Subblocks interleaving PTS technique with minimum processing time for PAPR reduction in OFDM systems

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Abstract: This study presents subblocks interleaving partial transmit sequence (SBI-PTS) technique having low complexity for reducing the peak-to-average power ratio in orthogonal frequency division multiplexing systems. In this technique, a new subblocks interleaver is proposed, in which each subblock is interleaved with the others. Moreover, a new optimisation scheme is introduced, in which the number of iterations is made to be equal to the number of subblocks only which results in reduced processing time and less computation that leads to reduced complexity. Simulation results demonstrate that the new technique can effectively reduce the complexity by up to 99.95% (for subblocks number $M = 16$, inverse fast Fourier transform size $N = 256$) compared with the conventional PTS and new existing PTS techniques and yield good bit error rate performance. The other salient feature of this scheme is that it does not require side information and thus it offers increased transmission efficiency.

1 Introduction

One of the major drawbacks of orthogonal frequency division multiplexing (OFDM) systems is its high peak-to-average power ratio (PAPR) in the time domain. The large PAPR causes the transmit power amplifier to enter the non-linear region, distorting the signal, and resulting in a significant increase in the bit error rate (BER) at the receiver. Clearly, it is important that the PAPR should be reduced to ensure efficient transmissions in OFDM systems. The main criterion when designing a PAPR reduction scheme is to have the PAPR reduced to its lowest possible value and maintain its performance at the same time. The two metrics to evaluate the system performance are low computational complexity measured by computational complexity reduction ratio (CCRR) and reduction of PAPR measured by complementary cumulative distribution function (CCDF) [1, 2]. Many techniques have been proposed to mitigate this problem, but all of them incur various costs in the form of transmission efficiency and/or average transmission power. Multiple signal representation methods such as partial transmit sequence (PTS) and selected mapping (SLM) have been among the most widely investigated techniques [1]. Of these techniques, PTS [3] is the most attractive because it offers good PAPR reduction performance, it is distortionless, it can be used with any modulation scheme, and it is a flexible approach with no restrictions on the number of subcarriers that can be used [1, 2]. However, there are certain drawbacks arising from this technique: namely, high computational complexity and reduced data rate.

In this paper, we propose subblocks interleaving PTS (SBI-PTS) technique that uses a new SBI approach which can be applied in the time domain [after inverse fast Fourier transform (IFFT)] or in the frequency domain (before IFFT). Additionally, we also propose a new optimisation scheme in which only M iterations and a single two-phase sequence need to be applied. The simulation results demonstrate that with the proposed technique the PAPR reduction is improved further while the computational complexity is reduced by up to 99.95% (for $M = 16$, $N = 256$). Moreover, no side information need to be sent, unlike in the conventional PTS (C-PTS), thus allowing for increased transmission efficiency.

2 Related works

Several techniques have been proposed over the past few decades to reduce PAPR, they are categorised into clipping and filtering, block coding, SLM, tone reservation, PTS, and many more [1, 4]. Clipping and filtering is the simplest method, wherein the signal is merely clipped at a specific power level to ensure that the PAPR is under the threshold value. However, this method suffers from band distortion and spectral spreading. With the block coding method, the PAPR can be reduced without inducing distortion, but this method is only applicable with short code words. In general, the most popular PAPR reduction techniques are PTS and SLM, both can reduce PAPR significantly without distorting the output. However, SLM is computationally more complex than PTS, thus limiting its implementation in OFDM having large number of carriers. While in PTS, even though the computational complexity increases exponentially with the number of subcarriers, it is easier to implement compared with the SLM. Therefore, this paper focuses on the simulation of this SBI-PTS approach to reduce PAPR.

In [5], a novel PTS approach was proposed, it repeatedly generates new candidates by moving the time-domain signal left (down) or right (up). The original signal is combined with the shifted signal, and the group of signals with the lowest PAPR is transmitted. The simulation result shows that this method reduces PAPR more effectively than C-PTS.

In [6], a PTS scheme using real-valued genetic algorithm was proposed. First, they determined the amount of PAPR, then they defined the cost function based on the examined PAPR. The lowest cost function was then selected as the phase factor for signal transmission. The simulation results suggest that this method can reduce PAPR significantly even though it is not highly suitable for practical implementation; this is because of its high computational complexity.

In [7], it was also claimed that PAPR can be lowered by separating the long frames of signal correlation data. These correlations can be split through a fixed set of interleaves that are deployed by a transmitter. $M-1$ (M is the number of subblocks) amounts of interleave are used to produce $M-1$ number of frames of input data, and the frame giving the smallest PAPR is transmitted. Moreover, the focus of the interleave that generates the lowest PAPR is transmitted as side information simultaneously.

3 System model

In OFDM systems, a data stream of rate R (in units of bits per second) is mapped to a phase shift keying or quadrature amplitude modulation (QAM) modulation scheme. A set of N mapped signals is converted into N parallel streams by means of a serial-to-parallel converter. These sets are referred to as the OFDM symbols. Afterwards, an IFFT of length N is applied to produce orthogonal data subcarriers. Then, all orthogonal subcarriers are transmitted simultaneously over a symbol interval T. A complex baseband OFDM signal $x(t)$ with N orthogonal subcarriers can be written as follows [1]

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \boldsymbol{X}_k e^{j2\pi k \Delta f t} \tag{1}$$

where $\Delta f = (1/T)$ is the subcarrier spacing and $\boldsymbol{X}_k$ is the kth frequency-domain signal.

PAPR is defined as the ratio between the maximum instantaneous power and the average power of the OFDM signals [3], that is

$$\text{PAPR}(x(t)) = \frac{\max_{0 \le t \le T} |x(t)|^2}{E[|x(t)|^2]} \tag{2}$$

where $E[.]$ is the expectation value operator.

We can also describe the characteristics of the above power in terms of their magnitudes by defining the crest factor (CF); $\text{CF} = \sqrt{\text{PAPR}}$. High peaks appear when N different mapped symbols phases in (1) are accumulated constructively [2].

4 C-PTS-OFDM technique

In the C-PTS technique as shown in Fig. 1, the incoming serial random data vectors at the transmitter are mapped into QAM symbols and then converted from serial-to-parallel streams

$$\boldsymbol{X} = [X_0, X_1, \ldots, X_{N-1}]^{\mathrm{T}} \tag{3}$$

Then, $\boldsymbol{X}$ is partitioned into M disjoint subblocks, which are represented by the vectors $\boldsymbol{X}_m$ $(1 \le m < M)$ of length V, where $N = MV$ for integers M and V. For $m = 1, \ldots, M$, let the matrix $\boldsymbol{D}$ be a zero-padded of X_m, which can be written as:

$$\boldsymbol{D} = \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1,M} \\ D_{21} & D_{22} & \cdots & D_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ D_{LN,1} & D_{LN,2} & \cdots & D_{LN,M} \end{bmatrix} \tag{4}$$

where L is the oversampling factor

$$\begin{aligned} \boldsymbol{D_l} &= [D_{11} = \boldsymbol{X}_1, \ldots, D_{N/M,1} = \boldsymbol{X}_{N/M} \quad 0, \ldots, 0]^{\mathrm{T}}, \\ \boldsymbol{D_m} &= [0, \ldots, 0\; D_{((((m-1)(N/M)+1), m))} \\ &= \boldsymbol{X}_{((m-1)(N/M)+1)}, \ldots, D_{((m.(N/M)), m)} \\ &= \boldsymbol{X}_{(m.(N/M))} \quad 0, \ldots, 0]^{\mathrm{T}} \end{aligned}$$

$$\boldsymbol{X} = \sum_{m=1}^{M} \boldsymbol{D}_m \tag{5}$$

Then, let matrix $\boldsymbol{R}$ be the zero-padded IFFT of $\boldsymbol{D}$, which can be written as

$$\boldsymbol{R} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1,M} \\ R_{21} & R_{22} & \cdots & R_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ R_{LN,1} & R_{LN,2} & \cdots & R_{LN,M} \end{bmatrix} \tag{6}$$

Next, the time-domain sequences can be combined to minimise the PAPR, this is done by applying the complex phase rotation factors $\boldsymbol{b} = [b_1, b_2, \ldots, b_M]^{\mathrm{T}}$. The resulting time-domain signal after combination can be written as

$$\boldsymbol{x}' = \boldsymbol{Rb} \tag{7}$$

where $\boldsymbol{x}' = [x'_1, x'_2, \ldots, x'_{LN}]$ is the block of optimised signal samples. Hence, the objective of PTS technique is to come out with an optimal phase factor for the subblock set that minimises the PAPR. The objective of the optimisation problem is to identify optimum phases $\hat{b}$ that satisfy

$$\{\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_M\} = \underbrace{\text{argmin}}_{\{b_1, b_2, \ldots, b_M\}} \left(\underbrace{\max}_{1 \le k < LN} \left| \sum_{m=1}^{M} b_m R_{k,m} \right| \right) \tag{8}$$

where $b_m \in \{\pm 1, \pm j\}$ and $(W = 4)$, where W is the number of phase weight factors. b_1 Can be set equal to 1 without loss of performance [1, 3]. Therefore, in the PTS technique, it is necessary to test W^{M-1} sets of distinct possible candidate vectors $\boldsymbol{b}$ to satisfy (8). Accordingly, the computational complexity of the PTS technique increases exponentially with M.

At the receiver, after the N-point FFT block, the frequency-domain sequence can be written as

$$\boldsymbol{D}' = \text{FFT}(\boldsymbol{x}') \tag{9}$$

Then, the vector $\boldsymbol{D}'$ is partitioned into M disjoint subblocks, which are represented by the vector $\boldsymbol{D}'_m$ $(1 \le m < M)$ of length V, where $N = MV$ for certain integers M and V. For $m = 1, \ldots, M$, let the matrix $\hat{\boldsymbol{D}}$ be the zero-padded version of $\boldsymbol{D}'_m$, which can be written as

$$\hat{\boldsymbol{D}} = \begin{bmatrix} \hat{D}_{11} & \hat{D}_{12} & \cdots & \hat{D}_{1,M} \\ \hat{D}_{21} & \hat{D}_{22} & \cdots & \hat{D}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{D}_{N,1} & \hat{D}_{N,2} & \cdots & \hat{D}_{N,M} \end{bmatrix} \tag{10}$$

So, using the inverse phase rotation vector $\boldsymbol{b}'$, we can recover the signal as follows:

$$\hat{\boldsymbol{X}} = \hat{\boldsymbol{D}} b' \tag{11}$$

As noted, in the PTS technique, only the phase information is changed. Accordingly, no out-of-band radiation occurs.

5 Analysis of the proposed technique

In this paper, a new SBI scheme for PTS-OFDM is proposed. As explained in Algorithm 1 shown in Fig. 3, the subblocks interleaver used in this technique is described next. As shown in Fig. 2, the subblocks interleaver can be applied in the frequency domain (before IFFT) or in the time domain (after IFFT). In other words, the input of the subblocks interleaver can be the matrix $\boldsymbol{D}^{\mathrm{T}}$ in (4) or the matrix $\boldsymbol{R}^{\mathrm{T}}$ in (6).

5.1 SBI-PTS technique

The use of subblocks interleaver offers a more constructive approach than that of the conventional interleaving technique.

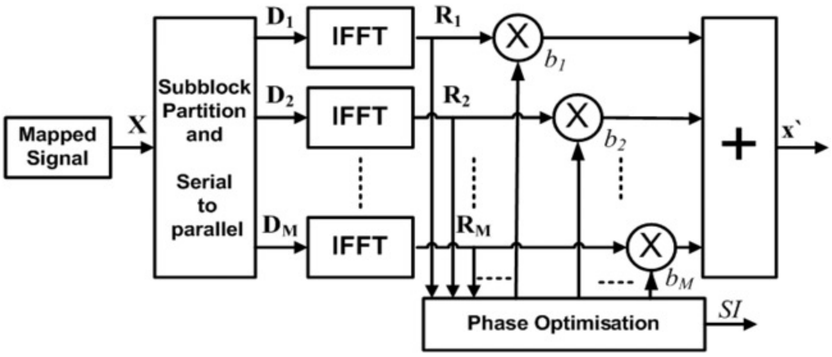

Fig. 1 *Block diagram of the C-PTS technique*

In the conventional interleaver, the interleaving process works on a single subblock of N symbols and record them or permute them serially [8], but in subblocks interleaver, the interleaving process operates on M subblocks comprising of $M \times N$ symbols and record them or permute them in parallel as will be described next. The subblocks interleaver technique in SBI-PTS is adopted mainly to reduce PAPR by limiting the probability that two peaks are combined which would increase the output envelope abruptly. In SBI-PTS, the data interleaving process combines some high peaks with low peaks which will neutralise each other and hence reducing the probability occurrence of high PAPR and at the same time decrease BER [7, 9, 10]. We begin with a matrix of $(M \times N)$ symbols/samples and write them column-wise into an $(N/B) \times (MB)$ matrix, where $B = N/(M/2)$ to ensure that only non-adjacent symbols occur. Then, we transpose the resulting matrix and read out the symbols/samples column-wise into an $(M \times N)$ matrix. Equations (13) and (14) present the matrices $\boldsymbol{n}$ for writing and $\boldsymbol{\pi}(\boldsymbol{n})$ for reading. Consider, as an example, $N = 8$, $M = 4$, $L = 1$, and $B = 4$. We write the matrix $\boldsymbol{D}^{\mathrm{T}}$ or $\boldsymbol{R}^{\mathrm{T}}$ as a (4×8) matrix as follows

$$\begin{bmatrix} dr_{11} & dr_{12} & dr_{13} & \dots & dr_{17} & dr_{18} \\ dr_{21} & dr_{22} & dr_{23} & \dots & dr_{27} & dr_{28} \\ dr_{31} & dr_{32} & dr_{33} & \dots & dr_{37} & dr_{38} \\ dr_{41} & dr_{42} & dr_{43} & \dots & dr_{47} & dr_{48} \end{bmatrix}$$

Then, we write the above matrix column-wise into a (2×16) matrix as follows

$$\boldsymbol{n} = \begin{bmatrix} dr_{11} & dr_{31} & dr_{12} & dr_{32} & \dots & dr_{17} & dr_{37} & dr_{18} & dr_{38} \\ dr_{21} & dr_{41} & dr_{22} & dr_{42} & \dots & dr_{27} & dr_{47} & dr_{28} & dr_{48} \end{bmatrix} \tag{12}$$

We then transpose the resulting matrix into a (16×2) matrix as follows

$$\boldsymbol{n1} = \begin{bmatrix} dr_{11} & dr_{21} \\ dr_{31} & dr_{41} \\ dr_{12} & dr_{22} \\ \vdots & \vdots \\ dr_{36} & dr_{46} \\ dr_{17} & dr_{27} \\ dr_{37} & dr_{47} \\ dr_{18} & dr_{28} \\ dr_{38} & dr_{48} \end{bmatrix} \tag{13}$$

Next, we transpose the resulting matrix and read out its contents column-wise into a (4×8) matrix as follows

$$\boldsymbol{\pi}(\boldsymbol{n1}) = \begin{bmatrix} dr_{11} & dr_{13} & dr_{15} & \dots & dr_{25} & dr_{27} \\ dr_{31} & dr_{33} & dr_{35} & \dots & dr_{45} & dr_{47} \\ dr_{12} & dr_{14} & dr_{16} & \dots & dr_{26} & dr_{28} \\ dr_{32} & dr_{34} & dr_{36} & \dots & dr_{46} & dr_{48} \end{bmatrix} \tag{14}$$

If the subblocks interleaver is applied in the frequency domain, the matrix $\boldsymbol{\pi}(n1)$ is the input to the IFFT blocks. If the subblocks interleaver is applied in the time domain, the matrix $\boldsymbol{\pi}(n1)$ is the input to optimisation process.

Next, a new phase optimisation scheme that obviates multiplicative operations is applied. Only a two phase sequences, where the possible phases are $\{0,\ 1\}$ are required. First, all phase sequence possibilities are generated using an encoder G of size $2^M \times M$. This encoder generates all possible phase sequences for a total of 2^M phase sequences. For example, if $M = 3$, the size of the encoder is 8×3, as shown in Table 1.

Then, the phase of each subblock is converted in accordance with the proposed weight of phase rotation as follows:

$$\boldsymbol{x} = \sum_{m=1}^{M} (-1)^{b_m} x_m \tag{15}$$

where $b_m \in \{0,\ 1\}$ and $\{x_m,\ m = 1,\ 2,\ \dots,\ M\}$ are the input elements of the optimisation block. As shown in Fig. 2, the comparator detects whether the phase factor is 0 or 1. If the weight of the phase factor is 0, the phase of the elements of the subblock does not change, and they are passed directly to the summation unit; if the weight of the phase factor is 1, the phase is rotated by passing it through the inverter and later by passing to the summation unit. As the first step, the PAPR of the combined signal is computed. We then check the b_1 of the phase sequence of Table 1; if the PAPR at {000} is lower than the PAPR at {100}, then all phase sequence possibilities with $b_1 = 1$ will be neglected (i.e. $\{b_1\ b_2\ b_3\}$ ={100, 101, 110, 111}). Hence, half of the phase sequence in Table 1 is eliminated. Next, we check the b_2 of the remaining phase sequences (i.e. $\{b_1\, b_2\, b_3\}$ ={000, 001, 010, 011}); if the PAPR at {000} is lower than the PAPR at {010}, then all phase sequences with $b_2 = 1$ will be neglected (i.e. $\{b_1\, b_2\, b_3\}$ ={010, 011}). Hence, half of the remaining sequences are eliminated. Then, one of the remaining two sequences (i.e. $\{b_1\ b_2\ b_3\}$ ={000, 001}) will be the optimal sequence giving minimum PAPR. Finally, the optimal sequence will then be converted to its index, as shown in Table 1. For example, if the optimal sequence is{000}, it will be converted to its index= 1. Then, we minimise the sample numbered 1 among N-IFFT samples with known factor (by dividing it by M) to give the minimum power among the first 2^M samples of the OFDM symbols.

At the receiver side, as explained in Algorithm 2 shown in Fig. 3, there is an encoder that is similar to that at the transmitter. The first 2^M samples of the OFDM symbol are tested to determine the minimum sample power among them, identify its index, and

Algorithm 1

Input: M, N

Output: $\tilde{\mathbf{x}}$, Index

Initialisation :

$\mathbf{X} = [\mathbf{X_0}, \mathbf{X_1}, \ldots, \mathbf{X_{N-1}}]^\top$

$\mathbf{X} = \sum_{\mathbf{m=1}}^{\mathbf{M}} \mathbf{D_m}$

$\mathbf{R} = IFFT(\mathbf{D})$

Subblocks Interleaver Process

write $\mathbf{R}^\top$ column-wise into an $(N/B) \times (MB))$ matrix, where $B = N/(M/2)$

transpose the resulting matrix and read out the symbols column-wise into an $(M \times N)$ **x** matrix

Optimisation Process

G= all phase sequence possibilities are generated using an encoder of size $2^M \times M$.

$\mathbf{b} = zeros(1, M)$

for $m = 1$ to M **do**

  $b_m = 0$

  $\mathbf{PF} = ((-1).^{\mathbf{b}})^\top$

  $\tilde{\mathbf{x}} = (\mathbf{x}^\top * \mathbf{PF})^\top$

  $temp$ =Calculate PAPR($\tilde{\mathbf{x}}$)

  $b_m = 1$

  $\mathbf{PF} = ((-1).^{\mathbf{b}})^\top$

  $\tilde{\mathbf{x}} = (\mathbf{x}^\top * \mathbf{PF})^\top$

  $temp1$ =Calculate PAPR($\tilde{\mathbf{x}}$)

  if $(temp < temp1)$ **then**

    $b_m = 0$

  end if

end for

$\tilde{\mathbf{x}} = (\mathbf{x}^\top * ((-1).^{\mathbf{b}})^\top)^\top$;

G(i)=b

Index=i

$\tilde{\mathbf{x}}(i) = \tilde{\mathbf{x}}(i)/M$

Algorithm 2

Input: $\mathbf{y} = \tilde{\mathbf{x}}$

Output: **b**

Initialisation :

G= all phase sequence possibilities are generated using an encoder of size $2^M \times M$.

min=**y**(1)

index=1

for $i = 2$ to 2^M **do**

  if $(\mathbf{y}(i) <$ min) **then**

    min=$\mathbf{y}(i)$

    index=i

  end if

end for

Index=index

b=**G**(Index)

$\mathbf{y}(Index) = \mathbf{y}(Index) * M$

Fig. 3 *Algorithms of the proposed SBI-PTS technique and side information detection*

insert this index into the encoder to generate the phase sequence. For example, if the index of the sample giving the minimum power is 1, then the input of encoder number 1 will be ON, and its output will be {000}. Thus, the SBI-PTS technique does not require the sending of side information. This detection process adds complexity at the receiver side as it increases with M. This detection process works only when $2^M \leq N$.

At the receiver, we discuss only the situation in which the interleaving of the subblocks is performed in frequency domain because this procedure is less complex than the corresponding procedure in time domain. The received baseband OFDM symbol such that

$$\boldsymbol{y} = \boldsymbol{x} \tag{16}$$

$\boldsymbol{y}$ is passed to the FFT block to produce the N-point FFT output such that

$$\hat{\boldsymbol{X}} = [\hat{X}_0, \hat{X}_1, \ldots, \hat{X}_{N-1}]^\mathrm{T} \tag{17}$$

Now, the deinterleaver takes the $(1 \times N)$ vector $\hat{\boldsymbol{X}}^\mathrm{T}$ and writes its elements column-wise into a $((B) \times (N/B))$ matrix. Then, the resulting matrix is transposed and its contents read out column-wise into a $(1 \times N)$ matrix. The output of the deinterleaver is then divided into M partitions; for example, if the output of deinterleaver is given as $\hat{\boldsymbol{Y}}$ and $M = 3$, then

$$\begin{aligned} \hat{\boldsymbol{Y}}_1 &= Y_{1,\ldots,k}, \\ \hat{\boldsymbol{Y}}_2 &= Y_{k+1,\ldots,2k}, \\ \hat{\boldsymbol{Y}}_3 &= Y_{2k+1,\ldots,3k} \end{aligned} \tag{18}$$

where $k = N/3$. Afterwards, if the detected phase sequence is $\{b'_1, b'_2, b'_3\}$, then the signal can be expressed as

$$\begin{aligned} \hat{\boldsymbol{X}}_{1,\ldots,k} &= \hat{\boldsymbol{Y}}_1 \times \hat{b}_1, \\ \hat{\boldsymbol{X}}_{k+1,\ldots,2k} &= \hat{\boldsymbol{Y}}_2 \times \hat{b}_2, \\ \hat{\boldsymbol{X}}_{2k+1,\ldots,3k} &= \hat{\boldsymbol{Y}}_3 \times \hat{b}_3 \end{aligned} \tag{19}$$

where $\hat{b}_m = (-1)^{b'_m}$.

Then, the reconstructed signal $\hat{\boldsymbol{X}}$ is

$$\hat{\boldsymbol{X}} = [\hat{X}_{1,\ldots,k}, \hat{X}_{k+1\ldots,2k}, \hat{X}_{2k+1\ldots,3k}]^\mathrm{T} \tag{20}$$

5.2 *Complexity analysis of SBI-PTS technique*

Despite its superior PAPR performance, PTS techniques are unsuitable and expensive for real hardware implementation. This is because of the necessity to do comprehensive search to identify the optimal phase factors and also because of the need to perform computation and comparison of the PAPR for W^{M-1} candidate phase sequences, where W is the number of phase weight factors.

In the C-PTS technique, the total complexity at oversampling factor of $L = 1$ can be given by [2]

$$T_{\text{C-PTS}} = (3MN/2)\log_2 N + 2MNW^{M-1} \tag{21}$$

As shown in (21), the most significant factors that contribute to the complexity of the C-PTS technique are the M-IFFT blocks, the N-point IFFT, and the calculation and comparison of W^{M-1} different PAPRs. In our SBI-PTS technique, there are only two-phase weight factors, {0, 1} and the calculation and comparison of PAPRs is performed only among M candidate phase sequences. These adaptations reduce the total complexity given as follows

$$T_{\text{SBI-PTS}} = (3MN/2)\log_2 N + 2M^2N \tag{22}$$

The first term of (22) shows that the complexity of the IFFT itself does not change. However, the complexity of the search algorithm, represented by the second term of (22) is significantly reduced.

The computational complexity of a PAPR reduction scheme is based on the number of iterations needed to complete the PAPR reduction process. Low computational complexity is important in such a way that the lower the computational complexity, shorter the time taken to perform PAPR reduction, and lower amount of hardware resources used to carry out design in hardware implementation environment; therefore, the system cost will be lower as well. Computational complexity of each reduction scheme can be calculated by adopting the following formulas:

In C-PTS

Complex addition = $N(M-1)U$; where U is the number of iterations; $U = W^{M-1}$.

In SBI-PTS

Complex addition = $N(M-1)U$; where $U = M$.

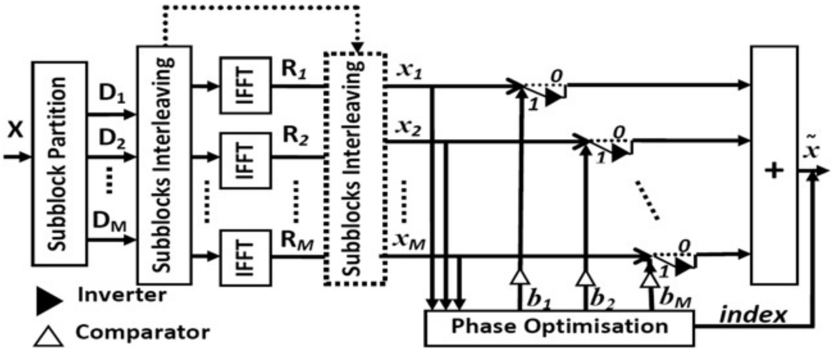

Fig. 2 *Block diagram of the SBI-PTS technique*

A measure of the complexity reduction of SBI-PTS against the C-PTS, called CCRR, can be defined as

$$\text{CCRR} = \left(1 - \frac{\text{complexity of SBI\_PTS tech.}}{\text{complexity of C\_PTS tech.}}\right) \times 100 \quad (23)$$

Table 2 presents a comparison of the CCRRs of the C-PTS technique [3], the optimal search [11], and our proposed technique (SBI-PTS) for $M = 16$, $L = 4$, $W = 2$, and $N = 256$. Since the complexity associated with the number of complex multiplication and addition operations is dependent on the number of iterations, the number of iterations is listed and considered in Table 2. This table illustrates that compared with the CCRR of the C-PTS technique, the optimal search gives a CCRR of 50%, whereas our proposed technique (SBI-PTS) achieves a CCRR of 99.95%. Clearly, our proposed technique offers the lowest computational complexity among the available low-complexity PTS techniques.

A measure of the complexity reduction of SBI-PTS against the new existing PTS can be defined as

$$\text{CCRR} = \left(1 - \frac{\text{complexity of SBI\_PTS tech.}}{\text{complexity of New\_PTS tech.}}\right) \times 100 \quad (24)$$

On the basis of (24), Table 3 is presented to provide a comparison of CCRRs of the parallel tabu search algorithm (parallel TS-PTS) scheme described in [12], artificial bee colony algorithm (ABC-PTS) scheme described in [13], and successive local search using sequences (SLS) scheme described in [14] with our proposed SBI-PTS technique when $M = 16$, $L = 4$, $W = 2$, $N = 256$, $T1 = 900$, $T2 = 900$, and $T3 = P_0 + (W-1)\sum_{m=1}^{M-1} P_m = 138$. The number of iterations is considered in Table 3 since the complexity associated with the number of complex multiplication and complex addition is dependent on the number of iterations. This table shows that SBI-PTS achieves a CCRR of 98.22% compared against the CCRR of the parallel TS-PTS; 98.22% compared against the ABC-PTS scheme and 88.40% compared against the SLS scheme. Clearly, our SBI-PTS scheme achieves the lowest computational complexity among all the compared low-complexity PTS schemes.

Table 1 Candidate phase sequences using an 8 × 3 encoder

Index	1	2	3	4
$\{b_1\ b_2\ b_3\}$	{0 0 0}	{0 0 1}	{0 1 0}	{0 1 1}
index	5	6	7	8
$\{b_1\ b_2\ b_3\}$	{1 0 0}	{1 0 1}	{1 1 0}	{1 1 1}

6 Numerical results

To evaluate the performance of the SBI-PTS technique and compare it with that of C-PTS and the original OFDM, simulations have been performed using MATLAB. We employed 16-QAM modulation with various IFFT lengths of $N = \{128, 256, 512, 1024\}$, an oversampling factor of $L = 4$, and subblocks $M = 4$.

The SBI is employed before and after IFFT and it gives the same PAPR performance. To obtain CCDF, 10^5 random OFDM symbols were generated. The CCDFs of the SBI-PTS, C-PTS, and the original OFDM for various numbers of subcarriers $N = \{128, 256, 512, 1024\}$ are presented in Fig. 4. The performance of PAPR reduction for 16-QAM modulation scheme can be evaluated in Fig. 4 for different lengths of IFFT. The range of PAPR reduction achieved by adopting SBI-PTS is 2.7–3.2 dB compared with original OFDM. The results also showed that shorter IFFT lengths will achieve higher PAPR reduction compared with longer IFFT lengths where in this case, for $N = 128$ the reduction is 3.2 dB while for $N = 1024$ IFFT length the reduction is only 2.7 dB. Next, a comparison of PAPR reduction performance between C-PTS and SBI-PTS was carried out to determine the extent of SBI-PTS PAPR reduction performance on an OFDM signal.

The result is shown in Fig. 4, from this figure, it is evident that the PAPR reduction of SBI-PTS technique is slightly more superior compared with that of C-PTS technique. The range of PAPR reduction achievement is from 0.18 to 0.75 dB. Among various IFFT lengths adopted, 128 IFFT lengths achieved the highest improvement which is as much as 0.75 dB. Thus, it is clear that the proposed technique yields 2.7−3.2dB reduction in PAPR with respect to the original OFDM transmission with only four iterations at a CCDF of 10^{-4}.

Fig. 5, from this figure, it is evident that the PAPR reduction of SBI-PTS technique is slightly more superior compared with that of interleaved C-PTS technique. The range of PAPR reduction achievement is from 0.41 to 0.52 dB. Among various IFFT lengths adopted, 128 IFFT lengths achieved the highest improvement which is as much as 0.52 dB. Thus, it is clear that the proposed technique yields 0.55−0.75 dB reduction in PAPR with respect to the C-PTS technique with only four iterations at a CCDF of 10^{-4}.

Table 2 CCRRs of the SBI-PTS technique compared with other PTS techniques

PTS schemes	Iterations	Complex addition	$M = 16$, $N = 256$	CCRR, %
C-PTS	$U = W^{M-1}$	$N(M-1)U$	125,829,120	0
optimal search	$U = (W^{M-1})/2$	$N(M-1)U/2$	62,914,560	50
SBI-PTS	$U = M$	$N(M-1)U$	61,440	99.95

Table 3 CCRRs of the proposed technique (SBI-PTS) compared with new existing PTS techniques

PTS schemes	Iterations	Complex addition	$M = 16$, $N = 256$	CCRR, %
parallel TS-PTS	$T1$	$N(M-1)T1$	3,456,000	0
SBI-PTS	M	$N(M-1)M$	61,440	98.22
ABC-PTS	$T2$	$N(M-1)T2$	3,456,000	0
SBI-PTS	M	$N(M-1)M$	61,440	98.22
SLS-PTS	$T3$	$N(M-1)T3$	529,920	0
SBI-PTS	M	$N(M-1)M$	61,440	88.40

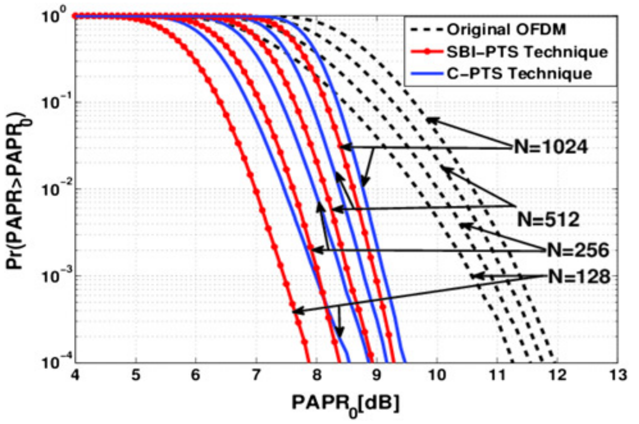

Fig. 4 *CCDFs of the PAPRs of the SBI-PTS technique compared with the C-PTS and original OFDM for 16-QAM, M = 4*

Fig. 6 shows that the PAPR performance of SBI-PTS technique is the same whether the SBI is performed before or after IFFT.

Fig. 7 shows comparison in PAPR reduction performance between the SBI-PTS, C-PTS, optimal search, SLS, parallel TS-PTS, ABC-PTS, and TS-PTS for $M = 16$, $W = 2$, $N = 256$, and 16-QAM modulations. As shown in this figure, the PAPR of the SBI-PTS at CCDF $= 10^{-3}$ is 6.4 dB. Meanwhile, the PAPRs of C-PTS, optimal search, SLS, parallel TS-PTS, ABC-PTS, and TS-PTS at CCDF $= 10^{-3}$ are 6.73, 6.73, 6.84, 6.91, 7.02, and 7.04 dB, respectively. Compared with C-PTS and new existing PTS techniques, the proposed (SBI-PTS) technique shows better PAPR reduction performance.

Tables 2 and 3 present CCRR of SBI-PTS compared with the C-PTS technique, the optimal search and the new existing PTS (parallel TS-PTS, ABC-PTS, and SLS) schemes. Clearly, our

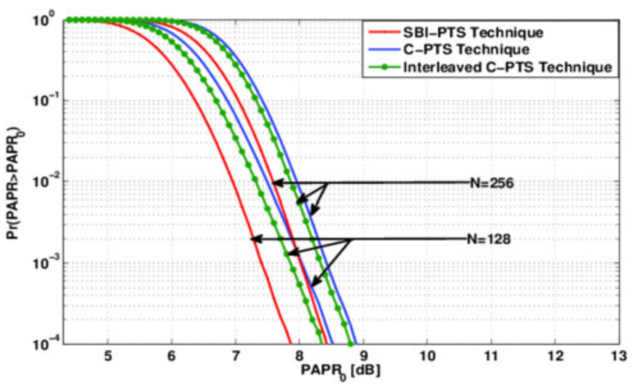

Fig. 5 *CCDFs of the PAPRs of the SBI-PTS technique compared with the C-PTS and interleaved C-PTS for 16-QAM, M = 4*

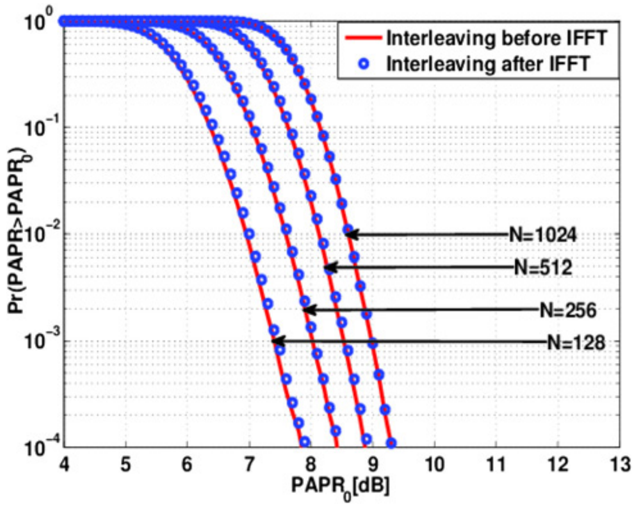

Fig. 6 *CCDFs of the PAPRs of the SBI-PTS technique with interleaving after and before IFFT for 16-QAM, M = 4*

SBI-PTS technique shows the lowest computational complexity among all the low-complexity PTS schemes evaluated here.

The analytical BER expressions for M-ary QAM signalling in additive white Gaussian noise (AWGN) and multipath Rayleigh fading channel are, respectively, given as [15]

$$P_e = \frac{2(M-1)}{M \log_2 M} Q\left(\sqrt{\frac{6E_b}{N_o} \cdot \frac{\log_2 M}{M^2-1}}\right) \quad (25)$$

$$P_e = \frac{M-1}{M \log_2 M}\left(1 - \sqrt{\frac{3\gamma \log_2 M/(M^2-1)}{3\gamma \log_2 M/(M^2-1)+1}}\right) \quad (26)$$

where γ and M denote $E_b/N_\circ$ and the modulation order, respectively, while $Q(\cdot)$ is the standard Q-function defined as

$$Q(\cdot) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2}\, dx \quad (27)$$

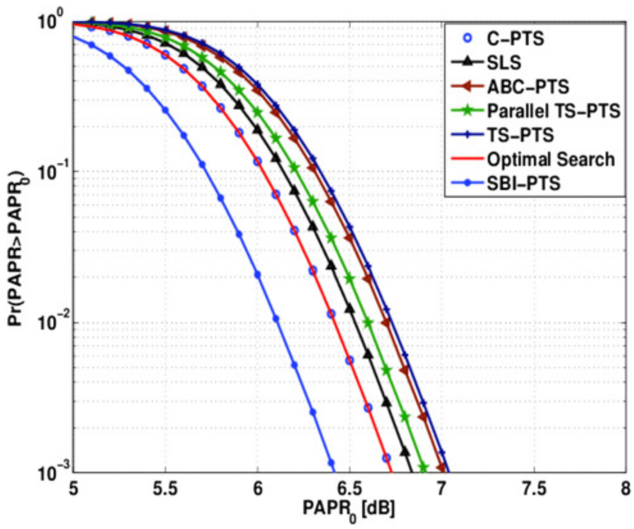

Fig. 7 *CCDFs of the PAPRs of the SBI-PTS technique compared with the C-PTS and new existing PTS techniques for 16-QAM, M = 16*

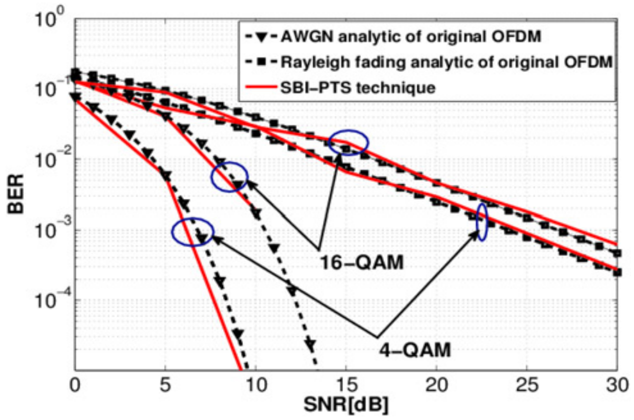

Fig. 8 *BER performance for the OFDM system and the SBI-PTS technique with 4-QAM, 16-QAM, and N = 256*

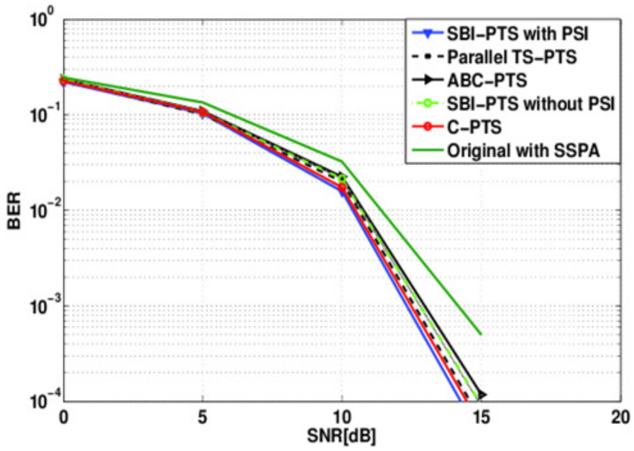

Fig. 9 *Comparison of BER performance of the SBI-PTS technique, C-PTS, and new existing PTS techniques with 4-QAM and N = 25*

We employed 4-QAM and 16-QAM signalling with an IFFT length of $N = 256$ to evaluate the BER performance under AWGN and multipath Rayleigh fading channel (with a maximum delay of 15 samples).

We employed the SBI in frequency domain (before IFFT) because it is less complex than in time domain at the receiver side. The results are presented in Fig. 8; here, the BER performance bounds are obtained by ignoring the effect of the high power amplifier (HPA) and directly transmitting the OFDM signals through AWGN and Rayleigh fading channels and using perfect side information (PSI). From this figure, it is clear that the BER performance in the AWGN channel and the Rayleigh fading channel of SBI-PTS technique is consistent with the analytical results.

Fig. 9 shows the BER performance with employing the Rapp's solid state power amplifier HPA model with the non-linearity parameter to be 2. This figure shows a comparison of BER performance of the SBI-PTS with and without PSI, C-PTS, parallel TS-PTS, and ABC-PTS for $M = 4$, $W = 2$, $N = 256$, and 4-QAM modulation over AWGN channel. Compared with C-PTS and new existing PTS techniques, the proposed (SBI-PTS) technique with PSI shows lower than C-PTS and other PTS techniques.

7 Conclusion

In this paper, a new PTS technique has been described. It uses a new SBI and optimisation scheme in which only a single two-phase sequence and M iterations are required. Both SBI and optimisation schemes are applied to reduce the computational complexity associated with weighting factors and also to reduce the PAPR and BER. In this manner, a minimal PAPR can be obtained with the need for M iterations, which conserves processing time and demands fewer computational resources, thus leading to lower complexity. Above all, this technique does not require side information, and therefore offers increased transmission efficiency. Hence, compared with other PTS techniques, SBI-PTS has been shown to be less complex and less resource consuming, while offering superior PAPR reduction performance.

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