# On a Filter Bank Correction Scheme for Mitigating Mismatch Distortions in Time-Interleaved Converter Systems

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*Abstract*— **This paper presents a multirate filterbank architecture that mitigates the distortion caused by non-ideal samplers in a time-interleaved system. Closed form fractional delay filters with finite impulse response (FIR) and infinite impulse response (IIR) type are employed to model the behaviour of the non-ideal converters. Based on a polyphase description of the system, the reconstruction filters are derived for the IIR case, which can be regarded as a generalization of the FIR design scheme. Furthermore, the achieved performance of filter banks for various fractional delay filters is compared. To investigate the numerical robustness, the impact of limited coefficient lengths on different figures-of-merit was explored. Finally, the reconstruction of a non-uniformly sequence was used as an example to verify the reconstruction scheme.**

## I. Introduction

Analog-to-digital converters are a crucial part in modern communication systems allowing subsequent stages to process the sampled signals. Technological progress, and the tendency to replace analog circuitry with digital signal processing, demands ever increasing performance in terms of conversion rate and accuracy. One example of this trend is the growing interest in software-defined radio [1] where converter performance is a primary determinent in overall performance. One solution of enhancing the performance is to use time-interleaved Nyquist rate converters. For these devices, the conversion accuracy is limited by the utilized sampler [2], but the conversion speed can be increased by using multiple converter in parallel [3]. Another motivation for the interest in time-interleaved systems is the prospect of exchanging expensive high performance converters by systems of low cost, low power converters. While the usage of multiple ADCs enables the designer to increase the overall sampling rate of the system quite easily, this technique entails a degradation of the spectral purity caused by the dissimilar characteristics of the individual A/D converters [4]. In particular, gain, offset and timing mismatch between the individual A/D converters cause spurious images of the input signal in the output spectrum of the converter system, thereby degrading its spurious-free-dynamic-range (SFDR) and signal-to-noise-ratio (SNR). Timing mismatch errors and techniques for their correction have attracted a lot of attention in recent years since its compensation has proven to be challenging for wide-band input signals [5]. In this paper, a multirate filterbank approach will be presented for mismatch correction that models the mismatch using IIR filters and the subsequent generation of stable FIR filter structures that offer superior performance.

## II. Time-Interleaved A/D converter

For A/D converters that operate in a time-interleaved manner, the overall sampling rate $f_s$ becomes a multiple of the sampling rate of a single A/D converter $f_{AD}$, depending on the number of parallel A/D converters $M$. The sampling instants of the individual converters, also called channel ADCs, are controlled by their dedicated clock signals, where $m = [0, 1, \ .., L]$ indicates the channel index. The converter outputs are zero-padded and consecutively filtered (see Fig. 1). If no post-correction is implemented these filters only comprise a delay of $z^{L-m}$ to serialize the converter outputs in combination with the upsampling stages.

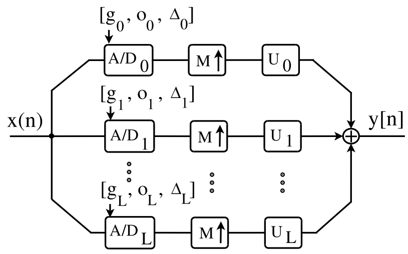


Fig. 1. Time-interleaved A/D converter system comprising $M$ mismatch affected channels with reconstruction of sub-sampled channel outputs.

The non-idealities of individual A/D converters, such as the mismatch of gain $g_m$, offset $o_m$ and timing $\Delta_m$ are static, or slowly changing due to component aging or temperature influence. Static errors such as gain and offset error induce a deviation from the ideal converter transfer characteristic. Timing mismatch arises from a combination of clock skew and individual circuit response times. It represents a constant deviation from the ideal sampling instant and results in significant signal distortion.

## III. Mismatch Correction Scheme

In this section, a methodology for the design of robust FIR multirate correction filters will be presented. To achieve this, the channel ADCs will be represented by an IIR analysis filter which more accurately models ADC timing mismatch than the more commonly used FIR filters. Directly utilising IIR filters to design the filterbank tends to leads to unstable designs. In our proposed methodology, the recursive elements of the IIR analysis filters will be extracted to a common preprocessing filter and the corresponding inverse postprocessing filter. The remaining elements of the channel ADC filter have a finite impulse response and a matching channel reconstruction filter can then be designed using the FIR perfect reconstruction technique [6]. The resulting reconstruction filters are of IIR type and typically unstable, however, their frequency characteristics can be approximated using least-mean-square design to produce stable FIR equivalents. The following sections will describe the individual steps in more detail.

### A. System Model

This paper proposes that the ADC characterization can be performed by Thiran allpass IIR filters whose general definition is given in (1) [7]. The coefficients for a Thiran fractional filter can be determined using (2), where $\Delta_m$ refers to the desired time delay of the $m^{th}$ channel, and $k = [0, 1, ...K]$ where $K$ is the filter order. In the

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original Thiran filter design, the parameter $N$ equals the order of the filter ($K$) however a truncated Thiran interpolator can be obtained by setting $N$ to be greater than the filter order [8].

$$T_m(z) = (1+g_m)\ z^{-K-m}\ \frac{D_m(z^{-1})}{D_m(z)} = \frac{B_m(z)}{A_m(z)} \tag{1}$$

$$d_{mk}(z) = (-1)^k \binom{N}{k} \prod_{n=0}^{N} \frac{n-\Delta_m}{k+n-\Delta_m} \tag{2}$$

Each channel ADC is modeled using an IIR Thiran filter. To construct the filterbank, the common demoninator of the Thiran filters is calculated and used to generate a preprocessing filter (as shown in Fig. 2). The individual ADC channel analysis filters are now multipled by this common demoninator (3) to construct FIR structures described by (4). This will allow the FIR perfect reconstruction technique of Vaidyanathan to be used to generate the matching channel reconstruction (synthesis) filters. To maintain equivalence, the output of the filterbank must then be filtered by the inverse of pre-processing filter. For a more efficient implementation it is possible to integrate the post-processing filter and the channel reconstruction filter into a single design.

$$A(z) = \prod_{m=0}^{L} A_m(z) \tag{3}$$

$$H_m(z) = B_m(z)\ D_m(z) = B_m(z) \prod_{n=0, n\neq m}^{L} A_n(z) \tag{4}$$

The polyphase representations of the analysis and synthesis filters that are employed in the polyphase filter bank formulation are shown in (5) and (6), respectively [6]. By using equations (5) and (6), it is possible to describe all sets of analysis and synthesis filters in the polyphase matrices $E$ and $R$, respectively. The required polyphase decomposition of $H$ and $F$ into corresponding subfilters is shown in Fig. 4.

$$H_m = \sum_{k=0}^{L} z^{-k}\ E_{mk}(z^M) \tag{5}$$

$$F_m = \sum_{k=0}^{L} z^{-(L-k)}\ R_{km}(z^M) \tag{6}$$

It is important to note that only the set of synthesis filter need to be implemented in an ADC sampling correction scheme as the derivation of the analysis filter is used solely for the modeling of the non-ideal A/D converters.

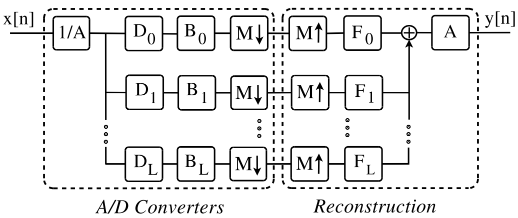


Fig. 2. Maximally decimated $M$ channel filter bank with analysis filter of FIR type. The filter bank output is post-processed by a FIR filter and the input pre-processed by its inverse filter.

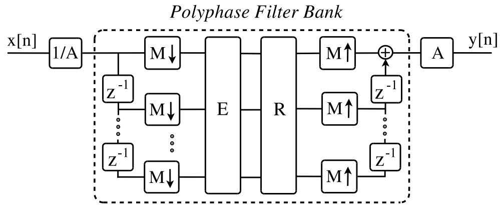


Fig. 3. Polyphase representation of a maximally decimated $M$ channel filter bank employing IIR analysis filters as sampler models.

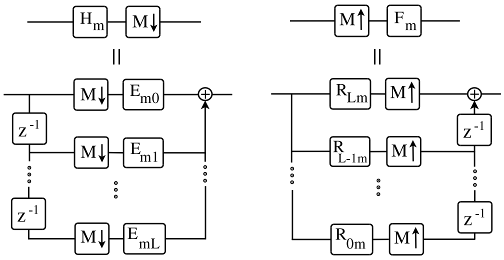


Fig. 4. Polyphase decomposition of analysis (left) and synthesis filter (right).

## B. Synthesis Filter Design

One possibility to design the synthesis filter would be to apply the perfect reconstruction criteria [6] to the polyphase filter bank as shown in Fig. 3 and process the filter bank output with the filter $A(z)$. In the following section, a design technique is described that results in synthesis filter that perform the necessary post-filtering of the filter bank. These filters reduce aliasing and distortion errors and achieve polyphase filtering at the same time. The synergy of these two functionalities results in a single system, whose implementation and computational complexity is more favourable. The perfect reconstruction criteria for system post-processed by the FIR filter $A(z)$ is given in (7), where $c$ is a scalar value, and $G$ is the polyphase formulation of $A(z)$ as described in (5).

$$R\ E = c\ z^{-d} G \tag{7}$$

By treating (7) as a least-squares problem, the polyphase synthesis filter matrix $R$ is obtained by (8).

$$R\ = c\ z^{-d} G \left( (EE^T)^{-1} E \right)^T \tag{8}$$

An equivalent formulation of (8) is shown in (9), where the synthesis filter matrix $R$ is given as the product of the polyphase matrix $P$ whose elements are of finite impulse response type and the recursive filter $q$ which is constituted by the determinant of the pseudo-inverse of $E$.

$$R\ = c\ \frac{z^{-d}}{q(z)}\ P \tag{9}$$

Using the coefficient vectors $b$ and $a$ to describe the forward and recursive part of the IIR filter, (10) is obtained, where $v = [1\ \ z^{-1}\ z^{-2}\ ...\ z^{-Q}]^T$ and $Q$ is the order of $q(z)$.

$$R\ = \frac{b^T v}{a^T v}\ P \tag{10}$$

Typically, the filters as described in (8) and (10) are unstable since the roots of the recursive part lie outside the unit circle in the $z$-plane. To ensure the stability of the polyphase filters, it is necessary to approximate the subfilter characteristics of $R$ using finite impulse response filters (FIR). The desired frequency response of this filter is given by equations (11) and (12). It is important to note that (11) itself is only an approximation of the ideal impulse response, since the recursive part of the filter lets the impulse response trail off to infinity.

$$H_d(e^{j\Omega_k}) = \frac{b^T\Phi}{a^T\Phi} \tag{11}$$

$$\Phi = [1\ e^{-j\Omega_k 1}\ e^{-j\Omega_k 2}\ \dots\ e^{-j\Omega_k (Q-1)}]^T \tag{12}$$

A column vector $\vec{H}_d$ representing the desired spectrum containing $W$ bins, can be calculated using $\Omega_k = [0, \frac{2\pi 1}{W}, \frac{2\pi 2}{W}, \ \dots\ ,2\pi]$. Furthermore, the squared error of the FIR filter can expressed using a phasor matrix of size $WxN$, shown in (14). By selecting $W$ greater than $Q$, the desired frequency spectrum of the IIR filter, that is to be approximated, can be modeled more accurately.

$$e(h) = [\Phi_M^T h - \vec{H}_d]^T [\Phi_M^T h - \vec{H}_d] \tag{13}$$

$$\Phi_M = \begin{bmatrix} 1 & 1 & 1 & . & . & 1 \\ 1 & e^{-j\frac{2\pi}{W}} & e^{-j\frac{2\pi 2}{W}} & . & . & e^{-j\frac{2\pi(Q-1)}{W}} \\ . & . & . & . & & . \\ . & . & . & & . & . \\ 1 & e^{-j\frac{2\pi(W-1)}{W}} & e^{-j\frac{2\pi(W-1)2}{W}} & . & . & e^{-j\frac{2\pi(W-1)(Q-1)}{W}} \end{bmatrix} \tag{14}$$

This error can be expressed in a quadratic equation and a global minimisation of (15) equals the least squares solution of $h$ with size $Q$ (16)

$$e(h) = h^T(\Phi_M\ \Phi_M^T)h - 2h^T\Phi_M\vec{H}_d + \vec{H}_d^{\ T}\vec{H}_d \tag{15}$$

$$\hat{h} = (\Phi_M\ \Phi_M^T)^{-1}\Phi_M\vec{H}_d \tag{16}$$

## IV. Simulation Results

To demonstrate the effectiveness of this methodology, the outcome of using IIR analysis filters have been compared to previously published results where an FIR analysis filter has been used to model the ADC nonidealities [10]. All filter coefficients were assumed to be 20 bit long unless stated otherwise. The FIR analysis filters that were selected to model the converter time delay were windowed fractional filters. For the respective cases, the Hann and Kaiser methods were used as window functions. In the IIR case, truncated Thiran filters were choosen to model the ADC behavior with a parameter $N$ (see (2)) of 92 and a filter order of 40. The augmented value of $N$ increased the normalized bandwidth of interest to 0.4 compared with a value of 0.34 for the ordinary Thiran interpolator. The design parameters were chosen in such a way that the distortion in the band of interest was less than -95 dB, a challenging requirement for near-Nyquist frequency signal reconstruction. The timing mismatch magnitude of the first channel ($\Delta_0$) was set to a constant value and the timing mismatch of the other channels were selected using a Gaussian distribution with a standard deviation of 5%. The presented figures of merit are the average of 500 outcomes to provide statistical significance.

Fig. 5 depicts the distortion error of a FIR based filterbank design using Hann and Kaiser windows respectively. Both architectures achieve good performance for small mismatch magnitudes but in both cases the error severely increases for larger delay values. The Kaiser window especially shows poor accuracy for large fractional timing mismatch magnitudes.

Fig. 6 shows the distortion error after the reconstruction by the Thiran interpolator based filter bank. The distortion error is significantly lower compared with the performance of Hann and Kaiser window as shown in Fig. 5.

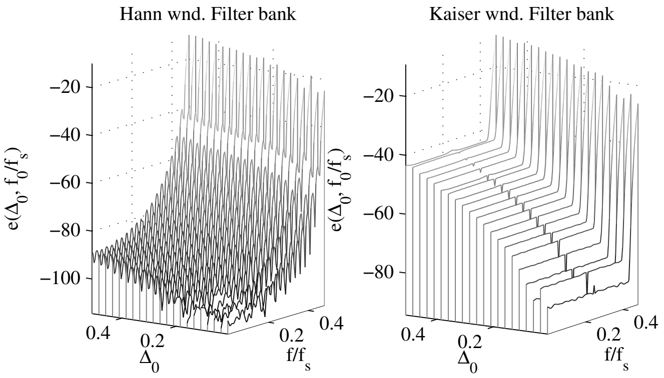


Fig. 5. Filter bank reconstruction error in dB employing Hann (left) and Kaiser (right) windowed FIR fractional delay filter.

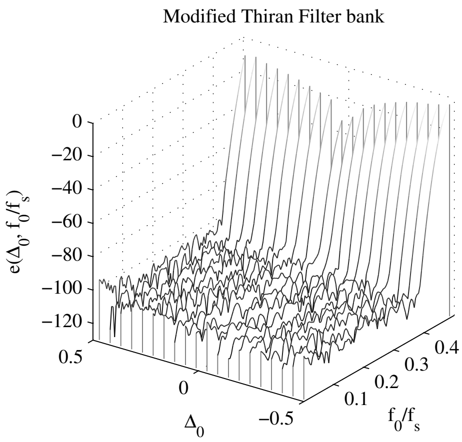


Fig. 6. Error in dB against the timing mismatch of the first channel and the normalized frequency obtained after reconstruction by the Thiran based filter bank.

Table I depicts the number of filter coefficients required for the reconstruction outcome as shown in Fig. 5 and Fig. 6. Coefficients smaller than $1.9\ 10^{-6}$ which is equivalent to the least significant bit for a 20 bit representation did not contribute to the filtering and were removed from the filters.

The impact that the finite coefficient length of the reconstruction filter has on the figures of merit is shown in Fig. 7. The SNR and SFDR performance over the coefficient length is shown for Thiran based reconstruction filters. The depicted SNR performance refers in this context to the average of the measured SNR values for $\Delta_0 = [-0.5, -0.475,\ \dots\ ,0.5]$ and the normalized bandwidth $f_0/f_s = [0.01,\ 0.02,\ \dots\ ,0.4]$.

To verify the function of the proposed reconstruction method, a non-uniformly sampled two tone signal was applied to a four channel filter bank with the channel delays $\Delta = [0.45,\ -0.35,\ 0.15,\ 0.05]$ and individual gain errors $g = [0.15,\ -0.25,\ 0.015,\ 0.05]$. The

TABLE I

NUMBER OF SYNTHESIS FILTER COEFFICIENTS FOR DIFFERENT TIMING MISMATCH VALUES $\Delta t$

| $\Delta t$ | -0.5 | -0.4 | -0.3 | -0.2 | -0.1 | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|---|---|---|---|---|
| *Hann* | 82 | 79 | 78 | 63 | 56 | 61 | 75 | 67 | 81 |
| *Kaiser* | 78 | 70 | 70 | 63 | 64 | 61 | 68 | 64 | 76 |
| *Thiran* | 62 | 53 | 60 | 54 | 48 | 55 | 64 | 70 | 82 |

frequencies of the sinusoidal components were chosen to be high fractional frequencies which is typically challenging for reconstruction filters ($f_0 = \frac{24320}{65536}\ f_s$ and $f_1 = \frac{25728}{65536}\ f_s$). Fig. 8(left) depicts the non-uniformly sampled spectrum where the tones marked with circles constitute the signal and the spurious tones are indicated by stars. Fig. 8 (right) illustrates the attenuation of the spurious tones by the Thiran based reconstruction filterbank, resulting in a SFDR improvement of around 93 dB.

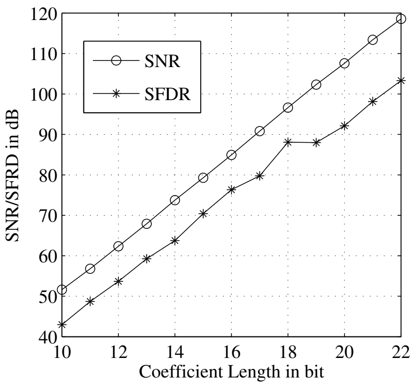


Fig. 7. Signal-to-noise ratio and spurious-free-dynamic-range against the coefficient length of the reconstruction filters in bit for the Thiran based filter bank scheme.

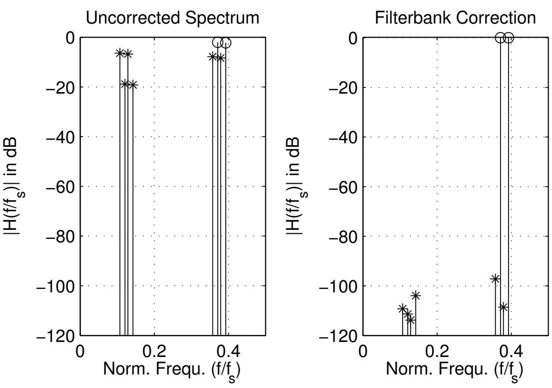


Fig. 8. Magnitude spectrum of a non-uniformly sampled signal (a) and the reconstructed spectrum employing the Thiran based filter bank scheme (right).

## V. CONCLUSION

In this paper, we presented a polyphase filter bank methodology that allows for the reconstruction of gain and timing mismatch affected channel ADCs in a time-interleaved data converter system. Closed form IIR Thiran filters were employed to model the non-ideal channel characteristics and an enhanced perfect reconstruction criterion based on the polyphase representation of the filter bank was utilized to derive reconstruction filters that showed good performance even for large mismatch magnitudes. Moreover, an FIR approximation in a least-squares sense was described to overcome the stability issues of the derived reconstruction filters. To demonstrate the efficacy of this design scheme, different filter banks based on Hann and Kaiser windowed fractional filters and the recursive truncated Thiran interpolator were compared in terms of their reconstruction performance and required computational complexity. The proposed Thiran-based design showed a superior reconstruction performance while requiring a similar number of filter coefficients compared with the FIR-based designs. Furthermore, the impact of limited coefficient length on the spurious free dynamic range and the signal to noise ratio was investigated to provide guidance to designers. Finally, the reconstruction of a non-uniformly sampled signal was employed to verify the proposed filter bank architecture.

## ACKNOWLEDGMENTS

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