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Local Forms of Spatial Analysis

Local forms of spatial analysis focus on exceptions to the general trends represented by more traditional global forms of spatial analysis. There is currently a rapid expansion in the development of such techniques but their history almost exactly parallels that of Geographical Analysis, *with the first examples of local analysis appearing in the late 1960s. Indeed,* Geographical Analysis *has published many of the significant contributions in this field. This paper reviews the development of local forms of spatial analysis and assesses the current situation. Following a discussion on the nature and importance of local analysis, examples are given of local forms of point pattern analysis; local graphical approaches; local measures of spatial dependency; the spatial expansion method; adaptive filtering; multilevel modeling; geographically weighted regression; random coefficients models; autoregressive models; and local forms of spatial interaction models.*

Throughout its history, quantitative human geography has been faced with a difficult question and a potential dilemma: are there any "laws" that govern spatial processes, and if there aren't, does the subject matter have any validity? The exuberances associated with a youthful, and to some extent, "revolutionary," subject matter perhaps blinkered some geographers into thinking that, just as in the natural sciences, there *were* laws awaiting discovery that would explain most, if not all, aspects of human spatial behavior. This was always going to be a very difficult position to defend, especially when it became clear that results derived from one system could generally not be replicated in another. While we could claim some success in deriving new types of spatial analytical procedures and new forms of spatial models, the application of these new techniques seemed to pull us back to our descriptive, nonquantitative roots by often yielding quite different results in different systems which demanded unique explanations. Vexingly, certain explanatory variables would be highly relevant in some applications and seemingly irrelevant in others; parameters describing the same relationship would sometimes be significantly negative and sometimes be significantly positive; and the same models would replicate spatial patterns extremely accurately in some systems but not in others. Quite understandably, in the light of such findings, quantitative geographers came under attack from their nonquantitative colleagues for their pursuance of global statements of spatial behavior [see Jones and Hanham (1995), for a fuller discussion of this issue

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and a link between realists and positivists through local analysis]. Partly as a response to such attacks, there has been a recent surge of interest in what are known as "local" forms of spatial analysis and modeling that involve spatial disaggregations of the more traditional "global" approaches (Fotheringham 1992, 1997; Fotheringham and Rogerson 1993; Openshaw 1993). However, the earliest works on local spatial modeling predate this recent interest considerably (Linneman 1966; Monmonier 1969; Casetti 1972; Greenwood and Sweetland 1972; Chisholm and O'Sullivan 1973).

Models of spatial processes and methods of spatial analysis have usually been applied at a global level, meaning that one set of results is generated from the analysis and these results, representing one set of relationships, are assumed to apply equally across the study region. Essentially, what is being undertaken in a global analysis, but is rarely acknowledged, is the generation of an "average" set of results from the data. If the relationships being examined vary across the study region, the global results will have limited application to specific parts of that region and may not, in fact, represent the actual situation in any part of it. Calibrating a global model is therefore akin to being given the information that "*the average precipitation in the United States last year was 32 inches.*" This is a "global" statistic in that it provides information about the study area in general but not necessarily about any specific part of it. Consequently, it is of little use if precipitation does vary locally.

In this paper, we review the powerful movement within spatial analysis, termed local analysis or local modeling, where the focus of attention is on identifying spatial variations in relationships rather than on the establishment of global statements of spatial behavior. The movement encompasses the dissection of global statistics into their local constituents and the concentration on local exceptions rather than the search for global regularities. This is important not only because it brings issues of space to the fore in spatial analysis, but also because it demonstrates that some forms of quantitative geography are not concerned with the search for global generalities and "laws," a point made eloquently by Jones and Hanham (1995).

THE NATURE OF LOCAL VARIATIONS IN RELATIONSHIPS

In spatial data analysis, the data about which relationships are to be examined are drawn from spatial units. In a univariate analysis, these data are used to estimate a single relationship, such as the degree of spatial autocorrelation within the data. In a multivariate analysis a set of relationships is estimated. In either case, the relationships being estimated are typically global and are assumed to be stationary over space. That is, for each relationship a single parameter estimate is implicitly assumed to depict the nature of that relationship for all points within the entire study area. Clearly, any relationship which is not stationary over space, and which is said to exhibit *spatial nonstationarity*, will not be modeled particularly well by a single parameter estimate and indeed this global estimate may be very misleading locally.

It is useful to speculate on why relationships might vary over space. In fact, this issue raises an interesting and as yet unsolved puzzle in spatial analysis. Are the observed spatial variations in relationships simply due to model misspecification or are they due to intrinsically different local spatial behavior? In a nutshell, can all contextual effects be removed by a better specification of individual level models (Hauser 1970)? The idea that human behavior can vary intrinsically over space is consistent with postmodernist beliefs on the importance of place and locality as frames for understanding such behavior. However,

is the role of place simply a surrogate for individual-level effects which we cannot recognize or measure? This view, more in line with the positivist school of thought, assumes a global statement of behavior could ultimately be made but that the structure of our model is not sufficiently well formed to allow us to make it. That is, the model from which the relationships are measured is a gross misspecification of reality and one or more relevant variables are either omitted from it or are represented by an incorrect functional form. When the nature of the misspecification is identified and corrected, the local variations in relationships will disappear.

Local forms of spatial analysis and local spatial models attempt to provide evidence on the nature of possible significant spatial variations in relationships. The spatial distribution of local relationships can inform on potential causes of misspecification bias so that ultimately all misspecification bias could, in theory, be eliminated. We can still only speculate on whether if one were to achieve such a state in human geography, all significant spatial variations in local relationships would be eliminated. The reason for this is that we can never be completely confident that our models are correct specifications of reality because of our lack of theoretical understanding of the processes governing human spatial behavior. In some ways, there is a chicken-and-egg dilemma here. We can never completely test theories of spatial behavior because of model misspecification but model misspecification is the product of inadequate spatial theory.

However, all is not lost: in specific applications of any form of spatial model, we can ask whether the current form of the model we are using produces significant local variations in any of the relationships in which we are interested. If the answer is yes, then an examination of the nature of the spatial variation can suggest to us a more accurate model specification or the nature of some intrinsic variation in spatial behavior. In either case, our knowledge of the system under investigation will be improved, in some cases dramatically.

Given the potential importance of local statistics and local models to the understanding of spatial processes, and given the development of what are known as variable parameter models (VPMs) in aspatial contexts (Maddala 1977; Casetti 1997), it is surprising that local forms of spatial analysis are not more frequently encountered. However, there have been some notable contributions to the literature on spatially varying parameter models which we now describe. These developments can be divided into three categories: those that are focussed on local statistics for univariate spatial data, which include the analysis of point patterns; those that are focussed on more complex multivariate spatial data; and those that are focussed on spatial patterns of movement.

MEASURING LOCAL RELATIONSHIPS IN UNIVARIATE DATA

Local Point Pattern Analysis

The analysis of spatial point patterns has long been an important concern in geographical enquiry (Getis and Boots 1978; Boots and Getis 1988). Until relatively recently, however, most applications of spatial point pattern analysis involved the calculation of some global statistic that described the whole point pattern and from which a conclusion was reached related to the clustered, dispersed, or random nature of the whole pattern. Clearly, such analyses are flawed in that interesting spatial variations in the point pattern are subsumed in the calculation of the average or global statistic. In many instances, particularly in the study of disease, such an approach would appear to be contrary to the purpose of the study, namely to identify any interesting local clusters.

One of the first developments for the local analysis of point patterns was the geographical analysis machine (GAM) developed by Openshaw et al. (1987) and updated by Fotheringham and Zhan (1996). These are the basic components of a GAM:

1. a method for defining subregions of the data;
2. a means of describing the point pattern within each of these subregions;
3. a procedure for assessing the statistical significance of the observed point pattern within each subregion considered independently of the rest of the data;
4. a procedure for displaying the subregions in which there are significant patterns as defined in 3.

The basic idea outlined in Fotheringham and Zhan (1996) is very simple and serves to demonstrate the interest in the local quite well. Within the study region containing a spatial point pattern, randomly select a location and then randomly select a radius of a circle to be centered at that location. Within this random circle count the number of points and compare this observed value with an expected value based on an assumption about the process generating the point pattern (usually that it is random). Ideally, the population-at-risk should be used as a basis for generating the expected value, as shown in Fotheringham and Zhan (1996) who use a Poisson probability model with the observed mean and the population-at-risk within each circle. Once the statistical significance of the observed count within a circle has been examined, the circle is drawn on a map of the region if it contains a statistically significant cluster of points. The process is repeated many times until a map is produced containing a set of circles centered on parts of the region where interesting clusters of points appear to be located.

The use of automated cluster detection techniques as described above assumes that not all parts of the study region have the same pattern of points even when the underlying at-risk population distribution is taken into account. This is quite different from classical approaches such as various neighbor statistics and quadrat analyses that produce global statistics (inter alia Dacey 1960; King 1961; Tinkler 1971; and Boots and Getis 1988). The GAM-style of analysis concentrates on spatial variations and spatial differences in the location of points and hence produces truly local rather than global statistics. It is meant to be an exploratory technique for highlighting interesting parts of the data, rather than as a formal significance testing procedure. As with all local statistics, GAM-generated statistics are enhanced by, and are even dependent on, the ability to map the results so that variations over space can be easily visualized.

Local Graphical Approaches

Since the beginnings of *Geographical Analysis* there have been marked advances in the speed and availability of computers. One aspect of computing that has changed dramatically has been the user interface. In the early years of quantitative spatial analysis, analysts interacted with computers using punched cards and paper tapes, and would see the outcomes of their requests in the form of textual printout. Currently computing is a highly interactive activity with input and output of information carried out through graphical user interfaces. One consequence of this for spatial analysis has been the rise of exploratory graphical techniques which tend to emphasize the local nature of relationships. In particular, using software such as MANET (Unwin et al. 1996), and XLISP-STAT (Tierney 1990; Brunsdon and Charlton 1996), it is possible to link maps of spatial data with other noncartographical representations (such as

scatterplots or dotplots). Selecting an object on one representation highlights the corresponding object on the other [an early example of this is Monmonier (1969)]. For example, if a scatterplot reveals a number of outlying observations, selecting these points will highlight the zones corresponding to these on a map. Similarly, selecting a set of points or zones on a map will highlight the corresponding points on a scatterplot. In this way, the spatial distribution of an attribute for a locally selected region can be compared to the distribution of the same attribute across the study area as a whole. Using techniques of this sort, combined with a degree of numerical preprocessing, it is possible to carry out a wide range of exploratory tasks on geographical data that are essentially local. For example, one can identify multivariate clusters in data and investigate whether these are also associated with geographical clusters. One can also identify spatial outliers—that is, cases that are *locally* unusual even if not atypical for the data set as a whole. More complex graphical techniques for depicting local relationships in univariate data sets include the spatially lagged scatterplot (Cressie 1984), the variogram cloud plot (Haslett et al. 1991), and the Moran scatterplot (Anselin 1996).

Local Measures of Spatial Dependency

Measures of spatial dependency have a relatively long history in quantitative geography (inter alia Cliff and Ord 1972; Haining 1979). These measures, though, have been applied globally so that one statistical measure is generally produced that describes an "average" degree of spatial dependency across the whole study region. However, local versions of these global univariate statistics have recently been developed by Getis and Ord (1992), Ord and Getis (1995) and by Anselin (1995). Getis and Ord (1992), for example, develop a global measure of spatial association inherent within a data set that measures the way in which values of an attribute are clustered in space. A local variation of this global statistic is then formulated to depict trends in the data around each point in space. There are two variants of this localized value depending on whether the point i around which the clustering is measured is included or not in the calculation. The local spatial association statistic allows that different trends in the distribution of one variable might exist over space. In some parts of the study area, for example, high values might be clustered; in other parts there might be a mix of high and low values. Such differences would not be apparent in the calculation of a single global statistic. In their empirical example, Getis and Ord (1992) find several significant local clusters of sudden infant death syndrome in North Carolina although the global statistic fails to identify any significant clustering.

In a similar manner to that of Getis and Ord (1992), Anselin (1995) has recently developed a local variant of the classic measure of spatial autocorrelation, Moran's I. Spatial autocorrelation is traditionally measured globally so that the statistic describes an average trend in the way a variable is distributed over space. Where spatial data are distributed so that high values are generally located near to high values and low values are generally located near to low values, the data are said to exhibit positive spatial autocorrelation. Where the data are distributed such that high and low values are generally located near each other, the data are said to exhibit negative spatial autocorrelation. Clearly these descriptions are global ones and may not adequately describe the relationships in all parts of the study area. Anselin's development of a localized version of spatial autocorrelation allows spatial variations in the arrangement of a variable to be examined. Anselin (1995) presents an application of the localized Moran's I statistic to the spatial distribution of conflict in Africa and Sokal, Oden, and

Thomson (1998) demonstrate its use on a set of simulated data sets. Other studies of local Moran's I include those of Bao and Henry (1996), Tiefelsdorf and Boots (1997), Tiefelsdorf (1998) and Tiefelsdorf, Fotheringham, and Boots (1998).

MEASURING LOCAL RELATIONSHIPS IN MULTIVARIATE DATA

The increasing availability of large and complex spatial data sets has led to a greater awareness that the univariate statistical methods described above have limited application and that there is a need to understand local variations in more complex relationships [see, for example, the attempts of Ver Hoef and Cressie (1993) and Majure and Cressie (1997) to extend the local visual techniques for autocorrelation described above to the multivariate case]. In response to this, several attempts have been made to produce localized versions of traditionally global multivariate techniques. Perhaps the greatest challenge, given its widespread use, has been to produce local versions of regression analysis. We now examine several different responses to this challenge.

The Spatial Expansion Method

The expansion method (Casetti 1972, 1997; Jones and Casetti 1992) attempts to measure parameter "drift." In this framework, parameters of a global model can be made functions of other attributes, including location, so that *trends* in parameter estimates over space can be measured (Brown and Jones 1985; Brown and Kodras 1987; Brown and Goetz 1987; Fotheringham and Pitts 1995; Eldridge and Jones 1991). Initially, a global model is proposed such as

$$y_i = \alpha + \beta x_{i1} + \cdots \tau x_{im} + \varepsilon_i \tag{1}$$

where y represents a dependent variable, the xs are independent variables, $\alpha, \beta, \ldots \tau$ represent parameters to be estimated, ε represents an error term, and i represents a point in space at which observations on the ys and xs are recorded. This global model can be expanded by allowing each of the parameters to be functions of other variables. While most applications of the expansion method (see Jones and Casetti 1992) have undertaken aspatial expansions, Brown and Jones (1985) and Eldridge and Jones (1991) show that it is relatively straightforward to allow the parameters to vary over geographic space so that, for example,

$$\alpha_i = \alpha_0 + \alpha_1 u_i + \alpha_2 v_i \tag{2}$$

$$\beta_i = \beta_0 + \beta_1 u_i + \beta_2 v_i \tag{3}$$

and

$$\tau_i = \tau_0 + \tau_1 u_i + \tau_2 v_i \tag{4}$$

where u_i and v_i represent the spatial coordinates of location i. Equations (2)–(4) represent very simple linear expansions of the global parameters over space but more complex, nonlinear, expansions can easily be accommodated.

Once a suitable form for the expansion has been chosen, the original parameters in the basic model are replaced with their expansions. For instance, if it is assumed that parameter variation over space can be captured by the simple linear expansions in equations (2)–(4), the expanded model would be

$$y_i = \alpha_0 + \alpha_1 u_i + \alpha_2 v_i + \beta_0 x_{i1} + \beta_1 u_i x_{i1} + \beta_2 v_i x_{i1} + \cdots$$
$$\tau_0 x_{im} + \tau_1 u_i x_{im} + \tau_2 v_i x_{im} + \varepsilon_i. \quad (5)$$

Equation (5) can then be calibrated by ordinary least squares regression to produce estimates of the parameters which are then fed back in to equations (2)–(4) to obtain spatially varying parameter estimates. These estimates, being specific to location i, can then be mapped to display spatial variations in the relationships represented by the parameters.

The expansion method has been extremely important in highlighting the concept that relationships might vary over space and that the parameters of regression models applied to spatial data might exhibit spatial nonstationarity. It does, though, have some limitations. One is that the technique is restricted to displaying *trends* in relationships over space with the complexity of the measured trends being dependent upon the complexity of the expansion equations. Clearly the maps of the spatially varying parameter estimates obtained through the expansion method might obscure important local variations to the broad trends represented by the expansion equations. A second is that the form of the expansion equations needs to be assumed a priori although more flexible functional forms than those shown above could be used. A third, and most problematic, is that the expansion equations must be assumed to be deterministic in order to remove problems of estimation in the terminal model.

Adaptive Filtering

Another approach to regression modeling that allows coefficients to vary is that of adaptive filtering (Widrow and Hoff 1960; Trigg and Leach 1968). When applied to multivariate time series data, this method is used to compensate for drift of regression parameters over time. Essentially, this works on a "predictor-corrector" basis. In the time series case we assume a model of the form

$$y_t = \Sigma x_{tj} \beta_{tj} + \varepsilon_t \quad (6)$$

in which t is an index of discrete time points (usually assumed to be regular). When a new multivariate observation occurs at time t, the existing regression coefficients, $\boldsymbol{\beta}_{t-1}$, are used to predict the dependent variable. However, if the prediction does not perform well, the values of the regression coefficient are "adjusted" to improve the estimate. The adjusted coefficients are referred to as $\boldsymbol{\beta}_t$. The degree of adjustment applied has to be "damped" in some way to avoid problems of overcompensation. One could in most cases find a $\boldsymbol{\beta}_t$ that gave a perfect prediction, but that also fluctuate wildly and not give a good indication of the true values of $\boldsymbol{\beta}$ at time t. A typical approach is to use an update rule of the form

$$\beta_{jt} = \beta_{jt-1} + |\beta_{jt-1}| \alpha_j (y_t - y_t^*) / |y_t^*| \quad (7)$$

where β_{jt} is the jth element of $\boldsymbol{\beta}_t$, y_t^* is the predicted value of y_t based on $\boldsymbol{\beta}_{t-1}$, and α_j is a damping factor controlling the extent to which correction is applied for coefficient j.

Gorr and Olligschlaeger (1994) suggest applying adaptive filtering ideas to spatial data—in an attempt to investigate the "drift" of regression parameters. With spatial data the predictor-corrector approach becomes iterative. With time series data one simply updates $\boldsymbol{\beta}_{t-1}$ in terms of its nearest *temporal* neighbor at time t. Here a given case has a unique neighbor, and the flow of updating is one

way. However, when considering a spatial arrangement of data, zones (or points) do not have unique neighbors, and one has to update coefficient estimates several times. In addition to this, the flow of updating is now two way between a pair of neighboring zones which requires the process to iterate between coefficient estimates until some form of convergence occurs. If this does occur, then the result should be a unique estimate of the regression coefficient vector $\boldsymbol{\beta}$ for each case. The fact that the casewise correction procedure is damped, and based on incremental corrections applied between adjacent zones, suggests that some degree of spatial smoothing of the estimates of the individual elements of $\boldsymbol{\beta}$ must take place. Thus, the method tends to produce models in which regression parameters slowly "drift" across geographical space. Local and regional effects may be investigated by mapping the coefficient estimates.

Multilevel Modeling

The typical geographical application of multilevel modeling attempts to separate the effects of personal characteristics and place characteristics (contextual effects) on behavior (Goldstein 1987; Jones 1991a, 1991b). Modeling spatial behavior purely at the individual level is prone to the atomistic fallacy, missing the context in which individual behavior occurs (Alker 1969), whereas modeling behavior at the aggregate level is prone to the ecological fallacy, that the results might not apply to individual behavior (Robinson 1950). Multilevel modeling tries to avoid both these problems by combining an individual-level model representing disaggregate behavior with a macrolevel model representing contextual variations in behavior. The resulting model has the form:

$$y_{ij} = \alpha_j + \beta_j x_{ij} + e_{ij}, \tag{8}$$

in which y_{ij} represents the behavior of individual i living in place j; x_{ij} is the ith observation of attribute x at place j; and α_j and β_j are place-specific parameters where

$$\alpha_j = \alpha + \mu_j^{\alpha} \tag{9}$$

and

$$\beta_j = \beta + \mu_j^{\beta}. \tag{10}$$

Each place-specific parameter is therefore viewed as consisting of an average value plus a random component. Substituting (9) and (10) into (8) yields the multilevel model,

$$y_{ij} = \alpha + \beta x_{ij} + (e_{ij} + \mu_j^{\alpha} + \mu_j^{\beta} x_{ij}), \tag{11}$$

which, because it contains three random components, cannot be calibrated by OLS regression unless μ_j^{α} and μ_j^{β} are zero, and specialized software is needed such as Mln (Rasbash and Woodhouse 1995). Place-specific parameter estimates can be obtained by estimating separate variance effects and substituting into equations (9) and (10).

Several refinements to the basic multilevel model described above have been suggested. These include adding place attributes in the specifications for α_j and β_j; extending the number of levels in the hierarchy beyond two (Jones, Gould, and Watt 1996); and the development of cross-classified multilevel models

where each lower unit can nest into more than one higher order unit (Goldstein 1994). Examples of the application of multilevel modeling to spatial data include those of Jones (1997), Verheij (1997), Duncan, Jones, and Moon (1996), Jones and Bullen (1993), Smit (1997), and Duncan (1997).

A problem with the application of multilevel modeling to spatial processes is that it relies on an a priori definition of a discrete set of spatial units at each level of the hierarchy. While this may not be a problem in many aspatial applications, such as the definition of what constitutes the sets of public and private transportation options, or what constitutes the sets of brands of decaffeinated and regular coffees, it can pose a problem in many spatial contexts. The definition of discrete spatial entities in which spatial behavior is modified by the attributes of those entities obviously depends on such entities being identified. It also implies that the nature of whatever spatial process is being modeled is discontinuous. That is, it is assumed that the process is modified in exactly the same way throughout a particular spatial unit but that the process is modified in a different way as soon as the boundary of that spatial unit is reached. Most spatial processes do not operate in this way because the effects of space are continuous. Hence, imposing a discrete set of boundaries on most spatial processes is unrealistic. There are exceptions to this, however, such as where administrative boundaries enclose regions in which a policy that affects the behavior of individuals is applied evenly throughout the region and where such policies vary from region to region. Examples where this type of behavior modification might occur include education districts and health districts. Still, the application of the multilevel modeling framework to continuous spatial processes awaits a development, perhaps akin to that of the competing destinations model in spatial choice. The latter is a continuous version of the discrete nested logit model of spatial choice [see Fotheringham and O'Kelly (1989) for a further discussion].

Geographically Weighted Regression

Consider the global regression model given by

$$y_i = a_0 + \Sigma_k a_k x_{ik} + \varepsilon_i. \tag{12}$$

In the calibration of this model, one parameter is estimated for the relationship between each independent variable and the dependent variable and this relationship is assumed to be constant across the study region. Geographically weighted regression (GWR) is a relatively simple technique that extends the traditional regression framework of equation (12) by allowing local rather than global parameters to be estimated so that the model is rewritten as

$$y_i = a_{0i} + \Sigma_k a_{ki} x_{ik} + \varepsilon_i \tag{13}$$

where a_{ki} represents the value of a_k at point i (Brunsdon, Fotheringham, and Charlton 1996, 1998; Fotheringham 1996, 1997a, 1997b, 1998). Note that the global model in equation (10) is a special case of the GWR model represented by equation (13) in which the parameters are assumed to be constant over space.

In the calibration of the GWR model it is assumed that observed data near to point i have more of an influence in the estimation of the a_{ki}s than do data located farther from i. In essence, the equation measures the relationships inherent in the model *around each point i*. Hence weighted least squares provides a basis for understanding how GWR operates. In GWR an observation is weighted in accordance with its proximity to point i so that the weighting of an observation is no longer constant in the calibration but varies with i. Data from

observations close to i are weighted more than data from observations farther away. Algebraically, the GWR estimator is

$$\hat{\mathbf{a}}_i = (\mathbf{X}^t\mathbf{W}_i\mathbf{X})^{-1}\mathbf{X}^t\mathbf{W}_i\mathbf{y} \tag{14}$$

where the bold type denotes a matrix, $\hat{\mathbf{a}}_i$ represents an estimate of $\mathbf{a}_i$, the place-specific parameters, and $\mathbf{W}_i$ is an n by n matrix whose off-diagonal elements are zero and whose diagonal elements denote the geographical weighting of observed data *for point i*. That is,

$$\mathbf{W}_i = \begin{matrix} w_{i1} & 0 & 0 & \cdots & 0 \\ 0 & w_{i2} & 0 & \cdots & 0 \\ 0 & 0 & w_{i3} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & w_{in} \end{matrix} \tag{15}$$

where w_{in} denotes the weight of the data at point n on the calibration of the model around point i. Clearly, these weights will vary with i which distinguishes GWR from traditional weighted least squares where the weighting matrix is constant. Typically, the weights are defined as continuous functions of distance so that the closer is a data point to the calibration point, the greater is its weight in the estimation of the parameters for that calibration point. Fotheringham, Brunsdon, and Charlton (1999) define different types of weighting functions for GWR including functions that can adapt themselves to the varying density of data points over space when the data do not have uniform density.

There are parallels between GWR and that of kernel regression (Cleveland 1979; Cleveland and Devlin 1988) and drift analysis of regression parameters (DARP)'(Casetti 1982). In kernel regression and DARP, $\mathbf{y}$ is modeled as a non-linear function of $\mathbf{X}$ by weighting data in attribute space rather than geographic space. That is, data points more similar to x_i are weighted more heavily than data points that are less similar and the output is a set of localized parameter estimates in x space. However, Casetti and Jones (1983) do provide a spatial application of DARP that is very similar to GWR although it lacks a formal calibration mechanism and significance testing framework and is treated by the authors as a rather limited heuristic method.

It should be noted that as well as producing localized parameter estimates, the GWR technique described above will produce localized versions of all standard regression diagnostics including goodness-of-fit measures such as r-squared. The latter can be particularly informative in understanding the application of the model being calibrated and for exploring the possibility of adding additional explanatory variables to the model. It is also useful to note that the point for which the GWR model is calibrated need not be a point at which data are collected: calibration of the GWR model can be undertaken for any location. Hence, in systems with very large numbers of data points, GWR calibration can take place at predefined intervals such as at the intersections of a grid placed over the study region. Not only does this reduce computing time but it can also be beneficial for mapping the results.

As with all methods of generating local parameters, it is useful to assess the question: "Does the set of local parameter estimates exhibit significant spatial variation?" That is, what is the probability that the observed variance could be due to randomness and does not reflect any underlying spatial variation in behavior? Theoretical significance tests have been established for GWR (Brunsdon,

Charlton, and Fotheringham 1999) and an experimental procedure is also available (Fotheringham, Brunsdon, and Charlton 1998).

The technique of GWR is a very generalizable one for spatial models. Any model which can be weighted, can also be spatially weighted. Hence the technique can be applied to many situations and nonlinear versions of GWR are possible, as are geographically weighted versions of principle components analysis, projection pursuit methods, and basically any form of linear or nonlinear relationship can be made local in this way. For instance, Brunsdon et al. (1998) use GWR as an alternative method of producing local spatial autocorrelation measures by calibrating a local version of a spatially lagged linear model. Brunsdon et al. (1999) also provide a series of localized versions of standard parametric and nonparametric descriptive statistics such as the mean, median, variance, and skewness as well local versions of the Pearson and Spearman's rank correlation coefficients. For instance, a spatially varying Pearson's correlation coefficient is

$$r_i = \frac{1/n \sum_j w_{ij}(x_j - x^*)(y_j - y^*)}{\left[1/n \sum_j w_{ij}(x_j - x^*)^2\right]^{1/2}\left[1/n \sum_j w_{ij}(y_j - y^*)^2\right]^{1/2}} \tag{16}$$

where r_i is the local correlation coefficient for point i, x and y are two variables with means x^* and y^*, respectively, and w_{ij} is the weight of the data at point j for the calculation of r at point i.

Random Coefficient Models

In geographically weighted regression and the spatial expansion method local variations in parameters are assumed to be smooth. An alternative approach allows coefficients to vary *randomly* for each case. For example, in a study of several house sales one might specify a regression model where the dependent variable is house price, and the independent variables are characteristics of the houses. The classical linear regression approach would assume that the regression coefficient for a given variable would be the same for all cases—so that, say, the presence of a second bathroom had an identical effect on house price for any house in the study. The form this model would take is the familiar

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{17}$$

where $\mathbf{X}$ is a matrix of predictor variables (in this case house characteristics), $\boldsymbol{\beta}$ is a vector of regression coefficients, $\mathbf{y}$ is a vector of response variables, and $\boldsymbol{\varepsilon}$ is a vector of independent random error terms with distribution $N(0, \sigma^2)$. In terms of individual cases, one might write

$$y_i = \Sigma_j x_{ij}\beta_j + \varepsilon_i. \tag{18}$$

In the random coefficients approach, coefficients are assumed to vary from case to case, and are drawn from some random distribution, typically the normal. In this case, our model becomes

$$y_i = \Sigma_j x_{ij}\beta_{ij} + \varepsilon_i \tag{19}$$

where β_{ij} is now a random variable. For each variable j there are i draws of the random regression coefficient from some distribution. Assuming this distribution

to be normal, we have

$$\beta_{ij} \sim N(\beta_j, \sigma_j^2). \tag{20}$$

Calibrating a random coefficients model is then a task of estimating the parameters of the distributions from which casewise parameters are drawn—in this case $\{\beta_j, \sigma_j^2\}$ for all j and σ^2, the error term variance. Then, using Bayes' theorem it is possible to estimate the value of the regression coefficient actually drawn for each case. A further, nonparametric, extension of the technique is to drop the assumption that the coefficients are drawn from a prespecified distribution and to estimate the distribution itself from the data (Aitkin 1997).

The random coefficient modeling approach is not intrinsically geographical—casewise regression coefficients are assumed to be drawn independently from some univariate distribution and no attention is paid to the *location* of the cases. Cases that are in close proximity to each other can have regression coefficients drawn from very different looking distributions. However, the ability to estimate the casewise regression coefficients does allow the possibility of mapping them —and investigating geographical patterns. Indeed, when the data are spatial, and a zone or point location can be associated with each case, then associating an individual coefficient to each case implies an association between coefficients and locations. In this way, local variability of certain types of models can be considered. Brunsdon, Aitkin, Fotheringham, and Charlton (1999) provide an empirical comparison of the application of GWR and the random coefficients model to a data set in which the spatial distribution of a health variable is related to the spatial variability of a set of socioeconomic indicators.

Autoregressive Models

Recognizing that spatial data are not generally independent, so that statistical inference in ordinary regression models applied to spatial data is suspect, a number of attempts have been made to provide a more geographical approach to regression that takes spatial dependency into account. These approaches may generally be described as *spatial regression models*. They extend the standard regression model by relaxing the assumption that the error terms for each observation are independent. In particular, if each observation is associated with a location in space, it is assumed that the error terms for observations close to each other are correlated. The vector of error terms, $\{e_i\}$, is assumed to have a multivariate Gaussian distribution with a zero mean and a variance-covariance matrix having nonzero terms away from the leading diagonal. This implies that although any given e_i will have a *marginal* distribution centered on zero, its *conditional* distribution will depend on the values of the error terms for surrounding observations. For example, if nearby error terms tend to be positively correlated, then *given* a set of positive error terms one would expect the error term of another observation close to these to be positive also. That is, its *conditional* distribution would be centered on some positive quantity rather than zero. Although the output from such models still consists of global parameter estimates, local relationships are incorporated into the modeling framework through the covariance structure of the error terms. In this sense, these models can be thought of as "semilocal" rather than fully local.

There are a number of approaches that work in this way. Perhaps the oldest such technique is that of *Kriging* (Krige 1966). Here, it is assumed that the geographical data are a set of measurements taken at n points in geographical space. Suppose one of these is a *dependent* variable and the others are *predictor* variables. Then one could proceed to fit an OLS regression model, but this

would ignore the spatial arrangement of the measurement points. An alternative would be to assume that the covariance between any two error terms will be a function of the distance between them. That is, if **C** is the variance-covariance matrix for the n error terms, and **D** is the distance matrix for the sampling points, then

$$C_{ij} = f(D_{ij}) \tag{21}$$

where f is some distance-decay function. There are a number of restrictions on the possible functional form of f, mainly due to the fact that **C** must be positive definite in order for the model to be well defined. Typical functions might be the exponential

$$C_{ij} = \sigma^2 \exp(-D_{ij}/k), \tag{22}$$

or the Gaussian

$$C_{ij} = \sigma^2 \exp(-D_{ij}^2/k^2) \tag{23}$$

where the parameter σ^2 determines the level of variation of the error terms, and k determines the spatial scale over which notable covariance between pairs of measurements occurs. Essentially, k controls the degree of locality in the model—small values of k suggest correlation only occurs between very close point pairs, whereas large k suggests that such effects exist on a larger geographical scale.

Calibrating such a model is typically treated as a two-stage problem. Firstly, one has to estimate σ^2 and k, and once this has been done, the regression model itself is calibrated using the formula

$$\boldsymbol{\beta} = (\mathbf{X}^T\mathbf{C}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{C}\mathbf{y} \tag{24}$$

where $\boldsymbol{\beta}$ is a vector of regression coefficients, **X** is a matrix of independent variables, $\mathbf{X}^T$ is its transpose, and **y** is a vector of dependent variables. **C** is the covariance matrix from the error term estimated using the parameter estimates described above. However, this approach is not without its theoretical shortcomings. In particular, it is assumed here that **C** is known exactly, whereas in reality it is itself estimated from the data. A good discussion of the estimation procedure is given in Bailey and Gattrell (1995) which reveals it to be something of a "black art." Although useful results can be obtained from this approach to modeling, caution should be exercised when drawing formal statistical inferences.

Recently, these objections have been addressed to some extent by Diggle, Tawn, and Moyeed (1998). Here, the analytically awkward form of the likelihood function for $\boldsymbol{\beta}$, σ^2 and k is dealt with in a Bayesian context. In particular, drawings from the posterior probability function for these unknown parameters are simulated using Monte-Carlo Markov chain (MCMC) techniques. This is an approach growing in importance in the field of applied statistics and is likely to have a strong influence on spatial modeling in the future [see, for example, Besag and Green's (1993) discussion on this topic].

The last quarter century has also seen the growth of other kinds of spatial regression models, particularly those applied to zonal data such as states, counties, or electoral wards. As with Kriging, one would expect that ordinary linear regression models applied to data aggregated in this way would fail to encapsulate any spatial interactions taking place because local relationships in the error terms are not represented in the simple nonspatial model. In spatial regression

models, zonal proximities are taken as surrogates for local relationships and are typically measured by a *contiguity matrix*, an n by n matrix whose (i,j)th element is one if zones i and j are contiguous, and zero otherwise. Clearly this matrix is symmetrical and encapsulates the *relative* spatial arrangement of the zones. Note that this approach does not take into account the size or shape or absolute of the zones—the information is solely topological. In most applications the contiguity matrix is standardized so that the rows sum to one and is referred to as $\mathbf{W}$. A number of such spatial regression models exist: for example, the spatial autoregressive model of Ord (1975):

$$\mathbf{y} = \mu\mathbf{1} + \rho\mathbf{W}(\mathbf{y} - \mu\mathbf{1}) + \boldsymbol{\varepsilon} \tag{25}$$

where μ is an overall mean level of the random variate $\mathbf{y}$ multiplied by $\mathbf{1}$, a vector of ones, and $\boldsymbol{\varepsilon}$ is a vector of independent normal error terms, and ρ is a coefficient determining the degree of spatial dependency of the model. The model can be extended so that the error term $\boldsymbol{\varepsilon}$ also exhibits spatial autocorrelation. In this case, the coefficient does not determine the distance decay rate of the spatial autocorrelation, but the degree to which the values at individual locations depend on their neighbors. In this case, neighborhood influence is not calibrated in terms of the data, but prescribed by the specification of $\mathbf{W}$.

One interesting problem with this approach is that the concept of proximity is subject to the *modifiable areal unit problem* (Openshaw 1984). By changing the zonal system, not only will the aggregated data alter, but also the points in the study area that are considered to be in adjacent zones. To overcome this, in some cases $\mathbf{W}$ is not described in terms of zonal *contiguity* but on some other concept of "nearness" such as the inverse square distance between zonal centroids. As with Kriging, recent advances in Bayesian statistical analysis techniques have been used to calibrate models of this kind (LeSage 1997).

As mentioned above, spatial regression models are really mixed models in the sense that although they recognize the impact of local relationships between data, such relationships are usually measured with a global autocorrelation statistic and the output of the model is a set of global parameter estimates. Brunsdon, Fotheringham, and Charlton (1998) provide an interesting example where GWR is applied to a spatially autoregressive model such as that in equation (25) so that the output from the model is a locally varying set of parameter estimates that includes a locally varying autocorrelation coefficient. GWR applied to spatially autoregressive models is therefore an alternative, and perhaps simpler, method of deriving local measures of spatial autocorrelation.

MEASURING LOCAL RELATIONSHIPS IN SPATIAL INTERACTION MODELS

It was recognized relatively early that global calibrations of spatial interaction models hid large amounts of spatial information on interaction behavior and that localized parameters yielded much more useful information (Linneman 1966; Greenwood and Sweetland 1972; Gould 1975). This was patently clear when distance-decay parameters were estimated separately for each origin in a system instead of a single global estimate being provided [see Fotheringham (1981) for a review].

Consider, for example, a typical global spatial interaction model of this form:

$$p_{ik} = S_k^{\alpha} d_{ik}^{\beta} \Big/ \sum_j S_j^{\alpha} d_{ij}^{\beta} \tag{26}$$

where p_{ik} is the probability that a person at i selects spatial alternative k; S_k is a measure of the size of alternative k; d_{ik} is the distance between i and k, and α and β are global parameters to be estimated. This model can be calibrated locally to provide separate estimates of the parameters for each origin or for each destination in the system. Usually, it makes more sense to calibrate the former because the behavioral characteristics depicted by the parameter estimates tend to be a product of the origin rather than the destination. The origin-specific form of (26) is

$$p_{ik} = S_k^{\alpha(i)} d_{ik}^{\beta(i)} \Big/ \sum_j S_j^{\alpha(i)} d_{ij}^{\beta(i)}. \tag{27}$$

Origin-specific parameters can be mapped to provide visual evidence of spatial variations and spatial patterns in their values. It was the consistent, but counterintuitive, spatial patterns of origin-specific distance-decay parameters from models of the form of that in equation (27) that led to the realization that such models were gross misspecifications of reality (Fotheringham 1981, 1984, 1986; Fotheringham and O'Kelly 1989; Meyer and Eagle 1982). This, in turn, led to the development of the competing destinations framework from principles of spatial information processing (Fotheringham 1984, 1991). It is worth stressing that such misspecification only came to light through an investigation of spatial variations in localized parameters that would be completely missed in the calibration of a global model.

SUMMARY

Interest in local forms of spatial analysis and spatial modeling is clearly not new. The recognition that the calibration of global models produces parameter estimates which represent an "average" type of behavior, and are therefore of very limited use when behavior does vary over space, dates back at least to Linneman's calibration of origin-specific models of international trade flows (Linneman 1966). A major advance in the development of local modeling techniques was provided by Casetti (1972) with the development of the expansion method. Further advances in local modeling have come in the form of multilevel modeling, spatial regression modeling, and geographical weighted regression, amongst others.

As Fotheringham (1997) notes, the current high level of interest in the "local" rather than the "global" and the emergence of a battery of techniques for local modeling is interesting for several reasons. Among these are that it refutes the criticism that those adopting a quantitative approach in geography are only concerned with the search for broad generalizations and have little interest in identifying local exceptions, an observation also made by Jones and Hanham (1995). The latter point out that local forms of spatial analysis are not dissimilar in intent to the "intensive" types of analysis in the realist paradigm and can act as a bridge between the two previously disparate fields. Local forms of spatial analysis also provide a linkage between the outputs of spatial techniques and the powerful visual display capabilities of GIS and some statistical graphics packages. Perhaps most importantly though, they provide much more information on spatial relationships as an aid to both model development and the better understanding of spatial processes.

The development of local statistics and local models is also symptomatic of a general maturing of quantitative geography. We are no longer embarrassed by

the fact that our subject matter typically exhibits problems such as spatial nonstationarity. Instead, we see these characteristics as opportunities for exploring the complexities of our data and the richness of our subject matter. Local statistics and local models provide us with the equivalent of a microscope or a telescope; they are tools with which we can see so much more detail.

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