Implementing Spectral Convolution for Complex FM signal generation for arbitrary signal lengths

Joseph Timoney, Antonio Goulart, Victor Lazzarini, Marcelo Queiroz, and Tom Lysaght

Maynooth University, Maynooth

jtimoney@cs.nuim.ie;antonio.goulart@usp.br;
victor.lazzarini@nuim.ie;mqz@ime.usp.br;tlysaght@cs.nuim.ie

Complex Frequency modulation (Complex FM) describes an advanced form of the standard FM equation where the modulation signal is non-sinusoidal ([1], [2]). Many relevant textbooks derive the frequency spectrum of FM for a sinusoidal modulator [3], as it sufficient to highlight the inherent properties of this modulation. Complex FM was introduced as a sound synthesis tool using a modulator composed of a fixed number of sinusoids [4]. The well-known DX7 synthesizer is an excellent example of a complex FM instrument [5].

The usefulness of Complex FM oscillators has been shown in relatively recent work, for the generation of almost bandlimited digital versions of the classical waveforms of analog subtractive synthesis ([6], [7]), for describing the bandwidth of Exponential FM signals [8], and for assessing the output spectrum of audio-rate modulated PLTV allpass filters [9].

In all cases, the theoretical spectrum derived from a Complex FM oscillator description has been required. However, the expression for the expansion of an arbitrary Complex FM equation can require significant computational resources for its implementation, especially as the number of modulators, and their modulation indices, increases. Previous work attempted to deal with this by optimising the implementation [10], and then modifying the approach by implementing the expression for the spectrum of a Complex FM signal as a series of spectral convolutions [11]. This was shown to result in a much faster and more flexible implementation of the algorithm. However, in [11] it was assumed that the spectra of the carrier and modulator were sampled such that they were purely line spectra. This did simplify the procedure. However, it must be acknowledged that it is rarely the case that the signals we have are the perfect length to produce line spectra. Most of the time we have a non-integer number of periods so the appearance of the signal spectrum will change to take account of the rectangular windowing that is taking place. The spectral convolutions will then involve spectra with Sinc functions, and it also likely they have phase shift parameters to account for signal phases. This paper will look at the using the derivation of the phase-shifted Sinc spectrum from [12] in the convolution expressions to demonstrate how the faster method can be applied to arbitrary length carrier and modulator signals for which a Complex FM spectrum is required. This will thus render this approach to be completely general purpose and become another useful program in the toolbox of sound analysis and designers, particularly for aliasing analyses.

[1] B. Schottstaedt, ‘The Simulation of Natural Instrument Tones Using Frequency Modulation with a Complex Modulating Wave’, *Computer Music Journal*, Winter 1977, Vol. 1, no. 4, pp.46-50.

[2] M. LeBrun, ‘A Derivation of the Spectrum of FM with a Complex Modulating Wave’,*Computer Music Journal*, Vol. 1, no. 4, Winter 1977, pp.51-52.

[3] S. Haykin, *San introduction to analog and digital communications*, Wiley, NJ, USA, 2006.

[4] J. Chowning and D. Bristow, *FM theory and applications – By Musicians for musicians*, Yamaha, Tokyo, Japan, 1986.

[5] *600 voices for the DX7*, Amsco Music, USA,1990.

[6] J. Kleimola, V. Lazzarini, J. Timoney and V. Valimaki, ‘Phaseshaping oscillator algorithms for musical sound synthesis,’ *Proc. Sound and Music computing 2010*, Barcelona, Spain, July 2010.

[7] J. Pekonen, V. Lazzarini, J. Timoney, J. Kleimola, and V. Valimaki, ‘Discrete-Time Modelling of the Moog Sawooth Waveform’, *EURASIP Journal on Advances in Signal Processing*, vol. 2011, Article ID 785103, 15 pages, 2011. doi:10.1155/2011/785103

[8] B. Hutchins, ‘The frequency modulation spectrum of an exponential voltage-controlled oscillator,’ *Journal. Of Audio Eng. Soc.*, Vol. 23(3), April 1975, pp. 200-206.

[9] J. Timoney, J. Pekonen, V. Lazzarini, and V. Valimaki, 'Dynamic Signal Phase Distortion Using Coefficient-Modulated Allpass Filters,’ *Journal of Audio Eng. Soc.*, 62(9), 2014, 596 - 610.

[10] J. Timoney, V. Lazzarini, T. Lysaght, J. Kleimola, and L. MacManus, 'Considerations for a Computational Estimation of the Complex FM Spectrum'. *Irish Sound Science and Technology Conference (ISSTC) 2011*, Limerick, Ireland, 2011.

[11] J. Timoney, and V. Lazzarini, 'A Homomorphic Interpretation of the Complex Fm Expansion'. Proc. of the 45th Audio Engineering Society Conference: applications of Time-Frequency Processing in Audio, Espoo, Finland, 2012.

[12] J. C. Burgess, ‘On digital spectrum analysis of periodic signals’, *The Journal of the Acoustical Society of America*, 58 (3), 1975, 556-567.