# Linear models for short term wave forecasting

Francesco Fusco[1], John V. Ringwood[2]
[1]Polytechnic University of Marche, Ancona, Italy, fuscof85@yahoo.it
[2]Dept. of Electronic Engineering, National University of Ireland Maynooth,Co Kildare, Ireland
Tel: +353 1 708 4766, Fax: +353 1 708 6027, john.ringwood@eeng.nuim.ie

## Abstract

In this paper a preliminary study about the problem of forecasting the wave elevation some seconds ahead, at a particular point on the sea surface, basing on past observations at the same point, is presented. It is discussed how, in the field of wave energy, short term wave forecasting plays a key role for energy extraction optimization. Some simple linear techniques from time series theory are analyzed and then tested both on linear simulated data and on real sea data. Along with these methodologies, a way to preprocess the data from a wave energy perspective, is proposed.

## 1 Introduction

The main application field for short term wave forecasting is in the implementation of optimum control algorithms for wave energy devices. In [1] and [2] full motivation, about the role played by future wave elevation knowledge in wave energy extraction optimization, is provided.

The optimum condition in order to achieve the maximum power absorption of an oscillating body from the waves is the following [2]:

$$u(\omega) = \frac{F_e(\omega)}{2R_i(\omega)} \tag{1}$$

where $u(\omega)$ is the oscillation velocity, $F_e(\omega)$ is the excitation force due to the incoming wave (i.e. the hydrodynamic force acting on the system) and $R_i$ is the intrinsic mechanical resistance of the system. This condition can be split up in a *phase condition* and an *amplitude condition*, respectively expressed as the following:

$$\phi_u = \phi_{F_e} \tag{2}$$

$$|u| = \frac{|F_e|}{2R_i} \tag{3}$$

where $\phi_u$ and $\phi_{F_e}$ are the phases of $u(\omega)$ and $F_e(\omega)$ respectively. An optimum control is achieved by satisfying both conditions (2) and (3), but it presents a number of difficulties, as condition (3) requires some energy to be supplied during part of the wave cycle (it is an *active control*) [3]. A sub-optimal control technique exists, which tries to accomplish the only phase condition (2) (it is a *passive control*), and it is termed, therefore, *phase control* or *latching control* (for details refer to [4]). While implementing either the optimal or the sub-optimal control law, however, two are the main reasons why next wave estimation is needed:

1. In the time domain, the excitation force acting on a body is obtained by the convolution of the incident wave amplitude with a kernel function depending on the geometry of the body itself, the latter function being non-causal, so that future wave elevation is needed for computing the excitation force (and so the optimal body velocity required to maximize the energy extraction), and

2. the system to control has some inertia (time lag) and optimization can only be achieved by knowing the wave excitation force some time in advance (and so the future wave elevation).

The approaches so far presented for short term wave forecasting concern the prediction of the wave elevation at a distant site from the observation point. Among the main solutions provided is a Deterministic Sea Wave Prediction (DSWP) technique (Belmon *et al* in [5]), which performs the forecasts at the prediction site by means of several linear filters (one for each dominant swell detected) estimated through the data collected at the observation sites. The numerical results presented show good accuracy for a few tens of seconds ahead in the case of a single long crested swell. When more than two swells are involved, however, the computational effort becomes really significant and make this technique less attractive. An alternative solution is presented in [6], where the observed wave elevations are fitted to a pure harmonic model (sum of sines and cosines) and then the wave propagation is assumed to be governed by the linear dispersion relation. However it is only preliminary work and the technique is tested in simplified conditions (mono-directionality, wavetrains consisting of a small number of harmonics). Some non-linear methods are presented in [7], where focus is placed on the estimation of the time from the present instant until some events of interest (e.g. the time until the next wave peak), whose knowledge is required to achieve the sub-optimal latching control.

This paper will focus on a slightly different problem: The prediction of wave elevation at a specific point of the sea surface based on past observations made at the same point. In this situation, simple linear models can be applied without any significant simplifying hypothesis.

The problem can be seen as a pure forecasting problem for its own sake, as well as relevant in a wave energy context. Since, to the best of the authors' knowledge, no relevant scientific results have been produced for this particular problem, some simple linear techniques from time series theory are analyzed and tested both on simulated and real sea data.

In the remainder of the paper, section 2 focuses on the simulated and real data on which the models presented have been tested, along with a possible preprocessing performed on them. Then, in section 3, the forecasting models are discussed and in section 4 the achieved results are shown. Finally, conclusions and possible further work are outlined in section 5.

# 2 Data availability

The forecasting models presented in this paper were tested both on linear simulated data and on real data.

The real data was provided by the Irish Marine Institute and comes from a data buoy located in Galway Bay, on the West Coast of Ireland (at approximately 53° 13′ N, 9° 18′ W). The data consists of 20 minute records sets for each hour, collected at a sampling frequency of 2.56 Hz, for the months of January, March and April 2008.

The simulated data were generated from one of the most popular spectral models, the Pierson-Moskovitz spectrum [8], parametrized as follows:

$$S(\omega) = \frac{Ag^2}{\omega^5} e^{-B\left(\frac{(g/H_s)^2}{\omega^4}\right)} \quad (4)$$

where $A$ and $B$ are constant parameters and $H_s$ is the significant wave height (defined as four times the square-root of the area under the spectral density function, or the mean of the one third highest waves).

Then, based on the assumption that irregular sea waves can approximately be considered as a superposition of many different regular (i.e. harmonic) waves [1], the wave elevation $\eta(t)$ may be generated by choosing a finite number of frequency components from the spectrum:

$$\eta(t) = \sum_{i=1}^{m} a(i) sin(\omega(i)t + \phi(i)) \quad (5)$$

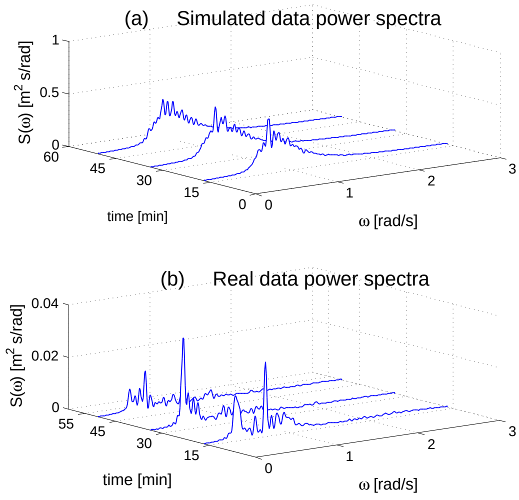


Figure 1: Power spectrum is quite constant for simulated data (a), while it changes over time for real data (b)

where $a(i)$ and $\phi(i)$ are, respectively, the amplitude and the phase of the $i$-th (of the $m$) harmonic component. The amplitudes are proportional to the spectrum envelope, while the phases are chosen randomly (the amplitude spectrum does not give any information about them). The choice of the frequencies, in particular their spacing, is crucial in making the simulated data as realistic as possible. In [9] it has been proposed a set of regularly spaced frequencies ranging from 0 up to $3.5 rad/s$, with maximum interval $d\omega = 0.00625 rad/s$.

Apart from full linearity, the further simplification introduced with these simulated data is stationarity. The amplitudes and phases of each harmonic component are, in fact, constants over all the simulation time. Real data, on the other hand, show a non-stationary behavior due to some changes over time in the physical properties (phases, amplitudes) or to unknown nonlinearities of the real process. One representative way to show this non-stationarity is by computing the power spectrum (distribution of the autocorrelation in the frequency domain) over different time intervals (figure 1). It is seen to be quite constant for simulated data, while it significantly changes over time for real data.

## 2.1 Data preprocessing

If one is interested in short term wave forecasting for specific wave energy optimization issues, it is useful to perform data preprocessing, where the high frequency components are removed. As discussed in [9], in fact, wave records containing high energy, which are the most relevant for wave energy applications, show a single significant peak, most likely a large dominant low frequency swell. On the other hand, most of the wave records containing two (or more) coexisting wave systems, either low and high frequency ones, occur when the total energy is low and are not very relevant for wave energy conversion.

Moreover, high frequency components are highly non-linear (Ochi [10], Falnes [1]) while this is not the case for low frequency swells, so that linear forecasting models are expected to behave better on pre-filtered data.

The most appropriate low pass filter, in the authors' opinion, is the Butterworth filter, as it minimizes the distortion in the band-pass. It requires, however, a high order (between 10 and 20) to perform a sufficiently sharp cut-off, but it is not a big problem, because preprocessing is an off-line procedure, so complexity is not a highly relevant issue. Great care should be taken, at this stage, in the choice of the cut-off frequency. It depends, of course, on which components of the wave we are interested in forecasting, which depends on any specific application needs.

The forecast accuracy can, however, be different depending on whether one is trying to predict all the frequency components or only the low-frequency ones. In section 4, a comparison between forecasting performed on pre-filtered and non pre-filtered data will be presented.

# 3 Forecasting models

As pointed out in section 1, irregular sea waves can approximately be considered as superpositions of many different harmonic components [1]. A general expression for a regular plane wave $\eta_i(x, y, t)$ with propagation direction given by $\beta$ (angle formed with the $x$ axis) can be written as:

$$\begin{aligned} &\eta_i(x, y, t) = \\ &= A_i \cos(\omega_i t - kx\cos\beta - ky\sin\beta + \phi_i) \end{aligned} \quad (6)$$

where $A_i$ and $\phi_i$ are the amplitude and initial phase of the wave, and $\omega_i$ is its angular frequency.

A general sea state (superposition of irregular sea waves propagating in different directions) $\eta(x0, y0, t)$ at some point $(x_0, y_0)$ of the sea surface is then represented as:

$$\begin{aligned} &\eta(x_0, y_0, t) = \eta(t) = \\ &= \int_0^{+\infty} A(\omega)\cos(\omega t + \phi(\omega))d\omega + \zeta(t) \end{aligned} \quad (7)$$

where the multi-directionality disappeared because a specific point of the sea surface is considered (note that $kx_0 \cos\beta$ and $ky_0 \sin\beta$ are constants and have been included in the initial phase). The disturbance term, $\zeta(t)$, has been added to take into account of any kind of error introduced by this approximation. No assumption, however, is made about the nature of the disturbance (it can not be said to be Gaussian, or white noise), as nothing is known about the neglected real system's components.

Based on these considerations, it is straightforward to define a forecasting model for the wave evolution at a point of the sea surface as a discretisation of expression (7), where only a finite number $m$ of harmonic components is chosen:

$$\eta(t) = \sum_{i=0}^{m} A_i \cos(\omega_i t + \phi_i) + \zeta(t) \quad (8)$$

Note that no simplifying assumptions have been made on the real physical process (mono-directionality of the wave is not required, eventual non-linearities can be incorporated in the disturbance term, etc...).

A more convenient expression for the model (8), which is non-linear in the phases, can be obtained by applying a trigonometric transformation:

$$\eta(t) = \sum_{i=0}^{m} a_i \cos(\omega_i t) + b_i \sin(\omega_i t) + \zeta(t) \quad (9)$$

where the parameters $a_i$ and $b_i$ take into account both the amplitude and the phase of the $i$-th component.

Basically, the number of degrees of freedom available in building up such a model is two, before it is fitted to the data: The choice of the frequencies and than the choice of a model for the amplitudes (they can be considered as constants or a dynamic for them can be allowed).

## 3.1 Choice of frequencies

The choice of the frequencies is the most crucial problem in applying a forecasting model of the form (9). It is a trade-off between complexity and modeling capability.

A first methodology could be a set of regularly spaced frequencies. The smaller the $dw$ the more the complexity of the model, but even the greater are the numerical problems that can be encountered in the estimation procedure. A good value for it has been found to be $d\omega = 0.01 rad/s$.

One alternative is to use a non-linear spacing, where more frequencies are concentrated in the central part of the spectrum (where the peak is supposed to occur). This kind of choice gives slightly better performance when tying to forecast real sea data, as it will be shown in section 4.

## 3.2 Models for the amplitudes

The simplest approach is to consider the amplitudes $a_i$ and $b_i$ as constants. Then, regular least squares can simply solve the estimation problem, but obviously the model will not be able to track possible non-stationary behavior of the system (changes in amplitudes and phases over time), which, as shown in section 2, is the case with real sea waves.

One first possibility is to allow a drift in the parameters. In this case, by introducing a state vector $x(k) = [a_1(k)\ b_1(k)\ ...a_m(k)\ b_m(k)]^T$, the system can be represented by the following state space form:

$$x(k+1) = x(k) + w(k) \quad (10)$$

$$\eta(k) = C(k)x(k) + \zeta(k) \quad (11)$$

where $C(k) = [\cos(\omega_1 kdt)\ \sin(\omega_1 kdt)\ ...\ \cos(\omega_m kdt)\ \sin(\omega_m kdt)]$, with $dt$ the sampling time, and $w(k)$ a Gaussian white noise which represents the drift. The state vector can then be recursively estimated by means of recursive least squares with a forgetting factor $\lambda < 1$, which represents a measure of the parameters variability (and the covariance matrix of $w(k)$). The closer $\lambda$ is to 1, the less the allowed variability.

A second, and more complex, possibility analyzed in this paper is the Dynamic Harmonic Regression (DHR) model, proposed by Young *et al* in [11]. Each parameter is modeled as a two-dimensional state vector:

$$x_i(k) = [a_i(k), s_i(k)]^T \quad (12)$$

$$x_{i+m}(k) = [b_i(k), s_{i+m}(k)]^T \quad (13)$$

for $i = 1, ..., m$. Here $a_i$ or $b_i$ represents the level and $s_i$ is the slope of each parameter. The two-dimensional space vectors dynamically evolve as an Integrated Random Walk (IRW):

$$x_i(k+1) = A_i x_i(k) + G_i \eta_i(k) \quad (14)$$

$$A_i = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad G_i = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad (15)$$

where $i = 1, ..., 2m$. The disturbances $\eta_i(k)$ are considered as Gaussian white noise. The overall state space model is then obtained by aggregating the subsystem matrices defined in (15). Once the hyperparameters (i.e. the disturbance variances) have been estimated (an efficient non-linear estimation procedure in the frequency domain is discussed in [11]), the Kalman Filter can recursively be implemented to obtain the estimate $\hat{x}(k+1/k)$ of the state vector at each step. The multi-step ahead forecasting is then obtained as:

$$\hat{\eta}(k+l/k) = C(k+l)A^{l-1}\hat{x}(k+1/k) \quad (16)$$

where $\hat{x}(k+l/k)$ is the estimate of $x(k+l)$, based on the observations $\eta(1)$, $...\eta(k)$, as for $\hat{\eta}(k+l/k)$, and $C(k) = [\cos(\omega_1 kdt)\ 0\ \sin(\omega_1 kdt)\ 0\ ...\ \cos(\omega_m kdt)\ 0\ \sin(\omega_m kdt)\ 0]$.

More complex models could be considered, or a different dynamical matrix for the DHR model could be chosen. This, however, leads to computational problems in the preliminary hyperparameter estimation procedure (it is a non-linear problem), which then prevents the convergence of the Kalman Filter estimates.

# 4 Results

In this section models and methodologies presented throughout the paper, particularly in section 3, are analyzed and tested on simulated data and real data. The N-steps ahead forecasting performance at instant $k$ is quantified through the mean error (ME) and the mean relative error (MRE), expressed by the followings:

$$ME(k) = \frac{1}{N} \sum_{i=k+1}^{k+N} |\eta(i) - \hat{\eta}(i/k)| \quad (17)$$

$$MRE(k) = \frac{1}{N} \sum_{i=k+1}^{k+N} \frac{|\eta(i) - \hat{\eta}(i/k)|}{|max_i\{\eta(i)\}|} \quad (18)$$

## 4.1 Simulated data

Since simulated data are stationary (the harmonic components have constant amplitudes and phases), the amplitudes of the forecasting model (the $a_i$ and $b_i$ coefficients of (9)) can be considered as constants and regular least squares can simply fit the model to the data. The model can then be used to forecast the simulated wave elevation at any future time instant.

The data is simulated as explained in section 2, with $d\omega = 0.005 rad/s$ and a sampling frequency of $2.56 Hz$, based on a Pierson-Moskovitz spectrum with significant wave height $H_s = 3m$.

The only degree of freedom in the model is represented by the modeling frequencies. Figure 2 shows the forecast ME resulting from models characterized by different frequency spacings (in the range $[0.5, 1.5]\, rad/s$). It's interesting to note how the forecasting performance substantially decreases when the forecasting model's $d\omega$ changes from the same value as the simulating model ($0.005 rad/s$) up to two or three times its value (0.01 and 0.015 rad/s). Further increases in $d\omega$ do not affect significantly the ME.

## 4.2 Real data

When dealing with real sea data, the most appropriate forecasting models are the ones, described in section 3, that include some dynamic in the parameters: a simple drift, estimated through recursive least squares (RLS) with forgetting factor and a Dynamic Harmonic Regression (DHR) model.

After training, at each step, the model estimate is updated with the current observation and

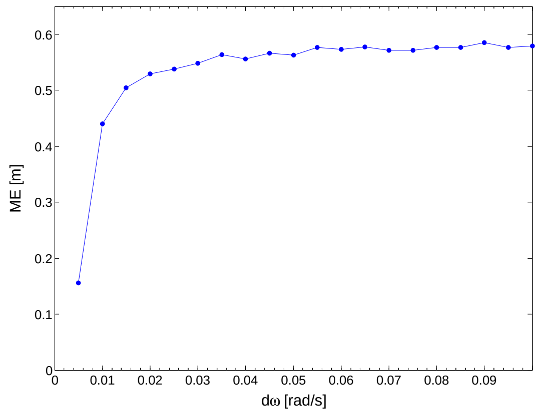


Figure 2: ME resulting from forecasting models built on different frequency spacing $d\omega$

| model | $d\omega$ [rad/s] | ME [m] | MRE [%] | m |
|---|---|---|---|---|
| drift | 0.01 | 0.185 | 33.96 | 91 |
| drift | 0.05 | 0.206 | 38.17 | 19 |
| drift | non-linear | 0.186 | 34.33 | 73 |
| DHR | 0.01 | 0.306 | 57.42 | 91 |
| DHR | 0.05 | 0.475 | 86.84 | 19 |

Table 1: Summary results obtained by setting up the two general models with different frequencies choices. The forgetting factor for the drift model is ($\lambda = 0.998$)

the forecast, with corresponding ME and MRE, is computed for 100 steps ahead (nearly 39 seconds).

Table 1 shows some summary results obtained by means of the two general models set up with different choices of the modeling frequencies. They have been chosen in the interval [0.3, 1.2] $rad/s$ (wider intervals did not improve the results). In each case the mean ME and MRE are showed, along with the number $m$ of frequencies included into the model.

It is obvious how the DHR model behaves quite worse than the simpler drift model. The main reason for this poor performance of the DHR model could lie in its complexity. For each frequency, in fact, it requires 4 parameters (while only 2 are required in the drift model) and, moreover, in the training procedure, a non-linear hyperparameters estimation procedure is required.

Model complexity has been experienced to be critical in this forecasting problem. When, for the drift model, in fact, a $d\omega = 0.005 rad/s$ is chosen (with consequently 181 frequency components), the accuracy decreases (while an increase should be expected) to a $ME = 0.199m$ and a $MRE = 36.85\%$.

The best choices for the frequency spacing, then, turned out to be a constant spacing with $d\omega = 0.01 rad/s$ and a non-linear spacing (where

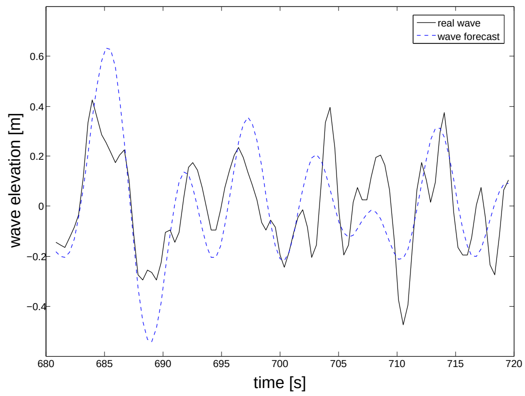


Figure 3: wave forecast obtained with a drift model, $d\omega = 0.01 rad/s$, $\omega \in [0.3, 1.2]$ $rad/s$

.

more frequencies are concentrated in the center of the interval), the latter allowing a decrease in the number of modeling frequencies by lightly affecting the results (as shown in table 1).

Figure 3 shows a forecasting sample obtained with the best model (drift model and $d\omega = 0.01 rad/s$). Note how the forecasts are more reliable in the early future instants, while they become less accurate later in the future.

In choosing the modeling frequencies, the best interval has been found to be $[0.3, 1.2] rad/s$. In such a situation, the higher frequency components do not give any useful information to the estimation algorithm (RLS or Kalman Filter). This consideration, as well as wave energy motivation outlined in section 2.1, make it interesting an analysis of the forecasting models behavior when a prefiltering is applied to the data.

By applying a Butterworth filter with cut-off frequency $\omega_c = 1.2 rad/s$ (mean wave energy neglected nearly 29%), a drift model with $d\omega = 0.01 rad/s$ gives an $ME = 0.144m$ (on the same validation set as results in table 1), which represents a substantial improvement. Fig. 4 shows a sample forecast computed at the same time instant and on the same data set as in fig. 3.

## 5 Conclusions

Short term wave forecasting is a relevant issue in the wave energy filed. This paper focused on forecasting the wave elevation at a point on the sea surface, by means of observations made at the same point. This problem allows the avoidance of simplifying hypothesis (e.g. mono-directionality, linear dispersion relationship, etc.) and permits to work with real data, even when dealing with simple linear models.

This was supposed to be preliminary work and a simple sines/cosines linear model was presented, where the degrees of freedom are two: the fre-

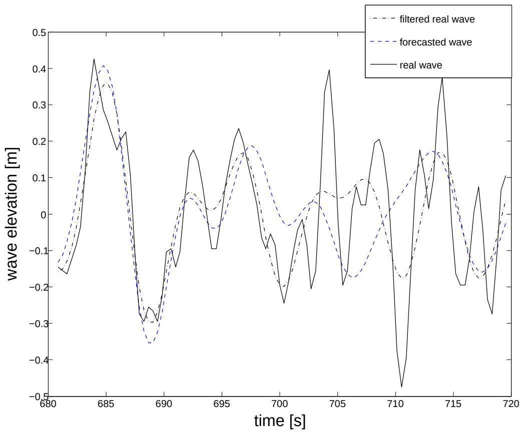


Figure 4: wave forecast obtained with filtered data

.

quencies and the amplitude models (to track non-stationarity of real sea waves).

Simulated data were useful in assessing the importance of choosing the modeling frequencies as thick as possible in a significant interval (i.e. an interval where most of the signal's energy is stored). When working with real data, however, it has been shown how high complexity, both when choosing the frequencies (small $d\omega$) and when building a model for the amplitudes, can seriously affect the performance. So a lower limit for the frequencies spacing has been found ($0.01 rad.s$) and the best choice for the amplitudes model proved to be a simple drift (a more complex DHR model presents too many numerical problems when dealing with nearly a hundred of harmonic components).

An improvement in the forecasting accuracy can also be achieved by pre-filtering the data and removing high frequencies components. This makes sense either in a wave energy context (where high energy low frequency components are much more relevant) and in a pure forecasting issue (including high frequencies in a linear model does not improve the forecasts).

The general results were encouraging, since sometimes the wave dynamics 20-30 seconds ahead is tracked by the forecasting models, and so it is authors' opinion that further work can significantly improve the relevance and the effectiveness of the techniques here presented. The models, however, were tested only on a poor variety of data (few months at a single sea site). That's why future work will involve a much wider validation (based on wave measurements made at different sea locations and in several weather conditions). Then other possible further developments will involve the introduction of some non-linearities in an attempt to model any non-linear component contained in real sea and possibly improve the forecasts.

# 6 Acknowledgments

The authors are grateful to the Irish Marine Institute for providing real sea data. In addition, the funding provided by Enterprise Ireland, under contract EI/CTFD/IT/325, and by European Union, under the Socrates-Erasmus exchange project, are gratefully acknowledged.

# References

[1] J. Falnes, *Ocean waves and oscillating systems.* Cambridge University Press, 2002.

[2] J. Falnes, "A review of wave-energy extraction," *Marine Structures*, vol. 20, no. 4, pp. 185 – 201, 2007.

[3] J. Ringwood, "The dynamics of wave energy," *Proceedings of the Irish Signals and Systems Conference - ISSC*, pp. 12 – ?, 2006.

[4] A. Clement and A. Babarit, "Optimal latching control of a wave energy device in regular and irregular waves," *Applied Ocean Research*, vol. 28, no. 2, pp. 77 – 91, 2006.

[5] M. Belmont, J. Horwood, R. Thurley, and J. Baker, "Filters for linear sea-wave prediction," *Ocean Engineering*, vol. 33, no. 17-18, pp. 2332 – 2351, 2006.

[6] V. Voronovich, B. Holmes, and T. Gareth, "A preliminary numerical and eperimental study of wave prediction," *European Wave and Tidal Energy Conference (EWTEC), Glasgow*, 2005.

[7] A. Price and A. Wallace, "Non-linear methods for next wave estimation," *Proceedings of the 7th European Wave and Tidal Energy Conference (EWTEC), Porto, Portugal*, 2007.

[8] W. Pierson and L. Moskowitz, "A poroposed spectral form for fully developed wind seas based on the similarity theory of s.a. kitaigorodskii," *Journal of Geophysical Research*, vol. 69, no. 24, pp. 5181–5190, 1964.

[9] G. Nolan, J. Ringwood, and B. Holmes, "Short term wave energy variability off the west coast of Ireland," *Proceedings of the 7th European Wave and Tidal Energy Conference, Porto, Portugal (EWTEC)*, 2007.

[10] M. K. Ochi, *Ocean waves. The stochastic approach.* Cambridge University Press, 1998.

[11] P. C. Young, Pedregal, and W. Diego J.and-Tych, "Dynamic harmonic regression," *Journal of Forecasting*, vol. 18, pp. 369–394, 1999.