Physica A 565 (2021) 125557

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Markets as networks evolving step by step: Relational Event Models for the interbank market[☆]

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ARTICLE INFO

Article history:
Received 7 April 2019
Received in revised form 3 October 2020
Available online 25 November 2020

Keywords:
Network structure of markets
Financial markets
Market dynamics
Relational Event Models
Time-weighted statistics
Interbank money market

ABSTRACT

We introduce a modeling framework for continuous-time relational data that allows detecting the fine-grained dynamics of network formation and change in financial markets. We propose newly derived Relational Event Models with time-weighting functions and corresponding time-weighted statistics as a suitable approach to capture the temporal aspects of observed network processes (i.e., memory) as well as a variety of extra-dyadic microstructures that include tie frequency (i.e., trading intensity) and tie value (i.e., traded amount). By specifying novel statistics and fine tuning weighting-function parameters, we show how this framework allows (1) obtaining more accurate representation of market dynamics, (2) disentangling competing micromechanisms of network formation and change, and (3) assessing their relevance. Also, by comparing alternative specifications of time effects, we emphasize the parsimony afforded by our approach. We illustrate the merits of our modeling framework in a study of the interbank liquidity market during the 2008 financial crisis.

1. Introduction

Economics [e.g., 1–3] and Finance [e.g., 4,5] have long conceptualized markets as networks of exchange relationships, shaped by specific relational structures, not exclusively by economic rules. The network framework has been applied especially to the modeling of financial markets. A few studies have focused on assessing the effects of an observed market structure on the resilience of the system and the risk of contagion [6–9]. Others have aimed to understand how this structure emerges and evolves [3,10] both naturally and as a consequence of events like market shocks. Because the interbank market is typically conceived as the most salient channel of contagion for the financial system, it has been repeatedly examined [11–13] and used as empirical setting for testing novel methodological approaches [14].

Papers interested in detecting the evolution of a market structure have frequently taken a macro-structural approach, and used descriptive measures to reveal the topological properties of interbank networks - e.g., network roles [15,16], core–periphery structures [17], or cohesive subgroups [18]. Though informative, this approach suffers from several limitations: (a) it does not exploit all information available in the data, (b) it examines a limited variety of network topological properties, (c) it captures each property separately from others, making it difficult to assess the relative

☆ **Acknowledgment:** Helpful remarks provided by the members of the Social Networks Lab at ETH Zurich, Alberto Caimo, Caterina Liberati, Alessandro Lomi and Paolo Zagaglia throughout the project, and comments of participants to the 2014 INSNA Sunbelt Conference, 2014 ERCIM Conference, 2015 Computational Social Science Conference and UCD Working Group on Statistical Learning seminar series are gratefully acknowledged.

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https://doi.org/10.1016/j.physa.2020.125557
0378-4371/

contribution to tie formation and persistence, and (d) it typically does not allow identifying which mechanism of exchange – within a set of competing ones – could explain an observed network property. For instance, clustering can result from distinct mechanisms of subgroup formation, which imply different relationships between banks [19]. Ignoring these mechanisms is potentially harmful, because this makes it difficult to evaluate and control systemic risk [20].

An attempt to alleviate these limitations has been made by papers that have taken a micro-structural perspective. Acknowledging that a market structure is the product of decentralized and repeated exchanges between lenders and borrowers, these papers have modeled sets of daily transactions with the purpose of uncovering patterns in counterpart selection [20–23]. For instance, Cohen-Cole et al. [21] have used game theory to model bank choice to form a tie, finding that selection of trading partners is affected by their network centrality. Iori et al. [22] have proposed a simulation model to examine the memory structure of the market. The model introduces the idea of a time window (T_M) during which external conditions remain stable and banks are assigned transactions to execute in an order determined by an event parameter (t). Potential borrowers are given a parameter of attractiveness. Also, a newly formed network is assumed to be conditioned on the past network structure, specifically on the number of times two banks have already traded with each other. The authors have concluded that network ties are shaped by inertia. Hence, banks that traded repeatedly in the past are likely to trade again unless external conditions change.

The existence of extra-dyadic microstructures, whereas a transaction between two banks depends on a previous transaction which involves at least another bank, has been repeatedly acknowledged. Yet, the modeling has been just partial. Specifically, Finger and Lux [23] have proposed the use of stochastic models [24,25] based on the myopic optimization underlying multinomial logistic regression [26] to model transaction data as panel data and capture the probability of specific changes in counterpart selection. These models allow specifying a large variety of trading microstructures and capturing several network nuances (e.g., the number of trading partners changed at each time point, the influence of actor characteristics on change frequency). Yet, because these models focus on relationships (i.e., states) rather than events, they also require to aggregate network data over convenient timestamps and dichotomize it [27,28]. In all these cases, data aggregation and simplification imply loss of information on ordered sequences, on entry and exit of actors, on frequency of activity of individual actors and interaction between pairs of actors. Also traded amount cannot be modeled directly. As we will demonstrate, these manipulations of the data structure could result in partially capturing key market dynamics as well as not fully revealing their complexity.

In this paper we show how the limitations of current models could be addressed by applying an event history framework. Relational Event Models (REMs) model the time to the next event – i.e., the next transaction in our case – conditional on the current context and a sequence of previous interactions [29,30]. REMs make it possible to account for the temporal aspects of observed network processes (i.e., memory) as well as a variety of extra-dyadic microstructures that include trading frequency (i.e., intensity) and traded amount (i.e., tie values). We introduce these novel network statistics and show how they allow revealing trading behaviors that could not be modeled otherwise. Indeed, we propose a unified scheme that weights past interactions in an informative and parsimonious way. In doing so, our modeling framework can test the statistical significance of specific hypotheses on network dynamics with a unique level of detail.

We demonstrate the merits of REMs by examining the trading dynamics of the interbank market during the recent global financial crisis. This is a sample period that has received substantial attention, mainly because of the policy relevance of the surrounding issues [31]. Comparing the trading dynamics observed in stable market conditions, in the turmoil period ending with the Lehman Brothers bankruptcy and in the following crisis period, we reveal changes in the trading behavior that banks adopt across the phases of the global crisis. We draw attention to characteristics of the trading dynamics that previous studies were unable to detect so clearly. The rest of this paper is organized as follows. Section 2 discusses the Relational Event framework. In Section 3, we introduce the empirical setting and the dataset, and we outline the research questions that we will explore. We describe how the model is specified to fit the characteristics of our data structure. Indeed, we introduce the unified weighting method for the computation of the network statistics and the different parametrizations for the weighting method. In Section 4 we compare the weighting schemes tested on the interbank lending dataset, demonstrate the advantage of the proposed time-weighted function and interpret the estimated statistics. Robustness checks include a fine-grained modeling specification that allows accounting for the seasonality in the data structure, and a direct comparison with a modeling approach based on aggregated network data. Section 5 concludes.

2. Relational event models

Relational Event Models are a recently developed class of models, which can be broadly described as event history models for relational data. REMs allow explaining the probability of observing an interaction between a pair of actors (i, j) at time t – a transaction between banks in our case – as a function of a set of individual and network statistics. This is done by using a multiplicative Cox function.

Individual statistics capture the characteristics of the two actors and the differences in these characteristics, while network statistics account for the history of past transactions involving i and j. Example of the first class of statistics is *Country match*. It captures the number of transactions prior to time t between bank i and other banks based in the same country which could make i more likely to trade with j based in the same country at time t. Example of the second class is *Reciprocity*, capturing the number of transactions from j to i prior to time t that could make i more likely to lend money to j at time t.

REMs were introduced by Butts [29] in the domain of social relationships. Since his pioneering work, a few variations of this methodological framework have been proposed [32–34]. Here we draw on and extend the stratified version proposed by Perry and Wolfe [35]. We argue and show empirically that this is most appropriate when the goal of analysis is to model the dynamics of partner selection. We briefly discuss this approach using counting process notation in event history analysis [36]. This counting process framework [37] will facilitate our next discussions on the weighting method for the computation of network statistics in Section 3.3.

2.1. Counting process framework

Let us consider:

- n actors, with $i = 1, \ldots, n$ and $j = 1, \ldots, n$, $i \neq j$.
- a tuple (t, i, j, w) where (i, j) is the event that we observe at time t, consisting in i sending a tie to j. In our empirical exercise, the observed event is bank i lending money to bank j. The amount of money traded is indicated as w.
- a counting process $N_{ij}(t)$, consisting in the cumulative number of events from i to j by time t.

To model relational events among actors, the counting process $N_{ij}(t)$ is placed on each directed tie between sender i and receiver j. The counting process $N_{ij}(t)$ increases by one when actor i initiates an event j at time t. Since we have n actors, the total number of independent counting processes is $n \times (n-1)$. Each counting process can be modeled by a conditional intensity function $\lambda_{ij}(t)$ of proportional hazard form [38]:

$$\lambda_{ij}(t|\boldsymbol{H}_{t-}) = R_{ij}(t)\lambda_{i0}(t)exp[\boldsymbol{\theta}^T \mathbf{s}(t, i, j)] \tag{1}$$

where $\boldsymbol{H}_{t-}$ is the network history right before time t, $R_{ij}(t)$ is the at-risk indicator, $\mathbf{s}(t, i, j)$ are network statistics of type $s(i)$ and $s(i, j)$ at time t, and θ are coefficients to estimate. The current history $\boldsymbol{H}_{t-}$ of all counting processes provides us a mean to construct network statistics $\mathbf{s}(t, i, j)$ that define not only how the past influences the future but also how these individual processes are interconnected. When all the actors are assumed to be at risk during the observation period, $R_{ij}(t) = 1$ for each t. Since this is the case in our study, the risk indicators will not be included in our further discussion.

We allow the baseline intensity $\lambda_{i0}(t)$ to vary across senders (i.e., lenders in our case study). We will assume a non-parametric or piecewise constant form of these baseline functions which helps to avoid misspecification of sender statistics. Since heterogeneity among senders is absorbed into baseline functions, only receiver statistics and dyadic statistics between senders and receivers can be estimated directly. This results in an estimation procedure which is less computationally intensive and, therefore, is suitable for large datasets. The hazard ratios resulting from model estimation can be interpreted as conventional hazard ratios from event history analysis or converted to predicted probabilities. Also, the proposed approach fits cases – and theoretical issues – where a variety of interaction behaviors need to be investigated, and therefore several network statistics need to be estimated. In the next sections we illustrate these statistics, pointing to those which are peculiar of the REM framework (Section 3.2), and explain how past events should be weighted to compute these statistics (Section 3.3).

3. Micro-dynamics of exchange in the interbank market

Money market transactions are a crucial funding vehicle for financial institutions. The money market includes two types of actors: the central bank – main provider of money – and private banks that lend to each other in the so-called interbank money market. Banks typically recur to the central bank to manage gross liquidity needs and access the interbank market to meet liquidity needs on a daily basis and reallocate deposit imbalances among themselves [39]. In the European money market – examined in this paper – private banks need to be listed as eligible counterparts by the European Central Bank (henceforth ECB) and sustain substantial administrative costs to participate in the liquidity tenders coordinated by the ECB. These characteristics prove a disincentive especially for smaller banks and contribute to make the interbank money market crucial for liquidity provision. Liquidity needs tend to change over time according to specific patterns. One relevant source of influence for liquidity transactions are the compulsory reserve requirements imposed by the ECB. All banks operating a reserve account with the ECB are subject to an average minimum reserve requirement. This is computed according to the quantity and quality of the bank's assets and calculated over a period of typically one solar month - i.e., the Reserve Maintenance Period (RMP). [22,39].

Several scholars have investigated the European interbank money market in the aftermath of the 2007–2008 global financial crisis [e.g., 10,21,23] to understand how the market dynamics changed throughout that period. Numerous studies conducted by the ECB have revealed that the interbank market was severely affected by the global financial crisis. Being unable to assess the depth of issues on balance sheets of potential counterparts, banks became unwilling to lend to each other on the interbank market without accommodation for counterpart risks. This resulted in a freeze of the trading activity and a decrease in the number of the transactions in the interbank money market. To compensate this lack of liquidity, the ECB increased the frequency of traditional operations and implemented a set of non-standard ones [39].

Drawing on this evidence at market level, scholars have tried to understand how the crisis affected the lending behavior of the banks that remained active on the interbank market. To show how the proposed methodological approach can contribute to this effort, we examine the period between January 1, 2006 and December 31, 2009 and reconstruct the micro-dynamics of exchange among banks trading on the interbank market. Specifically, we observe the transition from stable market conditions to the global crisis in order to reveal how banks traded with one another, and how their trading behavior changed when market stability declined. We split the observation period into three phases. Phase 1 captures the pre-crisis period and includes all transactions occurred between January 1, 2006 and August 9, 2007 when announcement of liquidity shortage by BNP Paribas shed light on the severe crisis that was affecting large part of the US and European banking systems. Phase 2 captures the turbulence which anticipated the crisis itself and includes all transactions occurred between August 10, 2007 and September 15, 2008. Finally, phase 3 captures the beginning of the global crisis and spans between September 16, 2008 and December 31, 2009. We investigate how selected topological properties [10,11,15,18] emerge over time as the result of bank choices of trading partners - i.e., market microstructures [40]. In doing so, we do not aim to test specific theoretical hypotheses. Rather, we observe emerging evidence on the adopted trading behavior. In detail, we ask: How do banks select their trading counterparts? How do past trades affect the decision to lend to a specific bank? How does this behavior change when market stability declines?

3.1. Data

Interbank transactions are split between an electronic segment and other channels for liquidity provision. These are mainly bilateral over-the-counter (OTC) trades which occur via brokers or phone [39]. Because OTC trades are not visible, related data is not publicly available. Following previous studies, we base our analysis on the electronic segment of the market [e.g., 27,39]. Specifically, we examine the Euro Electronic Market for Interbank Deposits (e-MID). The e-MID is the unsecured segment of the Euro money market and the only electronic market for interbank deposits in the Euro area. Indeed, the e-MID data is the only interbank data which can be accessed without any restrictions [27]. Transactions in e-MID take place through an electronic platform that is fully centralized and operates in Milan. E-MID is accessible to all banks active in the Euro area. Also, e-MID consists of two submarkets, with different transactions and rules. In one market, the focus is on buying liquidity: the transaction is started by the borrower (technically labeled as the aggressor on this platform) who asks for money and sets the interest rate. When a lender (technically labeled as the quoter) agrees, the transaction takes place. Hence, this submarket is called buy or ask market. In the other market, the focus in on selling liquidity: the transaction is initiated by the lender (the aggressor) who offers money and sets the interest rate. When a borrower (the quoter) agrees, the transaction takes place. Hence, this submarket is called sell or bid market. In both submarkets, banks can choose their trading counterpart, and the information on rates and amounts is made public. Each bank enjoys complete information on the structure of exchanges in the whole network. In addition, the minimum trade size is established a priori.

Among all euro interbank transactions, the e-MID held an estimated market share of 17 to 22 percent during the pre-crisis period [39]. In 2007 e-MID represented 20% of the overall interbank transactions in Europe [41]. This market share decreased slightly during the financial crisis, and so did the volumes traded. Average daily trading volumes were 24.2 billion Euro in 2006, 22.4 billion Euro in 2007 and only 14 billion Euro in 2008 [39]. This was the case because e-MID is a transparent market. Exchanges in the e-MID platform are organized in a way that allows market participants to be aware of counterpart characteristics, and evaluate past trading behavior. Since a bank reputation is key to building and maintaining lending relations, borrowing banks became reluctant to use e-MID as a source of money in order to avoid the potential stigma of being perceived as illiquid and damage their reputation [42]. Likewise, lending banks became more risk averse and willing to focus on bilateral operations [41]. In spite of these caveats on decreasing market share and representativeness, e-MID has proven capable of capturing the dynamics of the whole Euro interbank market [43] and has been examined in several papers aimed at investigating the consequences of the financial crisis [for a recent review, see 28].

Transactions in e-MID involve money exchange at various maturity structures, ranging from overnight to two years. Yet, most studies focus on the overnight segment of the interbank deposit market, because it is the largest segment of money markets [e.g., 27]. The overnight transactions accounted for roughly 80% of all trades in 1999 – when the e-MID platform was launched – and have constantly increased their fraction over time up to 87.7% at the end of 2009, when the increased uncertainty reduced the attractiveness of the longer maturity loans. This combination of factors imply that the e-MID Euro overnight market offers a comprehensive view of interbank lending during this crisis [39] and can be regarded as a benchmark of the euro area money market especially for the overnight maturity and up to the end of 2008 [41].

For each transaction executed, the system produces a string of data. This contains information on the identity of each bank, their nationality as well as interest rate, amount, date and time of occurred trade. Because of privacy concerns, bank identity is represented by a unique six digit code with the first two digits signifying country of origin and the following four being a 0001 to nnnn code. This system allows determining the nationality of a bank, but not its identity. No other individual attributes are available to the public. In e-MID the overnight transactions are classified in two subgroups: regular size transactions (labeled ON), for which the minimum involved amount is 1.5 million euros, and large transactions (labeled ONL), for which the minimum involved amount is 100 million euros. To ensure comparability, we excluded the overnight large size transactions which constitute a sub-sample on their own and were severely affected by the reduction in average loan size throughout the sampled period. Hence, from the 382,327 transactions of various maturities occurred on the platform from January 1, 2006 to December 31, 2009 we extracted and analyzed the 305,489 overnight regular sized transactions. Fig. 1 displays the number (i.e., rate) of overnight transactions occurred daily between January 2006

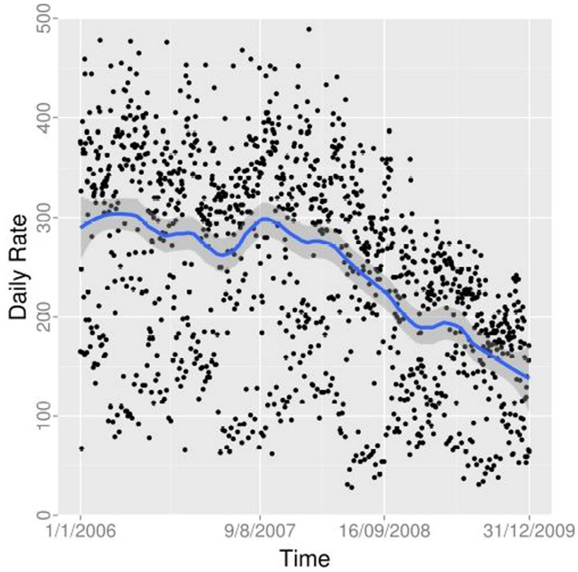

Fig. 1. Number of daily transactions. Time indicates the time point when the transaction occurred. Daily Rate in Fig. 1 indicates the number of transactions occurred at a specific time point (day in this case).

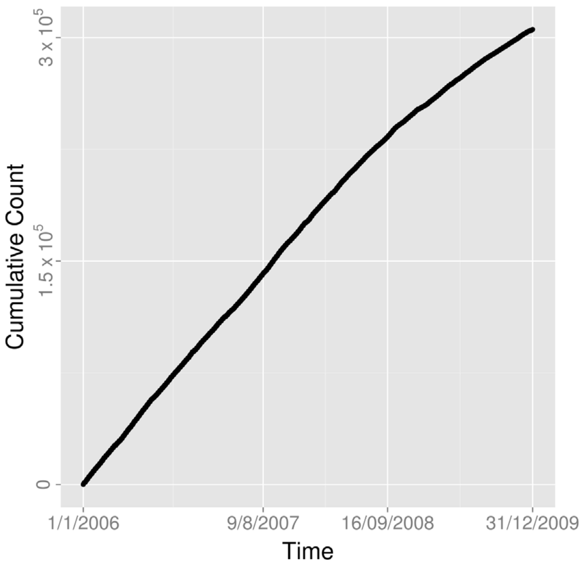

Fig. 2. Cumulative number of transactions. Time indicates the time point when the transaction occurred. Cumulative Count in Fig. 2 indicates the cumulative number of transactions occurred by the specific time point.

and December 2009. As the figure clearly shows, the number of transactions varied significantly across days but declined steadily over time. This tendency is confirmed by Fig. 2 which displays the cumulative number (i.e., cumulative count) of overnight transactions occurred throughout the observation period. Specifically, there were 90,368 ON transactions in 2006 (they represent the 76.2% of the overall transactions of various maturity occurred during 2006), 86,447 in 2007 (78.2%), 75,931 (81.6%) in 2008 and 52,743 (87.7%) in 2009.

We arranged the sampled transactions into three subsamples, one for each phase, and estimated the same model separately on each subsample. Then we standardized the parameter estimates and compared them to detect changes in trading behaviors. To fit the data structure, transactions were converted into the event form (t^e, i^e, j^e, w^e) where t^e is the time of lending event e, i^e is the lender or sender, j^e is the borrower or receiver, and w^e is the lent amount or tie value. Besides timestamped events, the dataset also includes country codes of the banks that were active throughout the observation period.

3.2. Network statistics

In this section we introduce the statistics that are used to detect the evolution of the interbank network structure. Since the purpose of our analysis is to illustrate the proposed approach, we do not test specific hypotheses. Rather, we introduce a wide range of statistics that capture different microstructures. Network statistics can be arranged in two categories. The first includes statistics capturing the topological properties which are most frequently observed in financial networks. The mathematical form of most statistics is based on the stochastic modeling approach mentioned in Section 1. We discuss how these statistics are defined and interpreted within the REM framework. The second – and more relevant – category includes statistics that are peculiar of the REM framework, and whose effect could not be tested otherwise. These are statistics based on (i) time or inertia and (ii) trading intensity. Most of these statistics can be specified to include the traded amount. Indeed, intensity statistics allow us to investigate competing mechanisms of tie formation and therefore illuminate the merits of the REM approach. Table 8 in the Appendix shows the mathematical definition of all included statistics. In our empirical exercise, i and j denote lenders and borrowers respectively while k denotes a generic bank in the sample.

Nodal statistics. They are used to model the influence of node (i.e., bank in our case) characteristics on the likelihood of sending or receiving ties. *Covariate sender* is a node attribute that affects the probability of sending ties. We specify this generic variable as *Size* of bank j which is defined as the difference between the total borrowing amount and the total lending amount before time t, in line with Finger and Lux [23]. We measure the lending activity of bank j by defining *Out-degree* as the number of banks to which j has lent money. A negative coefficient of *Out-degree* means that the higher number of borrowers that bank j has, the lower probability that it will borrow money itself. In other words, bank j is specialized in a lending role and is less likely to become a borrower. We measure the borrowing scope of bank j by defining *In-degree* as the number of banks from which j has borrowed money. A positive coefficient of *In-degree* is expected and can be interpreted as the higher number of lenders that bank j has, the higher probability that it will borrow from other lenders. This is a statistic that is used to test if banks are specialized in borrowing roles, so that a current borrower is more likely to borrow more in the future. The following statistics are specific of REMs. *Out-intensity* accounts for the lending intensity of bank j. Since intensity captures the number of trades, *Out-intensity* is measured by the number of times that bank j has lent to k, with $k = 1, \ldots, n$. A negative coefficient of *Out-intensity* points to specialization as lender - i.e., a bank that intensively lends money is less likely to borrow. Specialization is based on the frequency of trading in the market, not on the number of borrowers. To measure the borrowing intensity of bank j, we define *In-intensity* as the number of times that j has received money from the generic bank k. *Recency* is included to control for a bank activeness on the market. *Recency* is defined as the gap time since the last lending transaction in which the same bank has been involved. A negative coefficient of *Recency* statistic provides evidence for a higher likelihood of involvement with a new transaction following a recent one [44].

Dyadic statistics. Dyadic statistics are used to model distinctive attributes of current ties between banks that influence the likelihood of their future lending transactions. For interpretations, we further divide dyadic statistics into assortativity, flow, and clustering groups. *Assortativity statistics* capture mixing patterns in the lending network where banks with similar or dissimilar attributes are more likely to establish trading ties. *Covariate match* is a node-match indicator which equals 1 if sender and receiver nodes have the same value of an attribute and 0 otherwise. We specify it as *Country match*, capturing whether lender and borrower are from the same country. *Covariate mismatch* is a node-mismatch indicator which accounts for the difference between sender and receiver in the value of a continuous attribute. We include *Size difference* capturing the effect of bank's size (defined in terms of available resources) on trading between a lender and a borrower. A positive coefficient of *Size difference* means that the higher the difference between banks in availability of financial resources the higher the likelihood they trade with each other.

Degree assortativity measures assortativity in terms of number of trading partners. A negative coefficient of this assortativity statistic can be interpreted as propensity of banks which lend to many others to trade with banks that borrow from few others. This corresponds to a core–periphery structure. *Intensity assortativity* is specific of REMs. It measures assortativity in terms of trading intensities. A negative coefficient of *Intensity assortativity* implies that banks with high lending intensity (i.e., trading very often) are more likely to trade with banks with low borrowing intensity. This points to an alternative form of core–periphery structure based on the frequency of trading instead of the number of trading partners.

Flow statistics model the effect of past transactions between pairs of banks on the likelihood of future transactions. *Reciprocity* measures the propensity that bank i reciprocates the borrowing relationship with bank j. A positive coefficient of *Reciprocity* suggests that banks tend to reciprocate their trading relationships by lending back to their past lenders. *Inertia* is specific of REMs. It measures the propensity that bank i continues its current lending relationship with bank j by lending repeatedly. A positive coefficient means that banks tend to reinforce their trading ties by continuing to lend to their previous borrowers.

Clustering statistics model clustering patterns where banks tend to form tightly knit groups characterized by preferential trading. *Transitive closure* captures the tendency that banks lend to borrowers of their borrowing partners. In other words, banks transform indirect relationships into direct ones. *Cyclic closure* models generalized exchange of resources, which captures the tendency of bank i to lend to j if i has received money from a bank k in the past. A negative coefficient of

Cyclic closure when *Transitive closure* is positive implies a hierarchical structure in the lending network. Finally, *Balance closure* accounts for the effect that lending relationships of i and j can exert on the direct relationship between i and j themselves. If the coefficient of the *Balance closure* statistic carries a positive sign, banks i and j which lend to the same borrowers have higher probability of lending also to each other.

3.3. Temporal event weighting schemes

The temporal relevance of past trades between banks i and j (indicated as T^e_{ij}) needs to be included in the computation of the network statistics $s(i)$ and $s(i,j)$, especially when network processes are observed over a long period. In such circumstances, it is unrealistic to expect that transactions within the last three days – for instance – are as influential in predicting future transactions as those occurred three years ago. In the next section we propose a weighting scheme that provides many options to define the temporal relevance of past events. We start by showing the basic options and then we move to introducing our approach.

3.3.1. Basic scheme

The most basic weighting scheme consists in imposing a constant weight for all events regardless of their timestamps [30,45]:

$$f(t, T^e_{ij}, \boldsymbol{\nu}) = 1.0 \tag{2}$$

Each transaction T^e_{ij} occurred between banks i and j prior to time t is assigned a weight which is specified by the parameter $\boldsymbol{\nu}$ of the weighting function. In the uniform weighting scheme all events have weight equal to 1.0 regardless of when they happened. Intuitively, this weighting scheme implies that all past transactions between i and j are assumed to have the same impact on the likelihood of observing a transaction involving the two banks at current time, no matter how far away in the past these previous transactions are. We use this weighting scheme as the baseline (model M1) against which to test more meaningful schemes.

3.3.2. Short- and long-term separation scheme

Under this scheme, all events T^e_{ij} involving i and j occurred up to the current time t are weighted equally but separated into short-term and long-term windows (M2). This requires the researcher to identify a meaningful time point α prior to time t that divides the past into short term versus long term. E.g., if α is set equal to 30 days, all the transactions occurred within the 30 days prior to the current time t fall into the short-term window, while all transactions occurred from 31 days back to the beginning of the observation period fall into the long-term window. Within the long-term and the short-term windows, all observed transactions have the same weight (therefore, equal to 1). Hence, they exert the same influence on the propensity of observing a transaction involving i and j at time t. Network statistics for short-term variables are defined using the weighting function:

$$f(t, T^e_{ij}, \alpha) = \boldsymbol{I}[T^e_{ij} \geq (t-\alpha)] \tag{3}$$

while network statistics for long-term variables are defined using:

$$f(t, T^e_{ij}, \alpha) = \boldsymbol{I}[T^e_{ij} < (t-\alpha)] \tag{4}$$

Window parameter α can be varied to obtain the best fit [30,46] and, therefore, the best definition of short and long term. This weighting scheme is very detailed and allows a clear separation into short- and long-term effects. Yet, it suffers from lack of parsimony and increases the dependence in the data. A short-term and a long-term statistic need to be included for each original network variable, and a long- and short-term coefficient are estimated. The short-term one indicates the extent to which the network effect (e.g., reciprocity) is affected by the transactions occurred between i and j in the short-term window (e.g., the 30 days prior to time t). The long-term coefficient indicates the extent to which the same network effect is affected by the transactions occurred between i and j in the long-term window (e.g., from 31 days prior to time t to the beginning of the observation period). Interpretations of the long-term versus short-term coefficients are not straightforward when there is not a theoretically informed and meaningful way of identifying α.

3.3.3. Time-weighting scheme

To achieve model parsimony and account for the temporal relevance of short- and long-term events, we propose a weighting scheme which combines them. It operates on the principle that events within the recent time window should be weighted equally, while events far in the past should be weighted less than recent ones (M3). As for the previous weighting scheme, the researcher sets a parameter α which specifies the number of time units prior to time t used to separate short and long term. E.g., If α is set equal to 30 days, all the transactions T^e_{ij} occurred in the 30 days prior to the current time t fall within the short-term window. Within the short-term window, all observed transactions have the same weight. All transactions occurred from 31 days back to the beginning of the observation period fall within the long-term window. Within the long-term window, the transactions are weighted by a decay rate β which assigns them a decreasing influence on the network statistics $s(i)$ and $s(i,j)$ at time t. This decreasing influence is proportional to the distance in

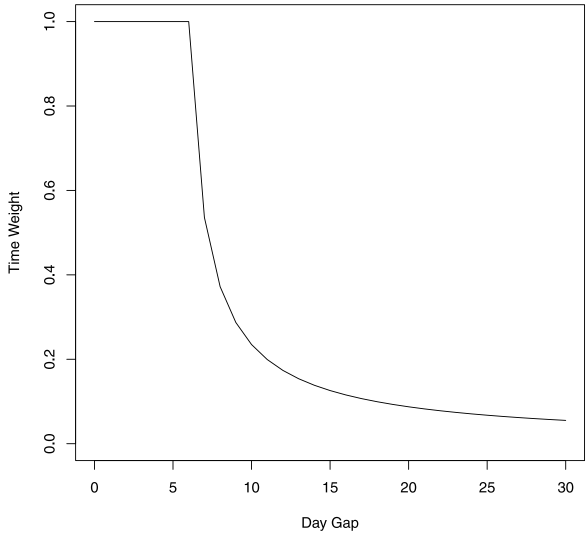

Fig. 3. Time-weighting function (Eq. (5)) with $\alpha = 5$ days and $\beta = 0.9$. Day Gap is the time unit (number of days in this case) prior to the current transaction. Time Weight indicates the coefficient by which a past transaction should be weighted in computing the network statistics. The parameter α is the length of the time gap in which all past transactions are weighted by 1.0, while β is the decay rate of past transactions occurred outside the α time window.

time between the current time t and the past transactions. This weighting scheme generalizes the previous one. If $\alpha = 0$ and $\beta = 1$ the time-weighting scheme equals the basis scheme (M1). If $\alpha > 0$ and $\beta = 1$ the time-weighting scheme equals the short- and long-term separation scheme (M2). The time-weighted function is of the form:

$$f(t, T_{ij}^e, \alpha, \beta) = \boldsymbol{I}[T_{ij}^e \geq (t - \alpha)] \times 1.0 + \boldsymbol{I}[T_{ij}^e < (t - \alpha)] \times \frac{1.0}{(t - T_{ij}^e)^\beta}, \tag{5}$$

where $\alpha > 0$ is the width of the recent time window and $\beta > 0$ is the decay rate for event weights. Under this scheme, transactions occurred within the latest α time units – whose length can range between few seconds and several weeks or months depending on the context – receive a weight 1.0 while weights of transactions further in the past are decayed starting from 1.0 depending on their timestamps.

Fig. 3 shows an example of the time-weighting function (5) with $\alpha = 5$ days and $\beta = 0.9$, for time gaps between 0 to 30 days. Event weights equal 1.0 for events during the latest 5 days and are decayed from 1.0 for events occurring further than 5 days in the past. The larger the gap between the event time and the current time, the less impact of that event on network statistics. By applying this weighting scheme, we obtain one coefficient only for each network statistic. Once defined the values – and interpreted the meaning – of the time-dependent parameters α and β, the interpretation of each estimated coefficient is straightforward and similar to the basic-scheme case.

We specify two versions of the proposed time-weighting scheme. In the first version (M3) the amounts traded are not included in computing the statistics. All w_{ij}^e are set equal to 1. In the second version (M4) also the amounts traded are included, so the values w_{ij}^e can vary across observations. Comparing the two schemes allows assessing the contribution of the traded amounts to explaining the network structure.

3.4. Model estimation and comparison

The parameters α and β can be defined *a priori* if there exist theoretical assumptions on how the past events should be weighted. Alternatively, the parameters can be searched for optimal values. We adopt the second approach, as it allows better illustrating the proposed weighting scheme.

The statistics computed according to the different weighting schemes enter the model specification. Thanks to the non-parametric choice of the baseline intensities $\lambda_{i0}(t)$, the model parameters θ can be estimated by maximizing partial likelihoods of the form [37]:

$$PL(\boldsymbol{\theta}) = \prod_{e \in E} \frac{\exp\left[\boldsymbol{\theta}^\top \mathbf{s}(t_e, i_e, j_e)\right]}{\sum_{j \neq i_e} \exp\left[\boldsymbol{\theta}^\top \mathbf{s}(t_e, i_e, j)\right]} \tag{6}$$

where E is the set of relational events during the observation phase. The covariance matrix of $\hat{\boldsymbol{\theta}}$ is estimated as the inverse of the negative Hessian matrix of the last iteration.

Table 1
Time span between transactions involving the same bank.

	1 day	5 days	10 days	15 days
Overall	95.26%	99.40%	99.73%	99.77%
Phase 1	95.31%	99.41%	99.75%	99.86%
Phase 2	95.52%	99.44%	99.75%	99.84%
Phase 3	94.84%	99.32%	99.69%	99.81%

Overall indicates the percentages of transactions made by the same bank within respectively 1 day, 5 days, 10 days and 15 days across the observation period. Phase 1, Phase 2 and Phase 3 indicate the same percentages within each phase of the financial crisis.

Table 2
Time span between transactions (by bank).

	Mean	St. dev.	Q1	Q2	Q3	1 day	5 days	10 days	15 days
Overall	9.09	28.30	0.26	0.61	3.88	56.92%	78.72%	84.04%	87.77%
Phase 1	6.28	20.43	0.26	0.62	3.02	60.23%	80.11%	89.21%	91.48%
Phase 2	13.56	40.00	0.22	0.54	4.03	59.39%	75.76%	81.82%	84.24%
Phase 3	28.72	104.02	0.30	0.64	9.00	59.60%	70.20%	77.48%	80.80%

The table reports mean, standard deviation and the quartiles (Q1 to Q3) of the distribution of time gaps between consecutive transactions involving the same bank. The following columns display the percentages of banks involved in consecutive transactions within respectively 1 day, 5 days, 10 days and 15 days across the observation period (Overall) and by phase (Phase 1, Phase 2 and Phase 3).

In practice, we first generate a dataset of network statistics $\mathbf{s}(t, i, j)$ over time, and then use conditional logistic regression to obtain $\hat{\boldsymbol{\theta}}$ and its standard errors. To compare different weighting schemes we consider two model selection criteria: Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Models with smaller AIC and BIC values are preferred because they indicate a better fit to the data.

4. Results

In total, 194 banks were active at least once on the overnight segment of e-MID during the observation period, with some variations across phases. The highest number of active banks was observed in phase 1 (180), followed by phase 2 (167) and phase 3 (151), consistently with the evidence on the decline of lending activity during the financial crisis. Previous studies have already shown that this decrease in number of active banks and transactions also corresponded to the exit of foreign banks from e-MID in phase 3 and the decline in activity of Italian banks [27]. Most banks used e-MID as a channel for both lending and borrowing liquidity: 176 banks (90.72% of all active banks) lent and 184 (94.85% of all active banks) borrowed at least once. Because we are interested in the influence of past transactions on current ones, we focus on the time span between transactions. Being e-MID a high-frequency market, the time span between consecutive transactions involving the same bank – in the role of lender or borrower – was very short. In 95.26% of the cases two transactions involving the same bank occurred within 1-day time span. This percentage increases to 99.40% for transactions occurred within a 5-day time span, and 99.73% for transactions within a 10-day time span. Finally, 99.77% of the banks were active within a 15-day span. Table 1 reports the percentage breakdown by phase, suggesting there were no significant changes over time.

To understand how transactions were distributed across market participants, we examine the trading behavior by bank. On average the time between a past transaction T^e_{ij} and a consecutive transaction involving i or j in lending or borrowing roles (this enters network statistics $s(i)$) was 9.09 days with significant differences across phases. The mean value is poorly informative in this case because the frequency distribution is skewed, as proved by the high values of the standard deviation (st. dev.= 28.30). For 50% of the banks two consecutive transactions occurred within 0.61 days. The percentage increases to 56.92% of the banks for 1-day time span, 78.72% for 5-day time span, 84.04% for 10-day time span. Finally, 87.77% of the banks were active twice within a 15-day period. This suggests that e-MID consists of a majority of banks that trade very frequently (at least twice any 5 days, but most even more often) and a minority of others that use this market just occasionally. Table 2 displays these statistics for the overall period as well as by phase.

Finally, Table 3 displays the distribution of the timestamps between a past transaction T^e_{ij} and a consecutive transaction between i and j (this enters network statistics $s(i, j)$). Banks exchanged repeatedly with the same counterpart across the sampled period (mean = 24.24 days, st.dev. = 31.57). This behavior was less frequent than the general activity by bank on the platform. Also, the time gap between consecutive transactions involving the same dyads increased from phase 1 to phase 3.

Given this evidence, what weighting scheme fits the data best, adds information on how past trading behavior of banks affects their current behavior, and helps achieving model parsimony? To answer this question, we compare different weighting schemes for network statistics.

All nodal and dyadic statistics described in Section 3.2 are used in our comparison experiment. We specify the statistics according to the different weighting schemes that we consider:

Table 3
Time span between transactions (by dyad).

	Mean	St. dev.	Q1	Q2	Q3	1 day	5 days	10 days	15 days
Overall	24.24	31.57	12.14	17.36	25.28	0.20%	1.61%	14.32%	40.52%
Phase 1	23.79	41.75	10.22	15.91	24.08	1.12%	3.81%	23.77%	46.64%
Phase 2	30.01	46.64	11.82	18.68	30.06	0.22%	3.06%	16.38%	37.34%
Phase 3	35.26	60.41	13.94	20.92	35.24	0.00%	2.35%	9.97%	14.94%

The table reports mean, standard deviation and the quartiles (Q1 to Q3) of the distribution of time gaps between consecutive transactions involving the same dyads (i.e., pairs of banks). The following columns display the percentages of dyads involved in consecutive transactions within respectively 1 day, 5 days, 10 days and 15 days across the observation period (Overall) and by phase (Phase 1, Phase 2 and Phase 3).

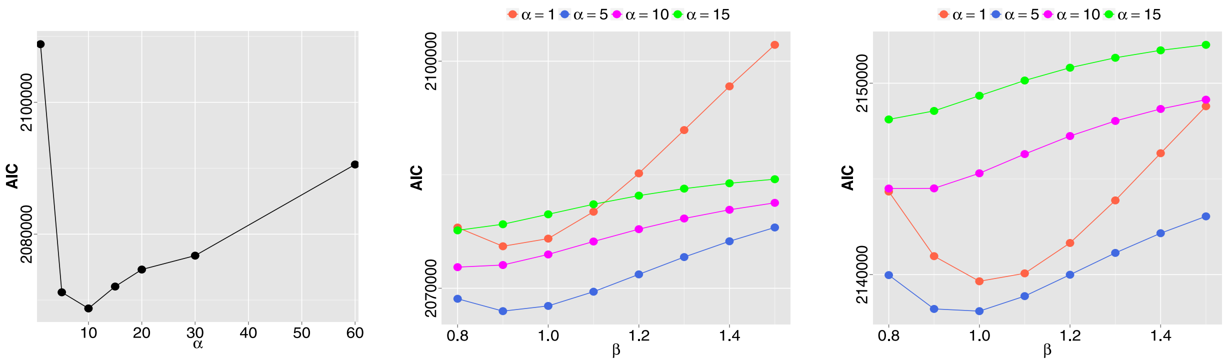

Fig. 4. AIC curves of models M2, M3, and M4 (respectively in the first, second and third column) at different values of event weighting parameters α and β. AIC is the Akaike Information Criterion coefficient for each model, which is defined as a combination of α and β values. For each model α is the length of the time gap in which all past transactions are weighted by 1.0 and β is the decay rate of past transactions occurred outside the α time window.

- Model with count-based statistics (M1) uses the weighting function (2) that assigns a constant weight 1.0 to all events no matter how far they are in the past.
- Model with short- and long-term statistics (M2) uses the weighting functions (3) and (4) to separate each counted-based statistic into short-term and long-term ones.
- Model with time-weighted statistics (M3) uses the weighting function (5) to account for the temporal relevance of events.
- Model with amount and time-weighted statistics (M4) uses the weighting function (5) similar to model M3 but also includes lending amounts w^e_{ij} in the computation of the statistics.

Since weighting functions of models M2, M3, and M4 are parametrized, we carry a grid search for each model to obtain its optimal set of parameters. Figs. 4 and 5 show respectively AIC and BIC of these models at different values parameters α and β. We specified α equal to 1, 5, 10, 15 days, while β was set in the range between 0.8 and 1.5. Lower values of the AIC and BIC coefficients outline a better fitting model. Accordingly, model M2 is best fit when short- and long-term events are separated by a 10-day window. Model M3 is optimal when events within the last 5 days are assigned a constant weight 1.0 while weights of events further in the past are decreased at an exponential decay rate $\beta = 0.9$. Model M4 is optimal at the recent window of 5 days and $\beta = 1.0$, which corresponds to a sharper decrease in the weight of past transactions.

Models M2, M3, and M4 with optimal parameter sets are compared with model M1 in Table 4 on both model selection criteria AIC and BIC. We observe that M2 and M3 which account for the importance of recent events fit data better than model M1 where all events are considered equal. This result is an empirical evidence for the temporal relevance in the computation of network statistics. Secondly, model M3 is better than model M2 on both model selection criteria. This means that compared to the short- and long-term separation scheme, the time-weighting scheme in M3 results in not only a more parsimonious model but also a better data fit, as indicated by the lower values of the AIC and BIC coefficients. Each network statistic corresponds to one coefficient only and value changes across phases can be interpreted more straightforwardly.

The temporal relevance of events is integrated as their continuous weights rather than being discretized into multiple time windows. Finally, the AIC and BIC values for M4 point out that the inclusion of lending amounts into the network statistics does not improve the fit. Interesting enough, this result outlines that the market structure is explained by the history of past transactions between banks, but not by the traded amounts. Also, the selected M3 suggests that the short-time window covers 5 days.

Parameters α and β are obtained by comparing the BIC and AIC values computed for the three phases together. However, it is reasonable to expect that the time-related dynamics might actually differ across phases. This might be particularly the case for the examined period, because the interbank market was affected by several changes. We repeat

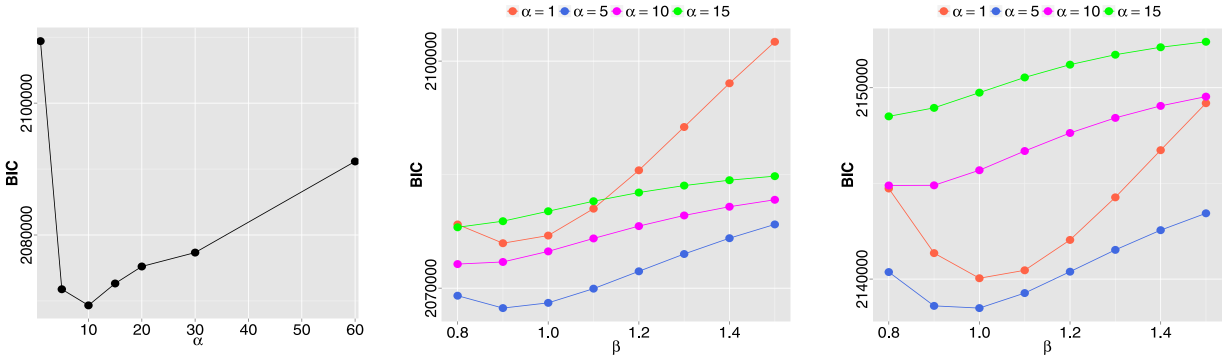

Fig. 5. BIC curves of models M2, M3, and M4 (respectively in the first, second and third column) at different values of event weighting parameters α and β. BIC is the Bayesian Information Criterion coefficient for each model, which is defined as a combination of α and β values. For each model α is the length of the time gap in which all past transactions are weighted by 1.0 and β is the decay rate of past transactions occurred outside the α time window.

Table 4
AICs and BICs of four models with different weight schemes for network statistics.

Model criterion	M1	M2 $\alpha = 10$	M3 $\alpha = 5$ $\beta = 0.9$	M4 $\alpha = 5$ $\beta = 1.0$
AIC	2,160,280	2,068,467	**2,066,456**	2,138,114
BIC	2,162,710	2,068,113	**2,066,877**	2,138,535

For basic scheme (M1), long- and short-term separation scheme (M2), time-weighting scheme (M3) and time-weighting scheme with traded amount (M4), the table reports the values of α and β parameters which correspond to the lowest values of AIC and BIC and therefore ensure the best model fit. AIC and BIC values of the best model are in bold.

the comparison on the three periods separately (Tables 9 to 11 in the Appendix). Patterns of results remain substantially unchanged. Only the β values are slightly different to suggest that the influence of past trades – outside the 5-day recent window – on future one declines sharper during the pre-crisis phase. This is possibly due to the higher frequency of transactions observed during this phase. We will interpret coefficient estimates of model M3 to reveal which mechanisms of counterpart selection explain the observed network structure.

Table 5 displays the results related to the best specification of M3. To streamline the discussion, we focus on salient parameter estimates. The negative estimates of the *Recency* parameter point to the likelihood of involvement with a new lending transaction following a recent one. In other words, transactions involving the same bank are clustered in time, suggesting that banks are likely to trade on the interbank market for limited periods only. This tendency decreases steadily during the observation period, possibly following a decline in the market activity that made transactions involving the same banks slightly further apart (although still in the same recent time window, as suggested by the analysis of the weighting schemes). Because this decrease is particularly marked between phase 1 and 2, it might be linked to the growing market uncertainty that led liquidity-rich banks to reduce their activity on the unsecured segment of the interbank market and liquidity-short banks to borrow less on the same market in order to preserve their reputation [42].

The parameter estimates of degree and intensity statistics are mostly in line with expectations, pointing to the centralization around few banks that specialize in borrowing or lending roles. The estimates of *In-degree* and *In-intensity* parameters are positive to indicate that borrowers tend to borrow more over time. The more lenders or the more borrowing transactions a bank has had, the higher likelihood of borrowing in the future. Both parameter estimates decrease throughout the observation period, though remaining positive, to further confirm a potential reputation effect on the borrowing side of the market. The estimates of the *Out-degree* parameter are very small, therefore variations in its values are difficult to interpret. The parameter estimates of *Out-intensity* are negative in all phases to suggest that lenders are less likely to borrow from others. Interesting enough, including newly specified statistics based on frequency of transactions allows comparing the commonly used centrality measures based on number of trading counterparts to centrality measures based on frequency of past transactions. The high values of the *Out-intensity* and *In-intensity* parameters suggest that specialization in lending or borrowing roles depends more on the frequency of transactions than on the number of partners. Once accounted for this frequency, the well documented skewness of the degree distribution [e.g., 23] appears less crucial to explaining interbank lending. This tendency becomes stronger across

Table 5
Repeated events Cox hazard regression of trades in the EU interbank market.

	Phase 1			Phase 2			Phase 3		
	Par.	(SE)		Par.	(SE)		Par.	(SE)	
Out-degree	0.001	(−0.001)		−0.001	(0.0002)	***	−0.002	(0.0002)	***
In-degree	0.049	(0.001)	***	0.039	(0.001)	***	0.020	(0.001)	***
Out-intensity	−0.763	(0.016)	***	−0.925	(0.018)	***	−1.710	(0.030)	***
In-intensity	0.871	(0.007)	***	0.799	(0.008)	***	0.650	(0.008)	***
Size lender	0.017	(0.001)	***	0.013	(0.001)	***	0.008	(0.001)	***
Recency	−0.275	(0.004)	***	−0.208	(0.004)	***	−0.193	(0.003)	***
Degree assortativity	−0.001	(0.00001)	***	−0.0003	(0.00001)	***	−0.0002	(0.00001)	***
Intensity assortativity	−0.335	(0.005)	***	−0.460	(0.007)	***	−0.706	(0.013)	***
Country match	0.850	(0.019)	***	0.967	(0.022)	***	1.993	(0.055)	***
Size difference	0.003	(0.001)	**	0.001	(0.001)		0.001	(0.001)	
Reciprocity	0.182	(0.010)	***	0.152	(0.005)	***	0.072	(0.005)	***
Inertia	0.106	(0.001)	***	0.234	(0.001)	***	0.255	(0.001)	***
Transitivity closure	0.012	(0.001)	***	0.010	(0.001)	***	−0.030	(0.001)	***
Cyclic closure	−0.065	(0.003)	***	−0.037	(0.003)	***	−0.027	(0.004)	***
Balance closure	−0.007	(0.001)	***	0.001	(0.001)		0.013	(0.001)	***
AIC	788,787.98			724,165.72			553,502.27		
BIC	788,929.98			724,307.10			553,640.10		

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.
For each phase, the table reports parameter estimates and standard deviations of the network statistics included in the best fitting M3; $\alpha = 5$ and $\beta = 0.9$. The AIC and BIC parameters reported in bold in Table 4 are displayed by phase here.

phases. The increasingly negative estimates of the *Out-intensity* parameters point to a stronger division of roles between lenders and borrowers during the crisis, while the decreasingly positive values of the *In-intensity* parameter suggest that particularly active borrowers seem to attract slightly fewer loans. The negative *Degree* and *Intensity assortativity* values can be interpreted as anti-assortativity tendencies. We speculate that active lenders are unlikely to lend money to active borrowers, so to distribute the risk involved in lending activity. The risk of loans not being repaid affects lending activity regardless of the level of market uncertainty, because it is mostly due to e-MID being an unsecured market. Of course, this risk is also likely to increase in phases of uncertainty, and so is tendency against assortativity. At macro level, this corresponds to a core–periphery structure based on frequency of trading, whereas active lenders are more likely to distribute their liquidity among many small borrowers. Parameter values are particularly high in phase 3. This is also the period when the ECB implemented a set of operations which induced banks to reduce their activity on the interbank market and turn more to the ECB [39]. The decision to switch to fixed-rate full-allotment monetary policy tenders increased the tendency of liquidity-short banks to obtain liquidity from ECB monetary policy operations, and of liquidity-rich banks to deposit their liquidity excess at the ECB overnight deposit facility [41].

The positive estimates of the *Reciprocity* parameter suggest that banks reciprocate trading relationships, lending back to their previous lenders. In line with the increasing division of roles across the sampled period, this tendency declines in phase 2 and even more in phase 3. *Inertia* – which is specific of REMs – is positive to indicate that banks tend to rely on established relationships, lending money to their previous borrowers [22]. These are the parameter estimates displaying the sharpest increase from phase 1 to phase 2. *Inertia* shows that banks did not simply reduce their activity on the unsecured segment of the market, but also modified their lending behavior in favor of previous borrowers.

The values of the clustering parameter point to densely connected subgroups. Interpreted together, the positive *Transitive closure* and the negative *Cyclic closure* outlines the existence of local hierarchical ordering, whereby few banks lend to several others. Controlling the overall flow of liquidity within sub-groups, these banks tend to behave as "local central banks". This behavior disappears when the market enters phase 3, as shown by the significantly negative estimates of the *Transitive closure* parameter. Remaining parameter estimates are in line with expectations.

4.1. Testing time-heterogeneity

For simplicity, we have assumed that salient network mechanisms operate similarly within each phase, and so does the memory structure of the market. This might hold true for several markets, but should be carefully checked in the case of the interbank market. Since banks are subject to an average minimum reserve requirement (RMP), needs for liquidity may differ over a RMP. Indeed, the RMP might generate a strong seasonality in the market with trading activity reaching a peak over the last days of the period [43]. Conversely, it might happen that some banks front load their reserve accounts at the beginning of the RMP and borrow less toward the end of it [39]. To investigate seasonality, we adopt the method illustrated in [47]. We use ECB official reports to identify the starting and ending date of the 48 RMPs occurred between

Table 6
Repeated events Cox hazard regression of trades in the EU interbank market (by RMP period).

	Phase 1			Phase 2			Phase 3		
	Start	Center	End	Start	Center	End	Start	Center	End
	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)	Par. (SE)
Out-degree	0.001 (−0.002)	0.002 (0.002)	0.001 (−0.001)	−0.002 (0.0003)***	−0.002 (0.0002)***	−0.001 (0.0001)***	−0.001 (0.0002)***	0.001 (0.0002)***	−0.002 (0.0002)***
In-degree	0.035 (0.002)***	0.039 (0.001)***	0.052 (0.001)***	0.036 (0.002)***	0.031 (0.002)***	0.044 (0.001)***	0.017 (0.001)***	0.014 (0.001)***	0.025 (0.001)***
Out-intensity	−0.645 (0.019)***	−0.648 (0.015)***	−0.771 (0.015)***	−0.884 (0.019)***	−0.893 (0.017)***	−0.928 (0.018)***	−1.702 (0.024)***	−1.696 (0.033)***	−1.713 (0.028)***
In-intensity	0.832 (0.008)***	0.808 (0.007)***	0.885 (0.008)***	0.752 (0.008)***	0.736 (0.007)***	0.804 (0.008)***	0.590 (0.008)***	0.600 (0.008)***	0.670 (0.007)***
Size lender	0.014 (0.002)***	0.010 (0.001)***	0.019 (0.001)***	0.009 (0.001)***	0.007 (0.001)***	0.015 (0.001)***	0.006 (0.001)***	0.006 (0.002)**	0.009 (0.001)***
Recency	−0.208 (0.003)***	−0.237 (0.004)***	−0.286 (0.004)***	−0.181 (0.004)***	−0.196 (0.004)***	−0.213 (0.004)***	−0.182 (0.004)***	−0.187 (0.003)***	−0.195 (0.003)***
Degree assort.	0.001 (0.00001)***	−0.001 (0.00001)***	−0.001 (0.00001)***	−0.0002 (0.00001)***	−0.0001 (0.00001)***	−0.0003 (0.00001)***	−0.0001 (0.00001)***	−0.0002 (0.00001)***	−0.0002 (0.00001)***
Intensity assort.	−0.325 (0.007)***	−0.328 (0.005)***	−0.343 (0.005)***	−0.445 (0.005)***	−0.448 (0.004)***	−0.471 (0.007)***	−0.698 (0.017)***	−0.686 (0.008)***	−0.709 (0.013)***
Country match	0.640 (0.021)***	0.930 (0.021)***	0.810 (0.018)***	0.964 (0.022)***	0.968 (0.018)***	0.972 (0.024)***	2.008 (0.052)***	2.004 (0.049)***	1.986 (0.056)***
Size difference	0.002 (0.001)*	0.004 (0.002)*	0.004 (0.001)***	0.001 (0.002)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)	0.001 (0.001)
Reciprocity	0.207 (0.030)***	0.191 (0.030)***	0.176 (0.010)***	0.162 (0.006)***	0.158 (0.008)***	0.147 (0.004)***	0.081 (0.004)***	0.073 (0.005)***	0.067 (0.005)***
Inertia	0.113 (0.001)***	0.098 (0.001)***	0.103 (0.001)***	0.249 (0.003)***	0.227 (0.001)***	0.226 (0.001)***	0.263 (0.002)***	0.261 (0.030)***	0.248 (0.001)***
Transit. closure	0.010 (0.002)***	0.013 (0.001)***	0.008 (0.001)***	0.007 (0.002)**	0.011 (0.002)***	0.009 (0.001)***	−0.026 (0.002)***	−0.034 (0.001)***	−0.029 (0.001)***
Cyclic closure	−0.060 (0.002)***	−0.071 (0.030)*	−0.064 (0.002)***	−0.034 (0.001)***	−0.035 (0.003)***	−0.04 (0.003)***	−0.02 (0.006)**	−0.031 (0.004)***	−0.026 (0.004)***
Balance closure	−0.009 (−0.001)***	−0.006 (0.001)***	−0.008 (0.001)***	0.002 (0.001)*	0.004 (0.001)***	0.001 (0.001)	0.009 (0.001)***	0.014 (−0.001)***	0.011 (0.001)***

*$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.
For each period within each phase, the table reports parameter estimates and standard deviations of the network statistics included in the best fitting M3.

2006 and 2009. Then, we split each RMP into three subperiods of equal length labeled as "RMP start", "RMP center" and "RMP end" and define the sets of events and non-events related to each subperiod. Finally, we replicate the analysis (i.e., searching for the best specification of the parameters α and β; estimating the model in Table 5) for each of the three subperiods of phases 1 to 3. Of course, data might be split in other ways within a RMP, for instance to separate the transactions occurred in the final few days of the RMP from the others. We find that $\alpha = 5$ is the optimal short-term window for all periods. Conversely, the decay parameter β varies within a RMP, with higher values toward the end of the RMP (β ranges between 1 and 1.40 for phase 1, between 0.70 and 0.95 for phase 2, and between 0.65 and 0.90 for phase 3. No meaningful differences are observed between the initial and central part of the RMP). This is possibly due to the increased market activity in the final days of the RMP that makes recent past transactions more relevant than those far away in the past.

Results (Table 6) seem to indicate that tendencies observed in Table 5 are best representative of the final part of a RMP, when a higher number of transactions occur. Indeed, the results confirm the existence of seasonality, with higher values in the centralization of transactions in the final days of a RMP, as shown by the positive and increasing values of *In-degree* and *In-intensity*. Likewise, the increasingly negative values of *Out-intensity* suggest that lenders are even less likely to borrow from others toward the end of a RMP. In other words, division of roles becomes stronger. This result appears to be strengthened by the lower level of *Reciprocity* indicating that a clearer division of roles emerges also within pairs of banks which traded in the past. It is worth noticing that REMs detect a more marked seasonality in phase 1 than in the following phases. This suggests that banks' trading behavior tends to become less differentiated across a RMP when the activity in e-MID decreases and, possibly, the market becomes less crucial for liquidity provision.[1]

4.2. Relational event models and dynamic network models

We have started this paper by claiming that extant network models typically do not use all information available in the data, require to aggregate transactions over convenient timestamps and assume a simplified memory structure of the market. Hence, these models may prevent exploring salient characteristics of actors' trading behaviors. Specifically, what are the differences between dynamic network modeling and relational event modeling? To answer this question, we compare the two approaches directly.

We select the stochastic actor-oriented model (SAOM) for comparison [23], because it is complex and allows examining many nuances of actor trading behavior. Indeed, among dynamic network models, SAOM is the closest to REM both conceptually and statistically. For each phase, we aggregate transactions over three-month periods [22,27] to obtain a panel network dataset. Then, we dichotomize each network. This results in losing information on traded amount. Network X has generic entry x_{ij} which equals 1 if i lent to j over the sampled period, and 0 otherwise. X_t is modeled as a function of the changes in microstructures made by the generic actors i and j between time $(t-1)$ and t. Past transactions cannot be weighted. To allow a direct comparison, we specify the network statistics that capture similar trading behaviors to those modeled by REMs and run the same model for each phase.[2] Because both SAOMs and REMs aim to model network microstructures, some network statistics are the same (or very similar) across models.[3] Others exist in one framework only. Table 7 displays the results. As mentioned above, network statistics based on memory and frequency cannot be specified. This has two main implications on the results. First, degree-related effects have significantly positive and large values, which point to centralization in borrowing and lending as well as to a core–periphery structure. As we have already shown, these effects become significantly weaker when we control for intensity-related effects. Second, because network statistics are interdependent, the impossibility of accounting for salient trading behaviors may affect sign and magnitude of the included parameter estimates.

Some network statistics are specific of SAOMs, and are due to their focus on states.[4] The *Rate* coefficient counts the number of ties changed between two consecutive periods. Rate assessment can be improved, specifying interactions with individual and network characteristics [23]. Yet, this does not allow capturing frequency properly. The model could be further refined to include the *Persistence* of observed network statistics - i.e., a trading behavior that did not emerge across consecutive periods, but was already existing in the prior period and banks decided not to change [23]. Also, it would be possible to account for *Past trade* between two banks. However, this captures inertia just partially. The model allows specifying *Past trade* only as symmetrical, making it impossible to distinguish between lending and borrowing roles. For

[1] Findings have to be interpreted carefully. [47] observe that comparing the magnitude of the network coefficients across models is not straightforward when there are several models and little variation across them. Models across groups might not predict the response variable equally well, and differences in the coefficients might actually be due to differences in unobserved heterogeneity.

[2] For comparability, we follow the approach used by [23]. They build on the hypothesis that trading behavior is heterogeneous across three-month periods, estimate separate parameter values for each pairs of consecutive periods and then compute the average across periods. This two-step procedure is based on [48]. It is worth noticing that it is most frequently used when modeling many periods.

[3] Stochastic actor-oriented models have limited capability to deal with continuous variables, unless they are grouped in few categories. The *Size* related statistics had to be simplified.

[4] This is the case because these models were introduced to study interpersonal relationships, and then successfully extended to other contexts. However, for market transactions this conceptualization requires to simplify data, as transactions are events by definition.

Table 7
Stochastic actor-oriented estimates of trades in the EU interbank market.

<table>
<tr><th></th><th colspan="4">Phase 1</th><th colspan="4">Phase 2</th><th colspan="4">Phase 3</th></tr>
<tr><th></th><th>Par.</th><th>(SE)</th><th></th><th>Sig. periods</th><th>Par.</th><th>(SE)</th><th></th><th>Sig. periods</th><th>Par.</th><th>(SE)</th><th></th><th>Sig. periods</th></tr>
<tr><td>Rate</td><td>59.146</td><td></td><td></td><td></td><td>109.927</td><td></td><td></td><td></td><td>30.134</td><td></td><td></td><td></td></tr>
<tr><td>Density</td><td>−4.146</td><td>(0.216)</td><td>*</td><td>|0| 0|5|</td><td>−2.635</td><td>(0.542)</td><td>*</td><td>|0| 0|3|</td><td>−5.895</td><td>(0.044)</td><td>*</td><td>|0| 0|4|</td></tr>
<tr><td>Out-degree</td><td>0.095</td><td>(0.048)</td><td>*</td><td>|3| 2|0|</td><td>−0.098</td><td>(0.128)</td><td></td><td>|1| 2|0|</td><td>0.255</td><td>(0.061)</td><td>*</td><td>|4| 0|0|</td></tr>
<tr><td>In-degree</td><td>0.223</td><td>(0.036)</td><td>*</td><td>|5| 0|0|</td><td>0.216</td><td>(0.079)</td><td>*</td><td>|2| 1|0|</td><td>0.364</td><td>(0.044)</td><td>*</td><td>|4| 0|0|</td></tr>
<tr><td>Degree assortativity</td><td>0.036</td><td>(0.009)</td><td>*</td><td>|4| 1|0|</td><td>0.029</td><td>(0.022)</td><td></td><td>|1| 2|0|</td><td>0.028</td><td>(0.010)</td><td>*</td><td>|4| 0|0|</td></tr>
<tr><td>Country match</td><td>0.028</td><td>(0.042)</td><td></td><td>|2| 2|1|</td><td>0.100</td><td>(0.050)</td><td>*</td><td>|2| 1|0|</td><td>0.956</td><td>(0.019)</td><td>*</td><td>|4| 0|0|</td></tr>
<tr><td>Size similarity</td><td>−0.334</td><td>(0.179)</td><td></td><td>|1| 2|2|</td><td>0.145</td><td>(0.118)</td><td></td><td>|1| 2|0|</td><td>0.001</td><td>(0.001)</td><td></td><td></td></tr>
<tr><td>Reciprocity</td><td>0.544</td><td>(0.080)</td><td>*</td><td>|5| 0|0|</td><td>0.076</td><td>(0.095)</td><td></td><td>|1| 1|1|</td><td>0.530</td><td>(0.134)</td><td>*</td><td>|4| 0|0|</td></tr>
<tr><td>Transitivity closure</td><td>−0.0001</td><td>(0.004)</td><td></td><td>|0| 5|0|</td><td>−0.020</td><td>(0.030)</td><td></td><td>|1| 2|0|</td><td>−0.002</td><td>(0.006)</td><td></td><td>|1| 2|1|</td></tr>
<tr><td>Cyclic closure</td><td>−0.019</td><td>(0.007)</td><td>*</td><td>|0| 0|5|</td><td>−0.037</td><td>(0.015)</td><td>*</td><td>|0| 1|2|</td><td>−0.031</td><td>(0.020)</td><td></td><td>|0| 2|2|</td></tr>
<tr><td>Balance closure</td><td>0.002</td><td>(0.001)</td><td>*</td><td>|1| 4|0|</td><td>−0.010</td><td>(0.011)</td><td></td><td>|0| 2|1|</td><td>0.006</td><td>(0.003)</td><td>*</td><td>|2| 2|0|</td></tr>
</table>

*$p < 0.05$. Sig. periods indicates in how many periods the parameter estimates are significant on the 5% level [positive sig.| lack of sig.| negative sig.]. No other significance level can be used in SAOMs. For each phase, the table reports parameter estimates and standard deviations of the network statistics included in SAOMs and comparable to the best fitting M3 estimated by REMs.

a similar reason, *Past trade* is also an imperfect proxy for frequency. Because REMs offer high flexibility, the same effect could be captured by replacing amount with intensity in the specification of the time-weighted function.

5. Conclusions

We introduce a modeling framework which allows bridging the macro- and micro-level structure in the analysis of financial networks. By focusing on the trading microstructures, our approach uncovers how the topological properties of the network emerge and change over time as a result of actors' selection of their trading counterparts. The empirical exercise that we present illustrates the building blocks of our approach. Examining the interbank market during different phases of the 2007–2008 global financial crisis, we demonstrate the capability of REMs to assess whether and how current transactions depend on past ones. Our empirical study outlines how the influence of past transactions unfolds, documenting how the market microstructures emerge and evolve when market uncertainty increases. We show that REMs allow testing microstructures that would not have been possible to model otherwise. In doing so, we better assess the relevance of trading behaviors that appeared salient in previous studies and propose additional – if not alternative – mechanisms of tie formation based on time and intensity. Also, we demonstrate how these statistics can be fine-tuned to account for the memory structure of the market, distinguishing between short- and long-term influence in a parsimonious way. Our empirical exercise highlights that e-MID is a market with short memory, where transactions occurring outside a five-days time frame have a sharply decreasing likelihood of affecting current transactions.

Because the purpose of our empirical analysis is mainly illustrative, we draw on the approach adopted by papers which have examined the interbank market in itself [e.g., 22,23]. Future work aimed at testing specific hypotheses could apply REMs to better assess the impact of exogenous factors on e-MID trading dynamics. For instance, it has been repeatedly claimed that the unconventional policy measures taken by the ECB with the aim of easing tensions in the market caused an excess liquidity which led banks to decrease their activity on the interbank market [e.g., 49]. In this paper we argue these measured could be a plausible reason for the reduced activity on the e-MID platform, and specifically the increasing tendency toward anti-assortativity. However, we can only speculate on these effects. Future research could test salient hypotheses directly. Because there is anecdotal evidence that exchanges clustered around the timing of ECB policy interventions [43], data could be stratified by period to compare trading dynamics before and after interventions as well as in the short and long aftermath of them.

Also, the model specifications could be refined in several directions. First, it would be possible to select a subset of particularly meaningful microstructures. Second, the parameters of the time-weighted statistics could be set *a priori* according to reasonable assumptions on the memory structure of the market. Third, the transaction value could be specified as the interest rate at which each transaction occurs instead of the exchanged amount.

Table 8
Network statistics.

Name	Formula	Description
Out-degree	$\sum_{k\neq j} \boldsymbol{I}[N_{jk}(t) > 0]$	$N_{jk}(t)$ is the number of lending transactions from bank j to bank k (with k denoting a generic bank) by time t, $k = 1, \ldots, n$. $\boldsymbol{I}(x)$ is an indicator function that equals 1 if the statement x is true, 0 otherwise.
Out-intensity	$\frac{\sum_{k\neq j}\sum_{e=1}^{N_{jk}(t)} f(t,T^e_{jk},\boldsymbol{\nu})}{\sum_{k\neq j}\boldsymbol{I}[N_{jk}(t)>0]}$	T^e_{jk} is time of lending event e between bank j and bank k. Each event is assigned a weight $f(t, T^e_{jk}, \boldsymbol{\nu})$ which accounts for the event temporal relevance as specified by $\boldsymbol{\nu}$.
In-degree	$\sum_{k\neq j} \boldsymbol{I}[N_{kj}(t) > 0]$	$N_{kj}(t)$ is the number of lending transactions that bank j received from bank k by time t.
In-intensity	$\frac{\sum_{k\neq j}\sum_{e=1}^{N_{kj}(t)} f(t,T^e_{kj},\boldsymbol{\nu})}{\sum_{k\neq j}\boldsymbol{I}[N_{kj}(t)>0]}$	The number of times that bank j received money from bank k. Each borrowing event is assigned a weight $f(t, T^e_{kj}, \boldsymbol{\nu})$.
Covariate sender	$c_{t,j}$	It is a value of a covariate for bank j at time t.
Recency	$t - max_{e\in E_j}(te)$	t_e is the time of transaction event from bank j.
Degree assortativity	$Out\text{-}degree(t, i) \times In\text{-}degree(t, j)$	It accounts for assortativity between i and j by number of ties.
Intensity assortativity	$Out\text{-}intensity(t, i) \times In\text{-}intensity(t, j)$	It accounts for assortativity between i and j by number of transactions.
Covariate mismatch	$c_{t,i} - c_{t,j}$	It accounts for the difference between sender i and receiver j nodes in the value of a continuous covariate.
Reciprocity	$\sum_{e=1}^{N_{ji}(t^-)} f(t, T^e_{ji}, \boldsymbol{\nu}) \times w^e_{ji}$	An event e from j to i at time t reciprocates a previous event from i to j. e is weighted by $f(t, T^e_{ji}, \boldsymbol{\nu})$. If $w^e_{ij} \neq 1$, the amount of money traded is included in the specification of the statistic.
Inertia	$\sum_{e=1}^{N_{ij}(t^-)} f(t, T^e_{ij}, \boldsymbol{\nu}) \times w^e_{ij}$	An event e from i to j at time t follows a previous event from i to j.
Transitive closure	$\sum_{k\neq i,j} m\left[a\left(t,i,k\right), a\left(t,k,j\right)\right]$	The harmonic mean function $m(x, y) = \frac{2xy}{x+y}$ is used to combine the trading flows $a(t, i, k)$ and $a(t, k, j)$ into one measurement of trading of the corresponding two-path, with $a(t, i, k) = \sum_{e=1}^{N_{ik}(t^-)} f(t, T^e_{ik}, \boldsymbol{\nu}) \times w^e_{ik}$ defined as the current number of transactions from i to k. Similar definition is applied to $a(t, k, j)$ from k to j. If $w^e_{ij} \neq 1$, traded amount is included.
Cyclic closure	$\sum_{k\neq i,j} m\left[a\left(t,j,k\right), a\left(t,k,i\right)\right]$	It models generalized exchange of resources, which captures the tendency of i to lend to j if i has received money from k in the past.
Balance closure	$\sum_{k\neq i,j} m\left[a\left(t,i,k\right), a\left(t,j,k\right)\right]$	It accounts for the effect that lending relationships of i and j can exert on the direct relationship between i and j themselves.

These examples are a demonstration of the reasons why we believe that the merits of REMs extend well beyond the empirical analysis of the e-MID dataset and the interbank market. The premises of REMs with time-weighted statistics, as well as their flexibility, make this modeling framework suitable for exploring the evolutionary dynamics of trading networks in general [50], and of those characterized by high-frequency transactions [51] – like the increasingly popular digital markets – in particular.

CRediT authorship contribution statement

Paola Zappa: Conceptualization, Data curation, Formal analysis, Methodology, Resources, Supervision, Writing - original draft, Writing - review & editing. **Duy Q. Vu:** Data curation, Formal analysis, Methodology, Software, Validation, Visualization, Writing - original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

See Tables 8–11.

Table 9
AICs and BICs of four models with different weight schemes for network statistics (Phase 1).

Model criterion	M1	M2 $\alpha = 5$	M3 $\alpha = 5$ $\beta = 1.3$	M4 $\alpha = 5$ $\beta = 1.4$
AIC	817,499	787,153	**786,611**	798,281
BIC	817,641	787,371	**786,753**	798,423

Basic scheme (M1), long- and short-term separation scheme (M2), time-weighting scheme (M3) and time-weighting scheme with traded amount (M4) applied to Phase 1. For each scheme, the table reports the values of α and β parameters which correspond to the lowest values of AIC and BIC and therefore ensure the best model fit. AIC and BIC values of the best model are in bold.

Table 10
AICs and BICs of four models with different weight schemes for network statistics (Phase 2).

Model criterion	M1	M2	M3	M4
		$\alpha = 10$	$\alpha = 5$ $\beta = 0.85$	$\alpha = 5$ $\beta = 1.0$
AIC	762,641	724,146	**723,926**	750,715
BIC	762,782	724,362	**724,068**	750,856

Basic scheme (M1), long- and short-term separation scheme (M2), time-weighting scheme (M3) and time-weighting scheme with traded amount (M4) applied to Phase 2. For each scheme, the table reports the values of α and β parameters which correspond to the lowest values of AIC and BIC and therefore ensure the best model fit. AIC and BIC values of the best model are in bold.

Table 11
AICs and BICs of four models with different weight schemes for network statistics (Phase 3).

Model criterion	M1	M2 $\alpha = 10$	M3 $\alpha = 5$ $\beta = 0.85$	M4 $\alpha = 5$ $\beta = 0.7$
AIC	580,140	555,574	**553,358**	585,973
BIC	582,287	555,785	**553,496**	586,111

Basic scheme (M1), long- and short-term separation scheme (M2), time-weighting scheme (M3) and time-weighting scheme with traded amount (M4) applied to Phase 3. For each scheme, the table reports the values of α and β parameters which correspond to the lowest values of AIC and BIC and therefore ensure the best model fit. AIC and BIC values of the best model are in bold.

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