Remote State Estimation With a Strategic Sensor Using a Stackelberg Game Framework

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***Abstract*—This article studies how to design the encoder and decoder in the context of dynamic remote state estimation with a strategic sensor. The cost of the remote estimator is the estimation error covariance, whereas the cost of the self-interested strategic sensor includes an additional term related to its private information. A Stackelberg game is employed to model the interaction between the strategic sensor and the remote estimator, where the leader (strategic sensor) first designs the encoder, and the follower (remote estimator) then determines the decoder. We derive the optimal encoder and decoder based on the mismatched cost functions, and characterize the equilibrium for some special cases. One interesting result is that the equilibrium can be achieved by transmitting nothing under certain conditions. The main results are illustrated by numerical examples.**

***Index Terms*—Estimation, privacy, Stackelberg game, strategic sensor.**

I. Introduction

Wireless sensor networks are prevalent and indispensable in a wide range of applications, including environmental monitoring, traffic control, health care, and manufacturing industries [1]. Wireless networking builds connections between various equipment nodes, but private and secure exchange of information is not easily guaranteed. For instance, private information, such as vehicle trajectories and personal health data, needs to be protected. However, different purposes of the involved users may cause potential conflicts of interest. Even without a malicious attacker or eavesdropper, users' private information can be fully accessed by others whom they do not completely trust.

Take privacy issues in crowdsensing as an example. Crowdsensing is a broad range of community sensing paradigms, where individuals with mobile devices capable of sensing and computing collectively share data and extract useful information to measure and map processes of common interests [2]. Specifically, personal drivers are sometimes financially incentivized and actively recruited to report their locations, accidents, police traps, or any other hazards along the way to a fusion center to improve the quality of everyone's daily driving. One famous application is *Waze*, where users contribute together to the "common good." However, some drivers do not trust the fusion center enough to tell the whole truth and, therefore, they may strategically alter the transmitted data to protect their privacy within a certain range. Motivated by this context, the encoding strategy and the corresponding decoding strategy need to be carefully designed when self-interested sensors are involved.

The strategic information transmission has been a hot research topic in the privacy field for many years. Different formulations and approaches have been proposed. Akyol *et al.* [3] considered a Stackelberg game where objectives are different for the transmitter (leader) and the receiver (follower). First, the transmitter, whose cost is additionally related to some private information, announces an encoding strategy. Second, the receiver determines the decoding strategy to minimize its cost based on the announced encoding map. The equilibrium and the associated costs are characterized in this static scalar state estimation problem. Similar results on Stackelberg game equilibriums for static state estimation with strategic sensors and further extensions can be found in [4]–[7]. For dynamic state estimation with self-interested sensors, Farokhi *et al.* [8] assumed that the private information of the strategic sensor evolves independently according to a linear update rule. A Stackelberg game is also employed to model the interaction between the sensors and the remote estimator. The equilibrium is characterized and the sensor's transmission policy is proved to be memoryless. However, the true state and the private information are assumed to be known by the strategic sensor. This may not be true due to the measurement noise in real applications. Sarıtaş *et al.* [9] investigated the dynamic quadratic Gaussian signaling games under Nash and Stackelberg equilibria. They showed that affine policies constitute an invariant subspace for Nash equilibria, and the Stackelberg equilibria admit linear policies for scalar cases. However, they considered a constant bias in the objective, and provided analysis on the properties of encoding and decoding policies, instead of constructing the optimal strategies.

In this work, we consider a dynamic state estimation problem with a strategic sensor in a Stackelberg game-theoretic framework. The leader (strategic sensor) first designs the encoder aiming to make the remote estimator compute the state estimate as the leader would expect, taking its private information into consideration. The follower (remote estimator) then determines the decoder to obtain

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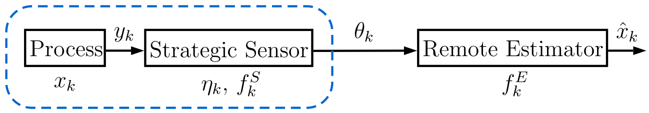

Fig. 1. System block diagram.

the estimate as close to the original true state as possible. Since the private information is correlated with the true source state, it becomes quite complicated to design encoder and decoder in dynamic remote estimation.

The main contribution of this work and comparison with existing work from the literature are summarized as follows.

1) In this work, we develop encoder and decoder, which can minimize their respective costs with a dynamic process in a Stackelberg game-theoretic setup, where the private information has correlation with the state. To the best of our knowledge, it is the first time that private information correlated with the source state has been studied for an encoder and decoder design problem in the context of dynamic state estimation.
2) One of the main results in our work indicates that the equilibrium can be achieved by transmitting nothing, which is in accordance with the results derived in [3]. To the best of our knowledge, we are the first to extend this interesting result in static scalar state estimation with strategic sensors to a dynamic vector case.

The remainder of this article is organized as follows. Section II introduces the setup of the strategic information transmission in a Stackelberg game-theoretic framework and the problem of interest. Section III derives the optimal encoder and decoder, and analyzes the equilibrium. Section IV provides simulations and interpretations. Section V draws conclusions.

Notations: $\mathbb{R}$ denotes the set of real numbers. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{S}_+^n$ ($\mathbb{S}_{++}^n$) is the set of $n \times n$ positive semidefinite (definite) matrices. When $X \in \mathbb{S}_+^n$ ($\mathbb{S}_{++}^n$), we simply write $X \succeq 0$ ($X \succ 0$). The identity matrix with size n is represented by I_n. The superscript $^\top$, †, $\mathrm{Tr}\{\cdot\}$, $\mathrm{r}\{\cdot\}$, and $\rho(\cdot)$ stand for the transpose, Moore–Penrose pseudoinverse, trace, rank, and spectral radius of a matrix, respectively. $\mathscr{R}(X)$ and $\mathscr{N}(X)$ denote the range space and null space of X. $\mathbb{E}[\cdot]$ denotes the expectation of a random variable. $\mathcal{N}(\mu, \Sigma)$ denotes Gaussian distribution with mean μ and covariance Σ. For functions f_1 and f_2, $f_1 \circ f_2$ is defined as $f_1 \circ f_2(X) \triangleq f_1(f_2(X))$.

II. Preliminaries

Consider the system in Fig. 1. The discrete-time linear time-invariant process is as follows:

$$x_{k+1} = Ax_k + w_k \tag{1}$$

$$y_k = Cx_k + v_k \tag{2}$$

where $x_k \in \mathbb{R}^n$ is the process state, and $y_k \in \mathbb{R}^m$ is the measurement collected by the sensor. The process noise $w_k \in \mathbb{R}^n$, the measurement noise $v_k \in \mathbb{R}^m$, and the initial state x_0 are mutually independent zero-mean Gaussian random variables with covariance $Q \succeq 0, R \succ 0$, and $\Pi_0 \succeq 0$, respectively. The pair (A, C) is assumed to be observable and $(A, \sqrt{Q})$ is controllable.

A. Strategic Information Transmission

The strategic information transmission was originally introduced in [10], which aroused wide attention in the economic field. In the strategic information transmission model, there are two players: a transmitter (e.g., strategic sensor) and a receiver (e.g., remote estimator). The mismatch between the transmitter and the receiver is modeled using different costs where the transmitter's cost is additionally affected by a single parameter, e.g., the private information η_k. Specifically, in this work, at each time k, the transmitter collects the measurement y_k and transmits a message $\theta_k = f_k^S(y_1, \ldots, y_k, \eta_k)$, where η_k is the private information kept between the process and the strategic sensor, and the encoder f_k^S is a stochastic mapping. The receiver observes θ_k and produces an estimate of the state x_k through a mapping f_k^E as $\hat{x}_k = f_k^E(\theta_1, \ldots, \theta_k)$. The objective of the receiver is to pick a decoder f_k^E so as to minimize the trace of the state estimation error covariance

$$D_k^E \triangleq \mathbb{E}\left[d^E\left(x_k, \hat{x}_k\right)\right] \tag{3}$$

where $d^E : \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ is defined as

$$d^E\left(x_k, \hat{x}_k\right) \triangleq \mathrm{Tr}\left\{\left(x_k - \hat{x}_k\right)\left(x_k - \hat{x}_k\right)^\top\right\}. \tag{4}$$

The objective of the transmitter is to minimize

$$D_k^S \triangleq \mathbb{E}\left[d^S\left(x_k, \eta_k, \hat{x}_k\right)\right] \tag{5}$$

using the freedom in choosing the mapping f_k^S, given the receiver's objective function. Similarly, for the transmitter, the function d^S : $\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}$ is given by

$$d^S\left(x_k, \eta_k, \hat{x}_k\right) \triangleq \mathrm{Tr}\left\{\left(x_k + \eta_k - \hat{x}_k\right)\left(x_k + \eta_k - \hat{x}_k\right)^\top\right\}. \tag{6}$$

In the strategic information transmission, the parameter η_k is introduced by the strategic sensor to blur the state estimation, and hence to protect the exact value of the system state x_k. To be consistent with the definition in [3], we call it private information.

B. Stackelberg Game

In this section, we introduce the Stackelberg game, which is a key supporting concept for our strategic information transmission.

Let Φ_S and Φ_E be the sets of admissible strategies for the player strategic sensor and the player remote estimator, respectively. Let the cost functions $J_S(\phi_S, \phi_E)$ and $J_E(\phi_S, \phi_E)$ be two functions mapping $\Phi_S \times \Phi_E \mapsto \mathbb{R}$ such that the player strategic sensor wishes to minimize J_S and the player remote estimator wishes to minimize J_E. In a Stackelberg game, the player (strategic sensor) who selects his strategy first is called *the leader*. The player (remote estimator) who selects his strategy second is called *the follower*. The definitions of a Stackelberg optimal strategy pair and the Stackelberg game equilibrium are stated as follows, mainly based on the work in [11]–[13].

Definition 1: If there exists a mapping $\mathcal{M} : \Phi_S \mapsto \Phi_E$ such that, for any fixed $\phi_S \in \Phi_S, J_E(\phi_S, \mathcal{M}(\phi_S)) \leq J_E(\phi_S, \phi_E)$ for all $\phi_E \in \Phi_E$, and if there exists $\phi_S^\star \in \Phi_S$ such that $J_S(\phi_S^\star, \mathcal{M}(\phi_S^\star)) \leq J_S(\phi_S, \mathcal{M}(\phi_S))$ for all $\phi_S \in \Phi_S$, then the pair $(\phi_S^\star, \phi_E^\star) \in \Phi_S \times \Phi_E$, where $\phi_E^\star = \mathcal{M}(\phi_S^\star)$, is called a Stackelberg optimal strategy pair. An equilibrium in the Stackelberg game is reached under this optimal strategy pair.

The remote state estimation with the strategic sensor is then formulated as a Stackelberg game.

Problem 1:

$$f_k^{S\star} = \arg\min_{f_k^S} \mathbb{E}\left[d^S\left(x_k, \eta_k, f_k^{E\star}\left(\theta_1, \ldots, \theta_{k-1}, f_k^S\left(y_1, \ldots, y_k, \eta_k\right)\right)\right)\right]$$

$$f_k^{E\star} = \arg\min_{f_k^E} \mathbb{E}\left[d^E\left(x_k, f_k^E\left(\theta_1, \ldots, \theta_k\right)\right)\right].$$

The strategic sensor aims to obtain the optimal encoder $f_k^{S\star}$ first, given that the remote estimator will design the corresponding optimal decoder $f_k^{E\star}$, as stated in Problem 1. The strategic sensor aims to drive the estimate $\hat{x}_k$ to a perturbed state $x_k + \eta_k$, whereas the remote estimator intends to obtain an accurate estimate $\hat{x}_k$ of the true state x_k. If the private information $\eta_k = 0$, the objectives of the strategic sensor and the remote estimator are exactly the same, and the situation degenerates to a standard remote state estimation problem. The private information η_k reflects the mismatch between both players' objectives.

C. Strategic Sensor

For the strategic sensor, the data packet θ_k transmitted contains the private information η_k of concern. The privacy mechanism is proposed to protect the process's states, for example, the real-time trajectory of the vehicle. For the quadratic Gaussian setting in the static strategic remote estimation problem [3]–[6], the source x_k and the private information η_k are jointly Gaussian. In the dynamic strategic remote estimation problem [8], [14], and [15], the private information η_k is assumed to evolve according to a linear update rule independently. Based on the existing literature work mentioned earlier, we focus on scenarios where the private information $\eta_k \in \mathbb{R}^n$ is an affine transformation of the source state x_k, i.e.,

$$\eta_k \triangleq (\Gamma - I_n)\, x_k + \beta_k \tag{7}$$

where $\beta_k \sim \mathcal{N}(0, \Sigma_\beta)$ is an independent identically distribute (i.i.d.) Gaussian variable independent of the process noise and measurement noise. The transformation matrix $\Gamma \in \mathbb{R}^{n\times n}$ and the covariance matrix $\Sigma_\beta \in \mathbb{S}_+^n$ are known by the strategic sensor and the remote estimator.

Remark 1: Notice that the affine-formed private information is composed of two parts, which is similar to a combination of the two forms in the existing work [3]–[6], [8], [14], and [15], where one is correlated with the dynamic process state, and the other is independent of all the random variables. For the correlated part $(\Gamma - I_n)x_k$, let us consider the special case when the dynamic matrix A is stable. According to (1), it becomes a stationary Gaussian process and the state x_k is therefore zero-mean Gaussian distributed. Then, it is obvious that x_k and η_k are jointly Gaussian, which is aligned with the setting in the static strategic remote estimation problem [3]–[6]. For the independent part β_k, a larger Σ_β results in a smaller correlation coefficient between x_k and η_k.

The reason why a random-type η_k is adopted, instead of a deterministic constant, is to capture situations where the strategic sensor wants to lie based on the present state. Actually, η_k can be designed using an arbitrary function of x_k and β_k. The affine transformation is chosen in this work to illustrate the effectiveness of the strategic information transmission. We shall consider general nonlinear functions in future work.

Since sensors nowadays are often equipped with memory buffer and on-board processors [16], the preprocessing capability can improve the system performance. The strategic sensor in Fig. 1 is capable of running a local Kalman filter. Its minimum mean-squared error (MMSE) state estimate $\hat{x}_k^\ell$ and the corresponding error covariance P_k^ℓ for $k \geq 1$ are denoted as

$$\hat{x}_k^\ell \triangleq \mathbb{E}\left[x_k \mid y_1, \ldots, y_k\right]$$

$$P_k^\ell \triangleq \mathbb{E}\left[\left(x_k - \hat{x}_k^\ell\right)\left(x_k - \hat{x}_k^\ell\right)^\top \mid y_1, \ldots, y_k\right]$$

which are computed via a Kalman filter as follows:

$$\hat{x}_{k|k-1}^\ell = A\hat{x}_{k-1}^\ell \tag{8}$$

$$P_{k|k-1}^\ell = AP_{k-1}^\ell A^\top + Q \tag{9}$$

$$K_k^\ell = P_{k|k-1}^\ell C^\top\left(CP_{k|k-1}^\ell C^\top + R\right)^{-1} \tag{10}$$

$$\hat{x}_k^\ell = \hat{x}_{k|k-1}^\ell + K_k^\ell z_k \tag{11}$$

$$P_k^\ell = \left(I_n - K_k^\ell C\right) P_{k|k-1}^\ell \tag{12}$$

where z_k is the local innovation

$$z_k \triangleq y_k - CA\hat{x}_{k-1}^\ell. \tag{13}$$

The initial states are $\hat{x}_0^\ell$ and P_0^ℓ. From Anderson and Moore [17], the estimation error covariance of the Kalman filter converges to a unique value $\overline{P}^\ell$ no matter what the initial values are. For notational brevity, we define the operators $h, \tilde{g} : \mathbb{S}_+^n \mapsto \mathbb{S}_+^n$ as

$$h(X) \triangleq AXA^\top + Q$$

$$\tilde{g}(X) \triangleq X - XC^\top\left(CXC^\top + R\right)^{-1} CX.$$

We assume that both the *a priori* and *a posteriori* error covariances at the strategic sensor have already reached the steady states and let

$$P_k^\ell = \overline{P}^\ell, P_{k|k-1}^\ell = P^\ell, k \geq 0$$

where $\overline{P}^\ell$ is the unique positive semidefinite solution to $\tilde{g} \circ h(X) = X$ and P^ℓ is the unique positive semidefinite solution to $h \circ \tilde{g}(X) = X$. This results in a steady-state local Kalman filter with fixed gain

$$K^\ell = P^\ell C^\top\left(CP^\ell C^\top + R\right)^{-1}$$

and the innovation z_k is i.i.d. zero-mean Gaussian distributed with covariance $CP^\ell C^\top + R$.

In existing literature [18]–[21], the sensor sends innovation z_k to the remote estimator. One reason is that transmitting zero-mean Gaussian variable z_k, instead of the raw measurement y_k or the local estimate $\hat{x}_k^\ell$, can reduce the communication bandwidth and save the sensor's energy consumption due to a lower average signal magnitude, which is communication efficient [22]. Besides, the innovation sequence $\{z_k\}$ contains all the information of the measurement sequence $\{y_k\}$, and thus z_k is informative enough. In this work, we suppose that the strategic sensor transmits a message θ_k based on the innovation z_k and the private information η_k. Motivated by the linear encoder proposed in [9, Th. 15] for a vector case dynamic Stackelberg game, we focus on the affine transformation, i.e.,

$$\theta_k = f_k^S\left(z_k, \eta_k\right) \triangleq T_k z_k + b_k \tag{14}$$

where $b_k \sim \mathcal{N}(0, \Sigma_{b,k})$ is an i.i.d. Gaussian variable. Both the transformation matrix $T_k \in \mathbb{R}^{m\times m}$ and the covariance matrix $\Sigma_{b,k} \in$

$\mathbb{S}_{+}^{m}$ need to be determined by the strategic sensor to minimize D_k^S in the Stackelberg game framework.

Remark 2: The affine transformation of the encoding policy is one of the possible choices of the strategic sensor. By choosing an affine transformation, the strategic sensor is able to derive the optimal encoding strategy with specific parameters, which ensures both players to achieve the game equilibrium. The goal here is to provide constructive directions on how to design encoding and decoding strategies. Thanks to the nice structural properties provided by the affine transformations, important insights into this strategic dynamic state estimation can be obtained. More general nonlinear transformations are left for future work.

D. Remote Estimator

An affine estimator is adopted at the remote estimator. Based on the receiver's measure (4), its MMSE state estimate $\hat{x}_k$ and the corresponding error covariance P_k are given by

$$\hat{x}_k \triangleq \mathbb{E}\left[x_k \mid \theta_1, \ldots, \theta_k\right]$$

$$P_k \triangleq \mathbb{E}\left[\left(x_k - \hat{x}_k\right)\left(x_k - \hat{x}_k\right)^\top \mid \theta_1, \ldots, \theta_k\right].$$

The affine remote estimator is in the form

$$\hat{x}_k = f_k^E\left(\theta_1, \ldots, \theta_k\right) \triangleq A\hat{x}_{k-1} + L_k\theta_k \tag{15}$$

where the estimation gain L_k needs to be optimized by the receiver. Since all the noises involved are Gaussian, this recursive affine estimator is also the best MMSE decoder possible under certain conditions. No nonlinear estimator can do better. More detailed discussions are presented in Lemma 2. Besides, we denote the error covariance S_k at the strategic sensor as

$$S_k \triangleq \mathbb{E}\left[\left(x_k + \eta_k - \hat{x}_k\right)\left(x_k + \eta_k - \hat{x}_k\right)^\top \mid \theta_1, \ldots, \theta_k\right].$$

E. Problem of Interest

In this work, we aim to tackle Problem 1 in a Stackelberg game framework. The two players collaboratively accomplish the state estimation task to serve their respective purposes. Specifically, at each time k, the strategic sensor derives T_k and $\Sigma_{b,k}$ to minimize D_k^S given the remote estimator's objective and its affine form but not necessarily the value of L_k. The leader, i.e., the strategic sensor, forward θ_k together with decision variables T_k and $\Sigma_{b,k}$ to the remote estimator. Then, the follower decides its estimation gain L_k to minimize D_k^E. Note that although the remote estimator does not disclose the value of L_k in advance, the strategic sensor can infer it. This is because the strategic sensor knows that the goal of the remote estimator is to obtain an accurate state estimate. With this capability, the strategic sensor can make its optimal decision among all the feasible choices. The motivation of forwarding the encoding strategy is to accomplish the state estimation in a collaborative manner since the performance of the strategic sensor relies on the remote estimator. Specifically, the goal of the strategic sensor is not to produce a freely chosen state estimate, but to drive the estimate to a designed perturbed value. Intuitively, far from degrading the strategic sensor's performance, broadcasting the encoding strategy helps the strategic sensor to minimize its objective function. By implementing this mechanism, the two players achieve a Stackelberg game equilibrium.

III. Equilibrium Analysis on Strategic Information Transmission

In this section, we characterize the equilibrium in this Stackelberg game with the strategic sensor. First, the optimal decoder $f_k^{E\star}$ is derived for the remote estimator and the recursive affine decoder is proved to be MMSE optimal among all the possible types of decoders under certain conditions. Second, for the strategic sensor, getting the optimal encoder $f_k^{S\star}$ is simplified to solving a concrete optimization problem, and it can be solved perfectly when the system parameters satisfy some conditions. For general cases, an algorithm is proposed to obtain the optimal encoder numerically. Finally, based on the optimal strategy pair $(f_k^{S\star}, f_k^{E\star})$, limiting costs at this equilibrium are presented for some special cases.

A. Optimal Decoder $f_k^{E\star}$

As a preliminary, the following Lemma 1 gives solutions to a normal equation, which is used to derive the optimal decoder. The proof is presented in Appendix A.

Lemma 1: The normal equation

$$L_k\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] = P^\ell C^\top T_k^\top$$

has feasible solutions, and they are all given by

$$\begin{aligned} L_k = {} & P^\ell C^\top T_k^\top\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger \\ & + B\bigg\{I_m - \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\ & \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger\bigg\} \end{aligned} \tag{16}$$

for arbitrary $B \in \mathbb{R}^{n\times m}$.

Then, one obtains the optimal decoder as follows.

Theorem 1: The optimal decoder is $\hat{x}_k = f_k^\star(\theta_1, \ldots, \theta_k) = A\hat{x}_{k-1} + L_k^\star\theta_k$, where

$$\begin{aligned} L_k^\star \triangleq {} & P^\ell C^\top T_k^\top\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger \\ & + B\bigg\{I_m - \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\ & \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger\bigg\} \end{aligned} \tag{17}$$

and $B \in \mathbb{R}^{n\times m}$ can be arbitrarily designed.

Proof: Based on (1) and (15), $x_k - \hat{x}_k$ can be computed as

$$\begin{aligned} x_k - \hat{x}_k = {} & A\left(x_{k-1} - \hat{x}_{k-1}\right) - L_kT_kCA\left(x_{k-1} - \hat{x}_{k-1}^\ell\right) \\ & + \left(I_n - L_kT_kC\right)w_{k-1} - L_kT_kv_k - L_kb_k. \end{aligned}$$

Then, the corresponding error covariance is

$$\begin{aligned} P_k &= \mathbb{E}\left[\left(x_k - \hat{x}_k\right)\left(x_k - \hat{x}_k\right)^\top\right] \\ &= AP_{k-1}A^\top + \left(L_kT_k\right)R\left(L_kT_k\right)^\top + L_k\Sigma_{b,k}L_k^\top \\ &\quad + \left(I_n - L_kT_kC\right)Q\left(I_n - L_kT_kC\right)^\top \\ &\quad + \left(L_kT_kC\right)A\overline{P}^\ell A^\top\left(L_kT_kC\right)^\top \end{aligned}$$

$$- A\mathbb{E}\left[(x_{k-1} - \hat{x}_{k-1})\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] A^{\top}(L_k T_k C)^{\top}$$
$$- (L_k T_k C) A\mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)(x_{k-1} - \hat{x}_{k-1})^{\top}\right] A^{\top}.$$

The correlation term can be computed as

$$\begin{aligned}
&\mathbb{E}\left[(x_{k-1} - \hat{x}_{k-1})\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] \\
&= \mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1}^{\ell} + \hat{x}_{k-1}^{\ell} - \hat{x}_{k-1}\right)\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] \\
&= \overline{P}^{\ell} + \mathbb{E}\left[\left(\hat{x}_{k-1}^{\ell} - \hat{x}_{k-1}\right)\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] \\
&= \overline{P}^{\ell}. \qquad (18)
\end{aligned}$$

The fact that $\mathbb{E}[(\hat{x}_{k-1}^{\ell} - \hat{x}_{k-1})(x_{k-1} - \hat{x}_{k-1}^{\ell})^{\top}] = 0$ is due to the orthogonality principle, i.e., all the random variables generated by the knowledge of the strategic sensor is independent of the estimation error $x_{k-1} - \hat{x}_{k-1}^{\ell}$ of the MMSE estimate $\hat{x}_{k-1}^{\ell}$. After some algebraic manipulation, the error covariance at the remote estimator is given by

$$\begin{aligned}
P_k &= AP_{k-1}A^{\top} + Q + L_k\Sigma_{b,k}L_k^{\top} + (L_kT_kC)P^{\ell}(L_kT_kC)^{\top} \\
&\quad - P^{\ell}(L_kT_kC)^{\top} - (L_kT_kC)P^{\ell} + (L_kT_k)R(L_kT_k)^{\top} \\
&= AP_{k-1}A^{\top} + Q - P^{\ell}(L_kT_kC)^{\top} - (L_kT_kC)P^{\ell} \\
&\quad + L_k\left[T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k}\right]L_k^{\top}. \qquad (19)
\end{aligned}$$

According to (optimal linear L.M.S. estimators) [23, Th. 3.2.1], any solution L_k to the following normal equation:

$$L_k\left[T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k}\right] = P^{\ell}C^{\top}T_k^{\top}$$

minimizes $\mathrm{Tr}\{P_k\}$. Thus, $L_k^{\star}$ is an optimal gain based on Lemma 1, which implies the optimal decoder $f_k^{E\star}$. ■

Lemma 2: The recursive affine decoder $f_k^{E\star}$ is MMSE optimal among all the possible types of decoders if

$$T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k} \succ 0. \qquad (20)$$

Proof: Since all random processes $(x_k, \theta_k, w_k, v_k, b_k)$ are jointly Gaussian, it follows that the conditional random variable at the remote estimator $(x_k \mid \theta_1, \ldots, \theta_k) \sim \mathcal{N}(\hat{x}_{k|k}, P_{k|k})$ and $(x_k \mid \theta_1, \ldots, \theta_{k-1}) \sim \mathcal{N}(\hat{x}_{k|k-1}, P_{k|k-1})$. Suppose at time k that $(\hat{x}_{k-1|k-1}, P_{k-1|k-1})$ is given. We shall compute $(\hat{x}_{k|k-1}, P_{k|k-1})$ and $(\hat{x}_{k|k}, P_{k|k})$ using the following two steps. First, $\hat{x}_{k|k-1} = \mathbb{E}[x_k \mid \theta_1, \ldots, \theta_{k-1}] = A\hat{x}_{k-1|k-1}$ and $P_{k|k-1} = AP_{k-1|k-1}A^{\top} + Q$ by recalling (1). Second, since $\theta_k = T_kC(x_k - A\hat{x}_{k-1}^{\ell}) + T_kv_k + b_k$, the conditional vector $(\left[\begin{smallmatrix} x_k \\ \theta_k \end{smallmatrix}\right] \mid \theta_1, \ldots, \theta_{k-1})$ is Gaussian with mean $\left[\begin{smallmatrix} A\hat{x}_{k-1|k-1} \\ 0 \end{smallmatrix}\right]$ and covariance

$$\begin{bmatrix} P_{k|k-1} & \left(P_{k|k-1} - \mathbb{E}\left[\left(x_k - \hat{x}_{k|k-1}\right)\hat{x}_{k-1}^{\ell\top}\right]A^{\top}\right)C^{\top}T_k^{\top} \\ * & T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k} \end{bmatrix}$$

where $*$ means the transpose of the second element in the first line of the augmented covariance matrix. We apply the formula for conditional expectation of Gaussian random variables preconditioned on $\theta_1, \ldots, \theta_{k-1}$. It follows that $(x_k \mid \theta_1, \ldots, \theta_k)$ is Gaussian with mean

$$A\hat{x}_{k-1|k-1} + \left(P_{k|k-1} - \mathbb{E}\left[\left(x_k - \hat{x}_{k|k-1}\right)\hat{x}_{k-1}^{\ell\top}\right]A^{\top}\right)C^{\top}T_k^{\top}$$
$$\times\left[T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k}\right]^{-1}\theta_k.$$

By algebraic manipulation, we have

$$\begin{aligned}
&P_{k|k-1} - \mathbb{E}\left[\left(x_k - \hat{x}_{k|k-1}\right)\hat{x}_{k-1}^{\ell\top}\right]A^{\top} \\
&= AP_{k-1|k-1}A^{\top} + Q - A\mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1|k-1}\right)\hat{x}_{k-1}^{\ell\top}\right]A^{\top} \\
&= A\mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1|k-1}\right)\left(x_{k-1} - \hat{x}_{k-1|k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right]A^{\top} \\
&\quad + Q
\end{aligned}$$

and further

$$\begin{aligned}
&\mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1|k-1}\right)\left(x_{k-1} - \hat{x}_{k-1|k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] \\
&= \mathbb{E}\left[x_{k-1}\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top} - \left(x_{k-1} - \hat{x}_{k-1|k-1}\right)\hat{x}_{k-1|k-1}^{\top}\right] \\
&= \mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1}^{\ell} + \hat{x}_{k-1}^{\ell}\right)\left(x_{k-1} - \hat{x}_{k-1}^{\ell}\right)^{\top}\right] \\
&= \overline{P}^{\ell}.
\end{aligned}$$

Finally, it turns out that $(x_k \mid \theta_1, \ldots, \theta_k)$ is Gaussian with mean

$$A\hat{x}_{k-1|k-1} + P^{\ell}C^{\top}T_k^{\top}\left[T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k}\right]^{-1}\theta_k$$

which implies that $\hat{x}_{k|k} = \hat{x}_k$, and the resulting optimal decoder is the same as the affine one as derived in Theorem 1 when condition (20) is satisfied.

Remark 3: Condition (20) holds when $\Sigma_{b,k} \succ 0$, or T_k has full rank.

B. Optimal Encoder $f_k^{S\star}$

For the strategic sensor, the best response can be obtained by solving a simpler optimization Problem 2, as provided in the following theorem.

Problem 2:

$$\min_{T_k \in \mathbb{R}^{m\times m}, \Sigma_{b,k} \in \mathbb{S}_{+}^{m}}$$
$$\mathrm{Tr}\left\{\Delta P^{\ell}C^{\top}T_k^{\top}\left[T_k\left(CP^{\ell}C^{\top} + R\right)T_k^{\top} + \Sigma_{b,k}\right]^{\dagger}T_kCP^{\ell}\right\}$$

where $\Delta \triangleq I_n - \Gamma^{\top} - \Gamma$. ■

Theorem 2: The optimal encoder $f_k^{S\star}$ is the solution to the optimization Problem 2 if Γ can commute with A.

Proof: Based on (7) and (15), the difference of $x_k + \eta_k - \hat{x}_k$ is given by

$$\begin{aligned}
&x_k + \eta_k - \hat{x}_k \\
&= \Gamma x_k + \beta_k - \hat{x}_k \\
&= A(\Gamma x_{k-1} + \beta_{k-1} - \hat{x}_{k-1}) + (\Gamma A - A\Gamma)x_{k-1} + \beta_k \\
&\quad - L_k^{\star}T_kv_k + (\Gamma - L_k^{\star}T_kC)w_{k-1} - L_k^{\star}b_k - A\beta_{k-1}
\end{aligned}$$

$$- L_k^\star T_k C A \left(x_{k-1} - \hat{x}_{k-1}^\ell \right).$$

Due to the commutativity between Γ and A, the corresponding covariance can be computed as follows:

$$\begin{aligned} S_k &= \mathbb{E}\left[(x_k + \eta_k - \hat{x}_k)(x_k + \eta_k - \hat{x}_k)^\top \right] \\ &= AS_{k-1}A^\top + \Sigma_\beta + (L_k^\star T_k) R (L_k^\star T_k)^\top + L_k^\star \Sigma_{b,k} (L_k^\star)^\top \\ &\quad + (\Gamma - L_k^\star T_k C) Q (\Gamma - L_k^\star T_k C)^\top - A\Sigma_\beta A^\top \\ &\quad + (L_k^\star T_k C) A \overline{P}^\ell A^\top (L_k^\star T_k C)^\top \\ &\quad - A\mathbb{E}\left[(\Gamma x_{k-1} - \hat{x}_{k-1}) \left(x_{k-1} - \hat{x}_{k-1}^\ell \right)^\top \right] A^\top (L_k^\star T_k C)^\top \\ &\quad - (L_k^\star T_k C) A \mathbb{E}\left[\left(x_{k-1} - \hat{x}_{k-1}^\ell \right) (\Gamma x_{k-1} - \hat{x}_{k-1})^\top \right] A^\top. \end{aligned}$$

The correlation term is equal to

$$\begin{aligned} &\mathbb{E}\left[(\Gamma x_{k-1} - \hat{x}_{k-1}) \left(x_{k-1} - \hat{x}_{k-1}^\ell \right)^\top \right] \\ &= \mathbb{E}\left[\left(\Gamma x_{k-1} - \Gamma \hat{x}_{k-1}^\ell + \Gamma \hat{x}_{k-1}^\ell - \hat{x}_{k-1} \right) \left(x_{k-1} - \hat{x}_{k-1}^\ell \right)^\top \right] \\ &= \Gamma \overline{P}^\ell \end{aligned}$$

where the last equation holds due to the same expression as given for (18). Hence, the covariance term S_k is simplified as

$$\begin{aligned} S_k &= AS_{k-1}A^\top + \Sigma_\beta - A\Sigma_\beta A^\top + \Gamma Q \Gamma^\top \\ &\quad - L_k^\star T_k C P^\ell \Gamma^\top - \Gamma P^\ell (L_k^\star T_k C)^\top \\ &\quad + L_k^\star \left[T_k \left(C P^\ell C^\top + R \right) T_k^\top + \Sigma_{b,k} \right] L_k^{\star\top}. \end{aligned} \tag{21}$$

Since $L_k^\star [T_k (C P^\ell C^\top + R) T_k^\top + \Sigma_{b,k}] = P^\ell C^\top T_k^\top$, we have

$$\begin{aligned} L_k^\star T_k C P^\ell \Gamma^\top &= L_k^\star \left[T_k \left(C P^\ell C^\top + R \right) T_k^\top + \Sigma_{b,k} \right] L_k^{\star\top} \Gamma^\top \\ \Gamma P^\ell (L_k^\star T_k C)^\top &= \Gamma L_k^\star \left[T_k \left(C P^\ell C^\top + R \right) T_k^\top + \Sigma_{b,k} \right] L_k^{\star\top}. \end{aligned}$$

Hence, minimizing the trace of the covariance S_k in (21) is equivalent to minimizing

$$\mathrm{Tr}\left\{ \Delta L_k^\star \left[T_k \left(C P^\ell C^\top + R \right) T_k^\top + \Sigma_{b,k} \right] L_k^{\star\top} \right\}. \tag{22}$$

By substituting the closed form in (17) for $L_k^\star$, one can verify that no matter what the value B takes, the term in (22) is equal to the objective function in Problem 2. ■

According to Theorem 1, the optimal decoder may be endowed with many forms due to the free choice of B in $L_k^\star$. Theorem 2 implies that whatever B the decoder chooses, the decision of optimal encoder is unrelated to it. Besides, since the optimization Problem 2 does not rely on time k any more, the optimal T_k and $\Sigma_{b,k}$ degenerate to constant matrices. The time-invariant property of the optimal T_k and $\Sigma_{b,k}$ is caused by the one-step optimization objective in Problem 1 and the assumption that the local Kalman filter at the strategic sensor has already reached the steady state. Additionally, due to the freedom in choosing Γ, it is possible for the strategic sensor to design the transformation matrix Γ such that it commutes with the system dynamic matrix A. In this Stackelberg game framework, the purpose of the strategic sensor is to solve Problem 2 to derive T_k and $\Sigma_{b,k}$. Though Problem 2 is more concrete and concise than the original Problem 1, the solution is not immediately obvious. The following Corollary 1 gives the optimal solutions in some special cases. The proof is presented in Appendix B.

Corollary 1: Consider two special cases for Problem 2 where Γ commutes with A.

1) If $\Delta \succeq 0$, then the optimal encoder is $f_k^{S\star} = b_k$.
2) If $\Delta \preceq 0$ and the strategic sensor's decision pair $(T_k, \Sigma_{b,k})$ needs to satisfy $\mathrm{r}\{T_k(CP^\ell C^\top + R)T_k^\top\} = \mathrm{r}\{T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}\}$, then the optimal encoder is $f_k^{S\star} = T_k z_k$, where T_k has full rank and can be arbitrarily designed.

Furthermore, the optimal encoders derived in both cases are also optimal in the sense of minimizing the long-run average cost, i.e.,

$$\min_{\{f_1^S, f_2^S, \ldots, f_k^S\}} \frac{1}{k} \sum_{t=1}^{k} \mathbb{E}\left[d^S(x_t, \eta_t, \hat{x}_t) \right]. \tag{23}$$

Case (1) in Corollary 1 is worthy of attention. When $\Delta \succeq 0$, its optimal strategy is to transmit only the noise b_k (i.e., $T_k = 0$), and the remote estimator updates only on independent noises, i.e., $\hat{x}_k = A\hat{x}_{k-1} + B(I_m - \Sigma_{b,k}\Sigma_{b,k}^\dagger) b_k$. Because the remote estimator can set $B = 0$ and, thus, does not utilize the received packet, this transmission strategy is equivalent to sending nothing, which also can save transmission energy. For example, let $\Gamma = 0$ and, hence, $\Delta = I_n \succeq 0$. The strategic sensor wants to drive the remote estimator's state estimate $\hat{x}_k$ to $x_k + \eta_k = \beta_k$, which is an i.i.d. Gaussian noise. Therefore, it is reasonable that the strategic sensor chooses to send only the noise b_k, without giving any additional information about the system state.

Algorithm 1: CCP Algorithm for the Optimal Encoder.

1: Input: Functions $\mathcal{F}, \mathcal{G}$, and $\nabla\mathcal{G}$;
2: Initialization: $\Lambda^0 \succeq 0$ and $i = 0$;
3: **repeat**
4: Form $\mathcal{G}(\Lambda; \Lambda^i) \triangleq \mathcal{G}(\Lambda^i) + \mathrm{Tr}\{\nabla\mathcal{G}(\Lambda^i)^\top(\Lambda - \Lambda^i)\}$;
5: Set the value of Λ^{i+1} as a solution to the convex optimization problem
$$\min_{\Lambda \in \mathbb{S}_+^m} \mathcal{F}(\Lambda) - \mathcal{G}(\Lambda; \Lambda^i); \tag{24}$$
6: Update iteration $i = i + 1$;
7: **until** stopping criterion (26) is satisfied.

Due to the intricacy of Problem 2 when Δ is neither positive semidefinite nor negative semidefinite, we propose an algorithm and solve it numerically, instead of giving a closed-form solution. In this case, Δ is an arbitrary matrix where Γ is required to commute with A, and $\Sigma_{b,k}$ is assumed to be positive definite. Since Δ is a real-valued symmetric matrix, it is always possible to write Δ as the difference $\Delta_1 - \Delta_2$ of two positive semidefinite matrices Δ_1 and Δ_2. For notational convenience, we define $\Lambda \triangleq T_k^\top \Sigma_{b,k}^{-1} T_k \succeq 0$, and define the functions $\mathcal{F}, \mathcal{G} : \mathbb{S}_+^m \mapsto \mathbb{R}$ as

$$\mathcal{F}(X) \triangleq -\mathrm{Tr}\Bigg\{ \sqrt{\Delta_2} P^\ell C^\top$$

$$\times\left\{X - X\left[\left(CP^{\ell}C^{\top}+R\right)^{-1}+X\right]^{-1}X\right\}CP^{\ell}\sqrt{\Delta_2}\Bigg\}$$

$$\mathcal{G}(X) \triangleq -\operatorname{Tr}\Bigg\{\sqrt{\Delta_1}P^{\ell}C^{\top}$$

$$\times\left\{X - X\left[\left(CP^{\ell}C^{\top}+R\right)^{-1}+X\right]^{-1}X\right\}CP^{\ell}\sqrt{\Delta_1}\Bigg\}.$$

According to [24, Lemma 1], both functions $\mathcal{F}$ and $\mathcal{G}$ are convex. When $\Sigma_{b,k} \succ 0$, we apply the matrix inversion lemma on $[T_k(CP^{\ell}C^{\top}+R)T_k^{\top}+\Sigma_{b,k}]^{-1}$ and we have

$$\left[T_k\left(CP^{\ell}C^{\top}+R\right)T_k^{\top}+\Sigma_{b,k}\right]^{-1}$$
$$=\Sigma_{b,k}^{-1}-\Sigma_{b,k}^{-1}T_k\left[\left(CP^{\ell}C^{\top}+R\right)^{-1}+T_k^{\top}\Sigma_{b,k}^{-1}T_k\right]^{-1}T_k^{\top}\Sigma_{b,k}^{-1}.$$

The objective function in Problem 2 is then transformed into $\operatorname{Tr}\{\Delta P^{\ell}C^{\top}\{\Lambda-\Lambda[(CP^{\ell}C^{\top}+R)^{-1}+\Lambda]^{-1}\Lambda\}CP^{\ell}\}$ by change of variable. Consequently, Problem 2 is equivalent to the following Problem 3, which is a difference of convex (DC) programming problem.

Problem 3:

$$\min_{\Lambda\in\mathbb{S}_{+}^{m}} \mathcal{F}(\Lambda)-\mathcal{G}(\Lambda).$$

The convex–concave procedure (CCP) [25] is a heuristic algorithm to find a local optimum for DC problems. To solve Problem 3 numerically, we propose Algorithm 1 based on the CCP. The gradient of $\mathcal{G}$ is provided as follows:

$$\nabla\mathcal{G}(X) = -\left\{I_m-\left[\left(CP^{\ell}C^{\top}+R\right)^{-1}+X\right]^{-1}X\right\}$$
$$\times CP^{\ell}\Delta_1P^{\ell}C^{\top}\left\{I_m-X\left[\left(CP^{\ell}C^{\top}+R\right)^{-1}+X\right]^{-1}\right\}. \tag{25}$$

One reasonable stopping criterion is that the improvement in the objective value is less than a nonnegative threshold δ [26]

$$\left(\mathcal{F}(\Lambda^{i})-\mathcal{G}(\Lambda^{i})\right)-\left(\mathcal{F}(\Lambda^{i+1})-\mathcal{G}(\Lambda^{i+1})\right)\le\delta \tag{26}$$

where the superscript i represents the ith iteration. After obtaining the solution $\Lambda^{\star}$ to Problem 3 by implementing Algorithm 1, we can recover the original variables by letting $\Sigma_{b,k}^{\star}=I_m$ and $T_k^{\star\top}T_k^{\star}=\Lambda^{\star}$, where $T_k^{\star}$ is an upper triangular matrix by Cholesky decomposition.

Remark 4: Algorithm 1, which replaces the concave term $-\mathcal{G}(\Lambda)$ with a convex upper bound, numerically searches an acceptable pair $(T_k,\Sigma_{b,k})$ for the strategic sensor. Other algorithms, i.e., disciplined convex–concave programming [27], also can be applied to solve the DC problem.

C. Equilibrium Analysis

Based on the aforementioned two sections, we provide results on the game equilibrium for some special cases.

Theorem 3: The strategic information transmission in a Stackelberg game framework achieves equilibrium in the following two special cases when Γ is designed to commute with A.

a) If $\Delta \succeq 0$, the equilibrium is achieved at

$$\theta_k = b_k$$
$$\hat{x}_k = A\hat{x}_{k-1}+B\left(I_m-\Sigma_{b,k}\Sigma_{b,k}^{\dagger}\right)\theta_k$$

where $\Sigma_{b,k}\in\mathbb{S}_{+}^{m}$ is determined by the strategic sensor and $B\in\mathbb{R}^{n\times m}$ is arbitrarily designed by the remote estimator.

b) If $\Delta \preceq 0$ and the strategic sensor's decision pair $(T_k,\Sigma_{b,k})$ needs to satisfy $\mathrm{r}\{T_k(CP^{\ell}C^{\top}+R)T_k^{\top}\}=\mathrm{r}\{T_k(CP^{\ell}C^{\top}+R)T_k^{\top}+\Sigma_{b,k}\}$, the equilibrium is achieved at

$$\theta_k = T_kz_k$$
$$\hat{x}_k = A\hat{x}_{k-1}+P^{\ell}C^{\top}\left(CP^{\ell}C^{\top}+R\right)^{-1}T_k^{-1}\theta_k$$

where $T_k\in\mathbb{R}^{m\times m}$ with full rank is determined by the strategic sensor.

Proof: Both cases directly come from the optimal encoder in Corollary 1 and the optimal decoder in Theorem 1. ■

Under the optimal strategy pair $(f_k^{S\star}, f_k^{E\star})$, the limiting costs for both players converge if $\rho(A)<1$ by observing the evolvements of error covariances S_k and P_k. For special cases when all the variables and matrices are scalar, the limiting costs when $|A|<1$ are given as follows.

1) If $\Gamma\le\frac{1}{2}$, the limiting costs under the optimal strategy pair derived in Case (a) are

$$D_{\infty}^{S}=\frac{\Gamma^2Q}{1-A^2}+\Sigma_{\beta} \tag{27}$$

$$D_{\infty}^{E}=\frac{Q}{1-A^2}. \tag{28}$$

2) If $\Gamma\ge\frac{1}{2}$, the limiting costs under the optimal strategy pair derived in Case (b) are

$$D_{\infty}^{S}=\frac{\Gamma^2Q}{1-A^2}+\frac{1-2\Gamma}{1-A^2}\frac{C^2P^{\ell 2}}{C^2P^{\ell}+R}+\Sigma_{\beta} \tag{29}$$

$$D_{\infty}^{E}=\frac{Q}{1-A^2}-\frac{1}{1-A^2}\frac{C^2P^{\ell 2}}{C^2P^{\ell}+R}. \tag{30}$$

One can calculate these limiting costs according to (19) and (21). When the system is stable, it turns out that both players' costs are bounded at the equilibrium.

IV. Simulation Results

In this section, we first take the linearized discrete-time model of a simplified longitudinal flight system (for more details, see [28]) as an example to illustrate the effectiveness of the proposed encoder and decoder with strategic sensor. The state variable $x_k\in\mathbb{R}^3$, representing the pitch angle, the pitch rate, and the normal velocity, respectively. The system parameter matrices are

$$A=\begin{bmatrix}0.99 & -0.12 & -0.43\\ 0 & 0.99 & -0.07\\ 0 & 0.82 & 0\end{bmatrix}, C=I_3$$

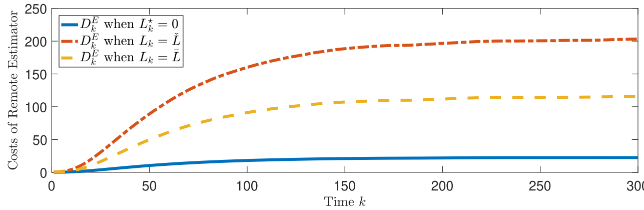

Fig. 2. Illustration of the optimality for the remote estimator, i.e., $f_k^{E\star}$, with the longitudinal flight system.

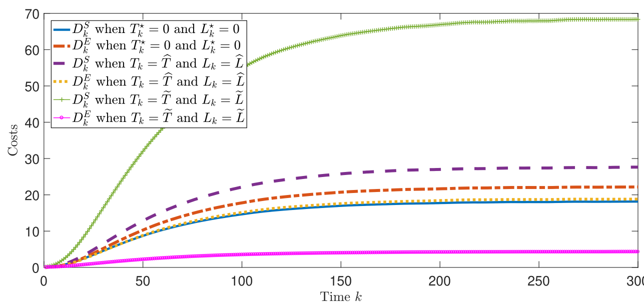

Fig. 3. Illustration of the optimality for the remote estimator, i.e., $f_k^{E\star}$, with the longitudinal flight system.

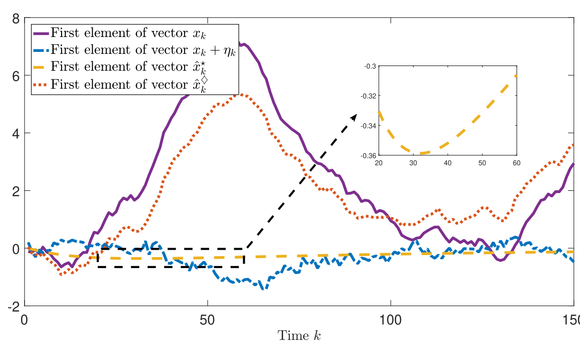

Fig. 4. Illustration of the optimality for the strategic sensor, i.e., $f_k^{S\star}$, with the longitudinal flight system.

$$Q = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix}, R = 0.01 I_3.$$

Let $\Gamma = \begin{bmatrix} -0.89 & 0.28 & 0.34 \\ 0 & -0.87 & 0.06 \\ 0 & -0.65 & -0.08 \end{bmatrix}$, which commutes with A and also implies $\Delta \succeq 0$, and $\Sigma_\beta = 0.01 I_3$. Fig. 2 illustrates the optimality of the proposed decoder $f_k^{E\star}$ in Theorem 1. When the strategic sensor sets $T_k = 0$ and $\Sigma_{b,k} = 0.1 I_3$, the optimal decoder is $L_k^\star = 0$ for $B = 0$. To compare the performance between different decoders, we randomly generate L_k's where $\check{L} \triangleq \begin{bmatrix} 0.40 & 0.43 & 0.72 \\ 0.77 & 0.62 & 0.18 \\ 0.56 & 0.27 & 0.72 \end{bmatrix}$ and $\bar{L} \triangleq \begin{bmatrix} 0.18 & 0.97 & 0.23 \\ 0.37 & 0.60 & 0.33 \\ 0.17 & 0.15 & 0.04 \end{bmatrix}$. After running 70 000 simulations, from Fig. 2, we can conclude that the optimal decoder derived in Theorem 1 leads to a lower cost D_k^E for the remote estimator since the blue solid line is below the red dash-dotted line and the yellow dashed line. Additionally, Fig. 3 illustrates the optimality of the proposed encoder $f_k^{S\star}$ in Corollary 1. Since $\Delta \succeq 0$, an optimal strategy pair is $T_k^\star = 0$, $\Sigma_{b,k}^\star = 0.1 I_3$, and $L_k^\star = 0$ when $B = 0$. Again, to compare the performance between different encoders, we randomly generate T_k's where $\widehat{T} \triangleq \begin{bmatrix} 0.81 & 0.91 & 0.28 \\ 0.91 & 0.63 & 0.55 \\ 0.13 & 0.10 & 0.96 \end{bmatrix}$ and $\widetilde{T} \triangleq \begin{bmatrix} 6.95 & 0.34 & 7.66 \\ 3.17 & 4.39 & 7.95 \\ 9.50 & 3.82 & 1.87 \end{bmatrix}$. The decoder is set as the optimal one with respect to the respective encoder, i.e., $\widehat{L} \triangleq P^\ell C^\top \widehat{T}^\top [\widehat{T}(CP^\ell C^\top + R)\widehat{T}^\top + \Sigma_{b,k}]^{-1}$ and $\widetilde{L} \triangleq P^\ell C^\top \widetilde{T}^\top [\widetilde{T}(CP^\ell C^\top + R)\widetilde{T}^\top + \Sigma_{b,k}]^{-1}$. After running 70 000 simulations, from Fig. 3, it can be observed that $T_k^\star = 0$ achieves a lower cost D_k^S for the strategic sensor in that the blue solid line is below the purple dashed line and the green solid line with plus signs. Moreover, due to the stability of the system dynamic matrix A, the limiting costs for both players are bounded. The blue solid line and the red dash-dotted line describe the costs for both the strategic sensor and the remote estimator at the Stackelberg game equilibrium under the optimal strategy pair.

In Fig. 4, the real-time pitch angle state (the first element of x_k), the target pitch angle state of the strategic sensor (the first element of $x_k + \eta_k$), and the remote pitch angle state estimates under two strategy pairs are shown. In this simulation, $\Gamma = \begin{bmatrix} -0.16 & -0.71 & 0.23 \\ 0 & -0.25 & 0.04 \\ 0 & -0.41 & 0.25 \end{bmatrix}$, which commutes with A and implies $\Delta \succeq 0$. The parameters A, C, Q, R, and Σ_β remain the same. According to Corollary 1, we let the optimal encoder and decoder pair be $T_k^\star = 0$ and $L_k^\star = 0$, and the resulting trajectory of $\hat{x}_k^\star$ is plotted by a yellow dashed line. As a comparison, $\hat{x}_k^\diamondsuit$ is computed using $T_k = \widetilde{T}$, $\Sigma_{b,k} = 0.1 I_3$, and $L_k = \widetilde{L}$, and the resulting trajectory of $\hat{x}_k^\diamondsuit$ is plotted by a red dotted line. The purple solid line shows the pitch angle state in one realization. The blue dash-dotted line is the target estimate for pitch angle as the strategic sensor expects. It can be seen that the pitch angle estimate $\hat{x}_k^\star$ under the optimal strategy pair is much closer to the blue dash-dotted line, whereas the red dotted line $\hat{x}_k^\diamondsuit$ deviates a lot from the blue dash-dotted line. Fig. 4 illustrates the effectiveness of the proposed encoder and decoder in real time.

When $\Gamma = \begin{bmatrix} 0.30 & 0.74 & 0.58 \\ 0 & 0.37 & 0.10 \\ 0 & -1.13 & 1.74 \end{bmatrix}$, the induced Δ is neither positive semidefinite nor negative semidefinite. Let $\Delta_1 = \Delta + 4I_3 \succ 0$ and $\Delta_2 = 4I_3 \succ 0$. We choose the initialized Λ^0 ($T_k^0 = 3I_n, \Sigma_{b,k}^0 = 0.1 I_3$), the computed Λ at the 5th ($T_k^5 =$

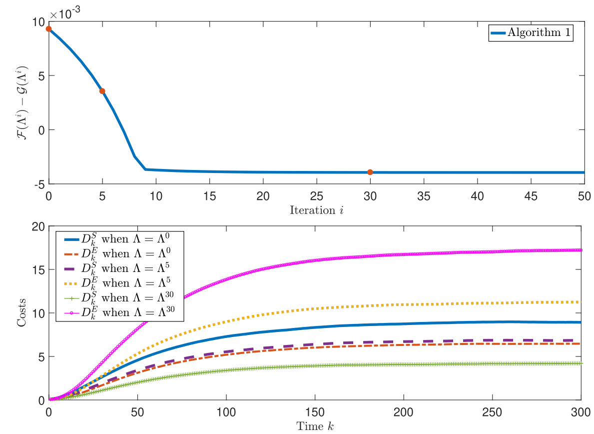

Fig. 5. Real-time states and remote estimates for the longitudinal flight system.

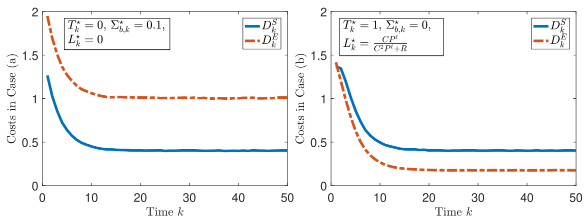

Fig. 6. Costs of strategic sensor and remote estimator based on Algorithm 1.

$\begin{bmatrix} 2.85 & 1.62 & 0.84 \\ 0 & 2.18 & -1.17 \\ 0 & 0 & 2.97 \end{bmatrix}$, $\Sigma_{b,k}^5 = 0.1 I_3$) and the 30th iteration ($T_k^{30} = \begin{bmatrix} 3.67 & 3.07 & 1.27 \\ 0 & 1.51 & -3.76 \\ 0 & 0 & 0.07 \end{bmatrix}$, $\Sigma_{b,k}^{30} = 0.1 I_3$) during the implement of Algorithm 1. As shown in Fig. 5, after 70 000 simulations, the strategic sensor's cost D_k^S induced by Λ^{30} is smaller than D_k^S induced by Λ^0 or Λ^5, which implies the effectiveness of the proposed Algorithm 1.

Second, we provide an example for a scalar system, where $A = 0.85$, $C = 1$, $Q = 0.28$, and $R = 0.31$. Let $\Gamma = 0.5$ and $\Sigma_\beta = 0.15$. From Theorem 3, one can find that both strategy pairs provided in Cases (a) and (b) can achieve the equilibrium since $\Delta = 0$. Due to the existence of random variables, we run 100 000 simulations for both cases. In Case (a), $T_k^\star = 0$, $\Sigma_{b,k}^\star = 0.10$, and $L_k^\star = 0$, and in Case (b), $T_k^\star = 1$, $\Sigma_{b,k}^\star = 0$, and

$$L_k^\star = \frac{CP^\ell}{C^2 P^\ell + R}$$

which are derived by Theorem 3. The costs D_k^S and D_k^E as defined in (5) and (3) are plotted in Fig. 6. The strategic sensor's cost D_k^S in both cases converge to 0.40, which is exactly the value calculated in (27) and (29) for equilibriums. Additionally, the limiting values of D_k^E in both cases are equal to the values as calculated in (28) and (30). Recall that the strategic sensor plays the leading role in this Stackelberg game and it takes actions first to minimize D_k^S with an affine-form strategy. It only cares about its own objective function D_k^S and pays no attention to D_k^E. Those are probably the reasons for $D_k^E < D_k^S$ in Case (b). The decisions it makes in both cases lead to the minimum D_k^S, and the equilibria are achieved in both cases.

V. Conclusions and Future Work

In this article, we investigated a dynamic remote state estimation problem with a strategic sensor in a Stackelberg game-theoretic framework. The mismatch of the cost functions between the strategic sensor and the remote estimator helps to protect the strategic sensor's private information. The optimal encoder and decoder were derived, and the equilibrium was characterized under certain conditions. Examples and simulations verified the theoretical results.

With the purpose of saving transmission energy, one possible future direction is to develop the encoder and decoder with an event trigger, since the result in this article indicates that sometimes, transmitting nothing is the optimal strategy. Besides, one may consider the scenario with multiple strategic sensors, which is essential in applications.

Appendix A
Proof of Lemma 1

First, we prove the existence of feasible solutions by proving that

$$\mathscr{R}\left(T_k C P^\ell\right) \subseteq \mathscr{R}\left(T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right).$$

Equivalently, we only need to focus on their null spaces and prove that

$$\mathscr{N}\left(T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right) \subseteq \mathscr{N}\left(P^\ell C^\top T_k^\top\right).$$

For some $\nu \in \mathscr{N}(T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k})$, there exists

$$\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]\nu = 0$$

which implies $\nu^\top[T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}]\nu = 0$. Furthermore, it implies $\nu^\top T_k(CP^\ell C^\top + R)T_k^\top \nu = 0$. Since $CP^\ell C^\top + R \succ 0$, $T_k^\top \nu = 0$ trivially. Therefore, $P^\ell C^\top T_k^\top \nu = 0$. Directly, this relation ensures the existence of the corresponding normal equation solutions.

Second, since there always exists a solution L_k such that $L_k[T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}] = P^\ell C^\top T_k^\top$ as proved earlier, all the solutions are given by (16) [29]. One can verify the normal equation's solution L_k as follows:

$$\begin{aligned}
&L_k\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&= P^\ell C^\top T_k^\top\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger \\
&\quad \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&\quad + B\Big\{I_m - \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&\quad \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger\Big\} \\
&\quad \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&= L_k\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&\quad \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger \\
&\quad \times \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&= L_k\left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right] \\
&= P^\ell C^\top T_k^\top
\end{aligned}$$

where both the second and third equations hold due to the definition of the Moore–Penrose pseudoinverse.

Appendix B
Proof of Corollary 1

Before presenting the optimality proof, we focus on the iteration of the error covariance S_k at the strategic sensor, as derived in (21)

$$S_k = AS_{k-1}A^\top + \Sigma_\beta - A\Sigma_\beta A^\top + \Gamma Q \Gamma^\top + g(T_k, \Sigma_{b,k})$$

where

$$g(T_k, \Sigma_{b,k})$$

$$\triangleq P^\ell C^\top T_k^\top \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger T_k C P^\ell$$
$$- P^\ell C^\top T_k^\top \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger T_k C P^\ell \Gamma^\top$$
$$- \Gamma P^\ell C^\top T_k^\top \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger T_k C P^\ell.$$

Furthermore, we have

$$S_k = A^k S_0 \left(A^\top\right)^k + \sum_{t=1}^{k} A^{k-t}\left(\Sigma_\beta - A\Sigma_\beta A^\top + \Gamma Q \Gamma^\top + g\left(T_t, \Sigma_{b,t}\right)\right)\left(A^\top\right)^{k-t}. \quad (31)$$

Due the commutativity of Γ and A, we obtain

$$\sum_{t=1}^{k} \mathrm{Tr}\left\{A^{k-t} g\left(T_t, \Sigma_{b,t}\right)\left(A^\top\right)^{k-t}\right\} = \sum_{t=1}^{k} \mathrm{Tr}\Big\{CP^\ell \left(A^\top\right)^{k-t} \Delta A^{k-t} P^\ell C^\top \times T_t^\top \left[T_t\left(CP^\ell C^\top + R\right)T_t^\top + \Sigma_{b,t}\right]^\dagger T_t\Big\}. \quad (32)$$

The proof is divided into two parts. Case (1) is $\Delta \succeq 0$. Since the Moore–Penrose pseudoinverse of a positive semidefinite matrix is still positive semidefinite [30] and the trace of the product of two positive semidefinite matrices is nonnegative, $T_k = 0$ and an arbitrary $\Sigma_{b,k} \succeq 0$ minimizes the objective function in Problem 2, where $CP^\ell \Delta P^\ell C^\top \succeq 0$ and $T_k^\top[T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}]^\dagger T_k \succeq 0$. Additionally, the encoding policy sequence $\{f_1^S, f_2^S, \ldots, f_k^S\}$ where $T_t = 0$ and $\Sigma_{b,t} \succeq 0$ for all $t \leq k$ also minimizes $\mathrm{Tr}\{S_k\}$ according to (31) and (32), and hence $f_k^{S\star} = b_k$ is also optimal in the sense of minimizing the long-run average cost (23).

For Case (2), we denote $\Psi \triangleq (-CP^\ell \Delta P^\ell C^\top)^{\frac{1}{2}} \succeq 0$ and $\widehat{\Psi} \triangleq \Psi(CP^\ell C^\top + R)^{-\frac{1}{2}}$ for notational brevity. The objective in Problem 2 is transformed into

$$\max_{T_k, \Sigma_{b,k}} \mathrm{Tr}\left\{\Psi T_k^\top \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top + \Sigma_{b,k}\right]^\dagger T_k \Psi\right\}.$$

First, we prove that $\Sigma_{b,k} = 0$ is an optimal solution for all values of T_k under the constraint $\mathrm{r}\{T_k(CP^\ell C^\top + R)T_k^\top\} = \mathrm{r}\{T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}\}$. According to Milliken and Akdeniz [31], we have the inequality $[T_k(CP^\ell C^\top + R)T_k^\top + \Sigma_{b,k}]^\dagger \preceq [T_k(CP^\ell C^\top + R)T_k^\top]^\dagger$ due to the rank constraint and $\Sigma_{b,k} \succeq 0$. The optimality of $\Sigma_{b,k} = 0$ is proved. After letting $\Sigma_{b,k} = 0$, the objective function becomes

$$\max_{T_k} \mathrm{Tr}\left\{\Psi T_k^\top \left[T_k\left(CP^\ell C^\top + R\right)T_k^\top\right]^\dagger T_k \Psi\right\}. \quad (33)$$

Second, we proof that $\Psi T_k^\top[T_k(CP^\ell C^\top + R)T_k^\top]^\dagger T_k \Psi \preceq \Psi(CP^\ell C^\top + R)^{-1}\Psi$ for all T_k. Due to the positive definiteness of R, we denote $X = T_k(CP^\ell C^\top + R)^{\frac{1}{2}}$ and reformulate the optimization problem by change of variable

$$\max_{X \in \mathbb{R}^{m \times m}} \mathrm{Tr}\left\{\widehat{\Psi} X^\top \left(XX^\top\right)^\dagger X \widehat{\Psi}^\top\right\}. \quad (34)$$

According to Ben-Israel and Greville [32], there exists $X^\top(XX^\top)^\dagger = X^\dagger$, and hence $X^\top(XX^\top)^\dagger X = X^\dagger X$. By singular value decomposition of X, one can obtain $0 \preceq X^\dagger X \preceq I_m$. Therefore, $\widehat{\Psi} X^\top (XX^\top)^\dagger X \widehat{\Psi}^\top \preceq \widehat{\Psi}\widehat{\Psi}^\top = \Psi(CP^\ell C^\top + R)^{-1}\Psi$ and the equality is reached when T_k has full rank. Additionally, the encoding policy sequence $\{f_1^S, f_2^S, \ldots, f_k^S\}$ where $\Sigma_{b,t} = 0$ and T_t has full rank for all $t \leq k$ also minimizes $\mathrm{Tr}\{S_k\}$ according to (31) and (32), and hence $f_k^{S\star} = b_k$ is also optimal in the sense of minimizing the long-run average cost.

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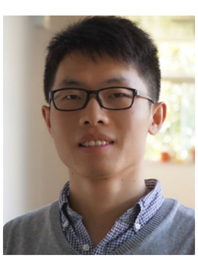

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