# Generalised Digital Predistortion of RF Power Amplifiers with Low-Rate Feedback Signal

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***Abstract*—Assuming there is an infinite periodically sinusoidal signal $x$ with known frequency, one can sample $x$ at Nyquist zone or sampling-rate to recover the signal. In terms of digital predistortion (DPD) with modulated signal as input, this paper will show the same concept that theoretically one can sample the feedback signal at arbitrary sampling rate to obtain the same coefficient of DPD or behavioural models as the one calculated from the high sampling rate feedback signal. The performance of a generalised DPD technique using a traditional memory polynomial with low-rate analog-to-digital device(ADC) is experimentally validated. The NMSE and ACPR results indicate the same performance by using an ADC sampling from 100 MHz to 10 MHz.**

***Index Terms*—DPD, ADC, Sampling rate, High power amplifier, Behavioural modeling, Probability**

## I. INTRODUCTION

Wider bandwidth signals will require more efficient means of dealing with bandwidth expansion. Since the conventional DPD system requires multiple times the signal bandwidth it has become a limiting factor for DPD techniques to follow the increase in signal bandwidth. There are two possible answers: direct RF-ADC converters operating at GHz sampling-rate or using low-rate ADC with additional recovering algorithms. Research into extending the DPD bandwidth by means of recovering high-rate feedback signal from an under-sampled signal is very active recently [1], [2], [3]. However, many issues arise by employing these methods. Firstly, the modeling accuracy using these methods degenerates to some degree. Considerably additional complexity is also required. Moreover, the minimal sampling-rate achieved by these methods is theoretically limited with respect to the bandwidth of transmitted signal.

In [4], a simple multi-rate identification method was introduced to calculate the coefficients of models with high-rate input samples and the equivalent of low-rate feedback samples. The procedure successfully demonstrated the ability in simulation to train a behavioural model for a PA using input and output signals where the output signal had effectively a lower sample rate. However, different number of training samples with respect to different sampling rate are used in the simulation which leads to a degeneration of identification accuracy. This paper analyzes this multi-rate identification method from the view of information carried by training sample [5]. It reveals that by utilizing the probability property of modulated signal, a generalized DPD technique with any time-series based models can be directly performed with any sampling-rate of ADC without additional complexity. The principle of a DPD using low-rate feedback samples is presented. And the experimental results of the DPD using a low-rate ADC is also demonstrated.

## II. DIGITAL PREDISTORTION SYSTEM WITH LOW-RATE ADC

A number of common DPD techniques utilize Least Square (LS) based techniques to determine the DPD coefficients from an over-determined equation. Based on a theory of the information carried by training samples used in LS which is introduced in [5], equation (1) indicates the relationship between the determined equation derived from LS and the joint probability information of the training samples,

$$\begin{aligned} R_{\phi\phi} &= E[\phi\bar{\phi}] = \int \phi \cdot \rho(\phi)\ \mathrm{d}\phi \\ R_{\phi y} &= E[\phi(\cdot)\bar{y}_n] = \iint \phi y_n \cdot \rho(\phi, y_n)\ \mathrm{d}\phi \mathrm{d}y \end{aligned} \tag{1}$$

,where $\rho(\cdot)$ indicates the probability distribution function, and $\phi$ indicates the generalised behavioural model which can be any time-series based models. According to this theory, in terms of the extraction of coefficients, the determined equation only depends on the value of feedback $y$ and behavioural model $\phi$, as well as the information of joint probability distribution function(PDF) $\rho(\phi, y)$. For the system with low-rate feedback signal, once the value of $y$ and the information of $\rho(\phi, y)$ is conserved to be the same as those obtained from high-rate system, the coefficient of the behavioural model or DPD from this data will also be the same.

### *A. Conserving the Information from Low-rate Feedback Samples by Cyclo-Stationary Process*

Assuming there is an infinite periodic sinusoidal signal $x$ with known frequency, one can sample $x$ at any Nyquist zone or sampling-rate to capture the sufficient information for the recover of signal, because of the repetition and no aliasing. This section will show a similar concept on the DPD with a modulated signal as input that one can sample the feedback signal at arbitrary sampling-rate to conserve the correct information $\rho(\phi, y)$ for the estimation of DPD coefficients.

Suppose input samples and feedback samples are $x_{nr}$ and $y_{nr}$ respectively, where $r$ indicates the under-sampling rate. The procedure of multi-rate identification method is to build a $N \times M$ behavioural model with high-rate input $x_n$ first, where $N$ is the length of high-rate input and $M$ is the number of coefficients. Then by only selecting the row of the model matrix which is corresponding with the low-rate feedback sample, a multi-rate $\hat{N} \times M$ model is achieved, where $\hat{N}$ is the length of low-rate feedback samples $y_{nr}$. Since there is no additional operation for employing this multi-rate modeling method compared to the conventional DPD procedure, it can be treated as a generalised DPD technique.

The simplified expansion of the multi-rate joint PDF $\rho(\phi, y_{nr})$ which includes the high-rate input and low-rate feedback in (2) can be rewritten as below,

$$\begin{aligned}
&\rho(\phi, y_{nr}) \\
&= \rho(x_{nr}) && \text{(2a)} \\
&\cdot \rho(x_{nr-1}|x_{nr}) \cdot \rho(x_{nr-2}|x_{nr}, x_{nr-1}) \\
&\qquad ...\rho(x_{nr-m}|x_{nr}, x_{nr-1}, ..., x_{nr-m+1}) && \text{(2b)} \\
&\cdot \rho(y_{nr}|x_{nr}, x_{nr-1}, x_{nr-2}, ..., x_{nr-m}) && \text{(2c)}
\end{aligned}$$

, where (2a) indicates the low-rate nonlinearity term, (2b) indicates high-rate bandwidth terms and (2c) indicates memory-length term. It should be noticed that (2a) is a low-rate term with respect to under-sampling rate $r$, and the terms in (2b) are high-rate. So the joint probability $\rho(\phi, y_{nr})$ in (2) represents the information carried by training samples pair, high-rate input samples and low-rate feedback samples, by using the multi-rate modeling method.

Now, the joint probability $\rho(\phi, y_{nr})$ can be conserved by the property of cyclostationarity of modulated signal. Modern communications signals, such as GSM, WCDMA, LTE-advance and beyond, exhibit high-order cyclostationarity due to underlying periodicities introduced through coupling stationary message signals, pilot sequences, spreading codes and repeating preambles [6]. One of the properties of high-order cyclostationary signal is the repetition of probability. Even though the magnitude of a LTE signal is random, the PDF is periodic. As shown in Fig. 1, the statistics distribution function of a one-frame, 245.76 MHz data-rate, 20 MHz bandwidth LTE signal is approximately constant over each $P = 8192$ samples.

Another property for cyclostationarity is the stationarity in each period P. Thus the probability information of a stationary signals does not vary with respect to time in terms of high-order stationary process. That means the probability distribution of a continuous-time modulated signal satisfying the high-order stationarity, is not a function of time, but actually a function of length of time occupied by the signal. So the equivalent PDF of low-rate consecutive samples will not lost information after reordering caused by undersampling. Now, knowing that the accuracy of PDF of stationary continuous signal is related to the length of time occupied by the signal, while for discrete-time signals, the accuracy of PDF is determined by the number of uniform-distributed samples. Fig. 2 shows the probability distribution of a discrete-time 16-bit 4-carriers LTE signal in different sampling rate and different number of samples. The

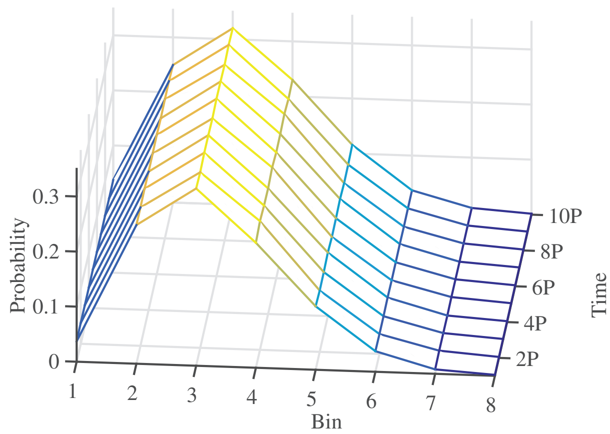


Fig. 1. The Probability repetition of one-frame 20 MHz LTE signal. P = 8192

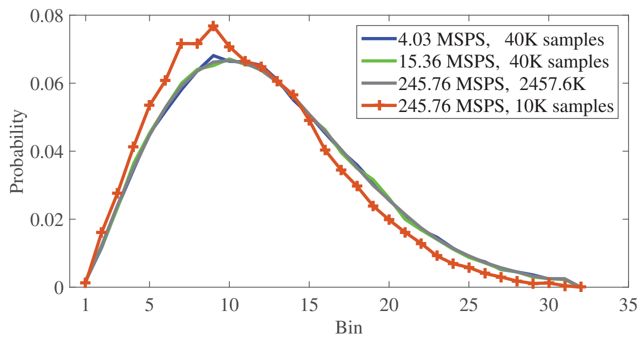


Fig. 2. Probability distribution of 4-carriers 80 MHz LTE signal $\rho(x_{nr})$ with different sampling-rate and different number of samples

probability distributions of LTE samples in different sampling-rates are almost identical given that the number of samples is the same. Furthermore, Fig. 2 shows that the accuracy of the probability distribution decreases by using less samples even though the sampling-rate stay high. In brief, as for a modulated signal satisfying the cyclostationarity, the nonlinearity term (2a) is only determined by the number of uniform-distributed samples, not the sampling rate.

Similarly, the conditional probability of previous samples and current samples in (2b) also satisfies the same property of cyclostationarity. Fig. 3 shows the conditional probability $\rho(x_{nr-1}|x_{nr})$ given $3/8$ of maximum power of the input signal. As a result, the accuracy of bandwidth terms (2b) is also determined by the number of uniform-distributed samples, not the sampling rate.

Moving to the last terms (2c) which is a model-based term related to the length of memory taps in behavioural models. If the behavioural model of DPD is sufficiently accurate which means the memory taps in the model is long and accurate enough to represent the memory-effects, the memory-length term (2c) can be approximated to 1.

To conclude, the information carried by multi-rate training samples, high-rate input and low-rate feedback signal, can be conserved to be the same as the the information carried by

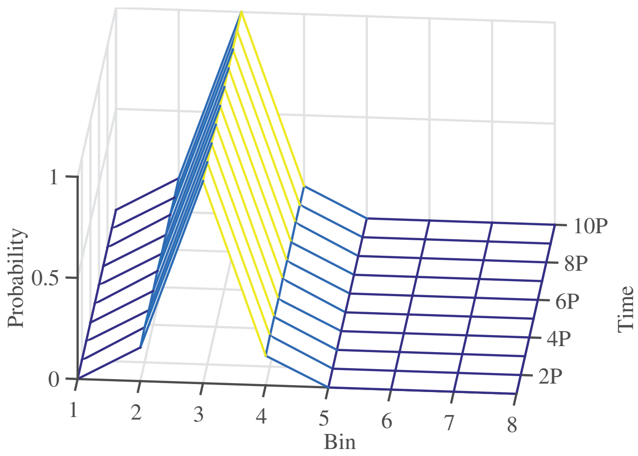

Fig. 3. The conditional probability $\rho(x_{nr-1}|x_{nr})$ given 3/8 of maximum power of $x_{nr}$

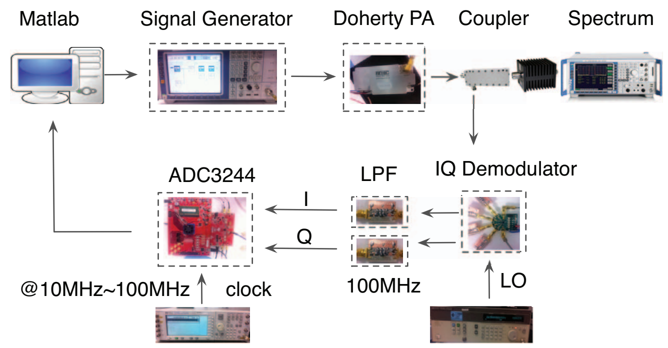

Fig. 4. Experimental setup used to evaluate the behavioural model and DPD

high-rate training samples by means of the multi-rate modeling method.

### B. Anti-aliasing Filter and the Accuracy of Feedback Samples

As previously mentioned in the beginning of section II, besides the information carried by training samples, the accuracy of feedback samples in time domain also have to be guaranteed. Since the accuracy of analog feedback signal is determined by the bandwidth of anti-aliasing filter preceding the low-rate feedback ADC, the bandwidth of anti-aliasing filter, as well as the ADC analog input bandwidth, have to be the same as the one preceding the high-rate ADC which is normally 5 times the input bandwidth

## III. Experimental Results

To verify the DPD linearization performance with low-rate ADC, an experimental test bench is assembled as shown in Fig. 4. It includes a Rohde & Schwarz signal generator, a 15-Watts GaN Doherty power amplifier(RTP26010-N1), an IQ demodulator LTC5585, 100MHz low-pass filters and a dual-input ADC3244 EVM board. Experiments are performed on a one-carrier 20 MHz LTE signal. The predistorted signal centred at 2.63GHz was sent from the signal generator running at 100 MHz bandwidth to the Doherty power amplifier operating at

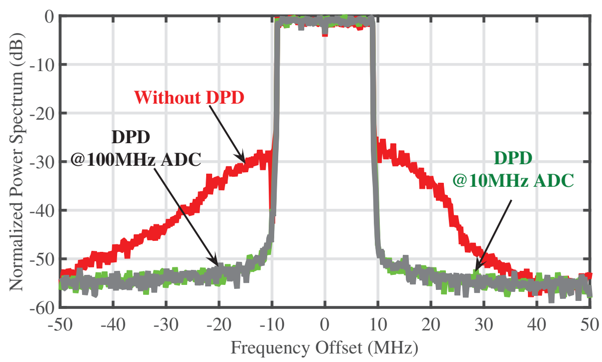

Fig. 5. Measured spectra of PA output before and after DPD for 1-carriers 20-MHz LTE signal.

TABLE I
NMSE With Different Sampling-Rate

| | ADC Sampling Rate (MHz) | NMSE (dB) | ACPR (dB) |
|---|---|---|---|
| Without DPD | — | –26.7 | –38.1 |
| Full-rate | 100 | –42.28 | –49.8 |
| Low-rate | 50 | –42.29 | –49.7 |
| Low-rate | 20 | –42.21 | –49.4 |
| Low-rate | 10 | –42.26 | –49.9 |

45-dBm peak power. The output signal of the power amplifier is attenuated and down converted to zero-IF, then 16K I/Q samples at zero-IF are recorded by ADC3244 to extract a GMP model [7] with 40 coefficients(10 order and 4 memory taps). The minimal sampling rate of ADC3244 is 10 MHz, so the feedback samples are captured at the sampling-rate from 100 MHz to 10 MHz. According to [4], the GMP model is constructed by full-rate input signal first. And then only select the row of the model matrix which is corresponding with the low-rate feedback samples. At last, obtain the coefficients from the multi-rate GMP model using LS algorithm. It is also worth noting that IQ imbalance correction and equalization method is employed in terms of zeero-IF structure.

The NMSE and ACPR are calculated to assess the DPD accuracy shown in Table I. Fig. 5 shows the spectra of PA output using different ADC sampling-rate. According to test results, the accuracy of DPD stay the same with respect to different sampling-rate.

## IV. Conclusion

This paper reveals that a generalised DPD comprised of a time-series based model and LS based algorithm can be directly performed by using arbitrary sampling-rate feedback signal. The limit factor of feedback bandwidth in DPD technique turns to the bandwidth of anti-aliasing filter and ADC input bandwidth. It relieves the requirement of the sampling-rate of ADC in DPD system. Experimental results using a 1-carrier 20 MHz LTE signal demonstrated the ability of a generalised DPD in achieving the same linearity performance by using different sampling-rates.

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