The Hamilton Walk and Its Positive Impact on Adults Learning Mathematics Outside the Classroom

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Abstract

This paper discusses how the annual Hamilton walk has a positive impact on adults learning mathematics outside the classroom. The walk takes place on October 16 and typically attracts about 150 people from a wide variety of backgrounds, including many from the general public. I will show how the walk has changed many adults' perception of mathematics for the better and enhanced their understanding, appreciation and awareness of mathematics. Also, the walk has inspired many artists to create art works in relation to Hamilton, including poems, paintings, a song and more.

How it All Began

On October 16, 1843, Ireland's greatest mathematician, William Rowan Hamilton, was walking along the banks of the Royal Canal in Dublin when he had a Eureka moment. Hamilton's creation of a strange new system of four--dimensional numbers called quaternions on that day was a major moment in the history of mathematics and would change the world of mathematics forever. Hamilton later described his Eureka moment to his son in a letter, as follows:

> *Although your mother talked with me now and then, yet an undercurrent of thought was going on in my mind, which gave at last a result, whereof it is not too much to say that I felt at once an importance. An electric current seemed to close; and a spark flashed forth, the herald (as I foresaw, immediately) of many long years to come of definitely directed thought and work ... Nor could I resist the impulse -- unphilosophical as it may have been -to cut with a knife on a stone of Brougham Bridge as we passed it, the fundamental formula...*

The annual Hamilton walk takes place on October 16 and commemorates Hamilton's creation of quaternions by retracing his steps from Dunsink Observatory to Broombridge where he created quaternions. Before I discuss the walk in more detail, I will tell more of the story of Hamilton's Eureka moment. This story is also part of the beginning of the walk and the end of the walk (and potentially during the walk).

Hamilton's creation of quaternions was his most celebrated contribution to mathematics. Number couples (or complex numbers) were important in mathematics and science when working in two dimensional geometry and Hamilton had been trying to extend his theory of number couples to a theory of number triples (or triplets). Hamilton hoped the triplets would give a natural mathematical structure and a new way for describing the three dimensional world. He was finding it difficult to define the multiplication operation in his search for a suitable theory of triplets. We now know why he was having such difficulty because it's impossible to create the suitable theory of triplets.

Then, in a spark of inspiration, Hamilton's mind gave birth to quaternions on the banks of the Royal Canal on October 16, 1843. In an act of graffiti, he scratched his quaternion formulas on the bridge as described in his letter above. This piece of mathematical vandalism would change the world of mathematics forever. Hamilton reckoned that if he worked with number quadruples and an unusual multiplication, then he would obtain all that he desired. He called his new system of numbers Quaternions because every number quadruple contained four components.

The mathematical community was stunned at his audacity in creating a system of numbers that broke the usual commutative rule for multiplication (ab=ba). Hamilton was called the Liberator of Algebra since his quaternions smashed the previously accepted convention that a useful algebraic number system should satisfy the rules of ordinary arithmetic. Quaternions opened up a whole new mathematical landscape in which mathematicians were now free to conceive new algebraic systems which were not constrained by the rules of ordinary arithmetic. I suppose one could say that it was:

One small scratch for a man, one giant leap for mathematics!

An important part of the Hamilton walk is to say, at the beginning of the walk, that Hamilton's motivation for doing mathematics was the quest for beauty in mathematics. I believe that any discussion of beauty in mathematics needs to start with the statement that mathematics essentially consists of an abundance of ideas. Number and circle are just some examples of the myriad ideas in mathematics. I find from experience in promoting mathematics with adults outside the classroom, that it can come as a surprise when they hear that number is an idea that cannot be sensed with our five physical senses. Yet, numbers are indispensable in society today and arise almost everywhere from football scores to the time of day.

The reason number arises practically everywhere is because number is an idea and not something physical. Many adults think that they physically see the number two on a blackboard but this is not so. The number two cannot be physically sensed because it's an idea. What appears on the blackboard is merely a symbol to represent the idea we call two.

Mathematical ideas like number can only be really 'seen' with the 'eyes of the mind' because that is how one 'sees' ideas. Think of the analogy with the sheet of music versus the music. The sheet of music is important and useful but it's nowhere near as interesting, beautiful or powerful as the music it represents. Similarly, mathematical notation and symbols on a blackboard are just like the sheet of music; they are important and useful but they are nowhere near as interesting, beautiful or powerful as the actual mathematics (ideas) they represent. When some adults outside the classroom first become aware that mathematics comprises ideas, it's as if they have just first become aware of the existence of music (that is not just a sheet of music). Consequently, this changes many adults' perception of mathematics for the better (in a major way).

Many adults say they do not see mathematics in the physical world and this is because they are looking with the wrong eyes. These adults are not looking with the eyes of their mind. As part of the walk, I ask adults to look with the eyes of their mind. They then begin to 'see' mathematics in places they never imagined.

As mentioned above, at the start of the walk I say that Hamilton's motivation for doing mathematics was the pursuit of beauty. Now, the beauty in mathematics typically lies in the beauty of ideas, because as stated earlier, mathematics essentially comprises an abundance of ideas. Our notion of beauty usually relates to our five senses, like a beautiful taste or a beautiful vision etc. The notion of beauty in relation to our five senses plays a very important part in our society. However, I believe that ideas (which may be unrelated to our five senses) can also have beauty and this is where you will typically find beauty in mathematics. Consequently, to experience beauty in mathematics, adults typically need to look, not with their physical eyes, but with the eyes of their mind because that is how one 'sees' ideas. I believe that beauty is the most important feature in mathematics (Ó Cairbre, 2009).

The notion of freedom in mathematics is also an important part of the walk because, as mentioned above, Hamilton was called the Liberator of Algebra. Hamilton was free to create a new system of

numbers that did not satisfy the rules of ordinary arithmetic, even though this broke the accepted convention in the mathematical community at the time. The notion of freedom in mathematics shocks many adults. However, as Cantor once said:

> *The essence of mathematics lies in its freedom.*

The reason freedom is an important feature of mathematics is because one is free to conceive of any new ideas one wants in mathematics. These new ideas may or may not lead to anything interesting or useful. Historically (and probably also in the future), the big breakthroughs in mathematics have typically occurred because the great mathematicians felt free to conceive of any new ideas they wanted even if their wild thoughts broke the conventions and seemed strange to other mathematicians and the general public. Hamilton's creation of quaternions is one such example. I believe there is great educational value in adults hearing about how freedom in mathematics has led to many crucial breakthroughs in mathematics.

The Annual Hamilton Walk

The annual Hamilton walk was initiated by Anthony G. O'Farrell in 1990. I organise it now and it typically attracts about 150 people from diverse backgrounds including staff and students from third level, second level and many from the general public. A Nobel Prize winner or a Fields Medallist often participates in the walk. The walk takes about forty five minutes and ends up at Broombridge in Cabra in Dublin, where a plaque on the bridge commemorates the creation of quaternions. I suppose one could call the annual walk a pilgribridge! The large number in attendance from the general public indicates a substantial public interest in the walk. Also, I get many calls from the media (television, radio and newspaper) and other bodies every year expressing an interest in doing a piece on Hamilton and the walk. Thus, the walk has appeared six times on television and an abundance of times on a variety of radio programmes and in many newspaper articles and I have given lots of talks on Hamilton. The walk has built up a large momentum of its own in the sense that it seems to be well known in the general public even outside mathematical circles. See Ó Cairbre, 2010 for a history of the walk. Anybody interested in coming on the walk should contact me.

The local Cabra Community Council make the annual walk into a very festive event with a large banner draped across the bridge and stalls along the canal. Broombridge is now a world famous location in the history of mathematics and science because of Hamilton's creation of quaternions. The word, Broomsday, is now often used in mathematical and public arenas to indicate October 16 and the walk. This word plays the same role as Bloomsday for literary groups and James Joyce fans. Actually, since I mentioned James Joyce, I may as well say that quaternions appear in Joyce's celebrated book, Finnegans Wake.

> *Wondering was it Hebrew set to himmeltones or the quicksilversong of qwaternions; his troubles may be over but his doubles have still to come.*

In this quote, Joyce is conflating Hamilton with two other Hamiltons. James Archibald Hamilton was the first astronomer at Armagh Observatory and he observed the transit of Mercury, i.e. quicksilver. James Hamilton was a clergyman who published a book on psalms.

Positive Impact of the Walk

The walk is a great example that relates to adults learning mathematics outside the classroom. It's also important to note that adults participate in the walk voluntarily and many of the adults on the walk are outside mainstream mathematics education. The walk brings to life a famous event in the history of mathematics and gives adults the opportunity to interact with Hamilton's story. In this way, adults feel much more connected to Hamilton's story than if they were just reading some mathematics in a book.

One of the main aspects of the positive impact of the walk on adults is that the walk changes many adults' perception of mathematics for the better. The walk can also lead to a very positive image of

mathematics in a whole community. Here are some examples of quotes related to the above comments:

> *On account of the walk, Hamilton is in the folk consciousness of the local people.*

Local Cabra resident, Jack Gannon, who was also inspired by the walk to write a ballad about Hamilton in 2003.

> *The walk has had a huge impact on the local community. In fact it has gone way beyond just being a walk because all the local school children and the community are extremely proud of Hamilton and their local connection with him. The walk really has touched the local community in a big way. The fact that famous mathematicians and Nobel Prize winners mingle with school children and the local community on the walk and at the bridge is a great experience. Also, not one but two local artists have been commissioned in recent times to do portraits of Hamilton which are then publicly displayed at the bridge during the walk.*
>
> (Aodhán Perry of Cabra Community Council)

> *The Hamilton walk was my licence to explore and express myself around the subject of mathematics. By the age of nine I had decided I couldn't do mathematics, but I had also developed a strong interest in things technical and scientific and this created a conflict that simmered in the background of my educational and professional career for forty years. That is until I took part on the Hamilton walk in 2005.That walk had a profound effect on me. Hearing not only a Nobel laureate and a professor of mathematics sing Hamilton's praises, but also local poets, school children, balladeers and the Cabra community council, spurred me to turn my desire to celebrate Ireland's Science Heritage into action. That action turned out to be a family run business called Science Heritage Ireland.*
>
> (Mick Kelly from Swords in Co. Dublin)

Mick also wrote the following:

> *By the 2007 walk I could sense flaws developing in the glass wall I had built around learning mathematics and found it strange but very uplifting to be answering queries from people about quaternion algebra. There was a special sense of magic at Broombridge on that fine Tuesday, October 16, 2007, when the canal bank was alive with children playing all kinds of mathematics games. I couldn't help but wonder how many bridges to the future the organisers of this walk and maths week had created for our children.*

I believe this change of perception can be crucial in any form of positive mathematics education, because after the change in perception, adults are then more likely to enhance their understanding, awareness and appreciation of mathematics. I see this happening frequently in relation to the Hamilton walk.

Many adults tell me that one of the main reasons their perception of mathematics changes for the better is because the walk introduces them to the fact that mathematics comprises an abundance of ideas and consequently beauty and freedom/creativity can be important features of mathematics. Another positive impact on adults is the storytelling approach to mathematics which happens naturally as part of the walk. Also, adults say that the sheet of music versus the music (analogy), mentioned earlier in this paper as part of the walk, has a big impact on changing their perception of mathematics for the better. When they realise, for the first time, that mathematics exists in the world of ideas (rather than the physical world), it's as if they become aware of music (rather than the sheet of music) for the first time. It really is that extreme in a positive sense.

I believe that this sheet of music versus the music (analogy) should be presented early when adults are learning mathematics. In other words, adults should be made aware early that mathematics comprises an abundance of ideas (rather than being just a load of symbols and notation), just like people are aware of music early (rather than music being just a load of notes on a sheet).

I have been promoting mathematics in the public arena for many years now. The annual Hamilton walk probably has the biggest impact on changing adults' perception of mathematics for the better. I believe this is because the walk (and the accompanying Hamilton story) has all, what I call, the "big picture of mathematics" features.

Here is a list of these "big picture of mathematics" features:

a) Storytelling

b) Drama

c) Humour

d) History of mathematics

e) Human element and famous characters

f) Beauty

g) Freedom and creativity/imagination

h) Practical power and applications

i) Motivation

j) Irish connections

k) Research and unsolved problems

l) Word origins

m) Outdoor activities

n) Cultural connections

o) Tricks/magic

p) Puzzles

a) Mystery

b) Wonder

c) Deductive reasoning

d) Abstraction

All of the above features can be relevant to adults learning mathematics outside the classroom. I will elaborate on some of the items on the list above. The drama related to the walk follows naturally from Hamilton's Eureka moment mentioned at the beginning of this paper. Beauty and freedom have

already been discussed in relation to the walk. Now I will mention a variety of examples of the practical power and applications of quaternions that can be mentioned at the beginning of the walk.

i. Quaternions play an important role in computer games. One example of this, which always appeals to journalists, radio hosts and students of course, is the fact that Lara Croft in Tombraider was created using quaternions!
ii. Continuing with the theme of entertainment, quaternions now play a significant role in computer animation and special effects in movies. For example, an Irish company called Havok used quaternions in the creation of the acclaimed new special effects in the film, The Matrix Reloaded, and also in the movie, Poseidon, which was nominated for an Oscar for its visual effects in 2007. Havok also received an Emmy award in the US in 2008 for pioneering new levels of realism and interactivity in films and games.
iii. Quaternions play a crucial part in space navigation. For example, they were fundamental in the Apollo 11 landing on the moon in 1969 and the Curiosity Rover landing on Mars in 2012.
iv. Quaternions played a part in Maxwell's mathematical theory and prediction of electromagnetic waves in 1864. This theory eventually led to the detection of radio waves by Hertz. Thus, the inventions of radio, television, radar and many other important products of our society are directly related to quaternions.
v. Vector analysis, which is indispensable in Physics, is an offspring of quaternions.

The walk and Hamilton's story has been an inspiration for a diversity of artists to create works of art related to Hamilton. Also, there has been a wide variety of public interactions with Hamilton's story and the walk. Here are some (of many) examples:

a) Sculptures (busts, bog oak and sand)

b) Paintings

c) Poems

d) Stamps

e) Plaques

f) Graffiti art

g) A song

h) A statue

i) A coin

j) A video installation in an art exhibition

k) Television news and documentaries

l) Radio shows

m) Newspaper articles

n) Housing areas

o) Institutes

p) T-shirts

q) A building

r) A moon crater

s) A golf hole

t) A banner

u) A bridge

v) References in James Joyce's Finnegans Wake, Flann O'Brien's work and a play by Sebastian Barry.

All of the above can be relevant to adults learning mathematics outside the classroom. Why all the inspiration? I believe the reason again is because all the "big picture of mathematics" features are present in the walk and Hamilton's story.

Now, I will discuss some of the examples above in more detail. The walk inspired Jack Gannon to write a song about Hamilton. This song has been played many times on radio and also on some television shows. Consequently, the song plays a role in adults learning mathematics outside the classroom. The walk also inspired the local community to commission two portrait paintings of Hamilton, which are publicly displayed at the bridge during the walk. Mick Kelly's T-shirts were also a consequence of his participation in the walk. The walk has appeared on television six times and on many radio shows and in lots of newspaper articles and thus the general media audience can hear about the walk in a variety of ways. The general public plays a significant role in mathematics education at all levels because parents, policy makers and the media are all members of the general public and can exert great influence on the attitude of young people and adults towards mathematics.

A current/future example of the impact of the walk on adults relates to Dublin City Council. In the last month I have been contacted by the City Council because they are doing an exhibition on Hamilton later this year. The exhibition will run from September to November and the main artist wishes to interact with me in relation to Hamilton. One example of this interaction will be to incorporate the walk into the exhibition.

There are a variety of places where one can read about Hamilton's life and works (Hankins, 1980; Ó Cairbre, 2000 & O'Donnell, 1983).

Conclusions

The annual Hamilton walk is a great example of something related to the history of mathematics that continues to inspire adults outside the classroom. Many of these adults are initially outside mathematics and then the walk changes their perception of mathematics for the better. Consequently, the walk has a positive impact on adults learning mathematics outside the classroom.

References

Hankins, T. (1980). *William Rowan Hamilton.* Johns Hopkins University Press.

Ó Cairbre, F. (2000). William Rowan Hamilton (1805--1865), Ireland's greatest mathematician. Riocht *na Midhe (Meath Archaeological and Historical Society)*, XI, 124-150.

Ó Cairbre, F. (2009). The importance of being beautiful in mathematics. *Irish Mathematics Teachers' Association Newsletter*, 109, 29-45.

Ó Cairbre, F. (2010). Twenty years of the Hamilton walk. *Irish Mathematical Society Bulletin*, 65, 33--49.

O'Donnell, S. (1983). *William Rowan Hamilton*. Boole Press Limited.