tasks along with peer collaboration (Boaler, 2003; Goos, 2002, 2006; Mason, 2004; Nolte, 2012; Sheffield 1999; Sriraman, 2021). Each of these will be examined in turn in the paragraphs below. Given the diverse learning opportunities provided by peer collaboration (such as opportunities for challenge, uncertainty, having the freedom to think and the learning of new ideas through group participation) collaboration will be discussed in a separate section, 2.6.

2.5.2 Importance of uncertainty

Sriraman's (2005) model to maximize creativity advocates giving students opportunities to experience uncertainty in order to cultivate perseverance. In his research he maintains that the same principles for fostering creativity in professional mathematicians also have significance for upper secondary school students such as those in my study. Uncertainty forces mathematicians at all levels to think outside the box and be more creative with their strategies to overcome difficulties. In a study on eighteen professional mathematicians, Sriraman (2021) further explored the role of the uncertainty in spurring creativity to prove his hypothesis that "Uncertainty is both a catalyst and a necessary condition for creativity" (Sriraman, 2021, p.6). Taylor (2009) supports this idea of the importance or uncertainty for challenging able mathematicians. He advocates building in an element of uncertainty to assessments to reward the student who can perform in new situations rather than those who can just memorise classwork. Whilst assessments may require the same techniques they can still be designed to appear different and force the student to use creativity in answering them.

2.5.3 Multiple solution tasks

Other significant literature on creativity includes a model for using Multiple Solution Tasks (MST) to evaluate relative creativity as proposed by Leikin, (2009). By a careful selection of such tasks, it proved possible to identify creative students in a classroom situation and to maximise the challenge level for them. By comparing conventional, individual and collective 'solution spaces' Leikin developed a scoring scheme based on the four components of fluency, flexibility, originality and creativity. When students worked on tasks, finding as many possible solutions as possible, their 'solution spaces' could then be scored to assess their level of creativity. Creativity was measured as the product of flexibility and originality so previously discovered solutions could not be classified as creative.

MST provide a challenge for students who are conditioned to believe that the aim of a task is to get 'a' correct answer because the focus is on exploring possible solutions. They also have the potential to increase student motivation and understanding of mathematical concepts more than procedural tasks, such as those found in the text book. MST are also an effective differentiating tool because, when engaging with MST, students no longer stop when they get the correct answer but are encouraged to extend the challenge themselves (Sheffield, 2003). For high ability students to achieve their potential they need to be challenged in such ways and teachers should be incorporating appropriate tasks into their classroom in order to do this (Kattou, 2013). Teachers who feel they lack resources should be encouraged by the potential to adapt textbook tasks to transform them into MST for developing creative skills in students (Matić & Sliško, 2022). It has been shown that creative tasks are a tool by which students can achieve their potential in a mathematically diverse classroom (Nolte & Pamperien, 2017; Mellroth 2018). This is particularly significant given that diversity is becoming more evident in most post-primary schools in Ireland.

2.6 Peer collaboration

The concept of learning through peer collaboration has been widely acknowledged in mathematics education (Boaler, 2002; Goos, 2002, 2006; Lave & Wenger, 1991; Sriraman, 2005; Wilkie & Sullivan, 2018). It is particularly relevant when examining how able mathematicians can be challenged because of how it facilitates such students to be pushed out of their comfort zone. As discussed, in section 2.5.1, it is one of the features that is embedded in all five of Sriraman's Principles for fostering creativity. The paragraphs below will discuss the various forms in which it has been advocated by researchers and its relevance as a tool for challenging highly able students.

2.6.1 Vygotsky's sociocultural theory

Vygotsky's sociocultural theory places much emphasis on the importance of culture on the cognitive development of students (Vygotsky, 1999). In particular, he maintains that students learn most of their skills from interactions with their peers and teachers, and that it continues throughout, not only their formal education, but their lifetime. Vygotsky suggests that, whilst much learning takes place in the pre-school years, school introduces important environments for students that are required for their cognitive development. When introducing the 'zone of proximal development', he proposed that "the only 'good learning' that takes place, is that which is in advance of development" (Vygotsky, 1978, p89). The ZPD was described by Vygotsky as:

> 'the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978, p86).

In this description he advocates the need for students to be pushed out of their comfort zone, which will inevitably mean that this zone is not homogeneous for a typical classroom of students. For able mathematicians it will require more advanced cognitive tasks in order to bridge the gap and facilitate them reaching the level of their potential development. The ZPD, and the collaboration that it implies, is an essential part of the learning process for Vygotsky because:

> learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once these processes are internalised, they become part of the child's independent developmental achievement.
>
> (Vygotsky, 1978, p.90).

2.6.2 Collaborative ZPD

A 'collaborative ZPD' is a learning environment that was evolved from Vygotsky's work on how students learn through 'adult guidance' or 'more capable peers' (Goos, et al., 2002). The intention of the term was to distinguish between an expert-novice relationship and one of equal status collaboration. In a collaborative ZPD students of similar ability can learn from each other by providing scaffolds and justifying each other's conjectures. It is described as "a reciprocal process of exploring each other's reasoning and viewpoints in order to construct a shared understanding of the task" (Goos et al., 2002, p.196). This type of interaction requires students to be equally involved in the process of problem solving and to engage in risk-taking dialogue with each other. There have been suggestions that student collaboration may not always produce the desired metacognitive outcomes (Stacey, 1992). However, such research also acknowledges that the type of task and teacher intervention is important to ensure that the students are engaging in constructive dialogue that includes refuting or challenging each other's ideas. This supports Sriraman's proposition that it is necessary to create a classroom environment that encourages risk-taking for student creativity to flourish.

2.6.3 Student agency

The idea of people learning from each other is not confined to education. Recent decades have seen mathematics educators (Boaler, 2002; Goos, 2002, 2006) look at the wider environment that students develop in to try to understand what learning mathematics entails. Lave & Wenger's support for socio-cultural theories can be seen in their community of practise concept, where individuals learn by acquiring mastery of knowledge and skills through 'legitimate peripheral participation' in a community of practice (Lave & Wenger, 1991). Lave & Wenger sought to understand the process of learning, rather than to suggest strategies for teaching, but the ideas they put forward have much relevance in mathematics education. 'Legitimate peripheral participation' offers an alternative way to look at learning which is in sharp contrast to the traditional teacher-led instruction discussed in section 2.3 earlier. It has been acknowledged by researchers that the role of the teacher should be to facilitate students in self-regulating their mathematical thinking, so that they can become agents of their own learning (Boaler, 2003; Goos, 1999; Mason, 1998). Mason called this ‘awareness-in-action’ where students learn to become independent problem solvers by making sub-conscious connections or various concepts for themselves. Through participation with peers, in the absence of the traditional teacher authority, students can develop this awareness and help themselves make sense of the mathematics together.

2.7 Irish Background

Entry into third level education in Ireland is based solely on a points system where students receive points for their best six subject grades in the Leaving Certificate Established Examinations at the end of the Senior Cycle, with most students studying a minimum of six or seven subjects. Almost all students study mathematics for the duration of their time in school. They sit state examinations at the end of their three-year Junior Cycle and then again after their two-year Senior Cycle level. Students can sit two different levels at Junior Cycle, ‘Ordinary Level’ or ‘Higher Level’. At Senior Cycle they have a more diverse assessment system which includes the Leaving Certificate Established, the Leaving Certificate Applied or a Leaving Certificate Vocational Programme depending on the preferred career path of the student. Most able mathematicians take the Leaving Certificate Established, which has three different levels of challenge. Students are offered additional points for studying mathematics at higher level which has resulted in many students who struggle with the concepts still opting for the higher level (O’Meara et al., 2021). A consequence of this is that there can be a wide range of ability in mathematics classes which can provide challenges for teachers to accommodate these students in the same classroom as the mathematically able regardless of how the classes are organised. Post-primary schools are free to choose whether they decide to have mixed-ability classes or not. A common procedure for schools is to use the ‘setting’ approach where the core subjects of mathematics, Irish and English are grouped according to ability. As discussed, in section 2.1, there will inevitably be diversity in mathematics classes even if schools choose a setting approach.

2.7.1 Project Maths Reform

The motivation behind the Project Maths curriculum reform, developed in 2010, was largely to address the findings in reports on international assessments such as The Programme for International Assessment (PISA) and Trends in International Mathematics and Science study (TIMSS). These reports identified that the students who participated in the studies had a conceptual understanding of mathematics below the OECD average, as discussed in section 1.2. The overall message from the assessment results was that as a country, Ireland performs well on ‘knowing’ procedures but higher order thinking and ‘reasoning’ have been neglected in favour of drilling students on procedural tasks which will be discussed in the next section. The focus on learning and applying routine algorithms, such as those required for the Leaving Certificate, neglects the importance of giving the students the opportunity to experience the creativity that is the essence of learning mathematics (Mann, 2006). As has been researched in other countries, this denies many creative students the opportunity to reach high achievement in school assessment since these assessments are traditionally exam focused and do not measure creativity (Kim, Cho, & Ahn, in Mann, 2006).

2.7.2 Reliance on procedural tasks

The evidence found in a report on the impact of Project Maths in Ireland (Jeffes et al., 2013), suggests that there is still too much emphasis on procedural tasks in the classroom at the expense of rich tasks that encourage cognitive development. This research also found that despite the introduction of reform measures “traditional approaches like using textbooks and copying from the whiteboard continue to be widespread” (Jeffes et al., 2013, p.6). The report recommended that more emphasis should be given to students engaging in problem-solving approaches and justifying or explaining their solutions. Reliance on textbooks may have a negative impact on students’ cognitive development given that it has been shown that all three textbook series have neglected tasks that require reasoning-and-proving and real-life applications (O’Sullivan, 2017). Given the influential effect that the classroom has on students’ perspectives on the purpose of mathematics, the content and reliance on textbooks merits research (Hourigan & O’Donoghue, 2007; Nosrati & Andrews, 2022). Comparing the traditional approaches to the five strands of mathematical proficiency recommended by Kilpatrick et al. (2001) mastering procedural fluency for examinations seems to be the main focus of teaching in Ireland. Research pre-Project Maths had shown that such exam-orientated Senior Cycle classes in Ireland, combined with a teacher centred didactic approach, has negative implications for students' preparation for tertiary level mathematics (Hourigan & O' Donoghue, 2007). Suggestions made from the research of Hourigan and O' Donoghue include moving away from the current didactic contract to focus on creating an environment that gives students opportunities to take risks and be creative, rather than focusing on getting the 'right answer'. However, the evidence found by Jeffes et al. (2013) on the impact of Project Maths seems to suggest that this has not yet been achieved.

2.7.3 Extra-curricular opportunities for high ability students

Rather than having specific strategies built into the school curriculum, Ireland has catered for 'gifted' students through summer schools and extra-curricular courses since the 1990's. Universities run the Irish Olympiad and Maths Circles courses for high-achieving mathematics students. Similarly, the Irish Centre for Talented Youths (CTYI), at Dublin City University, was established in 1992 to offer students in the 95^{th} percentile or above, the opportunity to enrol on enrichment courses. These courses have proved to be of much benefit to its participants (O'Reilly, 2018; Fitzsimon, 2021). However, entry onto these courses is limited to those students who have been identified as 'gifted' by standard tests or through achievement in national assessments. The lack of a specific education policy to challenge high-ability students reflects Ireland's egalitarian philosophy of focusing on low achievements rather than those of the high achievers in international tests such as those outlined in the following paragraph (O'Reilly, 2018).

2.7.4 Implementing the intended curriculum

The mathematics specifications, at both junior cycle and senior cycle, have a strong emphasis on teaching students conceptual learning, making connections across the specification and using differentiated strategies. Reflecting on the research of Skemp (1976), the intention of the specification is for the students to learn relationally, build on their knowledge, and learn to apply it in different situations rather than follow teachers' instructions (NCCA 2015, 2018). The differences in the intended versus the enacted curriculum need to be examined in light of the effect of teaching approaches on students' mathematical beliefs. Recent research examining the attitudes of pre-service teachers highlights the potential long-term consequences of students' mathematical experiences in school (Prendergast, et al., 2020). Instilling positive experiences in students, through engagement with creative tasks, may be the key to ensuring that the curriculum is taught with a less procedural approach in the future. The predicament faced by educators in Ireland is how to ensure that the curriculum is implemented in such a way that its aims are achieved and that learners 'develop a flexible, disciplined way of thinking and the enthusiasm to search for creative solutions' (NCCA, 2015, p.6). If students are continually waiting for instruction and verification from their teacher their creativity and their mathematical potential will be hindered (Schoenfeld, 1988). Instead, by incorporating the 5C's into mathematics instruction – curiosity, connection-making, challenge, creativity and collaboration – Boaler, (2016) believes that every student has the potential to reach higher performance levels. These strategies concur with Sriraman's five principles to maximise creativity (2005) and underpin the choice of tasks and methodology for the present study.

Summary

Catering for the educational needs of able mathematicians requires an understanding of what these students are capable of achieving without teacher guidance. An important aspect of this is recognising the diversity of ability within a classroom and executing educational reform measures that are introduced to accommodate this diversity. The research literature outlined here strongly supports the need for differentiation and the implementation of tools to promote creativity as a means by which able mathematicians can be challenged.

Chapter 3 Research Design

3.1 Introduction

This chapter outlines the research design behind the study. It explains the rationale behind the research which is an important factor in justifying the specific methodology and analysis employed. The focus of the research is on accommodating able mathematicians within a diverse classroom situation, such as is characteristic of Irish post-primary schools. The rationale behind this originates from a number of educational factors. In both mixed-ability classes and those that use setting, where mathematics classes are divided according to ability, there will naturally be a range of abilities in the majority of situations. The teaching and learning that able mathematicians experience often depends on the extent to which the teacher employs differentiation strategies. In addition to the diversity in classroom differentiation, the opportunities to engage in further mathematical challenges outside the school classroom can be limited for many students. The primary reason is that many able mathematicians may not be identified in standard intelligence tests so do not get selected for the external challenges offered to 'gifted' students. Furthermore, even for those who have been identified, geographically, it is not always feasible for students to travel to the extra-curricular courses that are available.

3.2 Ethical approval

Having decided upon the participants for the research and the proposed research questions it was necessary to apply for ethical approval to continue with the research design. The different stages of design that approval was sought for were: justification for proceeding with the research, the selection of schools and students, the design of the survey questions, the overall workshop design, and the design of the questions for the interviews that were conducted after the workshops. A number of research methods books were consulted for advice on the various stages of the design (Cohen et al., 2018; Robson, 2011). Other literature on more specific aspects, such as interviewing, will be outlined at the relevant stage of the design in the following sections.

3.2.1 Social Research Ethics Sub-Committee (SREC) protocol

Before embarking on the research, ethical considerations were made with regards to the selection of schools and students, the organising of the workshops and the collection of data. Maynooth University's protocol for this study involved an application to the Social Research Ethics Sub-Committee: Tier 2-3 Ethical Review of a Research Project Involving Participation of Humans. Hammersley (2009), argues that whilst institutions may use such applications to regulate research, it is ultimately the responsibility of the researcher to ensure that the ethical conduct of the research upholds the principles of informed consent. When making the decision to embark on this project, the research methods books mentioned above provided guidance on ethical considerations for all aspects of the study. Specialist educational ethics books which suggested considerations for carrying out research with school students were also consulted. As Brooks et al. (2014) advised, it was important to be conscious of the risks involved when the researcher has multiple responsibilities, such as teacher-researcher as is the case in this study. Furthermore, given that the research was carried out with school students there was the added dimension of the 'gatekeeper', in the form of the school teacher, principal, and parent through which the participants were selected. Consequently, careful consideration had to be given to ensure that informed consent had been given by each of the participants. The information presented to the students was written in clear English and they were given the opportunity to ask questions prior to, and during, the duration of the research. The considerations that preceded the ethical application are outlined in the paragraphs below.

3.2.2 Justification for the research topic

This research supports the claims that a broader criteria of characteristics are needed to identify able mathematicians as was discussed in Chapter 2 (Mann, 2009; Mellroth, 2014; Nolte & Pamerien, 2017; Reiss & Renzulli, 2010; Sheffield, 2003; Sternberg, 1996). Later in this chapter details on how the research rationale influenced the choice of methodology will be outlined. The method for selecting the participants will be explained in detail and how each phase of the research was implemented. This will include how the design of the workshops and the tasks that would be presented to the students were decided upon. It will also justify how the method of delivery of the tasks in the workshop was tied in with the recommendations in the research literature. The rationale behind the construction of the student questionnaires and interview questions, and how each of these test items were administered to the students, will be explained. Finally, the method of analysis for each phase of the research will be outlined.

3.2.3 Additional ethical considerations

A detailed account of each stage of the design, that was submitted to the SREC for approval, will be given in the following sections. Before commencing the research, consideration had to be given to factors such as the secure storage of the data. The student consent letter (Appendix A) reassured all participants that they would be given ID numbers to protect their identity and that all their data would be securely stored. This will be explained further in section 3.3.2.

3.3 Data collection

3.3.1 Selection of schools and participants

The research for this study was carried out in five schools in the east and south-eastern part of Ireland. The schools were selected by convenience sampling to facilitate travel to the workshops which formed the basis of the project. All five schools were non-fee-paying and either traditional, voluntary secondary schools that are common in Ireland or the newer community colleges. Together these types of schools represent 93% of the second level schools in Ireland. Schools 1 and 3 were all-girls schools, Schools 2 and 4 were mixed gender and School 5 was an all-boys school.

In total 92 Transition Year students, aged 15-16 years, took part in the project – 50 girls and 41 boys. The students who participated had been recommended by their class teachers as being of high mathematical ability. To avoid excluding any interested students it was emphasised to the teachers that the participants did not have to be exclusively high achievers in examinations. Hence, the teachers were made aware that the tasks would be challenging but were encouraged to offer the workshops to all students who showed enthusiasm towards being involved in the workshops and problem solving in general. The priority in the research study was to seek out motivated students who showed promise mathematically and wished to partake in the workshops out of an interest to challenge themselves. The mathematics teachers in each school were involved in explaining the project to the students, distributing the consent forms, organising the room, collecting names of those who agreed to participate in the group interviews afterwards and arranging the interview rooms. These consent forms can be seen in Appendix A and a very similar one was given to the parents/guardians of each student.

3.3.2 Summary of data collected

When deciding what sources of data to build into the research design, consideration was given to what was required to best answer my research questions. As stated in Chapter 1, the research questions addressed in this study are:

1. What is the perception of able students in post-primary schools in Ireland towards their mathematics classes?
2. To what degree do these students feel challenged by the experiences they encounter in the mathematics classroom?
3. What comparison can be made between the opportunities these students had, in school and in the workshops, to experience each of Sriraman's five principles to maximise creativity.

To answer the first two research questions, the students' current experience in school prior to their participation in the workshops needed to be examined. The third research question required the students to be given tasks in the workshops that provided opportunities for creativity. Data was collected from a number of sources over the duration of the research. This data consisted of a pre-workshop survey, a post-workshop survey, audio recordings of the students as they discussed the tasks during the workshops, worksheets of the students' solution methods and finally interview recordings which took place approximately one week after the second workshop in each school. There were several cases where this was not logistically possible: because of the school closures as a result of Covid-19 restrictions, when there was a period of adjusting to Zoom, the online platform used, and because of other school activities in the case of School 5. Each of these data sources will be discussed in more detail in the forthcoming sections.

Table 1 below gives a summary of the number of students who participated in the surveys and interviews per school. To provide anonymity the students were given ID numbers which they wrote on their surveys and worksheets. The first digit represented the school, from 0 to 4, and the remaining double digit was the individual student identifier within the school. Not all of the original students on the list of ID numbers participated in the workshops, for their own individual reasons, so the range of ID numbers in Table 1 does not represent the actual number of students who participated per school. Some students missed the second workshop so did not fill in the post-workshop survey. For example, in School 3, a school trip had clashed with the second workshop so it can be seen, in Table 1 below, that five students missed the second workshop. One student in School 1, even though she missed the first workshop still filled in the survey because it was based on her experience of school mathematics so it seemed appropriate.

Table 1 Summary of Data Collected

	School 1	School 2	School 3	School 4	School 5	Total N
Gender	F	M/F	F	M/F	M	
Participant No.	19	19	21	20	13	**92**
Workshop Type	Face-to-face	Face-to-face	Face-to-face	Online	Online	
ID Numbers	020-046	120-143	222-242	321-343	421-435	
Pre-Surveys Post- Surveys	19 19	19 18	21 16	20 20	13 13	**92** **86**
Number of group Interviews per School	3	2	1	3	2	**11**
Students per Interview	5, 4, 3	4, 2	3	2, 4, 3	3, 2	**35**
Percentage of group interviewed	75%	32%	14%	45%	38%	**38%**

3.3.3 Timeline

The workshops were designed to be run one week apart which happened in all but the last set of workshops. The delay in this case was because of school activities planned along with school exams so there was not much flexibility. Built in to the workshop design was the opportunity to give students time to incubate one of the more difficult tasks so see how they felt about such time-consuming tasks. To do this the students started the second task in the first workshop and continued it in the second workshop.

The interviews were planned to take place approximately one week after the second workshop. This was to give the students time to discuss their experience with each other yet not so long that they may have forgotten how they felt at the time of the workshops.

	January 2020				February 2020				March 2020				April 2020				May 2020			
School 1				31	7	14	21													
School 2								25	3									12		
School 3									6	12							7			
	Workshops				Interviews															

Figure 3 Face-to-face Workshop and Interview Timeline

- N.B. the government closed schools on 12th March 2020, because of Covid-19, so the interviews for schools 2 and 3 were delayed.

	March 2021				April 2021				May 2021			
School 4		12	19	22-4								
School 5									5		19	24
	Workshops				Interviews							

Figure 4 Online Workshop and Interview Timeline

3.4 Design of survey questionnaires

Before designing the surveys, research methodology books (Cohen et al., 2018; Robson, 2011) were consulted to decide how to conduct the survey and how best to achieve the desired aim of the surveys. Each survey was submitted for ethics approval to ensure that the questions met the required standards in terms of reliability and validity, anonymity, sensitivity, and lack of intrusion. As Cohen et al. (2018) emphasise, the ethics of the questionnaire must be considered throughout the whole research period: how the participants were approached, how the survey was explained to them and how the questions were analysed and written up. When deciding on the type of questionnaire, advice from the research methods books was sought regarding structured or unstructured surveys, and the caution required when analysing each. In the draft research design, the various types of data that were required in order to answer the research questions were also noted. With approximately 100 participants, closed questions were most suitable, given that interviews and audio recordings were planned to gather more open data. The decision to use a Likert scale questionnaire with ordinal data was taken because it would be less time-consuming to complete than open questions yet provided an opportunity for students to give a range of responses. It would also provide data that could be used for quantitative analysis. More details on the final Likert scale questionnaire used are given below. The draw backs of interpreting the responses of rating scales were outlined in the research methods books mentioned above. These included the nonuniform nature of each point in the scale from an individual participant's viewpoint, the effect of having 5 versus 7-, or 10-, points on the scale and how the order of the wording in the question can lead to different results. With this in mind, the addition of other data sources was intended to compensate for these drawbacks, by providing more open data which could be used to back up the survey responses.

The survey questions were designed to examine the students' attitude towards their experience of school mathematics and that of the workshops. Details of the rationale for the specific questions will be discussed in section 3.4.1. At the beginning of the first workshop the students were given a 'Pre-Workshop Survey' (see Appendix D) with 47 questions to be answered using a 5-point Likert scale. The items were rated from 1 to 5 where 1 represented 'strongly agree', 2 'agree', 3 'neutral', 4 'disagree' and 5 'strongly disagree'. After the second workshop the students were given a second Likert scale survey, with 30 questions. This was called the 'Post-Workshop Survey' (see Appendix F) and it was intended to gather data on the students' experience of the workshops. There was also an open question at the end of the post-workshop survey asking students to add any further comments on their experience of school mathematics or the workshops. For each survey the questions were grouped into scales where each scale described a particular feature of mathematics teaching and learning to be explore. Appendices D and F show the pre-workshop and post-workshop surveys respectively and Appendices E and G show the scales the questions were assigned to. The questions and scales will be discussed in more detail in section 3.4.1 below.

3.4.1 Design of survey questions and scales

In each of the surveys the questions were grouped into scales according to a similar theme. The scales were used so that the questions could first be tested for reliability using Rasch analysis, which will be explained in more detail in Chapter 4, and so that some of the similar scales from the pre- and post-workshop responses could be compared. The pre-workshop survey questions were grouped into scales called: *Enjoyment*, *Self-Perception*, *Feeling Challenged*, *Motivated by thinking, Teacher Methodology* and *How to Improve.* The post-workshop survey questions were grouped into scales called: *Enjoyment*, *Task difference, Feeling Challenged*, *Motivation, Multi-solution tasks* and *Self-Perception.* The intention had been to gather evidence on some student experiences in class and in the workshops, and then to compare what the students liked and disliked. Some of the topics in the scales were similar, such as, *Enjoyment*, *Challenge* and *Self-Perception* because it was feasible to compare the effect of class and the workshops for these traits. However, the tasks for the workshops were deliberately selected to be different to a typical classroom task, so other topics differed in each survey. For the pre-workshop survey the scales were testing for traditional features of teacher-led classrooms as suggested in the literature. In contrast, for the post-workshop survey the scales were testing for the perceptions of the students to MST because of the importance of these tasks for fostering creativity. Given that the aims were different for each, the same questions for each survey were not retained.

Research was carried out on a number of surveys and attitude scales, but none were really relevant to use for this specific research topic. Hence, most of the questions in the pre-workshop survey and all of those in the post-workshop survey were designed specifically for this study. These survey questions were influenced by what the research literature said about classroom features that hinder or foster creativity. The pre-workshop survey questions probed features that the literature suggested were common in second level classrooms. In addition, it included questions that would hopefully help gather data on what the students liked or disliked about their mathematics classes. These included questions to investigate whether the students were being given opportunities for uncertainty and creativity. They also intended to explore the opinions of students to 'learning oriented goals' (Dweck, 1986) by looking for evidence that the students "enjoy exerting effort in the pursuit of task mastery" (Dweck, 1986, p.1040).

The questions that were not specifically designed were either taken directly from questionnaires designed by Kloosterman & Stage, (1992) and Lim & Chapman, (2013) or were adapted from these (see Appendix C). In keeping with the nature of a Likert scale, the questions were required to be unambiguous and only measure one item. Even though only a small number of questions from the questionnaires of other researchers were used, others were helpful as guidance on the type of questions to ask. From the Kloosterman & Stage questionnaire, two of their 'beliefs' were adapted when designing questions to test the students' self-perception. These were, Belief 1: 'I can solve time-consuming mathematics problems' and Belief 5: 'Effort can increase mathematical ability'. From the Lim & Chapman questionnaire two questions were adapted for the '*Enjoyment'* scale, two for the '*Self-Perception'* scale and one for the '*Motivated by Thinking*' scale in the pre-workshop survey. Details of the exact questions that were derived from other surveys, and which surveys they came from, are listed at the end of Appendix D.

3.4.2 Implementation of Surveys

At the start of the first workshop the students were given some time to fill in the pre-survey based on their experience of mathematics class to date. The nature of the Likert scale was explained and that some of the questions were worded in reverse, so the students were advised to take their time reading them to ensure that their results were consistent. These questions can be seen in Appendix D and are labelled with (R) after the question number. They included questions such as Q17 'I am always confused in mathematics class', where an answer of '1' (strongly agree) on the Likert scale would imply low self-perception in relation to mathematics. In contrast, an answer of '1' in Q33 'I am confident in my ability to learn mathematics' would imply high self-perception. When analysing the data, the total score of each student would be used to generate a measure for each scale's trait so it was important the students understood how to answer the questions correctly. Rasch Analysis was used both to validate the scales and to create the scale measures. This process will be explained in detail in 3.4.3 below and in Chapter 4.

In Schools 1-3, the surveys were collected before the first task was presented to the students. For the online workshops the survey was uploaded to a secure online platform where the students could enter their ID number and answer the questions confidentially. After the second workshop, the students were asked to fill in the post-survey to examine their opinion of the multi-solution tasks presented and the peer collaborative nature of the workshops. There were also questions related to having to think about the method to solve tasks which would have been a feature of all of the workshop tasks. These questions were intended to provide data that could be compared with their responses in the pre-workshop survey to the question about being given the method first in class.

3.4.3 Validation of surveys

The survey responses were inputted by me and analysed using SPSS software. The initial frequencies were recorded for each question and Cronbach's alpha values were then recorded for each scale, details of which will be given in Chapter 4. To obtain meaningful results describing student perceptions the survey instrument used was validated using Rasch analysis (Bond & Fox, 2007). Rasch analysis allows us to investigate the reliability of the scales used in the survey and to compute a measure for each student and each item on each scale. The item reliability index shows how likely the item measure orders would remain the same if the scale was administered to a different group of students. The person reliability index shows how likely it would be for the person measure order to be the same if the same students were surveyed on different but similar items. For each of the scales the Infit and Outfit index was also recorded (these are goodness of fit measures of the items to the scale). When deciding upon the validity of each scale we used the recommendation that the Infit and Outfit measures should be within a range of 0.6 – 1.4 (Bond & Fox, 2007, p.243) to determine whether an item should be omitted from a scale. Details of the analysis and the validation of each scale will be given in Chapter 4. As mentioned above, some of the questions had negative wording so the results of these questions were reversed before analysis. These have been denoted by (R) in the survey questions in Appendix D.

Once a reliable scale was decided upon, person measures for each scale were computed using Winsteps (Linacre, 2021). In addition, SPSS was used to find a corresponding Cronbach's alpha value for each scale. The accepted figures used for deciding upon the scale's reliability in this analysis were those over 0.7. The Rasch analysis ranked all the items and students in terms of agreeableness creating a scale of measures for all participants that were then used for further statistical analysis. In the Likert scale surveys, the code '1' was used for strongly agree and '5' for strongly disagree. The lowest rating category therefore represents more agreement with the item. The person measures were centred about zero with the lowest measure corresponding to the lowest score and hence the most agreement with the scale items.

The mean measure per school was found using SPSS and a series of statistical tests was carried out with these means to test for differences between the schools and between genders on their opinions of classroom practices.

3.5 Design and analysis of the workshops.

Two workshops of approximately two hours long, each with a short break in the middle, were designed and delivered for each school. The workshops were run face-to-face, in three different schools, in early 2020, but the pandemic necessitated that those for the final two schools were run via an online platform using a similar group set-up in Zoom breakout rooms. The students worked in groups of three or four where they had selected the groups themselves. In each workshop, students were given opportunities to work on challenging problems and discuss their ideas and solutions with their peers. In order to collect good quality audio data on the solving problem strategies and experiences of each group, a small audio recorder was placed in the middle of each table. The students were encouraged to discuss openly what they were thinking and were reassured that everything they said was confidential.

3.5.1 Rationale for Task Selection

Key features of the workshop design, based on the chosen frameworks of Vygotsky (1986) and Sriraman (2005), were that the tasks selected were challenging, multi-solution tasks intended to encourage creative thinking, peer collaboration and perseverance. The rationale for task selection was based on the research literature on how to challenge able mathematicians, as outlined previously, and from my discussions with the NRICH team in Cambridge who had designed a number of multi-solution tasks. Given that the students had not started their Leaving Certificate course yet, it was necessary to be aware that their competency on topics such as trigonometry was relatively limited. Topics were selected on the basis that the students could be given tasks that they would be able to find multi-solutions to. Such tasks included those that could be solved by trial and error, algebra and basic geometry because this had been covered by the students in their school classes. The students were given one or two tasks per workshop that were designed to be unfamiliar and suitable for diverse classrooms. One of the essential classroom practices that Sriraman recommended for fostering creativity, was to provide opportunities for uncertainty for the students. When selecting the tasks, it was therefore essential that they were unfamiliar to the students. The research was guided by the definition of 'an unfamiliar task' as "one for which students have no algorithm, well-rehearsed procedure or previously demonstrated process to follow", (Breen & O'Shea, 2015).

3.5.2 Workshop outline

The two face-to-face workshops were carried out in large classrooms within both schools, in the case of School 1 it was after school and for School 2 it was during school time. All three of the online workshops were carried out during normal school hours. For both the face-to-face and online workshops the teachers were not present other than to give a brief introduction and check attendance. Once the researcher had been introduced the teachers left the room but were available for consultation if any issues arose regarding a student arriving late or not being able to sign in to the online workshops. It was felt that having teachers present might have had an influence on the atmosphere of the workshops. It was important to create an atmosphere that did not feel like a normal mathematics class and one where the students felt free to discuss the tasks openly.

The tasks that were finally chosen were those that would best enable students to work independently and would also encourage collaboration. Each group of students was given a laminated copy of the task; in the case of the online workshops the tasks were presented via a link to a Padlet in each breakout room. The students were given no guidance as to the method to employ and were required to make connections across various strands of the Leaving Certificate syllabus to solve the tasks. They were encouraged, as a group, to discuss the problem-solving strategies they employed and the merits of a number of different solution methods presented to the class. At the start of the workshops, the students were reminded that the workshops were being audio recorded. A recording device was placed in the middle of each group's table, and the students were encouraged to talk freely and use each other's names so that the individual students could be identified during the analysis phase. For the online workshops, the workshops were recorded on a number of different computers in different rooms in my house. By sitting in one central room, it was possible to move between breakout rooms to talk to each group as they worked. It was also explained to the students how they could contact the researcher via a 'chat message' if they needed help. It was easier to identify the students with the online workshops because, in most cases, their names appeared on the screen when they spoke.

The role of the researcher was that of a facilitator who only intervened to ask probing questions or to encourage a line of thinking that the students had suggested but lacked the confidence to pursue. It was important to avoid putting the students under time pressure to solve the tasks. However, given the limited time available, sometimes this role as a facilitator included encouraging the groups to pause for thought if they were going down a path that would be unproductive and time-consuming.

In addition to the audio recordings, the worksheets were collected from the students after the workshops to examine their solution methods. Samples of some of these will be shown in Chapter 7. Unfortunately, not very many were received from the students in the online workshops because this involved the students scanning the sheets and sending them at a time when the students were busy with school exams.

3.5.3 Breakdown of Tasks Presented to the Students in the Workshops

Four tasks were selected for the workshops. Each task, and the reasons for choosing them, will be outlined below.

Task 1: ***Field of Dreams*.**

Differentiated teaching is defined as a pedagogical approach where the teacher proactively selects tasks and teaching methods designed to maximise the learning opportunities for all students in the classroom (Tomlinson, 2018). The first task presented to the students was a task designed for differentiated teaching and learning. The task was called *Field of Dreams* and was designed by Sheffield (2003). The gradual increase in difficulty of this task, the introduction of problem posing, and the discussion built into the advanced stages make it ideal for this purpose. The task consisted of a series of seven different arrangements of fields where a certain number of children were playing (see Figure 5 below). Sheffield designed the task to divide the seven arrangements of fields into three levels of increasing difficulty so that it could be used as a differentiated task. In the level A arrangements, the number of combined students in each pair of fields was given and the students were asked to find out how many children were in each field and what the total number of children was.

The task got progressively more difficult, from Field 1, classified as level A, to Field 4, classified as level B, with more unknowns and more fields in the arrangement. Finally, in Field Arrangements 6 and 7, classified as level C, the students would be faced with scenarios that had multiple solutions and had to discuss why this was the case. The fields in Type C were designed to evoke comparison of answers, discussion, and justification of why the students could have different answers yet are all correct.

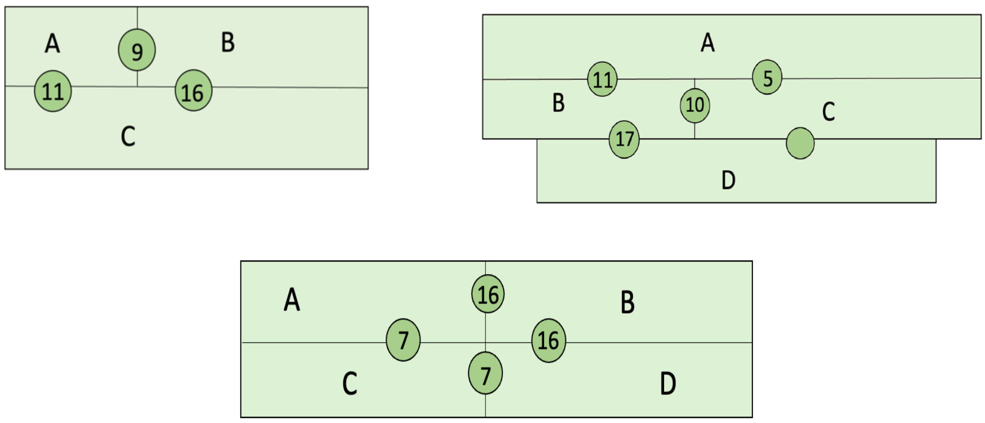


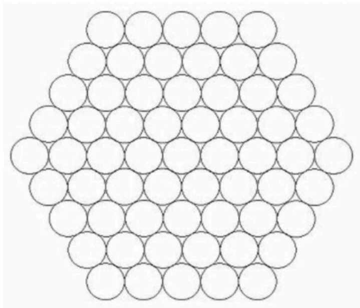
Figure 5 Field of Dreams Task: Field 1, Field 4 and Field 6. (Sheffield, 2003)

There was more than one way to solve the task and it was hoped that this, along with the multiple solutions to level C, would raise areas for discussion within the groups. It was possible to solve the initial fields by trial and error, so these problems were intended to be doable by all students in the study. As well as being a manageable task it was a good warm up task for the students to get accustomed to working in their groups. The most efficient way to solve the task was by using simultaneous equations but there were some interesting observations the students could make whilst doing so. At this stage in school all students would have dealt with mathematical equations with two variables in the Junior Cycle but would not cover more than two unknowns until starting Senior Cycle. Two of the schools had covered this in class by the time the workshops took place but the other three had not, and we will see that it took them longer to solve it. Despite these between-school differences, the speed with which the students solved the task varied greatly within schools and it can be seen that the groupwork dynamics had a noticeable impact on this. Within the task there were also questions asking the students to create their own problems for others in the group once they had solved the initial task.

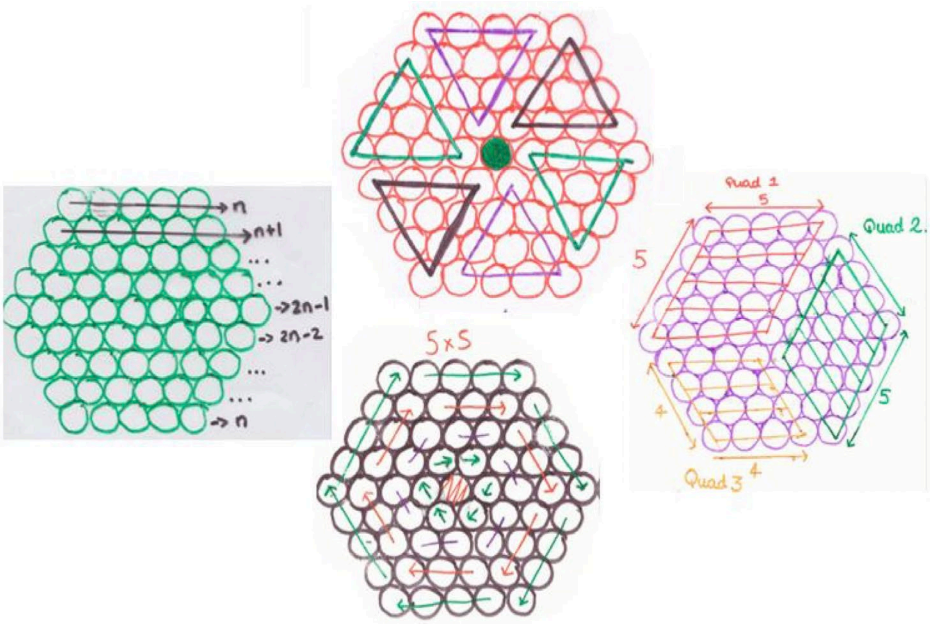
Task 2: ***Steel Cables***

The NRICH *Steel Cables* task (https://nrich.maths.org/steelcables), (shown in Figure 6 below) is an example of one of the tasks selected because of its suitability to challenge all students simultaneously in diverse classrooms.

Phase 1: Size 5 cable



Phase 2: Solution methods for discussion



Note. Reproduced from www.nrich.maths.org/7760

Figure 6 Steel Cables Task

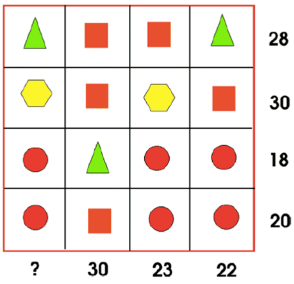
The task was divided into two parts, '*Phase 1*' and '*Phase 2*' (see Figure 6 above). This was quite a time-consuming task and was deliberately split over two workshops. The students were presented with '*Phase 1*' in the first workshop and then '*Phase 2*' in the second workshop. The rationale for this was that the students would have time between workshop 1 and workshop 2 to incubate, and possibly, discuss their ideas.

The task presented the students with a hexagon shaped cable of size 5, with 61 steel strands and asked them to figure out how many strands would be in a cable of size 10, then one of size $'n'$. An important aspect of the task was for the students to be able to explain their reasoning to the others in their group. The students were encouraged to think individually first, then to discuss their thoughts with each other. Only after they had found an answer for the cable of size 10 and discussed their methods were they to be presented with the second sheet which gave four possible solution methods for discussion (see Figure 6, *Phase 2*, above). For the online workshops *Phase 1* and *Phase 2* were presented in separate padlets and each group's progress was monitored to decide when to give them the second padlet link. As a group, they were asked to agree upon which solution method made most sense to them and to explain their reasoning. The solution methods included a variety of algebraic and visual techniques.

An important part of the research was to examine how students reacted to experiencing multi-solution tasks, where they were required to explore another person's method for solving a task. There has been considerable research done on the benefits of such tasks for challenging highly able students and providing much opportunity for creativity (Polya, 1973, Schoenfeld, 1982, Leikin & Lev, 2007; Leikin, 2008, 2011).

Task 3: ***What's it Worth***

What's it Worth? (Figure 7 below) was another multi-solution task designed by NRICH to be useful as a differentiation task for all students, not just those considered highly able. The task was relatively short compared to the other three tasks so it could be combined with *The Field of Dreams* or *Steel Cables* which took much longer for most of the groups to complete fully. The students were given this task on a sheet, while the reverse side had the five different solution methods. They were advised not to look at the solutions until they had tried as many possible methods as they could. The schools doing the online workshops had the task and solutions presented in separate padlets (see Appendix B), as for the *Steel Cables* task.



Note. Reproduced from www.nrich.maths.org/1053

Figure 7 What's It Worth? Task

In this task the students were asked to find the missing total, denoted by a question mark, in as many ways as possible. Once they had exhausted their ideas, they were told to look at the back of the sheet and discuss the approaches taken to arrive at each of the given five solutions and if possible, agree on which one they preferred (see Appendix B). One of the questions raised on the sheet was, 'What's the difference between an answer and a solution?'. This was added by NRICH in an attempt to encourage the students to discuss their reasoning with their peers rather than just find the final answer for the total of the question mark. It was up to the students to decide when they wanted to look at the solutions. Amongst the six solution methods suggested were a couple of simple solutions, often overlooked, that were intended to raise discussion within the groups. The simplest of these had no hint attached, unlike the first 5 diagrams given in Appendix B, to see how many students would notice it. This was labelled solution 'Method 6', and it involved adding, and equating, the totals on each row and column, so avoided finding any of the values for the shapes.

Task 4: ***Area of Circles***

The inspiration for including the *Area of Circles* task (see Figure 8 below) came from a Mathematical Creativity and Giftedness (MCG) conference in Hamburg in 2019. The research (Simensen & Olsen, 2019) was on the creative potential of tasks that were suitable for collaborative heterogeneous and homogeneous learning environments. The mathematics involved was designed to include a combination of algebra and geometry, not too difficult that the students could not solve it, and to provide opportunities for creative reasoning. The question posed in this *Area of Circles* task was to find the ratio of the area of the larger circle to the area of the smaller circle in each diagram. Diagram B was significantly harder because it required more geometry and visual awareness and was intended as an extension task for any groups working faster than the other groups. It was interesting to see how the students dealt with an unseen geometry task and what methods they used to approach it. This task was only used with the face-to-face workshops in the end, partly because of time restraints but also because it became clear that many students had forgotten much of the mathematics required for alternative methods, so it was not as suitable for a workshop examining multi-solutions. The final two schools had also missed quite a lot of school due to closures so it was difficult to know what they would have covered by the time of the workshops.

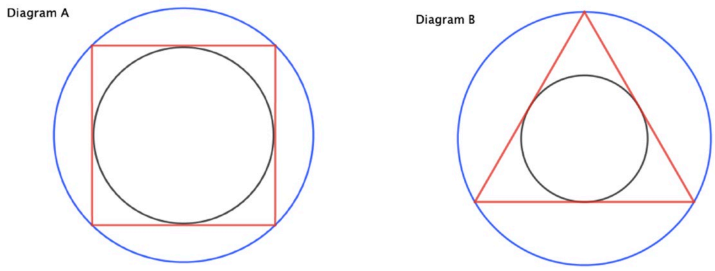


Figure 8 Area of Circles Task

3.5.4 Analysis of Workshop Data

As mentioned above, the workshops were audio recorded via a portable device placed in the middle of the table for each group. The students were informed that the audio recordings would be analysed to provide data on their problem-solving techniques and any interesting comments they made about their peer work. They were encouraged to verbalise their thoughts as much as they could and not to be afraid to say anything that may not be correct. Each student was given a worksheet with their ID on it and any necessary diagrams, such as the *Steel Cables* templates and the *Area of Circles* task, that might help them for each task. The audio recordings and worksheets were collected, or uploaded in the case of the online workshops, after the workshops for analysis. Before carrying out the analysis the audio recordings of each group of students were transcribed and collated with the relevant student worksheets per group. Initially an outside company was used to transcribe them, but when the first two came back it was apparent that it was too difficult for the transcribers to pick up the distinct group chat. Because the researcher recognised the voices better, it proved more reliable if the audio recordings of each group were transcribed by hand.

3.6 Design of interviews

When planning for the interviews educational research books (Cohen et al., 2018; Brinkmann & Kvale, 2015; King, Horrocks & Brooks, 2019) were consulted on the techniques for conducting and analysing different types of interviews. This included investigating the purpose of the interview, designing the questions, and how to plan, deliver and analyse them. Guidance was taken on thinking about the careful analysis of the students' words and having an awareness of the context in which they are said. Caution was also given about recognition of bias, personal experiences and assumptions interfering with the researcher's judgement. Whilst it is not possible to be completely free of these impediments in research, an awareness of the importance of standing back and trying to be objective and let the voices of the students be heard is essential.

3.6.1 Rationale for Interviews

The rationale for the interviews was to give the students the opportunity to discuss their thoughts on their experience of mathematics in school and the workshops. The interviews served as a technique for gathering information to help answer the research questions and also as a backup source of data to help justify the responses from the surveys. When planning the study, a considerable amount of research had been done to determine the key phenomena related to challenging highly able mathematicians in second-level schools. This consisted of extensive reading of academic literature on challenging highly able students, examination of government policy documents in Ireland (Education Act 1998; NCCA 2005, 2007, 2012), the researcher's own 25 years' experience teaching along with anecdotal evidence from discussions with students and teachers during this time. Together these sources provided a background knowledge base when deciding upon the questions for the interviews (see Appendix H).

3.6.2 Design of questions

All of the questions in the interviews were designed to probe the students about their perceptions of their experiences in school and the workshops (see Appendix H). It was important to encourage the students to talk openly so the questions were designed to be clear and unambiguous yet open enough to allow for a discussion if the students wished to digress. The interviews started off with questions probing how new topics were usually introduced in their mathematics classes in school, then developed into questions to find out if they had any experience of discussing different solution methods in school. Because the interviews were designed to back up the survey responses, questions related to the scales in the surveys were asked, such as, questions on enjoyment, challenge and being motivated by having to think of the method for themselves. After these, the students were asked questions that would hopefully bring about a discussion of the tasks presented in the workshops and how they compared with those given in school. The open style of questioning was designed to allow a level of flexibility where the students would feel free to input ideas rather than feel constrained to give a fixed response, such as are an inherent feature of surveys.

3.6.3 Planning and delivery of interviews

When signing up to the project the students had already been told that a series of interviews would be conducted. This was to ensure that they had time to consider volunteering for them during the workshops. To avoid the students feeling that they were being examined, prior warning allowed them time to ask questions about the nature of the interviews before they signed up for one. Consent forms were given out during the first workshop for them to sign and return should they wish to participate (see Appendix A). For the workshops that were carried out online the consent forms were part of the information pack that was sent to the schools. The coordinating teacher was consulted with regards to which students had agreed to participate. The interviews took place in person for the first two schools and on zoom for the last three schools because the school closures due to Covid-19 were scheduled to happen on the day of the final workshop for School 3. They were held a week after the second workshop of each school, were between 35 and 80 minutes long and varied from 2 to 5 participants. In total there were 11 group interviews involving 35 students (see Table 1). During the interviews the questions were put on a PowerPoint to give the students time to consider their answers. The same process was used for the face-to-face and online interviews. At the beginning of the interviews, the rationale for holding the interviews was explained and it was reinforced that they were completely confidential. Each student was asked their opinion in turn, but they were free to add further thoughts before we moved onto the next question.

3.6.4 Interview analysis methods

The interviews were audio recorded and transcribed verbatim. Having used an outside company to transcribe the data, each transcript was reread to check for inaccuracies due to language usage or colloquial interpretations. The transcripts were then edited to have wide margins with double spacing and each line was numbered for more convenient referencing during the coding. Template Analysis was used to analyse the interviews, basing the structure of my templates on Sriraman's five principles to tie in the coding with the theoretical framework behind my research. Template Analysis involved a series of iterations of templates, 14 in total, to develop my final template. To make the process easier to follow a number of the transitional iterations were combined into three different templates to describe the process of analysis. All of the details of the iterations involved in the development of the final template, and the analysis of the interviews using the template, will be given in Chapters 5 and Chapter 6. For reliability and validity, the possibility of having another researcher test out the interview template was built into the design of the interviews. The coding of a second researcher would hopefully validate the template and coding. This will also be explained in Chapter 5.

Chapter 4 Survey Findings

4.1 Pre-Workshop Survey Results.

The pre-workshop survey was delivered to the students at the beginning of the first workshop and contained questions on the students' perceptions of their classroom mathematical experience. As mentioned at the outset of Chapter 3, all of the students who participated in the study completed the pre-workshop survey. The first three schools completed a paper version which was collected and inputted to SPSS. The remaining two schools filled out an online survey of the same questions. The percentages of responses to each question in the pre-workshop survey can be seen in Appendix D and those selected for each scale are in Appendix E. The Rasch analysis results will be discussed below for the scales that had a suitable reliability index with infit and outfit values within the accepted range of 0.6 – 1.4 mentioned above in section 3.4.3.

4.1.1 Enjoyment Scale

There were originally 7 items in this scale, however Rasch Analysis showed that Question 6, 'I like spending time solving mathematics problems' had infit values outside of the acceptable range. When this item was removed the new scale behaved well and had infit and outfit values within the accepted range. The final Cronbach's alpha value for this scale is 0.814, the personal reliability index is 0.82 and the item reliability index is 0.98. These values show that the scale can therefore be used for reliable statistical analysis.

The responses to the items in this scale reflected that the majority of students who participated in the survey were keen mathematicians; for example, more than 80% of the participants enjoy doing mathematics problems (Q21) and find mathematics interesting (Q15). A t-test was carried out on the mean *Enjoyment* measures to determine whether there was any statistically significant difference in the mean measure on this scale between genders. The results show that there was no significant difference in the mean *Enjoyment* measures between males and females ($t (89) = -0.572$, $p = 0.569$). A one-way ANOVA was performed and showed no statistically significant difference in mean *Enjoyment* measures between the schools ($F(4, 86) = 1.30$, $p = 0.275$). We can conclude that there is no evidence to reject the hypothesis that students in all five groups appeared to enjoy mathematics to a similar extent. Given that the survey participants are seen to be keen mathematicians the data in the scales that follow can be taken as relevant for the purpose of the study into the experiences of able mathematicians.

4.1.2 Self-Perception Scale

The *Self-Perception Scale* relates to students' confidence in their own mathematical abilities. It originally had seven items however after conducting the Rasch analysis, we found that Question 2 (R), 'Studying mathematics makes me nervous' did not fit as well with the rest of the items, so it was removed from the scale. When the scale was reduced to six items a second Rasch analysis was carried out and all six items had infit and outfit values in the acceptable range. Table 2 below shows the fit statistics for the *Self-Perception Scale*. The Cronbach's alpha value for this scale is 0.801, the personal reliability index is 0.77 and the item reliability index is 0.95. These values indicate that the items in the scale are measuring the same trait for self-perception and the person measures can therefore be used for reliable statistical analysis.

A scale measure was computed for each participant, where a lower measure on this scale corresponded to a higher level of self-perception. Using these measures, a one-way ANOVA was run to test for any differences in the means between schools. We found a statistically significant difference between the group means ($F(4, 86) = 3.27$, $p = 0.015$). Tukey's HSD Test for multiple comparisons found that the mean *Self-Perception* measure was significantly lower for School 5, the all-boys school, than that for School 3, the all-girls school, ($p = 0.019$, 95% C.I. = [0.2219, 3.7184]). This reflects that the students in School 5 had a higher mean measure of self-perception in relation to their mathematical ability than their peers in School 3. Results from a t-test showed that there was a statistically significant difference between genders with the males showing that they felt more strongly than the females that they had confidence in their ability ($t(89) = 3.869$, $p < 0.01$).

Table 2 Item Fit for the Self-Perception Scale

	Survey Question	S.E	Infit MNSQ	Outfit MNSQ
Q7	I believe I have good mathematics problem-solving skills.	0.19	0.75	0.68
Q12	I am confident that I can solve problems that take a long time to solve.	0.18	0.99	1.03
Q17R	I am always confused in mathematics class.	0.2	0.93	0.97
Q33	I am confident in my ability to learn mathematics	0.21	0.91	0.84
Q35	I would rate my ability as a mathematician as above average in my school year.	0.18	1.35	1.28
Q45	I feel confident when presented with a problem that I have not seen before that I will be able to solve it.	0.17	1.05	1.08

The responses to these questions (see Appendix D) show that the majority of the students in the survey had high self-perception in mathematics with 93.1% in Q33 stating that they were confident about their ability to learn mathematics and 63.8% in Q35 believing themselves to be above average ability in their school year. The high percentages of students answering neutral for Q12 (34.5%) and Q45 (41.4%) shows that, despite their overall confidence, students do seem to be a little unsure about their ability to solve unseen or time-consuming problems.

4.1.3 Feeling Challenged Scale

Table 3 shows the final Rasch statistics for the *Feeling Challenged Scale*. There were originally 8 items in this scale. In the initial statistical analysis, Q3R 'I am often bored in class while the teacher explains the solution to other students' and Q41'What we do in mathematics class suits my ability' had item fit values outside the recommended range of 0.6 –1.4 so were removed from the scale. When the remaining 6 questions were analysed all 6 had infit and outfit values within the range 0.75 – 1.25. The final Cronbach's alpha value for this scale is 0.872, the personal reliability index is 0.86 and the item reliability index is 0.90. These values show that the items in the scale are measuring the same trait for *Feeling Challenged* and the person measures can therefore be used for reliable statistical analysis.

The Rasch model was used to compute person measures for each student in the *Feeling Challenged Scale* items, seen in Table 3 below. Higher measure scores represent those students who felt least challenged by school mathematics. There were 6 questions in the final *Feeling Challenged Scale*, therefore the maximum score in the Likert scales was 30. The student scores ranged from 7 to 30 and the corresponding person measures ranged from 7.27 to -5.55, where the highest measure of 7.27 represented the student who felt the least amount of challenge.

Table 3 Item Fit for the Feeling Challenged Scale

	Survey Question	**S.E**	**Misfit MNSQ**	**Outfit MNSQ**
Q8	I find the work in mathematics class demanding	0.17	1.01	0.99
Q22	I feel challenged to my maximum ability by mathematics problems given to me in class.	0.17	0.98	1.00
Q27	I am often given challenging problems in mathematics class	0.16	1.07	1.10
Q31	I find mathematics class challenging	0.17	0.75	0.78
Q36	We do challenging problems in mathematics class	0.16	0.80	0.75
Q46	I often get stuck in mathematics class	0.18	1.25	1.22

It can be seen from the responses to the *Feeling Challenged Scale* questions, in Figure 9 below, and Appendix D that the students in this study do not feel very challenged by their school mathematics. From the bar charts for the *Feeling Challenged Scale*, in Figure 9, we see that overall, there was a high percentage of students who felt under challenged. The results showed that 72.4% of students disagreed with the statement that they find the work in mathematics class demanding and 62.1% disagreed that they felt challenged to their maximum ability. Furthermore, even though about one third of the respondents agreed that they often encounter challenging problems in their mathematics classes, less than 9% of the group reported that they often feel stuck in class. This indicates that the level of challenge felt by students is quite low.

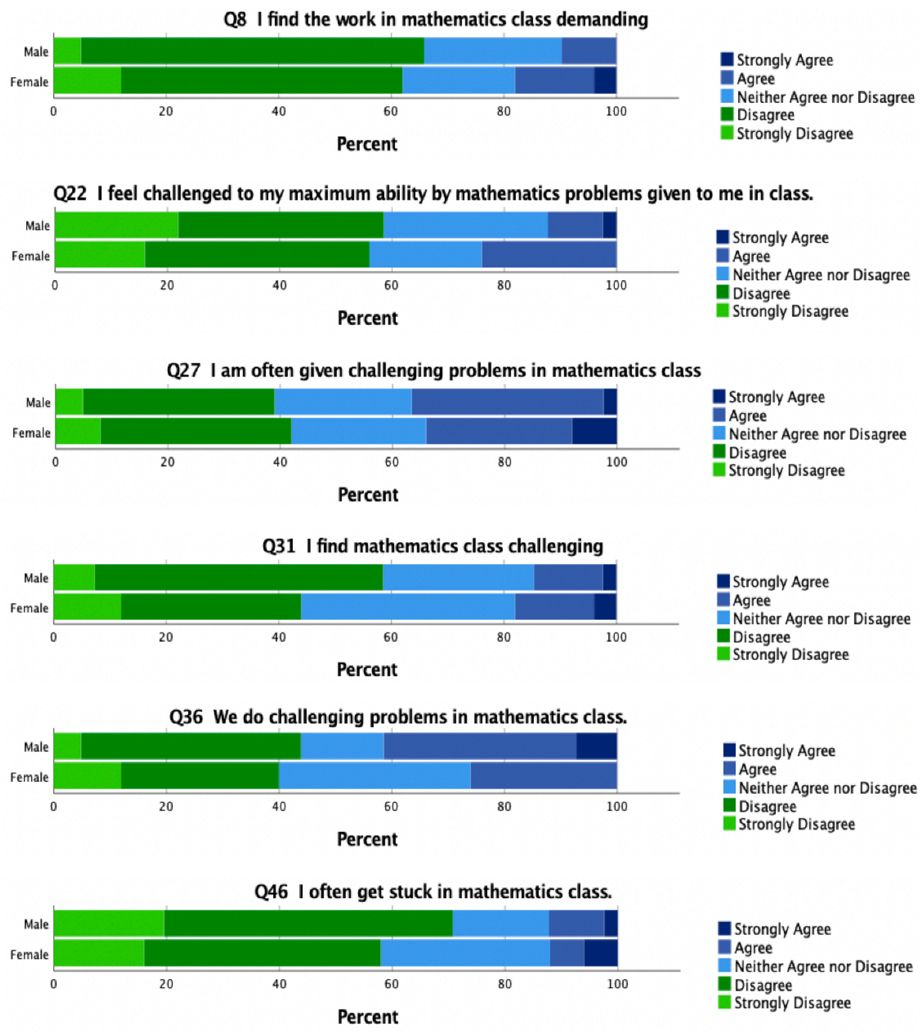


Figure 9 Percentages of Responses to Pre-Workshop Questions in the Final Feeling Challenged Scale

The mean *Feeling Challenged* scale measures for each of the five schools can be seen in Table 4 below. The table shows that School 2, a mixed gender school had the highest mean measure score meaning that they felt least challenged and School 1, one of the all-girls schools had the second highest score. A one-way ANOVA revealed that there was a statistically significant difference in mean *Feeling Challenge* measures between the schools ($F(4, 86) = 2.65$, $p = 0.039$). Tukey's HSD Test for multiple comparisons found some evidence of differences in *Feeling Challenged* measures between School 2 and Schools 3, 4 and 5 but they were not statistically significant at the alpha = 0.05 level, as shown in Table 5 below.

A further t-test on the mean *Feeling Challenged* measures was carried out to determine whether there was any statistically significant difference in the mean measures between genders. The results of this showed that a significant difference between genders was not found, ($t(89) = -0.235$, $p = 0.815$). From the t-test results, and the percentages of responses in the *Feeling Challenged Scale*, we can see that both males and females felt equally underchallenged by their current school mathematical experience.

Table 4 Mean of Feeling Challenged Scale Measures

School	Mean
1	1.3289
2	2.1132
3	0.6245
4	0.3035
5	0.1646

Table 5 Tukey HSD for multiple comparisons between the Feeling Challenged Scale Means of School 2 and the other schools

School	1	3	4	5
2	0.781	0.187	0.065	0.085

4.1.4 Motivated by Thinking Scale

This scale had 10 items in it initially but 4 of the items had large infit and outfit values when the Rasch analysis was conducted and were removed. These items can be seen in the miscellaneous questions in Table 6. The remaining 6 questions all had infit and outfit values within the accepted range. The final Cronbach's alpha value for this scale is 0.824, the personal reliability index is 0.76 and the item reliability index is 0.93. These values show that the items in the scale are measuring the same trait for feeling *Motivated by Thinking* and the person measures can therefore be used for reliable statistical analysis.

A one-way ANOVA on the person measures for each school did not find a statistically significant difference in the mean scores on this scale between the schools ($F(4, 86) = 1.99$, $p = 0.104$). A t-test was then calculated to test for a statistically significant gender difference in the mean measure. This test found that the males seemed to be more motivated by tasks that made them think, than the females ($t(89) = -0.235$, $p = 0.03$). From the responses to the individual questions on this scale, we can see that there is evidence that the majority of these students prefer to be given questions that make them think, rather than be shown the method first by their teacher. However, in Q38 it can be seen that 93.1% of students agreed that ‘Most problems I do in school can be answered by recall of examples and formulae’. This appears to contradict what the students state as their preferred type of question. The attitude of these students to such practices is evident in the results for Q37 which show that 70.7% of students want to work on questions with a higher level of challenge than those that they currently get in class.

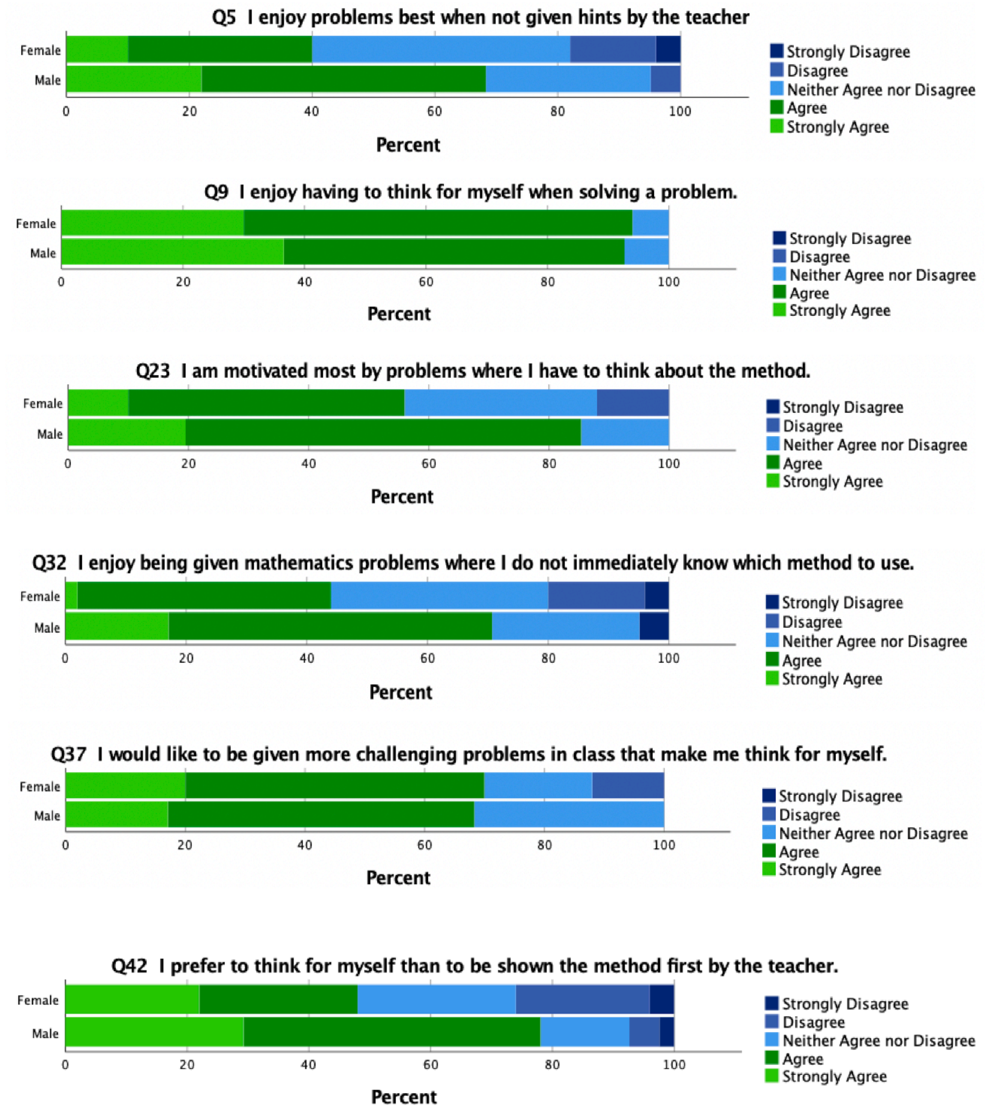


Figure 10 Percentages for the Pre-Workshop Motivated by Thinking Scale

Table 6 Item Fit for the Motivated by Thinking Scale.

	Motivated by Thinking Scale Questions	S.E	Misfit MNSQ	Outfit MNSQ
Q5	I enjoy problems best when I am not given any hints by the teacher.	0.17	1.11	1.14
Q9	I enjoy having to think for myself when solving a problem.	0.20	0.81	0.76
Q23	I am motivated most by problems where I have to think about the method	0.18	0.96	1.01
Q32	I enjoy being given mathematics problems where I do not immediately know which method to use.	0.17	0.88	0.95
Q37	I would like to be given more challenging problems in class that make me think for myself.	0.18	0.93	1.01
Q42	I prefer to think for myself than to be shown the method first by the teacher.	0.17	1.09	1.12

4.1.5 Remaining Scales

Two other scales were originally designed as part of this survey; these scales contained questions relating to teacher-/textbook-led pedagogy, and how students believed that their mathematical ability could be improved. When a Rasch Analysis was conducted on these scales, we found that neither of them was successful. There were a number of individual questions that did not fit a decisive scale yet provided information on student perceptions that was interesting for this study. When considered alongside the scales analysed above, these questions reinforced the significant aspects of the students' experience and what style of teaching they preferred. These items were retained for further analysis some of which can be seen in Table 7 below along with the original scales that they belonged to.

The results, shown in Appendix D, for Q16, Q34 & Q44 strongly support the findings for the *Feeling Challenged* and *Motivated by Thinking Scales*. The overall indication is that the students in this study enjoy more challenging questions and feel unchallenged in class. In Q13 and Q30, over 90% of students say that they are given examples before being set problems to solve. Similarly, over 93% of participants say that most of their mathematical tasks can be answered by recalling similar examples or formulae (Q38) and 86% of them say that these tasks are mostly taken from textbooks (Q24). From these responses there is strong evidence to indicate that these students experience traditional methods of instruction in their mathematics classrooms. However, the students themselves indicate that they would benefit from more challenge with 93% of them agreeing that they could increase their level of achievement if they worked on more challenging problems (Q44) and over 90% of them would like to work on these problems without a time limit (Q16).

Table 7 Sample of Results on Miscellaneous Questions of Interest.

	Original Scale	**Survey Question**	**Strongly Agree/ Agree**	**Neither**	**Disagree /Strongly Disagree**
Q13	Teacher - Text Book Led	I am usually given an example to explain a new topic before being given questions on it.	91.4 %	5.2%	3.4%
Q16	Challenge	I prefer to be given challenging problems with an unrestricted time to solve them.	89.7%	5.2%	5.2%
Q24	Teacher - Text Book Led	Most of the time we work on mathematics problems from the textbook.	86.2%	6.9%	6.8%
Q30	Teacher - Text Book Led	I am rarely given problems to do on new topics without being given an example first.	93.1%	5.2%	1.7%
Q34	How to Improve	I believe that I would become a better mathematician if school mathematics was more challenging.	59.6%	29.3%	12%
Q38	Teacher - Text Book Led	Most problems I do in school can be answered by recall of examples and formulae.	93.1%	3.4%	3.4%
Q44	How to Improve	I believe that I could increase my achievements in mathematics with more practice on challenging problems.	93.1%	5.2%	1.7%

4.2 Post Workshop Survey

After the workshops, the students were resurveyed to examine their opinion of the tasks presented and the peer collaborative nature of the workshops. As mentioned in Chapter 3, there were four students who could not attend the second workshop so did not complete the post-workshop survey. The percentages of responses to each question in the pre-workshop survey can be seen in Appendix F and the questions selected for each scale are in Appendix G. The results in this section are related to the questions which examine students' experience of the workshops. This will include an assessment of their enjoyment of the workshops, self-perception with regard to solving unfamiliar tasks, feelings of being challenged and their motivation for the tasks in the workshop.

As for the pre-workshop surveys person measures for each scale were computed using Winsteps (Linacre, 2021) and SPSS was used to find a corresponding Cronbach's alpha value for each scale. A mean measure per school was found for the successful scales using SPSS and a series of statistical tests was carried out with these means. These included testing for differences between the schools and between genders on their opinions of the workshops.

4.2.1 Enjoyment Scale

There were 6 items in this scale and a Rasch analysis of the responses showed that all had infit and outfit values inside the acceptable range of 0.6 – 1.4 (see Table 8 below). The final Cronbach's alpha value for this scale is 0.831, the personal reliability index is 0.72 and the item reliability index is 0.85. These values show that the scale can therefore be used for reliable statistical analysis.

The responses to the questions on this scale reflected that the majority of students who participated in the workshops enjoyed the experience. In QB6, 83.9% of the participants agreed or strongly agreed with the statement 'I found the experience of the workshop very enjoyable. Similarly, in QB21, 76.3% of the participants found the tasks in the workshops more interesting than what they do in school.

Table 8 Item Fit for the Post-Workshop Enjoyment Scale

	Survey Question	**S.E**	**Infit MNSQ**	**Outfit MNSQ**
QB1	I enjoyed the tasks of the workshops.	0.21	0.80	0.84
QB6	I found the experience of the workshops very enjoyable.	0.21	0.83	0.83
QB16	I enjoyed having to think hard during the workshops.	0.19	1.00	0.99
QB19	I enjoyed finding multi-solutions to problems more than just finding the answer in one way.	0.19	0.98	0.98
QB21	I found the tasks of the workshops more interesting than mathematics tasks I am used to doing in school.	0.20	1.25	1.38
QB23	These tasks reminded me about what I enjoy about mathematics.	0.18	0.92	1.03

The mean measures for each of the five schools in the Post-Workshop *Enjoyment Scale* can be seen in Table 9 below.

Table 9 Mean of Post-Workshop Enjoyment Scale Measures

School	**Post- Workshop Mean**	**Standard deviation**
1	−1.9005	2.24082
2	−2.7322	1.67327
3	−2.8381	2.02545
4	−2.8965	1.83852
5	−2.8192	1.72817

A one-way ANOVA was carried out on the mean *Enjoyment* measures to determine whether there was any statistically significant difference in the mean measure on this scale between schools. The results of the analysis showed that there were no significant differences in the mean *Enjoyment* measures between any of the schools ($F(4,81) = 0.550$, $p = 0.699$). These results reflect, what the percentages, in Appendix F, also indicate, that all five schools seemed to enjoy their experience of the workshops.

A t-test carried out to test for differences between genders showed that there was a statistically significant difference in measures between males and females ($t(85) = 2.182$, $p = 0.035$). The female measures had a mean of -2.2115 and the male measures had a mean of -3.0810. The significantly lower measure score of the males conveys that they were in higher agreement with the *Enjoyment Scale* items on the workshops. We can conclude that the students in each of the five schools appear to enjoy the mathematics of the workshops to a similar extent but that the males enjoyed it more than the females.

4.2.2 Self-Perception Scale

I deleted one question from the original *Self-Perception Scale* after conducting the Rasch analysis. Item, QB25 (R), 'When solving unfamiliar mathematics problems I give up trying when I feel uncomfortable' did not fit as well with the rest of the items, so it was removed from the scale. After the Rasch analysis showed that the questions in the multi-solution tasks scale did not fit the required reliability, I added QB29, 'I have learned some useful new skills for problem-solving by solving tasks in different ways' to the *Self-Perception Scale.* Since having more confidence in your problem-solving skills is an important aspect of self-perception this seemed like a valid decision. When a second Rasch analysis was carried out and for the final five items the infit and outfit values were in the acceptable range. Table 10 below shows the fit statistics for the *Self-Perception Scale*. The Cronbach's alpha value for this scale is 0.832, the personal reliability index is 0.75 and the item reliability index is 0.78. These values indicate that the items in the scale are measuring the same trait for self-perception and the person measures can therefore be used for reliable statistical analysis.

A scale measure was computed for each participant. For the five questions in the final *Self-Perception Scale*, the maximum score in the Likert scales was 25. Except for one score of 21 the remaining student scores ranged from 5 to 13 and the corresponding person measures ranged from -9.40 to -1.84, where the lowest measure of -9.40 represented the student who felt most confident in their ability to solve unfamiliar tasks.

Table 10 Item Fit for the Post-Workshop Self-Perception Scale

	Survey Question	S.E	Infit MNSQ	Outfit MNSQ
QB10	Solving unfamiliar mathematics problems improves My problem-solving skills.	0.28	0.94	1.29
QB15	When I solve unfamiliar mathematics problems, I feel more confident that, in the future, I will be able to figure out what other problems are asking me.	0.26	1.05	1.00
QB20	I feel more confident I will find a solution to unfamiliar mathematics problems with perseverance.	0.25	0.83	0.82
QB29	I have learned some useful new skills for problem-solving by solving tasks in different ways.	0.25	1.06	1.06
QB30	I feel more confident in my ability to solve unfamiliar mathematics problems.	0.25	1.03	0.95

Table 11 Mean of Self-Perception Scale Measures

School	Pre-Workshop Mean	Post-Workshop Mean
1	−1.8505	−5.6855
2	−2.7505	−6.0444
3	−1.5460	−6.5475
4	−1.9265	−5.9345
5	−3.5162	−6.1900

Using these measures, an ANOVA was run to test for any differences in the means between schools. It was found that there was no statistically significant difference for the mean *Self-Perception* measures between the of schools ($F(4,81) = 0.21$, $p = 0.933$). All schools had low measure scores which signified that they were in high agreement that they felt more confident about their ability to solve unfamiliar tasks after their experience in the workshops. It was interesting to note that in the pre-workshop surveys School 3 had the highest self-perception mean measure, hence the lowest self-perception, but in the post-workshop surveys this school had the lowest self-perception mean measure (see Table 11 above). Results from a t-test showed that there was no statistically significant difference between genders on the items in this scale ($t\ (85) = 0.583$, $p = 0.562$). This indicates that there is no evidence to reject the null hypotheses that both males and females felt more confident to solve unseen tasks after their experience of the workshops.

When the percentages for the responses to the items in this scale are examined (see Appendix F) we can see that most of the students in the survey felt much more confident about their ability to solve unfamiliar mathematical tasks. Examples of this can be seen in QB10 where 100% of the students agreed that ‘Solving unfamiliar mathematics problems improves my problem-solving skills’, and also in QB20 where 91.9% stated that they feel more confident they will find a solution to unfamiliar mathematics problems with perseverance. This contrasts the pre-workshop survey Q45 where only 39.6% agreed that they felt confident that they would be able to solve an unseen problem. Overall, the students were in agreement that the workshops had been a learning experience for them with 93% in QB29 agreeing that they felt they had learnt some useful new problem-solving skills by solving tasks in different ways.

4.2.3 Motivation Scale

This scale had five items in it initially, but I decided to move one of the items, QB23 ‘These tasks reminded me about what I enjoy about mathematics’ to the *Enjoyment Scale*. The remaining four questions all had infit and outfit values within the accepted range (see Table 12 below).

Table 12 Item Fit for the Post-Workshop Motivation Scale

	Survey Question	**S.E**	**Infit MNSQ**	**Outfit MNSQ**
QB4	I would be interested in doing the tasks in the workshop in my own time.	0.18	0.94	0.98
QB9	The tasks in the workshop encouraged me to persevere when stuck.	0.21	1.14	0.98
QB18	I would like to do more of these tasks.	0.20	0.72	0.72
QB28	I preferred doing the tasks in the workshops to those we do in school textbooks.	0.20	1.21	1.24

The final Cronbach’s alpha value for this scale is 0.733, the personal reliability index is 0.73 and the item reliability index is 0.95. The impact of moving QB23 on the person and item reliability was minimal, but I felt that QB23 suited the *Enjoyment Scale* best. These values obtained from the Rasch analysis show that the items in the scale are measuring the same trait for *Motivation* and the person measures can therefore be used for reliable statistical analysis.

The results of an ANOVA ($F(4,81) = 1.5115$, $p = 0.206$) found no significant difference between the means of the schools. A t-test was also carried out to test for a statistically significant difference of the mean measures between genders. This analysis found that there was no significant difference in motivation by the tasks of the workshops between genders ($t(85) = 0.583$, $p = 0.072$). This result indicates that we have no evidence to reject the hypothesis that both male and female students felt equally motivated by the tasks in the workshops.

4.2.4 Feeling Challenged Scale

There were originally five items on the *Challenge Scale*. The final Cronbach's alpha value for this scale is 0.762, the item reliability index is 0.86 but the personal reliability index was only 0.64. This personal reliability index meant that the items in the scale were not reliable for measuring the same trait for *Challenge* so the person measures could not therefore be used for reliable statistical analysis. Some of the answers seemed either contradictory or that the students had interpreted the word 'challenge' differently to the way it had been interpreted in the survey design. For example, two of the students who had given a response of strongly disagree to the QB8 'School mathematics is not as challenging as the mathematics in the workshops' had also answered strongly agree to QB27 'I had to think a lot more when solving the tasks in the workshops than in class'.

However, despite the low personal reliability of the scale the response frequencies to many of the questions, shown in Figure 11 below, are in themselves interesting pieces of data. These frequencies provide noteworthy back-up to the evidence that will be discussed in the interview and audio recording analysis in Chapters 6 and 7 respectively. In particular, the frequencies for QB13 showed that 88.4% of the students in the study agreed that the workshops pushed them more to their maximum ability. Similarly, the frequencies for QB27 showed that 94.2% of the students agreed that they had to think a lot more when solving the tasks in the workshops. The evidence from these frequencies supports the belief that the students found the workshops challenging.

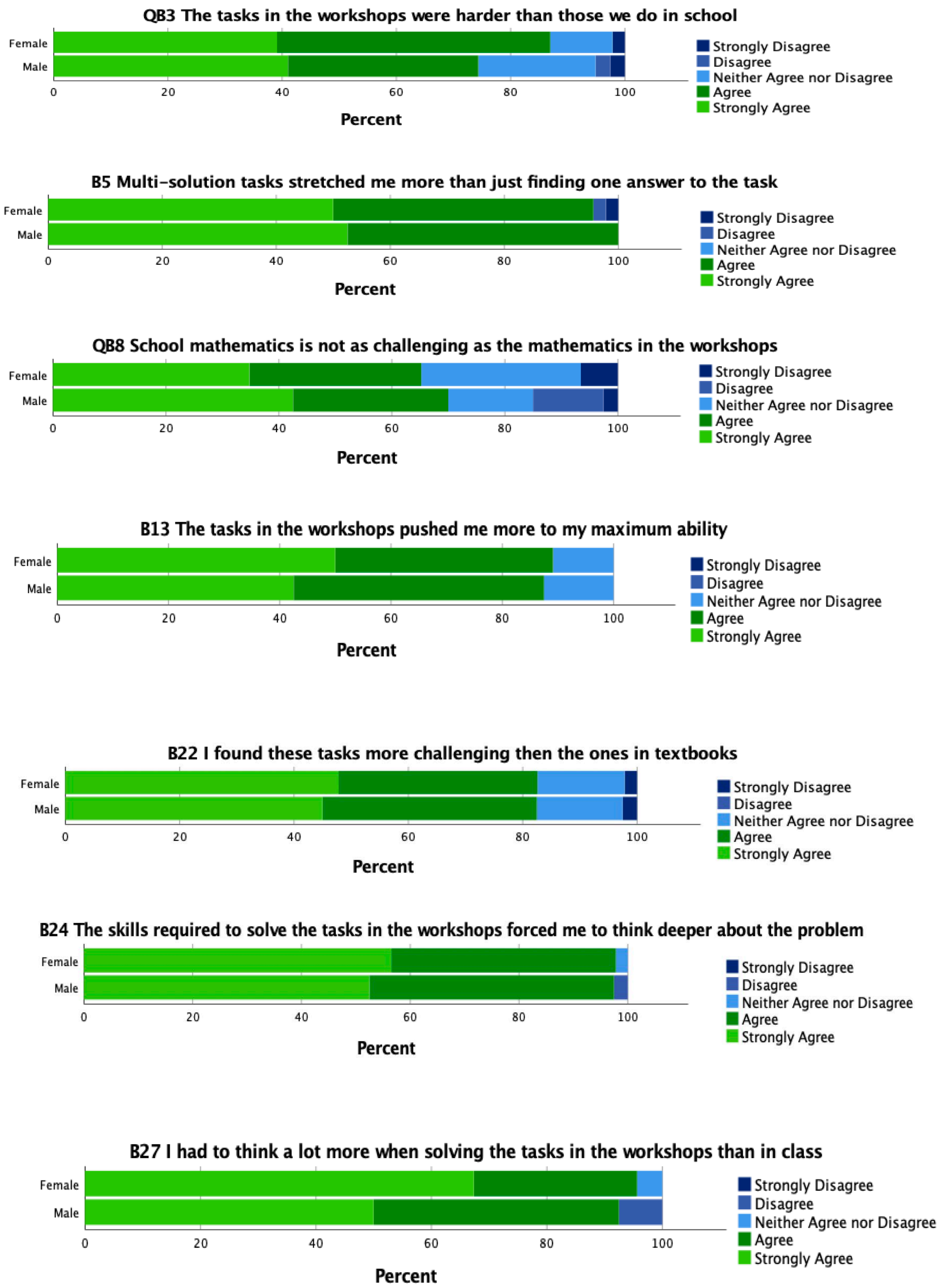


Figure 11 Percentages of Responses to Post-Workshop Challenge Scale

4.2.5 Remaining Items

Originally there were two scales focusing on *Multi-Solution Tasks* and *Task Difference* that were designed as part of this survey. When a Rasch Analysis was conducted on these scales, it was found that neither of them was successful. However, there were a number of individual questions that did not fit the intended scale yet provided information on student perceptions to the workshops that were interesting for this study. The survey was intended to highlight the different experiences of the workshops and for that reason the student responses to questions on task differences and using multi-solution tasks were very relevant. The items of interest were retained for further analysis and will be used in discussions in later chapters. The percentages of responses to the remaining questions can be seen in Appendix F and some are displayed in the Figure 12 bar charts below.

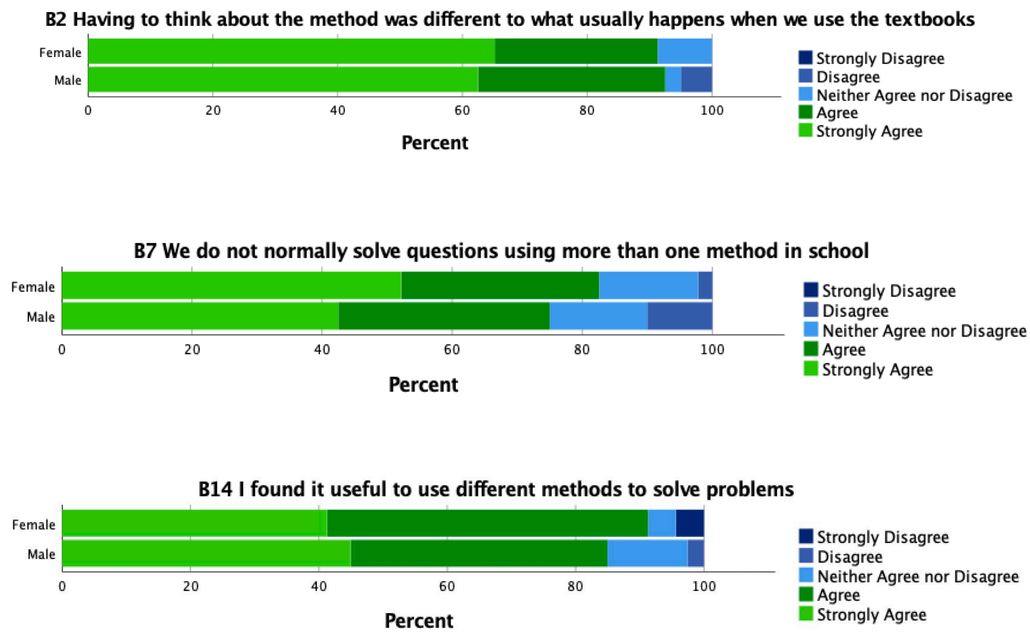


Figure 12 Percentages of Responses to some of the Post-Survey Questions in the Remaining Scales

The items in the two unsuccessful scales reflect the different experiences the students had in the workshops. It is evident that having to think of a method was an important feature of the workshops and contrasted the responses in the pre-workshop surveys which discussed the typical experience of these students in their classrooms. The responses to QB17 and QB26 also show that the students found the tasks very different in the workshops. These results highlight that exploring the multi-solution tasks in the workshops was definitely a novel experience for the students. The benefits of them for the students are apparent from the responses to QB14 that shows 87.3% of the students in the study found their experience of multi-solution tasks useful for solving problems.

4.2.6 Summary of Survey Findings

The survey findings show strong evidence that the students in this survey do not feel challenged by their school mathematics. In contrast, the results indicate that the tasks the students were presented with in the workshops provided much more of a challenge to them. It appears that a large part of this may be the type of task they are being presented with in each situation. The percentages to the questions on Task difference, (Appendix G), highlight that the students clearly found the workshop tasks very different to those that they experience in school. By examining the pre-workshop survey responses, it can be seen that the students are clearly motivated by tasks which make them think. However, it appears that they are not getting opportunities to do this in class because they are predominantly being given an example of the method to use first.

The above findings will be further discussed in conjunction with the interview and audio data from the workshops in later chapters. However, from the survey findings alone, there is evidence to support the belief that the students had a very different experience of mathematics in the classroom and the workshops. The question responses also showed signs that the lack of opportunities to experience unfamiliar questions may have an effect on their confidence to do so. The data that follows, from the interviews and audio recordings of the workshops, enabled the students to express their perceptions in more detail. The student discussions will draw attention to what the most noteworthy differences were, and which were of most importance to them from an enjoyment and learning perspective.

Chapter 5 Template Analysis

5.1 Introduction

This research is specifically focused on the students' perspective of their experiences in their classrooms and the workshops. Through a series of focus group interviews, the thoughts and perceptions of the students were explored using open questioning and discussion. Some of the questions were specifically related to features of classroom instruction that the research literature had recommended for challenging highly able mathematicians. Other questions were more open to allow for the emergence of new themes that may be relevant to help understand how the students perceived their learning experience of mathematics to date. In total eleven group interviews were held with 35 students, the first three interviews being face to face and the remaining on zoom.

The type of data that would transpire, and the rationale behind holding the interviews, were important when considering which of the numerous qualitative analysis methods would fit the study best. Given the open questioning and the researcher's unfamiliarity with most of the schools in the study, a flexible analysis technique was required. It was anticipated that the technique would need to be adapted as themes emerged from the discussions with the students. For this reason, it was decided upon a technique of analysis that could be tailored to the specific data set rather than one that was too prescribed. Even within one country, and a type of school within that country, schools can be very different. The perceptions of the students within each school tells a story of their mathematical beliefs and their personal experiences of mathematics. The interview data in this study may not reflect every school in the country but nevertheless it is important if it highlights student experiences that could be enhanced with further research.

The purpose of the interviews was to look for evidence of Sriraman's five Principles, and opportunities for students to experience Vygotsky's ZPD in their classrooms and workshops. From this perspective the realist approach of grounded theory, that the student opinions are awaiting discovery by the researcher, did not seem suitable for this study. The literature review had already identified many characteristics of classrooms that were recommended for challenging highly able students, including specific reference to those that hindered or fostered creativity. Selecting a technique that would allow the analysis to build upon these recommendations was the objective.

With these considerations in mind, Template Analysis was decided upon as a procedure to analyse the interview data. The details of this technique will be outlined below and how it was used to develop a template that incorporated the themes relevant to the research project.

5.2 What is Template Analysis

Template Analysis is considered to be a technique for thematically organising and analysing qualitative data rather than a methodology (Crabtree & Millar, 1999; King, 2004, 2012). The procedure involves analysing the textual data to look for evidence of the occurrence of themes. The themes identified are those that describe the phenomena of relevance to the research questions of a particular study. It has been widely used in management research (King & Brooks, 2017), more recently in psychology (Brooks et. al, 2015) but less frequently in educational research (Au, 2007). Template Analysis gives the researcher the flexibility to work with the participants' data, relevant to the specific research questions, and uncover the depth of what is there. The ability to tailor the template to highlight the areas where the data is richest enables the researcher to gain a better understanding of the nature of the experiences that appear most relevant to the participants. Template Analysis is also believed to be less time-consuming than other forms of qualitative analysis and can be used for larger data sets (King, 2004).

The techniques that give Template Analysis its unique character centre upon organising textual data into themes starting from a set of broad codes, defined by the researcher, which then develop into a template through a series of iterations. The process can be seen as two phases of analysis that eventually produce a set of codes of a hierarchical nature. It begins with a preliminary reading of a subset of data during which the researcher examines the text for recurrent words or phrases that characterise the perceptions of the participants. The phrases that are seen by the researcher as relevant to their research questions, are identified as a priori codes, from which the initial template with more specialised codes will be formed. The second phase of the analysis involves applying the initial template to the complete data set and continually modifying it until the researcher is satisfied that the template can be considered finalised. These phases will be outlined in more detail later.

5.3 Features of Template Analysis

King (2004) describes the technique of Template Analysis as the middle ground between 'top down' methodologies such as Framework Analysis and 'bottom up' inductive processes such as Grounded Theory or Interpretative Phenomenological Analysis (IPA). Unlike other forms of thematic analysis, Template Analysis does not have a prescribed method for data collection, presentation style of template, or manner in which it is modified. This flexibility allows researchers to investigate the validity of their ideas in a manner that suits their particular data set. Template Analysis has been described as most similar to Braun & Clark's Thematic Analysis (2006) or to Framework Analysis (Brooks et al., 2015). Like the former, Template Analysis does not require the researcher to use a prescribed template and they both use a hierarchical coding structure. However, it differs from Braun & Clark in several ways: it produces an initial template from a sub-set of data rather than the whole set; it describes themes early in the analysis; it uses more levels of hierarchical coding to describe rich areas of data. Both Framework Analysis and Template Analysis develop an initial template from a priori codes used on a subset of data but Framework Analysis uses a more prescriptive matrix style template. The main differences in the process are the emphasis Template Analysis puts on the iterative process and in describing the details of the whole coding process during the writing-up stage of the analysis.

5.3.1 When it is appropriate to use Template Analysis and when not

Template Analysis is suitable for many types of textual data which focus on the content of what the participants are saying such as interview scripts, focus groups (Brooks, 2014), responses to open ended questions, and comparisons of research articles (Au, 2007). There are many qualitative research studies for which Template Analysis is suitable but also some situations where it is not an appropriate technique. Its flexibility enables it to be used by researchers using a variety of approaches including both 'realist' and 'constructivist' positions and those somewhere in-between these. 'Realist' methodologies, such as grounded theory, believe that there is only one reality and that the perceptions of the participants would be discovered through analysis of the data. For the methodology to be consistent with this epistemological stance it should be objective and free from any internal influence from the researchers' values and beliefs.

In contrast, Template Analysis is also suited to research taking the 'contextual constructivist' approach, as outlined by Madill et al. (2000), which believes that the social and cultural interpretations of both researcher and participant are embedded in the data. The researcher seeks to explore and understand the world of the participants, from their perspective. Hence, there will be multiple interpretations of the data reflecting these perspectives rather than one 'reality'. Furthermore, the social context of the research topic will further influence the data and create differing outlooks on the knowledge that emerges from the analysis (King, 2012). Researchers using this type of 'bottom up' inductive approach in Template Analysis would not be as reliant on a priori codes (Brooks et al., 2015), preferring the themes to develop from the data rather than from existing theory. The social context of both participants and researchers should be clearly stated in the analysis to ensure rigor is maintained.

In between these two approaches is 'subtle realism' which, as described by Hammersley (1992), holds that whilst the phenomena being analysed are independent of the researcher, the researcher is inevitably part of the reality. Hammersley maintains that subtle realism also asserts that the researcher must make a distinction between reality and what can be proved beyond reasonable doubt. The philosophy states that all knowledge is constructed by the researcher, rather than being logically derived. The knowledge is acquired by answering questions about the phenomena which can therefore lead to multiple explanations of the same phenomena depending on the questions asked (Hammersley, 1992). It acknowledges that prior assumptions can lead to insights, but the researcher must be careful to avoid potential errors when doing so. By carefully explaining how the knowledge claim was produced, the researcher can increase the credibility of the knowledge.

Template Analysis is not suited to those social constructivist methodologies which claim that language constructs social reality. Similarly, it is not suitable for mixed qualitative and quantitative analysis where only the frequency of a code is assumed to determine the significance of it. In the latter case Template Analysis allows the researcher to make a judgement on the significance of a phrase even if it is not reiterated by others in the study. Whatever stance the research holds, it is important at the outset for the researcher to be consistent with epistemological assumptions and to make these very clear in the analysis write up. In approaches where it has been acknowledged that there may be multiple interpretations of the phenomenon, the various interpretations should be made clear.

5.3.2 Development of the Initial Template

Brooks & King (2014) suggest that the first stage of carrying out Template Analysis is for the researcher to make themselves familiar with the data by reading through as much of it as possible. They advocate that this will give the researcher a sense of what is being said and what the key underlying themes might be. The next stage they suggest is to carry out preliminary coding of the data by reading through a subset of the data and highlighting words or phrases that seem relevant to the research questions along with anything else notable which may not have previously been considered. The selected codes are themes recognised as important in the research literature or themes that the researcher has already identified as relevant to the research questions (Cohen et al., 2018). The overall aim of the coding is to categorise the data by breaking it into segments that describe similar themes to be analysed. The researcher may choose to organise the transcripts in a manner that assists coding, such as double spacing with wide margins. This enables codes to be clearly written in the margins during the analysis. There are some important terms that give Template Analysis its uniqueness and these are outlined below:

A priori codes.

When carrying out the preliminary coding the researcher may have an initial set of codes, a priori codes, prepared in advance, although this is not an essential element of Template Analysis. As the coding process begins, a list is made of these codes and kept visible throughout the reading stage. When a piece of textual data appears relevant to one of these codes a note is made in the margin. It can be easy to get distracted by such themes, and miss other vital pieces of data, so an awareness that these are merely a starting point to be modified, or even deleted if irrelevant, is crucial. Their overall purpose is to speed up the initial phases of coding by giving the researcher areas to focus on.

Initial template

After the initial coding the next phase is for the researcher to group the a priori codes into clusters of connected themes to create the initial template (Symon & Cassell, 2012). This template should be designed in a way that illustrates how the themes are connected with each other and how the codes within each theme are related. If the data from the participants is quite varied, it may be necessary to increase the subset of data selected for the preliminary coding to ensure that all the key experiences have been included in the initial template. As the data is read, themes that emerge are added in response to the data, either in the form of adapting existing codes or creating new ones (Cohen et al., 2018). Once the researcher feels that no new themes are emerging from the data the preliminary coding phase can be said to be completed. When to begin the construction of the initial template is up to the researcher's judgement and will depend on the amount of data collected and the depth of the subset of data analysed in the preliminary coding (Symon & Cassell, 2012).

Hierarchical coding

Template Analysis allows for a more in-depth study of those codes that seem to be important to the participants. Where the data appears richer, hierarchical coding of the template provides an opportunity to expand the themes accordingly to ensure that the finer details of the participants' experiences have been coded. The initial template is designed with a hierarchical structure of broader higher-order themes which are then sub-divided into more specific themes labelled second, third order etc. This enables coding of the participants experiences to be grouped together into similar themes at the outset but then broken down to identify distinct differences within these themes. In situations where there might be phenomena of specific importance to the researcher, hierarchical coding enables the researcher to explore this theme in as fine a detail as required.

Parallel coding

There may be phrases that appear to be relevant in more than one theme because of potentially different interpretations of the data. Where a piece of text seems to be relevant to two different themes, Template Analysis has the facility to parallel code. This enables the same segment of data to be recorded in two or more different codes of the same level.

Integrative codes

Where there are relationships that appear to connect several higher-order themes laterally integrative codes can be introduced to classify these experiences. These can serve to highlight themes that the participants may not have expressed explicitly but were identified through the language used in the description of their experiences in the data.

5.3.3 The Final Template

Once the initial template has been constructed, it can then be applied to additional data. As the researcher reads through further sets of data, the template is modified to accommodate new information that does not fit into any pre-established code in the template. This flexibility enables the researcher to ensure that the template is suited to the specific data emerging from the study. Modification of the initial template can include inserting new codes, deleting existing codes, changing the scope or hierarchy of a code and changing the classification, or higher-order theme (King, 2004). These modifications can be noted as the researcher reads through the data sets and then a new template is constructed in stages rather than after every modification. The above process is repeated through an iterative process until the final template has been established. To ensure the rigor of the analysis, it is essential that the researcher maintains an open mind throughout the analysis and is flexible to the emergence of new and unexpected data. Deciding when the template is 'complete' may depend on limitations such as time or funding. Once the template has been applied to the full data set and all the relevant data emerging has been coded, the template can be considered 'final'. At this stage the template can be used to interpret the data, and the reasons behind the various iterations should be described when writing up the research findings.

There are several techniques used by researchers to ensure the consistency and validity of the template. The first technique is, if possible, to carry out research in a team where the collaboration of team members can be used to discuss and verify codes before modifications are finalised. Another technique is to make notes on the template to explain the rationale behind codes that may be ambiguous. This can help the researcher, or research team, maintain consistency when assigning a code to a phrase. Finally, it is very useful to get an independent coder to analyse a subset of the data using the template. Once this has been done the percentage agreement can be determined and a decision taken by the researcher on whether the template is sufficiently accurate to be deemed 'final'.

5.3.4 Writing up and presenting the analysis

When describing the process of Template Analysis during the writing up stage it is important to outline the rationale behind the modifications made to the template. This can be done in several ways: by making comparisons of the data case by case; by approaching the analysis theme by theme; by presenting the themes with a selection of case studies to illustrate the main themes (Brooks & King, 2014). In each case, including participant quotes from the data transcripts are very useful for illustrating the codes and justifying the modifications made during the iterations. The insertion of such quotes also gives the reader the opportunity to evaluate the final interpretation presented by the researcher. Deciding how to present the final template is open to the interpretation of the researcher. It will depend on the context of the study and the purpose. The most typical presentations are either a list of codes and sub codes that can be easily displayed or else a mind map, which can be useful for showing integrative codes.

5.4 Applying Template Analysis to the Data

The suitability of Template Analysis for the educational research in this study lies in its flexibility to customise the format and depth of coding to suit the variety of responses of the students to their experience of mathematics. The semi-structured nature of the interviews meant that new themes inevitably developed throughout the discussion. Template Analysis provided the researcher with the flexibility to modify the template throughout the phases of coding to ensure that all relevant data was being recorded as it emerged. This facility ensured that the researcher's experiences and beliefs did not prevent the identification of new themes that emerged from the students' discussions. The hierarchical nature of Template Analysis also seemed to fit well with an examination of students' classroom experiences because of the variety and depth of these experiences. This feature allowed for the creation of higher-order codes representing themes that were more influential in determining student perceptions and therefore would provide a structure for answering my research questions more clearly. The hierarchical structure also enabled me codes related to Vygotsky's theories and each of Sriraman's principles to be assigned to the data. It is, however, important that, when using Template Analysis, the researcher keeps an open mind to view the data from the different perspectives of individual students and schools. It must also be considered that the data might reflect a classroom environment that might be viewed differently by the teacher in each case.

The 'contextual constructivist' approach of template analysis, outlined by Mandill et al. (2000), was considered an appropriate technique for analysing this set of data because it recognises that there is not one definitive interpretation of the data. From an educational perspective this technique is suitable for this study because it reflects the position of an experienced teacher within the education system as a researcher. From discussion with students over the years it has appeared clear to the researcher that certain aspects of the classroom experience were viewed differently by individuals in the same classroom. The participants of this study were students from diverse cultural backgrounds, with different teachers, experiences and learning needs. It was felt that because of this they may have different opinions on similar practises that occurred in school and in the workshops. The facility within template analysis to parallel code data further highlighted its suitability given this potential diversity of opinions on the same classroom situation. Parallel coding of student comments made it possible to analyse how different individuals are affected by various features of their mathematics education. By taking a cross section of students in terms of school, gender, relative ability, and personality, it was hoped that a rich variety of sub-themes would emerge from the interviews. The final decision was based on the belief that template analysis enabled me to look for evidence of teaching methods that might foster creativity and contribute to students feeling challenged without using a prescribed methodology that might hinder the emergence of data.

5.4.1 Role of A Priori Codes

The role of a priori codes in developing an initial template suited my methodology and made Template Analysis a suitable option. As mentioned above, it was important for the codes to be relevant to the research questions and the theories of Sriraman and Vygotsky. Before commencing the coding, the researcher read through several of the transcripts to get a feel for what the students were saying in response to the questions. During this reading phase an open mind was kept to look for themes that recurred, within and between schools, or those that the students seemed to have strong feelings on. From this seven a priori codes were constructed, shown in Table 14 below, to describe the data for the preliminary analysis.

Table 13 Initial a Priori Themes in School 1, Interview 1.

A Priori Theme	Description
Level of challenge	Student perception, preparation of work by teacher.
Being motivated by thinking about methods	Did students enjoy having to think, regularity, importance attached to thinking.
Attitude to mathematics	Enjoyment of maths and importance of maths.
Feeling bored	Student perception of feeling bored.
Methods employed by teacher	Includes using examples, procedural questions, differentiated instruction.
Role of textbooks	Frequency of use, as a guide for class work, as further examples.
Groupwork	Enjoyment of group work, regularity.

The codes were based on the main questions in the interviews where students had given detailed responses. The choice of these initial codes, and their descriptions, aligned with the themes in the surveys and it was hoped that they would give a deeper insight into how the students felt about their experiences in the classroom and workshops. A subset of my data, Interview 1, with five students from one of the schools, was then examined. I reread the interview and looked for evidence of the a priori codes in the comments made by the students on their classrooms and then on their experience of the workshops.

5.4.2 Aligning the a priori codes with my Theoretical Frameworks

After reading through two further interviews, namely interview 5 and interview 7, it became evident that there was much overlap in the a priori codes. For example, the '*Methods employed by teacher*' code included comments on following the teachers' method that could equally be coded as a comment on '*Level of challenge*' or '*Feeling bored*':

> Student 036: They would write up notes on the board, you'd take them down and then for homework that night you would get questions and then the next day you just constantly do questions until…Yawn.

It seemed that some of the a priori codes were overlapping with other a priori codes rather than being distinct codes in themselves. A method was therefore sought to find a way to separate them into more distinct entities to create an initial template with a better structure. It was decided to focus on the theoretical frameworks used for the overall research project, namely, Sriraman's Five Principles to Maximise Creativity and Vygotsky's Sociocultural Theory of Cognitive Development. This seemed logical given that these frameworks encompassed the features of mathematics instruction that the literature had recommended for encouraging creativity in highly able students. Furthermore, in the preliminary analysis of the interviews there were recurring comments describing teacher-centred classrooms. Such comments suggested that the theories proposing classroom environments that cultivate student autonomy and creativity were relevant themes for the initial template. Consequently, the a priori codes were revised and regrouped to parallel Sriraman's Principles, see Figure 13 below. How each of the codes were integrated into each Principal will be explained after these principles and how they were interpreted are expanded upon. By designing the template to reflect Sriraman's framework it was hoped that evidence of his five Principles in action in the classroom and the workshop would emerge. The process by which the initial template was formed is explained in detail below. Vygotsky's theories were integrated at a later stage.

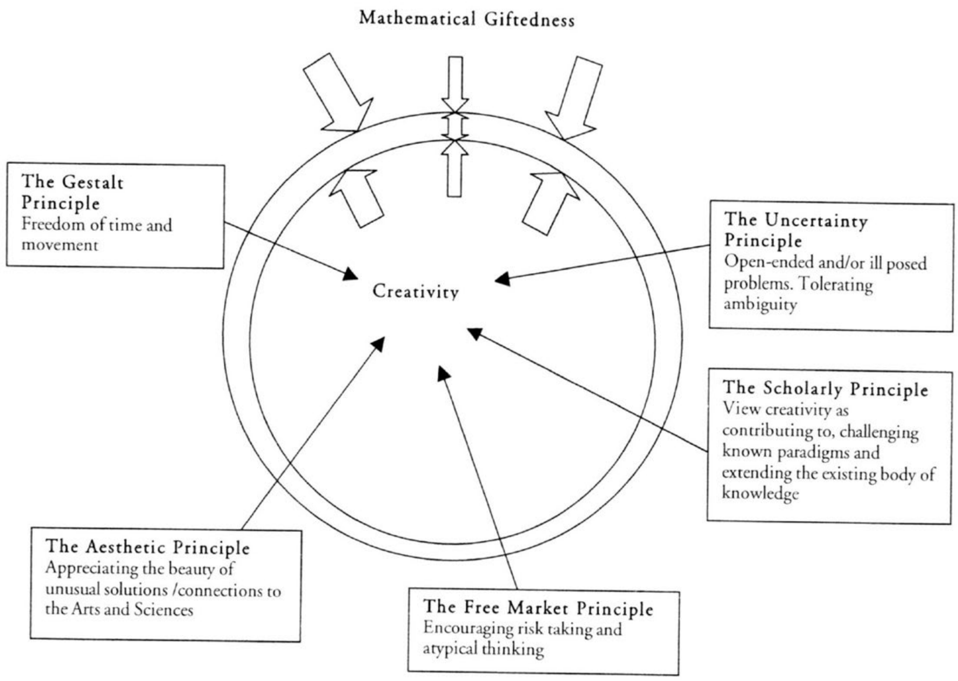


Figure 13 Sriraman's Five Principles to Maximise Creativity

Before designing an initial template from which to begin the coding on a larger scale, it was important to decide upon an interpretation of the five Principles in relation to second-level mathematics for the age group of the participants in the study. The Principles were initially designed for K-12 students (Sriraman, 2005) who would be typically two years older than the students in this study. For this reason, some of the practices recommended would be more applicable to an older class group, as will be explained later.

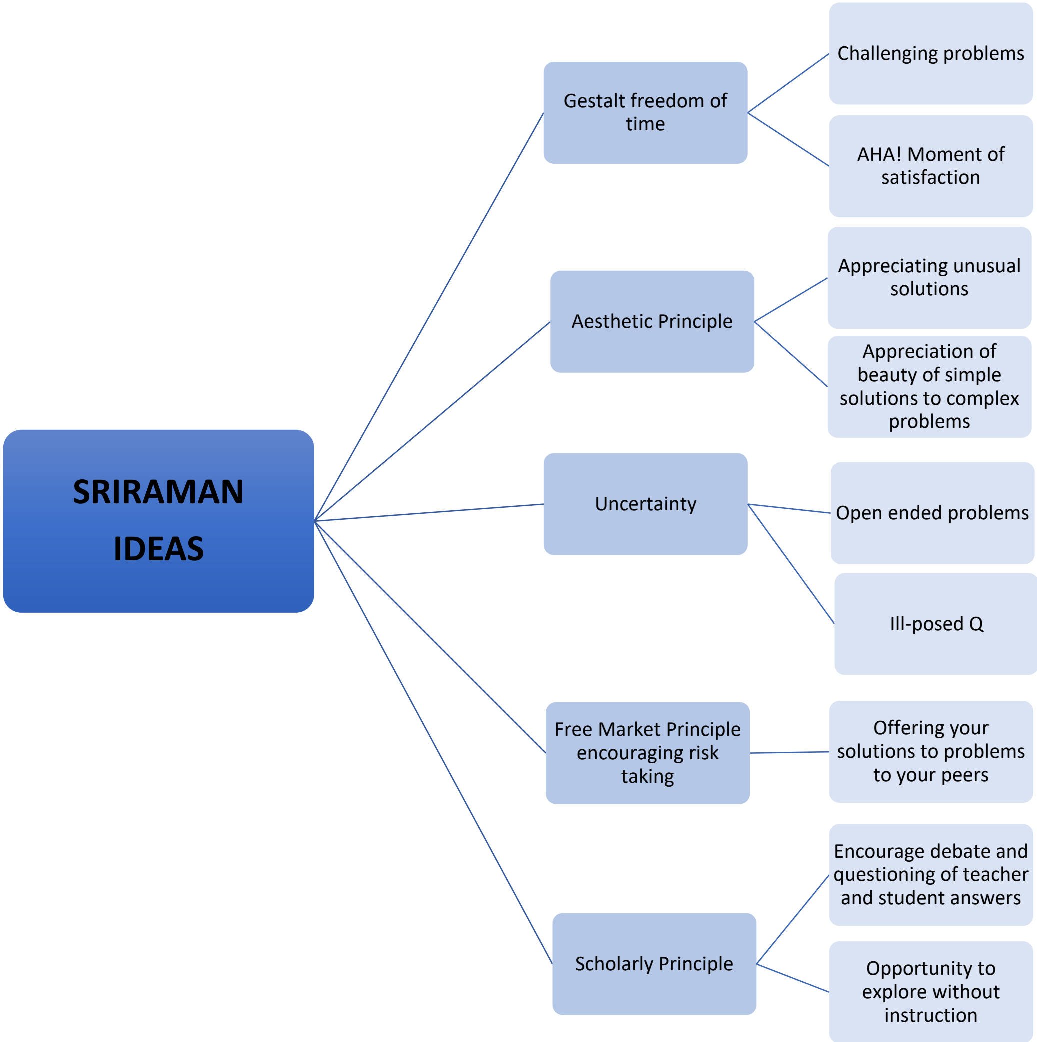


Figure 14 Outline of Template Themes Based on Sriraman's Principles to Maximise Creativity

5.4.3 Application of the Gestalt Principle to the template

The first Principle requires students to be given the opportunity to engage in the four-stage creativity process of the Gestalt psychology principle outlined by Wallas (1926) and Hadamard (1945): initiation-incubation-illumination-verification. This process advocates stages during problem solving that are believed to be central in order for mathematicians to develop their creativity and experience feelings of perseverance and satisfaction. In particular, providing challenging problems and allowing students to experience satisfaction, as outlined in Figure 14 above, is a key feature of this Principle. The initiation stage is when students read and familiarise themselves with the problem. Incubation requires a period of time to let the mind work on the problem, which would imply minimal teacher intervention in a classroom situation. Illumination is the AHA! experience, described by Liljedahl (2005) as the moment after a "lengthy, and seemingly fruitless, intentional effort" when there is a breakthrough in finding a solution (Liljedahl, 2005, p.220). In his research the AHA! moment has been shown to be instrumental in creating positive mindsets for students engaged in mathematics, not just for professional mathematicians. Verification is the final stage to confirm that the insight is, in fact, a correct solution. When interpreting the Gestalt Principle in a second-level mathematics environment the four stages are influenced by the setting up of class problems by the teacher and the student perception of the tasks set. If the students find the work too easy, they will not experience this Gestalt process that is believed to be essential for promoting creativity in students (Sriraman, 2004; Liljedahl, 2005, 2013; Lithner, 2008; Sternberg, 1996). Hence, the a priori codes related to feeling challenged or bored were integrated into the Gestalt Principle. The coding process in this project involved looking for evidence that the students were given challenging questions that made them think and consequently provided opportunities of incubation and perseverance, AHA! moments, and subsequent feelings of satisfaction when a solution was discovered.

5.4.4 Application of the Aesthetic Principle to the template

The second Principle in Figure 14, section 5.4.2, is the Aesthetic Principle which Sriraman interpreted as being about the beauty of mathematics as an entity and studying it for its beauty. He had specified the importance of appreciating unusual solutions to enhance students' creativity when defining it, so this was initially chosen as the main sub theme of this higher-order code. In a second-level school context the Aesthetic Principle includes comments on whether students had the opportunity to discuss other solutions and how they felt about seeing unusual alternatives. The careful staging of the discovery moment by the teacher is crucial for the student's appreciation of mathematics but it was decided to code the processes that led to the discovery moment in the Gestalt Principle, and the comments on the students' appreciation for mathematics as a subject in the Aesthetic Principle. The interpretation of the Aesthetic Principle also included codes that described how students felt about mathematics in general and what they felt its purpose was. For this reason, comments relating to the students' enjoyment of mathematics which had originally been under the a priori code '*Attitude to mathematics*' in this Principle were included here. Initially, comments on the importance attached to mathematics were also placed under the Aesthetic Principle.

5.4.5 Application of the Uncertainty Principle to the template

The Uncertainty Principle, as outlined by Sriraman in Figure 14, section 5.4.2, advocated posing open-ended and challenging questions that required students to persevere to be able to answer them. If the question presented is unfamiliar, they must make decisions about how to approach it. For this to happen Sriraman suggests that the questions should be ill-posed so that the students are faced with a dilemma. In a second-level context this was interpreted as allowing for more creativity by setting tasks where the students had to think for themselves how they should go about solving the task. If the questions are routine, or the method is given, then the opportunity to make decisions is removed from the student. It was decided that the comments previously under the a priori code '*Being motivated by thinking about methods'* could be described by the Uncertainty Principle. Similarly, comments relating to the students' enjoyment of having to think and the importance attached to thinking were put under this level-one code. The methods employed by the teacher and use of the textbook became another subtheme when they described phenomena that affected the opportunities students had to think for themselves. In general, this theme focused on the experience for the student, in terms of thinking, in the initial stages of facing a mathematics task.

5.4.6 Application of the Free market Principle to the template

Sriraman's Free Market Principle is concerned with encouraging risk-taking in mathematics. For professional mathematicians he suggested this implied presenting a new proof to an unsolved problem or a new solution method that may not be valid. However, it is not only professionals that risk damaging their reputation as a mathematician. For second-level students this same risk factor could be a concern when presenting their solutions to their peers in public forums or in the classroom. This was the interpretation, highlighted in Figure 14, emphasising the importance of risk taking at second level. Given that very few of the students in the study had had opportunities outside of school to discuss their mathematical ideas, it was decided to look for evidence of risk taking in the classroom through discussion. Sriraman recommended that teachers should create a classroom environment where students felt free to input their ideas and discuss their solutions with each other. A central feature of this was feeling free to take risks when solving problems and discussing the merits of a variety of solutions. For this reason, the a priori codes that related to student opinions of groupwork were inserted in the Free Market theme. Initially the discussion and debate in this theme were interpreted as being specifically related to discussing students' solutions, taking risks to offer their alternative solution and the students' enjoyment of doing so. As mentioned above, the Aesthetic Principle code was assigned to comments made on the appreciation of the mathematics in the solutions. However, the comments made on the enjoyment of discussing the actual solutions were placed under the Free Market Principle.

5.4.7 Application of the Scholarly Principle to the template

In his Scholarly Principle, Sriraman advocates that creative activities have the potential for contributing to new knowledge and discoveries by challenging accepted beliefs. Some aspects of what he described as contributing to the Scholarly Principle seemed appropriate for a more advanced stage of mathematics, such as contributing to the existing body of knowledge, as seen in Figure 13, section 5.4.2. The second column in Figure 14, section 5.4.2, outlines other aspects in Sriraman's definition that were chosen to focus on in the template because they relate more to the second level. The first of these was creating a classroom environment where students are encouraged to question the solutions of their peers and teachers, through debate and discussion. Elements of the a priori code, '*Groupwork*' fitted in here when the comments were related to learning new ideas. This was how it wass decided to distinguish the groupwork codes for the Scholarly Principle from those for the Free Market Principle. When interpreting the Scholarly Principle code, it is important to reiterate that the focus in this study is on second-level student experiences. Therefore, when coding, Sriraman's interpretation of student creativity as being relative to their peers (Liljedahl & Sriraman, 2006), as opposed to the creativity of professional mathematicians, was kept in mind. Hence, the discussion of new solutions were interpreted as having the potential to contribute to the body of knowledge within that class group rather than the global knowledge of mathematics.

The second aspect of the Scholarly Principle definition was encouraging the exploration of problems without instruction (Sriraman, 2004) as being a means of enhancing student creativity. In a second-level context this was interpreted as allowing the students time to work on a problem before being given a method by the teacher or from a textbook. Initially it was felt that this tied in with the a priori codes for students' perceptions of methods employed by the teacher, such as teacher instruction through examples and the use of procedural questions. Because of the similarities with the Uncertainty Principle, it was then considered that the focus here should be from a learning perspective where comments reflected beliefs that the teachers' methods helped or hindered learning.

5.5 Rationale for subthemes in the initial template

Once the a priori codes were attached to a relevant Sriraman principle, the 1st and then 5th interview were re-read carefully to assess whether these higher-order codes were appropriate. During this phase of reading, the codes categorised under each of the Principles relating to second-level schools evolved. These were then broken down to enable descriptions of the different classroom experiences to be coded within each main theme. Each theme was expanded by making notes in the margins of the interviews to signify which principle the comment related to, and how the level-one code might be further sub-divided. As the data was read further, multiple related codes that described features of the students' experiences evolved from the interview discussions. These became the level-two and level-three codes in the initial template. The first draft of the template, seen in Figure 15 below, was created and used to recode the three interviews, 1, 5 and 7, already examined.

Given the nature of the study, the data was coded in stages. The interview questions were concerned with pre- and post-workshop experiences, including a comparison of the two. Hence, the data was initially coded in two different phases to reflect experiences prior to the workshops and those during and after the workshops. The questions in the interviews relating to classrooms and then workshops were coded in this first phase of the analysis.

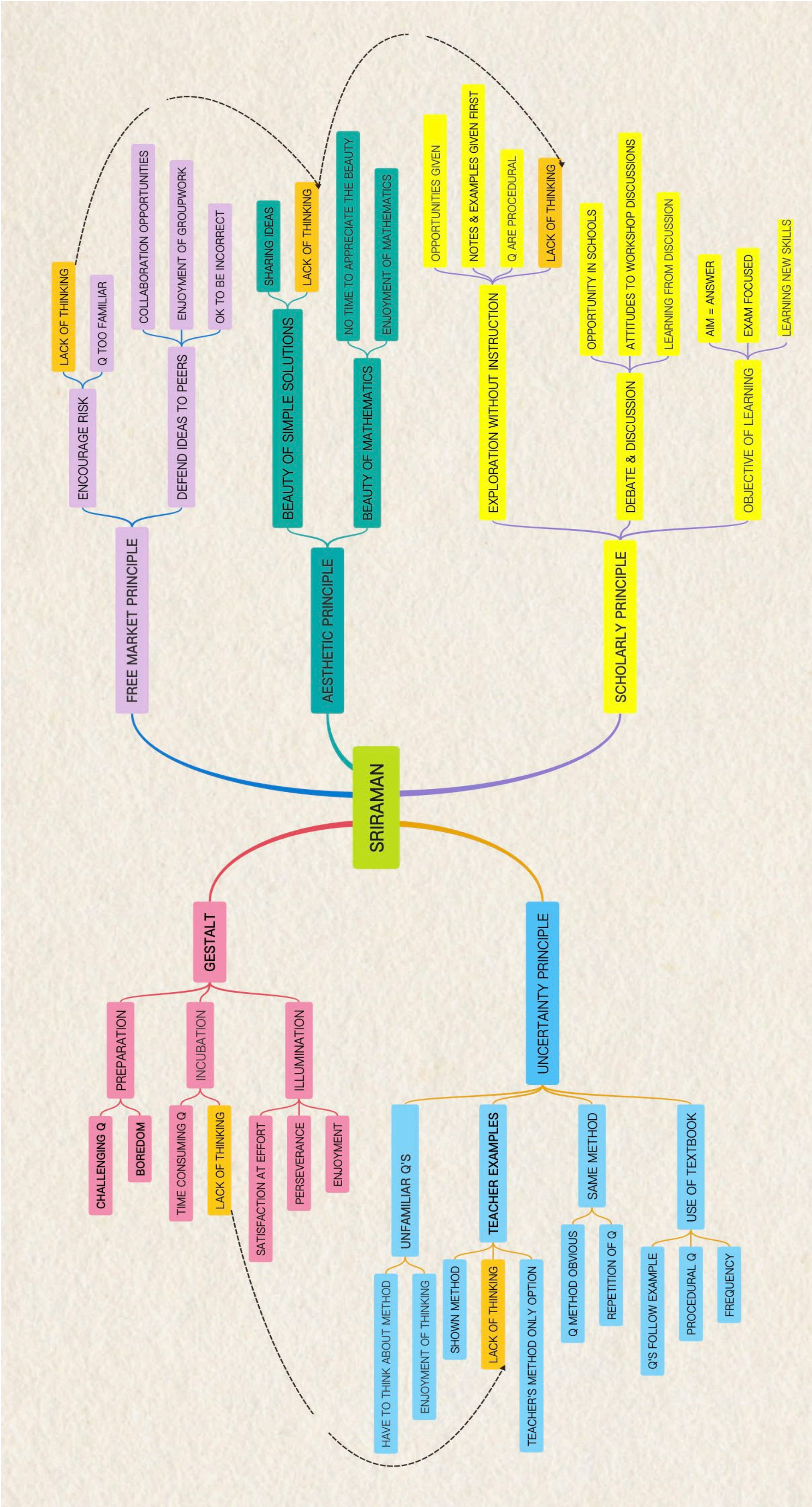


Figure 15 Template 1, Initial Template used for Preliminary Coding

A record was kept of the frequency of each code allocated and notes were made on any ambiguous comment that may need further investigation and on any potential new code that emerged. With the second- and third-level codes, the researcher hoped to fine-tune the distinctions between comments made in the text and capture the essence of what the students were trying to tell me. This also enabled acknowledgement of comments that may not have been very frequent but highlighted noteworthy experiences. The rationale for each subtheme is outlined in sections 5.5.1 to 5.5.5 below.

5.5.1 Gestalt Principle level-one code

In the Initial Template, in Figure 15, this theme was broken down into three level-two codes: the teachers' '*Preparation*', evidence of '*Incubation*' and evidence of '*Illumination*' opportunities. These were further expanded to distinguish what constituted each code. The '*Preparation*' code was broken down into codes that reflected challenging questions being given to the students or students' feelings of boredom from lack of challenge. '*Incubation*' was expanded to distinguish between time-consuming questions that made students think and questions that were classified by the lack of thinking involved to solve them. Finally, the '*Illumination*', or AHA!, code was broken down to illustrate comments that reflected student satisfaction at the effort made, the opportunity to persevere and the overall enjoyment of experiencing an AHA! moment. In general, the Gestalt theme was concerned with the ultimate feeling of satisfaction which depended on the tasks the teacher had prepared for the class. Hence, the coding was primarily related to how the class was organised by the teacher and how it was then perceived by the student in terms of challenge.

5.5.2 Aesthetic Principle level-one code

The Aesthetic Principle was split into two level-two codes: the '*Beauty of simple solutions*' and the '*Beauty of mathematics*'. This was intended to separate the data relating specifically to the discovery of simple, or enlightening solutions and the general comments on mathematics. Where the Gestalt Principle was more about the teachers' preparation and the process of problem-solving for the student, in terms of incubation opportunities, this Principle was concerned with the mathematics that emerged and how the students viewed it**.** The '*Beauty of simple solutions*' was further divided into '*Sharing ideas*' and '*Lack of thinking*' about the mathematics. This coding level included comments relating to whether students were given opportunities to appreciate mathematics solutions. The intention behind this was to highlight situations where students followed teachers' examples, where they did not think or discuss the merits of different methods. The opportunity for unusual solutions to present themselves and make an impact on students was therefore removed. Where students were sharing ideas and methods there was more potential for one of their peers in the group, or class, to suggest a solution that others found enlightening. The '*Beauty of mathematics*' code was subdivided into '*No time to appreciate the beauty*' and '*Enjoyment of mathematics*'. The former term was chosen to reflect comments read in the preliminary coding that mathematics in school was only concerned with getting the answer and 'moving on', implying that there was no time to appreciate the beauty of mathematics The intention of these level-three codes was to try and capture what students felt about mathematics as a subject in school and their appreciation of it in general. The '*Enjoyment of mathematics*' code encompassed comments where students spoke of having the opportunity to appreciate and enjoy the process of mathematics.

5.5.3 Uncertainty Principle level-one code

The Uncertainty Principle represented different ways in which uncertainty was either presented or removed for students. It was divided into four level-two codes: '*Unfamiliar questions*', '*Teacher examples*', '*Same method*' and '*Use of textbook*'. The first of these was further divided to code when students were required to think about the method for themselves, and their enjoyment of doing so. The remaining 3 level-two codes described the different ways in which the uncertainty was removed for students. The first of these, '*Teacher examples*', related to teachers giving examples first. This was then subdivided into third-level codes which reflected the effect on the student and how this removed uncertainty. The first of these, '*Shown method*' referred to comments made about students being shown the method they were to use, then specific comments related to not having the opportunity to think were coded under '*Lack of thinking*' and lastly student perceptions that by being giving an example first by the teacher, they were obligated to use that method, were coded under '*Teacher's method only option'*. The next second-level code, '*Same method*' was subdivided into codes relating to the question method being obvious and the repetition of questions presented. The intention here was to highlight comments on the removal of uncertainty by either giving questions that were too procedural or by giving too many questions using the same repetitive method. The final second-level code, '*Use of textbook*', described the ways in which the use of the textbook removed opportunities to experience uncertainty. This code was initially split to explain how the textbook did so: by providing an example first, by presenting procedural questions and by its overuse. This was intended to contrast with being given challenging, thought-provoking questions in the Gestalt higher-order theme.

5.5.4 Free- Market Principle level-one code

In the template this Principle focuses on the atmosphere in the classroom which either allows students to have, or inhibits them from having, the freedom to input their ideas. The Free Market Principle was initially divided into two second-level codes: '*Encourage risk*' and '*Defend ideas to peers*'. The former came about because preliminary reading had highlighted comments that there was no encouragement of risk because the students did not have to think and take chances when problem solving. Some students specifically mentioned that the 'questions were too familiar' hence this led to two subsequent third-level codes, namely '*Lack of thinking*' and '*Questions too familiar*'. The latter second-level code, '*Defend ideas to peers*', was further divided into '*Collaboration opportunities*', '*Enjoyment of groupwork*' and '*OK to be incorrect*'. These third-level codes related to comments made about student perceptions of collaboration in class and the workshops. The third code, '*OK to be incorrect*', was added to describe student remarks on feelings of freedom to share their ideas in a group or class. In general, the coding for the higher-order theme of a Free Market is more about the mathematical risk-taking involved for individuals and opportunities given to experience it which conflicts with a classroom where the students are told which method to employ.

5.5.5 Scholarly Principle level-one code

The Scholarly Principle code was broken down primarily to look for evidence of the two main definitions of Sriraman, as shown in Figure 11, section 5.4.2. A third second-level code was also added to investigate what the students felt was the main '*Objective of learning*' mathematics. The first second-level code, '*Exploration without* *instruction*', described when opportunities were given and situations students had spoken of that removed such opportunities, such as: the teacher giving notes and examples first, too many procedural questions or questions that did not require thinking. The next second-level code was '*Debate and discussion*' which was divided to record data on comments regarding opportunities for discussion being given, or not, along with references to learning from discussion. The initial template coded *'Enjoyment of groupwork'* in the Free Market Principle and '*Learning from discussion*' here in the Scholarly Principle. However, in later iterations, which are outlined below, these comments on discussion were combined into the Free Market Principle because they proved too difficult to distinguish from each other. Finally, the '*Objective of learning*' was split to reflect how the students viewed mathematics from their experience to date. It was originally intended to code comments on the learning aspect of mathematics in real life and how this was viewed in school. At this stage in the development of the template the only relevant comments appeared to be regarding too much focus on getting the answer and working towards examinations in school along with learning new skills in the workshops. Therefore, '*Aim = answer*', '*Exam focused*' and '*Learning new skills*' became the three third-level codes.

5.6 Ambiguities and Modifications to the initial template

As the subsequent interviews were analysed, coding ambiguities emerged, and the initial themes were then edited to make the categories more distinct and the emphasis of individual codes more reflective of the significance to the students. The initial template was modified a number of times by inserting new codes, deleting inconsequential codes, merging codes of the same level, changing the scope, or hierarchy of a code and moving a code to a different higher-order classification, or Principle as each higher-order theme in this study has been called (King, 2004). Examples of why and how this was done will be explained in detail as the ensuing iterations are described.

5.6.1 Integrative theme

Before outlining how each individual higher-order code was modified in subsequent iterations, an example will be given of the introduction of an integrative code because it concerned all five of the higher-order themes. As outlined earlier, an integrative code differs from parallel coding in that it represents an inference rather than a comment that means different things to different students. Because of this, it can feel inaccurate to code it within a theme because of the level of interpretation from the researcher that is involved. It became apparent that one of the most problematic codes was the third-level code '*Lack of thinking*', so it was the first to be deleted from all five higher-level codes. On coding it turned out to be too imprecise since each '*Lack of thinking*' subtheme means something quite different when viewed as an aspect of each of the five principles. It was initially felt each was explained well by the individual principles. However, as the coding developed, difficulties emerged trying to decide which higher-order theme to code comments on '*Lack of thinking*' under because of the possible multiple interpretations of the data.

5.6.2 Interpretations of 'Lack of thinking' code

Initially the Gestalt Principle '*Lack of thinking*' was used to code students being given questions that were too easy, so they did not require time to mull over. It was intended to describe the removal of incubation opportunities and subsequent AHA! moments, that is, it was viewed in terms of the students' satisfaction potential. The Aesthetic Principle '*Lack of thinking*' was related to the lack of an opportunity to discuss solutions and appreciate unusual solutions that emerged. Seeing the beauty in mathematics and its simple solutions from sharing ideas was seen as an unlikely occurrence if the students did not have to think to answer questions. The Uncertainty Principle '*Lack of thinking*' was initially used to code comments where students spoke of the question method being too obvious or when examples were given by the teacher initially, so the student knew what to do immediately. In general, this was concerned with the introduction and presentation of a new topic by the teacher and how it was perceived by the student rather than the lack of thinking in the problem-solving incubation stage in the Gestalt Principle or the discussion stage in the Aesthetic Principle. The Free Market Principle '*Lack of thinking*' was specifically to do with risk taking when students had to think about a method or solution and offer their idea to their peers or the class. It was associated with the atmosphere of the class and whether students felt the freedom to take such risks, and if not, why not. Finally, the Scholarly Principle '*Lack of thinking*' focused on not getting opportunities to learn new ideas. If students had to think about the method, rather than follow the teacher's method, it was more likely that new ideas would emerge through discussing these different methods and the benefits of each.

Each of these categories seemed distinct at the outset but the ambiguities arose when students did not express explicitly what they meant in each case. One option was to put all the '*Lack of thinking*' comments under a single code within one higher-order theme. However, it was felt that this would not portray the true meaning of what the students were saying. It seemed appropriate to look upon this '*Lack of thinking*' code as an integrative theme connecting each Principle laterally, so it was deleted as a third-level code in each higher-order theme and the template was modified accordingly.

5.7 Development of final template

As outlined above, there were quite a few modifications to the initial template as the interviews were read, some minor and some more substantial. The initial template was used to code interviews 1, 5 and 7 then modified in several stages before Template 2 was created which was used to code these and three further interviews. During the analysis of these six interviews, using Template 2, further iterations were made as each interview was read through. The final template was therefore developed in stages, modifying as felt necessary, and once 'completed' it was used to code the entire set of eleven interviews to ensure that all the relevant data had been included in the template. The decision to consider the template complete was taken when it felt that all the significant comments and reflections made by the students had been coded and each seemed to fit into the template without ambiguity. An independent researcher was also asked to test the template as it was being developed. An account of this coding will be provided after the explanation of how the template developed so that it can be read in context. In total 14 modified templates were made during the iteration process, some of which were only minor changes, but at the time it felt important to record them to be consistent with the coding. Rather than going through each modification the most significant changes made whilst moving from the initial template to Template 2 and finally to Template 3 have been outlined. As a result, there are several codes mentioned in the analysis that do not appear in any template presented, because they were merged into other codes before the final Template 3. The iterations that led to the development of Template 3 are outlined below, taking one theme at a time, to help maintain focus on the development of each higher-order theme.

5.7.1 Gestalt Principle Template 2

This template can be seen below, in Figure 16. As mentioned in section 5.1.1, the focus for this part of the template were the students' opinions on the teacher's preparation for the class. Codes were inserted here to allow for refinement of student perceptions of the teachers' preparation and how it affected them.

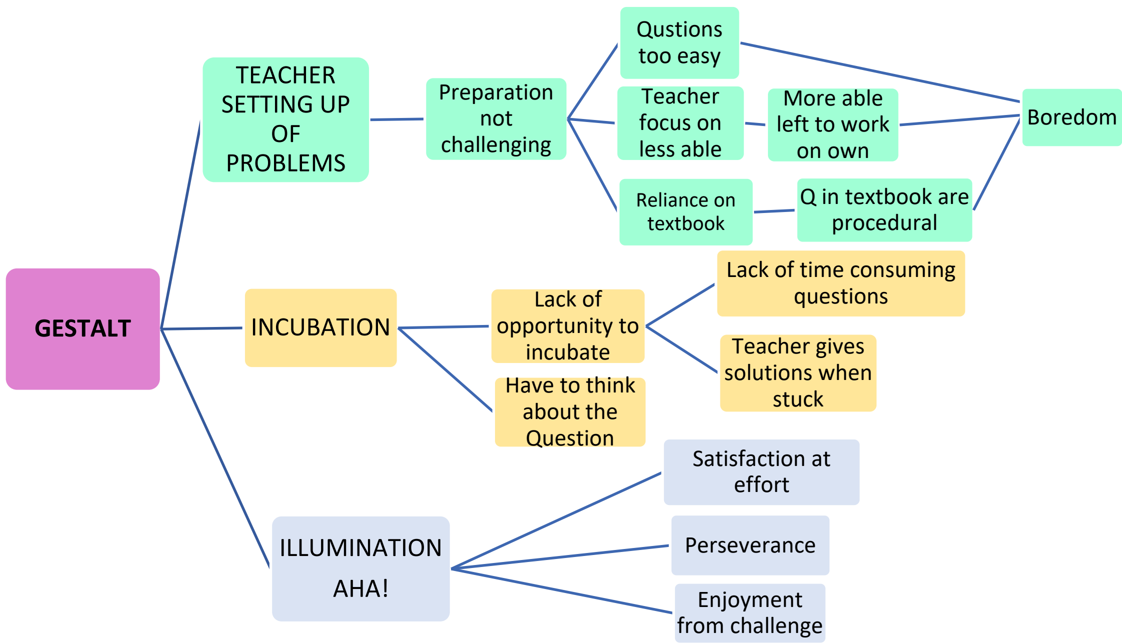


Figure 16 Template 2 of Gestalt Principle Codes

As several more interviews were read it became apparent that the students were intermittingly making comments about aspects of learning mathematics that challenged them and made them think and others that did not. A distinction between codes for challenging questions that required incubation and those that lacked challenge was made to distinguish between such comments so that these specific aspects could be recorded. Further levels of codes were necessary to identify the different reasons for students feeling unchallenged or experiencing a period of incubation. This was modified further in later iterations, but it was clear at an early stage that there were many reasons affecting how challenged students felt. It became apparent that the scope of the code '*Boredom*' needed to be changed because there were many reasons given by students for feeling bored. At this stage it felt more appropriate to code specific comments referring to being 'bored' or 'waiting around' separate from those describing teachers' actions such as using the textbook or focusing on the less able. The reason for this was that a sense of frustration was evident from a number of students which differed from comments where students merely acknowledged, or accepted, that the less able students required more time.

5.7.2 Gestalt Principle Template 3- Addition of codes relating to Vygotsky's theories

This template can be seen in Figure 17 below. Many of Sriraman's theories reflected those advocated by Vygotsky such as peer collaboration and opportunities to experience the ZPD (zone of proximal development), so these were woven into the template after several more interviews were read. Comments emerged that displayed an awareness of diversity in the classroom so providing codes related to this enabled me to explore how the students felt about challenging opportunities being provided for everyone.

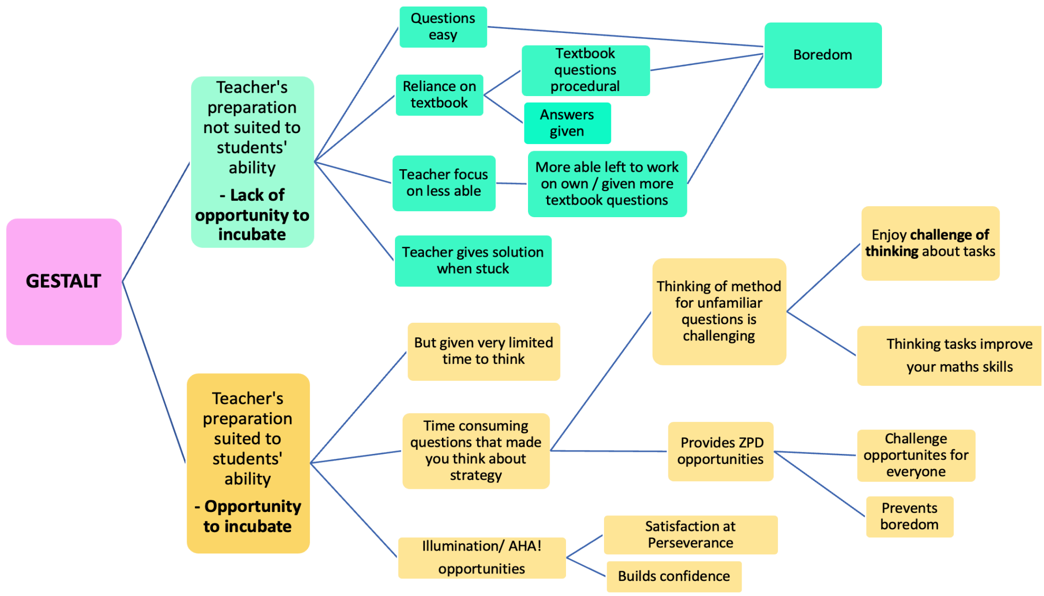


Figure 14 Template 3 of Gestalt Principle Codes

Codes related to Vygotsky's theories were embedded within the Gestalt, Free Market and Scholarly themes as they emerged. Being given opportunities to incubate mirrored Vygotsky's theories of students experiencing the ZPD so it tied in with the Gestalt theme. The enjoyment felt by the students' collaboration with their peers to discuss their ideas suited the risk-taking principles of the Free Market theme. Finally, learning from debate and discussion seemed to reflect the Scholarly Principle advocated by

Sriraman whereby students could debate solutions and contribute to the overall knowledge of the group.

The higher-order theme of 'Gestalt' in the initial template was divided into subthemes to depict the four-stage creativity process from the literature. There was, however, some difficulty deciding on whether to interpret comments related to having to think about the question as Gestalt or Uncertainty. Initially, in the second template, '*Have to think about the question*' in Gestalt was intended to code comments that led to a period of incubation. In Uncertainty, '*Have to think about method*' was intended to code comments on situations where the students had not been given the method in advance. From reading through the interviews, it was sometimes difficult to interpret the student's intended meaning behind a comment without the researcher having to make assumptions. The comment below was one where it proved difficult to decide whether to assign it to Gestalt or Uncertainty:

> Student 430: We're introduced to a question or an example, maybe a simplified version. Then we're given maybe a little bit of time to think about it, maybe two or three minutes. Then we are given how, we are shown, then we progress onto harder questions.

Because of this ambiguity, and my wish to retain the Gestalt theme given its historical position in the literature for fostering creativity, it was important to think carefully about the precise coding in this theme. Reflecting on this it was felt that this Principle should remain focused on the students' perception of the teachers' preparation. The second-level codes were divided so that the phrases could be coded into those that implied preparation that suited the students' ability and those that implied preparation that did not. These became the two second-level codes which were given subheadings of '*Lack of opportunity to incubate*' and '*Opportunity to incubate*'. Rather than viewing these as a different level of coding they were reworded to emphasise the focus on the teacher preparation as the facilitator of incubation opportunities to reinforce the focus of this hierarchy of coding. How the class developed, due to the preparation and teacher adaptation, or lack of, to student needs, became the main characteristic of this theme. As outlined above, Vygotsky's theories of the ZPD are interwoven into this because the opportunity to incubate is essential for providing zones of proximal development for students. For similar reasons, also tied in with this theme are codes relating to the fact that students found thinking challenging. Given the number of references to routine questions, having to push yourself because the questions presented were challenging seemed an integral part of the Gestalt stage of incubation.

The codes on illumination in the initial template were merged into this new, second-level '*Opportunity to incubate*' code because incubation was the overriding experience that produced the AHA! moment. It was also important at this stage to try to record how the students viewed AHA! moments. There is much talk in the literature about the importance of illumination for the development of positive student attitudes and beliefs (Liljedahl, 2005, 2013; Liljedahl & Sriraman, 2006). Although these sources acknowledge that the progress in the student is slow to emerge, recording what the students said was of interest because of the potential effect these moments might have on their self-perception. Comments relating to *'Satisfaction at perseverance*' were frequent as was a recognition that such challenges '*builds confidence*' in the student, so these were labelled as fourth-level codes to record the acknowledgement of the benefits students felt from the illumination opportunities.

5.7.3 Aesthetic Principle Template 2

The next phase of iterations to the Aesthetic Principle template can be seen in Figure 18 below. The initial template attempted to examine opportunities to explore, discuss and appreciate unusual solutions in class and in the workshops. After the '*Lack of thinking*' was deleted, the code '*Beauty of simple solutions*' was simplified so that it would be possible to identify when there were opportunities to discuss solutions and when not. The intention remained to keep this specifically for the discussion of the mathematics rather than the enjoyment of discussion in general. As the analysis developed, the comments on mathematics previously coded as third-level codes in the '*Beauty of mathematics*' seemed more appropriately viewed as comments relating to the '*Objective of mathematics*'. When reflecting on these comments it felt that this was one of those cases where the age of the students required a slightly different interpretation of that which Sriraman had intended for K-12 students. Hence, the

'*Beauty of mathematics*' code from the original template was replaced with the new code '*Objective of mathematics*'. It appeared that students were differentiating between enjoying the process of exploring problems and learning mathematics merely for functional reasons, so these became the third-level codes for '*Objective of mathematics*'.

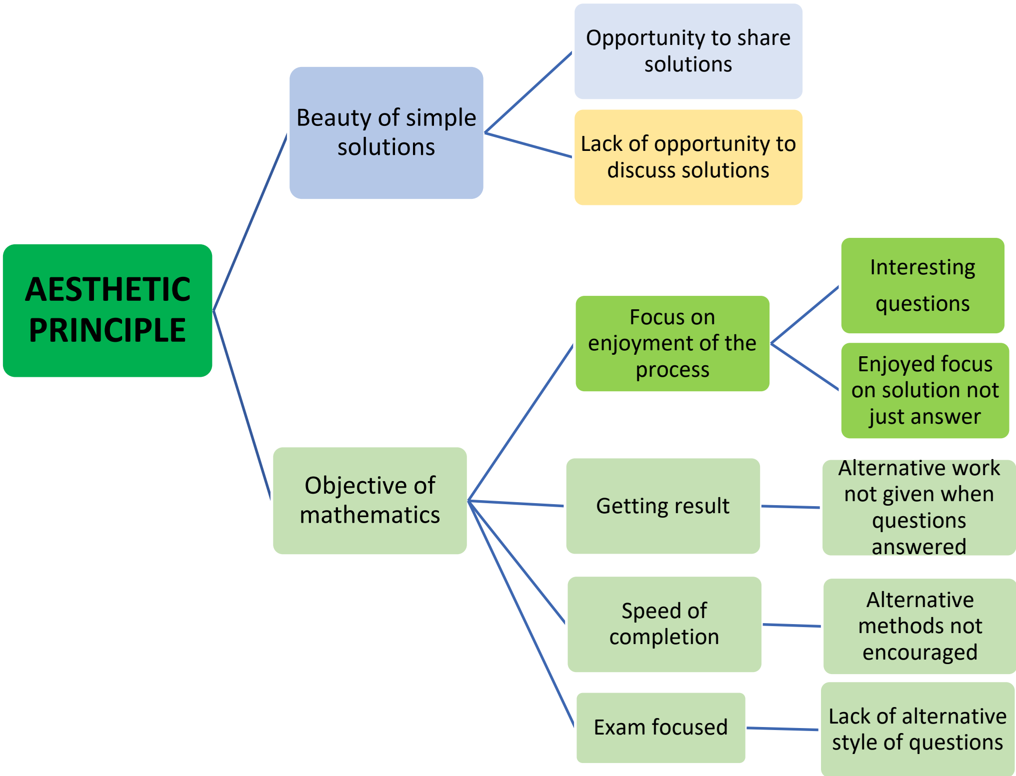


Figure 18 Template 2 of Aesthetic Principle Codes

Comments supporting the idea that students viewed mathematics as a combination of ideas and explorations rather than as sets of routine questions were combined under the third-level code 'Focus on *enjoyment of the process*'. This was further split to try and identify what it was that the students enjoyed. The two codes that came from the analysis at this stage were looking at interesting questions and the enjoyment of focusing on the different solution methods rather than just the answer. There was also an element of frustration relating to what the students saw as the objective of mathematics in the classroom. The remaining three more functional third-level codes in '*Objective of mathematics*' were further clarified by fourth-level codes. This was to try and capture the essence of remarks made by the students when critiquing the purpose of tasks set in the classroom.

5.7.4 Aesthetic Principle Template 3

The modifications to the Aesthetic principle for Template 3, see Figure 19 below, were in the descriptions of codes probing the student perceptions of the '*Objective of mathematics*' from their experience in school and then in the workshops.

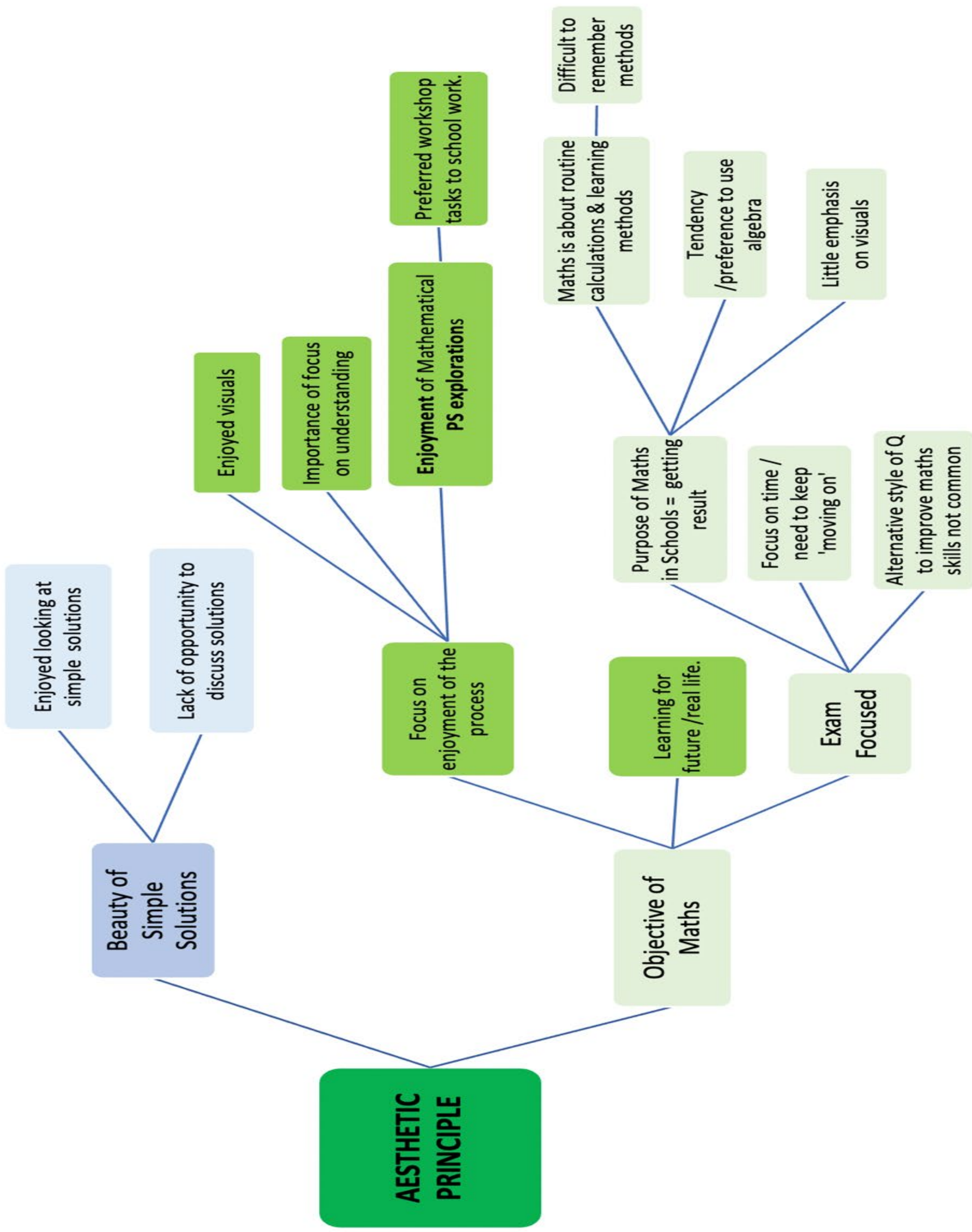


Figure 19 Template 3 of Aesthetic Principle Codes

The ‘*Focus on enjoyment of the process*’ was made slightly more specific to accommodate comments on using visuals to solve problems and focusing on the understanding of mathematics. The level of the code remained the same, since it described an enjoyment of the process of mathematics itself as the ‘*Objective of mathematics*’, rather than just being a necessary subject requirement for school exams. It became necessary to change the scope of some of the third-level codes because the comments emerging appeared to be all related to the focus on exams in school rather than as distinct ‘*Objectives of mathematics*’. For example, reading deeper into the student reflections that had been coded as ‘*Speed of completion*’ and ‘*Getting result*’, it felt that they were intended to give examples of practices used in school because of the focus on the importance of exams.

> Student 034: They don’t really have time to do that (try different methods) with us.
>
> Student 043: Sometimes when people ask questions, because they don’t understand it, the teachers look nearly annoyed because they can’t move on.

Because of the frequency and variety of phrases related to a focus on ‘*Getting result*’, an extra level of coding was inserted to try and capture the different comments that were connected to that code. Creating a specific code to highlight the use of algebra seemed useful because it was voiced as the preferred choice of method for many students in the interviews. Similarly, the idea that mathematics was all about memorising methods was repeated throughout the interviews and it appeared to have negative connotations for the students:

> Student 339: A lot of the time we're just given the methods. A lot of the time they don't give you the opportunity to try and work it out yourself.
>
> Student 430: I think these problems (in the workshop) would be far more beneficial to students than just being taught to be a robot.

Such comments were recorded with the intention of investigating the effect of rote learning on the motivation of these students in general.

5.7.5 Uncertainty Principle Template 2

Unlike in Gestalt, where the primary coding was on the teachers' preparation for the class, the Uncertainty Principle focused on the initial choices that the students were required to make when presented with mathematics problems. Whilst coding with the initial template there were a considerable number of phrases relating to students not having to persevere in class when approaching a question, because the method was obvious. In contrast, many students made specific reference to enjoying having to think about the method for themselves. When modifying the initial template to create Template 2, (see Figure 20 below) this became the focus of the second-level codes which changed the structure of the theme quite significantly.

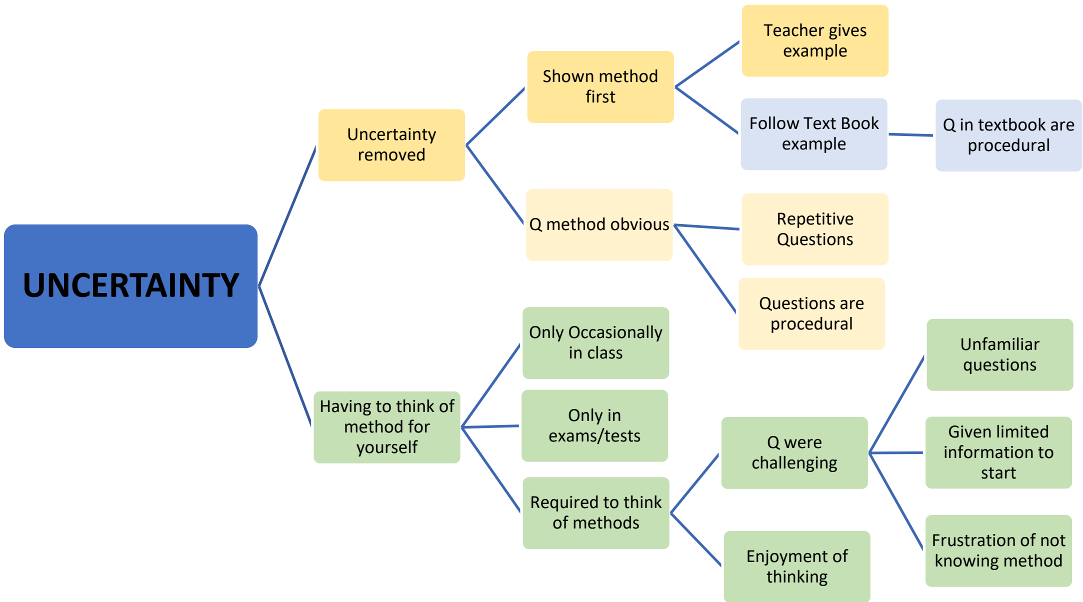


Figure 20 Template 2 of Uncertainty Principle Codes

The first of these level-two codes, '*Uncertainty removed*', was subdivided further to show different ways in which this was done, such as being '*Shown method first*' by the '*Teacher gives example*' or by '*Follow textbook example*'. This second-level code also included phrases that highlighted situations where '*Repetitive questions*' and '*Questions are procedural*' also took away the uncertainty for the students. The overall intention behind these new fourth-level codes was to identify specific situations raised by the students when they were presented with tasks where there was no decision making involved.

The references to the textbook in this theme were specifically to do with removing uncertainty for the student, in contrast to those in Gestalt which referred to the teachers' planning. In this theme, the coding '*Follow textbook example*' referred to comments students made where they did not have to think about the method when approaching a question because they were just required to replicate the method used in the textbook example. The use of the textbook in Gestalt was specifically for when the work set by the teacher had not been challenging because it centred around textbook questions, which were procedural. Although there did seem to be a degree of ambiguity at this stage, specific codes were retained for each different situation where students were given examples first because of the frequency with which these codes had recurred in the preliminary coding.

The expansion of the other level-two code, '*Having to think of method for yourself*', tried to encapsulate the diverse comments students made on how they felt about such questions and why they were different to other procedural questions. The first two third-level codes '*Only occasionally in class'* and '*Only in exams/tests'* were for phrases suggesting minimal opportunities to think being given to the students. The remaining third-level code '*Required to think of methods*' incorporated the data on why the questions or tasks required thinking. This included '*Questions were challenging'*, which in turn was split into further codes of '*Unfamiliar questions'* and '*Given limited information to start*'. Students' reactions to having to think about the method for themselves were recorded here, including their '*Enjoyment of thinking'* and their '*Frustration of not knowing method'*. As mentioned previously, the overall level-two code concerned with '*Having to think of method for yourself*' proved too similar to '*Have to think about the question*' in the Gestalt Template 2. For this reason the iterations evolved quite considerably over the course of the analysis before the final template was decided upon.

5.7.6 Uncertainty Principle Template 3

Because of the ambiguity of the code '*Having to think of method for yourself*', it was necessary to change the higher-order classification of the comments relating to the challenges and benefits of having to think for yourself. It seemed more appropriate to combine these with the incubation and ZPD aspect in Gestalt. So the fourth and fifth-level codes associated with '*Required to think of methods*' were removed from the Uncertainty theme in the final template, as shown in Figure 21 below. The codes in this section of the Uncertainty theme were now more focused on the opportunities students had to think for themselves and their initial reactions at having to do so. The code for '*Preference for thinking of methods*' was added just to record students who made a specific preference. This was to tie in with the new codes related to self-perception and additional codes added to '*Repetitive questions*' that are outlined below.

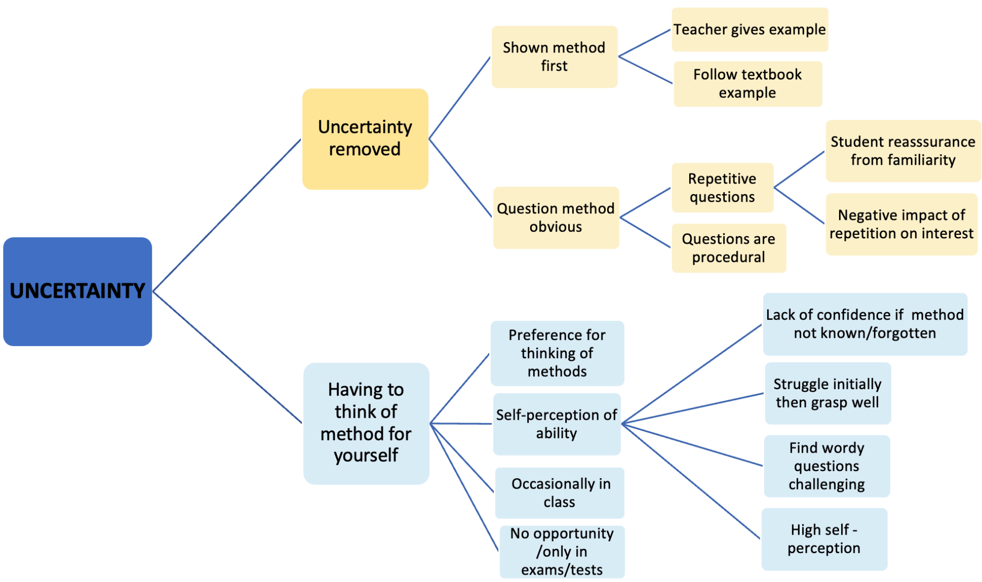


Figure 21 Template 3 of Uncertainty Principle Codes

As the interviews were read in the subsequent iterations, there were mixed responses to questions relating to uncertainty. Many students spoke about repetition with an element of frustration, but some seemed reassured by its familiarity. The Uncertainty codes were extended to accommodate these negative and positive reactions to repetitive classwork and hopefully provide a richer interpretation of this theme. Whilst reading these reactions to repetition there was also an awareness that the self-perception of the student had a strong influence on their opinions. To accommodate this, '*Self-perception of ability'* was added to the level-three codes on '*Having to think of method for yourself*' because the comments hinted that mathematical confidence was an important factor to consider when analysing student opinions on this issue. Having it recorded here would enable me to investigate connections at a later stage if necessary.

5.7.7 Free Market Principle Template 2

The changes to the initial template of the Free Market principle can be seen in Figure 22 below, the most significant modification to this theme being the insertion of the third-level code '*Enjoyment of freedom to think outside the box'*. This came about after reading an interview specifically using the word 'freedom'.

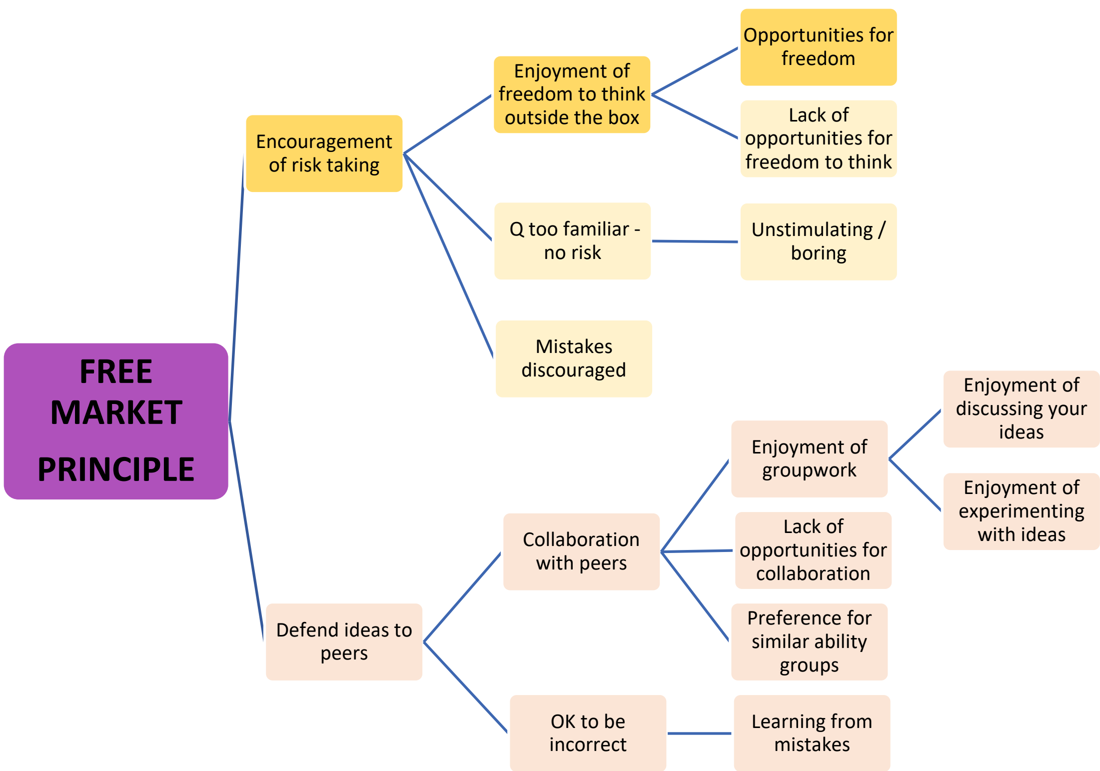


Figure 22 Template 2 of Free Market Principle Codes

The other third-level codes in Template 2, '*Questions too familiar - no risk*' and '*Mistakes discouraged*', tried to capture situations where the risk factor was removed. It also brought to light an element of frustration, evident in the students, over a lack of differentiation and stimulation which was important to be able to capture. The fourth-level codes '*Opportunities for freedom*', '*Lack of opportunities*' and '*Unstimulating/boring*' were designed to reflect this frustration. Another reason it seemed important to record this was because creating a differentiated classroom, where highly able students would have risk-taking opportunities, is recommended in the literature for developing creativity (Sternberg, 2017; Schoenfeld, 2016; Tomlinson, 2018; Lithner, 2008; Mann, 2006; Sriraman, 2005). There were only minor additions at this stage to the coding on defending ideas to peers through collaboration. These highlighted comments where students emphasised the enjoyment of working with peers of the same ability.

5.7.8 Free Market Principle Template 3

After reading the next set of three interviews, comments had emerged implying frustration at having to follow teachers' methods along with additional ones emphasising the enjoyment of having the freedom to think, and it was necessary to probe these thoughts deeper. The codes within '*Encouragement of risk-taking*' were further expanded to reflect the importance students attached to having the freedom to think and the negative thoughts on following a prescribed method. It seemed that the students felt the teacher had a significant role to play in creating a classroom environment where they had opportunities to explore their own ideas. However, the students seemed to be voicing much more about a restrictive atmosphere in the classroom than just following a given method. Given the research literature recommending that teachers should encourage creative deviance and alternative student approaches to enhance the creativity of individuals (Sternberg, 2017; Sriraman, 2005; Leikin & Lev, 2013) it felt appropriate to look for evidence of this aspect of the classroom. More levels of code in the final template were needed to describe the type of atmosphere that prevents risk taking and creative deviance, such as was unfolding from the accounts of the students.

Figure 23 below shows how the second-level code '*Encouragement of risk taking*' was reorganised and split to record comments where students felt they had freedom to take risks and those describing a classroom atmosphere that prevented it. An extra level of codes was then added to the new subtheme '*Teacher's method encouraged*' to reflect some of the negative undercurrents that had been voiced on the class atmosphere resulting from this. These included '*Disregard of need for alternative explanations*' which referred to teachers not feeling obliged to explain a topic differently when students were stuck. The other fifth-level code, '*Lack of trust in students*' to do it their own way, could also have been inserted within the '*Lack of opportunity for freedom to think*' code but it seemed an appropriate place here because of the strong negative connotations.

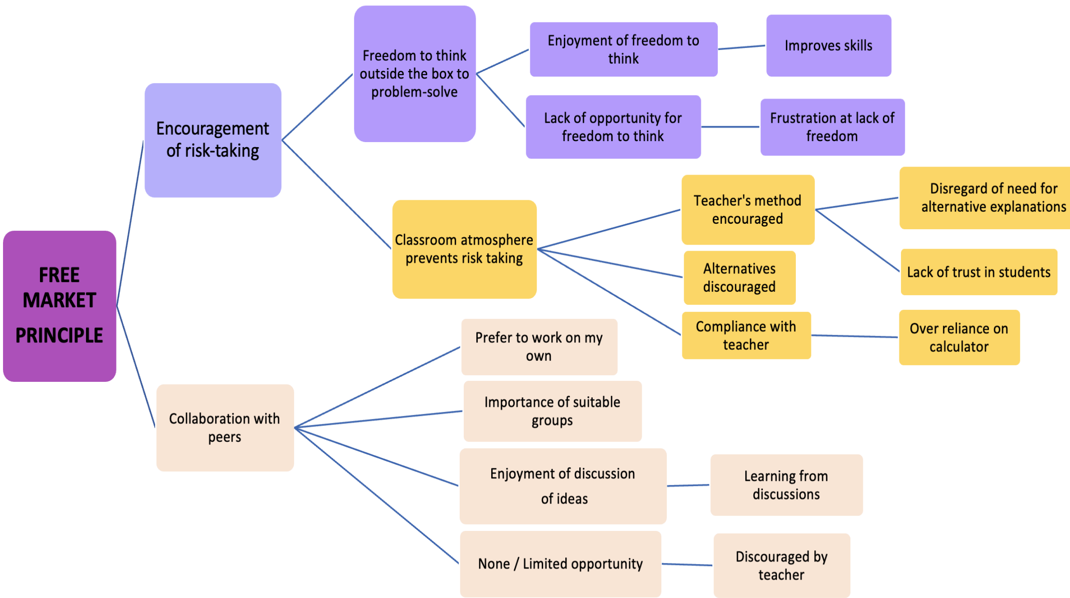


Figure 23 Template 3 of Free Market Principle Codes

There were also personality issues that emerged and have been tied in with the theme of a classroom atmosphere that prevents risk taking. For example, there is evidence of both student frustration combined with a desire for more independence as well as student compliance with what is the 'norm' in classrooms, the latter clearly shown in comments such as:

> Student 020: That's just the way we're taught so we just do it that way… Just accept it for what it is.

This comment suggested the perception of a didactic contract typical of traditional teacher-centred classrooms where students passively follow their teachers' instruction (Boaler, 2003; Wagner, 2007; Brousseau, 1978). The presence of this compliance was evident in several schools as an important contributory factor to the frustration expressed by those students at their lack of freedom. Hence, an extra level-four code '*Compliance with teacher*' was inserted to distinguish such comments from comments where the students stated specifically that the teacher's method was encouraged. Examples of the latter included:

> Student 046: Here's two methods but do it this way.
>
> Student 131: She kinda asserts her dominance and just says her way is right, so it's the easiest way to do it.

The importance of student collaboration is a key element of both Sriraman's and Vygotsky's theories regarding the development of the individual learner. In the early templates collaboration was seen as an essential element of learning and reaching your potential, so it was viewed as an integral part of the Scholarly Principle. However, after independent coding, as mentioned earlier, the ambiguities of interpreting student comments on the merits of collaboration became apparent. The Free Market template was modified to encapsulate all the students' comments on the benefits of collaboration both from enabling students to feel good about explaining and defending their ideas, and from learning from others. Because students had often made general comments about collaboration that were difficult to distinguish, the code '*Defend ideas to peers*' was amalgamated into the '*Collaboration with peers*' code. The code regarding student opportunities to learn from other people's ideas is now included here, as a fourth-level code, '*Learning from discussions*' along with reflections on situations where discussion is actively discouraged by the teacher.

5.7.9 Scholarly Principle Template 2

After reading several interviews it became apparent that the themes within the Scholarly Principle code were very diverse and overlapped with other higher-order themes. There was some ambiguity in the initial template on where to code '*Procedural questions'*. It was coded, in early iterations of the initial template, as a '*Reason for uncertainty being removed*' in the Uncertainty Principle and in the Scholarly Principle as a '*Feature of classrooms that prevented student opportunities to explore mathematics without instruction*'. So in the second template for the Scholarly Principle, see Figure 24 below, it was deleted as a subtheme of the Scholarly higher-order theme and instead keep the Scholarly Principle coding focused on the importance of mathematics and the opportunities for learning through multi-solution tasks (MST) and discussion.

MST had been a key feature of the workshops, so it was important to specifically reference it in the coding rather than just embedding it in the code '*Exploration without instruction*'. Research (Leikin & Lev 2013; Leikin, 2017) has shown that giving students the opportunity to engage in multi-solution tasks not only highlights creative activities, but it can improve the students' thinking skills. To investigate how the students felt about this, codes were inserted that were specifically related to the learning opportunities from MST and the lack of such opportunity when following teachers' methods. For several iterations there was a code to record comments on the lack of exploration in general, to distinguish from lack of exploring MST.

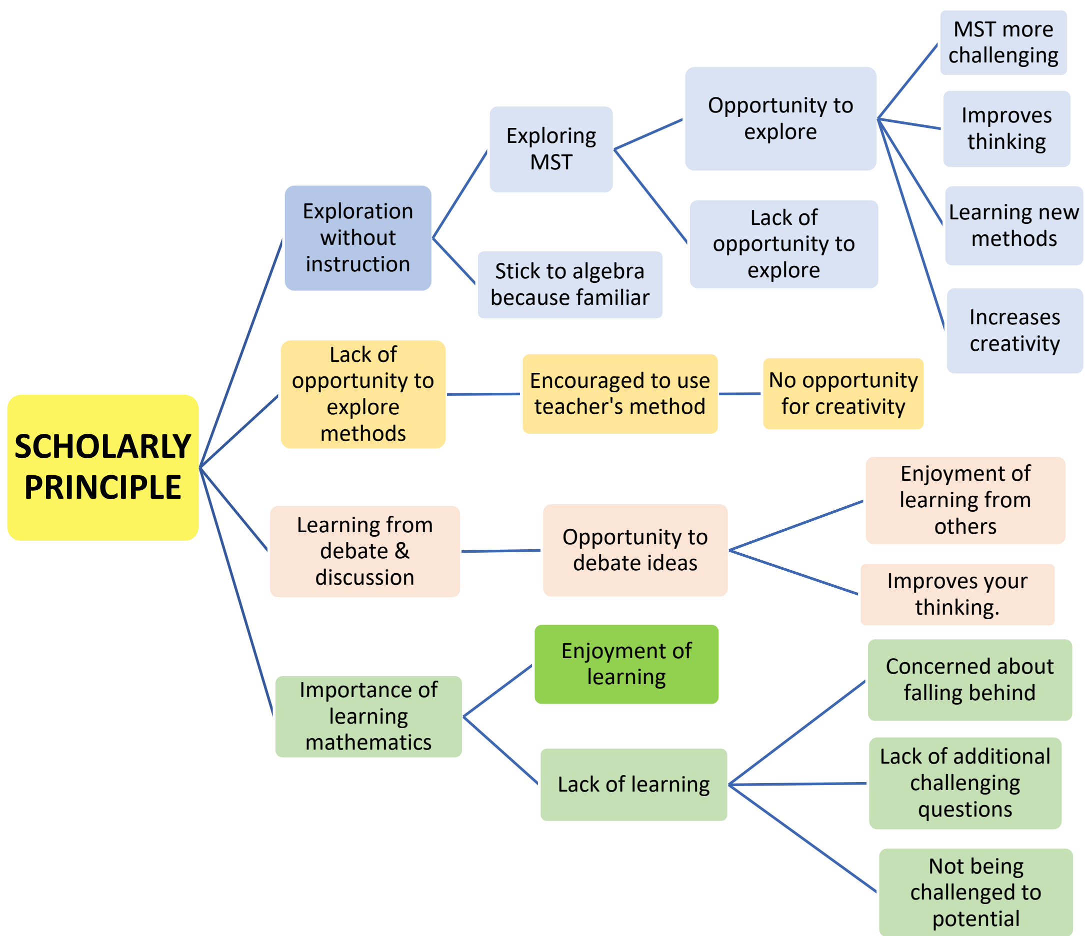


Figure 24 Template 2 of Scholarly Principle Codes

As mentioned previously, encouraging debate and discussion is also an important aspect of the Scholarly Principle and there were many student references to learning from the discussion with other students. At this stage the enjoyment and freedom of discussion was incorporated into the Free Market theme but comments relating to learning from others due to discussion were coded in this theme. The distinction was not so clear as the analysis progressed, which will be outlined in the rationale for template 3 modifications.

There was a strong sense of the importance students attached to mathematics, so '*Importance of learning mathematics'* was added as a new level-two code replacing the '*Objective of learning'* code. As mentioned earlier, it had decided to rename the '*Beauty of mathematics'* to '*Objective of mathematics'* in the Aesthetic Principle. It then felt appropriate to change the higher-order classification of the third-level codes in the Scholarly '*Objective of mathematics'* and move all such comments on mathematics into the Aesthetic Principle. The only codes that were retained here were those that made specific reference to the importance of learning mathematics which were intended to describe the students' focus on advancing their own knowledge. Codes describing students' concerns about the *'Lack of learning'* were then added as a fourth-level, such as their feelings of not being given opportunities to achieve their potential in mathematics in school and being *'Concerned about falling behind'*.

5.7.10 Scholarly Principle Template 3

The final template for the Scholarly Principle, seen in Figure 25 below, was much more focused on specific comments related to exploring MST and to learning. After deleting the second-level code regarding debate and discussion, the overall theme now related to student opinions on learning new skills and knowledge. Given research literature that states that an increase in knowledge has the potential to improve the creativity of students (Tabach & Friedlander, 2013) the final second-level codes can be seen to be interconnected. In a second-level context, they describe the key aspects of what Sriraman outlined as essential for contributing to the body of knowledge and ultimately to maximising creativity.

The overall higher-order code is subdivided into two second-level codes describing the students' perceptions of exploring tasks without instruction and to the importance of learning mathematics. The former centred upon the students' reflections on the merits of MST and the latter on feeling challenged in class or concerns about not being pushed to their potential. Under the '*Lack of learning*' code, the fourth-level code '*Lack of additional challenging questions*' was specifically for recording comments that reflected students feeling that this was affecting their learning potential. The interpretation of this code differed to the Gestalt code when the more able students were left to work on their own rather than being given extra questions. In Gestalt the focus was on preparation of the teacher that prevented incubation opportunities and in Scholarly it was to code phrases where the students specifically mentioned a lack of learning and achieving their potential.

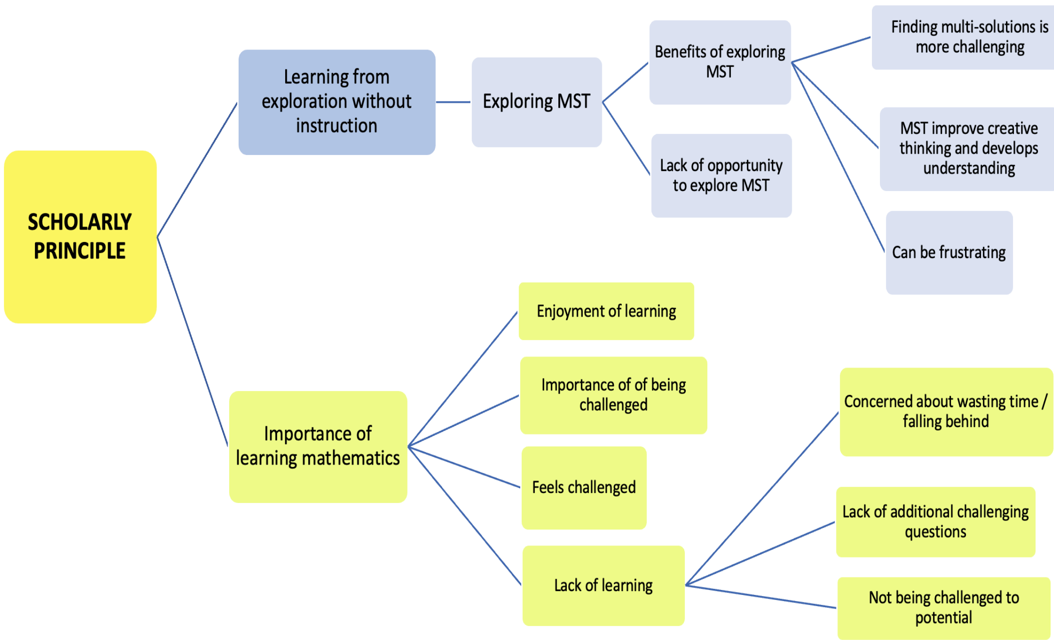


Figure 25 Template 3 of Scholarly Principle Codes

The final change to this template was the deletion of the second-level code on '*Lack of opportunities to explore methods*' because of an overlap with other themes. All general comments on exploring methods, not specifically related to MST, could be incorporated into the Uncertainty Principle and the second-level code, '*Teacher's method encouraged*', had been already incorporated into the Free Market theme in Template 3.

5.8 Checking validity of template

To ensure consistency and reliability in the development of the template an independent researcher used templates at two stages of iterations to code two different interviews. The first interview was coded shortly before Template 2 was finalised. The main ambiguity raised at this stage was the recurrence and interpretation of the '*Lack of thinking*' code in the initial template which encouraged the consideration of this as an integrative code. There were also other minor changes but ones that had been difficult to interpret such as where to put '*Repetitive questions'*. Initially this had been coded in the Aesthetic principle as a sub-code to '*Alternative work not given*', in an early iteration, but it caused too much confusion with '*Repetitive questions*' within the Uncertainty theme. The percentage of agreement was not recorded at this stage because it was only the preliminary stage of developing the template. Having considered ambiguities such as these the template was revisited before completing what became Template 2. There were other discrepancies that emerged, primarily, where to code comments on collaboration. At the time, the interpretation of these was clarified through memos on the individual Free Market and Scholarly higher-order templates.

After carrying out sufficient iterations to feel that the template was very near completion another interview was coded by the independent researcher. This analysis raised the issue of the positioning of codes on collaboration and discussion again, so it needed to be addressed more carefully in the template. The origins of these codes were reconsidered from the interpretation of Sriraman's Principles. From his framework (Sriraman, 2005), collaboration is seen as an essential feature of any classroom for fostering creativity. Its importance as an opportunity for students to take risks and question ideas, but also as a basis for scholarly learning, was highlighted in a more recent article: "they are not only learning how to engage with others and disagree respectfully, but they are also deepening content knowledge" (Luria, Sriraman & Kaufman, 2017, p.1036). Because of the multiple benefits of discussion, the interpretation had created some ambiguity and it was difficult to deduce the exact student meaning when they spoke of the enjoyment, freedom and learning that they felt occurred from discussion. Reflecting on Sriraman's Free Market Principle, classroom discussion appeared to be the only realistic opportunity for risk taking for this age group, as opposed to more senior students or professional mathematicians. It seemed more logical to code all discussion comments within the Free Market Principle, and expand where necessary, rather than to consider it as an example of parallel coding where the code could be considered under both the Scholarly and Free Market Principles. Despite these ambiguities in coding the percentage agreement with the researcher's own coding was counted for this interview because the modifications required afterwards appeared to be clear cut at this stage. The overall percentage agreement was 85%. This was a sufficiently high percentage to consider the template completed after these modifications.

In this chapter I have explained what template Analysis is and the reasons I felt it was suitable for my study. I have described the development of my coding template, in detail, from the Initial Template, through the various iterations, until a final template was decided upon. In the next chapter, I will report on the results of the analysis of the interview data using the coding template described in this chapter.

Chapter 6

Analysis of Interview Data

6.1 Introduction

The results of the interview analysis using the coding template that was explained in Chapter 5 will now be presented. The interview questions can be seen in Appendix H. The student responses to questions in the interviews were coded into the five themes that formed the structure of the final template. Each of the five higher-order themes will be examined in detail in the paragraphs that follow. Student comments on their school classroom practices will be looked at first followed by those related to the workshops. In this way, it will be able to investigate similarities or differences between the students' impressions of their mathematics classrooms and the workshops. In addition to presenting the frequency of codes per theme, students' quotes on their personal experiences in the classroom and the workshops will be examined. By doing so, a more accurate picture of the depth of their feelings towards both their classroom and the workshops can be revealed.

As an introduction to the data that emerged from the analysis, Table 15 below gives a brief overview of the distribution of results in each of the five Principles, or higher-order themes. The table displays the numbers of comments that were either specific to classrooms or workshops, as well as the ones that were difficult to distinguish which of the two they related to specifically. Such comments were more general statements about mathematical preferences, so were added a separate column for them. The 47 general comments in the Uncertainty Principle were all classified as the code *'Preference for thinking of methods'.* These comments were related to both the classroom and the workshops, so it was clearer to put them in a separate column. Similarly, the 39 general comments in the Aesthetic Principle were those classified as the code *'Tendency / preference to use algebra'*. The 8 in the Free Market Principle were those classified as codes related to student preferences for the make-up of groups, which were not specific enough comments to allocate them to the classroom or workshop. Finally, the 25 general comments in the Scholarly Principle included 14 classified as the code *'Importance of learning mathematics'* and 11 as the '*Importance of being challenged'* which referred to a combination of classroom and workshop situations.

Table 14 Number of Responses in Each of the Five Principles

Five Principles	Classroom Comments	Workshop Comments	General Comments
Gestalt	162	298	0
Uncertainty	291	57	47
Aesthetic	144	120	39
Free Market	112	144	8
Scholarly	102	123	25

6.2 Gestalt Principle

The overall number of student comments with regards to having the *'Opportunity to incubate'* can be seen in Table 16 below. The table summarises the data on the codes related to teacher preparation that affected students having the *'Opportunity to incubate'*. The subcodes represent situations given by the students that resulted in such opportunities being removed or just not experienced. The table just gives the data for the third-level codes, the comments coded using any fourth-level codes are included in the corresponding third-level code numbers. Each of these fourth-level codes can be seen in the final Gestalt Principle template in Figure 17. For example, the data on *'Reliance on textbook'* includes the fourth-level codes '*Textbook questions procedural'* and '*Answers given'*. Similarly, '*Teacher focus on less able'* also includes specific comments made on '*More able left to work on own / given more textbook* *questions'*. The next second-level code describes preparation that led to the students having to think about tasks presented and reflections made by the students on how these tasks brought about a different experience. In this section of the table, '*Time-consuming questions that made you think about strategy*' included comments from the fourth-level code, that '*Thinking of method for unfamiliar questions is challenging*'. The *'Provides ZPD opportunities'* code incorporated comments made that providing ZPD opportunities gave '*Challenge opportunities for everyone'* and that it '*Prevented boredom'*. Finally, the third-level code *'Illumination / AHA! opportunities'* included student comments on the *'Satisfaction at perseverance'* as well as those stating that such experiences *'Builds confidence'*.

Table 15 Number of Responses Coded in the Gestalt Principle

GESTALT PRINCIPLE	**Classroom comments**	**Workshop comments**
Lack of opportunity to incubate (general):	**9**	**0**
• Questions easy	**18**	**2**
• Reliance on textbook	**24**	**0**
• Teacher focus on less able	**36**	**0**
• Teacher gives solution when stuck	**26**	**0**
• Boredom	**50**	**0**
Opportunity to incubate (general):	**1**	**8**
• Time-consuming questions that made you think about strategy	**0**	**25**
• Enjoy challenge of thinking about tasks	**0**	**98**
• Thinking tasks improve your mathematics skills	**0**	**19**
• Provides ZPD opportunities	**0**	**65**
• Illumination/ AHA! opportunities	**0**	**81**

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6.2.1 Gestalt Principle Classroom Findings

As mentioned above, most data related to the classroom in the Gestalt Principle was concerned with an opportunity or lack of opportunity to incubate. Table 17 shows that the classroom seems to provide very few opportunities for incubation while the students perceived the workshops as having lots of opportunities to incubate. There was only one comment in the whole study from a student that indicated that his mathematics class were sometimes given time to think:

Student 430: Yes, there might be the odd time that we're given it. We're maybe given two or three minutes to think of the method.

The first subcode '*Questions easy'* refers to students being given tasks that were not suited to their ability because the student felt they were too easy. For this higher-order theme of Gestalt, the focus was on the teacher's initial preparation (here the choice of question), which had the result of removing opportunities to incubate because the student knew how to do the task immediately. The 18 comments coded here were for when the student specifically said that they found the work 'easy'. For example, the following are samples of student quotes in response to the interviewer's question "How would you describe your experience of maths in school?":

Student 231: I don't find it too difficult. I find it easy enough… It's easy enough to think of the methods because they're always the same.

Student 432: I found it quite good. I found it very easy. A lot of the time questions are almost identical to the examples you're given so there's no real challenge involved. It's just copy and paste in what you're already given.

Related to this first subcode on '*Questions easy*' were some of the comments on '*Reliance on textbook'*. The focus here was on the teacher's preparation that appeared to rely on the textbook and the consequence for the students' incubation opportunities. A typical example of the students' descriptions of how a mathematics class was introduced in school is given below:

Student 231: They're just read through the book and then talk us through and bring us through a couple of examples and then they just give us questions to do, and we just do those.

The code '*Reliance on textbook'* also included comments referring to the procedural nature of questions in the textbook:

Student 425: If you look chapter by chapter, it's basically the same question in different words over and over and over again.

The above comments were among the 24 that were coded as '*Reliance on textbook'***.** For such comments, it felt that the emphasis was on preparation that removed opportunities to incubate but did not appear to directly imply boredom like many others did. Comments that mentioned the textbook yet seemed to specifically express boredom or lack of stimulation, are included in the 49 comments coded as '*Boredom*'. Details on these will be given later.

Similar responses on the procedural nature of the textbook were given when the students were asked if they ever do mathematics problems not set by the teacher. The students did not seem to have used anything other than the textbook:

Student 324: If you get loads of work anyway, you're not going to want to do more of the same thing. There's not really any place where we can get the type of questions that we'd want to do ourselves. It would all be from the same book we did, and the same kind of questions.

Student 340: Even still *the book*, there's only so much you can do with *the book*. I feel like it would be better to be getting other sources of learning.

As mentioned earlier, there was considerable overlap between the themes as the final template was developed. For this reason it was decided more appropriate to keep the Gestalt Principle codes focused on the comments related to the initial teacher preparation and whether this allowed or prevented opportunities for incubation. Of these 24 comments on the *'Reliance on textbook'*, many have also been parallel coded with another higher-order principle because the focus was on more than just teacher preparation. For example, the comments below have been parallel coded in the Uncertainty Principle because they indicated reliance on the textbook by the teacher but also suggested that this made the method obvious for the student:

Student 036: So in class, most of the problems are given from the textbook and follow the same method, so when you see them if we've been doing, say, algebra or something … you can see immediately you look at it, "Oh, they all follow the same formula or way to do it." It doesn't really take that long to do it.

Student 333: You're just not given enough time in the class to think of other methods because most of the questions we are given are textbook questions, stuff that has been explained in the book.

Another predominant code in the Gestalt Principle was the feeling towards the '*Teacher focus on less able'*. There were 36 responses making reference to this, some of these quotes from individual students were just statements of fact describing their experience, such as the two comments below:

Student 126: You are kind of left to do your own thing, the ones who are struggling, the teacher can spend most of their time with them, making sure they are figuring it out. And you are just left off by yourself.

Student 131: If one person doesn't get it, the teacher can just …

Student 126: She keeps going for the whole class.

Student 425: I don't think I'm very challenged because when you finish a question … you just sit there until the class moves on to the next question.

Student 021: I'd understand what she's talking about … other people wouldn't. And she kind of takes time for them and no one else, you know?

Student 326: Everybody, regardless of ability, is all in the same class, so you could be waiting like a while.

There were other situations where students also voiced being annoyed with how the teacher was responding to queries from students who were struggling. Like the above comments, these were coded under Gestalt Principle because they reflected a lack of advance preparation on the teachers' part to have alternative explanations ready and to keep the more able students engaged while they worked with students who were having difficulty. The comments referred to a teacher not seeming to understand where in the problem the student was stuck, or offering an alternative explanation to help them:

Student 036:	There might just be one thing that they (the student) didn't understand, and … you know where they've gone wrong and then the teacher goes on for 20 minutes, and then you're like "They got that bit, but they haven't actually explained … they just did the question again."
Student 023:	Oh, yes, that's so annoying.
Interviewer:	**What do you lot do when that's happening, if somebody couldn't do it and you can do it?**
Students 020 & 023:	Just sit there and do nothing.

When students commented that they were just left waiting in class, the researcher enquired if they would ever be given something to do, something totally different maybe? The response to this question brought similar frustrated comments:

Student 023:	We'd be given one to do, then you wait for her to finish something else she's doing, and then she comes to that one question then there's another question.
Student 429:	For the most part, it is just not a whole lot to do once you get the questions you were asked.
Student 429	When you finish a question … you just sit there until the class moves on to the next question.

Student 330: Sometimes, our teacher will say, "Okay. If you've done that, go to this page, and do these questions." Usually, it's more waiting than doing that.

As in '*Reliance on textbook'*, there were also examples of parallel coding with the Uncertainty Principle in '*Teacher focus on less able'*. These included comments where the students spoke of being given repetitive questions. The following comment was an example of student frustration with teacher preparation that had not provided differentiated questions:

Student 036: Like there's no point in just getting another 5 questions about what you've just got right.

However, some students showed an element of empathy for their teacher, who they realised was trying to make sure that nobody was lost and consequently they did not seem to mind waiting around:

Student 333: No. Probably not, because the teacher is probably too busy trying to get the whole class to understand.

In addition to the comments that were coded in this theme as '*Reliance on Textbook'* and '*Teacher focus on less able'*, many appear to be directly connected to feelings of *'Boredom'* or lack of stimulation because of the teacher's class preparation. Boredom was a key thread in this Gestalt Principle, with 49 specific responses, and seems to be related to several factors that are associated with teacher preparation. The more general traits that removed incubation opportunities and led to student inactivity have been detailed above. However, the 50 comments relating to the code *'Boredom'* were additional comments that mention boredom or indicated that the students were bored. In some cases the students did not react negatively, but other feelings of frustration that they are not being catered for are noticeable:

Student 023: There's a lot of waiting around in maths and doing nothing. giggle.

Student 340: Yes, I find myself bored in the classroom sometimes, and I love maths.

Student 430: Honestly, you're just really sat there twiddling your thumbs. You might go ahead with the next couple of questions, but otherwise, you're not really doing much.

Student 340: That's why a lot of the time I'd be falling asleep, because the majority of the classes are actually the teacher trying to catch up the students that didn't get it right or didn't understand it.

The final subcode describing the students' classroom experiences, the '*Teacher gives solution when stuck*' arose out of student comments on the extent to which they were encouraged to persevere with problems in class. Enquiries regarding what happens when students could not do a question set by the teacher received diverse responses. In total, there were 26 comments coded under '*Teacher gives solution when stuck'* and the overall impression was that the teacher usually did the question on the board, rather than allowing the students the '*Opportunity to incubate'* and figure it out over time. The following excerpt is typical of the answers to this question:

Student 231: The teacher will just explain the question on the board and then we just move on to a new topic.

Interviewer: Would you ever be given a problem and not told the answer for about a week and made to go back and think about it again and again?

Student 231: No, not really. It's just when you ask, they just tell you how to solve it.

In another interview, all of the group were in agreement with one student's response which was similar to the one above:

Student 036: Never… the answer is always given.

One group of students who had similar experiences also hinted at teachers not always giving explanations as this conversation indicates:

Student 020:	The teacher would do it on the board.
Student 023:	You'd have to ask. It depends on the day and the form the teacher's in because they won't always go over it, even if people don't understand. It just depends.
Student 020:	Yes, they might leave it … or then it could be the opposite. They might spend nearly the rest of the class just going through that one question explaining it.

There were no examples given in these interviews of teachers allowing the students time in class to engage with the problem which they later came back to, as can be seen in Table 17. All but one of the codes for '*Time-consuming questions that made you think about strategy*' were for student reflections on the workshops and are detailed in the paragraphs that follow.

6.2.2 Gestalt Principle Workshop Findings

The lower section of the Gestalt Principle Table, labelled Table 17, is reproduced below with the data on *'Opportunity to incubate'*, to make it easier to compare the table data with the comments and quotes below.

Table 16 Number of Responses Coded in the Gestalt Theme under Opportunity to Incubate

Gestalt Principle	Classroom Comments	Workshop Comments
Opportunity to incubate (general):	1	8
• Time-consuming questions that made you think about strategy	0	25
• Enjoy challenge of thinking about tasks	0	98
• Thinking tasks improve your maths skills	0	19
• Provides ZPD opportunities	0	65
• Illumination/ AHA! opportunities	0	81

All but two of the findings for the workshop are coded under '*Opportunity to incubate*'. The two exceptions were comments made by two students in one group that they found, for the first task in the workshops, the initial method 'wasn't too challenging’ and ‘easy enough’. However, they both commented that the task did get more challenging:

> Student 131: When you had told think more in-depth and you have to find a different way, that was when it got a bit more challenging.

This comment referred to a differentiated task and, had the researcher realised how fast this group were working, it would have been wiser to move them on to a higher level sooner, as was done with other groups. It is worth mentioning here as an example of the importance for students to be able to move at their own speed and how a carefully designed differentiated task can satisfy a range of abilities.

In contrast, there were a significant number of comments describing the feelings of challenge students had in the workshops. There were 98 comments coded as ‘*Enjoy challenge of thinking about tasks*’. These 98 comments included those acknowledging that having to think of the method was challenging and when students said that they enjoyed the challenge of thinking of a strategy in the workshops rather than following a teacher’s method. The selection of quotes below are representative of the overall student reactions to how they perceived the workshop tasks to be different to those set in school:

> Student 326: I'd say it's definitely more challenging when you're not told the strategy straight off.

> Student 020: You have to be thinking the whole time. You're constantly thinking, you can’t switch off because they’re a lot more challenging.

> Student 036: You don’t necessarily start writing straight away. You might have to think for a few minutes, and say, "Would that work?" rather than just tearing straight into the question.

> Student 340: It's so much more rewarding to come up with your own strategy.

During these reflections students often made references to how more challenging questions that made them think improved their mathematics skills. These 19 phrases were coded separately as ‘*Thinking tasks improve your maths skills’***.** The distinction was made here for phrases stating that it was the incubation period that brought about an improvement in skills, and not specifically having to come up with multi-solutions which were coded in the Scholarly Principle.

Student 429: Whereas if you could be given even if it's a similar topic, or just a more complicated question, or a question that would take more time. I feel like that would be more rewarding and it would result in better skills in that topic in the future.

Student430: Do I think it would be helpful for revision and developing understanding? Yes, 100%. I agree. When you're doing all this, you can see why everything works rather than just being told that this works, and you should do it this way.

Student 020**:** It kind of trains your brain to think in a different way. When you go into the exam, then, you won’t have seen any of the questions before, and some people get a shock then that they don’t really know. If we were doing questions like problem-solving and stuff … I think we’d find it a lot easier to think about it and understand the questions.

Student 430: I'd say the difference between the two of them is one, you have to think. In the workshop, you have to be looking, you have to be aware of everything that's going on. You need to really process the information. But in school you have your formula, you have the answer, and you just need to find whatever you're looking for.

The subcode ‘*Provides ZPD opportunities’*, which came about as a result of student responses to ‘*Time-consuming questions that made you think about strategy’*, had 65 occurrences. When asked if they thought the tasks in the workshops would be beneficial in school the responses suggested that students appreciated the '*Challenge opportunities for everyone'* aspect of such time-consuming tasks:

> Student 126: No one's really sitting around waiting. Everyone is working their way to their own level. I think they'd work.

> Student 020: Yeah, definitely because everyone gets to work at their own pace as well and if you can't do the harder questions, you can just work on the first few and that may be challenging for you.

> Student 131: Everyone just gets to work away at their own ability. No one's being held back. Everyone's thinking, no matter how good you are at maths, you're still thinking.

> Student 425: I think that they would work great. It's like that everyone is learning the same basics of it, but then depending on your level, you can branch out and do so then people aren't just separated from higher to ordinary. It's all the same topic with just different ways of doing it depending on your skill level.

> Student 023: It would also benefit all students in the class, so the people who mightn't like maths, they might find it difficult, they'll still have something to work towards, as well as the people who would like more challenging questions.

There were other benefits that emerged from tasks that enabled students to experience the ZPD. One was the belief that the style of task in the workshop could have a positive influence on students' interest in mathematics. Comments that reflected this opinion were coded under '*Prevents boredom'*:

> Student 432: It would change a lot of attitudes towards math. People who originally said like, "Oh, maths is boring," might actually find maths interesting now. I think that's good.

All the comments related to Vygotsky's theory on the benefits of students experiencing the 'zone of proximal development were grouped together. Aspects of peer collaboration were included here if it referenced a challenge arising from thinking within a group. Also within this code, were those comments that acknowledged how differentiated tasks with multi-solutions can challenge all students simultaneously. There was evidence of significant consideration for the less mathematically able students to deserve to feel challenged too which highlighted a depth of thought in the student responses. Most of the above were responses to questions enquiring whether or not the students enjoyed being given tasks where they had to think for themselves first. From the quotes below you can get a feel for the enjoyment experienced and the recognition of '*Challenge opportunities for everyone'* which was a fifth-level code in this Gestalt Principle:

> Student 326: I did find it challenging, but the challenge was enjoyable, and working in a group and kind of bouncing ideas off each other and saying like, "Oh, does this work?" and then trying loads of different things. I enjoyed that challenge.

> Student 231: It pushes students who want the extra challenge and are able for it, but at the same time it accommodates people who aren't really sure what they're doing as well.

> Student 326: The students that will be more inclined in maths could like go get the answer and then try do more solutions for that question, and then the students that wouldn't achieve the answer or that would fall short, they're not feeling that they've underachieved anymore because they're shooting as far as they can and getting maybe one solution to a problem that someone else might get five solutions to.

Within this ZPD code was also a fourth-level subcode for students who felt that challenging tasks that made you think '*Builds confidence*' in your own ability. The following quotes show this sense of pride in their achievement:

Student 330: Yes. I do enjoy that because when it works, you feel like you've done something that you should be proud of without any other help.

Student 341: It is nice if you do get it correct and you do your own strategy because it makes you feel very confident in yourself that you can complete something without anyone else's help, or that you thought of an originally different way to do it. It makes it more challenging.

The final subcode, '*Illumination / AHA! opportunities'* had 81 comments. The tasks in the workshops were designed to try and provide a different experience to those the students have in the classroom. A key feature was to try and provide opportunities for the students to experience the satisfaction of perseverance. The ultimate objective of this was an attempt to bring about the illumination and AHA! experience students can have from reaching a solution through exploration without instruction from the teacher. For this reason, given the frequency of comments on students following their teacher's example, illumination opportunities did not appear to be a feature of classroom experiences. The results breakdown showed very positive feelings from the students in all interviews on the satisfaction and confidence that came about from persevering on a task. The following quotes were responses to questions probing whether students enjoyed having to think of a strategy for themselves:

Student 326: Once you finally get a solution that works, the satisfaction is far greater than a question that you can just learn off in a textbook.

Student 034: They were so different. They weren't even that confusing. Once you got it done once and you were like, "Oh, I got it," you got the sheer sense of achievement.

Student 034: It's more enjoyable when you can think about yourself and get more satisfaction out of it, when you actually think about the method yourself and you come up with a way to do it. When it is more challenging, I think it's better.

Student 129: I did because it took a long time to get to really think about it and then when you did it was rewarding, you feel accomplished when you actually get it right.

Overall, the student responses in the interviews point towards very different experiences in the classroom and in the workshops with regards to having opportunities to incubate. There was definitely a feeling of enjoyment and satisfaction from the experience of thinking and pushing yourself. The students were able to articulate what it was they liked about this challenge and the benefits they felt it brought them. In addition, there were elements of enjoyment of the independence such questions brought which will be further examined under the Free Market Principle.

6.3 Uncertainty Principle

The codes in the Uncertainty Principle theme were grouped into two main second-level codes. The first one was for situations that described where the uncertainty was removed for students. The second one was coded as '*Having to think of method for yourself'* which encompassed all general comments the students made on not knowing the method when they first looked at a task. The results in Table 18 below give evidence of diverse experiences for the students in the classroom and in the workshops. Overall, the students did not appear to experience uncertainty in their classroom. In contrast, there were no comments by students on being shown the method first in the workshops. The workshops had been specifically designed so that the students were not given any hints about which method to use.

6.3.1 Uncertainty removed - Classroom Findings

There were 243 comments in the second-level code *'Uncertainty removed'*. The focus here is on looking for situations describing how the uncertainty was removed for the students. Three of these were general comments on not having to think and the remaining ones have been divided into the third-level codes '*Shown method first'* and '*Question method obvious'*. The first of these, '*Shown method first'*, represented comments made that implied students did not experience uncertainty by having to think about which method to use because they were told how they should do the question in advance. There was a total of 93 comments under this code. There were 149 comments coded as '*Question method obvious*'. These included the data where the questions posed to the students were routine or repetitive. As a result, the students did not have any thinking or decision making to do upon starting a task.

Table 17 Number of Responses Coded in the Uncertainty Theme

UNCERTAINTY PRINCIPLE	Classroom comments	Workshop comments
Uncertainty removed	3	0
Shown method first:	33	0
• Teacher gives example	53	0
• Follow textbook example	7	0
Question method obvious:	27	0
• Questions are procedural	34	0
• Repetitive Questions	53	1
• • Student reassurance from familiarity	9	1
• • Negative impact of repetition on interest	24	0
Having to think of method for yourself	0	57
• Occasionally in class	14	0
• No opportunity / only in exams / tests	33	0
Preference for thinking of methods	-47-	

There were 33 general comments made by students on being '*Shown method first'* where no explicit details on how this was done were given, such as:

> Student 425: We rarely get to go at it ourselves first without given instructions.

The other comments were included under two subcodes of '*Shown method first'*. The comments coded here were generally in response to the students being asked in the interviews how a new topic was usually introduced to them. These subcodes were to distinguish between when the students specified that an example was given by the teacher or, less commonly, taken from the textbook. The first was '*Teacher gives example'* and there were 53 such comments classified using this code. In each case the students described similar situations where the teacher in their class gave them an example to show them which method they should use, hence there was no uncertainty for the student*:*

Student 330: The teacher, on the board, has already written down an example question and goes through how it works, and we do questions on it.

Student 129: They just show us examples on the board, explaining how to do it and then they let us do it ourselves.

Student 046: You’re just shown the method and then you just do it and then they'll check if you do it right, and then you'll just keep practicing the same questions.

Student 027: Usually we're like, given a method or if not, we are taught which method you should use in this situation.

The second subcode of ‘*Shown method first’* was ‘*Follow textbook example’* and there were 7 such comments in this code. However it was unclear when students said they followed an example from the teacher if it was taken from the textbook. In these 7 cases the students explicitly said they were either guided through a textbook example by the teacher or used the book independently if they were working ahead:

Student 121: Usually, the teacher would explain it to you to do it before, then let us go at it. Sometimes I could probably, just look at an example in the book and do it myself without the teacher.

Student 126: It's just usually like, we go through the example in the book and the teacher does an example on the board … and we go off and do our own questions.

Student 121: Sometimes I could probably, just look at an example in the book and do it myself without the teacher.

Another way in which the uncertainty was removed for the students was because of the question method being obvious to them and they felt they did not really have to spend time thinking about it. The third-level code *'Question method obvious'* included 27 general comments in response to being asked how long it takes to think of the method for a question, such as:

> Student 326: It's not really difficult that you have to spend ages thinking about a method or anything.

> Student 226: They're easy to do and it doesn't take long to know what method to use for the question.

> Student 429: To think about the methods is just, if you have a logbook to find the method for it, it's not too intense or anything.

To accommodate more specific comments on *'Question method obvious'*, this code was further divided into '*Questions are procedural'* which had 34 comments and '*Repetitive questions'* which had 88 comments. '*Questions are procedural'* referred to the student perception that they did not have to think about the method because the question was familiar or that it only required the application of a formula which they had already used before, such as:

> Student 226: The only challenging questions, like you might find one in the exam papers, but in the book, they don't really get you to think. You just use a procedure to get the answer.

> Student 121: With say textbook questions, you know exactly what methods you have to use … and the chapter is based on a certain method.

> Student 036: Yes, so in class, most of the problems are given from the textbook follow the same method, so when you see them if we've been doing, say, algebra or something, "The difference of two squares", you can see immediately you look at it, "Oh,

they all follow the same formula or way to do it." It doesn't really take that long to do it.

Student 126: The questions we have in the books … they're not like the ones we did in the workshop … In the book, they were in lines and the problems all looked the same.

Throughout all the interviews there were many comments from students reflecting on how their classroom mathematical experience was comprised of '*Repetitive questions'*. As shown in the table, there were 53 comments suggesting the recurrence of repetition. Most of these showed an element of frustration:

Student 129: It's all just repetition.

Student 046: We're doing stem and leaf diagrams where you just type in the numbers and it's just so tedious, and she gives us loads of them to do. It's just the same thing typing in the numbers, and it just takes so long.

Student 131: I find once they have done the example and if you get the example, the rest of the questions, they are just kind of repetitive and you're just sat there dealing the same thing over and over again for the whole class.

Student 123: (The workshops were) just more fun than anything you do in class. Class is just more repetitive.

During the development of the final coding template, comments emerged describing the recurrence of repetitive questions that seemed to specifically impact student interest levels in mathematics. To distinguish between this inherent frustration and the experience of those students who seemed to enjoy repetition '*Repetitive questions'* was split further into two subcodes. The subcodes were '*Negative impact of repetition on interest'* in which there were 24 comments coded and '*Student reassurance from familiarity*' in which there were 10 comments.

Out of the total of 86 comments coded as repetition in the classroom, there were 24 that were specifically implied a negative effect of repetition on the students' interest. These comments were dispersed throughout various sections of the interview, and it appeared that the students had strong feelings about its impact on their classroom experience and their learning.

> Student 023 Like there's no point in just getting another 5 questions about what you've just got right

> Student 425: You're not focusing on what you're good at. You're good at maths but you need to have more challenging questions. Sitting there, doing the same thing over and over again isn't really challenging.

> Student 325: Then, it gets more repetitive. I think it might even dull your mind. It might make you bored of it, and you might no longer want to do maths that much.

> Student 429: I'd feel like especially with how some of the classes are set up with the repetition. I would be fed up with it by then, but it could be useful in the future, yes.

The other subcode of the fourth-level code '*Repetitive questions'* was the code '*Student reassurance from familiarity'*, in which there were 10 comments. These comments reflected students who seemed to feel comforted by repetitive questions, rather than frustrated:

> Student 225: Maths is not one of my favourite subjects, but I like doing it because once you are given the questions, there's just always the same.

Student 225: I think I liked doing the tasks in the workshop more, but then in class sometimes I like learning stuff for exams and stuff and doing them over again.

Student 043**:** I feel like I prefer questions with a formula because then you can use the formula for all the questions so it's similar.

Student 021: I prefer school maths just because it's been so drilled into my head.

6.3.2 Having to think of a method – Classroom Findings

The comments made by students with regards to *'Having to think of method for yourself'* either suggested this was an occasional occurrence or that it only happened in tests or exams. There were 14 responses that indicate that students were given an opportunity to think of a method '*Occasionally in class*' and 33 comments suggesting that students were given '*No opportunity / only in exam/test*' situation. The comments indicating that students were specifically asked to think of a method '*Occasionally in class*' included:

Student 333: We've had several topics where we're just given a question first and then we are first to think about it and then, explain it to our teacher what we were thinking. Then, he would show us the real example on the board and how we would do it.

Student 430: Yes, there might be the odd time that we're given it. We're maybe given two or three minutes to think of the method, but otherwise, we're given it more or less straight away.

Among these 14 '*Occasionally in class*' responses were those to the question: How often do you experience not being able to identify a method required to solve the problem immediately? The students' responses to these seemed more related to thinking how to solve a question, mid topic, as opposed to thinking of the method at the outset of a topic:

Student 020: I think sometimes if they're worded problems, then they'd be different each time, so that makes you think a bit more because each of them are different.

Student 330: Maybe if it's revision for something we did a long time ago.

There were many responses of 'Never" when asked: How long does it take you to think about the method required to solve the problems you're given? Other comments with more explanation that were coded under '*No opportunity / only in exam / test*' situation included:

Student 027: Yeah. Like, in class, we'll never actually have to, like, think for ourselves because she'll tell us like, what kind of method.

Student 429: It wouldn't be too often, now. More in tests than in regular class … when you need to figure out what you're doing first.

Student 226: Normally, you wouldn't really know if you were doing exam papers.

Student 121: There is a chance a teacher might say, "What do you think you do here?" It would have something to do with another chapter you did.

Student 023: They never encourage you to look at it and just think.

The *'Preference for thinking of methods'* code was added as a third-level code here to record student preferences for thinking about strategies in contrast to following a teacher's method. There were 47 comments coded that revealed that students preferred to think of a strategy for themselves. It was difficult to determine whether the students were responding specifically to the classroom or workshop experience in many cases, so all the codes relating to having a preference about thinking about the method were combined into a general column. The comments below give a good impression of the overall attitude of the students in this survey to having the opportunity to think for themselves. Typical questions included:

> **Interviewer:** **Do you enjoy thinking of the strategy yourself, if so, what makes it interesting?**

And various responses were:

> Student 340: Yes, definitely. I enjoy that. That's the fun in maths, thinking for yourself. It's so important because that's what makes you improve on it.

> Student 046: If you come to a conclusion yourself about a question, you’re going to remember it more than if you're just given one.

> Student 023: You understand it more than know the method. If you understand it, you'll remember it more than, "Okay, you just learnt this method, but you don’t know where its coming from."

> Student 425: I like to choose a strategy myself because it's like, I don't want to just do the question for the sake of it. I want to understand it. I don't want to just follow a basic, whatever they're telling me to do.

6.3.3 Uncertainty removed – Workshop Findings

The only comment related to the ‘*Uncertainty removed*’ code in the workshops was with regards to the repetitive nature of the first task, Field of Dreams:

> Student 126: Some of them got a bit repetitive like the Field of Dreams one. But apart from that one.

It was from a student in the same group that had mentioned that this task was ‘easy enough’, previously coded in the Gestalt Principle. As commented upon in the Gestalt

Principle findings, this task was a differentiated task, and its purpose was to get gradually more challenging. This particular group of students had moved ahead before the researcher had the opportunity to notice. Other groups of fast paced students had been asked to skip one or two fields which reduced the repetitive nature. This accelerated them towards the more challenging aspect of the task which was finding alternative methods.

6.3.4 Having to think of a method - Workshop Findings

There were 63 comments coded in this theme as '*Having to think of method for yourself*'. Of these, 16 related to situations where the students spoke about experiences where they had to think for themselves in the workshops. When coding here the focus was on the initial uncertainty for students being removed or not. There were many different aspects to having to think for yourself throughout the interviews. As shown earlier, many of the comments relating to where students had to think for themselves were coded in the Gestalt Principle because the students had given further details on the challenge posed by time-consuming questions being set. The comments relating to the multi-solution tasks in the workshops were coded separately to try and distinguish student thoughts on thinking in general as opposed to comparing school tasks to those in the workshops. Student reflections on their enjoyment of thinking about the specific workshop tasks were also coded in the Free Market and Scholarly Principles. However, there were still a considerable number of general comments coded under '*Having to think of method for yourself*' that were related to the workshops, often in response to questions similar to:

Interviewer: **Did you find the problems in the workshop different to those you see in school? If so, in what way were they different?**

Student 432: I like that I actually had to think about what I was doing before I started. Normally, you just see the question, almost immediately you would know exactly what you'd have to do, because you've been shown an example. You can just get into it and you're not really thinking about it. You can just switch off your brain and do it.

Student 231: The problems in the workshop were really different. There were loads of different ways to do it and you had to think of the method yourself. It wasn't really like algebra or something like that, you had to think outside the box for it.

Student 045: Yeah. I enjoyed it because it was different than what we do in school and there's a lot more thinking in it, and like, we weren't really told how to do it so it was, like, you had to think for yourself.

Student 034: I think in the textbooks, like, you kind of, just have to learn off how to do it and then do it but like here, you kind of were just thinking like you were really actually thinking about it.

Having to think of a method in the workshops did prove to be more challenging for two or three students out of the 93 in the study. The contrasting experience of the workshop problems to their school maths was evident in their comments:

Student 041: I was nearly crying … I just found it really challenging … I did enjoy them when I knew what I was doing, but I wouldn't have known what I was doing for ages.

Student 037: I would be wanting the formulas back. About an hour in and I was like, "I would love a formula right now."

6.3.5 Self-perception thoughts

The 10 comments on '*Student reassurance from familiarity'* in the classroom findings were from students who did not appear to be as confident in mathematics as the majority in the study. This prompted the addition of a code relating to self-perception to this theme in order to cross reference comments made when doing the analysis. Because of the voluntary nature of the workshops, the students who participated were not necessarily all very confident mathematicians, particularly when working in groups with peers of higher ability. It was apparent that this code could not be used to provide any concrete evidence on the individuals' level of self-perception. The intended purpose of it was to be able to cross reference comments made with the responses in the survey just to be able to put their comments into context. It was, however, possible to confirm that those who made comments coded as '*Lack of confidence if method not known / forgotten*' had responded similarly in their pre-workshop surveys.

Table 18 Number of Responses Coded under Self-Perception of ability

Self-perception of ability:	• Lack of confidence if method not known / forgotten	**8**
	• Struggle initially then grasp well	**10**
	• Finds wordy questions challenging	**3**
	• High self-perception	**18**

'*Self-perception of ability'* codes were added to put into context comments where some students seemed to enjoy the reassurance from the familiarity of repetitive questions. It was interesting to get an overall feeling for how these students perceived their mathematical ability. The data in Table 19 shows the number of comments related to student *'Self-perception of ability'*. These were cross-referenced with other comments on classroom and workshop experiences to get a clearer picture of the student perceptions in the distribution. There were 8 comments signifying '*Lack of confidence if method not known / forgotten'*:

Student 041: I've kinda always like, hated maths. Like I find it … I find it really difficult.

Student 034: I am not particularly good at maths, so I think that's why I enjoyed it (the workshops).

Student 021: I can’t really like, think for myself. I need to be told what to do. I can’t really see the connection. Like what we’ve learned and the question that comes up on the exam … Once you kinda learn off the methods and stuff, it's okay.

Student 043: They give us the same questions to practice but then they’ll give us another question that was in the chapter, but we didn’t do it on a test and then you don’t really know what to do.

Of the remaining 28 comments indicating the self-perception of students, 10 implied that they *‘Struggle initially then grasp well only’* and 18 implied *‘High self-perception’*. The comments coded under ‘*High self-perception’* were useful when reflecting on the level of challenge felt by the individual students. It can be seen that the students who perceived themselves as having high ability in mathematics also felt noticeably underchallenged in school:

Student 045: I found it relatively easy, some things I kinda find hard, but … I didn't really have to think that long about the method, I know how to do it most of the time. But the teacher would have taught us that before, and then she'd give us questions to do.

Student 429: I feel like in class the level of difficulty isn't too high, especially if you get questions done.

Student 027: I didn't find maths that hard … And you don't really have to think. Usually, there's some questions, that, like, you might have to, like, decide which formula but usually you don’t.

Student 231: I think sometimes you get a bit bored. I don’t think I am ever in maths thinking, "Oh my, this is so hard. I don’t know what I am doing. I haven’t a clue."

In summary, there was a major difference between the opportunities that students had to encounter uncertainty in school and in the workshops. The experience of having to work on an unfamiliar task and devise a solution method was new to the students, and even though they sometimes found this difficult they seemed to relish the opportunities.

6.4 Aesthetic Principle

The results of the coding on the Aesthetic Principle have been grouped to distinguish between comments that highlight different perceptions students have about the purpose of mathematics. In the Table 20 below there were 14 comments related to enjoying seeing simple solutions and the lack of opportunity to do so in class. Similar comments were made regarding discussing other solutions, but these were coded in the debate and discussion code in the Free Market Principle.

There are 130 comments related to mathematics being *'Exam focused'* in school. Within this subtheme fourth-level codes were added to record specific aspects of how the focus on exams affected the students' experience. These fourth-level codes included students' comments on school mathematics being focused on the *'Purpose of maths in schools =getting result'* which had 14 comments classified under this code. The most frequent comment relating to the focus on exams was that it was more about calculations and the memorisation of methods required for exams. This became a fifth-level code '*Maths is about routine calculations and learning methods'* and had 63 individual comments coded.

The final two fourth-level codes were a focus on '*Focus on time / need to keep moving on'* which had 25 comments, and '*Alternative style of questions to improve maths skill not common'* which had 10 comments. There were other comments made by the students on the lack of alternative questions. These were coded in other higher-order themes because they specified the effect of teacher preparation, coded in the Gestalt Principle, or overall learning, coded in the Scholarly Principle, rather than only because of a focus on exams. The importance of '*Learning for future / real life'* was mentioned 13 times and 70 comments were made with reference to the importance or a '*Focus on enjoyment of the process*'. These comments tie in with the negative feelings students showed towards repetition which were coded in the Uncertainty Principle. Students made 39 comments on having a *'Tendency / preference to use* *algebra'*, which, as mentioned earlier, were about both classroom and workshops so it felt more appropriate to code them as general comments.

Table 19 Number of Responses Coded in the Aesthetic Theme.

AESTHETIC PRINCIPLE	Classroom comments	Workshop comments
Beauty of simple solutions		
• Enjoyed looking at simple solutions in workshop	0	11
• Lack of opportunity to discuss solutions in class	3	0
Objective of maths		
• Focus on enjoyment of the process	0	70
• Learning for future / real life	1	12
Exam focused: General comments	18	0
• Purpose of maths in schools = getting result	14	0
• Learning methods	63	0
• Focus on time /need to keep moving on	25	0
• Alternative style of questions to improve maths skill not common	10	0
Tendency / preference to use algebra		39
Visuals		
• Enjoyed visuals	0	25
• Little emphasis on visuals	10	0

6.4.1 Classroom Findings

Discussing the '*Beauty of simple solutions*' was not a feature of the students' classroom experience. Three students made direct reference to the fact that they lacked opportunities to do so. One such response was during a discussion about what the students would like to change about school mathematics, and it showed a preference to have more discussions on solutions in school mathematics:

> Student 425: Focusing on the practical side of maths like maybe using real-life examples like, "This was the problem that these people faced whenever and so how would you do it?" Then maybe at

the end of it like, "Okay, this is how they did it," and compare like that.

There was one student comment that understanding was important as well as getting the answer, during a discussion on opportunities to look at alternative methods. Other than the comment below there was no discussion on enjoying the process of mathematical problem solving in the classroom:

Interviewer: ***How often are you specifically asked to solve problem using more than one method?***

Student 325: The majority of the time, it's just the one straight orthodox way and that's it … Yes. It's getting the answer and understanding the topic.

Student 333: Yes. I would say it's the exact same. You're just not given enough time in the class to think of other methods

There was only one comment suggesting that school mathematics will be useful for the future:

Student 430: You do obviously learn something that might be beneficial to you in the future. However, at this stage, at this point in our lives, we actually don't know.

In contrast to the above observation, the most dominant theme in the Aesthetic Principle was the focus on exams. The 130 quotes related to aspects of school mathematics, that were outcomes of teachers teaching towards an exam, give a clear description of the students' classroom experience. Some of the comments were general comments mentioning the focus on exams:

Student 046: I hate that it's so exam-focused usually. Like you're doing something and you're trying to ask why. Teachers are like it doesn't matter …
in school you're just learning maths just for exams, like that's your only reason, the teachers are just teaching for that.

Student 425: When I'm doing questions in school, I feel like there's no purpose to it like it's just for the exam.

Other comments specified '*Learning methods'* for routine questions as being the primary objective of mathematics in school. Some of these quotes also mentioned exams, however they were coded here if they highlight memorisation as a key feature:

Student 324: I don't think it's so much skill anyway, it's your memory more. If you can remember the methods, remember the numbers, remember how to do everything then you'll be grand. Whereas for the problem-solving ones they're more skill based.

Student 027: Routine answering. Learning how to do it, doing it. And, like, they show you and then you do it.

Student 342: I think they're teaching more about you just have to learn stuff off just for exams. It's not really actually testing your individual skills.

Student 326: I think that we're more taught to just learn off as many formulas as we can and then spit them off in the exam paper. We're more in class to prepare us for an exam rather than to actually use maths and think for ourselves and stuff.

Student 339: I agree … on the skill bit being based on your memory. They don't give us a lot of problem-solving questions. It's more like methods that they give and you have to remember.

The comments coded as '*Focus on time / need to keep moving on'* appeared to vary between those indicating that the teachers' primary objective was the students getting the correct answer and those showing an element of frustration that teachers were moving too fast:

Student 231: Say if you did a method that was quick enough, they wouldn't ask you to look for another method if that takes the same amount of time.

Student 043: I feel like sometimes though the teachers need to be a little bit more helpful because sometimes when people ask questions because they don't understand it, the teachers look nearly annoyed because they can't move on.

Interviewer: How often are you asked to solve it using more than one method? So, if you get it right, they say, "Try a different method."

Student 037: No. I can't really think of one.

Student 021: No. They don't have time really to do that with us … They just want you to learn.

Other students showed empathy for the teachers trying to cover the course in the allocated time. They suggested that the teachers did not have time to allow students time to think about the answer to a task or to do problem solving, time-consuming questions:

Student 226: I think they don't have enough time so in fairness, they just have to give the answer. They don't have time to be waiting for us to think of those methods and solutions.

Student 129: Because that would take a lot more time. There's only 45 minutes in a class.

Student 037: I feel math's also very rushed. I know they have deadlines they have to meet and I understand that.

Throughout the interviews the students frequently expressed having *'Tendency / preference to use algebra'* and that it was the predominant method used in school. In total there were 39 comments relating to the use of algebra. Many of these also described how they approached the workshop problems with algebra first because of its familiarity. For this reason they were noted as general comments in Table 21 above, rather than as specifically classroom or workshop. This seemed like an interesting code to explore given the variety of methods the students were asked to explore in the workshops and the references in the literature to the overuse of algebra. Some of the comments in the interviews suggested that the students were not given the opportunity to explore different methods:

Student 023: Well, I suppose in school, they always say do it the algebra way. They never encourage you to look at it and just think.

Student 036: Yes, and I feel like it's easier for teachers to teach algebra because they can't really teach someone to see it differently, whereas algebra it's just like there's a method and you can teach it. If they do that, if they know the methods, they'll get it right.

Other comments were more general expressions of having a *'Tendency / preference to use algebra'* but still hinted that the possible reason behind it might be its frequent use in school.

Student 341: I think it's just algebra's fun. It's not that I find it any easier to find an answer, but I just find it more enjoyable. I think that's because there's such an emphasis on algebra. I don't know if it was just my class, but we were constantly doing algebra and we spent very little time on geometry.

Student 333: Yes. It has to be algebra. I think it's mostly because most of second year, we were only doing algebra, so it's just beaten into our heads to use algebra and only algebra, which is perfectly fine, but maybe sometimes I'd use visual.

A code on '*Use of visuals'* was added after reading some of the comments made on the students' preference for algebra. There were 10 comments that there was *'Little emphasis on visuals'* in school. Some students highlighted that the lack of visuals in school may have had an influence on the preference for algebra:

Interviewer: Would you have a preference for solving problems using visuals or algebra?
Student 034: We're not given visual ones
Student 043: We're not given visual ones so it never would have even crossed my mind to have said visual.

Student 425: If there's a way to do it without the visual, you'd expect that they would teach you the non-visual way.

Student 020: I feel like sometimes when you see it, it's more satisfactory because you're like, "Oh, I saw that." I know that sounds bad but … do you know? Whereas algebra, it's like, "Oh, I just did the method."

Student 043: If you look at a maths book, it is just all numbers and just questions.

The above responses were interesting considering what students found challenging in school:

Interviewer: What would you describe as a challenging mathematics problem?
Student 342: I say spatial awareness questions because I just find them really confusing about all the different dimensional shapes.
Interviewer: Would you do much visual stuff in school?
Student 342: Not really. We don't really do much of that.

6.4.2 Workshop Findings

In contrast to the classroom experience, students seemed to enjoy looking at different solutions in the workshops. There were 11 comments coded as the '*Enjoyed looking at simple solutions in workshop*':

Student 021: I was trying to do things a really hard way. And then I just realized, "Oh, this is so much easier, this other method." So, I thought it was, like, more enjoyable.

Student 429: I feel like it's a lot more engaging because you won't get fed up near as much as just doing the same question over and over with a formula you've been given rather than trying to find the most efficient way to go through a question.

Student 023: When I saw the others, I was like, "Oh, wow, there's so many different ways of getting the answer." It opened my eyes to trying different methods.

Student 341: I liked the shape one with the algebra. I liked it how instant it was because you could see that it wasn't that complicated, but there was many different solutions that were obvious so you could work it out fairly quickly.

Student 020: Once I saw it, I was like, "Oh, my god, how did I not see that?"

Student 041: I didn't really know what I was doing but then like when I found that easier way, of putting the columns or whatever. It was like, I don't know. [chuckles] I just actually enjoyed it.

There were no comments related to the workshops being exam focus and 70 comments related to enjoying the explorations of the process of mathematics in the workshops. Many of these were answers given to questions such as:

Interviewer: **Did you find the problems in the workshop different to those you did in school, and if so, in what way, and do you have a preference?**

Student 325: Very, very different. I found personally the problems in the workshop were more engaging because … you're not just looking at numbers and numbers and letters, you're looking at shapes and it's still maths, but I guess more parts of your brain thinking more creatively.

Student 429: I would say I'd prefer the ones in the workshop as they're a lot more engaging and it's not just repetitive

Student 329: Yes, I thought they were a lot different to what we do in school … In school I feel like we just do more like algebra, and then just a few other stuff, but it wouldn't be really any problem-solving like that. Like how they show you different ways to do it.

Student 324: Yes, I think it's the same, because I don't think you're going to use, unless you're like a scientist or something, you're not going to use algebra in life, so I don't think there's enough other stuff …

I prefer the problem-solving.

Student 325: If you do it on your own, you're kind of bringing something to maths.

Student 429: Overall, the workshop shines a good light on what maths can be in the classroom in an ideal world and how much fun it can be.

Student 046: It's just interesting and fun. Those two hours went so fast.

There were 12 comments on the workshops findings that emphasised the relevance of the tasks to the future and real life, which was coded as '*Learning for future / real life'*:

Student 036: I think that the questions we did were more real-life sort of stuff. Sometimes if you put maths into real-life situations, it helps people to understand it more.

Student 038: I feel like it's applied it more to real life. It's not just doing a method anymore, it's thinking about how maths is related to art, stuff that you see, you now would see it in maths more.

Student 226: Maths in school is just numbers on a page but the problems were like things in real life.

Student 425: The questions in the workshop, definitely they felt like they were much more practical.

Most of the students seemed to enjoy the opportunity to use visuals in the workshops with 25 comments on the '*Enjoyed visuals'*:

Student 038: Yes, I feel like the questions are catered for everyone. Some people would see things visually. I don't really know how to say this, but if you saw it visually and got the answer, then I feel like when you got explained the other way like you'd already solved the answer, you'd understand it more than if you just did the algebra way then.

Student 034: I definitely liked the visuals. I like when you see the triangles or whatever, stuff like that.

Student 333: Yes. I really enjoyed it because if they were and how they're all in shapes, I could still visualize them still like big numbers and lists.

There were a couple of students who preferred algebra for solving tasks and whilst they enjoyed the tasks they were not as keen on the visual aspect:

> Student 045: Like, you kind of have to visualize these ones instead of writing numbers down on the page which is definitely different 'cause I'm used to, like, numbers.

> Student 021: I probably didn't like how visual they were … Other than that, I liked the thinking in them.

The overall impression from the data in this theme is that the students enjoyed the different approach to mathematics in the workshops. Their experience in school appears to be very exam focused with little time being devoted to the exploration of different mathematical methods and solutions. It should also be noted that the participants were TY students and so the focus on examinations is likely to be even stronger in their 5th and 6th year classes. However, their experience in the workshops brought enjoyment at exploring solutions and mathematical approaches.

6.5 Free Market Principle

Table 21 below shows a breakdown of the codes in the Free Market Principle theme. The two main areas the researcher wanted to investigate were the opportunities for students to take risks, and the opportunities for students to collaborate and share ideas with their peers. These became the second-level codes, and the table shows how they were further divided to give a more in-depth view of how the students described their experiences.

There were 166 comments classified under the code '*Encouragement of risk taking'*, such as offering solutions to unseen tasks, and having the freedom to decide upon the method for solving tasks. There were 122 comments classified under the code '*Opportunity for collaboration with peers'*. Interview questions on the freedom to think in the students' classroom raised 12 comments on the *'Lack of opportunity for freedom to think'*. The third-level code *'Class atmosphere prevents risk taking'* had

77 comments in total, which included 35 comments on *'Teacher's methods encouraged'*, 24 on *'Alternatives discouraged'* and 18 on *'Compliance with teacher'* or the perceived classroom norm. The other second-level code '*Opportunity for collaboration with peers'* had 92 comments related to discussions in the workshops and 23 that stated there was *'None/limited opportunity'* in school.

Table 20 Number of Responses Coded in the Free Market Theme.

FREE MARKET PRINCIPLE	Classroom comments	Workshop comments
Encouragement of risk taking		
Freedom to think outside the box to problem solve		
• Enjoyment of freedom to think	0	52
• Lack of opportunity for freedom to think	12	0
Class atmosphere prevents risk taking		
• Teacher's method encouraged	35	0
• Alternatives discouraged	24	0
• Compliance with teacher	18	0
Opportunity for collaboration with peers		
• Enjoyment of discussion of ideas	0	92
• None / limited opportunity	23	0
• Importance of suitable groups	-7-	
• Prefer to work on my own	- 1-	

6.5.1 Classroom Findings: Encouragement of risk

The risk-taking codes in this theme described situations where students had the opportunity to decide upon the method themselves and to suggest solutions to tasks that they may not have been 100% sure of. This could be in a full class situation in school or to their peers in groupwork either in school or in the workshops. There were 12 comments classified under the code *'Lack of opportunity for freedom to think'* in the classroom. These included general comments when asked to describe the differences between the workshop tasks and school mathematics:

> Student 340: First of all, in school, we're not given any room to think for ourselves … you don't actually have to use any of your own creative thinking to find the answers.

Student 325: If you're just given a solution, I feel like sometimes that can restrict your thinking, like, "Oh, that's the only way," whereas before, you're looking, "Oh, I can do it this way, this way, and this way." I enjoyed them.

The subcode to having the freedom to think, '*Improves skills'*, was referred to by students when they spoke of how they would like to change school mathematics. It was coded under *'Lack of opportunity for freedom to think'* because it felt as though it was contrasting their current experience:

Interviewer: If you were able to choose how you could change the maths you do in school, what changes would you make?

Student 339: I think that if they gave us more opportunity to work things out for ourselves then you'd develop better skills for that rather than just being given what you're meant to do.

Further subcodes were added to identify specific situations that contributed to the lack of risk-taking opportunities. The third-level code *'Class atmosphere prevents risk taking'* highlighted features of the classroom that students spoke about which seemed to restrict risk taking. The first one was '*Teacher's method encouraged'*. The Uncertainty Principle theme highlighted that a typical classroom experience for the students appeared to be knowing which method to use when presented with tasks. The focus in the Uncertainty Principle had been on the effect of removing feelings of uncertainty, when approaching a question, on the student. This had included situations where the teacher had shown the students the method first. In the Free Market Principle theme the coding for '*Teacher's method encouraged'* was for comments with a stronger connotation that the teacher was inhibiting or, in some cases, discouraging, the students from exercising the freedom to think for themselves. This theme was more concerned with how the lack of opportunity to make your own decisions made the student feel. General quotes that were classified under the code '*Teacher's method encouraged'* included:

Student 046: Well sometimes you'd be like shown two methods but they're kinda like, so here's two methods but do it this way. It's like, 'cause they like one way better than the other.

Student 324: In some areas there might be another way, and even if you come up with your own way to do it, the teacher will be like, "No, don't use that. Use this one," because they want you to stick to the same method.

Student 325: We're taught to use only one method, which was the way that the teacher wants us ... When we do have our own methods, not all teachers, … but, sometimes, it's discouraged to use your own method because sometimes, they're inaccurate and they only work on some answers, but sometimes, when I'll say it does work a few times, it's just a little bit different. The teacher will say, "Use this method." Sometimes, that can be more confusing to the students.

Student 134: Well, in my experience, the teacher gives you their method and you have to stick to their method.

For some students the tone of their comments indicated that they were annoyed by the lack of freedom:

Student 340: We're just given a one single method and we're told to learn that off. That's how you do it. Even at times you'd be demonized for doing something a different way. I remember one time, the teacher was like, "No, that's not how you do it." Even though I knew that-there was another way of getting an answer, but because the book says, "This is the way you do it".

Student 131: She kinda asserts her dominance and just says her way is right, so it's the easiest way to do it.

Student 325: If someone tries to force it upon you, it also might just make you not enjoy maths anymore because it's still to one way instead of exploring and finding methods on your own.

Similar feelings were evident in this response to questions on the benefit of using tasks similar to those in the workshops:

Interviewer: Do you think those types of problems would be beneficial in school, and why do you think so?

Student 341: I think these would suit a lot because it's not just direct or you feel like you wouldn't be able to memorize stuff very well, it would be better for people like that because they would feel comfortable maybe because you wouldn't be told "No, that's not the way of thinking", or slapped on the wrist, you'd be able to just think.

The code '*Teacher's method encouraged'* also included a subcode '*Disregard of need for alternative explanations'*. This subcode consisted of 24 comments where the students felt that the teacher was reluctant to use alternative methods if the students were stuck. The discussion below outlines a group of students in one school describing the teacher's response when a student asks the teacher a question about a method that was just explained to the class:

Student 036: The teacher goes on for like 20 minutes and you're like "they got that bit" but they didn't actually explain what the student wanted; they just did the question again.

Student 036: If they changed the method then the student might get it, but if they just keep doing the same thing then.

Student 046: The teachers have it in their head if they're teaching a method, they think they'd probably get stuck on this part, and then if the student's asking a question, they just assume it's that part in their head, "Oh, they're going to get stuck on this part," and they just start explaining that part when really it's not that question at all.

Student 023: That happens all the time.

Student 342: Sometimes they'll just give you the answer. They won't actually explain it fully, they'll just tell you this is the answer when they're correcting it, and they won't actually give you an explanation.

Two students in another group made reference to teacher responses that showed elements of frustration when a student could not understand the question:

Student 034: "Could you just explain this one more time to me," and she would be like, "Come on. You should get it at this point."

Student 037: If it's only one or two people, they would be like, "Oh, ask the person beside you."

'Alternatives discouraged' was chosen as a code for comments that spoke about the students being discouraged from trying different methods, but had not specified that it had to be the teacher's method they used:

Student 430: I think we were taught just to find the one formula that works and remember that.

Student 023: Well, I suppose in school, they always say do it the algebra way. They never encourage you to look at it and just think.

Student 425: If you don't do it the way that you were told it's counted as wrong. If you don't do the method that was in the book word for word, number by number.

There were 4 comments on the belief that the teachers had a "*Lack of trust in students*" to find an alternative correct method themselves. These were also classified as the code *'Teacher's method encouraged'* because the students were not given the opportunity to take risks to try out their own method and see what happens. These comments also included suggestions that mistakes were discouraged in class:

Student 123: They use really long, confusing methods and then you just find an easier way. Then they say, "Don't do it that way because you might make a mistake if you do it that way."

Student 034: They wouldn't. It's like they don't trust us or something to do it without an example.

The final subcode to '*Classroom atmosphere prevents risk taking'* consists of more subtle comments on following the teacher's method. The quotes below suggest that the atmosphere in the class was one of *'Compliance with teacher'* when it came to using the teacher's method. The comments coded here are for those where the teacher did not compel the students to use a particular method. The students just seemed to go along with the classroom norm, which was to use the teacher's method:

Student 043**:** Then depending on the teacher, you do it a certain way but then that might not be the easiest way for you. You just have to do it that way because that's the way you were taught.

Student 231: We are … told how to do it and we just stick with that method throughout.

Interviewer: If you presented a different idea to the teacher, would that happen often?

Students**:** No, not really.

Student 036**:** See, because we follow the teacher's teaching, then everyone's just like, "Okay, this is what the teacher said." Then, there's like, "Okay-

Student 020**:** Just accept it for what it is.

Student 036**:** We have this in our notes. Let's do the question like this then."

6.5.2 Opportunity for collaboration with peers

Under the code '*Opportunity for collaboration with peers'* there were no comments describing '*Enjoyment of discussion of ideas*' in the classroom. However, there were 23 comments coded as *'None / limited opportunity'* for discussion with their peers. Collaboration is seen in this theme as an opportunity for students to take risks by proposing possible solutions to their peers. The lack of encouragement to discuss mathematics will therefore prevent the students from availing of these risk taking opportunities. Overall, discussion seemed neither frequent nor encouraged in the classroom:

Student 324: Some classes, you can talk, discuss things, whereas in others you can't. In maths, you can't really.

Student 046: You're not allowed to talk in class

Student 034: I just feel my teacher wouldn't … really enjoy us in the group discussing it. It was nice to discuss it with other people.

Student 425: Generally, in maths class, if you're talking to each other, you get in trouble. It's not only that you're not talking, it's like you're discouraged to talk which is terrible.

Student 339: For me anyway in my class whenever I'd even try to discuss it with someone else, I'd get told by the teacher that I need to ask him if I'm going to ask anything.

Student 432: In school, I'd say very rarely, because of the type of questions we'll be doing where there's examples, there isn't really much to discuss. It's straightforward

Student 340: In my class, we don't really ever discuss math and I wish we did because … all it takes is a little spark off someone, one word someone would say that leads you to find the answer.

The only opportunities for discussion with a peer seemed to be when a student was asked to show another student how to do a task:

Student 023: More like when the teacher is like, "Oh, you show them."… When she just doesn't want to explain it.

Student 326: The only time you'd really discuss a problem with another student in class would be if the teacher said to discuss your answers or if you finished the problem before, let's say the person sitting beside you and they ask for your help and you explain it to them, I think that would be the only time you'd actually discuss the problem with the someone in your class.

Comments where students acknowledged the benefits of discussion were also classified under the code *'None / limited opportunity'* if the implication was that they did not get the opportunity in class:

Student 340: I'd love if we had discussions before even attempting any questions because I think that's pushing us to our full capabilities instead of just learning straight from the book where, yes, broadening our mind.

6.5.3 Workshop Findings

There were 52 comments relating to the workshops, coded using the '*Encouragement of risk taking*' code. The comments included here described the students' *'Opportunity* *for freedom to think outside the box to problem solve'*. This code was further divided to incorporate the subcodes of '*Enjoyment of freedom to think'* and that it *'Improves skills'*.

Quotes on *'Enjoyment of freedom to think'* were largely responses to being asked if they enjoyed the workshops and, if so, what it was that they liked about them. The students had in some cases used the word 'freedom' to express how the workshops made them feel in comparison to what they had been used to:

> Student 231: It's just you enjoy thinking outside the box and thinking of other ways to do stuff instead of just the normal repetitive ways.

> Student 046: It's kinda like more like free. You are only kinda given a little bit and then you can go in any direction that you want … the freedom of just doing what you want to.

> Student 134: You have more freedom in the workshop. We had more freedom to go and solve them ourselves. In school, you have to sit and listen to a teacher explain for a full class.

When the students expressed their enjoyment of the tasks in the workshops, the comments often made comparisons to the tasks they experienced in school. Student preferences for opportunities to take risks and explore their own methods was evident in their specific reference to having the independence to choose a method for themselves:

> Student 325: Being given the question without a method, that was very refreshing because it gives you a bit of time to explore the question on your own. You might be able to find multiple solutions.

> Student 123: It's good the way you can use whatever methods you want and you don't have to do it like a really confusing way.

Student 425: Then the difference to school is like you actually have to find out how to do it yourself. It isn't just right or wrong. There's different ways of doing it.

Student 339: I definitely preferred what we did in this rather than what is covered in school … You just had to think more … the way you wanted to do it. Your own methods … a lot of the time you're just not allowed. You just have to do it the way your teachers taught you.

Within the 92 comments coded as *'Enjoyment of freedom to think'* were 9 specific references that this freedom *'Improves skills':*

Student 339: I just think it makes you think more, and it just would improve your skills more trying to think of it yourself rather than it just being on the page.

Student 043: I feel that way we learned different ways to do it as well because other people might have done it different ways.

Like '*Encouragement of risk taking'*, many comments classified under the code *'Opportunity for collaboration with peers'* were made when comparing the classroom experience to that of the workshops. There were 92 responses describing both feelings of '*Enjoyment of discussion of ideas'* and '*Learning from discussion'* and many students combined both feelings in their comments. The question asked was:

Interviewer: ***Was there anything in particular that was different about the workshops that you liked?***

The responses from the students illustrated the variety of ways in which the benefits of discussion were perceived by the students. One of the most common reactions was that the experience of communicating with their peers was enjoyable and engaging:

Student 020: When we were doing these, it didn’t feel like we were doing maths at all. It just felt like we were having a discussion about just a topic and working it out together or on our own, it didn’t feel like we were doing maths.

Student 430: You just engaged a lot more with everyone else and seen everyone else's method, and then I think you can learn and improve with everyone else's help.

Student 432: What we were doing in the workshop I found the day after we were in school both days we were all talking about different ways we had done it.

Student 329: It just reminding me of how enjoyable maths can be when you're working in groups solving or doing some problem solving questions, it's just the way I like it.

Student 339: It reminded me why I like doing maths because when you're able to think of things for yourself and give your ideas to other people it's just a lot more enjoyable.

An important aspect of the opportunity to discuss ideas with their peers appeared to be learning new ideas from each other. Through discussion, individuals develop mathematically through bouncing ideas off each other and the students seemed very aware of these benefits. They also specifically commented on the learning aspect of hearing the ideas of their peers and comparing different methods. The comments reflected a feeling of a mathematical community developing within the group:

Student 037: But when we were just doing those problems you had the time to discuss it and really understand. It took me a while to understand some of the things that they were saying. Once I got it, I was like, "Yes."

Student 131: I liked it because you get to see how other people think and that can cause you to think differently around different things.

Student 429: Just working in groups and comparing ideas, I would say was an enjoyable part of it. It gives you a different perspective on some questions that you might have just looked at the same way as before.

Student 341: It's different people giving their different opinions, and then you have to listen. It's like a discussion and you have to hear bits of information from different people, and then you piece it together yourself as well, and it's just trying to find a solution through different people's ways of thinking, and that was really nice to do.

Student 430: I find that it's so much more satisfying when you get the question right after coming up with your own method and being able to explain it to everyone else as well is a good feeling.

Student 430: What I liked about the questions in the workshop were that it seemed that all three of us always come up with different solutions and we could compare and contrast, which I found really interesting.

Student 429: It was really interesting to see what you could come up with and when working with others, how you compare and how you can use that line of thinking that they might have been using in the future.

The 92 comments on the enjoyment of groupwork highlighted the positive reaction of the students to the discussions in the workshops. However, it was important to also record the perceptions the students had about working in carefully selected groups, of which there were 7 comments. These appeared to be more general statements by the students on the ideal classroom and were most likely a combination of their experience in both the classroom to date and the workshops. For this reason they have been included in between the classroom and workshop columns in Table 22. The *'Importance of suitable groups'* code was used to encompass a variety of comments made about the ideal composition of the groups for discussion. The initial template had the code '*Preference for similar ability groups'* but several comments were not so specific, so the code was made more general to apply to all such comments:

Student 131: Working in groups, having the people who work best in a group because you can give them all a hard question because you know that they will work together and the less capable people, the less hard questions.

Student 330: Group work could be helpful, but it depends on how the group is. Maybe people can choose what group they can be in. Yes. It would be helpful.

Student 131: Sometimes your group … someone might not get it, and then they'll just keep asking questions and they might slow you down as a group because they don't understand, and they keep telling you to explain it to them.

Student 226: I think it's who you work well with, then you kind of enjoy group work. Like say if a teacher just puts you in groups, it makes more difficult to enjoy it.

Student 325: If you put one good student with the others, they might have some ideas, but there can be times where they're pulled back by the rest of the group because they're thinking too fast, but they can't really explain it to them ... In this case, the group work was enjoyable.

Overall, the interview data describes a very different experience of collaboration in the workshops compared with the experience the students had in school. The most notable features were the students' reflections on the lack of freedom to take risks they have in school versus the opportunities to explore in the workshops. The students suggest that the class atmosphere prevents them from taking risks in school. The workshops seemed to provide students with more freedom, and this led to a very enjoyable learning experience for the students in all the groups.

6.6 Scholarly Principle

Table 22 below shows a breakdown of the codes in the Scholarly Principle theme. The primary aim of these codes was to record the students' comments related to learning. The researcher wanted to investigate how the students found the experience of learning from MST and what they felt about the importance of learning mathematics in general. These two themes became the second-level codes. There were 159 comments on *'Learning from exploration without instruction'*. The subtheme to this was "Exploring MST" which was further divided into the codes '*Benefits of exploring MST', for* which there were 125 comments on the student perception of their benefits and the *'Lack of opportunity to explore MST'* which contained 34 comments. '*Benefits of exploring MST'* was further divided into ' *Finding multi-solutions is more challenging*', '*MST improves creative thinking and develops understanding'* and that it '*Can be frustrating'* having to find more than one solution method. The aim of these codes was to get an idea of the student reaction to such tasks and whether they thought they would be beneficial in school. The 92 comments within the second-level code '*Importance of learning mathematics'* included codes to gather data on different aspects of learning and challenge such as the '*Enjoyment of learning*', the *'Importance of being challenged'*, '*Feels challenged'* and '*Lack of learning'*. The third-level code '*Lack of learning'* was further divided to include students' comments on '*Concern for wasting time / falling behind'*, not being given additional challenging questions and not being challenged to achieve their potential.

Table 21 Number of Responses Coded in the Scholarly Theme.

SCHOLARLY	**Classroom comments**	**Workshop comments**
Learning from exploring without instruction		
Exploring MST		
• Benefits of exploring MST	**0**	**6**
• Finding multi-solutions is more challenging	**0**	**32**
• MST improves creative thinking and develops understanding	**0**	**69**
• Can be frustrating.	**0**	**12**
• Lack of opportunity to explore MST	**34**	**0**
Importance of learning mathematics	**-14-**	
Enjoyment of learning		
Importance of being challenged	**-11-**	
Feels challenged	**5**	**4**
Lack of learning		
• Concern about wasting time / falling behind	**23**	**0**
• Lack of additional challenging Q	**18**	**0**
• Not being challenged to potential	**19**	**0**

6.6.1 Classroom Findings

All the findings, related to the classroom, in the second-level code *'Learning from exploration without instruction'* were described by the *code 'Lack of opportunity to explore MST'*. They were mostly in response to the following question:

Interviewer: How often are you asked to solve problems using more than one method, when you get the answer to do it in a different way?

Students: Never

Student 341: No one would give you multi-solutions … I don't feel like the teacher really presents more than one method for a math question.

Student 027: If someone did in a different method, like, she'd be like, "Oh, … that counts as well," but she won't explain it to the rest of class, she'd just go, "Oh, yeah, you can do it that way either."

There were 14 comments classified under the code *'Importance of learning mathematics'*. These were quite general comments that did not necessarily give a specific reason. The comments appeared to be a combination of reflections on their current classroom experience and their recent experiences in the workshops. The theme behind this code was to capture the feeling that mathematics was important for the students, and that they were keen to do their best in school to improve. For example, when asked their opinion of having different types of tasks in school that are going to stretch students while the others are catching up with the basic stuff:

Student 340: I think that's a great idea what you're saying because I feel like even those type(s) of questions are even broader to what we learn in schools, so may be improving our maths abilities.

Sometimes they just reflected that they were keen to improve, or that mathematics was important for improving their general education:

Student 425: Then also people who are saying that "Oh, maths," like, "Why am I learning maths? When will I use this in my life?" When there's more real-life examples used to educate people, then they'll see the more useful side of maths.

Student 340: There's a lot of emphasis on it (algebra and coordinate geometry), but we don't actually learn a lot about it. We just spend a lot of time doing it which I don't think is great because coordinate geometry is so important in other aspects like physics and chemistry.

Student 432: Last year, that my teacher gave me a few little things that I could do in my free time, because he could see that I was good at maths.
I found them quite good. I would like to do it more, but I don't really see them around.

Other times, the attitude that mathematics was important to them was evident in their answers to specific questions such as:

Interviewer: If you had to change something in school about school maths, what would you do?

Student 121: Keep proper maths going through Transition Year … we were all really rusty.

Like the *'Importance of learning mathematics'* code, comments classified under its second subcode *'Importance of being challenged'* appeared to apply to both their current classroom experience and their recent experiences in the workshops. When discussing the workshop tasks the students made reference to how they could be used to challenge them in school:

Student 432: Yes, I'd much rather be given something more challenging, just find it to be a lot more stimulating. Doing the same things over and over isn't really broadening your knowledge of maths.

Student 341: If you don't feel satisfied just doing those questions, it's nice if you get more and just like work on your own abilities.

Student 131: It's doable, but it challenges your brain, so you get the most out of it.

Student 340: I think it's important to consider all abilities. We feel rewarded when we find the answer to difficult questions, but people with less of an ability from us should still feel good about themselves when they thought one day to answer a question that's more to their abilities. I think that's important.

The third subcode to *'Importance of learning mathematics'* aimed to identify how challenged the students felt by their mathematics in school and in the workshops. There were five comments classified under the code *'Feels challenged',* related to their experience in school:

Student 333: For the most part, I feel perfectly challenged because most of the time, we're just trying to move on. Sometimes, it can just slow down quite a bit because maybe majority of the class still don't know. So, I'll end up sitting there just staring at my textbook because I've already gone through all the questions. Majority of the time, yes, I feel perfectly challenged.

Student 041: I do find maths really challenging in school as well but like, yeah. As long as I know what I'm doing, I'm grand.

However, for some of these students they clarified their comment by following up that it was only challenging at the start of the year:

Student 129: Most of the time yes, I think yes … Maths on its own is quite difficult but once you get the hang of it, towards the end of the year it's always easy. But at the start of the year, … it's a big jump, first to second year and second to third year, big difference in maths.

There were 60 comments related to the final third-level subcode '*Lack of learning'* in school. The students had different concerns that contributed to them feeling that they were not learning as much as they could. These concerns were added as fourth-level codes: *'Concerned about wasting time / falling behind', 'Lack of additional challenging questions'* and *'Not being challenged to potential'*.

Comments related to *'Concerned about wasting time / falling behind'* in school hinted at students being frustrated that they were not doing more work, or that the teaching of less able students was holding them back:

Student 023: So, it's a waste of time then if you are doing 5 questions that are a bit easier and you know that you know how to do them but you're not really being challenged.

Student 226: They might spend nearly the rest of the class going through that one question, explaining it.

Student 134: You can fall so far behind if one person doesn't understand one thing.

Student 340: I think we just spend too long on things that we don't need to. When you've grasped it, I don't think there's any need to spend so long as we do.

Student 341: Like Higher Level, it's just more points for Maths, and then people go there just because they want more points even if they don't feel comfortable there. It may hold you back or it may hold someone else back, and it's not the nicest thing to do if someone feels uncomfortable there or you feel uncomfortable there.

There was also, like in the Gestalt Principle, an understanding that the teacher had to try and keep mixed ability students together. Sometimes the consequences of the teacher's methods to do so made the less able students feel that they were holding others back:

Student 034: They do have to accommodate for other people in the class as well. It is a little bit unfair, but it is unfair on everybody because other people who might take a little bit longer and they feel bad for maybe keeping the class back. It's hard to find a balance between everyone being happy.

There were some quotes recorded in the Gestalt Principle referring to not being given additional questions when the teacher was helping the less able students. Comments coded here under *'Lack of additional challenging questions'* were concerned with the students feeling they could be learning more if they were given different, more challenging, questions:

Student 131: It's like you’re just you're just doing the same thing over and over again for the whole class. You’re not really learning much.

Student 340: That's pretty much the same. Most of it is fairly easy and the same thing over and over again. Obviously, some parts get a bit more challenging but yes, it's pretty repetitive. It's easy to get a good score in the exam, but it isn't very good for actual education.

Student 023: I feel like alternative ways of challenging yourself other than the textbook would be beneficial. When you have the same book all the time, and you're just like, "Oh, it's so predictable," but then with something different, who knows.

Student 225: I think just giving more challenging questions and questions that are mixed with other things.

Although it may be difficult for students to know what their true potential is, it was worth asking their opinion on whether they felt challenged to their potential. Hence, comments classified under the code *‘Not being challenged to potential’* were mostly in response to the question:

Interviewer: Do you feel that you are challenged to your maximum ability in maths class, if not, why do you feel this?

Student 134: No, it is not that challenging because if one person doesn't understand the question then it just holds back the whole class, and you can't learn more.

Student 123: No, I don't think you are challenged to your maximum ability.

Student 131: It can just be falling behind if one person doesn't get it, the whole class has to struggle with them.

Interviewer: Do you feel that it affects your own maths potential?

Student 131: Yes, because you're not doing things that you know you're capable of. You're not even getting the opportunity to do, and the teachers doesn't even let you.

Students from other schools had similar comments to the same question regarding feeling challenged to their maximum potential in school:

Student 430: Never do I come out of a maths classroom really thinking, "Well, that challenged me and I'm a better student for it." In terms of my skills being tested fully, definitely not in maths ... It's just a case of memory.

Student 429: If you want to do a few extra questions you can but it's never really the limit of what your abilities would be. I think that'd be true for anyone in school in maths … No matter what your level is, it's never really pushed … For the most part, I would say that challenge doesn't accurately describe maths in school at the moment.

Student 225: I think that some questions you have to try your very best. The best you can do is 100% on tests and stuff. You just narrow it down to that whatever is on a test and that it's based on, whereas I think you'd have more potential if you are given harder sums to do and things like that.

Student 340: I don't know, I don't think we're very stimulated in school, some of us anyway.

6.6.2 Workshop Findings

All the comments related to learning in the Scholarly Principle were positive comments about what the students could learn from the workshops, as will be shown below. Most of the comments on exploring multi-solution tasks gave specific reasons for what it was the students liked, or felt about them, but there were six more general comments on '*Benefits of exploring MST'* and the enjoyment of the experience:

Student 038: You'd know lots of different ways. Even if you forgot one, you know that there's another way to do it.

Student 023: It brings you on and it just encourages you to try different ways, even if you're not as comfortable with the different ways. It would probably stand to you in exams when you're faced with the question and there's no obvious method.

Student 430: What I liked about the questions in the workshop were that it seemed that all three of us always come up with different solutions and we could compare and contrast, which I found really interesting.

Student 432: What we were doing in the workshop I found the day after we were in school both days we were all talking about different ways we had done it.

The specific comments the students made on the multi-solution tasks became the fifth-level subcodes of '*Benefits of exploring MST'*. There were 32 specific comments acknowledging that *'Finding multi-solutions is more challenging':*

Student 429: I enjoyed the challenge. It was a lot of fun going through it and comparing ideas. …The main thing I liked about them was that there was multiple solutions at the end. It was not limited to one way through it, especially with the last one we did where we found a solution that was meant to be one you wouldn't think of straight away. It was the first thing we tried, and when we found that that could actually work, that was a big boost.

Student 020: You're constantly thinking, you can't switch off because they're a lot more challenging so, and you're trying different ways to do it.

Student 339: I think we were challenged a lot more with the tasks we were given.

Student 231: Once you make up your own method, it's fine, but to make up the method sometimes it was a bit challenging, but you enjoy it because you get a satisfaction out of it.

Student 329: They were challenging but they were enjoyable at the same time so it would be a lot easier to do because I'd actually enjoy doing it. It just makes it easier for me.

There were 69 comments stating that the tasks in the workshops are beneficial for learning. These comments were coded as '*MST improve creative thinking and develops understanding'* to try and incorporate all the different student reflections. The majority came in response to being asked to compare the workshop tasks with those they usually do in school:

Interviewer: Did you find the problems in the workshop different to those you did in school, and if so, in what way, and do you have a preference?

Student 429: I'd say the main way they were different was just they were inspiring more creative thinking and not just translating a formula into a question repeatedly.

Student 329: I think it just gets you thinking a different way than you're used to thinking. In maths, we're usually just thinking, not normally, but you're just trying to remember what to do whereas with this you're thinking of how to solve it.

Student 038: It just helps to think about there's other ways to do it.

Student 020: You're trying different ways to do it, so it's not like once you find the answer you can just go to sleep, [chuckling] you have to stay thinking the whole time.

Student 325: "Hmm, maybe I can do it this way, this way," taking shapes into account. Then, it makes it more interesting and creative.

Student 326: I'd say definitely, I wouldn't have thought of math before as the way you would do it in the workshop. I've always thought of math as you just get the answer and then you move on to the next question whereas with the questions we were doing in the workshop, you get a solution, and you get an answer and then you try do it again a different way and it forces you to think more about the question.

Student 126: I just think it makes you more creative.

Student 038: It's not even just like the numbers; you have to visualize it a bit as well

Student 123: It's just different in the way that for these problems you have to use your common sense kinda of instead of just copying it from the book.

In addition to all the positive comments some students mentioned that MST *"Can be frustrating'*. In general it was because they got frustrated by how long it took or by having to find another solution after they had got the answer. Such responses usually came when the researcher specifically enquired how they found the tasks in the workshops:

Interviewer: Is there anything you did not like about the problems in the workshop?

Student 046: Sometimes it can be a bit frustrating if you just can't think of it.

Student 121: If the question has say, multiple methods, possible methods could be almost frustrating that you go, "Oh I could have just done that."

Student 129: The only thing is I did enjoy them, but I was getting frustrated as well. It’s not that I disliked them, but I was just getting kind of annoyed.

Student 036: Once I found this one way, I kept seeing this one way, and then I was like, which other way? As I kept trying, then I came out, but it was hard at once, trying to train your brain to keep looking for a different way other than the one you just did.

There were many comments showing clearly that the students said that they felt challenged by the workshops and they were very positive towards the enjoyment of the challenge:

Student 326: I did find it challenging, but the challenge was enjoyable, and working in a group and kind of bouncing ideas off each other and saying like, "Oh, does this work?" and then trying loads of different things. I enjoyed that challenge.

Student 341*:* It was challenging because usually, I find math sums. It's more relaxing and cathartic to just do it. I just see it instantly, but here I have to think for myself, and in the end, it was very rewarding if you got the right answer, or you got a solution that was along the lines because you had to think for yourself, and it wasn't instant. It was over time you were building it up.

Student 429: I would describe some of the tasks that we were doing as part of the workshop as challenging where you're not given any clear methods to do it. You have to come up with your own method yourself … something that you wouldn't normally be coming across in maths.

Student 326: The questions in the book do get progressively harder, but they're not really challenging that you have to think about like the ones that were in the workshop.

In the Scholarly Principle the students' reactions to the MSTs were coded. I found that these types of tasks were new to students and that they usually did not try to find alternative solution methods or compare solution methods at school. In the workshops students found this challenging but beneficial. They also spoke about their worries about lack of learning at school because of the pace of classes or lack of challenge.

6.7 Summary of Findings

In the Gestalt Principle the overall results convey that the students in this study have little opportunity to incubate when engaged in solving problems in school and in comparison, felt considerably more challenged by the preparation of tasks presented in the workshops. Given the strong connection between the teacher's preparation and the thinking required by the students the results in the Uncertainty Principle follow a similar pattern. The students made frequent references, when discussing their classes in school, to rarely having to think about the method along with experiencing very repetitive work. In contrast, they showed a preference for having to think about the tasks presented in the workshops. In the Aesthetic Principle, comments on classroom experiences reflected the view that mathematics in school was focused on learning methods for the exams. The data related to comments on the workshops showed an enjoyment of exploring mathematics and a different style of task. In the fourth higher-order theme, the Free-Market principle, the students contrasted the freedom they felt to think and discuss in the workshops versus being obliged to follow the teachers' methods in the classroom. Finally, in the Scholarly Principle, the comments on exploring different solution methods were confined to the workshops and concerns over a lack of learning dominated comments on their classroom experiences. The students clearly found the MST beneficial and a useful tool for providing learning opportunities and challenge, as the research literature had argued.

Chapter 7

Analysis of Data from Audio Recordings of Workshops

7.1 Purpose of audio recordings

The main focus of the analysis of the workshop data was on how the students would react to being given differentiated tasks for which they had not been given a method to solve. As outlined previously, the tasks were considered unfamiliar, challenging, and time-consuming for most students. The discussions the students had within their group, as they tried to solve the tasks, were recorded via an audio recording device placed on each desk. The recordings were transcribed and were analysed to look for evidence that the experience provided opportunities for challenge and mathematical creativity such as those suggested by Sriraman (2005). The four tasks, as presented to the students, are attached in Appendix B and segments of these will be included in this chapter to help visualise what the students were discussing in the audio recordings.

The features of interest for this research included the extent of collaboration between the group, the perseverance, individually and as a group, and the willingness of the students to take risks and offer solution methods to each other. Those that collaborated more as a group provided rich information on how the students approached the tasks in terms of the shared understanding, the tactics chosen, the thinking processes and the group dynamics. Also of interest, was any evidence relating to how they viewed the mathematics involved and their engagement with the multi-solution methods of each task. The researcher used the coding template developed for the analysis of the interview data when studying the workshop data. The excerpts taken from the audio recordings have been broken down and categorised to illustrate opportunities for students to experience each of Sriraman's Principles. As in the interviews, the students have been given codes where the first of the three digit number refers to their school, ranging from 0 – 4 and the last two digits represent the student identification number within their individual school.

7.2 Gestalt Principle

7.2.1 Examples of students being given tasks that were challenging

The tasks selected for the workshops were designed to be time-consuming so that the students would have incubation opportunities. Naturally, the time taken to solve each task ranged for different groups of students. However, by choosing differentiated tasks, with extension questions at the end, those students who solved the main task quicker than others had extra challenges to consider. It was hoped, if they engaged with these challenges, that this would avoid students left sitting waiting for others to finish. It was clear from the time it took the students to reach solutions and the comments in the audio recordings that the students did not find these tasks easy and that they were very different to the textbook tasks that many had described as procedural in the interviews. A sample of such comments is given below:

Student 020: Oh, now I am really confused.

Student 329: I don't feel too confident, … I'll write it down.

Student 240: We are drawing a Venn diagram, but from there we don't really know.

Student 340: I don't know, I just don't know how to do it.

Student 030: I literally don't have a clue

Student 127: I don't know. This is getting way to complicated.

Student 140: So it (*the Leaving Certificate*) wouldn't be as hard as this would it?

Student 021: The answer is 271, I don't believe that. It's not possible. Oh I don't know.

Student 434: This one's tough now.

7.2.2 Role of teacher helping student regulation

To encourage student-led thinking the role of the teacher in the workshops was observational, only providing scaffolding if the students were going down a path that would cause very time-consuming difficulties. Care was taken not to remove the students' opportunities to think, feel perplexed, and possibly frustrated from not knowing what to do. In some of the situations the researcher intervened to point out where students had made simple arithmetic errors that they had not noticed. There were other cases where one of the students had suggested a correct method, but it got lost in the discussion as others were mentally engaged in alternative, but equally valid, methods. This happened in cases where the groups were particularly vocal, especially on day one, and there was a danger of much good work getting overlooked. By encouraging the students to write everything down, and pause for thought, it might prevent them from missing key points. In one of the groups, several of the students were verbalising their thoughts at the same time, with excellent ideas emerging. On this occasion the researcher intervened to try and encourage the students to focus on what they had achieved rather than losing the moment, hoping that each of the ideas suggested would be looked at by the group at some stage:

Student 023: Could you not just do simultaneous equations?
Student 027: Trial and error is so much quicker, but maybe not if very big numbers.
Student 020: Is there more than one possible answer?
Student 038: We have got answers, but not why.
Teacher: Why don't you list the answers down and have a think about them.

On returning to this group when they had reached Field 6 and were managing to find the answers fairly quickly. It was possible to check how they had found them from reading their worksheets. They had not yet focused on simultaneous equations as suggested earlier by one of the students:

Teacher: What method are you using? Substituting numbers in?
Try another method because you've sussed it. Is there any other method you could use?

Immediately one of the girls responded with:

Student 036: Like, **is** it simultaneous equations? If you said, $x + y = 9$?
Student 038: Let's go back to Field 1.

In another situation, the teacher's intervention was to try and help the students monitor their work before dismissing it. The fear of making mistakes or showing up incorrect work can manifest itself in a number of ways. Students in several of the online groups had suggested in the audio recordings they should write in pencil in case they make mistakes. In the face to face workshops the researcher had discouraged this at the outset of the workshop so that evidence of how the students were thinking would not be lost. However, students were often reluctant to write in pen because they said they were not used to it, or their teacher told them not to. In one situation during a face to face workshop the researcher intervened when students showed evidence of not trusting themselves and their thinking by erasing their work. The students had been encouraged to draw various construction lines to inspire ideas in the Area of Circles task:

Teacher: No rubbing out! What was that you just rubbed out?
Student 224: Just that (*Horizontal line labelled 'r' in small circle*).
Teacher: Mmm, that was important, so put it back and have a look at it. It's good.

Other than interventions similar to the above, the students were left to make mistakes and discuss and reason ideas with each other. In this way, the students were given opportunities to create their own scaffolds, assist each other and feel ownership of their work.

7.2.3 Opportunities for Collaborative ZPD

In the interviews, students had made reference to the enjoyment of being challenged and learning from each other during the workshops. Students also commented that the tasks in the workshops provided something for everyone. The audio recordings from the workshop provided similar evidence whilst also offering insights into some of the benefits for students experiencing Vygotsky's zone of proximal development. When analysing the audio recordings the researcher has chosen to use the interpretation of a 'collaborative ZPD' (Goos et al., 2002) which seemed to best suit the processes that were emerging from the student discussions. Rather than focusing on the establishment of the ZPD as being guided by a teacher or more experienced individual, this interpretation highlights the important role of peers in constructing the ZPD for each other. Peers can be seen to provide scaffolding for others in the group, regulate each other's thinking and encourage self-regulation. There were many examples in the audio recordings where this interpretation appeared an accurate description of what was happening within the groups. In some instances, one student seemed to be guiding the thinking, but even within such groups it can be seen that the group dynamics change. An example of this will be outlined in section 7.2.5 when Student 340 raises a concern about the current method being employed by the group. As students gained confidence their input increased and there was evidence of more egalitarian relationships providing the key for progress. The students can be seen to learn from each other and at times to require input from other group members in order to make progress with the tasks. There is evidence of peer validation of each other's knowledge but also of more advanced students encouraging reasoning and proof. There were a variety of aspects of collaboration that arose in the audio recordings and the episodes that illustrated students having the opportunity to take risks will be dealt with as examples of experiencing the Free Market Principle section 7.5.

7.2.4 Peer Scaffolding

Feeling challenged and experiencing the ZPD is a major component of the Gestalt Principle. By having to figure out a method and discuss their ideas with each other, the students had opportunities in the workshops to create their own scaffolds rather than waiting for the teacher to give instructions when they were stuck. As the students progressed in the initial Field of Dreams task, there was evidence that they were increasing in confidence and seeing themselves as knowledge builders rather than relying on the teacher's authority. They were conjecturing, refuting irrational answers, and organising a systematic approach to find a solution for themselves:

Student 233: What would happen if C was empty, could C be empty?
Student 234: I don't think that will work

Student 233: Let's try it.

After a discussion of the numbers resulting from substituting zero for C:

Student 240: That would give us a minus child … the child passed away, mmm.

Student 233: I think C must be quite big because when added it's 13, 15 and 17. OK, shall we all just try different numbers?

Student 234: OK, Let's all try different numbers and see. I'll do B.

Often one student, as in the above example, took the lead making a conjecture, but this role changed as the tasks progressed and the others felt more comfortable engaging in dialogue.

7.2.5 Students regulating each other's thinking

The audio recordings gave an excellent insight into the important role peers can play in scaffolding the tasks for each other. In the previous excerpt, Student 233 was guiding the thinking, but in most situations the students contributed as a team and seemed to feel comfortable challenging each other's ideas. As Goos et al. (2002) found in their research, this scaffolding was instrumental in helping other students regulate their thoughts. Within their peer groups, students were experiencing a collaborative ZPD that helped them monitor their own thinking and combine ideas together to make sense of the task.

The Field of Dreams task, a section of which is seen below in Figure 26, was the first task given to each school, and the lack of guidance meant the initial interpretation of the task brought about a wide range of suggestions:

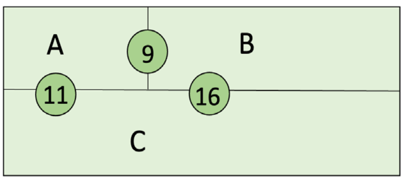

Figure 26 Field of Dreams, Field 1.

Student 342: Some people are switching pitches.

Student 329: How do you know they are switching pitches.

Student 342: I'm just guessing.

Student 329: You can't just guess if it doesn't say it.

Student 323: There's 27 in C and some of them go into A and some go into B.

Student 342: But, that's what I said, some of them are switching pitches.

Student 329: They're not switching pitches.

Similarly, students in another group were challenging each other to think differently. Having figured out answers to just the first field in the Field of Dreams task, one student was already questioning their method and starting to challenge the others to generalise:

Student 316: So are we saying A is 2, B is 7 and C is 9?

Student 340: Yes, but are there more possibilities?

Student 316: Are you saying we should be looking at other solutions?

Student 340: No, it's definitely 2, 7 and 9, but like what's the thought process that would be able to give us that answer if it was harder. You know like, if we couldn't guess it. So, we looked at 9, 11 and what links them all. But say that number was 252, 396 and 400 and blah dee blah. How would we do it then? … Anyway, it's not relevant right now maybe.

After five minutes of silent working individually, one of the students responded with:

Student 341: Can you solve it like simultaneously, so like A + B = 12, B + C = 7 and A + C = 11. And then you move B = 12 – A.

Student 340: Yeah, that is the only way that makes sense, yeah.

This type of differentiated task proved to be both time-consuming and one that provided different challenges for different students. Some were happy, initially, to have found the correct answers but others in the group were determined to find a more systematic approach that could be used for all levels of field. It was interesting to listen to the students acknowledging their good work and praising each other for what they had achieved. Particularly in cases where self-doubt was developing as a student found it difficult to progress in their work. Seventeen minutes into the task, and after another five minutes of silence, the different levels of thinking were evident when one of the students enquired into everyone's progress:

Student 331: Have we all got it?

Student 340: No, I'm still stuck.

Student 341: It doesn't make sense cause … I think I was wrong what I suggested

Student 340: No, but it makes sense to do it that way. It's just not working.

Student 331: I have the answer. I said A = 8, B = 4 and C = 3

Student 340: How did you do that though?

Student 331: I just thought what could the numbers be to make up the numbers …

Student 340: So you didn't use simultaneous equations?

Student 331: No.

Student 340: Yeah, but that's it. I feel like the simultaneous thing should work on a larger scale if you know what I mean, without having to sub in. I don't know, I just don't know how to do it.

7.2.6 Seeking clarifications and justifications

In situations where students may not have fully understood an explanation, there is evidence of students asking their peers to clarify and justify their reasoning which had a significant effect on their progress as a group. In the situation below a group had just figured out the answer to the first field but their reasoning was not fully correct.

Student 323: Will I show you what I have?

Group: Yeah.

Student 342: I've just got A equals 2, B equals 7 …

Student 323: Yeah, that's the right answer, and A + B + C = 18

Student 329: Wait, that makes so much sense cause half of 36 is 18 as well.

Student 323: Oh yeah, so that does work out, that has to be the right answer.

Moving on to Field 2, the group had presumed that this method would work. It was not clear at this stage of listening that their reasoning for the total number of children being half the sum of the three circles was based on the size of the fields in the picture:

Student 342: Will we just do the same thing as last time?

Student 329: We can't cause it's not half this time.

Student 339: Is it three quarters, or what?

Student 323: Actually I don't think we should be looking at the size (*of the field*), I don't think that matters.

Student 329: Mmm,

Student 342: That was the only thing we had.

Student 329: Yeah, but we shouldn't even have done that. Let's go back.

In the Steel Cables Task, shown in Figure 27 below, the extent to which the students could use trial and error was more limited than in Field of Dreams, and the students were challenged to generalise and find an n^{th} term earlier in the solution process.

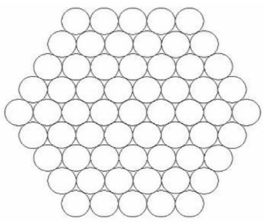

Figure 27 Steel Cables Task, Size Five Cable

Most students had been using a counting method to use the size 5 cable to figure out how many strands would be in the size 10 cable, stage 1 of the task, as shown in Figure 28 below. After the students had figured out the number of strands in a size 10 cable there were many examples of students persevering to find a general solution.

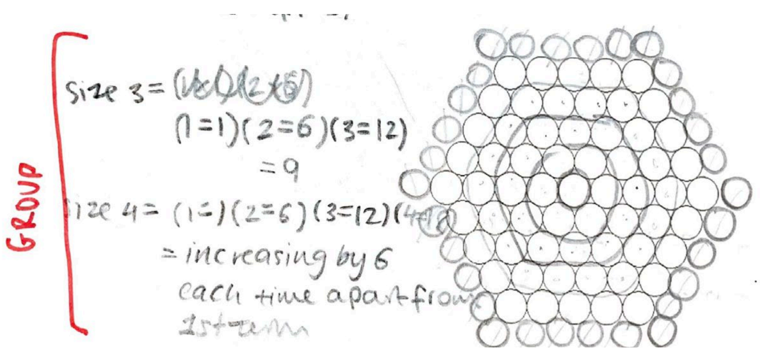

Figure 28 Sample of Student Work in Steel Cables Task, Stage 1

There were many situations where one student had a conjecture and others added their support to verify it. Students could be seen to work as a group to build on their ideas and make sense of what they had discovered already. There were periods of silence, for up to 5 or more minutes at a time, where the students worked on ideas individually. For one such group, quoted below, there were sound issues with a group member, but one of the other students managed to drive the group's thinking and communication via text comments. They showed excellent peer scaffolding:

Student 429: I have something that does work but it's not pretty. There definitely is a more complex way to do it (*than counting*) but I don't know how you'd write it out.

Student 428: So what do you have?

Student 429: 6 by $n-1$ in brackets, plus 6 by $n-2$ plus 6 by $n-2$, the whole way on until it gets to zero, and then plus 1. But you can't go further than the value n.

Student 430: Mmm, I feel like there's a way to use that 'equation'. So, T_5 should add up to 61 cause that's how many circles in size 5 cables.

Student 429: (*Checking for size 10 which they had figured out was 271*). Yeah, it works but it's very long. The length of the formula depends on the size of n.

Student 430: Can you show me the formula? I had been focusing on geometry.

(Student 429 holds up his sheet to the camera to reinforce his earlier verbal explanation quoted above)

Student 428: Yes, that makes sense because all the answers so far have 1 higher than a multiple of 6, like 19, 37 and then 61.

Despite having the solutions on a Padlet wall, (shown in Appendix B) to check when they felt ready to, the students showed determination to challenge themselves to find the most efficient method. They chose to give themselves periods of individual incubation and then discuss their thoughts together. The answer was not enough to satisfy them and the challenge of finding "the best way" or a "more compact way" was driving their perseverance:

Student 429: I don't know a more compact way of saying it … So, we have one method, but it's like the bigger the number the longer the formula.

Student 428: What's the answer for size 10 again?

Student 429: 271.

Student 428: Oh yeah, that's great.

Student 429: Factorial is a multiple, isn't it? Mmm … I feel like we're close enough to really getting something. Like the long way we have is a base we can go on to figure out a way to shorten it.

7.2.7 Inclusion and confidence building

Each of the above comments emphasised important aspects of peer work that were reflected in the audio recordings. However, for the researcher, one of the most poignant comments in the post-workshop survey was on how the peer work made an individual student feel. It also suggested that this may not have been a typical feeling, for her, in mathematics class:

Student 234: I really enjoyed it. I felt included. I am not exactly a good maths student, so I just signed up to see what I would be like, and I loved it ☺.

Throughout the audio recordings, this student's participation and enjoyment was evident. Through collaborative work she had the opportunity to express herself, seek explanations and ultimately gain confidence in her maths by feeling that she understood the group's work and was an active member of that team:

Student 234: So how did you get B to be 8?

Student 233: I just guessed, trial and error.

Student 234: Your communication is terrible, (*chuckle*). So what have we done so far? We wrote it out, the question, then we structured what to do out and then we just put in random numbers is that right? …

Student 233: Yes, but I feel B needs to be quite big because A must be small because A and C is only 5.

Student 234: Ah, OK.

Student 233: Could we try and use 9 for B because it will work out.

Student 234: I can't do group work in maths, I'm really bad at explaining …

(*Group discussion of final answers*)

Student 234: I'm so confused that I understand this one now and I'm so bad at maths.

There is evidence of the effect on the process of finding a solution, due to the confidence students gained from working through the tasks themselves. The feeling of satisfaction at their perseverance and feeling good about themselves for having solved it, gave them a more determined attitude to proceed. An example of such can be seen in the excerpts below where the development of the group's thinking was evident. Initially the approach was just random addition:

Student 242: OK, so A equals 11 plus 9.

Student 230: Why.

Student 242: Because I'm trying to think as if I was 10. What she just told me to do.

Student 230: OK. Then just plus all of them and divide them by 3. You would know that I was never in your math class.

However, through trial and error and comparing the difference between the totals in each pair of fields, the group figured out the first two fields and seemed proud of themselves:

Student 230: 8 then 3 then 4. Yes, you got it. So, let's write it out official.

Student 227: Yes, that was good work.

Student 230: It was you who kind of figured it out.

Having figured out Field 2, they immediately decided to try and use a formula when they saw the empty circle in Field 3, but sounded much more confident in their approach than they did at the outset:

Student 227: We need to put in an unknown and it will all add up in the end if we get it right.
Student 230: B to D is x. OK, so it's a bit complicated but that's OK.
Student 227: OK guys so let's just figure out all of the ones that we can know.
Student 242: This is going to be really…
Student 229: Just keep our heads down and just do it.

After a few minutes of silence, the group found the numbers for Field 3 but were beginning to question their method and seek justification:

Student 227: That worked.
Student 230: So, C and D are 8 so then 9 and 6 equals 15 so x equals 15.
Student 227: Does that work every other way?
Student 230: Yes. How did we just guess?
Student 239: That was you, you just guessed a perfect number, well done.

The group proceeded in a similar way to get the answer to the next field after which they began to look for a more systematic way once they had gained confidence in themselves:

Student 230: I can't believe we have just guessed the number. Maybe there's a pattern?

7.2.8 Illumination and the AHA! Moment

Having the opportunity to experience peer scaffolding, rather than teacher direction, could be seen to have the effect of creating student ownership of the solutions they arrived at. There were many comments from students in the interviews, regarding the feeling of satisfaction from finding their own solutions. This gave an insight into the individual preferences of students. However, the audio transcripts, whilst backing this up, gave a richer meaning to such comments. Whilst listening to the student progress towards a solution, the perseverance, determination, and satisfaction could be felt clearly. One particular student expressed their view in the interview, in response to being asked if they preferred to find their own method rather than being given the method by the teacher:

> Student 340: It's much more satisfying to find an answer in the workshops than in school because you know you solved it yourself.

The audio revealed that it had taken 54 minutes for the group, and largely this student, to find a general solution method to the Field of Dreams task. They had nearly reached the solution much earlier but had got distracted by over complicating the algebra, which will be outlined later in section 7.6.5 of the analysis. However, with several long periods of 3 or 4 minutes of silence, the above student figured out the solution:

> Student 340: I found out how to do it.
>
> Student 341: Oh my god, how did you do it?
>
> Student 340: I went back to the first question and, hold on, I'll hold up my sheet.

The researcher happened to interrupt the AHA! moment by walking into the room, just as this student was talking the rest of the group through her work and asking how the group were getting on. Hearing they had just solved the task, but that "it had taken a while to do it", the student was asked if it was satisfying working it out when not told the method:

> Student 340: Yeah, … that's the best feeling.

Another episode, illustrating the opportunities to incubate, showed how the discoveries of one student were able to support the others in making sense of the task. This in turn triggered off an AHA! moment in another student, which eventually led to the group finding a solution:

Student 236: Hey, I've found another way you can do the totals. 2A + 2B + 2C = 36, then you multiply across by a half, because it's two, and you get A + B + C = 18.

Student 236: And that's them added up together.

Student 225: Oh yeah, and that gives you what the total is.

Student 236: Yes, and then you can just do ... I don't know how to do the other bit. That's only the first bit.

Student 231: Oh yeah, so the total is always … Do you want to try and use that to go onto field 6 then?

Student 224: What do you do, add them all together?

Student 236: Yes, you do like, A + B + B + D + C + D + A + C, like you add them all up to get and multiply all across by whatever, by a half?

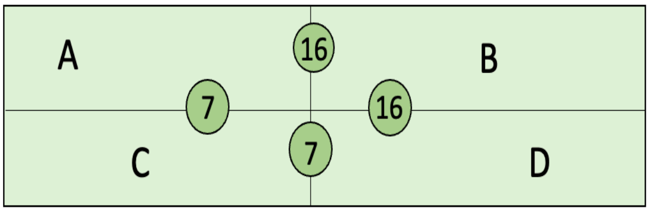

Figure 29 Field of Dreams, Field 6.

After a pause in discussion, student 231 suddenly realized the key to solving the task:

Student 231: Hey, wait lads, sorry for a second, if A + B = B + D and A + C = C + D, do you get me?

Student 236: Like, that just gives you the total and then you'd have to …

Student 231: Lads, look, A + B = B + D and A + C = C + D, do you get me?

Student 224: Ok, does that mean … can you cross off the B's?

Student 236: Emm, they just cancel.

Student 231: Yes, so then A = D and D = A.

Feelings of enjoyment and satisfaction at what they had achieved were evident in comments from other students on completing this task. One such comment was from the student, mentioned above, who said she had enjoyed the workshops because she "felt included":

Student 234: Well done guys, we did it. Yeah.

The other groups seemed pleased and proud of themselves too when they figured out each task. There was a clear element of excitement in their voices:

Student 227: I am so delighted we got the very first one.
Student 120: Look into your mind, your parents must be proud.
Student 342: Oh my god, wait, wait, wait…
Student 323: I think I'm after figuring out the second one too. Will I explain it?
Student 127: Oh my god, it actually works. Wait, wait.

Others were more reserved in their expressions but there was a definite tone of quiet satisfaction at their achievement. These tended to come from the male students whose discussions, in these groups, were more reserved by nature:

Student 343: Cool, I think we have it sorted.
Student 425: Hey, that's us done. That was good.

The audio recordings of the Steel Cables Task showed similar student perseverance and satisfaction at having solved it. One group had been struggling for some time and were really determined to find the n^{th} term, calling me over to check if there was an $(n-1)$ in the answer, to which the researcher replied "maybe, why do you say that". They kept discussing it for another ten minutes and then three minutes before the end of workshop one of the group figured it out:

Student 045: I got it, I got it, I got it, it's …
Student 032: What the formula?
Student 045: Yes!
Student 021: I'm so glad you worked that out.

Student 045: Its $2n^2$, plus in brackets so, $2n^2 + (n-1)^2 - n$

Student 021: Good job [Student 045].

Student 021: Thank you.

Similarly, one student in another school very casually said after the group had been discussing trial and error with numbers for 20 minutes:

Student 432: I think I have figured out a formula for the n^{th} term.

Then when asked what he did, he described one of the more complicated methods used in the solutions, which they had not been given yet. His calm response was well matched to the modesty at having found such a method:

Student 432: So I drew it out and divided it into six triangles with one in the middle, and they're all equilateral triangles. So, you get the formula for each of those, it's like, the formula for a triangular number is $\frac{n(n-1)}{2}$ and you can multiply that by six for the six of them, then add one. I think that's how you do it.

Student 432: What did you say you got [Student 434] for the ten?

Student 434: 271.

Student 432: If I got, … so ten is the n number, I get ten by ten minus one, I need my calculator… yeah 271. That's it.

Student 434: Yeah, you must have the formula.

Student 434: Yeah, I'm pretty sure I have it then. Alright, great.

The opportunity for incubation did seem rewarding to these students, through the tone of their voice, even if they did not verbalise it as much as others.

Some groups bounced off each other more effectively than others, perhaps because they did seem to be of more equal ability, given the quality and frequency of the contributions they each made. Having the opportunity to discuss methods enabled these students to provide the necessary stimulus to build each other's confidence and bring about that feeling of satisfaction as a group.

Student 036: Like, is it simultaneous equations? If you said, $x + y = 9$?

Student 038: Let's go back to field 1 …

Student 036: Oh yes, it is. Look, eliminating C. So say A is x, If we put C equals … so then we continue with A's and B's and get rid of C's?

Student 023: But how can you do simultaneous equations with 3 variables?

Student 036: You can, but it's very complicated. You do the first one and you get an answer, then you do the second one and you get an answer and then you do those two and you get…

Student 038: But we can't because $A + B + C$ is unknown, we've no answer like?

Student 036: Ok, I get you.

After several minutes it suddenly came together through the combination of several students' ideas:

Student 038: Oh my god, so then $11 - C\ is$ 9 ... and then … I don't really know what I'm doing here but …

Student 036: Wait, wait, if they both equal A then are they not equal?

Student 023: Yeah! so that means that $(11 - C = 9 - B)$ … [Student 036], you were on the right track!

7.2.9 Summary of audio recordings on Gestalt Principle

From listening to the students discussing their ideas, and the processes by which they came to a solution, there appears to be sufficient evidence to say that the workshop allowed for challenge, incubation, peer scaffolding and support, illumination, satisfaction, and confidence building. The tasks presented to the students and the facilitating role of the teacher combined to provide opportunities for incubation and compelled the students to think about strategies and seek clarifications from each other.

7.3 Uncertainty Principle

The tasks in the workshops were specifically chosen so that they would be unfamiliar to the students hence, in most cases, the students had to think about which methods to try. One school had just recently done simultaneous equations with three variables, so found the Field of Dreams task easier than the students in the other schools. Other than that, the audio recordings contained evidence that having to think of the method was unusual and challenging for the students. Listening to these conversations within the groups backed up what had been said previously, in the surveys and interviews, about being either given the method first by the teacher or following an example in a textbook. Without an example to follow the students had to make decisions for themselves. The most common method employed for the first task was trial and error, but students were suspicious from the outset that this probably was not the only way and proceeded to explore other methods. The open-ended question, in the post-workshop survey, included responses showing that the students acknowledged the benefits of having to think of the methods for themselves in the workshops:

> Student 129: I enjoyed the workshop because we had to think more about the questions before doing them. This is because we weren't given an example / method that showed us how to do the problem.

> Student 046: If you come to a conclusion yourself about a question, you're going to remember it more than if you're just given one.

> Student 020: Because it's from the book, … we know what to do so then we're not asking because that's just the way we're taught and we just do it that way.

7.3.1 Style of question

The unfamiliarity of the questions brought much uncertainty for the students possibly because, as they had voiced in other areas of data, they were most frequently given questions from the textbook. Even though some of the students had covered simultaneous equations with three variables, which helped with the Field of Dreams task, and elements of sequences, which helped with the Steel Cables Task, the presentation of the tasks still looked unfamiliar, and the students had to scrutinise and explore options before making a decision on how to proceed. One student commented specifically on the style of questions:

Student 121: It's completely different to what we get in the textbooks.

7.3.2 Opportunities for having to think of a method

In addition to having unfamiliar tasks, not being given the method, or not being shown an example, was evidently an unusual procedure for the students.

Student 335: Has she not even told us what to do?
Student 324: No, that's the whole point!

Similarly, the unfamiliarity of not having answers at the back of the textbook was also causing uncertainty for the students:

Student 323: How do we know we have the right answer?
Teacher: You should feel so confident that you all agree on it, that it has to be right.

The audio recordings backed up what the students had said in the surveys and interviews with regards to their classroom mathematics being predominantly routine questioning. So, by being presented with tasks that the students had not done before, they had no set method to rely upon and were therefore forced to come up with a strategy for themselves. For all groups there was much uncertainty at the outset of each task and throughout the discussions on the possible methods required to find solutions. The students can be seen to seek help from each other when feeling bewildered:

Student 120: Sorry I'm lost. How did you do that?

Student 423: What did you get for Field 3?

Student 431: I don't know I made a 'mess' of it anyway, so.

Student 421: Could you just explain your method because I haven't got a clue?

The thinking required to come up with strategies was challenging and for some groups there were periods of up to 10 minutes of silence while the students worked individually at the start of a task. In one of the interviews, one of the students, who had been challenging herself to find a strategy for the Field of Dreams from the outset, expressed the difficulty of thinking of a method very honestly:

Student 340: No, honestly, I think the first question was the hardest for me because I didn't have a clue what I was going to be doing.

Finding the n^{th} term was also particularly challenging for all the groups. Silences followed by comments similar to the one below were frequent for this task:

Student 433: Have you guys made any progress?

At times some of the students may have found it frustrating adapting to having to regulate their thinking and persevere to get an answer:

Student 339: I'm so confused. I don't get the n part.

The two excerpts below outline discussions between the students where they were guessing with numbers, experimenting with ideas, and trying out concepts they had learnt in school. Whichever way they decided upon it was clear there was a lot of decision making to be made by the group to find a strategy with which to proceed. The first example was from the first field in The Field of Dreams:

Student 020: $11 - x$. Would that work?

Student 038: But we don't know what x is so, that doesn't really help.

Student 036: I feel like you just have to guess then?

In the second example the group were trying to figure out the n^{th} term in the Steel Cables Task. From this early discussion they seemed fairly confident that it was something they had done recently in school. However, despite the early confidence, they had still not found a solution by the end of the first workshop twenty minutes later:

Student 421: Do you know how to do it?
Student 423: Kind of.
Student 421: Could you not just keep adding them on.
Student 423: Well you could but that would take forever.
Student 421: Is there like an equation, because it goes up and up.
Student 423: It's 1 then 7 then …
Student 421: That's easy enough isn't it.
Student 423: Yeah, it's exponential isn't it. It's one of them sequences, isn't it?

7.3.3 Overcoming uncertainty

The most common approach by the students to the first task was to use trial and error, or guessing as some students called it, to see what numbers might fit in each field. It was clear that a systematic approach was not obvious to the students at first. However, as they worked through the task, the students began to explore alternatives having gained confidence once they had found answers as a basis:

Student 038: Is there a way that you can solve it without trial and error though? There must be.

Student 132: I get what you're saying, so let's try this method and see if we get the same answer. If we don't … well that's life.

Student 325: But I'm just trying to see if we can do it a way without guessing.
Student 343: Can't we just guess the numbers and then figure it out at the end by backtracking.

Having found a method the student above, student 325, seemed pleased with himself which was important for giving them confidence in themselves for the remainder of the task:

> Student 325: We got an actual method rather than just guessing, so that was good.

This example ties in with similar comments outlined in the Gestalt Principle analysis, where having time-consuming tasks that made the students think contributed to feelings of satisfaction and confidence as individuals and as a group.

7.3.4 Preference for thinking of methods

Overall it is evident that the students liked having to think for themselves, even though the audio recordings may not have any direct comments stating so. The vibrant discussion, which permeated the audio recordings of the workshops, backed up what had been said in the interviews about the enjoyment of thinking. There has not yet been a discussion, in the analysis, on how the students felt with regard to individual tasks. The reason being that it seemed to fit well in this chapter because the interview comments correspond with the atmosphere of the workshops which is very difficult to describe in words. In the interviews, the students were asked specifically what they thought about thinking for themselves, and their responses are reflected in the excerpts of the audio recordings given throughout this chapter.

> Teacher: Do you like having to think of your own strategy?
>
> Student 325: Yes. It does make maths more stimulating, challenging but, yes, because doing it on your own with less help, trying to find your own way feels good.
>
> Student 432: I think being able to think for yourself and work things out without having to be told how to do it is very beneficial.

In addition to making the tasks more challenging, it was clear that thinking of the method was also a key feature of the enjoyment of the workshops for the students. You could hear lots of laughter and chatting in the face to face workshop audio recordings but even in the online ones students commented afterwards in the that they really enjoyed the thinking aspect of the workshops. Because it was difficult to determine, through the audio recordings, which task the students preferred, they were asked specifically in the interviews. Students made particular reference to enjoying the having to think aspect of the workshops when asked which was their favourite task:

> Student 046: I liked the last one (*Area of Circles*). I liked where you were not given numbers and you could kind of put in whatever you want and you just kind of go with it. It's really satisfying if you like, get it by the end.

> Student 340: I liked the first question, like Field of Dreams ... I had to think, "Will algebra work for this?" and I had to do lots of trial and error.

Student 340's comments, already given in section 7.2.8, backed up that the thinking and not knowing what to do was a motivating factor for her. Other responses to the interview question on their favourite task made similar reference to the enjoyment of uncertainty and having to think of a method:

> Student 231: The Field of Dreams ones, at the start it was like you hadn't a clue what you were doing, but as you progressed, you made up your own method.

> Student 333: Even though it was getting a bit frustrating at the end (*my favourite*) was Steel Cables because they got me thinking a lot.

7.3.5 Summary of audio recordings on Uncertainty Principle

The audio recordings can be seen to illustrate a high level of uncertainty for all groups of students. In having to think for themselves the students were compelled to experiment and collaborate to decide upon a successful method, which they appeared to enjoy. The tasks took much longer to solve than traditional classroom questions where the students know exactly how to approach a question. This uncertainty faced by the students provided opportunities for a 'collaborative ZPD', outlined previously in the Gestalt Principle. Further examples of uncertainty encouraging students to explore possibilities and take risks within their peer groups will be discussed in the Aesthetic and Free market Principles later.

7.4 Aesthetic Principle

Giving students the opportunity to experience the beauty of solutions, and an appreciation for the mathematics involved in tasks, was seen by Sriraman (2005) to be an essential element for promoting creativity. By presenting students with multi-solution tasks it was inevitable that they would not think of all the solutions. This in turn brought an element of surprise when they saw the variety of methods, especially when they were very different to traditional algebra techniques that many were accustomed to and in some case unexpectedly simple. Through the audio recordings and interviews, the students can be seen to enjoy the exploration of methods and seemed less focused on, and satisfied with, just getting the answer.

7.4.1 Beauty of simple solutions

When asked in the interviews which tasks the students preferred many said it was the *What's It Worth Task* where they were asked to find the value of the question mark in as many ways as possible (see Figure 30 below). From these responses in the interviews the reason for this was the variety of solutions and the simplicity of them:

Student 020: I liked the table and numbers one. Yeah, What's it Worth. Em, just because once again there were so many different ways to do it and, like, when I did it at the start, I could only see like 1 way.

Student 023: I didn't think about them but when I saw it (*the quickest solution*), I thought oh my god how could I not have seen that

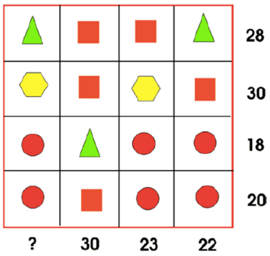

Figure 30 What's it Worth Task.

One of the two students who found this quick solution to finding the value of the question mark, without finding the value of any of the symbols, very casually explained her reasoning when the researcher enquired how the group was getting on:

> Student 032: Because the shape is a square, I added up this row with this column and they should be the same.
>
> Student 040: What did you say they add up to on the side?
>
> Student 032: 96. So the missing number is 21.

There was a momentary silence from the group and a look of surprise at how this student had seen the answer so quickly. Even though the group now had the answer to the question in the task they continued to look for other methods. When discussing each other's methods to find the values of the symbols the group were inquisitive about how each other did it. One of the group was explaining to another how she compared the two rows with three circles to figure out that the square was two more than the triangle and continued to use comparisons:

> Student 040: Hey, that's really smart. I'm gonna try that. I was just using trial and error.
>
> Student 021: So, (*after finding the value of all the symbols*) … the missing number is right, great.

This was another example of the effect a 'collaborative ZPD' can have on raising the potential of students within a group situation.

When Student 032 was asked to explain her quick method to the full group at the end of the workshop, she got a big round of applause and a student in one of the groups could be heard to say “she’s so smart”. This was repeated in one of the interviews for this school. The only other group to notice this was an online group, although it was evident in the worksheets of one student that she had started this method but did not finish her workings. For the online schools, this simple solution was brought to the attention of the whole group at the end of the workshop, and there were similar reactions, along with several “well done” comments addressed to the student who looked very pleased with himself.

The Steel Cables Task presented the most diverse methods for the students to grasp, and it was interesting how many students commented in the interviews that this was their favourite task. As mentioned in section 7.3.3, having to think of the methods proved challenging and seeing how other people had solved it appealed to the students.

> Student 023: I liked the Steel Cables because I found it hard at the start to see the spatial answers but when I worked them out and when I saw the others I thought ohh, whoa there’s so many different ways of getting the answer, it kind of opened my eyes to trying different methods.

> Student 021: Yeah, that’s the one I liked seeing all the methods for.

Some students wrote messages on their worksheets to tell me which methods they liked best, such as are shown in Figure 31 below.

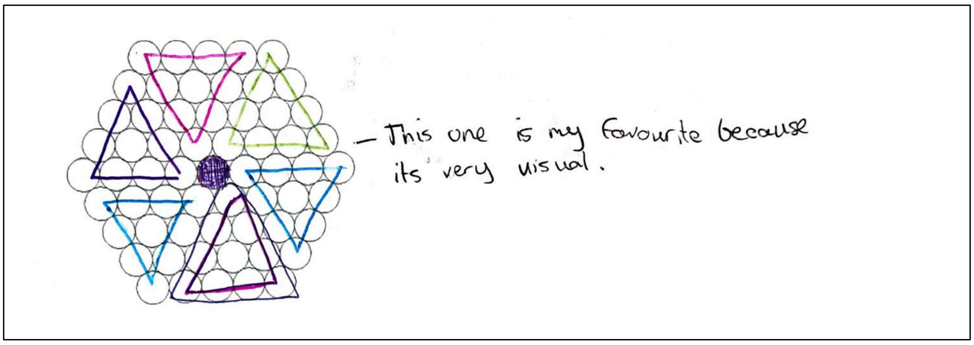

Figure 31 Sample of Student work in Steel Cables Task, Stage 2

To many students the time-consuming aspect of the multi-solutions made the task rewarding:

> Student 429: I feel like my favourite was probably the Cable one because I spent the most time thinking about it and I felt like each solution to it was very rewarding no matter what way you went through it … coming up with your own way, your own method was very rewarding and that's probably the bit I liked most.

However, to one group the Steel Cables Task was their least favourite because of how time-consuming it was, but the benefits of such thinking were acknowledged by the group:

> Student 129: I was getting really annoyed because it just took so long, I just couldn’t get it … it definitely helps you learn to think more … (*to*) think outside the box.

7.4.2 Focusing on the answer rather than the importance of understanding

Another noteworthy area identified in the interview analysis template was the focus on the answer in school mathematics rather than exploring the problems. The audio recordings clearly showed that the students seemed to enjoy the experience of exploring problems and could see their value. When I asked the students if the tasks in the workshop changed their attitude to what mathematics is at all, there were recurrent responses similar to the ones below:

> Student 225: Maths in school is just getting the answer. With these problems and stuff, it’s more of how you get it and the different ways you can do it.

> Student 432: Yes, I'd say they have, I think. What we did in the workshop was much more beneficial than what we would do in the

classroom. If some things like that could be integrated into the classroom, into textbooks, I think it'd be great.

There were many examples in the audio recordings where the students showed an interest and determination to explore the purpose of the task rather than just finding an answer. For the Field of Dreams Task this was common because it was easy to guess the answers to the initial fields, but the students did not seem happy with just getting the answers out:

Student 333: I don't feel that this is the way that she wants us to do it. I feel like there should be a maths way of solving this rather than just guessing.

One group had guessed the answer to the first two fields and after 7 minutes were asking:

Student 429: There's no formula is there? Could we come up with some sort of method for it?

Student 430: I think there should be some sort of easy formula for it once you realise the difference between them.

Student 429: I can't think of what it would be though, rather than just guessing.

Following these comments they used the difference between the numbers to find the solution to Field 5, seen in Figure 32 below, but did not pursue looking for a formula because they were able to do each field quickly by working together:

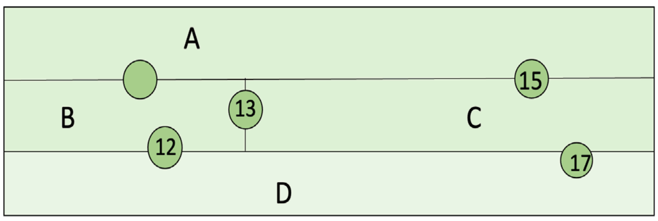

Figure 32 Field of Dreams Task, Field 5

Student 429: We can use D and see if there is a high difference between B and C. Is that how you do it?

Student 430: Yes, so you find the difference between $C - B$. So C is the bigger one, so $C - B$ is 5, no, -5. Does that work?

Student 429: I kept thinking that it would be 4 and 9, what did you get for B.

Student 430: I actually haven't got B yet, … so is B 4 and C 9, yes, that's grand.

Student 428: So D is 8, and A is 6 and then $6 + 4$ is 10.

Another group had suggested the use of a formula but had over complicated it and consequently it took the group 54 minutes before finding an appropriate method. They had found the answers fairly quickly but, even after Field 1, they raised the idea of a method to find a solution for all fields. Then they actually stopped trying to guess the answers after Field 3 and got bogged down in finding an algebraic method:

Student 331: Wait should we be looking for other solutions?

Student 332: Maybe.

Student 331: Oh, god no.

Student 340: No, it's definitely 2, 7 and 9 but like, what's the thought process that would be able to give us that answer if it was harder and we couldn't guess it … say that number was like 252, 196 and 400 blah, blah, like how would we get the answer? … I don't know, it's not relevant right now I suppose.

Student 340: I'm gonna try.

The challenges posed for this group, by the multi-solutions to this task, will be discussed in the Scholarly Principle analysis. However, it is fascinating to note here that despite how long it took the group to solve this task it was still seen, by Student 340, quoted earlier, as her favourite task.

In the Steel Cables Task the students were given solutions on the back of their sheet to consult when they felt they had the answer and had exhausted their solution methods. It was interesting to hear the dynamics of the group who had found the Field of Dreams Task fairly easy when faced with this new task. In the Field of Dreams Task the group did not really engage with the discussion questions, it appears as though they were very confident, they had found 'the method' so did not feel they had to keep going. Their attitude completely changed later in the same workshop when they were challenged and seemed to enjoy the explorations.

Student 140: Shall we just check the back?

Student 121: No, that would be cheating. It's not about the answer, it's about the methods.

7.4.3 Importance of focusing on understanding

The audio recordings of the workshops highlighted some interesting student discussions that emphasised the focus on the importance of understanding. In one example a group were in the initial phase of trying to understand the Area of Circles Task, shown in Figure 33 below.

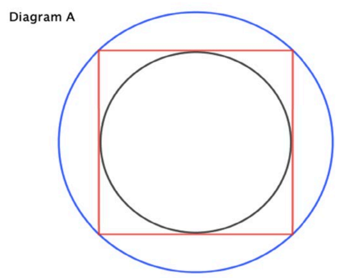

Figure 33 Area of Circles Task A

They had found the required ratio through measurement and then one of the students had suggested looking at other random numbers. The comments below show that this student was not happy that one of the group was just going to copy down her theory without understanding what she was doing:

Student 227: (*Explaining how to use their random number to find the areas of each*).

Student 230: Ok, I'll write that on my page.

Student 227: No!

Student 230: I don't get it!

Student 227: OK … let's do the big circle first. So, πr^2. Come on are you following.

Student 230: No, cause I was doing it wrong.

Through a determination to explain her reasoning this student was helping create a group atmosphere where the importance of understanding, rather than just getting the answer, was the 'norm'.

Similarly another group had different ideas about what was important in solving the Field of Dreams Task. They had got answers for the first 5 fields and used the difference between the totals to figure out a logical method. However, for Field 7 Student 325 and Student 336 were just using trial and error, Student 343 was only concerned about the answer, but Student 338 was unconvinced:

Student 325: Yeah, I think you're right, I'm just wondering if there's any other possible numbers, or if that's it.

Student 338: What way did you get the numbers?

Student 336: I just guessed that A is 3 and the rest kind of fell into place.

Student 338: But why did you guess it was 3 though?

Student 325: Let me just guess if A equals 4.

Student 338: Yes, that will work too, but that's just trial and error again. I'm just wondering because we have not used one formula like for two questions.

Student 343: Like, if we're getting the answers right, does it really matter?

Student 338: Tuh, well yeah …

The group went on to get a method for finding the values of the variables after about 5 minutes. Eventually the student looking for a method convinced them to work on it.

7.4.4 Problem posing: Creating their own field

To give the students the experience of looking for more than just the answer, having solved the first two fields in the Field of Dreams Task the students were asked to make up one for their friends to solve. This was an introductory step towards problem posing which the researcher had guessed the students would not have experienced. From the comments in the audio recordings, as anticipated, this was not something they were used to in school or seemed to have much confidence in initially:

Student 020: Oh, no

Student 038: Ahhh.

Student 023: Ok, I'll try … We'd usually skip these things in class like.

Student 038: Do we just write down the box here and we just know what it is?

(*Giggles*)

Student 023: I don't know how to do this.

Student 038: I just find it really hard.

Student 036: Do you just pick three random numbers?

Student 038: Yeah, mine doesn't work either

Student 023: Did you just make them up on the spot, or did you do like a formula to work it out?

Student 038: I just made up the numbers, A is this, B is this and I just added them.

Student 023: Ahhh.

Student 023: But wait, how do you know they all add up, how do you know they correspond?

The experience did bring about much discussion and highlighted the different ways students can view a task. When the group were asked how they had done it the students gave several different methods which seemed to amuse the group and immediately raised comparisons between methods such as:

Student 023: I did mine a really complicated way.

In general, it appears from other comments that this type of problem posing was not something the students felt comfortable doing or grasped the purpose of:

Student 331: Make up your own *Field of Dreams* problem and swap with a friend, are you serious?

Student 339: We have to make up our own? I'm not doing that cause I don't even understand what's going on with these.

Student 342: Wait, is that an extra question or is that just something we could do in our spare time, if you want to?

Student 339: We could ask for help?

Another group were equally intimidated initially, but also did really not grasp the purpose of the task. One of the students in this group wrote the numbers, that the group were supposed to be trying to figure out, at the top of the page:

Student 227: Make up your own field of dream problem and swap with a friend.

Student 239: Oh god.

Student 230: Oh no. I'm not in the mood for doing that.

Student 227: Don't make them too big. No that's fine.

Student 239: Does it have to have a method behind it? Does it need a method?

Student 227: Yes, because it mightn't work. It does have to have a method behind it. You can't just make up random…

Student 239: I don't know how to make up my own question.

Student 230: This is harder than I thought it would be. Jesus.

Student 227: I'm going to scrap that one. I am just going to make up a total and then go from there.

Student 230: I think I have it handled. Wait I'm going to write out my answers on this side, don't look at it OK.? All my answers are up here so you're just going to have to not look at it.

Despite not knowing what to do, initially the students were willing to give it a go and were enthusiastic at trying something new:

Student 231: Is there any pattern we could try that would make it easier to do? Do you get me?

Student 224: Probably not.

Student 236: Well, there probably is but …

Student 225: How do we make up our own field?

Student 236: I don't know.

Student 225: Let's try it.

Another group didn't read the question properly initially, but their comment did reflect a lack of familiarity with such explorations:

Student 329: Do we have to do the second one?

Student 334: What do you mean?

Student 329: Field 2, cause that's like make up your own field, it's not actually solving that field.

Student 324: That's like a side thing, I'm guessing we don't have to do that.

It was interesting to hear comments from groups that showed a genuine engagement with the whole problem-solving process. From listening to the audio recordings the students really seemed to enjoy the explorations and were not as concerned with rushing ahead to just find the answers. There were constant observations being made on what the groups beside them were doing and a feeling that they really were focused on the process:

Student 228: They're making a field up.

Student 233: Yeah, we were meant to as well, we skipped it.

Student 234: Oh, we skipped it.

Student 240: It's hard to do.

They then just went back to the discussion on Field 6 as if the creating their own field was a side issue. However, after feeling pleased with themselves for finding answers to the final field they immediately went back to creating fields for each other:

Student 240: But we didn't make the fields for each other.

Student 234: OK, I'm gonna make you a field.

7.4.5 Summary of audio recordings on Aesthetic Principle

In comparison to the responses in the surveys and interviews, the mathematical experience of the workshops for the students was very different to what they had in school. The only comment in all of the audio recordings that mentioned examinations or the Leaving Certificate was the query, mentioned in the Gestalt Principle analysis, over whether the questions in the Leaving Certificate would be as hard. This was also directed to his schoolteacher who was in the room at the time, rather than a query that arose out of discussions within his group. Another noticeable difference was the lack of satisfaction at just getting the right answer, which also became more noticeable as the workshops progressed. The students were looking for the "best" or "most efficient" solutions and were more focused on understanding what was involved. There was obvious enjoyment at having the opportunity to discuss different solutions and see simple methods that they would not have thought of themselves. Overall, the audio recordings and interview comments on the tasks illustrate a rich appreciation of the mathematics in the tasks and potential learning from such explorations.

7.5 Free Market Principle

A key feature of Sriraman's Free Market principle was providing students with opportunities to take risks either in a full class situation or through peer work. In doing this, the students get to explore alternative methods for themselves without feeling the fear of exposure if their suggestion is incorrect or irrelevant. The audio recordings backed up what students had said, in other areas of the data, with regards to the freedom and enjoyment associated with peer work. In the open question at the end of the post-workshop survey, students had expressed positive reactions to the groupwork, as had been the case in all the interviews. The comments covered a variety of characteristics associated with peer work including having the freedom to think for themselves, the dynamics of teamwork, enjoyment, and learning different methods through discussion. Below are some quotes from the surveys which will be seen to reflect what was said in the audio recordings of the workshops:

> Student 135: I preferred working in groups rather than on my own as we can try and work out many other solutions.

> Student 024: I enjoyed this workshop. It tested my mathematical ability … It was cool to learn a different type of problem solving and

ways to approach questions. It was really stimulating, and I would enjoy doing it again. We had a great group dynamic.

Student 329: I thought it was very fun and enjoyable and is my favourite type of way to do maths.

7.5.1 Teacher Authority

In traditional classrooms the teacher plays a dominant role and students are conditioned to see the teacher as the source of knowledge. (Boaler, 2003; Lampert, 1990; Lithner, 2008; Schoenfeld, 1992). As outlined earlier, the workshops were designed to encourage independent thinking with minimum teacher influence. The analysis of the audio recordings of the workshops relating to the Uncertainty Principle, show that the students were unfamiliar with this style of working where they had to think of the methods for themselves. During the first workshop the students seemed to get more confident to try out their methods, rather than relying on the teacher. However, there were undercurrents in the audio recordings that the students were still thinking of what the teacher wanted them to do despite trying to solve things their own way. This was particularly noticeable in the first task the students were presented with, the Field of Dreams. Having successfully guessed the answers to the first few fields the influence of the teacher's authority was contributing to self-doubt:

Student 331: I feel like we're trying to do what we think that she might want, which is trying to make it maths, but really, maybe it is just playing a guessing game. I doubt that, but maybe it is.

Student 323: I feel like we should be doing it a 'mathsy' way.

Student 231: She wants us to do it without using numbers.

7.5.2 Taking Risks

Enjoying having the freedom to think was a recurrent theme in the interviews. The lack of teacher direction that this entailed brought with it opportunities for the students to take the initiative and suggest methods for solving each task. There are examples already mentioned in the analysis relating to the Gestalt Principle where student collaboration provided peer scaffolding and regulation of each other's thinking. In addition to providing opportunities for a collaborative ZPD, the peer discussions were instrumental in encouraging the students to take risks that they may not have felt as free to do in a full classroom environment. By being part of a small group responsible for solving the tasks presented, the students viewed themselves as active participants and had opportunities to input their ideas without restraints. There is an example, given in section 7.5.8, where this may have caused elements of friction, but the students can also be seen to find a way to express themselves in a way that contributed to progressing the group's thinking. This aspect of peer dynamics can be viewed in terms of situated learning theories where the participation of students in the group discussions was instrumental for their learning. (Lave & Wenger, 1991). Such learning cannot take place unless the conjectures are voiced and evaluated by the group. The audio excerpts illustrate many episodes of spontaneous discussion where students freely offered their ideas for critique by peers. As the discussions progressed, the conjectures and justifications can be seen to contribute to overall learning within the group.

7.5.3 Dynamics of Teamwork

The audio recordings illustrate how working in a group can generate a variety of team dynamics, both positive and sometimes negative. There were instances where some team members participated less than others, but the participation of such students did fluctuate, and they could be seen to become more involved as they acquired more confidence. An example of this was given in the Gestalt Principle section where the confidence building aspect of a 'collaborative ZPD' was outlined. The positive aspects of team dynamics included peer support and encouragement, the development of a variety of strategies and a responsibility to ensure there was a shared understanding of the tasks.

7.5.4 Peer Support and encouragement

There were many general comments in the audio recordings that describe the feeling of freedom that the students spoke about in the interviews. Below are a few of these that illustrate episodes when a conjecture was made, and other group members can be seen to offer support and encouragement:

Student 023: I don't know if what I'm saying makes any sense but …
Student 038: Go on, go for it and see.

Student 329: Why not? It worked for the last one, it'll work this time. Let's just try it.

Student 434: Yeah, I've figured it out, I think. I'm not sure if it's right though.
Student 432: Do you want to go through it with us and see.

7.5.5 Development of strategies

Having the freedom to think of methods for themselves also provided the opportunity for a variety of methods to emerge. In other situations students had been working individually first, as was part of the brief, and then came together to compare what they had done. This provided an opportunity for a variety of methods to emerge for comparison and the students can be heard to weigh up and make decisions regarding which method appears the most viable. The first example is from the early stages of discussion of one group on the Field of Dreams Task:

Student 038: Yeah, I feel like it has something to do with $(A \cup B) \cap (B \cup \ldots)$ but I don't remember how to do it. It has to be something to do with algebra.
Student 036: I think it's x and y
Student 038: Ok, will we try x and y and see.

Another similar situation occurred within a group trying to find the n^{th} term in the Steel Cables Task where a compromise between two different methods was instinctive:

Student 132: I just tried out some numbers to check.

Student 127: We'll try that out and see if it works, because this (*method*) is getting way too complicated.

Student 127: Why don't we try n equals 6 and see if there is a pattern between the two.

Student 136: OK I'm just gonna try n equals 1 and do you want to try n equals 6?

Risk taking can have many benefits, including provoking spontaneous discussion in a class or group. Sometimes instinctive student conjectures provided the necessary catalyst for the group to develop a strategy with which to approach the task. An example of this can be seen from one group who had previously highlighted in an interview that discussion was not a key feature of their mathematics class:

Student 038**:** You're not allowed to talk in class.

In the workshops they can be seen to work together, bouncing ideas off each other in a constructive way that involved risk taking with conjectures on the Field of Dreams Task, Field 1, that they were not sure about. There was evidence of much constructive chat and the students seemed very comfortable whilst engaging in conversation with each other. They were taking gambles on ideas they were not entirely sure about but felt free to voice them from the outset of the task. Figure 34 below, also shown in section 7.1, is repeated here to improve clarity when reading the student comments that follow.

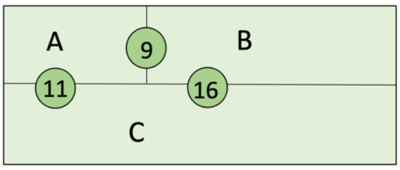

Figure 34 Field of Dreams, Field 1

Student 020: This mightn't be right at all but … There can't be more than 11 in field A, like cause that's not possible or more than 9 in Field B.

Student 038: Oh yeah, yeah, I see what you mean so there has to be at least 5 in B.

Student 020: Yeah.

Student 023: 5, in B?

Student 038: Yes, because there is maximum would be 11 in C so there has to be at least 5 in B.

Student 036: Can you just say that?

Student 038: You'll have different answers, but you'll still get the same total, do you know what I mean?

Student 036: Could you say there's 4 in A and just work it out? But you're not for certain …

Student 020: We might be able to work it out if we just try … if we use trial and error.

This discussion seems like an accurate illustration of their experience of, and attitude to, the mathematics in the workshops, which two of the group had articulated in the interview:

Student 020**:** When we were doing these, it didn't feel like we were doing maths at all. It just felt like we were having a discussion about a topic and working it out together or on our own, it didn't feel like we were doing maths.

Student 036: I think some things, as well, with the discussion, you would see something and then someone else would see another part, and then collaborating just makes it so much easier.

7.5.6 Possibility of misleading ideas because of spontaneity

In other situations, the conjectures may have been misleading but the freedom the students felt to voice them was important both for encouraging participation and for provoking discussion. There were a number of impulsive suggestions made with regards to Field 1, shown in Figure 34 above, in the Field of Dreams Task:

Student 227: 11 plus 9, then 9 plus 16 and then 11 plus 16 and then divide them 3.

Student 242: Go on let's try that.

Student 230: It's 24 and there's not 24 in all of them. We're adding more people than there actually is in the whole thing. It's 11 plus 9 plus 16. We can't use 11 twice and 9 twice.

Student 227: 11 plus 9 plus 16 and divided by 3. That goes. That's 12.

Student 230: That's what I was thinking so there's 12 in each field. That's just the simple way we would think to do it. If there's 12, but I think the size (*of the picture*) is something. I think if 1 is to 2 is to 3. So, the ratio of 12 is 1 is 2 is 3.

The group proceeded to discuss how they used ratios in school and how the 12 would fit in with the idea of using ratios. This continued for five minutes, until one of the group, who was the least vocal so far, raised the question:

Student 239: Why do we need ratios again, I forget?

Student 227: Because A looks like it is 1, B looks like it is 2, C looks like it is 3. Maybe that's wrong.

Student 230: Maybe it's right.

It may have distracted the group from solving the task for fifteen minutes, but it did highlight the danger of making assumptions and get all of the group involved in the discussion at an early stage. They realized their error and abandoned considering ratios for the second field.

Another group made the same assumption of dividing by three, but Student 342 interjected early on before the group got distracted by the assumption. The freedom with which all the students raised and defended their ideas until being convinced by a peer is evident:

Student 342: Why don't we just add A plus B plus C and divide it by 3

Student 339: Yeah, but they're not all even.

Student 329: See the way last time it was half, do you think this time it will be too?

Student 329: We could say that A plus B plus C equals 15 and work it out from there.

Student 339: Yeah, but there's nothing to prove that.

Student 323: But the last one could.

Student 339: But the last one is a different question altogether, isn't it?

Student 342: Wait how did we even get the other one cause I was lost at the time?

Field 6 in the *Field of Dreams Task* provided many opportunities for discussion. Again, Figure 35, shown below, is a repeat of an earlier Figure of Field 6, to improve clarity. The students by this stage had figured out a method to find answers, either by trial and error, examining differences between the totals or by using simultaneous equations. However, the multiple solutions did appear to confuse the students and lead them to freely offer up ideas to the group:

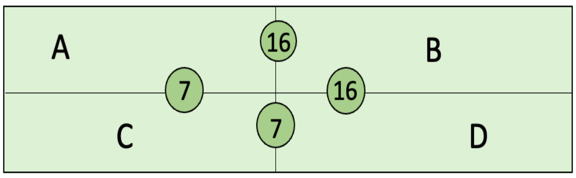

Figure 35 Field of Dreams, Field 6

Student 233: B can be huge, it could be 16 and it wouldn't make a difference. That's interesting.

Student 233: But can that work the other way? Say if A is 4, C is 3 and B is 12 and that still works out.

Student 233: I wonder if A and C could switch what would happen

Student 234: Ohh.

Student 240: That's what I think, if you switch A and C it might still work.

There was a brief discussion on this idea and an experimentation with numbers before it was temporarily abandoned with a sense that they should keep going:

Student 233: Oh god no, I don't know what I'm doing. So if you keep D is 12 and then B is 4, and you switch the A and the C it still works. Mmmm OK, scrap that idea, you know we've figured it out we don't need to. So A is 4 , B is 12, C is 3 and D is 4.

Student 233: Is there more than one possible answer, oh god, I don't know. OK, Field 7. Last Field, it's our last Field!

The determination of the group and the positive effect of earlier discussions was evident after they found solutions to the final Field 7. Having felt they had completed the task, they were starting to suspect they had missed something when they were asked how they were getting on. Following this, they proceeded to look for alternative ways to find the solution they had got from using a combination of examining the differences between the totals and trial and error:

Student 233: We think there's another way of doing it but we didn't figure out what it was.

Student 233: Shall we try and find out other ways to do field 6 then. I think there's something strange about it.

7.5.7 Shared understanding of task

Listening to the communication between the group members showed how the students ensured they had a shared understanding of the task. This included disagreements or conflict over both the interpretation of the initial task and the solution methods. This required the student who had mastered the task to articulate their thoughts in a way that the others could grasp and contributed to overall progress for the group as a whole. As expected, many of the most diverse suggestions were made during the first field in the Field of Dreams Task, when the students were experimenting and adjusting to the team dynamics:

Student 335: The first one, it gives you the answer does it not?

Student 324: No

Student 335: I swear it does

Student 324: No, you have to find what's in each one, it gives you the combined one.

Student 335: Wait can you explain that one then?

Student 324: Yeah, you get so you go $A + B = 9$, $A + C = 11$ and then $B + C = 16$. Here, I'll show you (*holds up sheet*).

Student 335: I get it now. You have to find A, B and C given that you know what the combined are. My head is not prepared for this.

On creating their own Field of Dreams, the group dynamics often showed a combination of group work, to help each other understand the task, but also a determination by those in difficulty to give it a go:

Student 129: I have no idea how to do it.

Student 142: But that's not how it works. Here let me show you.

Student 129: Let me try and figure it out.

There were also one or two instances where a student seemed happy to just get the answer from his peers. This tended to only happen in the first task, when these individuals seemed a bit bewildered by the unfamiliarity of the whole group work experience:

Student 335: Who solved the first one? Can you just give us the answer so that we can all move on to the second one.

Sometimes the students were even happy to vocalise their uncertainty and fear of getting something wrong. However, working in smaller groups with peers provided the students with opportunities to take these risks that they may not have felt as free to do in a full class environment:

Student 340: I just want to try something and see if it works then …
because I don't want to sound stupid if this doesn't work.

Such comments may also be seen as evidence of students having pride in their own abilities but being willing to express the fear of being wrong amongst their peers. Another student, in this same group, that had been taking some time to fine tune their thinking, commented after the researcher had just left their room:

Student 341: I feel so disappointing … I think maybe it's so easy that it's difficult.

This type of pressure has been seen to be common amongst highly able students (Scager et al., 2014) and can lead to frustrations when facing a challenging task. The side effects of this can sometimes become an obstacle for such students which will be discussed further in the analysis of the audio recordings related to the Scholarly Principle.

7.5.8 Potential conflict in group dynamics

Within the groups in the workshops, the audio recordings showed an aspect of peer dynamics that would not be typical of a full class situation, where students are not involved in small group discussions. It was evident that some students felt free to speak their mind, to an extent just thinking out loud, and others felt comfortable to express elements of frustration in a constructive manner. In the Area of Circle Task, one group had found the ratio of 2:1 by measuring and then two of the group, Student 227 and Student 230, were trying to confirm it by trying random numbers before realising that this did not prove anything. Student 242 had been remaining quiet during this discussion but clearly knew that it was not going to be productive. The different personalities enabled the group to help each other verbalise their thinking and strategy which eventually led to the solution:

Student 227: OK so πr^2, big *R*. How do we do that though. How do we multiply by *R*.

Student 230: We can't. Mmmm.

Student 227: Come on, contribute.

Student 242: I'd rather say something smart than just repeat ... We knew that was the formula.

Student 227: Yeah, but you have to actually do something.

Student 242: I am doing something, I'm thinking. My brain is working and not just babbling about nothing.

Student 227: I'm not babbling about nothing.

Student 242: You were …

Student 227: It was still right and at least I did something.

Student 242: Well, it kinda went backwards, well you didn't go forwards. (*Chuckles*).

Student 227: I don't get where we are meant to go next.

Student 242: We need to show the ratio using r's.

There was an element of frustration in the above excerpt, that all members of the group did not appear, according to one student, to be pulling their weight. However, the group were able to use it constructively to assess the validity of their discussions.

7.5.9 Learning from each other

There were many situations where one student took the lead with making a conjecture, and it was interesting to listen to the ways in which these students brought along those in their group. Through collaboration, there was evidence of not only students taking risks suggesting their ideas, but also of them feeling an element of responsibility to ensure that they brought their group along with them. This in turn became an episode of learning from peers for the other students in the group. The learning and enjoyment aspect of group work had been a recurrent theme in the interviews through comments such as:

Student 326: Working in a group and kind of bouncing ideas off each other and saying like, "Oh, does this work?" and then trying loads of different things. I enjoyed that challenge.

Below is an excerpt from a group commencing their discussion on the Area of Circles Task that illustrates the learning potential that was evident. One of the group was not contributing much in terms of ideas at this stage, and what she did say was highlighting that she was not grasping those being offered by her peers. The task, as mentioned earlier, did prove to be a challenge for most students. A ratio was found through measurement but having to prove the same using variables brought about much discussion and opportunities for peer scaffolding. This excerpt was an excellent example of the strong sense of collaborative learning that was evident in the workshops. A sample of Student 231's work is shown in Figure 36 below the excerpt:

Student 231: So, if we divide the square into 2 right angled triangles. Do you get it? So, the radius of the small circle is the length of it and the radius of the large one is the hypotenuse. Do you get me?

Student 224: Yeah.

Student 231: So, if you use Pythagoras', what's Pythagoras', $h^2 = o^2 + a^2$, so small circle radius is equal to, say it equals just 'O' ...

Student 224: But, is the radius not half of it?

Student 231: Oh yeah, half of it, so O divided by 2.

Student 224: We could measure these first and then we wouldn't be …

Student 231: No, we're not allowed to measure it.

Student 224: She said we could, to check? Do you have a ruler?

Student 225: Wait now, how are you getting the small circle is 'O' over 2?

Student 231: The diameter of the small circle is the same as the length of one side of the square. I might change that to x over 2 because $'O'$ is confusing.

Student 224: If we measure it, we can just sub in the number.

Student 231: Do you get it?

Student 224: Yeah.

Student 231: Do you get it [Student 225]?

Student 225: Like, if you know it's over 2, for the bigger circle … OK. I get it now.

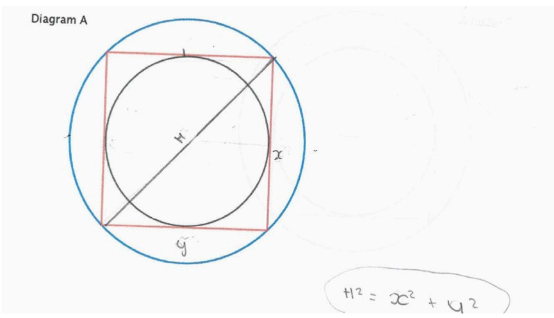

Figure 36 Area of Circles Task, Student 231

7.5.10 Similar Ability Grouping

The groups for the workshops were deliberately self-selected to allow students the opportunity to decide who they think they might work well with. By being in a group of similar ability, peer scaffolding will be interchangeable, and the group members will hopefully feel more inclined to contribute. This was a key aspect of the workshops because the aim was to provide opportunities for peer collaboration, rather than for one student to tutor the others as a teacher might. The benefits of this for high ability students had been highlighted in the literature (Scager et al., 2014) and it had been built into the research design as mentioned in section 3.5. The structure of the group did seem to be important for some students, as highlighted in the interviews previously and from the post survey comments:

> Student 325: The group work was great fun, and I was glad that everyone contributed.
>
> Student 340: It was very enjoyable to be surrounded by people who have the same interest in maths as me, I liked hearing how other people brains worked and how different think to how I do.

From the above audio excerpts, selected to illustrate examples of the Free Market Principle, students could be seen to ask their peers questions and view each other as equals when it came to finding solutions to the tasks. Within the groups very different contributions were made, and although some may not have been acted upon immediately, the students could be seen to give all members an opportunity to be heard and feel valued.

7.5.11 Online Versus Face to Face Workshops

Running the final two workshops online took much coordinating and attention to logistics, to ensure that the desired cooperation was taking place. In most cases the online groups managed the discussion productively and seemed very comfortable ‘chatting’ to each other. Some groups were more pro-active than others, for example holding their work up to the camera for the others to see, but the unfamiliarity was evident in their queries about whether “it will show backwards” to the rest of the group. However, the typical technological issues that accompany online work, such as visuals, sound interference and lack of a physical presence, were evident in the online workshops and were commented on by the groups at the time. Two of the groups had difficulty hearing a student's microphone which hindered discussion until they decided to use the chat box. At times, with groups who were particularly vocal, the students talked at the same time and some useful points were lost. In other cases it felt as though the students who did not use their cameras and could not see each other, were less inclined to discuss strategies than in the face to face workshops. They did make conjectures but there was definitely more individual work done than with other groups. Interestingly, one of these groups included Student 326 who was quoted earlier as enjoying "bouncing ideas off each other", so care should be taken in reading too much into students working more quietly than others. There were more obvious disadvantages of not being able to view each other's work and possibly see errors or good ideas. The importance of visuals in these tasks made this particularly difficult but the camera did help those who engaged with it. The benefits of having the workshops face to face could be seen in simple comments such as:

> Student 242: At least we're going somewhere with it. Can I just see your sheet?

In one group, the details of which will be discussed in the Scholarly Principle section, there was evidence of instances where collaboration online may have hindered progress because the students were moving too fast from an idea that was valid and struggled to verbalise complicated conjectures. There were other dangers of a student getting left out, or not contributing as much, which was more difficult for a teacher to monitor. There were two instances where the researcher could be heard in the audio recordings of the face to face workshops "make sure you are discussing your ideas as a team" or "is everyone contributing" because it was possible to scan the room better to observe what was going on. The equivalent in the online workshops involved quietly entering a room and listening for chat to make sure everyone was there if a student was silent. The levels of student participation were not as easy to identify this way and may have contributed to some students not feeling as involved.

7.5.12 Summary of audio recordings on Free Market Principle

The recordings show that students did have opportunities to take risks in the workshops and that they availed of them. There was evidence that they experimented with different solution strategies and were able to put forward their ideas for discussion. We have also seen how influential the dynamics of teamwork within a group can be. Peer collaboration could be seen to be both beneficial for problem solving and an enjoyable aspect of the workshops for the students.

7.6 Scholarly Principle

7.6.1 Opportunity to explore MST

The key feature of the workshop audio recordings that described aspects of Sriraman's Scholarly Principle was the students' experience of multi-solution tasks. The other aspect in this section of the template, regarding the importance of learning, was emphasised more specifically in the interviews and surveys rather than during the audio recordings of the workshops, where the discussion was focused on solving the tasks. The evidence outlined below shows that the multi-solution tasks were a novel experience, had benefits for their thinking and learning, provided a challenge and were also enjoyable. Below are three of the comments, made in the post-workshop surveys, in response to being asked if they had any further comments to make on the workshops:

> Student 029: The workshop was very enjoyable as it was challenging and different from the maths we do in school. It was very interesting to see that there were many methods to the same question as in school we are only taught one method. I feel more confident with problem solving.

> Student 020: I thought the workshop gave me a completely different view on problem solving in maths. It was different to what we usually do in class and made me think in different ways I'm not used to. I also enjoyed talking about the problems with others and sharing our ideas.

Student 126: I found the workshops very interesting as the problems we were given were different to the ones I am used to getting in school.

7.6.2 Novelty of MST for students

There were many episodes throughout the audio recordings showing that the students were not used to exploring multi-solution tasks in school. When designing the workshops one of the key features was to provide a novel experience that could be compared with the classroom experiences of the students. Most of the comments that emphasised the contrast with school were made during the interviews or in the surveys in response to questions enquiring on the differences. However, during the audio recordings of the workshops, one of the students, who was struggling to get started in the Steel Cables Task, announced without any prompting regarding a comparison with school:

Student 129: It's completely different to what we get in the textbooks.

This feeling of novelty was reinforced in many ways throughout the audio recordings. In the Field of Dreams Task, when the students first experienced the idea of multi-solutions, in Field 6, they were confused that they had different numbers but the same total answer. The idea of doing a task in different ways, but getting the same overall answer, seemed to bewilder some of the students:

Student 239: I got the wrong numbers as well.
Student 242: Did we get them the wrong way around, but we got the right number?
Student 227: OK. Let's check that out.
Student 239: No, I got it wrong.
Student 230: There are different ways of doing it.
Student 242: Did we do the whole thing the wrong way?
Student 230: No, there's just different ways it can be solved.

Student 242: OK. So, in conclusion of that … if you think of different numbers you will have different answers but always get the same total. Oh my god. That's so funny.

Fluency, flexibility and novelty are described in the literature as the 'core dimensions of creativity' (Leikin & Lev, 2013). Multi-solution tasks have been used to distinguish alternative methods for solving tasks, in terms of their level of creativity, by examining the above three facets (Leikin, 2009; Leikin & Lev, 2013). For the students in the workshops, who had not as yet experienced the idea of multi-solutions, it was interesting to listen to their interpretation of what a different solution method entailed. Following the progress of one particular group, solving the What's it Worth Task brought to light the diverse aspects of learning that can be embedded in one task. It did not take the students long to find answers to this task using trial and error but having to find alternative methods raised concerns amongst the group over what was a viable and novel method:

Teacher: How are you all getting on?

Student 127: We're trying to do it a different way. We've done it by simultaneous equations and it worked.

Student 136: Now we're trying something different but it's not working.

Teacher: That's good, keep going.

Student 132: *Begins to suggest a method starting with letting a triangle equal 6, which they had discovered already.*

Student 132: But we can't use the answers from the first method to find another method. We're meant to pretend we don't know anything.

Student 136 then discovered the simple way to solve the task by adding the total columns and rows, which was outlined in the Aesthetic Principle analysis. In addition to the obvious satisfaction the group felt, it opened up a discussion on what the whole point of the task actually was :

Student 136: Couldn't we just add up these totals and these totals and take one from the other.

Student 127: But, the end goal was to find the value of each shape.

Student 136: No it's not. It just says to find the question mark.

Student 127: Oh my god, we are such idiots.

Student 127: So what's the total of the column …

Student 136: This is 96 and this is 75 so the question mark ...

Student 135: The question mark is 21, Oh, my God.

Student 127: We are actually so dumb, I didn't realise.

Student 127: We did it. We got two ways though. I'm proud of it. I can't believe we didn't spot it.

The final part of the What's it Worth Task was to discuss five different methods, for which the first step of each had been given. The students had to explain how the task was solved using each method and to try and agree on which solution they preferred. This looked more straightforward than it actually was and it really brought the whole group into the discussion process. The students really engaged with this aspect of the task and took care justifying the steps involved in each solution. The solutions presented to the students, in the second phase of this task, are shown in Figure 37 below:

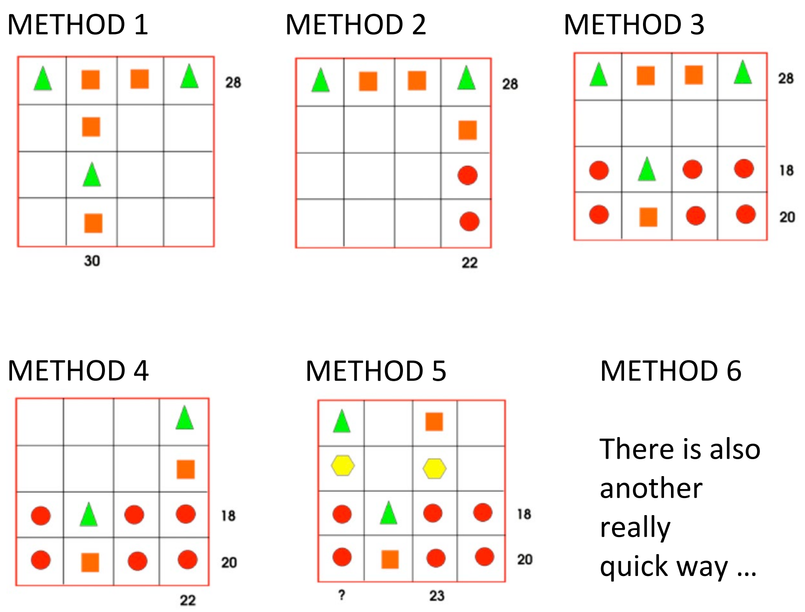

Figure 37 What's it Worth Task, Solutions

Student 136: I think method 5, and we did method 1.

Student 127: Our counting method isn't there.

Student 135: How is this the method to find 21?

Student 127: How do you do method 2, how does that work?

Student 136: It's like our simultaneous equations one.

Student 132: Mmm, maybe it's similar to the first one.

Student 127: No, but there's three unknowns.

Student 135: Dudes, I've got number two. Probably. The square and the triangle is half of that one, so 14. 22 take away 14 is 8 and the circle then, there's two of them so divide by 2 gives 4.

Student 127 was still confused about where their 'counting method', as they called the simple solution, was:

Student 127: But what method is the counting one?

Student 136: Just logic.

Student 127: No, … the method where we just counted them all, the one that we did. Is it not shown here?

Student 132: No. She must have printed this before she realised you could do that.

The novelty, enjoyment and satisfaction of experiencing multi-solutions was evident when student 132 swiftly moved on to conclude, before finding an explanation for the final method:

Student 132: We're like accidental geniuses. I love it. I love discovering things by accident.

7.6.3 Benefits of MST

The audio excerpts show many examples of the benefits of working through multi-solution tasks that the students had emphasised in the interviews and post-workshop survey. There was a definite feeling that working on multi-solution tasks was a learning experience for the students. Observing the students over the course of the workshops supports the research that has shown students will improve through the experience of MST (Matić & Sliško, 2022). In the above episodes, understanding what a novel method is can be seen as a learning experience in itself, but there was also an awareness that the tasks improved the students' skills which will be discussed below. This reinforced what students had said in the interviews about different methods arising from their discussions, and how beneficial it would be to students if they did similar tasks in school:

> Student 231: People created different methods ... I think if we can have a discussion about it and everyone just takes their own favourite method from the discussion it would be way better than just sticking to the same one for everyone.
>
> Student 121: I thought the workshops improved my maths by challenging me to think on my own and not be spoon-fed an answer… In my opinion maths can be interesting for everyone but at the moment we are taught to be robots and learn formulas off by heart instead of learning why a problem is solved that way.
>
> Student 023: I think that these activities forced me to think in a different way. They helped me reason different solutions that I would not usually think of ... Before, I was not encouraged to solve problems without seeing the answers first.

An example where student discussion consolidated one group's understanding of simultaneous equations, can be seen in their progression from the Field of Dreams Task to the What's it Worth Task. The group had, after lengthy discussion, recognised that they could use simultaneous equations to solve the Field of Dreams Task but were confused by the third variable which was new to them. Their eventual method to solve the task was by focusing on the differences between two pairs of fields. However, in doing so they were discussing memorised techniques such as suggesting this was where they "multiply across by something to get zero". They knew they had a variable they did not want but were unsure how to "remove it". The idea of eliminating a common variable to reduce the task to two variables had not yet been grasped. However, in the following episode from the What's it Worth Task, the students used their different representations to build on their Field of Dreams work and help them understand the objective of eliminating variables:

Student 038: I was trying to put everything into terms of T (*where T represented the triangle*). So, $T = S - 2$

Student 036: Oh yeah, that's like we were doing before.

Student 038: Then you can find out what S is.

Student 023: I did mine a really complicated way. So, $S - T = 2,$ the $S - H = 1\ldots$ but I don't know how to get numbers.

Student 038's explanation that followed led to an understanding of how they could reduce the system of equations down to 1 variable which they could use to solve the original equations they all had.

Student 038: … so if you have what T is, then you can find what the others are.

Student 023: Oh, that's really good.

In addition to learning new methods of solving tasks, having to find multi-solutions also developed the students' ability to participate in mathematical dialogue. By having to discuss different solutions, the students were given opportunities to justify solution methods and engage in the process of reasoning. In the Uncertainty Principle and Free Market Principle analysis the students can be seen to explore ideas and make conjectures that they were often unsure about. The audio episodes outline a very different experience to how the students described the mathematics they did in school, where they were usually given a method to follow by the teacher. In addition to providing the students with opportunities to have the freedom to think, the tasks in the workshops were selected to also try and offer wider mathematical learning opportunities. In the audio recordings discussed above, the unfamiliar nature of the tasks meant that the students were forced to think for themselves and reason with their peers. It can be seen in these, and the written worksheets, that for many, the first choice of method was to use trial and error to find solutions. The learning benefit of multi-solution tasks was that it forced the students to think of alternative solutions that were often novel. Even if these were relatively simple solutions in some cases, there were opportunities for creativity that they would not have had if they had a ready-made algorithm for each task.

7.6.4 Opportunities for Creativity

There were also episodes in the audio recordings illustrating that students seemed to embrace the challenge of finding different methods to their peers. Research has shown that multi-solution tasks are a useful tool for both encouraging and measuring the creativity of students (Leikin 2009; Leikin & Lev, 2013, Sheffield, 2013). In the discussions, students can be seen to ask "why" and "what if" to look for unique ways to solve tasks, to persevere and not to be afraid to try a novel method out. All of these have been seen to be characteristics of creativity (Chamberlin & Mann, 2022; Sriraman, 2004; Sheffield, 2013; Sternberg, 2017). The Steel Cables Task in particular had a variety of very different methods so students could find one that appealed to them and encouraged them to persevere. One student, whilst working within his group, was simultaneously trying to find an alternative method (see Figure 38 after the dialogue):

Student 142: Will I try the area of a Trapezium?

Meanwhile, the rest of the group were engaged in discussion about how many would be in each row of a size 10 cable.

Student 129: There's gonna be 10, 11, 12, 13, 14, 13, 12, 11, 10 …
Student 121: Call those out again?
Student 129: Hold on, [student 142] are you doing a size 10 or size 5?
Student 142: I'm just making sure this method [Trapezium] works for a size 5 first. So in this area here there should be 30.5 cables.
Student 121: Student 140, we are stupid. A size 10 cable, it's not gonna be 9 down it's gonna be more than that. It's bigger in every direction.

Fifteen minutes later the group figured out the answer for a size 10 cable, using the second difference between the number of strands in each consecutive size of cable. The above student was still working away on his alternative method in parallel:

Student 142:	I'm trying to do it differently. I want to figure out my trapezium. We need to get to 271.
Student 129:	Your triangle idea was right, but you just didn't cross out the middle.
Student 142:	But the trapezium wasn't right. That's annoying.

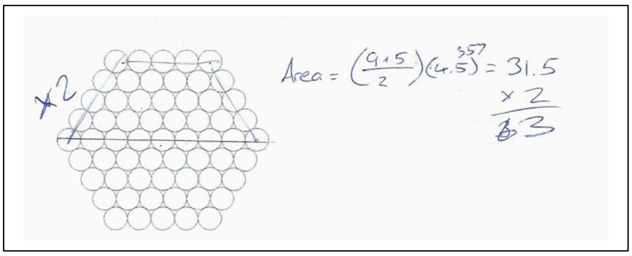

Figure 38 Steel Cables, Student's Trapezium Method

This was a lovely method, that did actually work, and was not one of the solutions given on the back of the sheet. Unfortunately, from Student 142's worksheet it looks like he got distracted and started looking for a sequence rather than checking why he had 63 instead of 61 for the 'Size 5' cable. He had assumed that the hexagon was comprised of two identical trapeziums and overlooked that the height of one trapezium was $n - 1$. However, being asked to try and find different solutions did seem to be the motivator for him to persevere. He was the only student who came up with this idea.

There have been many examples already outlined of students asking, "what if" and thinking of unique ways to approach a task that their peers had not thought of. The students were showing much more perseverance and thinking beyond routine algorithms to find solutions. The multi-solutions in Field 6 of the Field of Dreams Task, seen below in Figure 39, brought all sorts of spontaneous suggestions:

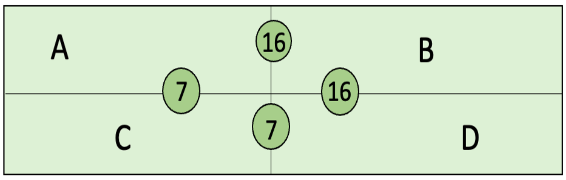

Figure 39 Field of Dreams Task, Field 6

Student 230:	A and B are 16, B and C. There's no B and C. There's zero people. Maybe there's zero people in A, there's zero people in D and there's just 7 in C and then there's zero in the others.
Student 239:	[Student 230] what are you talking about?

When the others realized that this did actually work, the suggestion became the key to the group solving the task and figuring out that C could have any number less than or equal to 7.

Similarly, in the Area of Circles Task, the students were trying to think of a number of methods they could use from their Junior Certificate mathematics. They struggled to remember the required geometry proofs but there was evidence that they were exploring different possibilities rather than just following a method given to them. One group had an interesting approach and just decided to make up a number for the diameter of the large circle and work from there. They had initially been calculating perimeters and areas of the square when one student came up with an idea:

Student 120:	Here, so we said the diameter of this is 12 so this is 6.
Student 133:	Did you just make that up?
Student 120:	Yep.
Student 141:	So like, that's the radius of the small circle. So, … that is 4.2 to 12 is the ratio.
Student 120:	Sorry I'm lost. How did you do that?
Student 141:	So, … diameter is 12 and since it's 90° and 45° I put it into Pythagoras'.
Student 120:	Ohhh, Pythagoras'!

They went on to find the correct ratio working from the "what if" the larger circle had a radius of 6 and working backwards, using Pythagoras', to prove the ratio of the areas must be 2:1.

7.6.5 Multi-Solution Tasks are more Challenging

In the post-workshop surveys students acknowledged that the multi-solution tasks were time-consuming and challenging and something they were not used to in school:

> Student 420: It was really good to challenge my brain in thinking outside of school type maths.

> Student 027: I thought the workshop was good to get us thinking about different things. It was challenging to think of more than one way to think of something. This was because I usually don't need to solve it twice.

> Student 120: Made me think of maths in a different way. I now have many different methods from other people. I enjoyed thinking about the questions for so long.

> Student 122: I enjoyed participating in the workshops. I found them very challenging, but I think they will help me to think about maths problems in a logical way in the future.

There was evidence from the audio recordings to back up the post-workshop survey and interview data that the students found the multi-solution tasks challenging. In the Gestalt Principle analysis we saw that the tasks took a longer time to solve than the students were used to, requiring perseverance and effort. The multi-solution aspect was a key factor in the challenge because there was usually a straightforward method to find an answer initially. Having to restart the thinking process and find another way to solve each task was a challenge for many. In some instances, small errors may have made the tasks frustrating, depending on whether a solution was eventually found or not. You could sense in the recordings that the students felt they were having to work hard to answer what was required of them. After much perseverance and constant discussion for over an hour, in the Area of Circles Task, one group sounded drained but did keep going for another 45 minutes. They were trying to prove the ratio for Diagram A but had a small surd error and didn't manage to find it because of the error.

Student 136: I'm exhausted.

Student 127: I'm so tired … I still think the thing about Pythagoras' is gonna work though.

Another group were struggling with the nature of the Field of Dreams Task*,* where finding the answer was not all that was involved. First, they debated the interpretation of what it meant to justify their answers:

Student 233: I don't know how I got 7 for B, I just kept going through numbers. It's not that high of a number. There must be like an actual way though.

Student 240: You're meant to prove it though aren't you?

Student 234: She said there's a way a 10 year old could do it?

Student 233: As a 10 year old I'd just leave it though, I wouldn't be trying to prove it, so that must be ok.

Then, on reading Field 6, the idea of more than one answer seemed perplexing to the whole group initially:

Student 233: Oh lads … there's more than one possible answer.

Student 234: But there mightn't be.

Student 240: But there probably is.

Student 234: That could be a trick question.

As mentioned in the Gestalt Principle analysis, in some situations students made the tasks more difficult than they could have been by missing key points other students had made. However, the challenge seemed to make the task more enjoyable for the groups rather than less so. In the group discussion below, Student 340 had decided, after solving Field 1, that there had to be a systematic formula to solve the Field of Dreams Task. She was determined to find it so much so that she created very complicated algebra for herself. During the discussion one of the group mentioned simultaneous equations but did not pursue it for some time:

Student 341: Can you solve it simultaneously, so, like $A + B = 12$, $A + C = 11$, $C + B = 7$ and then you move $B = 12 - A$

Student 340: Yeah, that's the only way that makes sense.

There followed five minutes of silence before Student 331 asked "have we all got it?" to which Student 341 and 340 replied "it doesn't make sense".

Student 341: I think I was wrong what I suggested.
Student 340: But it makes sense to do it that way. It's just not working.
Student 331: I have the answer.
Student 331: What did you say?
Student 331: I said $A = 8, B = 4$ and $C = 3$.
Student 340: How did you do that, though?

When Student 331 explained he had used trial and error it was obvious that Student 340 had been trying to find a solution by simultaneous equations. Her worksheets showed that she had been working relentlessly with three variables but had not managed to eliminate one of them which had been causing the confusion.

Student 340: So you didn't do simultaneous equation? … But that's it. I feel like the simultaneous thing should work on a larger scale if you know what I mean, without having to sub in. Take off from here…
Student 340: Wait, what if we multiply $A + B$ by $A + C$? And then … It's something to do with simultaneous equation with A and B and C all in the same question. I'm just fiddling around with things trying to get an answer.

The multiplication suggested by Student 340 resulted in a quadratic equation, which was then combined with one of the linear ones to produce an expression that the student could not solve. It was another 30 minutes before the technique became apparent and this student figured out how to eliminate one of the variables systematically, which would enable her to find the method for solving 'larger numbers' as had been her goal throughout.

7.6.6 Enjoyment of Multi-Solution Tasks

Although there were no specific comments in the audio recordings relating to enjoyment, as they were concentrating on solving the tasks, it was clear from the general good-humoured chatting and laughter that they were enjoying the experience. There were, however, many comments in the interviews indicating that the students were enjoying having the opportunity to explore multi-solution tasks:

> Student 425: When I asked my friends, the number one thing that they don't like about maths is that it's boring. I think adding these questions will be really beneficial because it'll just be more interesting. Being able to learn or being able to think for yourself, work in groups, and work together, I think that would be really beneficial.

> Student 430: I think that this one gives you more freedom to say what you think rather than just one thing to by. You can do it in different ways which, I preferred that just the more direct one.

When asked which tasks they preferred there was a wide range of answers but no one task stood out as the most popular. Some people preferred the Steel Cables Task because they found it more challenging whereas for others the enjoyment was looking at the different solutions shown.

7.6.7 Summary of audio recordings on Scholarly Principle

The main focus of the Scholarly Principle in the analysis of the workshop recordings was on MST. The recordings show that these tasks were new to the students and that they found them both challenging and enjoyable. In particular, they observed that seeing other people's solutions and discussing different methods led to learning opportunities for them.

7.7 Summary of data from workshop audio recordings

The audio recordings provided much evidence that the students had opportunities to develop their creativity in the workshops. Because of the nature of the discreet recording devices, the students felt free to speak and engage in discussion in a way that they may not have in a full classroom environment. The relaxed manner in which the students spoke, the laughter, the uninhibited discussion and conjecturing that was evident, all contributed to providing a transparent account of their positive experience in the workshops. That this data was aligned with the data from the surveys and interviews gives support to the belief that the students found the workshops a more challenging, enjoyable, and rewarding experience than they had had in school previously. The essence of this research had been to try and capture the perceptions of able mathematicians of their experiences, an area that has had limited research in Ireland to date. These audio recordings provide strong evidence that the students felt they had opportunities to experience incubation, challenge, uncertainty, peer collaboration, and creativity in the workshops and that it was an unfamiliar, but enjoyable, experience for them.

Chapter 8 Discussion

The data presented in the surveys, interviews, and audio recordings from the workshops highlights many interesting features of the experiences of the students in this study. The discussion below will examine the evidence from the data to support the research questions:

1. What is the perception of able students in post-primary schools in Ireland towards their mathematics classes?
2. To what degree do these students feel challenged by the experiences they encounter in the mathematics classroom?
3. What comparison can be made between the opportunities these students had, in school and in the workshops, to experience each of Sriraman's five principles to maximise creativity.

The discussion will assess the students' perceptions of the differences between their experience in mathematics class and that of the workshops. The results will be discussed in relation to the research literature on these topics.

The findings suggest that the majority of students in the study are highly able and thus provide evidence suitable for the intended research. When deciding upon how to select the students for the survey the researcher wanted to ensure that students who were keen mathematicians but had not necessarily been the highest achievers in their school year, were free to participate. Perhaps because of this decision there were a few students who seemed to struggle with the level of difficulty of the tasks. However, overall the data does appear to reflect the perceptions of students who were both able and motivated. The pre-workshop survey revealed that 63.8% of the total 92 stated that they rated their mathematical ability as above average in their school year and 25.9% were undecided, 79.3% enjoyed the challenge of mathematics and 91.3% were confident in their ability to learn mathematics.

The workshops had been designed to provide opportunities to challenge the students using multi-solution tasks that encourage creativity. The choice of tasks was guided by research on challenging high-ability students that emphasised the importance of creativity (Leikin, 2009, 2010; Liljedahl, 2009; Luria et al., 2017; Mann, 2006; Pehkonen, 1997; Schoenfeld, 1985; Sriraman, 2005; Sheffield, 2003). It was evident from the analysis of the post-workshop survey and the interviews that the students enjoyed the workshops and felt challenged by the tasks. The discussion on the first two research questions will examine what the students said about their classroom experiences and compare it to that of the workshops. This will include an evaluation of what they said about the learning opportunities and the level of challenge provided by each. In the discussion on the third research question, the evidence will be broken down to compare the opportunities the students had, in school and in the workshops, to experience each of the five principles to maximise creativity as outlined in Sriraman's framework (Sriraman, 2005).

8.1 Research question one: What is the perception of highly able students in post-primary schools in Ireland towards their mathematics classes.

As outlined in the literature review there has been much research on the needs of able mathematicians and how they might be met in the classroom (Ellerton, 2013; Leikin & Lev, 2013; Mellroth, 2008; Pehkonen, 1997; Sheffield, 2003, 2017; Tomlinson et al., 2003; Tomlinson, 2018). By examining the perceptions of the students to their classroom experience we get an insight into what they enjoy and what they feel they need.

When we examine the research literature in this area it can be seen how influential the classroom experience can be on the students' views on the objective of mathematics (Nosrati & Andrews, 2022). Students are conditioned by their experiences in mathematics, and unless they have been involved in mathematics outside of school, their experience in class is all that they know of what mathematics is (Boaler, 2016; Lampert,1990; Schoenfeld, 1985, 2016). In addition to shaping their perceptions about mathematics, Hiebert (2003) claims that the activities students are engaged in during class are instrumental for determining what they can actually get to learn. Given the importance of fostering creativity and conceptual understanding (Schoenfeld, 1988; Sheffield, 2009; Sternberg, 2017), activities that foster these should therefore be an integral part of all classroom instruction.

8.1.1 How the students view mathematics

The Aesthetic Principle analysis included several important aspects of what mathematics meant to the students. There was a clear distinction in the emphasis of the students' interview discussions on their school experience and that of the workshops. The data revealed much evidence to suggest that, for these students, mathematics is all about learning methods for the exams in school. However, in the context of the workshops their discussions were focused more on understanding the process and the exploration of tasks. One student's interview comment on school mathematics seemed to sum up many others: 'if you can remember the methods, remember the numbers, remember how to do everything then you'll be grand' (Student 324). The data from the audio recordings also provided episodes showing how reliant students were on formulae at the outset of the workshops. In the workshop audio recordings there were frequent comments similar to 'there's no formula is there?' and 'there has to be a maths way of doing this' (Student 429), that reflected a procedural outlook to solving tasks. The paragraphs below outline the predominant characteristics of the classroom that were described by the students in this study.

8.1.2 Reliance on the textbook

The analysis of data relating to the Gestalt Principle highlighted the importance of the teacher's preparation being suited to the ability of the students. In both the pre-workshop surveys and interviews, students spoke of the teachers' reliance on the textbook in class. Research literature shows that current textbooks in Ireland predominantly encourage the routine learning of algorithms at the expense of higher order tasks (O'Sullivan, 2019). Given that in Q24 (see Appendix D), 86.2% of students agree or strongly agree that they work mainly from the textbook in class, it appears that their mathematics experience may be overly focused on routine tasks rather than complex ones. It is clear from the student responses to Q37 that they felt a strong connection between thinking for themselves, and being challenged, and that having opportunities to think was important to them. Both the survey and interview data support the view that the overuse of textbooks plays a key role in removing both challenge and opportunities to think for the students.

Reliance on the textbook may also contribute to the evidence suggesting that the classroom practices appear traditional in both methodology and content. This raises concerns when we consider that researchers such as Pólya (1990), Schoenfeld (1988) Boaler (2002) and Leikin and Lev (2007) have advised of the inherent dangers for students of features of traditional mathematics teaching such as a reliance on procedures. Schoenfeld claimed that 'most textbooks present "problems" that can be solved without thinking about the underlying mathematics, but by blindly applying the procedures that have just been studied' (Schoenfeld, 1988, p.163). Schoenfeld found that the most significant risk attached to such procedural learning is that because students can get the correct answer without understanding the mathematics, they do not feel the need to try. The results discussed below suggest that this was a familiar experience for the students in this survey. Given that they expressed a preference for more challenging tasks, that made them think, the lack of stimulation and subsequent boredom correlates with what the literature had warned.

8.1.3 The importance of uncertainty

Providing students with appropriate uncertainty is often a challenge for teachers. It has been acknowledged that highly able students have different needs to those with average ability (Diezmann et al., 2002; Freeman 1998; Gallagher et al., 1997; Kanevsky et al., 2003; Tomlinson, 2003; Scager et al., 2014; Sousa, 2009). Despite these recommendations, research on the experience of high-ability students in school mathematics class shows that they are under-stimulated by most tasks presented to them (Mann, 2006; Reis & Renzulli, 2010; Sheffield, 2003). This is not surprising given that meeting the needs of able mathematicians in diverse classrooms can be problematic for teachers (McGrath, 2017; Cross et al., 2018). The data revealed that having to think of a method to approach the tasks in the workshops was considered challenging for the students in this study and could therefore be a useful differentiating tool for teachers. For Sriraman (2005), the importance of the Uncertainty Principle is that getting frustrated creates challenges that make students better mathematicians. In addition to providing challenge, Sriraman (2021) believes that "uncertainty is both a catalyst and a necessary condition for creativity" (Sriraman, 2021, p.6). The findings from the data provide evidence that the students experience traditional teacher-led mathematics instruction with minimal opportunities for uncertainty contrary to what is recommended in the literature above. The data relating to how uncertainty was removed for the students will be discussed below. The discussion in section 8.3 will consider how the uncertainty provided by the tasks and the environment in the workshops differed greatly to the students' experience in school.

8.1.4 Uncertainty being removed

The analysis of the survey data and the workshop audio recordings showed that students were aware of the importance of opportunities for encountering uncertainty but that they lacked opportunities to do so in their classes. This lack of thinking has important consequences because students may not consider alternative methods or engage in reasoning and justification of their work (Lithner, 2007). This in turn will have an effect on their mathematical potential because learning how to apply mathematics to unseen situations will be difficult for students trained in this way (Cooney, 2001; Schoenfeld, 1985). There were no comments in the interviews relating to the removal of uncertainty in the workshops and the surveys and audio recordings from the workshops indicated the same.

8.1.5 Shown method first

One of the most noticeable findings that emerged from the data was that the students felt their mathematics class was predominantly example led. This goes against advice in the literature that highly able students need to approach learning by exploration where they are discovering new knowledge rather than learning procedures (Gruber & Mandl, 2000; Milgram & Hong, 2009; Mc Clure & Piggott, 2007; Sousa, 2009). We saw that 93.1% of the students in the pre-workshop survey said that they were rarely given problems to do on new topics without being given an example first and 86.2% stated in the post-workshop survey that they were normally given an example of a task first, in school, so did not have to think of the method. This survey data was backed up by the 243 comments recorded in the interviews under the lack of uncertainty in school. Out of this total, 93 comments were related to being shown the method first by the teacher before they had the opportunity to try the questions. This was either in the form of the teacher giving an example to explain the method or else going through an example in the textbook. The quote from Student 046 that ‘you are just shown the method, and then you just do it, and then you’ll just keep practising the same questions’ summed up how many other students described a typical mathematics class. The only variation seemed to be Student 126’s description of going ‘through an example in the book and then the teacher does an example on the board … and we go off and do our own questions’. The outcome of each was that the students often did not have to think for themselves and were therefore denied the opportunity of having to make choices they were unsure of. The evidence supporting the lack of uncertainty in class was reinforced by the students’ unfamiliarity of having to think of a method in the workshops. In one of the online workshops, Student 335, who joined slightly late, sounded very surprised as he asked his group ‘has she not even told us what to do?’. The challenge provided by having to think of methods will be discussed under research question 3, where the data from the audio recordings highlighted how unfamiliar this was for the students.

As mentioned in section 8.1.2 students from all schools agreed with each other that the textbook was the primary resource in class. ‘We go through an example in the book and the teacher does an example on the board’ describes a pedagogy that is more teacher led than grounded in the importance of students thinking for themselves. The use of the textbook seems to have been a major factor in removing uncertainty in school. The students seemed to agree that the textbook questions were procedural, lacking in uncertainty, and did not encourage students to think of creative methods themselves. They used expressions like ‘they don’t really get you to think’, ‘you just use a procedure’ and ‘the chapter is based on a certain method’ to describe their experience of using the textbook. Such comments display little evidence of students experiencing uncertainty as is recommended by the research literature above. Instead, the data from the classrooms displays an environment with little opportunity for students to experience mathematical enquiry, uncertainty or creativity.

8.1.6 Question method obvious

The students’ descriptions of the questions that they work on in class bear the hallmarks of tasks that require imitative reasoning (Lithner, 2008) and aim to foster procedural fluency (Kilpatrick et al., 2001). There was little evidence of higher-order components of mathematical proficiency such as adaptive reasoning or strategic competence. There was also much evidence to reflect that after having started a new topic the students felt that the tasks posed in class were procedural and did not require thinking. In this case the style of question presented to the students was responsible for removing opportunities for uncertainty. It is recommended that providing a sufficient level of uncertainty for highly able mathematicians will require tasks that are open ended (Mason, 2004; Milgram & Hong, 2009; Nolte, 2012; Pehkonen, 1997; Sheffield, 1999, 2003) and less procedural in nature than those that might cause uncertainty and challenge for less able students. If the tasks are procedural and without uncertainty it has been acknowledged that highly able mathematicians may find them too easy and might struggle to maintain concentration and motivation (Korte, 2019; Pehkonen, 1997).

In the pre-workshop survey 93.1% of the students stated that most of the problems they do in school can be answered by recall of examples and formulae. In the interviews there were 147 comments describing situations where the question method was obvious to the students either because it was routine or that they had done so many of the same type that it did not require thinking. These included 53 specific comments on the 'repetitive' nature of the students' classwork, many of which expressed frustration and reflection on the danger of repetition 'dulling your mind' (Student 340) because of 'doing the same thing, over and over again for the whole class' (Student 131). Such comments were also aligned with students feeling that they were wasting their time and not achieving their potential.

8.1.7 Boredom

Research suggests that the needs of different students vary within diverse classrooms, and that able mathematicians have different needs to average ability students (Diezmann et al., 2002; Freeman 1998; Gallagher et al., 1997; Kanevsky et al., 2003; Wallace 2000; Tomlinson, 2003; Scager et al., 2014; Sheffield, 2003, 2017). This literature maintains that these students differ from their peers in that they work faster, have better memories, prefer to use a variety of problem-solving strategies and are much more likely to get bored in class if they perceive the work as unstimulating. Boredom in the school classroom was one of the key areas highlighted during the analysis of the interviews in this study and it also featured in the pre-workshop surveys. The research in this area highlights the need to challenge able mathematicians because of the evidence that they will be more creative and motivated if they find the work interesting, (Brunner, 1961; Collins & Amabile, 1999; Torrance, 1995). Csikszentmihalyi et al., (1993) write that students who have enjoyed challenge in their early years tend to seek similar motivation in adolescence. For this reason, getting a balance between student interest and readiness should be an essential part of class planning to maximise creativity as they progress through school.

The recurrence of the interview comments relating to boredom suggests that the students in this study were very aware of the diversity in their school classes. The language they used indicated elements of frustration that the pace of the class was slow, and that they had to wait 'until the class moves on' as though it was one homogenous body. As mentioned previously, the students showed empathy for their teachers in diverse ability classes, and were reflecting on how it could be improved. There were suggestions that whilst 'waiting around' for the teacher to explain solutions to less able students, the more able students could be given extra challenges. This enjoyment and preference for thinking will be examined in more detail in the next section, but it can be seen that the students feel that the class preparation does not suit their abilities. There were 18 quotes in the interviews that the questions given in school were easy and no student spoke of being given time-consuming questions that made them think or about having moments of illumination and satisfaction from perseverance.

8.1.8 Focus on exams

In contrast to enjoying the process of mathematics in the workshops, there were 130 comments describing the focus on exams in school. These included an emphasis on getting an answer, 'moving on' and a lack of alternative methods in school. There was no evidence in the data to suggest the students felt that they explored mathematics related to real-life in school. The interview comments related to focusing on exams often showed an element of frustration by the students that there seemed to be 'no purpose' to mathematics in school. Several students used phrases such as 'you just have to learn stuff off for exams' (Student 046) to describe their classroom experience yet the word 'exam' was never mentioned in any data relating to the workshops. It was clear that while the students appreciated the importance of the exams they wanted to understand the mathematics behind the classes better. One particular comment expressed rather harshly what others had also implied: ‘I hate that it’s so exam focused … the teachers are only teaching for that’ (Student 046).

There is much literature that supports the belief that if students focus too much on certainty and the importance of getting the correct answers to tasks their mathematics potential for the future will be sacrificed (Schoenfeld, 1985; Haylock, 1997; Diezmann & Watters, 2002). The multi-solution tasks in the workshops were selected to help students move away from focusing on ‘the correct answer’ or exams by encouraging inventiveness (Goldin, 2009). It was satisfying to hear a response, in the workshop audio recordings, to a student suggesting they look at the answer first: ‘No, that would be cheating. It’s not about the answer, it’s about the methods’ (Student 121). This comment summed up the definite shift in focus of the students as the workshops developed. When asked in the interviews what changes the students would like to see in school mathematics, making it less exam focused and more related to real life was a frequent suggestion. There were 12 comments in the interviews acknowledging that the workshop tasks were more related to real-life and several comments suggesting that more focus on the practical side of maths in school that could help you in later life would improve their learning experience.

8.1.9 Focus on learning methods

From the data it appears that the classroom instruction for these students takes the form of what Lithner (2008) described as ‘imitative reasoning’ where the students either memorised complete methods or recalled algorithmic strategies. In both cases there was no ‘creative reasoning’ involved, where getting the answer would require the student to pose arguments and justify solutions. For tasks to provide opportunities for creative reasoning it is important that the students have not practised a similar task and have to make decisions about strategy choice (Lithner, 2008). There was no evidence of this in the classroom data analysis as discussed in section 8.1.5 and 8.1.6. Cooney (2001) also writes of the ‘reductionist view’ of education where teachers take on the responsibility of learning and focus too much on making the topic easy to memorise rather than ensuring the students understand the concepts. Hourigan &

O'Donoghue (2007) concur with the dangers of this 'reductionist view' of mathematics and the limitations it can put on students' learning potential and beliefs. The data from this study suggests that these classroom practices are still embedded in post-primary mathematics education.

In the interview there were 63 comments related to learning off methods in school mathematics and none for the workshops. For a small number of students there was a sense in the interviews of reassurance from the familiarity of routine questions and memorisation rather than having to think for themselves. However, this tended to be from students who appeared to have low confidence in their ability and said they 'preferred to be told what to do' and liked school maths because 'the questions are always the same' and 'because it's been so drilled into my head' (Student 043). Such comments are also important because they reflect what most students in the survey were saying about their classes in school. It also mirrors the warning mentioned earlier from Schoenfeld (1988) of the danger that a teacher's instruction may be seen as a recipe for completing tasks. Tasks where students have to think about which method to employ are crucial for the development of their metacognition, confidence, and ability to transfer the knowledge to different situations (Boaler, 2009; Schoenfeld, 1985; Stillman et al., 2009).

The data from the survey responses did show high levels of indecision, by the students, on the usefulness of learning off methods. If this is how the students were being taught in school they most likely did not have much experience of alternative tasks and the associated suitability for able mathematicians. While 69% believed they would do better in mathematics assessments if given more problems that did not rely on memorizing notes, 43.1% agreed that they would like to be given more assessments which did not rely on learning classwork by rote but 36.2% were undecided. The idea that learning off methods is beneficial can also be seen by the 64.2% in the pre-workshop survey who believed that 'the best way to improve at mathematics is to do lots of similar questions on the same topic until I perfect the method for solving them.' This sheds interesting light on what the students believe it means to improve at mathematics.

8.1.10 Lack of alternative methods

Developing creativity through exploring alternative methods did not feature in the evidence on school mathematics. Discussions on the use of different methods raised 39 comments suggesting that students preferred, or tended to use, algebra and 10 comments indicating that there was little emphasis on visuals in school. It appeared from the interview comments that teachers placed much emphasis on algebra, to the extent that one student used the phrase 'it's just beaten into our heads to use algebra and only algebra' (Student 021). The over emphasis on learning algorithmic techniques has been seen to be detrimental for students' creative potential, although it has been acknowledged that procedural competency and knowledge is required to expand solution methods available to students (Tabach & Friedlander, 2013). Research carried out by Pehkonen (1997) suggested that the key to developing spatial awareness in the right hemisphere is allowing students to engage in tasks where they must explore problems, use visuals and come up with their own solution methods. Finding a balance between tasks that test procedural knowledge and creativity is crucial for motivating students. They require both the procedural fluency to ensure problem solving success along with opportunities to engage in creative mathematical explorations, and these should be taught in parallel (Pehkonen, 1997; Sternberg, 2017).

8.1.11 Classroom atmosphere preventing risk-taking

The lack of opportunities to conjecture in the classroom was evident in the 77 comments the students made in the interviews that suggested the class atmosphere was not conducive to risk taking. The underlying reasons for the lack of freedom seemed to be the style of instruction and lack of uncertainty discussed in sections 8.1.4, 8.1.5 and 8.1.6. We saw, for example in Q38, that 93.1% of students agreed that most of the problems they were given in school could be answered by recall of examples. The lack of uncertainty in school mathematics classes meant that there were no opportunities for the students to have the freedom to think for themselves and take risks. This is backed up by the previously mentioned classroom features that point towards teacher-led instruction. In the interviews, Student 324 described how the teacher would give an example of their method to solve a question and even if students come up with their own way to do it, 'they want you to stick to the same method' that the teacher showed.

This was mirrored in similar comments by students from other schools all suggesting that only one method to solve a task was allowed, and that had to be the teacher's method. Students were not only encouraged to follow the teacher's method but there was also considerable evidence to show that they were discouraged from experimenting with other ways to solve tasks. Examples in the interviews to illustrate frustration over this style of instruction included 'she kinda asserts her dominance' (Student 131), 'no, that's not how you do it … this is the way you do it' (Student 340) and 'if you don't do it the way that you were told it's counted as wrong' (Student 425).

8.1.12 Compliance

There were other situations where the class atmosphere prevented risk taking because of the compliance that seemed evident in the teacher-student relationships of the schools in the study. One of the dangers of traditional mathematics instruction, where students are taught to learn off algorithms, is the resultant expectation of the students that providing the method is the responsibility of the teacher (Schoenfeld, 1992). Even though the students spoke strongly of not being allowed to try alternatives there seemed to be an established tradition of the teacher taking the lead in the classroom. There were 18 comments related to this in the interviews and they were spread across all five schools. Rather than posing questions to the teacher or suggesting alternative methods, it came across in the interviews, through comments such as: 'we just stick with that method' (Student 231) and 'we just accept it for what it is' (Student 020), that there was a 'norm' that the students complied with. A lack of teacher-student interaction has been shown to be a common feature of traditional classrooms (Boaler, 2003; Hourigan & O'Donoghue, 2007) where the students' role is seen to be as receivers of knowledge passed on by the teacher. The evidence in both the interviews and audio recordings from the workshops reflects this view that the students see the teacher as the provider of knowledge and the students see themselves as 'learners'. One of the responses to a question on the opportunities to discuss mathematics really highlighted this view of mathematics: 'the type of questions we'll be doing where there's examples, there isn't really much to discuss. It's straightforward'. Similarly, comments such as 'we have this in our notes. Let's do the question like this then' or 'we were taught just to find one formula that works and remember that' (Student 430) show a very different type of thinking to that which the students described in the workshops. This atmosphere of compliance creates an environment that does not foster creativity and limits both the learning opportunities for the students and the potential positive affect that comes from exploration (Sriraman, 2005; Boaler, 1999, 2003; Goldin, 2009).

8.1.13 Summary of discussion on research question one

We have found evidence in the data analysis, in relation to research question one, that the participants in this study described feeling that they are missing out on learning opportunities in their school classroom. They reported that their mathematics classes are preparing them for examinations but are not giving them the chance to develop their full potential. They also commented that they feel held back by aspects of their classroom experience and view the objective of school mathematics to be success in state examinations.

The data highlighted the students' observations that the approach to teaching and learning mathematics in their classrooms was not conducive to an in-depth understanding of the concepts. In the interviews there were 60 comments related to the lack of learning in school mathematics class. One example of these, like others, was spoken with a tone of frustration: 'at the moment we are taught to be robots and learn formulas off by heart instead of learning why a problem is solved that way' (Student 121). Skemp (1978) described this type of learning as instrumental understanding, or 'rules without reasons' and emphasised that it limited students' potential and confidence to approach unseen tasks. Other student comments backed up this concern about the repetitive learning of rules in mathematics class versus what they required for real life: 'It's easy to get a good score in the exam, but it isn't very good for actual education' (Student 340) and 'you are not really learning much' (Student 131).

Sriraman (2021) advocated the need to provide high ability students with tasks that promote uncertainty to encourage freedom of thinking and risk taking which are all features of creativity. The data arising from this study does not reflect classrooms where students are being encouraged to think for themselves and take risks. In order for students to experience such freedom they require more unseen tasks where they do not have a readymade memorised solution method to turn to. They also need to be in a classroom environment that encourages discussion and the exploration of ideas and provides the time to do so. In contrast, the survey and interview data show that students have very few opportunities for collaboration with their peers in the classroom.

The tasks prepared by the teacher are example-led and heavily reliant on the textbook. They do not seem to suit the ability of these students and offer no opportunities for uncertainty, risk-taking, incubation or illumination. The result is that the participants sometimes find their mathematics classes boring, and that they are often waiting with no work to do. In the interview discussions there was concern that the teacher's focus on the less able students in the class was responsible for them 'falling behind': 'You can fall so far behind if one person doesn't understand one thing' (Student 134) and similarly, 'they might spend nearly the rest of the class going through that one question, explaining it' (Student 226). The students saw falling behind as preventing them learning because 'if one person doesn't understand the question, then it just holds back the whole class, and you can't learn more' (Student 134). This lack of challenge experienced by the students will be discussed under research question two.

8.2 Research question two: To what degree do these students feel challenged by the experiences they encounter in the mathematics classroom.

With regard to how challenged the students in this study feel in class, the frequencies from the pre-workshop surveys show strong evidence that both the girls and boys in each of the schools had an acute self-awareness of feeling under challenged. Individual survey questions and interview responses highlight the repetitive nature of their lessons which appear to be limiting the challenge opportunities for these highly able mathematicians. The pre-workshop survey showed that only 12% of participants found their schoolwork demanding, with 70.7% stating that they were often bored while the teacher explained the solution to other students. In the pre-workshop survey, Q37 (see Appendix D), 70.7% of the students said they would like to be given more challenging questions that made them think for themselves. The high person measure scores in the Rasch analysis for this survey show that this lack of challenge was consistent across all five schools, with School 2 feeling this most acutely. The data from the interviews suggests similar thoughts with the students making 50 comments relating to feelings of boredom. These statements on boredom were connected to 36 other comments in the interviews which indicated that the students believed the teacher focused more on the less able students than ensuring that the highly able were being challenged. They spoke of being left to work on their own while the teacher spent most of the time helping those who were struggling. Phrases such as 'I find myself bored', 'just sitting there', 'twiddling your thumbs' and even 'falling asleep' were used to describe the experiences of these able students whilst they waited for further instruction.

8.2.1 Importance of incubation and illumination

Time-consuming questions that are unfamiliar to the students play an important role in class preparation that aims to provide challenges for able mathematicians (Diezmann & Watters, 2002; Sheffield, 2003). Research has shown that "it is only when the task becomes sufficiently problematic that gifted students have the opportunity to engage in productive mathematical activity through higher-level cognition, and to develop and demonstrate intelligent behaviors" (Diezmann & Watters, 2002, p.15). Such questions encourage perseverance which can lead to illumination and subsequent feelings of reward and satisfaction for the students. The importance of students experiencing the AHA! moment after overcoming difficulties has been well researched (Krutetskii, 1976; Liljedahl, 2004, 2009; Mellroth, 2004; Taylor, 2009). Liljedahl claimed that AHA! moments can be instrumental because they had the 'power to transform attitudes and beliefs towards the learning of mathematics' (Liljedahl, 2004, p.232). For this reason, selecting tasks that challenge students sufficiently to push them into their ZPD where they can experience illumination can be seen as an important aspect of a teacher's class preparation for able mathematicians.

8.2.2 Traditional teaching methods

The Feeling Challenged scale data showed strong evidence that the students in all five schools concurred with the belief that they were under-challenged in mathematics class. In response to Q31 in the pre-workshop survey, only 13.8% of the students said they find mathematics class challenging. When viewed in parallel with their comments on the reliance on the textbook it suggests that the students do not feel challenged by the questions in the textbook because of the style of question and repetition. There were 24 interview comments relating to reliance on the textbook to back up the survey data, all of which suggest that the textbook questions do not force these students to think. The students described a typical mathematics class as going through an example from the book then being given ‘loads more questions’ which were ‘basically the same question in different words, over and over again’. There was a feeling throughout the interviews that the questions in the textbook ‘all follow the same formula or way to do it' and do not provide opportunities for perseverance and illumination. When asked in the pre-workshop survey if they felt the mathematics they did in class suited their ability only 36.2% agreed.

8.2.3 Comparison with challenge in the workshops

In contrast, the post-workshop survey results show that the students felt much more challenged by the tasks posed with 88.4% of the students feeling that the workshops pushed them more to their maximum ability and 96.6% feeling that the multi-solution tasks stretched them more than just finding one answer to a task. Two students mentioned that the first task of the workshops, the Field of Dreams, was easy but there were no interview comments referring to boredom in the workshops. One of the most noticeable results from the interviews in this aspect of the data were the 98 comments referring to enjoying the challenge of thinking in the workshops. It was evident that the students recognised how these challenges improved their thinking skills with 97.7% believing that the skills required to solve the tasks in the workshop forced them to think deeper about the problem. The students spoke of having ‘to be much more aware of everything that’s going on’ in the workshops, that the tasks ‘trained your brain to think differently’ and that you had to be ‘constantly thinking’ and were not just ‘tearing into a question’ like in school. These comments described a much more active learning experience than school mathematics appeared to be for highly able students.

8.2.4 Literature on MST

One of the key features of the workshops was the use of MST to provide challenges for the participants. MST have been shown to both stretch students in having to think of more than one method and provide opportunities for creativity by doing so. The originality of the students' work, relative to their peers, is an essential component of creativity (Liljedahl & Sriraman, 2006; Leikin, 2009; Mann, 2006). For students to get opportunities to be creative they must therefore be given the freedom to explore their own solutions to open ended problems rather than following a prescribed method from their teacher (Hashimoto, 1997; Schoenfeld, 1991; Sriraman, 2005; Milgram & Hong, 2009). This is one of the ways in which MST provide challenges for students. MST, by their nature, provide such opportunities because there is no one correct solution to the task. Research has shown that MST are also a powerful tool for measuring the relative creativity of students by providing opportunities for students to come up with their own ideas that can be compared with those of their peers (Leikin, 2009; Leikin & Lev, 2013). One of the obstacles to providing more creative opportunities for students is often teachers' own fixation with certainty which can obscure their view that mathematics is a creative subject (Haylock, 1997; Mann, 2006; Schoenfeld, 1985; Sheffield, 2017). This may be an influential factor in the lack of opportunities the students in this study had to explore MST in their class at school, as will be discussed below.

8.2.5 MST are more challenging

One of the most common techniques teachers use to reduce the difficulty of a mathematics task is to teach procedures that the students can memorise (Skemp 1978; Schoenfeld 1988; Tall, 1996; Lithner, 2008; Hourigan & O'Donoghue, 2007). By presenting students with unfamiliar tasks where they had no memorised algorithm to rely upon, the MST in the workshop forced the students to engage in the process of understanding the concepts embedded in the task in addition to finding several methods to solve it. From the data it was clear that the students found this aspect of the MST challenging. As mentioned in the discussion on the Gestalt Principle, in the post-workshop survey 96.6% of the students agreed that MST stretched them more than just finding one answer, which was backed up by the data in the open question where students were asked to add any further comments of their experience of the workshops of mathematics in general. Students said they found the MST 'very challenging', and that the MST made them think about mathematics in different ways. Student 121 said that he 'thought the workshops improved my maths by challenging me to think on my own and not be spoon-fed an answer'. Another observation on the challenging aspects provided by MST was that 'it was challenging to think of more than one way …because I usually don't need to solve it twice' (Student 027).

The above discussion reflects research by Sheffield (2003), showing that MST were particularly suitable for able mathematicians in diverse classrooms, because 'the real mathematics begins after a solution has been found' (Sheffield, 2003, p.7). Similar comments were made by the students in the interviews, one saying that 'you're constantly thinking, you can't switch off because they're a lot more challenging' (Student 020). The workshop audio recordings illustrated the effort the students had to put in to solve the tasks. Sometimes this was the 15 minutes of silence in the case of the online workshops, as the students were thinking individually. In other instances you could hear the frustration as groups tried to find alternative methods: 'it makes sense to do it that way, it's just not working' (Student 340). The difficulty experienced by the students reflected a lack of exposure to such tasks but their perseverance and satisfaction gives support to the research on the necessity of using MST to enhance creativity (Matić & Sliško, 2022).

8.2.6 Literature on the importance of learning

As will be discussed below, the students in this study appear to be aware that they were missing out on opportunities which allow them to reach their full potential. They spoke about 'falling behind' and 'not being pushed' in a way that implied they felt they could be, and were capable of, learning more. Research has shown that every student has the potential to improve in mathematics and schools should therefore be presenting students with challenging tasks that encourage them to be creative, and to have an open mindset that they can enhance their own learning (Boaler, 2016; Dweck, 1986; Mangels et al., 2006; Kattou, 2013; Sheffield, 2017). Vygotsky's (1978) idea of the ZPD is also rooted in this need for teachers to consciously provide such opportunities for highly able students to maximise their learning. The data in the surveys in this study show clearly that the students do not feel pushed to their maximum with only 36.2% believing that what they do in mathematics class suits their ability. Sheffield (2017) disputed one of the ‘myths about gifted students’ that they could reach their potential by themselves. She recommended that schools need to provide positive support and recognition for academic excellence by fostering opportunities for able mathematicians to be creative and experience success. The evidence above suggests that this is not being done in the schools of these students. What is also of concern are the comments related to ‘compliance’ in the Free Market data, where students feel they ‘must’ go along with what the teacher tells them to do. Research is consistent in this belief that teachers must deliberately put strategies in place rather than expect these students to reach their potential by themselves (Brousseau, 1997; Lithner, 2008; Schoenfeld, 1991; Hourigan & O’Donoghue, 2007). As discussed in Chapter 2, Brousseau (1997) claims that for meaningful learning to take place in a classroom, teachers must transfer responsibility to the students so that they learn to adapt prior knowledge and make the problem their own. Instead of presenting students with an algorithm, Brousseau describes this ‘adidactical situation’ as necessary if students are to learn how to transfer the mathematics they learn in school to solve problems that will occur in their real life.

8.2.7 Not being challenged to achieve potential

There were a number of comments in the interviews showing the students’ concerns about not achieving their potential because of the lack of learning in class. They showed their awareness of their own ability and skillset and emphasised the lack of opportunities to push themselves. Such comments included ‘I don’t think you are challenged to your maximum ability (Student 123), ‘you're not doing things that you know you're capable of’ (Student 131)’, ‘in terms of my skills being tested fully, definitely not in maths ... It's just a case of memory’ (Student 430) and finally ‘I don't think we're very stimulated in school’ (Student 340). The suggestion that ‘I think you’d have more potential if you are given harder sums to do’ (Student 225) ties in with the survey responses on the benefits of the MST, discussed above, where the students were in agreement that they provided them with more of a challenge.

8.2.8 Summary of discussion on research question two

With regards to research question two, which sought to examine the level of challenge experienced by the students, the data appears to suggest that the students felt under challenged by their school mathematics. There was evidence that the reasoning required of students in school was mostly imitative with little opportunity for them to employ creative reasoning, which led to a lack of challenge. There were frequent comments on the repetitive nature of the textbook tasks that were assigned to them. The participants also identified the lack of opportunity to encounter uncertainty and the predictability of tasks and solution methods as reasons for feeling under-challenged in school. The students described their classroom atmosphere as one where they do not need to take risks, in fact it seems to be actively discouraged. As a result, they remain mostly within their comfort zone which leads to a lack of incubation and challenge. Providing specific tasks designed to challenge high ability mathematics students does not seem to be a key feature of classroom preparation. This discussion on class preparation that promotes, or prevents, challenge and incubation opportunities has important implications for Irish education which will be discussed in the final conclusion to this research.

8.3 Research question three: What comparison can be made between the opportunities these students had, in school and in the workshops, to experience each of Sriraman's five principles to maximise creativity.

In comparison to the classroom focus on algorithms and learning methods for exams the objective of the workshops was to explore problems and multi-solutions with a view to enjoying the mathematical process. The tasks were specifically selected to provide opportunities for the students to experience creativity. In doing so, it was intended that the students would notice a significant contrast to the mathematics they were used to in school and would feel challenged and possibly out of their comfort zone. These are all features incorporated in Sriraman's five principles to maximise creativity.

8.3.1 Opportunities for incubation and illumination

Experiencing the Gestalt Principle was essential for maximising creativity because it pushes students into their ZPD where they are exposed to challenges and learn to appreciate the benefits of perseverance. In the analysis of the interview data we saw that students mentioned experiencing illumination and satisfaction in the workshops 81 times. They specifically mentioned how rewarding it was when they found their own strategy that worked, Student 326 emphasising that 'the satisfaction is far greater than (for) a question you can just learn off in the textbook'. They spoke of the 'sense of achievement' and how 'accomplished' they felt after spending a long time thinking about a task. 84.3% of the students said that the tasks in the workshop encouraged them to persevere when stuck.

An important outcome of this perseverance and feeling of accomplishment was the confidence that it inspired in the students. The findings show that completing a task 'without anyone else's help', 'feeling proud' and thinking of an original way to find a solution all contributed to confidence building. There were also episodes in the audio recordings showing how perseverance was exhilarating for students who did not have much self-esteem with regards to their mathematics. Another student in the audio recordings described the feeling of satisfaction at finding a solution as 'the best feeling'. The enthusiasm of the peer collaboration and praise for the teams once a solution was reached exemplifies the belief that a single AHA! experience can change attitudes and beliefs (Liljedahl, 2004). There were many instances of this in the audio recordings of the workshops, from simple 'well done guys, we did it, yeah!' to 'oh my god, wait, wait, wait', when the enthusiasm and elation was evident. This was seen to give the students confidence, which supported what was discussed above in relation to the interviews. In contrast there was no discussion in the interviews about illumination opportunities in school. Instead the students found the work repetitive and seemed to know what the question was about before starting it. In the post-workshop survey, 91.9% of the students said that they feel more confident they will find a solution to unfamiliar mathematics problems with perseverance. The percentages in this response showed a considerable change from the 39.6% in the pre-workshop survey who said they were confident they would be able to solve a problem they had not seen before. In addition to this, the data supports the view that uncertainty is importance for creating feelings of frustration that challenge highly able mathematicians, as suggested by Sriraman (2005, 2021). Student 333 spoke in the interviews about the Steel Cable task ‘getting a bit frustrating’ but said it was his favourite task because it got him thinking a lot. Other students acknowledged the challenge brought about by not knowing the method but also said that ‘trying to find your own way feels good’ (Student 325).

8.3.2 Opportunities for experiencing a collaborative ZPD

The challenge and confidence building that resulted from the workshop was both a product of the tasks selected, and the peer collaboration built into the design. Research has emphasised the importance of the role of the teacher to facilitate peer collaboration that enables students to create their own scaffolds (Goos et al., 1999). The teacher’s preparation in selecting appropriate tasks and the role monitoring the students’ progress to ensure that their debates are mathematically sound are essential. The style of the tasks in the workshop were therefore deliberately very different from the textbook questions that the students would have been used to. The intention behind presenting the students with more complex tasks that take time to solve was to provide them with opportunities to both experience their ZPD and to be creative. Sriraman (2005) and Lithner (2002) recommend that the fostering of creativity should be an essential element when selecting tasks for high ability students. They claim that this will provide opportunities for students to be challenged appropriately and exposed to their ZPD to a similar extent as the less able mathematicians in their class. There is also evidence to suggest, (Mellroth, 2018; Nolte & Pamerien, 2017 and Sheffield, 2003) that this can be accomplished in a diverse classroom, such as is common in Ireland.

The findings from the workshop support this research that differentiated learning is important for these students by allowing them to explore the tasks and try out creative strategies. The interview comments showed that the students understood the challenging position facing the teacher in diverse classrooms. They spoke with empathy on the difficulty of teaching mixed-ability classes and having to finish the course, yet they were very aware that the more able were not being catered for. When asked specifically about what they saw as the benefits of the workshops, providing challenges for all students was a common theme in all five schools.

8.3.3 Impact on students' views of mathematics

Having an appreciation for the objective of mathematics and exploring solutions is at the core of the Aesthetic Principle for Sriraman. The workshops gave the students a different impression of mathematics, which could be seen by descriptions in the interviews that 'you're not looking at numbers and letters, you're looking at shapes and it's still maths …more parts of your brain thinking more creatively' (Student 325). Through workshop audio comments like 'could we come up with some sort of method for it?' (Student 429) the students could be seen to gain confidence in their own abilities to explore the tasks and were taking responsibility for developing their own approach. If this is not their experience in school, then they are being denied opportunities to explore and experience challenge, illumination and the positive emotions that Liljedahl (2005) claims are so affective on student beliefs.

The data on the student experiences of the workshop also showed a very different idea of how they could improve. There was evidence that the experience had changed their view of what mathematics was and the effect engaging tasks could have on students. There were 86 interview comments on repetitive questions in school, with 24 of these highlighting the negative impact repetition had on the students' motivation. When discussing the workshops in the interviews the students said that the tasks were 'a lot more engaging … and you had to try and find the most efficient way to go through a question' (Student 429). They spoke of the benefits of 'bringing something to mathematics by doing it on your own' (Student 325) and how it wasn't just 'doing a method anymore' (Student 038) but showed what maths can be in the classroom in an ideal world and how much fun it can be' (Student 429). These comments reflected the opportunities to experience what Sriraman saw as the Aesthetic Principle of mathematics.

8.3.4 Enjoyment of the process

Enjoyment of the mathematical process, rather than focusing on the answer, was an important feature of Sriraman's Aesthetic principle. Much of the research literature on this topic encourages teachers to consider differentiating tasks and instruction to align with student interests in order to maximise student motivation, creativity and feelings of satisfaction (Amabile, 1996; Sriraman, 2004; Sternberg, 1985, 2017; Tomlinson, 2018). Enjoyment of the exploration of problems was a key feature of the interview discussion on the workshops with 70 comments on enjoying the process of doing mathematics. There were no comments on exploring mathematical problems or the process of doing mathematics in school. The survey responses to the students' overall enjoyment of mathematics were interesting and raised some ambiguities that were reinforced in the interview comments. In the pre-workshop survey, 72.4% said that 'mathematics is one of my favourite subjects', 69% agreed that they enjoyed going to mathematics class but only 22.4% said that they were happier in a mathematics class than in any other class with 44.8% undecided. When we consider the interview comments raised in the discussion on 'boredom' in class we might ask whether mathematics class could be more enjoyable for these students? If challenge brings enjoyment and motivation for able mathematicians, as has been suggested in the literature above, it should be an important consideration for teachers. The interview comments on the workshops give a very different impression of the students' experience: 'It's just interesting and fun' (Student 046), the 'problems were like things in real life' (Student 226), 'it makes you think about how maths is related to art and stuff' (Student 038). These quotes describe a much wider view of what doing mathematics is all about.

8.3.5 The beauty of simple solutions

The importance Sriraman (2005) attached to seeing the beauty in simple solutions can be seen to have wide implications for school students, not just professional mathematicians. Some of the simplest solutions can be found by exploring non-algorithmic methods such as using visual techniques and modelling real-life situations. Discussing different solutions methods was clearly not common practice in school for these students and the discovery of particularly simple solutions in the workshops encouraged the students to explore alternatives rather than just being content to find one answer. The time was limited in the workshops to discuss alternative solutions in great detail but by giving the students a taste for it the effect on their motivation was evident. Audio comments such as 'hey, that's really smart. I'm gonna try that' (Student 040) and 'I don't think this is the way that she wants us to do it' (Student 333) showed that the students were starting to think about experimentation for themselves. The multi-solution aspect of tasks was an attractive feature for the students and in the interviews, they specified this as a reason why a certain task was their favourite. This was particularly noticeable in the What's it Worth task, where the students were able to find so many simple solutions and they clearly enjoyed discussing them with each other. The simplest solution to this task, of adding the rows and columns, really surprised so many of the students. In addition to showing their appreciation of the simplicity of mathematics, voicing their surprise to the whole class became a great confidence trigger for the students who discovered it.

The overall data reveals the student enjoyment of the explorations of the workshops and an appreciation for mathematics. They spoke negatively about having to learn off methods, the focus on examinations in school and not being given opportunities to discuss alternative solutions. The interview comments also highlighted many benefits the students saw in the multi-solution aspect of the tasks and how influential MST can be on student beliefs on what doing mathematics is all about. This was evident in reactions such as: 'It opened my eyes to trying different methods' (Student 036), 'it definitely helps you learn to think more' (Student 121), 'it's a lot more engaging … than doing the same question over and over again' and 'each solution was very rewarding' (Student 430). By only focusing on getting an answer, as quickly as possible, the mathematically able students often miss out on so many opportunities to explore their creativity. Sriraman (2005) maintains this has a 'butterfly effect' on others with less creative potential because the more able do not get to impart their ideas, thus the knock-on effect is diminished global creativity. He viewed the emergence of unorthodox insights in school mathematics as instrumental for determining what mathematics can bring to society in the future.

8.3.6 Having to think of a method for yourself

Encouraging students to think for themselves by providing them with uncertainty was one of the most noticeable features of the workshops. The data showed that the students had very different experiences in school and the workshops in relation to having to think of a method to solve tasks. The contrast in experiences, and the student enjoyment of the workshops, backs up what research has said about highly able students needing to learn by exploration rather than by learning procedures (Gruber & Mandl, 2000; Milgram & Hong, 2009; Mc Clure & Piggott, 2007; Sousa, 2009). The lack of scaffolding from the teacher, which was built into the methodology of the workshops, provided the students with the opportunity to experience mathematical enquiry. By thinking of their own methods, highly able mathematicians rely on their own conceptual understanding and creativity rather than applying memorised algorithms without thinking (Schoenfeld, 1988; Silver & Stein, 1996). Similarly, Sheffield (2003) believes that in addition to having opportunities to be creative, tackling difficult problems enhances the students' perseverance. Chamberlin and Mann's research on The Five Legs of Creativity (2022) goes even further to emphasise how the two-way relationship between creativity and affect can have such a positive influence on students' problem-solving and belief system.

In the post-workshop survey 92% of responses agreed that 'having to think about the method was different to what usually happens when we use the textbook'. The students seem to have had a very different experience in the workshops when it came to uncertainty. In the workshop audio recordings, one student specifically said that the workshop tasks were completely different to the textbooks. The level of uncertainty that existed during the workshops could be heard in other recordings of the students' discussions where they spoke of being 'so confused', 'lost' and 'not having a clue'. It was evident from the data that the students enjoyed the experience of thinking about a task. In the pre-workshop survey 92.9% of the students said that they enjoy having to think for themselves when solving a problem. The interviews further revealed that the students preferred thinking for themselves and saw the fun and benefits of it for helping you 'understand it more' and 'to make you improve'. The students clearly recognised that the questions were very different in the workshops, shown by the 94.2% of students in the post-workshop survey who said they had to think a lot more when solving the tasks in the workshops than in class. The students made the distinction in the interviews that for textbook questions you were just 'learning off how to do it' but in the workshop tasks 'you were really actually thinking about it'. If schools are aiming to provide an education that suits the needs of all students, it would be worth remembering that, in Q38 of the pre-workshop survey, 93.1% of students reported that 'most problems in school could be answered by recall of examples and formulae'. However, this seems to contradict what they say they would like, in Q37, where 70.7% say they would like more challenging questions that make them think for themselves.

Having experienced the feeling of uncertainty accompanied by satisfaction in the workshops, the evidence suggests that students were thinking differently about the importance of having to think for yourself. From analysing the survey responses in parallel with the interview data the workshops appear to have been instrumental in bringing about this change in attitude, or perhaps awareness. The students were very vocal in the interviews that being shown the method first by the teacher was the norm in school mathematics classrooms and they seemed much more decisive about their preference for thinking, such as the tasks in the workshops required. In the pre-workshop survey question, 43.9% agreed that they enjoy problems best when not given any hints by the teacher, 39.7% were undecided and 15.5% disagreed. Despite 63.8% agreeing, there was a similarly high level of 25.9% undecided in the responses to 'I am motivated most by problems where I have to think about the method'. We have seen above that the students agreed that they had to think a lot more in the workshops. Results from the post-workshop survey show that 76.3% said they enjoyed having to think hard and the same percentage said they found the tasks in the workshops more interesting than those in school. When asked which task was their favourite in the interviews, most responses were related to the fact that their preference was one that 'you had to make up your own method' or that 'got me thinking a lot'. Overall, the experience of the workshops seems to show that tasks that made them think were a motivating factor for the students and the majority preferred thinking over being shown the method first by the teacher. Had the post-workshop survey included the same questions as the pre-workshop survey on thinking about the method I believe the responses would have been significantly different.

8.3.7 Lack of exploration of alternative methods

There is much evidence that the feeling of uncertainty when approaching tasks was not something that was common in school. Similarly, experimenting with different solution methods does not seem to be a feature of school mathematics. The students seemed to be almost conditioned to believe that following the teacher's method was the norm: 'that's just the way we're taught, and we just do it that way'. Given that they appear to have had very little experience of thinking for themselves in class, being asked if they had a preference for not being shown the method first may have appeared obscure to them. The unfamiliarity of having to think for yourself in the workshops was also shown in the audio analysis by comments, mentioned earlier, reflecting surprise at not being told the method first and by the long periods of silence when they had to think for themselves. Appreciating the beauty of different and simple solutions was seen by Sriraman as instrumental for encouraging creativity but was not a feature of school mathematics class for the students in the study.

In the workshops the tasks selected were deliberately more visual and 25 interview comments specifically addressed the opportunity to use visuals in the workshops. The lack of visuals in class ties in with the overuse of procedural questions in school and the 93.1% of students who agreed that most questions in school can be answered by recall of examples and formulae. It also reinforces the student comments above that they tend to focus on the answer in school rather than exploring alternative methods. Only 22.4% of students in the pre-workshop survey said they were often asked to solve a problem in more than one way and 63.8% disagreed. However, this 22.4% did not necessarily mean that the students were exploring alternatives having got the answer out using another method. Several students commented in the interviews that they would only be asked to do the question an alternative way if their way had taken too long, or if the teacher had a preference for a certain way. A lack of time was suggested by several students, as a reason for not exploring alternatives. There were 25 comments related to the focus on exams in school that specified teachers wanting to 'move on'. The speed of answering questions seemed to be a priority over exploring the understanding and possible solutions to the questions.

8.3.8 Effect of the workshops on student perceptions of mathematics.

When each section of the data is combined together the students seem to have enjoyed the different experience in the workshops. In the post-workshop surveys, 93.1% said that the tasks in the workshops were very different to those we do in school and 91.7% agreed that it they found the workshops fun, which seems to imply they enjoyed the tasks. These results were backed up by the audio recordings of the workshops, where you could hear the enjoyment within each group, and by interview comments confirming that the students enjoyed having to think of a method to try. Liljedahl's (2005) research on the impact of positive emotions on changing the attitude and beliefs of students focuses on the importance of the whole classroom experience for students.

Liljedahl claims that the AHA! experience can not only improve the students' beliefs about their own ability but that it can change their ideas of what it means to 'do mathematics'. The interview comments concurred with this and showed the effect the workshops had on the students' view of mathematics. One comparison to school mathematics that was made was that 'you actually have to find out how to do it yourself. It isn't just right or wrong' (Student 425). Given the focus on the answer in class, the workshops can be seen to have had an influence on the students' view on the 'objective' of mathematics. One interview comment, 'it didn't feel like we were doing maths at all' (Student 020), summed up how positive experiences can transform students' attitudes (Liljedahl, 2004).

The students' transition from the unfamiliarity at the outset of the tasks to feelings of illumination and certainty may also have led them to reconsider the benefit of not being given a method or formula by the teacher. The analysis of the data seems to suggest that these positive emotions had a noticeable influence on the students' belief about how mathematics tasks can be introduced. They acknowledged that they needed to 'really process the information' in the workshops but that 'in school you have your formula'. There was a definite feeling that they enjoyed this autonomy to decide upon a strategy. From a preparation point of view, they could see the benefits of being given tasks that could be solved using a variety of strategies to suit everyone's needs. In the interviews the students spoke about the fact that the tasks provided different levels of challenges for everyone, with 65 comments specifically relating to the workshop problems providing opportunities to experience their ZPD. The use of phrases such as 'everyone's working to their own level', 'no one's being held back' and 'everyone's thinking' seemed to reflect the diverse needs of students in their classes. In the post-workshop survey, 88.4% of the students said that the tasks in the workshops pushed them more to their maximum ability. Similarly, the audio recordings reinforced the survey and interview findings that the tasks in the workshops stretched the students and pushed them into their ZPD to an extent that it even created frustration at times. There were many episodes where the students expressed that they were confused or stuck combined with long periods of silence in some of the online workshops as the students explored individual conjectures. They were most definitely challenged but said the challenge and bouncing ideas off each other was enjoyable. Such reactions of the students to the peer work in the workshops provides evidence that the aim of providing them with an experience of the ZPD was being met.

8.3.9 Opportunities for risk-taking

The survey, interview and workshop data collected were all consistent in highlighting very contrasting student experiences in the workshops and school under the Free Market Principle for maximising creativity. The key findings discussed below relate to opportunities to have the freedom to take risks and opportunities to collaborate with peers. It can be seen that the classroom atmosphere contributed to the lack of risk-taking opportunities in school and to the students' response to having the freedom to think for themselves in the workshops. Evidence that this was important for the students can be shown in the 166 interview comments that discussed having the freedom to think outside the box in the workshops and the lack of it in school. The dangers of rote learning methods, suggested by Schoenfeld (1988), were discussed earlier as having a significant effect on students' lack of focus on understanding mathematics. Not providing students with opportunities to take risks may have a similar detrimental effect on their willingness to explore and their confidence to put forward their ideas, especially if they are unsure about them (Boaler, 2016; Sriraman, 2005). Sheffield (2003) argues that allowing students to take risks and make mistakes will enhance their enjoyment and understanding of the mathematical concepts embedded in the task. Risk-taking is also seen in research (Sriraman, 2021) as being one of the catalysts for creativity in mathematics. For this reason, allowing students to adapt existing methods, explore new methods and put their ideas forward to their peers is essential if we want to foster creativity in school.

There was evidence that the students felt this was being done in the workshops but not in school. The data from the workshop audio recordings showed students experimenting with ideas: 'Let's just try it', 'I've figured it out, I think. I'm not sure if it's right though' and 'this mightn't be right at all, but'. They emphasised the skills and opportunities to use 'your own creative thinking' that developed from having the freedom to explore methods in the workshops. They contrasted having the opportunity to 'work things out for ourselves' in the workshops with the belief that school did not allow room 'to think for ourselves' and that being given a solution can 'restrict your thinking'. In the post-workshop survey there was 100% agreement from the students that 'solving unfamiliar mathematics problems improves their problem-solving skills'. In addition to improving their mathematical skills, 93% also agreed, in QB15, that solving unfamiliar problems improved their confidence that they would be able to solve similar ones in the future.

The interview comments backed up the survey data showing that in school the students were generally taught by examples with no freedom to experiment. They spoke of 'enjoying being able to think outside the box … instead of the normal repetitive ways' and of being free to 'go in any direction you want'. The workshop experience was definitely seen as very beneficial in that it 'makes you think more' (Student 339), 'it pushed us to our full capabilities instead of just learning straight from the book' (Student 340) and was 'refreshing' (Student 325).

8.3.10 Enjoyment of having the freedom to take risks and collaborate

From the post-workshop survey, the interviews, and the audio recordings from the workshops, there is clear evidence that the students enjoyed the collaboration and freedom to think for themselves in the workshops. In the post-workshop survey 83.9% said that they enjoyed the experience of the workshops and, by examining the different sections of the data, we can see the various aspects of the workshops that they enjoyed. When asked in the interviews, students made 92 comments on the enjoyment of discussion of ideas with their peers and many of the 52 comments on having the freedom to think spoke about the enjoyment it brought compared to school. The student responses relating to enjoying the freedom to think in the workshops supports the findings of Middleton et al. (1992), that high achievers prefer activities where they have more control over their learning. Research on intrinsic motivation for highly able students (Middleton & Spanias, 1999; Wilkie & Sullivan, 2018) emphasises the enjoyment of perseverance, challenge and risk taking. In this regard, engaging in multi-solution tasks enables students to decide how they want to approach a question and might explain why they enjoyed such tasks. You could hear the satisfaction and enjoyment coming through the interview discussions, and expressions such as: 'once I got it, I was like, Yes' (Student 037) emphasised the elation success brought to the students.

8.3.11 Benefits of opportunities to collaborate

The importance of collaboration has been emphasised in the research literature (Boaler, 1999, 2009; Lave & Wenger, 1991; Goos, 2004) because of the learning that takes places within a community of experts and novices (Lave & Wenger, 1991). However, despite the potential for collaboration in classrooms, the traditions that have become accepted as the 'norm' in schools do not always allow them to take advantage of this learning environment (Boaler, 1999). The evidence revealed that collaborating with peers was not a feature of school mathematics. There were 23 specific comments in the interviews that spoke about not having opportunities in class, or if they did, they were limited discussions to confer on an answer. 'You're not allowed to talk in class' (Student 046) was the theme that came across in all of the comments on opportunities to collaborate with peers in class. They definitely acknowledged the benefits of discussion: 'all it takes is a little spark off someone, one word someone would say that leads you to find the answer' (Student 340). However, given the comment above, from Student 046, it is not surprising that there were no comments made in the interviews on discussion in mathematics class. The open question at the end of the post-workshop question backed up the above comments where many of the students commented on working in groups being one of the most enjoyable aspects of the workshops. Evidence of the enjoyment was also very clear in the 91.7% of students who said that the 'mathematics was fun in the workshops'.

The data on collaboration in the workshops was in agreement that the students saw many benefits to it and found it a novel experience. Learning different ways of doing mathematics from their peers was one of the most frequent comments made along with how much they enjoyed discussing mathematics with their peers. The freedom that came from having opportunities for uncertainty was evident in many interview comments: 'I enjoyed the workshops and particularly enjoyed discussing the problems in groups when we were unsure of how to solve the problem' (Student 043). Others spoke of the novelty of learning 'a different type of problem solving and ways to approach questions' (Student 024). The comments in the open-ended question in the post-workshop survey also spoke enthusiastically about collaboration and having 'a great group dynamic' (Student 034). This positive attitude towards peer work was backed up by the audio recordings from the workshops which showed students engaging with enthusiasm in the exploration of the tasks. They showed much consideration for each other and praised good conjectures which is one of the learning potentials associated with collaboration in the literature (Boaler, 2006; Goos et al., 2002; Tomlinson et al., 2003). The comment from Student 340 above about 'a little spark' from someone described much of the conversations in the workshop audio recordings.

8.3.12 Effect of classroom environment

Middleton & Spanias (1999) found evidence that the level of teacher support and classroom environment can have a significant effect on student motivation. In the case of the able mathematicians in this study, excessive repetition and lack of teacher disposition to pose challenging activities can be seen to have a negative effect on students. In contrast the tasks in the workshops seemed to really motivate the students to the extent that they found themselves 'talking about the different ways they had done it' in school the next day (Student 432). They felt 'more engaged' and looking at other methods enabled them to 'learn and improve with everyone else's help' (Student 430). Having the freedom to work with students of a similar ability also emerged as an important motivating factor. They seemed to like that they were allowed to choose which group they were in. Being 'surrounded by people with a similar interest in maths' (Student 340) was one perceived benefit. The idea of learning more was another benefit because they felt they could be 'slowed down' by less able students (Student 131). The practise of putting one good student in a group raised concern that 'they're pulled back' because they are 'thinking too fast' (Student 325). This aspect of groupwork was not integrated into the surveys but it was definitely a feature that was important to some students in this study and would have been interesting to research further.

8.3.13 Opportunity to explore MST

The benefits of MST for challenging high ability mathematicians and providing creativity was discussed in section 8.2. However, all of the evidence is in agreement with regard to the unfamiliarity of using MST in the school for the students. The post-workshop survey data showed 79.3% agreement with the statement 'we do not normally solve questions using more than one method in school'. It was also clear from this survey that the novelty of experiencing MST was a positive experience for the students with 81.6% agreeing that they enjoyed finding multi-solutions more than just finding one answer. This unfamiliarity with MST could also be deduced from the workshop audio recordings where students sounded surprised by the different techniques that proved viable for finding the answer. When one group of students had all been trying to solve the Field of Dreams task, there was initial confusion because of the unfamiliarity of being presented with a task that had different solutions. The students thought they had the 'wrong numbers' before one of the group announced, 'there's just different ways it can be solved' (Student 230), the surprise of which was summed up by the simple response of 'that's so funny' (Student 242). The interviews also showed evidence of the lack of opportunity to explore MST in school through the 34 interview comments under this code. The students were specifically asked if they were ever given MST and comments similar to 'no one would give you multi-solutions … I don't feel the teacher really presents more than one method for a maths question' (Student 341) were expressed in several of the interviews. Others said that they would only be asked to use a different method if their way was taking too long. However, the most frequent response when asked in the interviews if they would ever be asked to solve a problem in more than one way was simply, 'never'.

8.1.14 Improving creative thinking and developing understanding

The importance of tasks that foster creativity has been widely researched in recent decades (Leikin, 2007; Mann, 2006; Sheffield, 2017; Sriraman, 2005, 2021). An exploration on the effect of MST by Tabach & Friedlander (2013) showed that over focusing on algorithms and rote learnt methods inhibited student willingness to experiment with creative solutions. Sheffield (2009) also emphasised the importance of students learning to ask exploratory questions to increase their depth and interest in a task. It can be seen from the data discussed below that the student interest and skills were both enhanced by the tasks in the workshop.

The exploration of MST in the workshops seemed to develop the students' ability to reason about conjectures and learn new skills that gave them confidence. This was evident from comments in the open question in the post-workshop survey such as: 'I think that these activities forced me to think in a different way. They helped me reason different solutions that I would not usually think of' (Student 023). Similarly, comments such as: 'It was very interesting to see that there were many methods to the same question … I feel more confident with problem solving' (Student 029) emphasised the benefits of the whole workshop experience. In the post-workshop survey, the students were consistent in their recognition of the benefits of MST shown by the 93% agreement to 'I have learned some useful new skills for problem-solving by solving tasks in different ways'. When asked in the interviews which tasks they preferred the students in all schools expressed their preference for the MST, over those in school, because they were more satisfying and enjoyable. However, they also identified the benefits in terms of learning new skills and improving their understanding, for example: 'if you were interested in actually learning maths and enjoying to learn maths the workshop tasks were better' (Student 231).

The belief in the research that MST promote creativity could be seen by student reflections that 'they were inspiring more creative thinking' (Student 429) and 'I now have many different methods from other people. I enjoyed thinking about the questions for so long' (Student 120). The discussions in the workshop audio recordings backed up this perseverance to experiment and learn. You could hear the students discussing their strategies: 'We're trying to do it a different way. We've done it by simultaneous equations and it worked' (Student 127), 'Now we're trying something different but it's not working' (Student 136). There was also evidence that the students were developing a flexibility to use techniques they had learnt from one task to help them solve another one: 'oh yeah, that's like we were doing before' (Student 036). Another important skill learnt was an understanding of how multi-solutions were defined: 'but we can't use the answers from the first method to find another method' (Student 132). This showed the gradual confidence the students were building in taking ownership of their own mathematical reasoning.

8.3.15 Enjoyment of learning

The tasks in the workshop were intentionally designed to create mathematical learning opportunities where the students would feel they had achieved something for themselves that would be of benefit in the future. Interview comments on the workshops were very positive with regards to their success: 'No one's being held back.

Everyone's thinking, no matter how good you are at maths, you're still thinking' (Student 131). The workshop audio recordings did not reveal any additional statements relating to the importance of mathematics, but it could be seen that the students were motivated by the determination with which they persevered with the tasks. Listening to the conversations about exploring MST, discussed above, also showed that the students were taking responsibility for their learning as Brousseau (1997) recommended. There was very little idle chat during each 2-hour workshop, showing that the students were focused on the task presented to them. There were no comments made on time wasting, instead hearing the students say, 'I'm exhausted' and 'I'm so tired' showed how focused they were for the 2 hours. When the above post-workshop responses and interview comments on MST are considered, it can be seen that the students found the workshops useful and an important learning experience.

8.3.16 Summary of discussion on research question three

As regards research question three, the students found the workshops both challenging and enjoyable. There was much evidence in relation to research question three that the students had a very different experience in school and the workshops with regards to each of Sriraman's principles to maximise creativity. They spoke about the opportunities for challenge, incubation and illumination afforded them in the workshops and how this boosted their confidence levels. Students contrasted the types of tasks that they encountered at school and in the workshops and called for teachers to assign more unfamiliar tasks to them. They mentioned that uncertainty was removed (by the teacher or the textbook) in school but not in the workshops. In fact, in both the surveys and the interviews, students frequently mentioned that they needed to think for themselves and search for solution methods in the workshops.

The Aesthetic Principle data also provided much information in relation to research question three. The opportunity to explore different solution methods (for example, using visualisation instead of algebra) was not evident in the data relating to school classrooms but was appreciated by students in the workshops. The use of these different methods seemed to expand their ideas of what mathematics is. They also showed an appreciation of the beauty of mathematics when considering alternative solutions. Overall, the data coded under the Aesthetic Principle thus points to major differences between students' opportunities to develop creativity in school and in the workshops.

The data shows that there were more opportunities to take risks and to work in groups in the workshops than was in case in classrooms. Participants emphasised the necessity of thinking outside the box in the workshops and the lack of opportunities for mathematical exploration in school. Learning that it is OK to be wrong will persuade students to try out new ideas. By giving students the opportunity to collaborate with their peers they can build confidence within that small group community to express and defend their ideas. The workshop data showed how this can facilitate them to develop their own methods, rather than blindly complying with the method shown by their teacher or the textbook. Hence, in answer to the third research question, there seemed to be a very apparent difference between school and the workshops.

From a learning aspect, there were several aspects of the comparison between school and the workshops that answer the third research question. Students said that in the workshop situation they felt able to move at their own pace and did not feel like they were being held back. This was largely due to the nature of the MST. The students seemed to like the concept of MSTs and mentioned that these tasks helped them developed problem-solving skills and understanding. They also spoke about the lack of these types of tasks in school. Thus, it seems, from this analysis, that there was a difference in the types, and extent, of learning opportunities afforded to students in the workshops compared to school classrooms.

8.4 Discussion Chapter Summary

Having examined the classroom and workshop experiences of the students in this study, through the lens of Sriraman's five principles, there is strong evidence to suggest that the opportunities to be creative are not being maximised in classrooms. It is also evident that the students have clear ideas about what it is they would like to see happening in school. These findings are not only important for able mathematicians, but also for all those students who enjoy mathematical explorations. The evidence discussed brings to light the many benefits of challenge, collaboration, perseverance, making mathematics class about the process instead of the answer and giving students the freedom to think for themselves. The study has also examined the role of multi-solution tasks, which has been suggested by many researchers as a useful tool for differentiation and creativity and shown that the students both enjoyed the experience of working on this type of task and found it very beneficial for their learning.

Chapter 9 Conclusion

Through the research, it was aimed to answer three research questions on the mathematical experiences of able mathematicians. The key findings will be summarised here, in the context of wider research, and make recommendations for what could be done within a diverse classroom to enhance the experience of this group of students.

Research question one:

What is the perception of able students in post-primary schools in Ireland towards their mathematics classes?

The data collected in the pre-survey shows that the participants in this study enjoy mathematics and feel relatively confident about their own ability in the subject. However, it can be seen from the survey and interview data that the students feel that the class preparation does not suit their abilities in terms of content or the way it is presented. There is strong evidence of an emphasis on the use of routine algorithms and example-led, teacher-centred classrooms in each of the five schools in this study. The students' reactions to traditional teaching methodologies, such as the encouragement of repetitive rote learning and memorisation of procedures, highlighted that they felt they lacked the freedom to think for themselves and explore solution methods without teacher instruction. The students suggested in the interviews and surveys that they were often bored in class and felt that whilst 'waiting around' for the teacher to explain solutions to less able students, the more able students could be given extra challenges. They reported that they are being predominantly taught from the textbook which is not providing them with the complex tasks they require to really feel challenged. The students may have mastered the work required for the examinations, but they are not really learning how to deal with uncertainty to enable them to learn the self-regulating skills advocated by mathematics educators (Goos, 1999; Mason, 1998).

From the data it is also evident that the students see school mathematics as examination focused, rather than as an exploration of mathematical problems. They emphasised this with frustration and expressed a desire to experience mathematics that did not rely on memorisation of algorithms. They clearly feel that they are not really reaching their potential with this style of instruction.

Implications for Irish post-primary mathematics education

The data in this study supports the research of Sheffield (2017) that it is a myth that able mathematicians 'can teach themselves'. This belief ties in with the focus on high stake examinations that the students confirmed was a dominant feature of their classrooms. These able mathematicians may be well able to achieve top grades in their examinations but from the evidence we can see that they themselves are searching for more in terms of mathematics education. The numerous comments in the interviews revealed the widespread discontent felt about teachers only teaching for the examinations and this needs to be addressed in schools. Having to sit an examination for entry into college should not mean that students only get to experience 'exam-style' questions in class. On the contrary, exploring more open-ended problems in school, without instruction, should enable students to cope better when they meet an unfamiliar question in an assessment. As mentioned in the interview analysis section, these students had not yet reached 5th and 6th year when the focus on examinations will most likely become much more intense. Giving students opportunities to see the beauty of mathematics and enjoy the exploration of it would provide much more motivation for highly able students.

Similarly, the lack of understanding of what these able students both need and want was evident in the data and needs to be examined further. As outlined in Chapter two, research (O' Sullivan, 2017) has shown that the tasks in the current Irish textbook offer very few higher order tasks where the students are given opportunities to conjecture and reason, which would provide more of a challenge than algorithmic tasks do. The procedural nature of the textbooks may be suitable for many students, but they are not sufficient to stimulate high able mathematicians such as those in this study. More importantly, they need opportunities to develop their creativity and spatial skills, rather than focusing on left hemisphere activity, such as memorising rules and algorithms, as this will enable them to reach higher levels in mathematics (Pehkonen, 1997).

Teachers also need to take note of the 1993 SERC report's recommendation that 'exceptionally able' students should be facilitated to participate within mixed-ability groups but that teachers must also find ways to enable these students to work with peers of similar ability. The findings in this study concur with these recommendations and suggest that highly able students are recognised as students with 'special educational needs' and that these needs should be further examined. Enjoying working with 'like-minded people' was quoted in the findings and many students expressed a desire to have more of it in school.

Research question two:

To what degree do these students feel challenged by the experiences they encounter in the mathematics classroom?

The word 'challenge' is defined in the Cambridge Dictionary as: (the situation of being faced with) something that needs great mental or physical effort in order to be done successfully and therefore tests a person's ability. This research question was motivated by the desire to know if able mathematicians are faced with such situations in their mathematics classes. From the *Feeling Challenged* scale in the pre-survey data, we see that the participants in this study mostly felt under challenged by school mathematics. It was striking that under 10% of them reported ever being stuck in class. The students' responses to the *Motivated by Thinking* scale we see that these students appreciate opportunities to think for themselves and would like to be given more opportunities to do so. This data was backed up by the participants' comments in the interviews. They frequently spoke about being bored in class. The boredom stemmed from multiple sources but two were prominent in the interviews. Firstly, the majority of students found the reliance on the textbook and in particular on repetitive tasks tedious. In addition, we saw evidence that the more able students often have nothing to do in class and spend class time waiting while the teacher helps other students. They expressed sympathy for the teacher in this situation, but they also felt that they were being ’held back’.

From the data, as discussed above, it is thus evident that the students in this study felt under stimulated by their school mathematics class, because of its repetition, procedural approach, and lack of challenge. We have seen confirmation that they have a high self-perception with regards to mathematics and would prefer to be given more opportunities to experience independence and perseverance. They specified that they found it challenging to have to think of a method for themselves in the workshops, as all school groups concurred that they are always given the method first by the teacher in class. Hence, the removal of this uncertainty was thus removing the challenge for the students. It seems evident that these students are not benefitting from the recommendations of the SERC that exceptionally able students ‘need to be challenged in tasks pushing into the frontiers of their competence’ (SERC, 1993, p.160).

Implications for Irish post-primary mathematics education

Many of the observations from the data reflect an educational environment in school that appears to differ from that laid out in the Irish Education Act, 1998. This Education Act stated that schools were obliged to ensure that the educational needs of all students, including those with special educational needs were provided for. Providing an education appropriate to the abilities and needs of all students was specified as a function of schools. Given that these highly able students all agree that they are being underchallenged in school, schools might need to consider how best to provide an appropriate education for such students. The evidence from the workshop data suggests that the tasks presented provided more of a challenge than school mathematics and that the perseverance required was instrumental for building confidence. Extra-curricular opportunities have been found to be very rewarding and effective for challenging and improving the skills of highly able mathematicians (Fitzsimons, 2021). To implement the terms of the Education Act, schools have a duty to examine what has proved successful, and to aim to provide such opportunities for the students who cannot avail of such external interventions.

Research question three:

What comparison can be made between the opportunities these students had, in school and in the workshops, to experience each of Sriraman's five principles to maximise creativity.

The data was analysed using a template based on Sriraman's framework of principles to maximise creativity. The survey, workshop and interview data allowed investigation on the extent to which each of the principles are evident in mathematics classrooms and in the workshops. This data has been discussed extensively in the previous chapter but a short summary of findings will be given here.

Gestalt Principle

We saw in the interview data that students spoke frequently about the lack of opportunities to incubate. The interview data and the pre-survey data showed that incubation opportunities were rare in class because of teaching methods that involved showing students solution methods instead of allowing them to experiment and explore possibilities on their own. As in the summary of the findings on Research Questions 1 and 2 above, the repetition of questions for which the solution was obvious was cited by students as both boring and frustrating. It also meant that students were deprived of experiencing the state of being stuck and of achieving an AHA! moment in class. The data from the interviews and the audio recordings of the workshops showed that students experienced both of these states in the workshop. There were many comments in the interviews that showed that students had lots of incubation opportunities in the workshops and that they appreciated them. The audio recordings backed this up and showed how students worked for long periods of time to find solutions. There was also evidence of their delight when they had AHA! moments. In addition, the nature of the MSTs allowed students to experience tasks in their ZPD and provided challenges at different levels. The workshop recordings showed how important the group dynamics were here.

Uncertainty Principle

As in the Gestalt Principle, we saw that the main reason why students did not have incubation opportunities in school was because there was usually little uncertainty when they worked on a problem. In the interviews and the pre-survey, the students spoke about following solution methods given to them by their teacher or in the textbook and only rarely having to think of a method themselves. In contrast they spoke about having to come up with a method themselves to solve the workshop problems and this was corroborated by the audio recordings. The post-survey comments and the students' comments during the workshop showed that even though this method of working was unfamiliar and possibly uncomfortable for them at the beginning of the workshops, the students embraced the uncertainty and enjoyed the process.

Aesthetic principle

The interview data showed a sharp contrast between the comments coded under this principle in relation to school and to the workshops. We saw that students' view of mathematics and its purpose in school was closely related to achieving good examination grades, while the comments on the workshops referred to an emphasis on enjoyment and understanding of the subject. Students also spoke about appreciating the beauty of mathematics especially in relation to looking at different solution methods in the workshops. The audio recordings showed the students beginning to be less concerned with getting the right answer than gaining understanding.

Free Market Principle

The interview data once again showed a clear difference between the students' experiences in school and in the workshops. We saw that students appreciated the freedom to take risks and to think outside the box in the workshops. However in school they felt that risk-taking was actively discouraged. They commented that teachers often had preferences for one particular solution method and wanted the class to use that exclusively. They also mentioned that they liked having opportunities to discuss mathematics with their peers in the workshops and that this rarely happened at school.

The workshop recordings showed students taking risks by trying new methods and sharing their work with the group. Once again, we observed the importance of the class atmosphere and group dynamic in creating an environment where students were willing to do this.

Scholarly Principle

We saw that the students in this study appreciated the importance of learning mathematics, however they frequently expressed the view that they were not learning as much as they could in school. As we have seen previously, they cited the use of routine tasks and the classroom atmosphere and organisation for this. In contrast they expressed the belief that the workshops gave them opportunities to learn and develop problem solving skills because of the unfamiliarity of the tasks. In particular the interview comments and workshop recordings showed that the participants appreciated that MSTs, although challenging, gave them opportunities to explore and develop understanding. The participants also liked the fact that these types of tasks allowed everyone to work at their own pace.

The students made many comparisons with their school experience and with the workshops and others were evident from their behaviour identified in the audio recordings. One of the key findings was their enjoyment of group work, where the students got the opportunity to work with like-minded peers to debate and discuss strategies for solving the tasks. The satisfaction and confidence gained from their experience of the AHA! moment of illumination supports the belief that highly able mathematics thrive when given opportunities to experience their ZPD without teacher instruction. The experience of MST was also spoken about with much enthusiasm by the students as being both novel and challenging. They seemed to really engage with the whole workshop experience of finding multi-solutions and discussing their methods with each other.

The feelings of satisfaction evident in the students, when they solved time-consuming MST, certainly showed that they were enjoying the learning experience rather than focusing on the end goal of an answer. This was well supported by the survey data when we look at responses to some of the questions on student self-perception. In the pre-workshop surveys, 34.5% of students responded with neutral to Q12 ‘I am confident that I can solve problems that take a long time to solve’. However, the confidence of the students seemed to have definitely improved after the workshops with 91.9% agreeing in QB20 that ‘I feel more confident I will solve a solution to unfamiliar problems with perseverance’. If this was the effect of two workshops, it would be very interesting to see how students felt after a prolonged period in school of solving such tasks.

Implications for Irish post-primary mathematics education

The research presented here details teaching and learning strategies that can be implemented in diverse classrooms to challenge highly able mathematicians. The framework that this study was built upon is grounded in the belief that creativity is an essential aspect of providing challenge for them. MST could be seen in the workshops to provide such opportunities and could be implemented by teachers, as ways of including differentiated learning in their classrooms.

From the data it appears that able mathematicians require more complex tasks than they seem to be exposed to normally in school, in order to feel challenged and be forced to think creatively. The importance of giving students challenging tasks cannot be underestimated. Research into factors that contribute to the failure of students to engage in high level cognitive processes showed that the removal of challenging aspects of a task played a significant role (Henningsen & Stein, 1997). By incorporating Sriraman’s five principles into the planning of the workshops, the students were seen to have a rich learning experience that they said challenged them more than the mathematics they did in school. It was not that the content of the mathematics was more difficult than what they do in school, it was the selection and implementation of the tasks that differed greatly. The students seemed to agree with the research that providing challenge meant ‘complexity, lack of teacher direction, and high expectations of both teachers and students’ (Scager et al., 2014, p.659).

Recommendations for the future.

The overall message from the findings is that, by hearing how the able mathematicians in this study feel, teachers might reflect on their individual classroom practises to plan activities that are in more alignment with the interests and needs of these students. Examining how the students in this study responded to being presented with non-routine tasks requiring collaboration and discussion in advance of teacher instruction sheds light on how they can be stimulated. Designing suitable tasks can be time consuming but there are many excellent ready-made resources such as those in the Maths Circles handbook (www.mathscircles.ie), the NRICH website (www.nrich.maths.org), Irish Mathematical Olympiad (www.irmo.ie), the UK Mathematical Challenges (www.ukmt.org.uk) and many others. Using open-ended, challenging tasks to promote creativity and exploration in the classroom also needs to be followed up when considering assessments. Sheffield (2009) argues that teachers need to move away from assessments that are marked either right or wrong and provide assessments where students expect to be required to think creatively. In this way their belief about what mathematics is will become less rigid. Recognising the ability and motivation of highly able students to solve tasks independent of formal instruction and providing the opportunities to do so would be of great benefit to them.

One of the key factors for change requires teachers to examine their beliefs on “the appropriate locus of control in the teaching process” (Thompson, 1984, p.120). This, for many, involves having a more flexible plan and encouraging more discourse and student questioning that may change the direction of the class. If teachers in Ireland can encourage a culture where students feel confident to verbalize tasks it will help them acquire more agency to develop their own individual technique for solving non-routine problems. However, as Boaler (2003) acknowledges, teachers need to know more than what strategies are effective; many may also need guidance on how to make it work in the classroom (McGrath, 2017). O’Neill’s research (2022) on ‘recipe-based instruction’ serves as an example of what students can achieve when the classroom is more student centred.

One of the suggestions in the literature was that a lack of resources and knowledge, on how to best challenge high ability students, was an obstacle in providing differentiated instruction. Providing teachers with such resources, and guidance on how best to implement them, would encourage them to experiment and therefore alleviate the pressures on them to find suitable resources. Developing a website of such tasks, as Cambridge University did with NRICH, and the Norwegian Centre for Mathematics Education did with Mattelist, would have been a worthwhile conclusion to this research had time permitted. This type of website would also serve to provide students with opportunities to think for themselves without teacher instruction.

Limitations of the study

There were a number of limitations of the study that advocate further research to enhance the findings. The first of these was the size of the sample of students. It would have been interesting, had there been time and resources, to carry out a more widespread survey of the whole country to investigate if there were different experiences in other schools and if so, how did the students feel it influenced their mathematics education.

The use of self-reporting surveys was another limitation because of the difficulty measuring the reliability and validity of student answers when they report their own experiences. Whilst the data from all three data sources seemed to concur there are inherent limitations of this method of collecting data such as individual interpretation of the language, social desirability bias and reference bias. Every effort was made to give the students time to answer the questions carefully and the language used was appropriate to the reading age of the students to avoid ambiguity. However, there were some questions that may have been interpreted differently by some students because of their mathematical experiences, such as some of those on how best students can improve their ability (see Scale 6, Appendix E). This may have been responsible for some of the Rasch results where several of the scales did not meet the reliability threshold. Social desirability and reference bias are more difficult to eliminate. In an effort to avoid social desirability bias it was stressed several times that the survey was confidential so the students' answers would be anonymous and were only being used to gather data on how the participants felt as a whole. In the face-to-face workshops the researcher deliberately gave the students privacy to complete the surveys to reinforce this. Having no in-person contact during the online surveys may have helped reduce this potential for bias. Despite this, it is difficult in self-reporting surveys to be fully confident that they are free from student desires to present themselves in a certain way. Similarly, reference bias, where students are influenced by individual standards of comparisons, may have affected their opinions of their mathematical ability and what tasks they found a challenge.

The outbreak of the pandemic in 2020 prevented expanding the research as was originally planned. Originally the intention was to carry out further iterations of the workshops with the students when they had commenced their Leaving Certificate programme. This would have enabled the exploration, in more detail, of the effect the increased focus on exams had on the students' classroom experience. The more advanced level of mathematics the students would have had at the end of 5th year would also have given the opportunity to present them with tasks that were more obviously connected to the Leaving Certificate syllabus yet also provided the students with opportunities of uncertainty and the freedom to think.

Another limitation was the different experiences the students may have had as a result of having to move the final two schools' workshops online. Whilst every effort was made to try and replicate the face-to-face experience, the collaboration aspect was most definitely hindered. As mentioned in the analysis of the audio recordings, not being able to see each other's work did remove the extent of collaborative learning that was possible. Consequently, not all of the five schools in the research study would have had the same experience.

Conclusion

Most importantly the research has confirmed the benefits of MST which are not typically used in post-primary classrooms in Ireland. They have been shown to enhance the students' learning experience including increasing their skills and providing opportunities for discussion. MST can be used with all levels of students but are particularly useful for high ability ones.

Another noteworthy finding is that highly able students do actually like working with like-minded peers and that the practice of using high ability students to teach the weaker student is not fulfilling the potential of the more able.

The traditional style of teacher instruction, aimed at teaching for the exams, is not what highly able students want or need. These students have shown that they prefer to think for themselves rather than be shown the method by the teacher. It is not all about the examinations for these students.

A comment that was highly inspirational for undertaking this research was 'we need not worry about the bright ones; they will get their A's anyway'. This research has shown that there is much more to mathematics education, for these students, than getting an 'A' grade.

The findings from this study are important when we consider Dweck's research on mindsets, discussed in Chapter 2 (Dweck, 1986). It was confirmed in her research that experiencing challenging tasks can have a positive effect on students' confidence and perseverance by fostering student focus on learning oriented goals rather than performance-oriented goals. For this to happen in mathematics, teachers need to convey that it is not only OK to make mistakes, but that they are necessary for learning to be achieved. The holistic education of the individual has been high on the agenda for recent reforms on the Junior Cycle and Senior Cycle in Ireland (NCCA, 2015, 2018). The focus is on learning for the future and providing students with the skills necessary to carry them forward into adulthood. Nurturing student confidence that they know how to think and learn for themselves, can only bring positive rewards for society.

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Appendices

Appendix A: Student Consent Form

Maynooth University
National University of Ireland Maynooth

INFORMATION AND CONSENT FORM FOR STUDENTS

Purpose of the Study. I am Judi Mills, a doctoral student, in the Mathematics and Statistics Department, Maynooth University. As part of the requirements for a doctoral degree I am undertaking a research study under the supervision of Dr. Ann O' Shea.

The study is an investigation into the extent to which enthusiastic and mathematically promising students are challenged by mathematics at secondary school level in Ireland. I plan to investigate what problems interest you, whether you enjoy the challenge of non-textbook style problems and what level of challenge is suitable for you.

What will the study involve?
The study will involve two 2-hour workshops over a period of approximately 3 weeks. You will be given some non-textbook style problems to solve individually and as a group. I will ask you to take notes on what you are thinking as you solve the problems. This will hopefully include what you thought upon reading the question, where you were stuck, what ideas you tried when stuck, when and why you realized what you needed to do and how you felt when you solved or did not solve the problem. Following this you will also be asked to try to adapt problems to try and make up ones of your own. You will be asked to fill in a questionnaire, before and after the workshops. This will involve questions on what you enjoy about mathematics and how you find mathematics in school and during the workshops. I will make an audio recording of your discussions in both workshops and examine how you went about solving the problems. I will also be asking for 5 or 6 students from the class to volunteer to participate in a group interview on their experience of the workshops. This will take place within 1-2 weeks after the workshops and will be audio recorded for my records. Your decision to take part in the interview will be made after the workshops and you will be given a separate consent form for this.

Why have you been asked to take part?

You have been asked because you have shown an interest in problem-solving and challenging yourself in Mathematics. The aim of the project is to test the experience of enthusiastic and promising mathematicians. Given your interest in Mathematics your opinion is very valuable to me.

Do you have to take part?

No, you are under no obligation whatsoever to take part in this research. However, I hope that you will agree to take part and give some of your time to participate in the workshops and group discussions. I understand that it is a busy time of year for you and it is entirely up to you to decide whether or not you would like to take part. If you decide to do so, you will be asked to sign a consent form and will be given a copy of it and the information sheet for your own records. If you decide to take part you can withdraw at any stage of the project without feeling you have to give a reason.

What information will be collected?

You will be asked to take notes as you carry out your problem-solving and there will be an audio recording of the workshops to show me the methods you used during the problem solving and the discussions you had about the problem. This will help me to understand the level of questions you enjoy and to develop more challenging tasks if needed. I will be asking for a small number of students to take part in a group interview afterwards which will be audio recorded. There will be no specific questions addressed to you in the interview and any contribution you make to the discussion will be voluntary.

Will your participation in the study be kept confidential?

Yes, all information that is collected about you during the course of the research will be kept confidential. No names will be identified at any time. At the start of the first workshop you will be given a Student Identity number and only you will have access to this number. When the interviews and audio recordings are over I will be replacing all names with the ID numbers so that everything you say will be anonymous. Before any information is written up you will have the opportunity to read my transcript and use your ID number to check my comments and make sure you are happy with what has been written. If you are not happy with any statement you may ask for it not to be included in my final writings. All hard copy information will be held in a locked cabinet in Maynooth University and will be accessed only by Judi Mills and Dr Ann O'Shea. No information will be distributed to any other individual. If you so wish, the data that you provide can also be made available to you after the project has been completed.

What will happen to the information which you give?
All the information you provide will be kept at Maynooth University in such a way that it will not be possible to identify you. On completion of the research, the data will be retained on University files for ten years, after which all data will be destroyed.

What will happen to the results?
The research from these workshops will be analysed and used to provide me with a better insight upon what type of tasks motivate mathematically promising students at second level in Ireland. This will be written up in my doctoral dissertation for Maynooth University and may be presented at National and International conferences and may be published in mathematics educational journals. A copy of the research findings will be made available to you upon request.

What are the possible disadvantages of taking part?
I think this will be a positive experience for you rather than a negative one. The only area that may cause any concern would be the possibility of you feeling worried, or upset, if the tasks set are too challenging. I would like to remind you that the purpose of these workshops is to help me assess the level of tasks suitable for challenging students and to observe how their problem-solving strategies evolve. Consequently, it is just as important to know if the tasks are too difficult as too straightforward and you should not be afraid to say that you need help to get started or continue with the questions.

What if there is a problem?
At the end of the workshop, I will discuss with you how you found the experience and how you are feeling. If you have any worries following the workshops, I can arrange a time to discuss the issues or mathematics with you individually and go through the areas that have caused you concern. Alternatively, I can arrange for your mathematics teacher to talk to you or for you to contact the student liaison person in your school if you are more comfortable with that. You may contact my supervisor if you feel the research has not been carried out as described above.

Any further queries?
If you need any further questions about this research you can contact me, Judi Mills at judi.mills@mu.ie

If you agree to take part in the study, please complete and sign the consent form overleaf.

Thank you for taking the time to read this.

Maynooth University
National University of Ireland Maynooth

Consent Form for Students

I …………………………………… agree to participate in Judi Mills's research study titled A Study of the Experience of Promising Mathematics Students at Second level in Ireland.

Please tick each statement below:

The purpose and nature of the study has been explained to me verbally & in writing. ☐

I have been able to ask questions, which were answered satisfactorily. ☐

I am participating voluntarily. ☐

I give permission for the following aspects of the project:

- To complete a questionnaire before and after the workshops relating to my experience of mathematics in school and during the workshops. ☐
- To participate in the problem solving of tasks in the workshops. ☐
- To participate in the problem posing of tasks in the workshops. ☐
- To my written problem solving being retained by Judi for her research. ☐
- To the making of an audio recording of our discussions within the group of students as we problem solve. ☐

I understand that I can withdraw from the study, without repercussions, at any time, whether that is before it starts or while I am participating. ☐

I understand that I can withdraw permission to use the data right up to submission of Judi's thesis in November 2022. ☐

It has been explained to me how my data will be managed and that.
I may access it on request. ☐

Please sign the form overleaf

Participant Name in block capitals ……………………………………………..

Participant's signature …………………………………….

Date……………….

I the undersigned have taken the time to fully explain to the above participant the nature and purpose of this study in a manner that they could understand. I have explained the risks involved as well as the possible benefits. I have invited them to ask questions on any aspect of the study that concerned them.

Researcher's name (in block letters) JUDI MILLS

Researcher's Signature Judi Mills……………………………………. Date: 8th January 2020

Researcher Name in block capitals: JUDI MILLS

If during your participation in this study you feel the information and guidelines that you were given have been neglected or disregarded in any way, or if you are unhappy about the process, please contact the Secretary of the Maynooth University Ethics Committee at
research.ethics@mu.ie or +353 (0)1 708 6019.
Please be assured that your concerns will be dealt with in a sensitive manner.

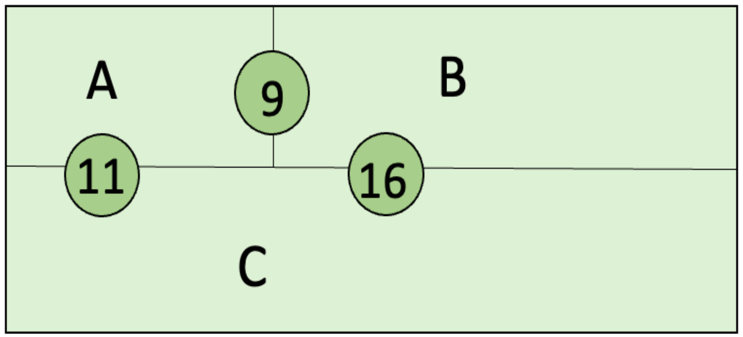
Appendix B: Workshop Tasks

TASK 1

TASK 1 FIELD OF DREAMS – LEVEL A

FIELD 1

Several children are playing different ball games in fields that are next to each other. On a fence around the fields is a sign with a number on it that gives the total number of students in the two fields. For example, this diagram shows that there is a total of 9 children playing in Fields A and B and a total of 16 playing in fields B and C.



- How many students are playing in the fields?

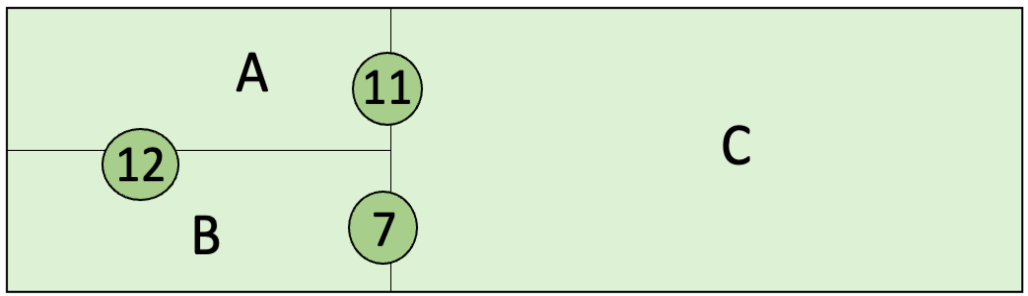
A ________ B ________ C ________ Total of $A + B + C =$ ________

FIELD OF DREAMS

FIELD 2

- How many students are playing in the fields?

A ________ B __________ C _____________ Total of $A + B + C =$ _________

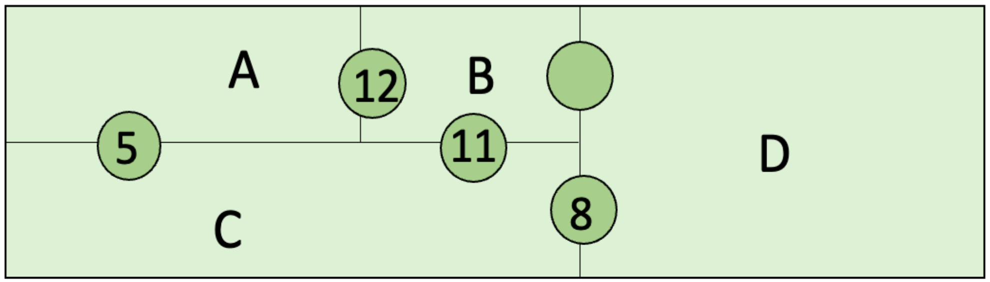


- Make up your own Field of Dreams problem and swap with a friend.

FIELD OF DREAMS – LEVEL B

FIELD 3

Several children are playing different ball games in fields that are next to each other. On a fence around the fields is a sign with a number on it that gives the total number of students in the two fields. For example, this diagram shows that there is a total of 12 children playing in Fields A and B and a total of 5 children playing in Fields A and C.

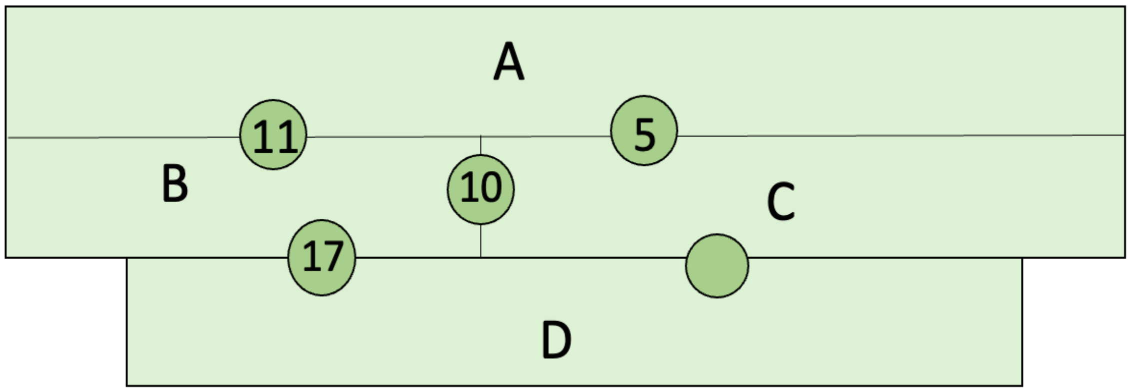


- What number should be the empty circle?
- How many children are there in the four fields?

FIELD OF DREAMS

FIELD 4

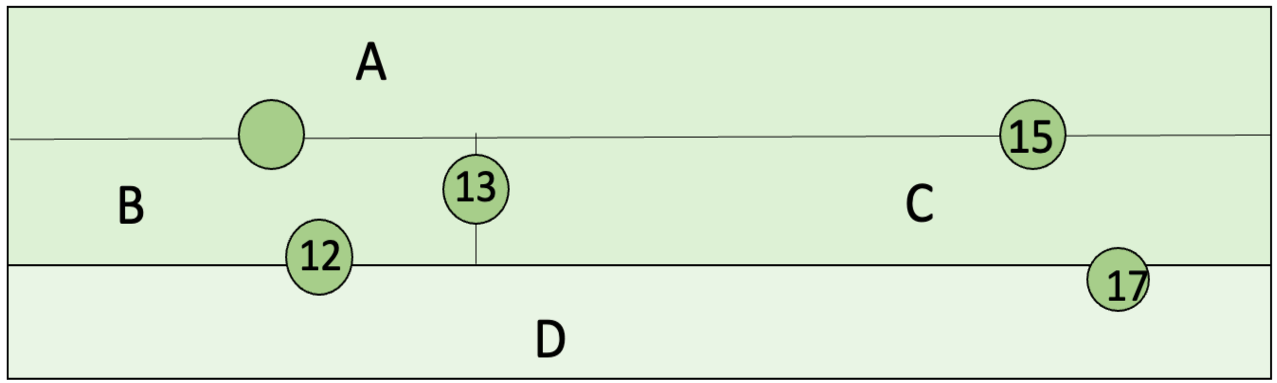
Several children are playing different ball games in fields that are next to each other. On a fence around the fields is a sign with a number on it that gives the total number of students in the two fields. In the following diagrams, there are four fields with signs telling the total number of children playing in the two fields next to the sign.



- What number should be the empty circle?
- What is the total number of children in each of the fields above?

FIELD OF DREAMS

FIELD 5



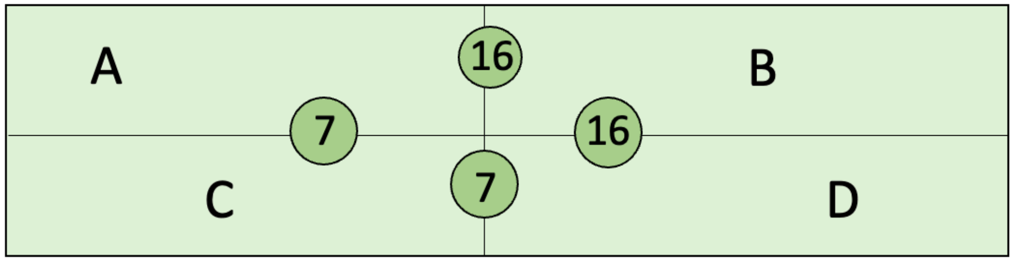
- What number should be the empty circle?
- What is the total number of children in each of the fields above?
- Make up your own Field of Dreams problem and swap with a friend.

FIELD OF DREAMS – LEVEL C

FIELD 6

Several children are playing different ball games in fields that are next to each other. On a fence around the fields is a sign with a number on it that gives the total number of students in the two fields. For example, this diagram shows that there is a total of 16 children playing in Fields A and B and a similar total of 16 in Fields B and D. There is a total of 7 children playing in Fields A and C and a total of 7 in fields C and D.

- How many children are playing in each of the fields?



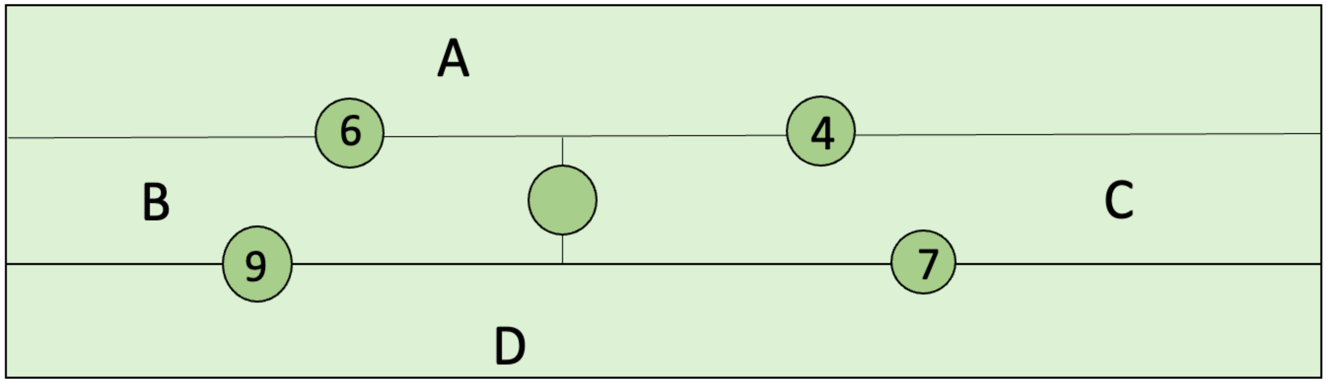
- Is there more than one possible answer? ___________ Why? ________________
- How many total children are playing in the four fields? ___________
- Is there more than one possible answer? _______________ Why? ________________

FIELD OF DREAMS

FIELD 7

In the following diagram, there are 4 fields with signs telling the total number of children playing in the two fields next to the sign.

- What number(s) might be in the empty circle? ______________



- Is there more than one possible answer? ___________ Why? ________________
- What is the total number of children that might be playing in the fields above? ___________
- Is there more than one possible answer? _______________ Why? ________________

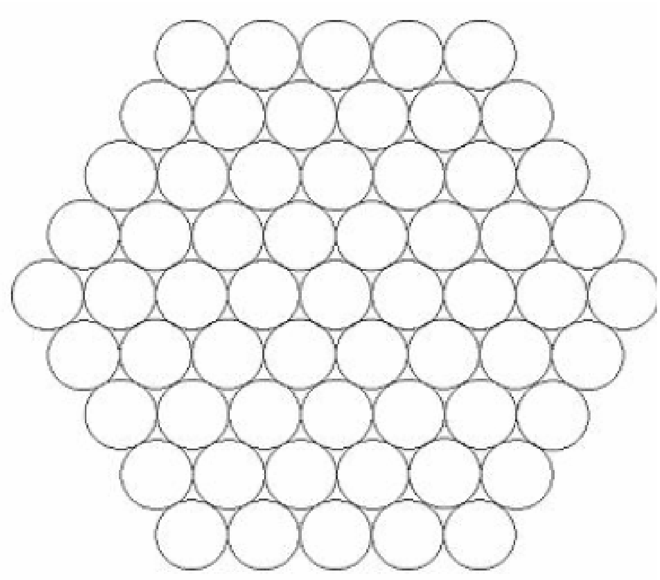
TASK 2

STEEL CABLES

Phase 1

Cables can be made stronger by compacting them together in a hexagonal formation.

Here is a 'Size 5' cable made up of 61 strands:



How many strands are needed for a 'Size 10' cable?

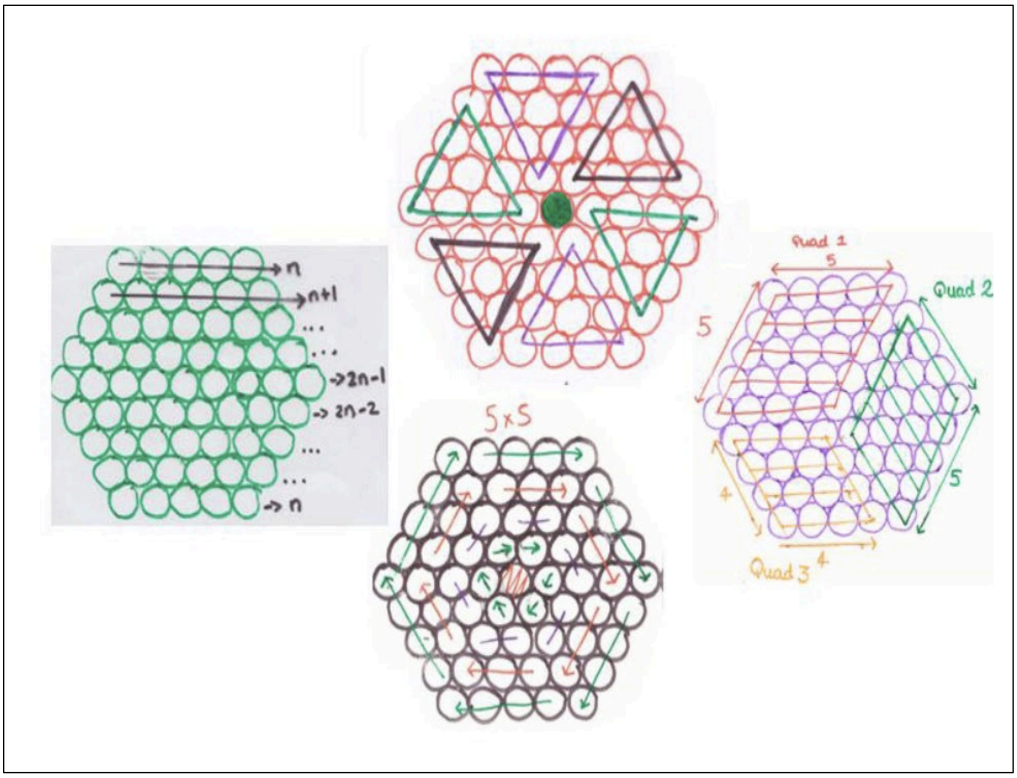
How many strands are needed for a 'Size n' cable?

Can you justify your answer?

Phase 2

Once you have thought about the problem yourself, look at the diagrams below which show 4 possible approaches.

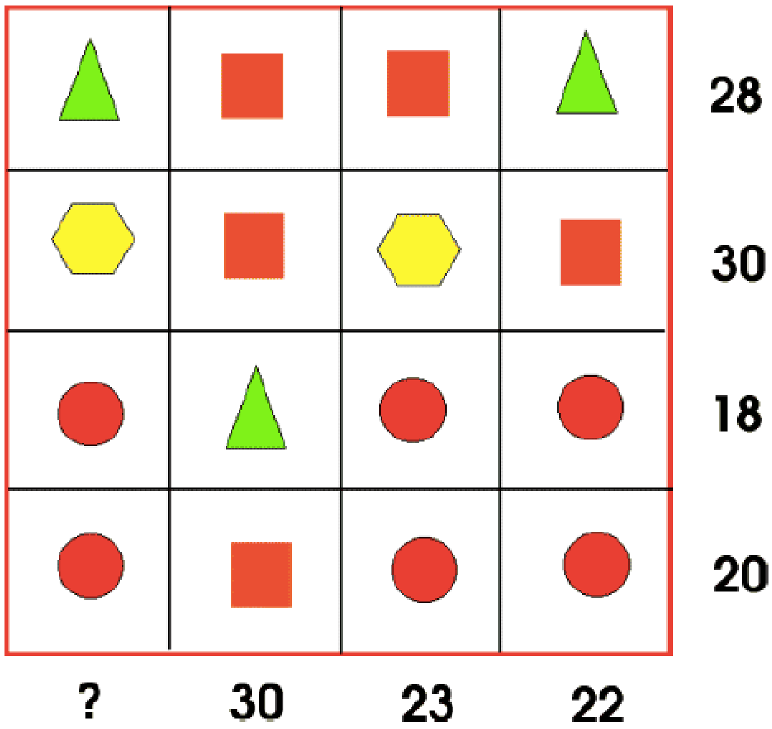
Can you explain how you would use each of the diagrams below to work out the number of strands for size 5, size 10 and size n cables.



TASK 3

What’s it Worth?

Each symbol has a numerical value. The total for the symbols is written at the end of each row and column.



- **Can you find the missing total that should go where the question mark is?**

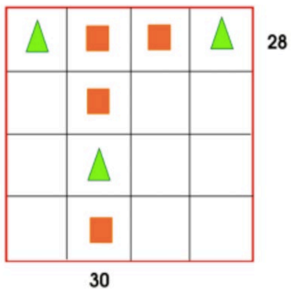
- **Can you find more than one way to do it?**

WHAT IS THE DIFFERENCE BETWEEN AN ANSWER AND A SOLUTION ?

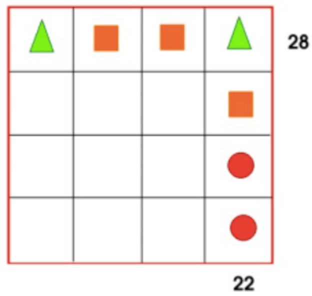
All of the following methods got the same answer of 21 for the question mark.

- Examine each of them and decide which one you prefer
- Which one did you find easiest to understand?
- Which method would you use if you were asked to solve a similar problem?

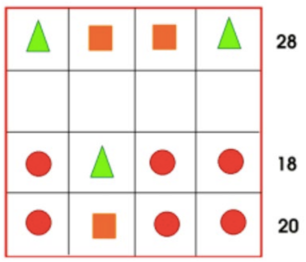
METHOD 1



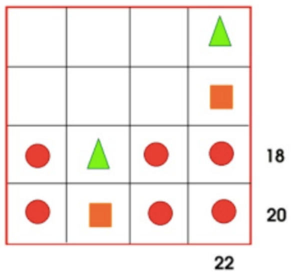
METHOD 2



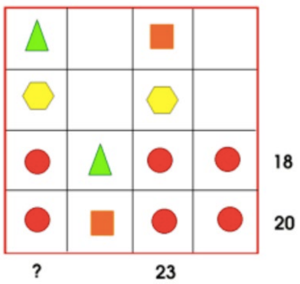
METHOD 3



METHOD 4



METHOD 5



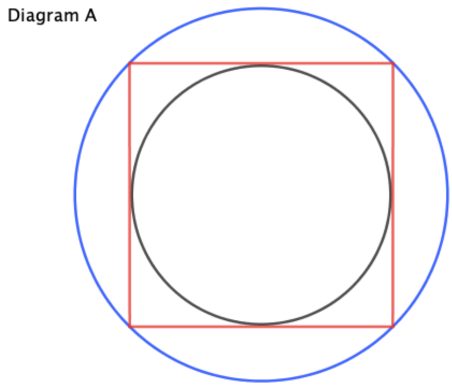
METHOD 6

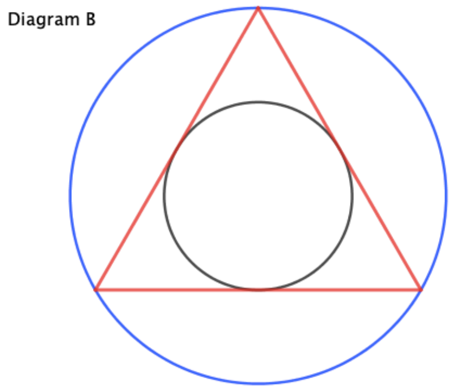
There is also another really quick way …

TASK 4

AREA OF CIRCLES

What is the ratio of the area of the larger circle to the area of the smaller circle in each diagram?





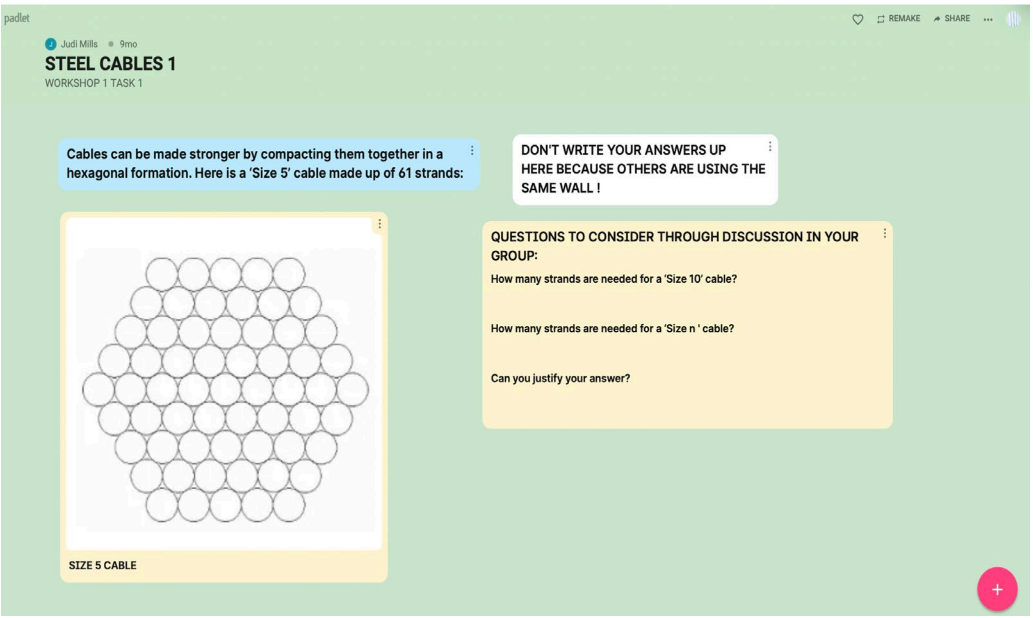
- This is the basis of the Method of Exhaustion credited to a famous Greek Mathematician Eudoxus (c408-355bce).

- How did the ancient Greeks use this to find an approximation for π?

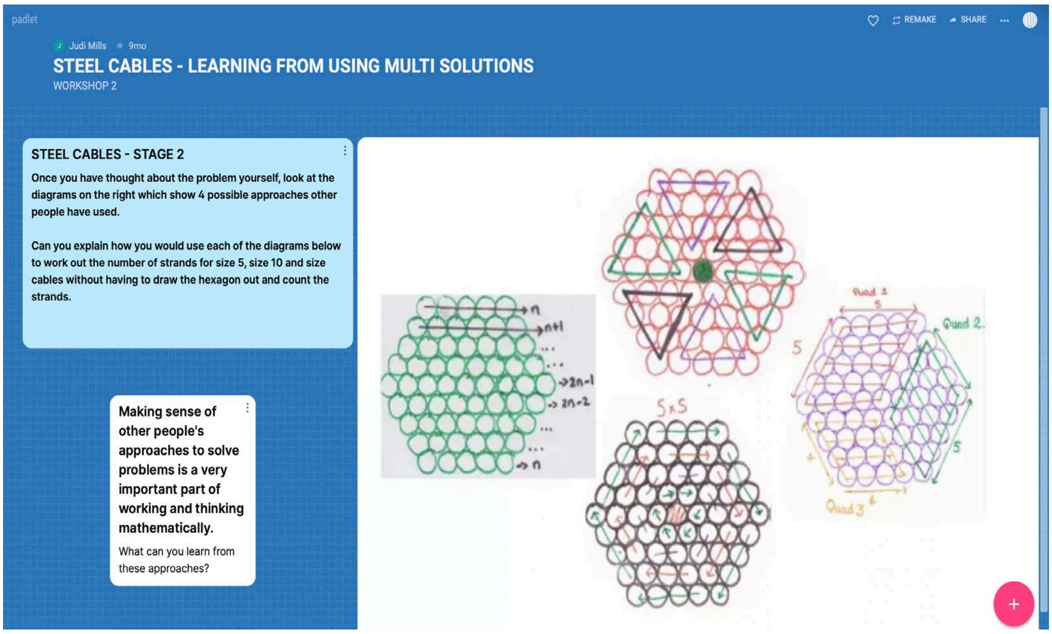
PADLETS FOR ONLINE WORKSHOPS

Task 2: Steel cables

Phase 1

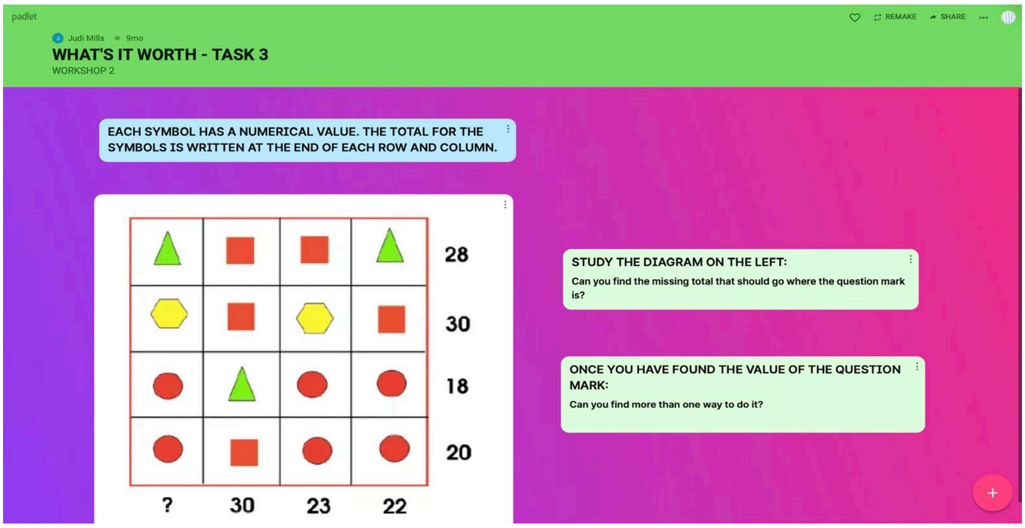


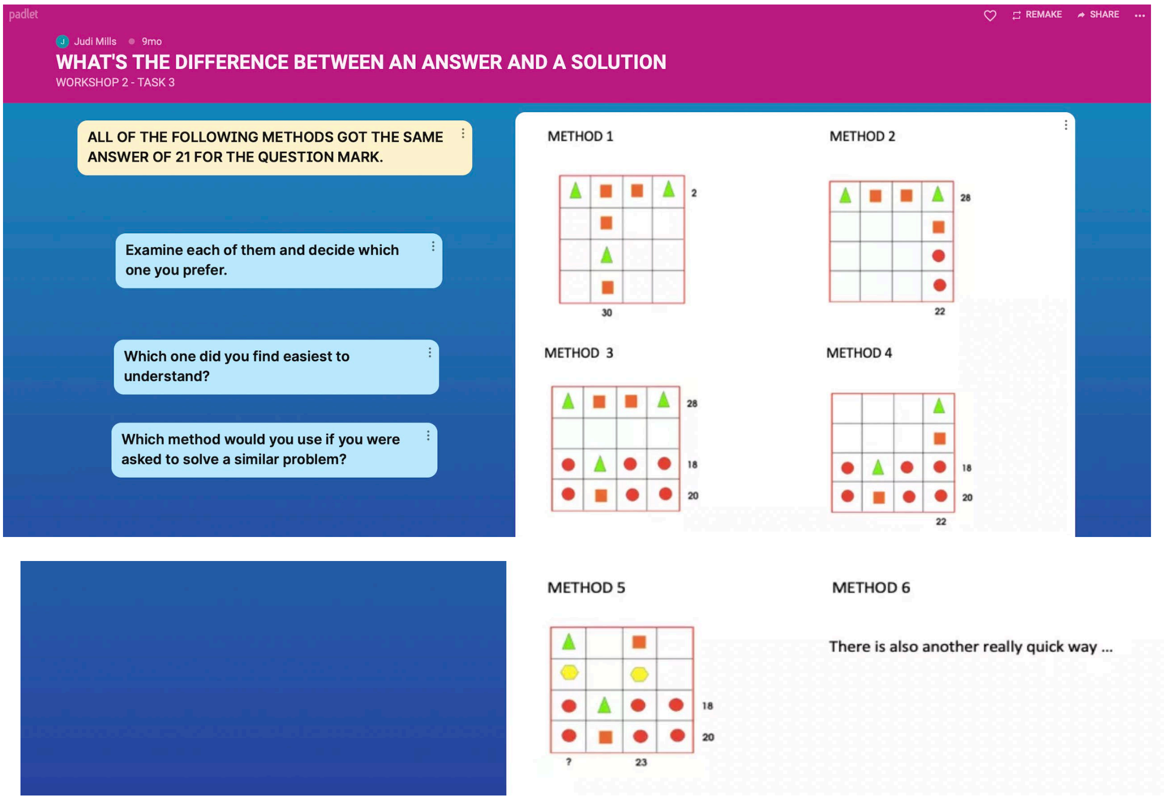
Phase 2



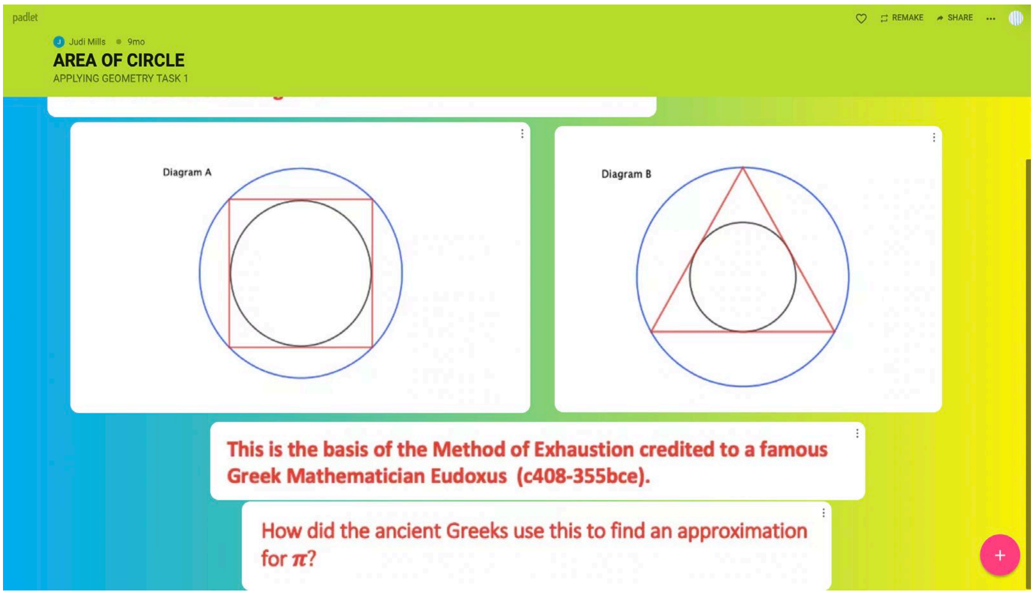
Stage 1 Stage 2

Task 3: What's It Worth





Task 4: Area of Circles



Appendix C: Researchers' Scales

Kloosterman, P., & Stage, F. (1992)

Indiana Mathematics Belief Scales

Belief I: **I *can solve time-consuming mathematics problems.***

- \+ **Math problems that take a long time don't bother me.**
- \+ **I feel I can do math problems that take a long time to complete.**
- \+ **I find I can do hard math problems if I just hang in there.**
- \- If I can't do a math problem in a few minutes, I probably can't do it at all.
- \- If I can't solve a math problem quickly, I quit trying.
- \- I'm not very good at solving math problems that take a while to figure out.

Belief 2: ***There are word problems that cannot be solved with simple, step-by-step procedures.***

- \+ **There are word problems that just can't be solved by following a predetermined sequence of steps.**
- \+ **Word problems can be solved without remembering formulas.**
- \+ **Memorizing steps is not that useful for learning to solve word problems.**
- \- Any word problem can be solved if you know the right steps to follow.
- \- Most word problems can be solved by using the correct step-by-step procedure.
- \- Learning to do word problems is mostly a matter of memorizing the right steps to follow.

Belief 3: ***Understanding concepts is important in mathematics.***

- \+ **Time used to investigate why a solution to a math problem works is time well spent.**
- \+ **A person who doesn't understand why an answer to a math problem is correct hasn't really solved the problem.**
- \+ **In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.**
- \- It's not important to understand why a mathematical procedure works as long as it gives a correct answer.
- \- Getting a right answer in math is more important than understanding why the answer works.
- \- It doesn't really matter if you understand a math problem if you can get the right answer.

Belief 4: ***Word problems are important in mathematics.***

- \+ **A person who can't solve word problems really can't do math.**
- \+ **Computational skills are of little value if you can't use them to solve word problems.**
- \+ **Computational skills are useless if you can't apply them to real life situations.**
- \- Learning computational skills is more important than learning to solve word problems.
- \- Math classes should not emphasize word problems.
- \- Word problems are not a very important part of mathematics.

Belief 5: ***Effort can increase mathematical ability.***

- \+ **By trying hard, one can become smarter in math.**
- \+ **Working can improve one's ability in mathematics.**
- \+ **I can get smarter in math by trying hard.**
- \+ **Ability in math increases when one studies hard.**
- \+ **Hard work can increase one's ability to do math.**
- \+ **I can get smarter in math if I try hard.**

<u>Questions take from researchers' surveys that were used for my pre-survey questions:</u>

Kloosterman & Stage:	Beliefs 1 and 5 above were used as a basis for some of my pre-survey questions.
Lim & Chapman:	ENJ 6 ENJ 7 MOT 1 MOT 5 SC 3 SC 10

Lim & Chapman, (2013). Survey Questions

Short form of the attitudes toward mathematics inventory

Table 1 Descriptive statistics for the ATMI

Label	Full item statement	Subs le 1		Subsample 2	
		M	SD	M	SD
ENJ1	I get a great deal of satisfaction out of solving a mathematics problem.	3.63	1.09	3.74	0.97
ENJ2	I have usually enjoyed studying mathematics in school.	3.44	1.04	3.47	1.02
ENJ3	Mathematics is dull and boring.	3.51	1.15	3.53	1.12
ENJ4	I like to solve new problems in mathematics.	3.30	1.03	3.40	1.00
ENJ5	I would prefer to do an assignment in mathematics than to write an essay.	3.66	1.33	3.62	1.33
ENJ6	I really like mathematics.	3.14	1.10	3.18	1.07
ENJ7	I am happier in a mathematics class than in any other class.	2.72	1.06	2.82	1.06
ENJ8	Mathematics is a very interesting subject.	3.28	1.00	3.31	0.96
ENJ9	I am comfortable expressing my own ideas on how to look for solutions to a difficult problem in mathematics.	3.04	0.97	3.12	0.93
ENJ10	I am comfortable answering questions in mathematics class.	3.13	0.95	3.19	0.91
MOT1	I am confident that I could learn advanced mathematics.	3.34	1.08	3.32	1.05
MOT2	I would like to avoid using mathematics in university.	3.32	1.20	3.27	1.21
MOT3	I am willing to take more than the required amount of mathematics.	2.89	1.07	2.93	1.07
MOT4	I plan to take as much mathematics as I can during my education.	2.76	1.03	2.78	1.07
MOT5	The challenge of mathematics appeals to me.	3.05	1.11	3.12	1.06
SC1	Mathematics is one of my most dreaded subjects.	3.59	1.31	3.56	1.30
SC2	My mind goes blank and I am unable to think clearly when working with mathematics.	3.69	1.09	3.69	1.08
SC3	Studying mathematics makes me feel nervous.	3.62	1.06	3.59	1.03
SC4	Mathematics makes me feel uncomfortable.	3.61	1.05	3.60	1.07
SC5	I am always under a terrible strain in a mathematics class.	3.57	1.03	3.57	1.03
SC6	When I hear the word mathematics, I have a feeling of dislike.	3.72	1.06	3.67	1.09
SC7	It makes me nervous to even think about having to do a mathematics problem.	3.63	0.98	3.65	0.96
SC8	Mathematics does not scare me at all.	2.81	1.07	2.83	1.03
SC9	I expect to do fairly well in any mathematics class I take.	3.24	1.03	3.27	1.03
SC10	I am always confused in my mathematics class.	3.51	0.98	3.53	0.97
SC11	I have a lot of self-confidence when it comes to mathematics.	2.83	0.99	2.90	0.98
SC12	I am able to solve mathematics problems without too much difficulty.	2.97	0.95	2.97	0.91
SC13	I feel a sense of insecurity when attempting mathematics.	3.38	1.07	3.40	1.05
SC14	I learn mathematics easily.	3.02	0.92	3.04	0.92
SC15	I believe I am good at solving mathematics problems.	3.00	0.96	3.07	0.94
VAL1	Mathematics is a very worthwhile and necessary subject.	3.64	0.96	3.73	0.92
VAL2	I want to develop my mathematical skills.	3.59	0.93	3.67	0.88
VAL3	Mathematics helps to develop the mind and teaches a person to think.	3.66	0.93	3.72	0.92
VAL4	Mathematics is important in everyday life.	3.46	0.98	3.52	0.97
VAL5	Mathematics is one of the most important subjects for people to study.	3.54	0.95	3.65	0.94
VAL6	College mathematics lessons would be very helpful no matter what I decide to study in future.	3.43	0.96	3.47	1.00
VAL7	I can think of many ways that I use mathematics outside of school.	2.89	1.04	2.92	1.03
VAL8	I think studying advanced mathematics is useful.	3.28	1.03	3.29	1.02
VAL9	I believe studying mathematics helps me with problem solving in other areas.	3.37	0.95	3.42	0.93
VAL10	A strong mathematics background could help me in my professional life.	3.54	0.95	3.57	0.94

Appendix D: Percentages for pre-workshop survey questions.

	Please tick the box that best suits your thoughts	1 ☺	2	3 😑	4	5 ☹
1	I enjoy going to mathematics class.	27.6%	41.4%	5.9%	1.7%	3.4%
2 (R)	Studying mathematics makes me nervous.	1.7%	10.3%	13.8%	36.2%	37.9%
3 (R)	I am often bored in class while the teacher explains the solution to other students.	31%	39.7%	12.1%	17.2%	0%
4	I get frustrated when given multiple versions of the same question to do.	10.3%	37.9%	25.9%	20.7%	5.2%
5	I enjoy problems best when I am not given any hints by the teacher.	13.8%	31%	39.7%	12.1%	3.4%
6	I like spending time solving mathematics problems.	17.2%	48.3%	25.9%	5.2%	3.4%
7	I believe I have good mathematics problem-solving skills.	20.7%	51.7%	20.7%	5.2%	1.7%
8	I find the work in mathematics class demanding.	3.4%	8.6%	15.5%	60.3%	12.1%
9	I enjoy having to think for myself when solving a problem.	32.8%	62.1%	5.2%	0%	0%
10	I like to be given hard problems to solve rather than easy ones.	20.7%	51.7%	20.7%	6.9%	0%

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
11	Mathematics is one of my favourite subjects.	27.6%	44.8%	10.3%	13.8%	3.4%
12	I am confident that I can solve problems that take a long time to solve.	17.2%	41.4%	34.5%	6.9%	0%
13	I am usually given an example to explain a new topic before being given questions on it.	72.4%	19%	5.2%	3.4%	0%
14	I do not enjoy being given lots of similar questions to do.	22.4%	32.8%	20.7%	22.4%	1.7%
15	I find mathematics interesting.	31%	51.7%	13.8%	1.7%	1.7%
16	I prefer to be given challenging problems with an unrestricted time to solve them.	48.3%	41.4%	5.2%	5.2%	0%
17 (R)	I am always confused in mathematics class.	0%	3.4%	8.6%	60.3%	27.6%
18	My favourite types of questions are those that I know how to do immediately.	13.8%	41.4%	29.3%	13.8%	1.7%
19	I believe that hard work will improve my mathematics grades.	56.9%	36.2%	3.4%	3.4%	0%
20	The best way to improve at mathematics is to do lots of similar questions on the same topic until I perfect the method for solving them.	29.3%	37.9%	17.2%	12.1%	3.4%

	Please tick the box that best suits your thoughts	**1** ☺	**2**	**3** 😐	**4**	**5** ☹
21	I enjoy doing mathematics problems.	32.8%	51.7%	12.1%	1.7%	1.7%
22	I feel challenged to my maximum ability by mathematics problems given to me in class.	0%	20.7%	17.2%	41.4%	20.7%
23	I am motivated most by problems where I have to think about the method.	15.5%	48.3%	25.9%	10.3%	0%
24	Most of the time we work on mathematics problems from the textbook.	50%	36.2%	6.9%	3.4%	3.4%
25	I can improve in mathematics if I work hard.	65.5%	29.3%	1.7%	3.4%	0%
26	I enjoy the challenge of mathematics.	36.2%	43.1%	17.2%	3.4%	0%
27	I am often given challenging problems in mathematics class	6.9%	25.9%	17.2%	43.1%	6.9%
28	My ability in mathematics will increase with more studying.	36.2%	46.6%	13.8%	3.4%	0%
29	I am happier in a mathematics class than in any other class.	6.9%	15.5%	44.8%	25.9%	6.9%
30	I am rarely given problems to do on new topics without being given an example first.	56.9%	36.2%	5.2%	0%	1.7%
31	I find mathematics class challenging	5.2%	8.6%	31%	44.8%	10.3%

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
32	I enjoy being given mathematics problems where I do not immediately know which method to use.	5.2%	41.4%	36.2%	13.8%	3.4%
33	I am confident in my ability to learn mathematics.	36.2%	56.9%	5.2%	0%	1.7%
34	I believe that I would become a better mathematician if school mathematics was more challenging.	34.5%	24.1%	29.3%	8.6%	3.4%
35	I would rate my ability as a mathematician as above average in my school year.	25.9%	37.9%	25.9%	6.9%	3.4%
36	We do challenging problems in mathematics class.	0%	24.1%	25.9%	39.7%	10.3%
37	I would like to be given more challenging problems in class that make me think for myself.	19%	51.7%	17.2%	10.3%	1.7%
38	Most problems I do in school can be answered by recall of examples and formulae.	37.9%	55.2%	3.4%	1.7%	1.7%
39	It is possible to improve your mathematics aptitude with hard work.	43.1%	37.9%	8.6%	5.2%	5.2%
40	I am often asked to solve a problem in more than one way.	3.4%	19%	13.8%	48.3%	15.5%

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
41	What we do in mathematics class suits my ability.	8.6%	27.6%	22.4%	32.8%	8.6%
42	I prefer to think for myself than to be shown the method first by the teacher.	24.1%	25.9%	24.1%	20.7%	5.2%
43	I would like to be given more assessments which did not rely on me learning classwork by rote.	19%	24.1%	36.2%	19%	1.7%
44	I believe that I could increase my achievements in mathematics with more practice on challenging problems.	43.1%	50%	5.2%	1.7%	0%
45	I feel confident when presented with a problem that I have not seen before that I will be able to solve it.	3.4%	36.2%	41.4%	19%	0%
46	I often get stuck in mathematics class.	6.9%	1.7%	24.1%	50%	17.2%
47	I believe I would do better in mathematics assessments if I was given problems that did not rely on memorizing notes.	36.2%	32.8%	19%	8.6%	3.4%

Appendix E: Scales for pre-workshop survey questions

Scale 1: Enjoyment of mathematics		
Enjoyment 1	I enjoy going to mathematics class	**Q1**
Enjoyment 2	I like spending time solving mathematics problems.	**Q6**
Enjoyment 3	Mathematics is one of my favourite subjects.	**Q11**
Enjoyment 4	I find mathematics interesting.	**Q15**
Enjoyment 5	I enjoy doing mathematics problems.	**Q21**
Enjoyment 6	I enjoy the challenge of mathematics.	**Q26**
Enjoyment 7	I am happier in a mathematics class than in any other class.	**Q29**
Scale 2: Self-perception of ability in mathematics		
Self-Perception 1	I believe I have good mathematics problem-solving skills.	**Q7**
Self-Perception 2	I am confident that I can solve problems that take a long time to solve.	**Q12**
Self-Perception 3	I am always confused in mathematics class.	**Q17**
Self-Perception 4	I am confident in my ability to learn mathematics	**Q33**
Self-Perception 5	I would rate my ability as a mathematician as above average in my school year.	**Q35**
Self-Perception 6	I feel confident when presented with a problem that I have not seen before that I will be able to solve it.	**Q45**
Scale 3: Level of challenge felt in secondary school mathematics class		
Challenge 1	I find the work in mathematics class demanding.	**Q8**
Challenge 2	I feel challenged to my maximum ability by mathematics problems given to me in class.	**Q22**
Challenge 3	I am often given challenging problems in mathematics class	**Q27**

Challenge 4	I find mathematics class challenging	**Q31**
Challenge 5	We do challenging problems in mathematics class	**Q36**
Challenge 6	I often get stuck in mathematics class	**Q46**
Scale 4: Motivation by problems that involve thinking.		
Motivated by Thinking 1	I enjoy problems best when I am not given any hints by the teacher.	**Q5**
Motivated by Thinking 2	I enjoy having to think for myself when solving a problem.	**Q9**
Motivated By Thinking 3	I am motivated most by problems where I have to think about the method.	**Q23**
Motivated by Thinking 4	I enjoy being given mathematics problems where I do not immediately know which method to use.	**Q32**
Motivated by Thinking 5	I would like to be given more challenging problems in class that make me think for myself.	**Q37**
Motivated by Thinking 6	I prefer to think for myself than to be shown the method first by the teacher.	**Q42**
Scale 5: Teacher / Textbook led Methodology		
Teacher Textbook Led 1	I am usually given an example to explain a new topic before being given questions on it.	**Q13**
Teacher Textbook Led 2	Most of the time we work on mathematics problems from the textbook.	**Q24**
Teacher Textbook Led 3	I am rarely given problems to do on new topics without being given an example first.	**Q30**
Teacher Textbook Led 4	Most problems I do in school can be answered by recall of examples and formulae.	**Q38**

Scale 6: Do students believe they can improve their mathematical ability.		
How To Improve 1	I believe that hard work will improve my mathematics grades.	**Q19**
How To Improve 2	I can improve in mathematics if I work hard.	**Q25**
How To Improve 3	My ability in mathematics will increase with more studying.	**Q28**
How To Improve 4	I believe that I would become a better mathematician if school mathematics was more challenging.	**Q34**
How To Improve 5	It is possible to improve your mathematics aptitude with hard work.	**Q39**
How To Improve 6	I believe that I could increase my achievements in mathematics with more practice on challenging problems.	**Q44**
How To Improve 7	I believe I would do better in mathematics assessments if I was given problems that did not rely on memorizing notes.	**Q47**
Miscellaneous questions removed from scales		
Ambiguous Q1	My favourite types of questions are those that I know how to do immediately.	**Q18**
Ambiguous Q2	The best way to improve at mathematics is to do lots of similar questions on the same topic until I perfect the method for solving them.	**Q20**
Ambiguous Q3	I am often asked to solve a problem in more than one way.	**Q40**
Ambiguous Q4	I would like to be given more assessments which did not rely on me learning classwork by rote.	**Q43**

Appendix F: Percentages for post-workshop survey questions

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
B (1)	I enjoyed the tasks of the workshops.	41.9%	45.2%	5.4%	0%	0%
B (2)	Having to think about the method was different to what usually happens when we use the textbook.	64.4%	27.6%	5.7%	2.3%	0%
B (3)	The tasks in the workshops were harder than those we do in school.	40.0%	41.2%	15.3%	1.2%	2.4%
B (4)	I would be interested in doing the tasks in the workshop in my own time.	16.1%	33.3	40.2%	10.3%	0%
B (5)	Multi-solution tasks stretched me more than just finding one answer to a task.	50.6%	46.0%	1.1%	1.1%	1.1%
B (6)	I found the experience of the workshops very enjoyable.	40.9%	43.0%	8.6%	0%	0%
B (7)	We do not normally solve questions using more than one method in school.	48.3%	31.0%	14.9%	5.7%	0%
B (8)	School mathematics is not as challenging as the mathematics in the workshops.	38.4%	29.1%	22.1%	5.8%	4.7%
B (9)	The tasks in the workshop encouraged me to persevere when stuck.	43.7%	46.0	8.0%	2.3%	0%
B (10)	Solving unfamiliar mathematics problems improves my problem-solving skills.	57.0%	43.0%	0%	0%	0%

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
B (11)	The mathematics was fun in the workshops.	41.1%	50.6%	6.9%	0%	1.1%
B (12)	We are normally given an example of a task first, in school, so we do not have to think of the method.	60.9%	25.3%	11.5%	2.3%	0%
B (13)	The tasks in the workshops pushed me more to my maximum ability.	46.5%	41.9%	11.6%	0%	0%
B (14)	I found it useful to use different methods to solve problems.	42.5%	44.8%	9.2%	1.1%	2.3%
B (15)	When I solve unfamiliar mathematics problems, I feel more confident that, in the future, I will be able to figure out what other problems are asking me.	46.5%	46.5%	7.0%	0%	0%
B (16)	I enjoyed having to think hard during the workshops.	37.6%	38.7%	12.9%	3.2%	0%
B (17)	The tasks in the workshops were very different to those we do in school.	52.9%	40.2%	4.6%	1.1%	0%
B (18)	I would like to do more of these tasks.	28.0%	49.5%	14.0%	1.1%	1.7%
B (19)	I enjoyed finding multi-solutions to problems more than just finding the answer one way.	34.5%	47.1%	13.8%	0%	4.6%
B (20)	I feel more confident I will find a solution to unfamiliar mathematics problems with perseverance.	40.7%	51.2%	8.1%	0%	0%

	Please tick the box that best suits your thoughts	1 ☺	2	3 😐	4	5 ☹
B (21)	I found the tasks of the workshops more interesting than mathematics tasks I am used to doing in school.	43.0%	33.3%	14.0%	2.2%	0%
B (22)	I found these tasks more challenging than the ones in textbooks.	46.5%	36.0%	15.1%	0%	2.3%
B (23)	These tasks reminded me about what I enjoy about mathematics.	28.7%	43.7%	20.7%	4.6%	2.3%
B (24)	The skills required to solve the tasks in the workshop forced me to think deeper about the problem.	54.0%	43.7%	1.1%	1.1%	0%
B (25)	When solving unfamiliar mathematics problems, I give up trying when I feel uncomfortable.	9.2%	20.7%	32.2%	24.1%	13.8%
B (26)	I have not done problems like these in school.	46.0%	34.5%	12.6%	5.7%	1.1%
B (27)	I had to think a lot more when solving the tasks in the workshops than in class.	59.3%	34.9%	2.3%	3.5%	0%
B (28)	I preferred doing the tasks in the workshops to those we do in school textbooks.	35.5%	34.4%	17.2%	5.4%	0%
B (29)	I have learned some useful new skills for problem-solving by solving tasks in different ways.	40.7%	52.3%	7.0%	0%	0%
B (30)	I feel more confident in my ability to solve unfamiliar mathematics problems.	45.9%	42.4%	10.6%	1.2%	0%

Appendix G: Post-Workshop Survey Scales

Scale 1: Questions regarding liking the tasks. **ENJOYMENT**		
	Q 1, 6, 11, 16, 21	Survey Question Number
1	I enjoyed the tasks of the workshops.	**1**
2	I found the experience of the workshops very enjoyable.	**6**
3	The mathematics was fun in the workshops.	**11**
4	I enjoyed having to think hard during the workshops.	**16**
5	I found the tasks of the workshops more interesting than mathematics tasks I am used to doing in school.	**21**
Scale 2: Questions regarding whether they found the tasks in the workshops different to those usually done in school. **TASK DIFFERENCE**		
	Q 2, 7, 12, 17, 26	Survey Question Number
6	Having to think about the method was different to what usually happens when we use the textbook.	**2**
7	We do not normally solve questions using more than one method in school.	**7**
8	We are normally given an example of a task first, in school, so we do not have to think of the method.	**12**
9	The tasks in the workshops were very different to those we do in school.	**17**
10	I have not done problems like these in school.	**26**

Scale 3: Questions regarding level of challenge felt during the workshops. **FEELING OF CHALLENGE**		
	Q 3, 8, 13, 22, 27,	Survey Question Number
11	The tasks in the workshops were harder than those we do in school.	**3**
12	School mathematics is not as challenging as the mathematics in the workshops.	**8**
13	The tasks in the workshops pushed me more to my maximum ability.	**13**
14	I found these tasks more challenging than the ones in textbooks.	**22**
15	I had to think a lot more when solving the tasks in the workshops than in class.	**27**
Scale 4: Questions regarding motivation. **MOTIVATION**		
	Q 4, 9, 18, 23, 28	Survey Question Number
16	I would be interested in doing the tasks in the workshop in my own time.	**4**
17	The tasks in the workshop encouraged me to persevere when stuck.	**9**
18	I would like to do more of these tasks.	**18**
19	These tasks reminded me about what I enjoy about mathematics.	**23**
20	I preferred doing the tasks in the workshops to those we do in school textbooks.	**28**

Scale 5: Questions regarding multi-solution tasks. **MST**		
	Q 5, 14, 19, 24, 29	Survey Question Number
21	Multi-solution tasks stretched me more than just finding one answer to a task.	**5**
22	I found it useful to use different methods to solve problems.	**14**
23	I enjoyed finding multi-solutions to problems more than just finding the answer one way.	**19**
24	The skills required to solve the tasks in the workshop forced me to think deeper about the problem.	**24**
25	I have learned some useful new skills for problem-solving by solving tasks in different ways.	**29**
Scale 6: Questions regarding changes in attitude to mathematics ability **SELF-PERCEPTION**		
	Q 10, 15, 20, 25R, 30	Survey Question Number
26	Solving unfamiliar mathematics problems improves my problem-solving skills.	**10**
27	When I solve unfamiliar mathematics problems, I feel more confident that, in the future, I will be able to figure out what other problems are asking me.	**15**
28	I feel more confident I will find a solution to unfamiliar mathematics problems with perseverance.	**20**
29	When solving unfamiliar mathematics problems, I give up trying when I feel uncomfortable.	**25 (Use R)**
30	I feel more confident in my ability to solve unfamiliar mathematics problems.	**30**

Appendix H: Interview Questions

Maynooth University
National University of Ireland Maynooth

Interview Questions for Students – Post Workshop

Questions for groups of 3-4 individuals who volunteer to participate in the interview.

1 How would you describe your experience to date of mathematics? Can you describe it in terms of level of difficulty, type of questions given, how long it takes you to think about the method required to solve the problems given?

2 What way is a new topic usually introduced in your class in school?

3 How often you experience not being able to identify a method required to solve the problem immediately?

4 What happens in your school class when students cannot solve a problem given for homework?

5 How often are you asked to solve problems using more than one method?

6 Did you find these problems different to those you see in school?
If so, in what way? Do you have a preference?

7 Did you enjoy doing these tasks? If so, what was it about them that you enjoyed?

8 Did you find the skills needed to solve these tasks different to those needed for solving tasks you do from textbooks? If so what is the difference in the skills?

9 What would you describe as a challenging mathematical problem?

10 Did you find the tasks in the workshop challenging? If so, did you enjoy the challenge? What did you like about them?

11 Is there anything you did not like about the problems given in the workshops?

12 Is there any particular problem you enjoyed more than others? What was it you liked about it?

13 Do you have a preference to solving problems using algebra/geometry/visual/other methods?

14 Would you like to try more of these problems?

15 Do you think these type of problems could be beneficial to students in school?

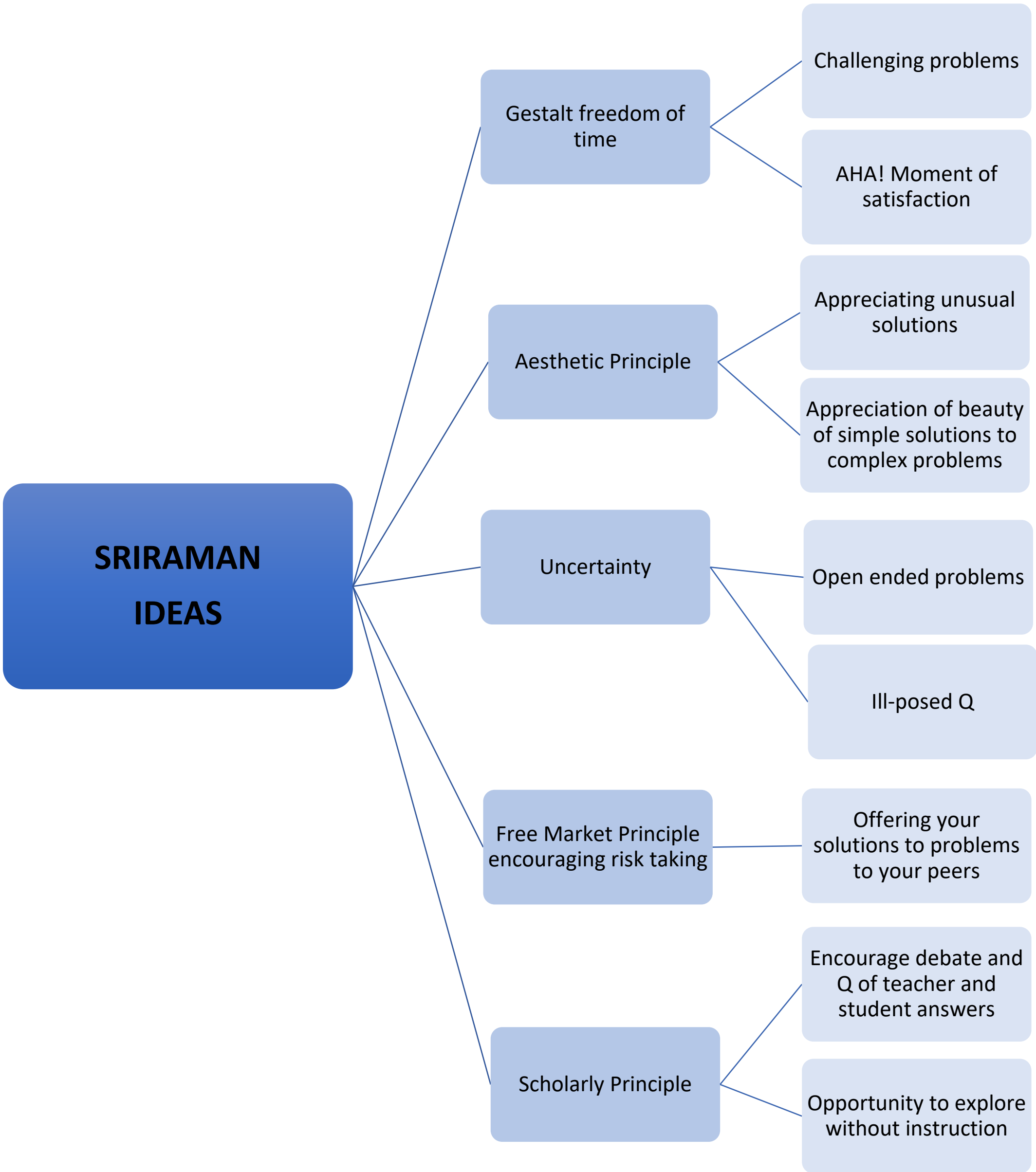
If so, can you explain what benefits you think they would have?
Do you think they would be helpful for revision and/or for developing understanding.

16 If you do not think they would be beneficial to students in schools can you explain why not?

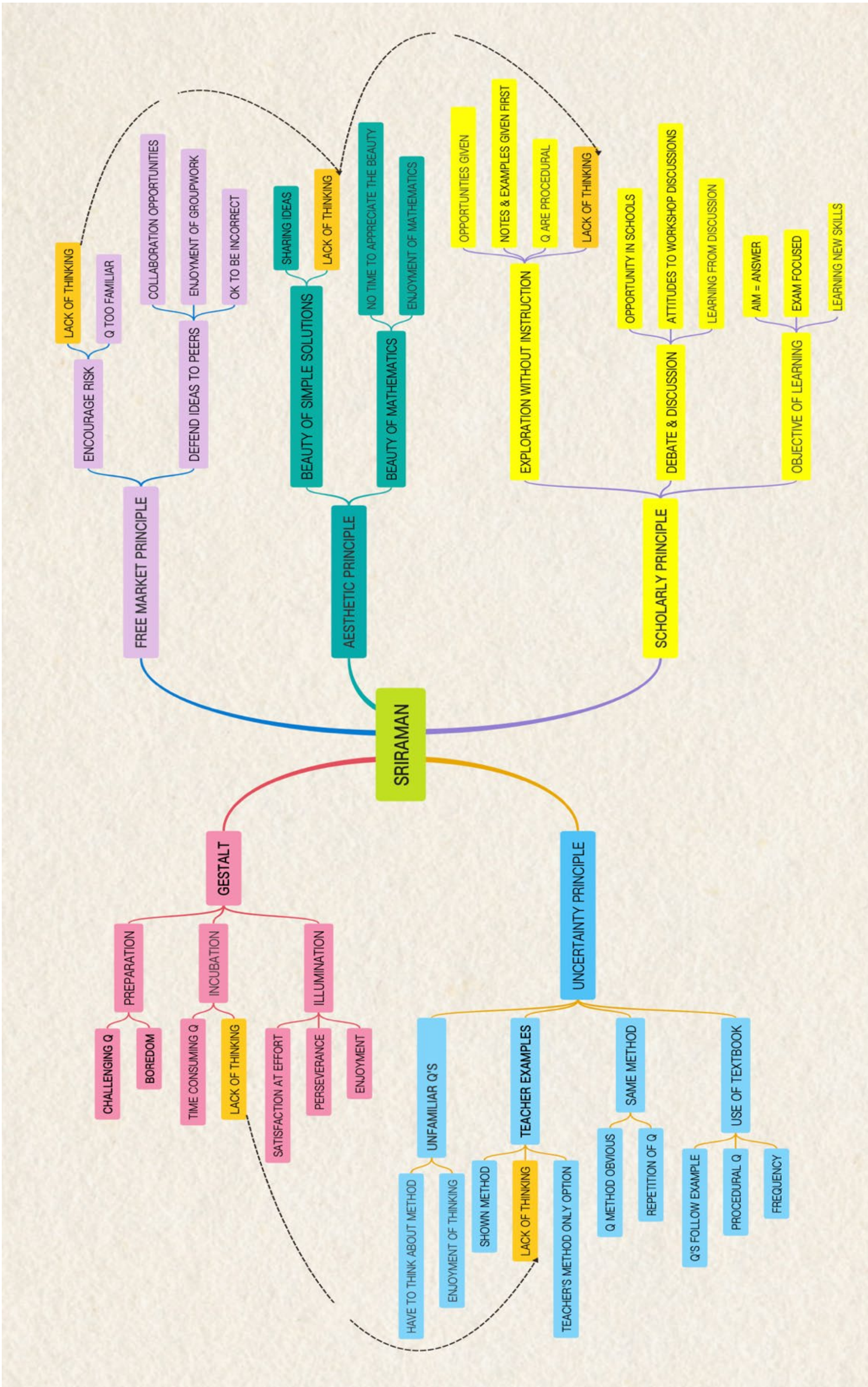
17 Have the tasks from the workshops changed your attitude to what mathematics is in any way?

18 Do you ever work on tasks not set by your teacher? If so, where do you find them and why do you do them?

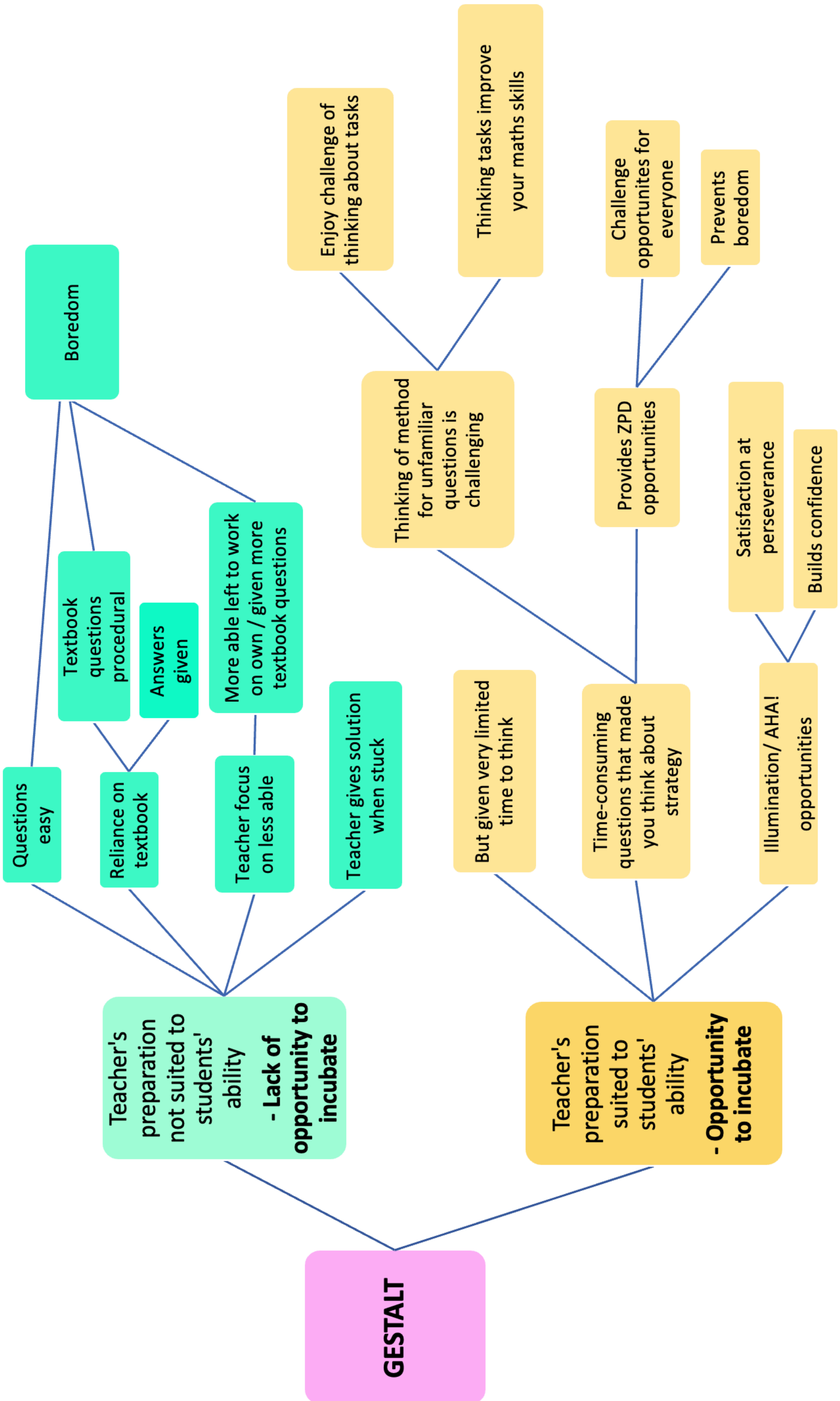
Appendix I: Interview Analysis Templates



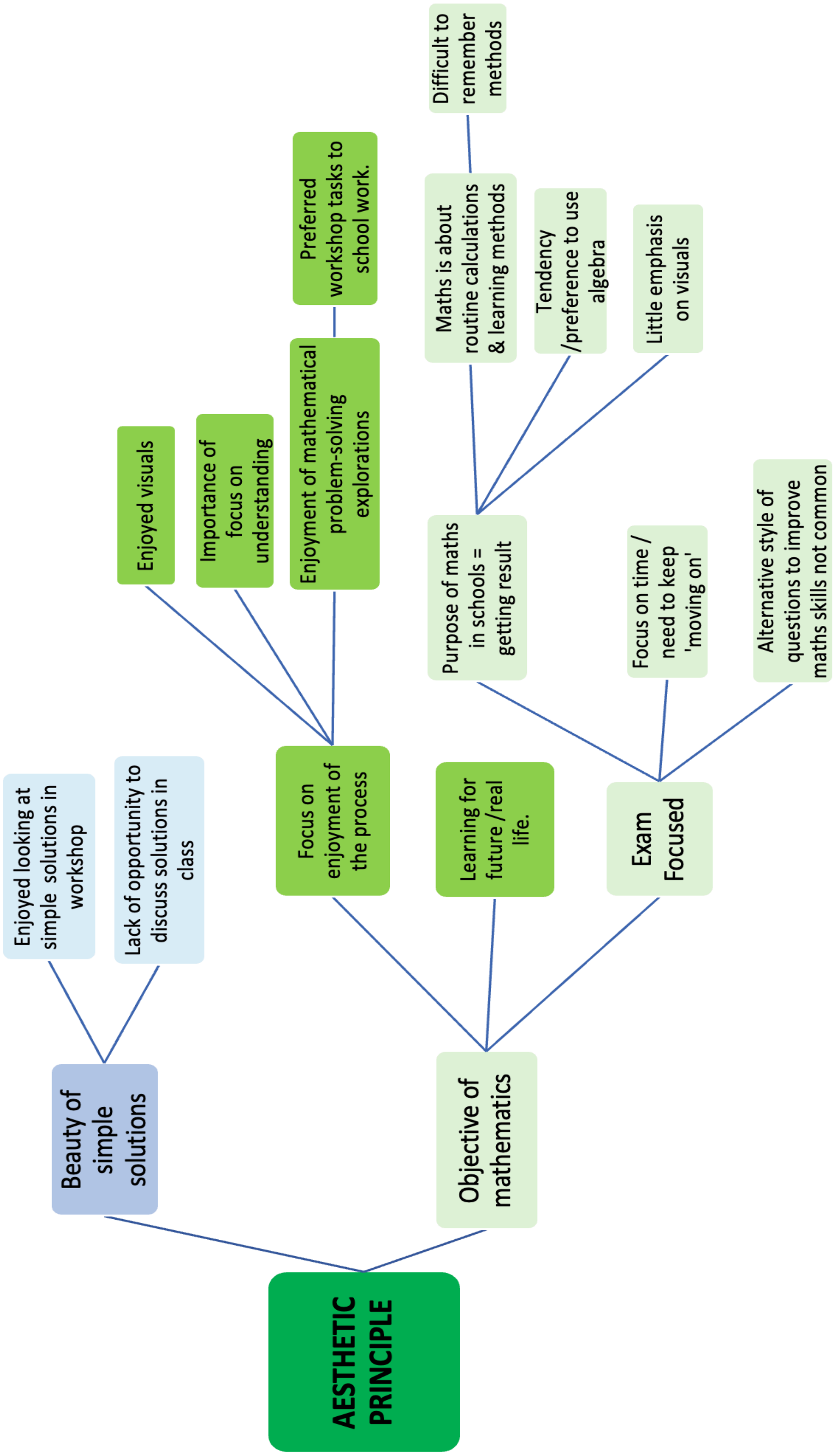
Initial Template



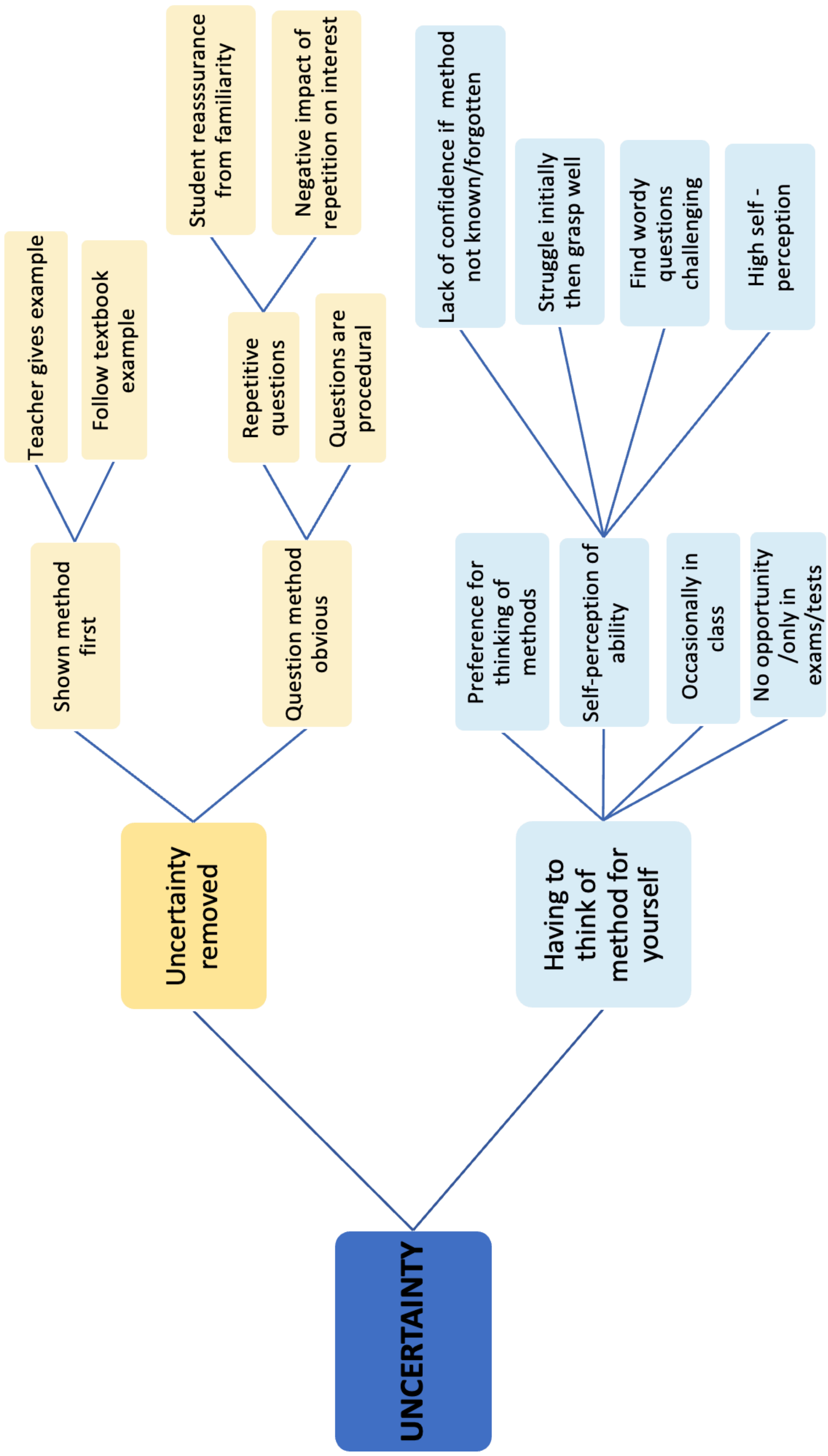
Final Gestalt Principle Template



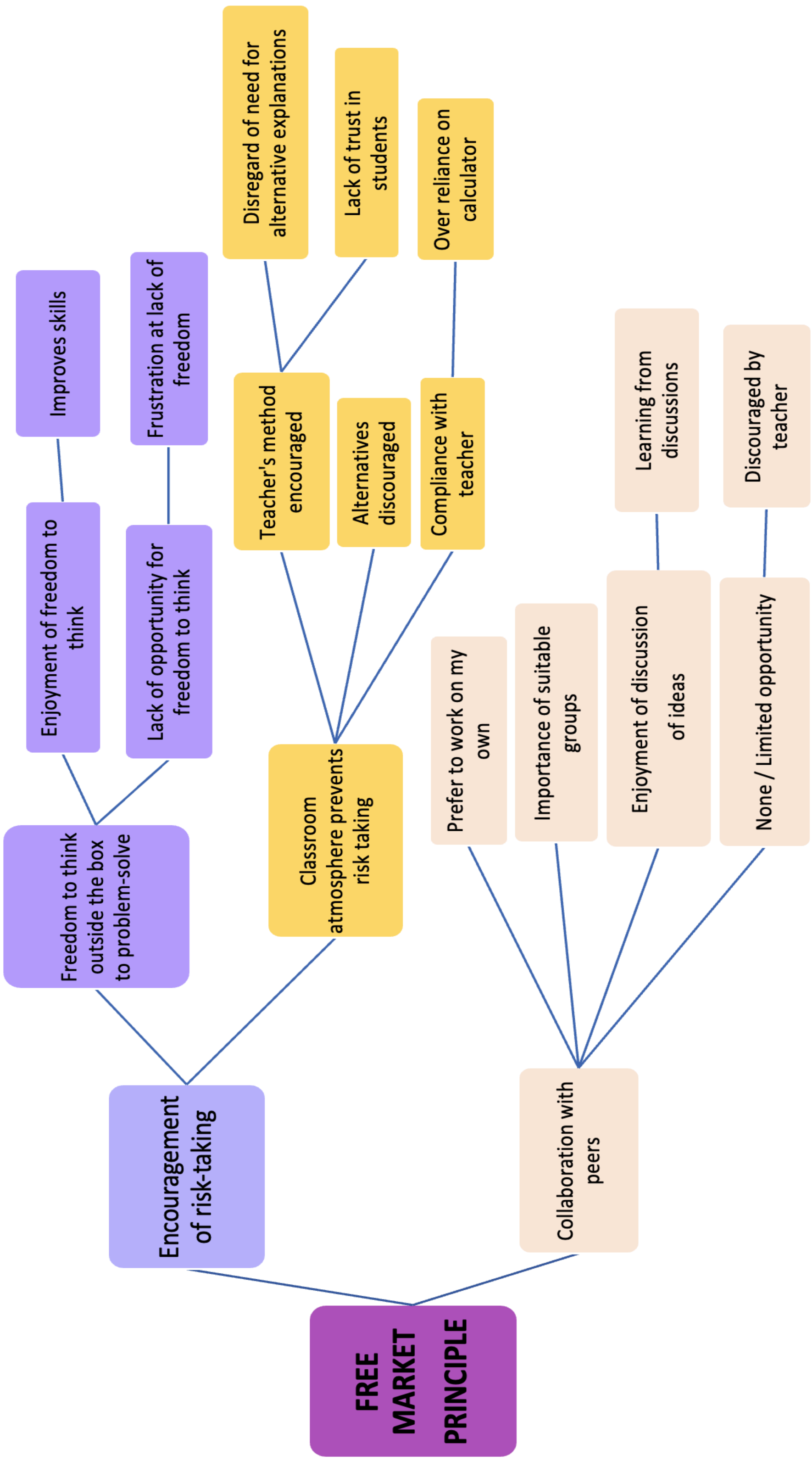
Final Aesthetic Principle Template



Final Uncertainty Principle Template



Final Free Market Principle Template



Final Scholarly Principle Template