

## Investigation of an optical potential model for post-collision interaction in ejected-electron spectra

To cite this article: P J M van der Burgt and H G M Heideman 1987 *J. Phys. B: At. Mol. Phys.* **20** 4605

View the [article online](#) for updates and enhancements.

### Related content

- [A model for post-collision interaction based on an optical-potential description](#)  
W van de Water, H G M Heideman and G Nienhuis
- [Orbital angular momentum exchange in post-collision interaction](#)  
P J M van der Burgt, J van Eck and H G M Heideman
- [Shape resonances and the excitation of helium autoionising states by electrons in the 57-66 eV region](#)  
P J M van der Burgt, J van Eck and H G M Heideman

### Recent citations

- [REVIEWS OF TOPICAL PROBLEMS: Post-collision interaction in atomic processes](#)  
Kuchiev, M. U. and SheĀnerman, S. A.
- [Excitation of atomic states of high angular momentum by near-threshold electron impact](#)  
P Hammond *et al*

## Investigation of an optical potential model for post-collision interaction in ejected-electron spectra

P J M van der Burgt<sup>†</sup> and H G M Heideman

Fysisch Laboratorium, Rijksuniversiteit Utrecht, 3584 CC Utrecht, The Netherlands

Received 23 January 1987, in final form 13 April 1987

**Abstract.** An optical potential method is used to formulate a model for the post-collision interaction that occurs in electron-helium scattering after excitation and decay of autoionising states. The model is an extension of an earlier description given by Nienhuis and Heideman, and is used to study the effects of post-collision interaction in ejected-electron spectra. We present a calculation of angular distributions of ejected electrons from the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state. It is argued that study of angular momentum exchange during the post-collision interaction requires detailed knowledge of the interfering background of the direct ionisation process.

### 1. Introduction

The long-range Coulomb interaction between the two electrons in the exit channel of a collision process, following the excitation and decay of an autoionising state, manifests itself in an exchange of energy between the two electrons. When the autoionising state is excited near its threshold by electron impact, the slow scattered electron is still close to the atom at the moment of autoionisation. After autoionisation the fast ejected electron acquires some extra energy as the field of the residual  $\text{He}^+$  ion is screened by the scattered electron, which simultaneously loses an equal amount of energy. This interaction, which has been termed post-collision interaction (PCI), has led to the formulation of models, which describe the energy exchange between the two electrons.

Two models have been studied extensively. The shake-down model, proposed by Read (1977) and also studied by Read and Comer (1980), is based on the idea that the scattered electron makes a sudden transition from a plane wave state to a Coulomb wave state in the field of the  $\text{He}^+$  ion. According to this model the transition amplitude can be expressed in the form of an overlap integral between the initial state and the final state of the scattered electron.

A semiclassical model was formulated by Morgenstern *et al* (1977) and Niehaus (1977), and used for the analysis of post-collision interaction in ion-atom, photon-atom (inner-shell ionisation) and electron-atom collisions. Recent developments concerning this model are described by Russek and Mehlhorn (1985). The semiclassical model uses time-dependent Born-Oppenheimer potential curves for the initial and final states in the autoionisation process. The transition amplitude is approximated by using first-order perturbation theory. For electron-atom collisions the semiclassical model

<sup>†</sup> Present address: Atomic Collisions Laboratory, Physics Department, North Carolina State University, Raleigh NC 27695, USA.

leads to an expression of the PCI transition amplitude that is equivalent to the shake-down transition amplitude (see van de Water and Heideman 1980).

As the ejected electron moves in a time-dependent dipole field formed by the scattered electron and the  $\text{He}^+$  ion, it may well be that small exchanges of angular momentum occur between the ejected and the scattered electron due to the non-radial forces exerted on both electrons. Recently van der Burgt *et al* (1985) have shown that angular momentum exchanges occur in a particular case. In the shake-down model angular momentum exchanges are excluded; in the semiclassical model angular momentum exchange may be taken into account by including a centrifugal term in the model potentials (see Niehaus and Zwakhals 1983).

The purpose of this paper is to reformulate the optical potential model of Nienhuis and Heideman (1976) and van de Water *et al* (1981). The model leads to an expression for the transition amplitude containing two terms, the first of which is equivalent to the shake-down model, and the second term describes angular momentum exchange during PCI. The present version of the model will be applicable to situations where the scattered electron remains in a continuum state in the field of the  $\text{He}^+$  ion, instead of being captured into a bound state. We present an outline of the model, the result of a calculation concerning the experimental study of van der Burgt *et al* (1985), and a discussion on the feasibility of a detailed comparison between theory and experiment.

## 2. Optical potential description

An optical potential description of PCI for the case where the post-collision interaction causes the scattered electron to be captured into a singly excited state was given by Nienhuis and Heideman (1976) and van de Water *et al* (1981). This description can in a straightforward way be reformulated for post-collision interaction in ejected-electron spectra (i.e. the scattered electron remains in a continuum state). The model employs the formalism of Feshbach (1962) by choosing the operator  $P$  to project on the subspace of helium states with at least one electron in the  $1s$  orbital. This subspace contains both the initial state of an electron incident on a helium atom in the ground state, and the final state of two electrons receding from a  $\text{He}^+$  ion in the ground state. The complementary operator  $Q = 1 - P$  projects on the subspace which contains the autoionising states together with a scattered electron of low energy. According to the formalism of Feshbach the transition amplitude can be expressed as the sum of two parts:  $T = T_P + T_{\text{opt}}$ , where  $T_P$  describes the direct ionisation process and  $T_{\text{opt}}$  contains the effect of autoionising states on the ionisation process. The equation for  $T_{\text{opt}}$  is (analogous to Nienhuis and Heideman (1976), equation (3.10))

$$T_{\text{opt}} = \int d\mathbf{k}_s \langle \mathbf{k}_\alpha \mathbf{k}_\beta^P | PHQ | \mathbf{k}_s a^{Q-} \rangle \frac{1}{E - \frac{1}{2}k_s^2 - E_a + \frac{1}{2}i\Gamma} \langle \mathbf{k}_s a^{Q-} | QHP | \mathbf{k}_i 0^{P+} \rangle. \quad (1)$$

Here the initial state  $|\mathbf{k}_i 0\rangle$  represents an electron with wavevector  $\mathbf{k}_i$  incident on a helium atom in the ground state, the state  $|\mathbf{k}_s a\rangle$  represents an autoionising state (energy  $E_a$ , width  $\Gamma$ ) and a free electron and  $|\mathbf{k}_\alpha \mathbf{k}_\beta\rangle$  represents the ejected electron  $\alpha$  and the scattered electron  $\beta$  in the field of the  $\text{He}^+$  ion. The final state  $|\mathbf{k}_\alpha \mathbf{k}_\beta\rangle$  replaces the state  $|\mathbf{k}_\alpha f\rangle$  in the equations of Nienhuis and Heideman (1976) and van de Water *et al* (1981).

We now explicitly introduce the Coulomb interaction  $V_{\alpha\beta} = |\mathbf{r}_\alpha - \mathbf{r}_\beta|^{-1}$  between electron  $\alpha$  and electron  $\beta$  in the transition amplitude by writing down a second-order

Born approximation for the final state:

$$|k_\alpha k_\beta^{P-}\rangle \approx |[k_\alpha][k_\beta]\rangle + \frac{1}{E^- - PH_{\alpha\beta}P} PV_{\alpha\beta}P|[k_\alpha][k_\beta]\rangle. \quad (2)$$

Here  $|k_\alpha k_\beta^{P-}\rangle$  denotes an eigenstate of  $PHP$ , the full Hamiltonian of the system projected on the  $P$  space, whereas  $|[k_\alpha][k_\beta]\rangle$  is an eigenstate of  $PH_{\alpha\beta}P = P(H - V_{\alpha\beta})P$  and represents electrons  $\alpha$  and  $\beta$  receding independently of each other in the  $\text{He}^+$  field. The operator  $(E^- - PH_{\alpha\beta}P)^{-1}$  can be expanded using an approximate closure of eigenstates of  $H_{\alpha\beta}$ :

$$P' \approx \iint dk'_\alpha dk'_\beta |[k'_\alpha][k'_\beta]\rangle \langle [k'_\alpha][k'_\beta]|. \quad (3)$$

This closure is approximate as the part of the spectrum of states of  $H_{\alpha\beta}$ , with one or both of the electrons being bound in the field of the  $\text{He}^+$  ion, is ignored.

The resulting expression for the transition amplitude is

$$T_{\text{opt}} = T_{\text{opt}}^{(1)} + T_{\text{opt}}^{(2)}$$

$$T_{\text{opt}}^{(1)} = \int dk_s \langle [k_\alpha][k_\beta] | PHQ | k_s a^{Q-} \rangle \frac{1}{E - \frac{1}{2}k_s^2 - E_a + \frac{1}{2}i\Gamma} \langle k_s a^{Q-} | QHP | k_i 0^{P+} \rangle \quad (4)$$

$$T_{\text{opt}}^{(2)} = \iiint dk_s dk'_\alpha dk'_\beta \langle [k_\alpha][k_\beta] | PV_{\alpha\beta}P | [k'_\alpha][k'_\beta] \rangle \frac{1}{E^+ - \frac{1}{2}k_\alpha'^2 - \frac{1}{2}k_\beta'^2 - IP}$$

$$\times \langle [k'_\alpha][k'_\beta] | PHQ | k_s a^{Q-} \rangle \frac{1}{E - \frac{1}{2}k_s^2 - E_a + \frac{1}{2}i\Gamma} \langle k_s a^{Q-} | QHP | k_i 0^{P+} \rangle \quad (5)$$

where  $IP$  is the ionisation energy of helium. The first part  $T_{\text{opt}}^{(1)}$  of the transition amplitude gives the sudden approximation on which the shake-down model (Read 1977) is based. The second part  $T_{\text{opt}}^{(2)}$  contains angular momentum exchange between the scattered electron and the ejected electron during the post-collision interaction.

Our intention is to study the final step in the total scattering process when the post-collision interaction takes place. Accordingly we have calculated only the matrix element describing the final step, and we have made suitable approximations for the excitation and decay of the autoionising state. We have not included the effects of exchange or spin in our formalism, since these effects are not likely to play a role during the post-collision interaction due to the large difference in energy between the scattered electron and the ejected electron. In the derivation leading to (4) and (5) we have assumed that after decay of the autoionising state the scattered electron and the ejected electron interact via intermediate states with both electrons in a continuum state in the field of the  $\text{He}^+$  ion. Furthermore we have used a second-order Born approximation for the final state.

If we now make the assumption that the decay of the autoionising state is not affected by the presence of the scattered electron, and that the interaction between the scattered electron and the autoionising state corresponds to an average potential interaction, it follows that

$$\langle [k'_\alpha][k'_\beta] | PHQ | k_s a^{Q-} \rangle \approx \langle [k'_\beta] | k_s^- \rangle \langle [k'_\alpha] | PH_\beta Q | a \rangle \quad (6)$$

where  $|k_s^- \rangle$  indicates a distorted wave. In the actual evaluation of the PCI matrix element, post-collision interaction is assumed to occur at a large distance from the  $\text{He}^+$  ion and the distorted wave is replaced by a plane wave.

Before calculations based on (4) and (5) can be performed, the transition amplitude has to be expanded in partial waves. The resulting integrals over radial wavefunctions can subsequently be evaluated numerically. Expansion in partial waves proceeds exactly as before (van de Water *et al* 1981) and leads to the equations

$$T_{\text{opt}} = \sum_{l_i m_i} \sum_{l_\alpha m_\alpha} \sum_{l_\beta m_\beta} Y_{l_\alpha m_\alpha}(\hat{k}_\alpha) Y_{l_\beta m_\beta}(\hat{k}_\beta) Y_{l_i m_i}^*(\hat{k}_i) t_{\text{opt}} \langle l_\beta m_\beta l_\alpha m_\alpha | l_i m_i \rangle \quad (7)$$

where

$$t_{\text{opt}} = \sum_{l_s} t_{\text{opt}}^{(1)} \delta_{l_\alpha L_\alpha} \delta_{l_\beta l_s} + \sum_{l_s} \sum_l t_{\text{opt}}^{(2)}$$

$$t_{\text{opt}}^{(1)} = \chi(k_\alpha \leftarrow a) T(K_s l_s, a L_\alpha \leftarrow k_i l_i, 0) \frac{-2 \exp(-i\sigma(l_s, k_\beta))}{k_\beta} \int_0^\infty F_{l_s} \left( \frac{-1}{k_\beta}, k_\beta r \right) \hat{h}_{l_s}^+(K_s r) dr \quad (8)$$

$$t_{\text{opt}}^{(2)} = \int_0^\infty k_\alpha'^2 dk_\alpha' \chi(k_\alpha' \leftarrow a) T(K_s l_s, a L_\alpha \leftarrow k_i l_i, 0) C_{L_\alpha l_i l_\alpha}^{l_\beta l l_s}$$

$$\times \frac{8(-i)^{l_\alpha + l_\beta} (i)^{L_\alpha + l_s}}{k_\alpha k_\beta k_\alpha' K_\beta'} \exp(-i\sigma(l_\alpha, k_\alpha)) \exp(-i\sigma(l_\beta, k_\beta)) \exp(i\sigma(L_\alpha, k_\alpha'))$$

$$\times \int_0^\infty dr_\beta \int_0^\infty dr_\alpha \int_0^\infty dr_s F_{l_\alpha} \left( \frac{-1}{k_\alpha}, k_\alpha r_\alpha \right) F_{l_\beta} \left( \frac{-1}{k_\beta}, k_\beta r_\beta \right) \frac{(r_\alpha^{\alpha\beta})^l}{(r_s^{\alpha\beta})^{l+1}}$$

$$\times F_{L_\alpha} \left( \frac{-1}{k_\alpha'}, k_\alpha' r_\alpha \right) F_{l_s} \left( \frac{-1}{K_\beta'}, K_\beta' r_s \right) H_{l_s}^+ \left( \frac{-1}{K_\beta'}, K_\beta' r_s \right) \hat{h}_{l_s}^+(K_s r_s). \quad (9)$$

Here  $\hat{h}_l^+(kr)$  is a Ricatti-Bessel function,  $F_l(-1/k, kr)$  and  $H_l^+(-1/k, kr)$  are radial Coulomb wavefunctions and  $\sigma(l, k) = \arg \Gamma(l+1-i/k)$  is the Coulomb phaseshift. Also  $K_s$  is the root, with positive imaginary part, of  $\frac{1}{2}K_s^2 = E - E_a + \frac{1}{2}i\Gamma$  and  $K_\beta'$  is given by  $K_\beta' = [2(E - IP) - k_\alpha'^2]^{1/2}$ . The geometrical factor is

$$C_{L_\alpha l_i l_\alpha}^{l_\beta l l_s} = (-1)^{L_\alpha + l_i - l_s} [(2L_\alpha + 1)(2l_\beta + 1)]^{1/2} \langle l_0 L_\alpha 0 | l_\alpha 0 \rangle \langle l_\beta 0 l_0 | l_s 0 \rangle \left\{ \begin{matrix} l_\beta & l & l_s \\ L_\alpha & l_i & l_\alpha \end{matrix} \right\}. \quad (10)$$

Finally  $T(K_s l_s, a L_\alpha \leftarrow l_i m_i, 0)$  and  $\chi(k_\alpha' \leftarrow a)$  are the matrix elements for excitation and decay of the autoionising state, respectively (see equation (3.8) of van de Water *et al* (1981) and equation (4.10) of Nienhuis and Heideman (1976)). The most important difference between the present formulae and the formulae of van de Water *et al* (1981) is the replacement of a hydrogen wavefunction by a Coulomb wavefunction. In (9) another Coulomb wavefunction replaces the Ricatti-Bessel function in equation (3.10) of van de Water *et al* (1981). It is seen that the radial integral in (8) is equivalent to the shake-down model of Read (1977) and that no angular momentum exchange occurs between the scattered electron and the ejected electron in the shake-down model.

### 3. Cross sections

If the quantisation axis is chosen along the incident electron beam the triply differential cross section for scattering of electron  $\alpha$  with energy  $E_\alpha$  in the solid angle element

$d\Omega_\alpha$  and of electron  $\beta$  in  $d\Omega_\beta$  can be written as

$$\frac{d^3\sigma}{d\Omega_\alpha d\Omega_\beta dE_\alpha} = (2\pi)^4 \frac{k_\alpha k_\beta}{k_i} \left| \sum_{l_i} \sum_{l_\alpha m_\alpha} \sum_{l_\beta m_\beta} Y_{l_\alpha m_\alpha}(\vartheta_\alpha, \varphi_\alpha) Y_{l_\beta m_\beta}(\vartheta_\beta, \varphi_\beta) \right. \\ \left. \times \left( \frac{2l_i + 1}{4\pi} \right)^{1/2} \{t_{opt} + t_{dir}\} \langle l_\beta m_\beta l_\alpha m_\alpha | l_i 0 \rangle \right|^2 + |B|^2 \quad (11)$$

where  $t_{opt}^{(1)}$  and  $t_{opt}^{(2)}$  are given by (8) and (9), respectively. The interfering part of the direct ionisation amplitude is contained in  $t_{dir}$  in (11). When a partial wave expansion of the transition amplitude  $T_P$  for direct ionisation is made, an expression similar to (7) results, the interfering part of which is represented by  $t_{dir}$ . The non-interfering part of  $T_P$  is contained in  $B$  in (11).

When the scattered electron is not detected, the differential cross section for detection of ejected electrons at angles  $\vartheta_\alpha, \varphi_\alpha$  is

$$\frac{d^2\sigma}{d\Omega_\alpha dE_\alpha} \sim \frac{k_\alpha k_\beta}{k_i} \sum_{l_\beta m_\beta} \left| \sum_{l_i} \sum_{l_\alpha} P_{l_\alpha - m_\beta}(\cos \vartheta_\alpha) \{t_{opt} + t_{dir}\} \langle l_\beta m_\beta l_\alpha - m_\beta | l_i 0 \rangle \right|^2 + |B|^2 \quad (12)$$

where  $P_{lm}(\cos \vartheta)$  is a normalised associated Legendre polynomial. If we write the interfering part of the direct ionisation amplitude simply as  $A$  and retain only the shake-down part of  $t_{opt}$ , the shake-down cross section can easily be derived:

$$\frac{d^2\sigma}{d\Omega_\alpha dE_\alpha} \sim \frac{k_\alpha k_\beta}{k_i} \sum_{lm} |aq e^{ix} + A|^2 + |B|^2 \quad (13)$$

where

$$a = \sum_{l_i} \langle lm L_\alpha - m | l_i 0 \rangle P_{L_\alpha - m}(\cos \vartheta_\alpha) \chi(k_\alpha \leftarrow a) T(K_s l, a L_\alpha \leftarrow k_i l_i, 0) \quad (14)$$

and

$$q e^{ix} = \frac{-2 \exp(-i\sigma(l, k_\beta))}{k_\beta} \int_0^\infty F_l\left(\frac{-1}{k_\beta}, k_\beta r\right) \hat{h}_l^+(K_s r) dr. \quad (15)$$

In the past there has been some discussion as to whether scattering amplitudes for autoionisation and for direct ionisation should be added coherently or incoherently. Equation (12) shows that PCI structures in ejected-electron spectra are partly coherent as only terms with equal  $l_\beta$  are added coherently. Note that the parameter  $A$  in (13) is a function of the quantum numbers  $l$  and  $m$  of the scattered electron and of the angle  $\vartheta_\alpha$  of the ejected electron. In fits of the shake-down and semiclassical models to ejected-electron spectra (e.g. Morgenstern *et al* 1976, Read 1977) interference was taken into account by merely including one variable for the interfering part of the direct ionisation amplitude, thereby ignoring the summation over  $l$  and  $m$ . These fits provided a qualitative understanding of PCI structures in ejected-electron spectra, but it is clear that angular distributions of ejected electrons can only be properly analysed when this summation is taken into account.

We have calculated only the part of the total transition amplitude that describes the post-collision interaction, and we have made approximations for the matrix elements for excitation and for decay of the autoionising state,  $T(K_s l_s, a L_\alpha \leftarrow k_i l_i, 0)$  and  $\chi(k'_\alpha \leftarrow a)$ , respectively. As a consequence we have to restrict calculations to cases where only one partial wave  $l_i$  of the incident electron and one partial wave  $l_s$  of the scattered electron dominantly contribute to the excitation of the autoionising state. Such a

situation occurs when the autoionising state is excited via a negative-ion resonance. As the calculation has been carried out for a fixed incident electron energy the amplitude for excitation of the autoionising state is merely a proportionality constant, which has been taken equal to 1 in the calculations. Calculations involving more partial waves would require evaluation of the matrix element for excitation of the autoionising state. Also, we have approximated the decay amplitude by a step function, the step occurring at  $k'_\alpha = 1$  au. The same approximation was made by van de Water *et al* (1981). Lipsky and Conneely (1976) have calculated the decay amplitude for some autoionising states. Their results show that the decay amplitude rapidly rises around  $k'_\alpha = 1$  au and is approximately constant for higher  $k'_\alpha$ .

The radial integrals in the transition amplitudes  $t_{\text{opt}}^{(1)}$  and  $t_{\text{opt}}^{(2)}$  contain integrands that are products of radial electronic wavefunctions behaving asymptotically as sums of sine functions with slowly decreasing amplitudes. Therefore our basic technique to evaluate these radial integrals was integrating up to a finite radius by repeated Gauss-Legendre quadrature or automatic quadrature, and estimating the remainder of the integral by applying a Euler transformation (Wynn 1971). The Euler transformation can be applied to the alternating series that results when repeated integration between adjacent zeros of a regularly oscillating function is performed.

An asymptotic expansion of the radial wavefunctions occurring in the multiple integral  $t_{\text{opt}}^{(2)}$  in (9) shows that a singularity occurs on the energy shell, i.e. when  $k'_\alpha$  equals  $k_\alpha$ . For that reason we have made a numerical approximation of the principal value integral  $\mathcal{P} \int dk'_\alpha$  by applying an even-point Gauss-Legendre quadrature on a symmetrical interval around  $k'_\alpha = k_\alpha$  (see Piessens 1970). This singularity may be of the same nature as the singularity in the Born series for the direct ionisation process (see Popov 1981). Thus our model is not suitable for a very accurate calculation and needs further improvement so that the singularity is removed. We have performed a model calculation to be able to illustrate the difficulty that arises when a detailed comparison with experiments is sought.

#### 4. Results and discussion

A practical case which lends itself to a model calculation concerns the electron impact excitation of the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state via the  $\text{He}^-(2s2p^2)^2\text{D}$  resonance. By measuring the angular distribution of ejected electrons from the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state we were able to show that the ejected electrons exchange orbital angular momentum with the resonant scattered electrons (van der Burgt *et al* 1985). We briefly summarise the argument.

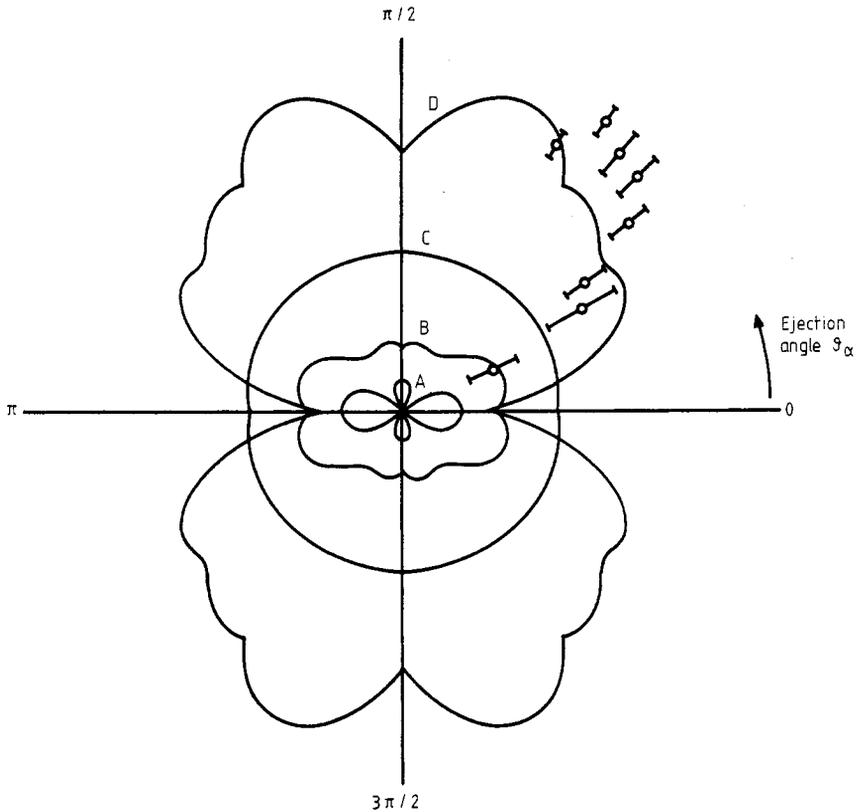
At an energy of 58.3 eV the excitation of the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state is enhanced by the  $\text{He}^-(2s2p^2)^2\text{D}$  resonance, and the slow scattered electron recedes with  $l_\beta = 2$ . As the slow scattered electron in the direct ionisation channel has a very low energy we assume that it only occurs significantly in  $l'_\beta = 0$  or 1 partial waves. Because the scattered electrons are not detected in our experiment, interference with the direct ionisation background vanishes unless angular momentum exchange results in  $l_\beta = l'_\beta$  (this can be verified from (12): only terms  $t_{\text{opt}}$  and  $t_{\text{dir}}$  with equal  $l_\beta$  interfere). As the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state decays to the ground state of the helium ion  $\text{He}^+(1s)$ , the initial angular momentum of the ejected electron is  $l_\alpha = 0$ . If there is no angular momentum exchange during PCI, and consequently no interference, clearly the distribution of the resonance in ejected-electron spectra has to be isotropic. Conversely, an

anisotropic distribution of the resonance provides an indication that angular momentum exchange takes place. The measurements clearly showed an anisotropic distribution. Angular momentum exchange results in  $l_\beta < 2$  and an anisotropic distribution is observed due to both  $l_\alpha \neq 0$  and interference.

The results of a calculation are shown in figure 1. The experimental position  $E_a = 57.82$  eV and width  $\Gamma = 0.138$  eV of the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state have been used in the calculation. We only consider resonant excitation of the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state and accordingly have used  $E_i = 58.3$  eV and  $l_s = l_i = 2$  in the calculations. We have not carried out the summation over  $l_\beta$  in the differential cross section given by (12), but we have only obtained angular distributions of ejected electrons. The angular distributions have been calculated using the formula

$$S(l_\beta) \sim \sum_{m_\beta} \left| \sum_{l_i} \sum_{l_\alpha} P_{l_\alpha - m_\beta}(\cos \vartheta_\alpha) t_{\text{opt}}(l_\beta m_\beta l_\alpha - m_\beta | l_i 0) \right|. \quad (16)$$

This amounts to the assumption that the interfering part of the direct ionisation



**Figure 1.** Calculated angular distributions are presented of ejected electrons from the  $\text{He}^{**}(2s^2)^1\text{S}$  autoionising state following its electron impact excitation via the  $\text{He}^-(2s2p^2)^2\text{D}$  resonance. After post-collision interaction the scattered electron recedes with  $E_{sc} = 0.21$  eV in different partial waves  $l_\beta = 0, 1, 2$  and  $3$  (see below). A measured angular distribution is also plotted in the figure. The radial scales of the measured and calculated angular distributions are different so that only the shapes of the curves can be compared with the measurements (see text). Measurements:  $\odot$ ; calculations: A,  $l_\beta = 0$  ( $\times 1$ ); B,  $l_\beta = 1$  ( $\times 2$ ); C,  $l_\beta = 2$  ( $\times 4$ ); D,  $l_\beta = 3$  ( $\times 8$ ).

amplitude in (12) is isotropic and much larger than the autoionisation amplitude  $t_{\text{opt}}$ . The angular distributions are plotted for fixed values of  $l_\beta$  in figure 1.

The results of the experiment by van der Burgt *et al* (1985) are also included in figure 1. However, care needs to be exercised in comparing the measurements with the calculations, as the measurements fully include interference with the direct ionisation but the calculations merely include an isotropic interfering background. There are two possibilities of reconciling the calculations with the measurements.

(i) In our analysis of the measurements we have assumed that only  $l'_\beta = 0$  and  $l''_\beta = 1$  partial waves of the slow electron in the direct ionisation process contribute significantly to the total differential cross section. Accordingly we may compare the measurements with the  $l_\beta = 1$  calculated angular distribution, and suppose that the differences with the measured angular distribution are caused by interference. The  $l_\beta = 0$  calculated angular distribution clearly is not satisfactory as it has a minimum at  $54^\circ$  in contrast with the measurements.

(ii) Our assumption may not be correct, i.e. also  $l'_\beta = 2$  and  $l''_\beta = 3$  partial waves in the direct ionisation cross section contribute significantly. In that case most probably the measured angular distribution arises for the major part through interference of the  $l_\beta = 2$  angular distribution with the direct ionisation. According to the calculation the  $l_\beta = 2$  partial wave dominantly contributes in the autoionisation cross section. The difference between the measured angular distribution, which is anisotropic, and the  $l_\beta = 2$  calculated angular distribution, which is nearly isotropic, may be assumed to be caused by interference.

The above discussion shows that a comparison between theory and experiment cannot yet give information about angular momentum exchange during PCI. Only a detailed analysis of the interference with the direct ionisation process can provide such information. However in our opinion it is unlikely that PCI in electron-atom collisions can be described by a sudden process only. In the final part of the collision process the scattered electron suddenly feels the attraction of the  $\text{He}^+$  ion at the moment of autoionisation, but after that a long-range Coulomb interaction occurs between the ejected electron and the scattered electron. In addition the polarising effect of the scattered electron on the autoionising state might determine the initial situation from which after autoionisation the post-collision interaction evolves. An indication that such an effect exists is given by the existence of shape resonances very close to the thresholds of the autoionising states (van der Burgt *et al* 1986).

The long-range Coulomb interaction between both outgoing electrons also manifests itself in the direct ionisation process and leads to observable effects (Klar and Franz 1986, Klar *et al* 1986). Therefore it might well be that this interaction does not contribute to the difference in phase between autoionisation and direct ionisation, but that the interference between autoionisation and direct ionisation is primarily determined by the sudden step in the autoionisation process. However a full understanding of the autoionisation process can only be achieved by a study of both autoionisation and direct ionisation, including a detailed analysis of the interference between both processes.

## 5. Conclusion

We conclude that the optical potential model for post-collision interaction, which is described in this paper, can be used for the calculation of effects of post-collision

interaction in ejected-electron spectra. Comparison of calculated angular distributions of ejected electrons with the measurements is greatly hampered by the fact that interference with the direct ionisation process cannot be taken into account. Further study of the post-collision interaction would therefore greatly benefit from calculations of differential cross sections for direct ionisation in the relevant energy ranges.

### Acknowledgments

This work was performed as a part of the research programme of the 'Stichting voor Fundamenteel Onderzoek der Materie' (FOM) with financial support from the 'Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek' (ZWO). We are indebted to Dr H F Arnoldus for providing us with a program for the second-order wkb approximation of Coulomb wavefunctions.

### References

- Feshbach H 1962 *Ann. Phys., NY* **19** 287-313  
Klar H and Franz A 1986 *Phys. Rev. A* **33** 2103-5  
Klar H, Franz A and Terhagen H 1986 *Z. Phys. D* **1** 373-80  
Lipsky L and Conneely M J 1976 *Phys. Rev. A* **14** 2193-205  
Morgenstern R, Niehaus A and Thielmann U 1976 *J. Phys. B: At. Mol. Phys.* **9** L363-7  
— 1977 *J. Phys. B: At. Mol. Phys.* **10** 1039-58  
Niehaus A 1977 *J. Phys. B: At. Mol. Phys.* **10** 1845-57  
Niehaus A and Zwakhals C J 1983 *J. Phys. B: At. Mol. Phys.* **16** L135-42  
Nienhuis G and Heideman H G M 1976 *J. Phys. B: At. Mol. Phys.* **9** 2053-62  
Piesens R 1970 *BIT* **10** 476-80  
Popov Y 1981 *J. Phys. B: At. Mol. Phys.* **14** 2449-57  
Read F H 1977 *J. Phys. B: At. Mol. Phys.* **10** L207-12  
Read F H and Comer J 1980 *Coherence and Correlation in Atomic Collisions* ed H Kleinpoppen and J F Williams (New York: Plenum) pp 243-55  
Russek A and Mehlhorn W 1985 *J. Phys. B: At. Mol. Phys.* **19** 911-27  
van de Water W and Heideman H G M 1980 *J. Phys. B: At. Mol. Phys.* **13** 4663-74  
van de Water W, Heideman H G M and Nienhuis G 1981 *J. Phys. B: At. Mol. Phys.* **14** 2935-50  
van der Burgt P J M, van Eck J and Heideman H G M 1985 *J. Phys. B: At. Mol. Phys.* **18** 999-1009  
— 1986 *J. Phys. B: At. Mol. Phys.* **19** 2015-37  
Wynn P 1971 *Comput. J.* **14** 437-41