A New Approach for Analysing Income Convergence across Countries.[a]

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Abstract

In this paper we develop a coherent framework that integrates both traditional measures of β-convergence and σ-convergence within a study of cross-country income dynamics. To do this we exploit the close links that exist between studies of income mobility and studies analysing the progressivity of the tax system. We also develop a welfare interpretation for the concept of β-convergence, which distinguishes it from the more general form of σ-convergence and which also suggests that the β-process may be worthy of independent study. We illustrate our approach using data for the period 1960-2000.

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1. Introduction

The degree to which income or productivity levels have converged across countries over time has been the subject of extensive research. The initial studies tended to be descriptive in nature, highlighting the key trends in inequality over time (Abramovitz (1986), Baumol (1986)). However in recent years this research has become more closely connected with research on the theory of economic growth. Two theories have come to dominate the literature on economic growth. The traditional Solow growth model (Solow (1956)) predicts that countries that are furthest away from their steady states will tend to grow more quickly than countries closer to their steady state. For countries with the same steady state, this implies that incomes will converge along the transition path to the steady state. In contrast endogenous growth models (Romer (1986)) can generate patterns of growth that do not exhibit any tendency towards convergence.[1] It was suggested that the presence or otherwise of convergence across countries could form the basis of a test of the neo-classical growth model versus more recent endogenous growth models. As a result, several papers have been written examining the nature of the convergence process across countries (Barro and Sala-I-Martin (1992), Mankiw, Weil and Romer (1992)). However, this subsequent literature has in turn generated a lot of controversy, debate and confusion regarding how to measure and interpret income convergence in general.

The dominant approach in the early literature is characterised by the work of Barro and Sala-I-Martin (1992). This involves regressing income growth rates on initial income in order to test whether poor countries grow faster than richer countries.[2] However, several authors (Friedman (1992) and Quah (1993)) have argued that although these regressions may detect mobility within a distribution they tell us little about convergence in the sense of a reduction in income dispersion across countries. It is possible to observe poor countries growing faster than rich countries and yet incomes diverging. For this to happen it must be the case that the initially

[1] The key distinction between these two models is the presence or otherwise of diminishing returns to capital. For a more detailed discussion of alternative growth models and their implications for the evolution of the international distribution of income see de la Fuente (1997).

[2] Essentially one considers a regression model of the form $\log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) = \alpha - \beta \log(y_{i,t}) + \varepsilon_{i,t}$. Values of $\beta>0$ are taken as evidence of convergence. In practice a non-linear version of this equation may be estimated but this makes little difference to the final results. It can be easily shown that β measures how rapidly an economy's output approaches its steady state.

poorer country overtakes/leapfrogs the richer country so that the rankings of both countries are reversed.[3,4] To distinguish between these different forms of convergence Sala-I-Martin (1996a)[5] coined the term *β-convergence* to capture situations where "*poor economies tend to grow faster than rich ones*". The term *σ-convergence* is then defined as a situation in which "*a group of countries are converging in the sense..that the dispersion of their real per capita GDP levels tend to decrease over time.*". While Friedman (1992) has argued that the real test of convergence should focus on the consistent diminution of variance among countries, Sala-I-Martin (1996a,b) argues that both concepts of convergence are interesting and should be analysed empirically.

In this paper we establish the close links that exist between existing measures of *β-convergence* and measures of tax progression used in the public finance literature. We exploit this relationship in order to develop a coherent framework for studying realised cross-country income dynamics that integrates existing measures of convergence. Our approach allows us to identify the relative contributions of *β-convergence* and *leapfrogging* to overall *σ-convergence.* It also allows for the possibility of incorporating varying degrees of inequality aversion into the measure of *σ-convergence.* We also develop a welfare interpretation for the concept of *β-convergence* that illustrates why the concept may be worthy of independent study. We illustrate our approach by examining income dynamics across countries from 1960-2000.

2.Decomposing Inequality

2.1 Measuring the Progressivity of Income Tax

In this sub-section of the paper we briefly outline the fundamental concepts and techniques used by economists studying the progressivity of the tax system. In the

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[3] Furthermore as noted by Friedman (1992) the presence of measurement error may bias the mobility estimates obtained from these regressions (see also O'Neill, Sweetman and Van de gaer (2002)) The approach we outline in this paper does not address the measurement error bias. However since our approach avoids the need for regression analysis it does provide a measure of *β-convergence* which is free of the potential biases involved in estimating dynamic data models. For a discussion of these biases in the context of Barro-Regressions see Lee, Pesaran and Smith (1997).

[4] Tamura (1992), Brezis et al (1993) and Sugimoto (2003) present examples of growth models in which leapfrogging/overtaking occurs. Tamura (1992) and Sugimoto (2003) emphasise the role of inequality within countries in generating differential growth paths, while Brezis et al (1993) focus on the disadvantage of leading countries in adopting new technologies.

[5] In that paper Sala-I-Martin dates the first use of this term to his Ph.D thesis in 1990.

next sub-section we illustrate how these techniques can be adapted to study cross-country income dynamics. In particular we develop a coherent framework for understanding the nature of income convergence across countries.

The starting point for much of the work on income redistribution is the concept of a Lorenz curve and the associated Gini-coefficient.[6] The Lorenz curve plays an important role in welfare economics[7] and is constructed by first ordering individuals by income, starting with the lowest. Once this is done, the cumulative proportion of total income received by the income units is plotted against the cumulative proportion of the population represented by these individuals. By definition the first ordinate on the curve corresponds to zero percent of the population which by definition must account for zero percent of total income. At the other extreme the final ordinate relates to 100% of the population which, as a group, must account for 100% of total income. The ordinates for the points in between these extremes are given by $(p,L_x(p))$ where :

$$p=F(x) \text{ and}$$

$$L_x(p) = \frac{1}{\mu_F} \int_0^x s dF(s)$$

μ_F is the mean of the income distribution whose cumulative distribution function is denoted by F. In situations where every member of the population receives the same income the Lorenz curve corresponds to the 45 degree line. In situations of extreme inequality, where one member of the population has all the income, the Lorenz curve will run along the horizontal axis before jumping up to the 45-degree line. In situations between these two extremes the Lorenz Curve will be a convex curve lying beneath the 45-degree line. The difference between the observed Lorenz curve and the 45-degree line is thus a plausible measure of inequality. This in turn provides the intuition behind the Gini coefficient which is equal to twice the area between the straight 45 degree line plot and the Lorenz curve L_x. Formally the Gini coefficient can be written as:

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[6] A more detailed discussion of what follows can be found in Lambert (1993). For an application of the use of Lorenz curves in studies of regional income convergence in the U.S see Bishop et al (1992, 1994).

$$G_x = 1 - 2\int_0^1 L_x(p)dp$$

A tax-system t(x) is said to be progressive if the average tax rate $\frac{t(x)}{x}$

is increasing with income (x). For our purposes it will be helpful to use an equivalent formulation of progressivity based on the redistributive power of the tax system. To do this we introduce the idea of a concentration curve, which we denote as $CC^x_z(p)$. The concentration curve for Z (with respect to X) plots the cumulative shares of Z against quantiles in the X-distribution. It is important to note that $CC^x_z(p)$ will differ from $L_z(q)$ in situations where the rankings of individuals based on X and Z differ and we will make use of this in our later analysis. Having defined what we mean by a concentration curve we can now state the Jakobsson-Fellman theorem which establishes the relationship between progressivity and redistribution.

Jakobsson-Fellman Theorem:

$$\frac{d[t(x)/x]}{dx} \geq 0 \text{ for all x} \Leftrightarrow CC_{X\text{-}T}{}^{x} \geq L_X \geq CC^{x}{}_{T} \text{ for every pre-tax income distribution } F(x).$$

For proof see Lambert page 150.

Intuitively this theorem states that the tax system is progressive if and only if the distribution of post-tax income (holding fixed rankings) is distributed more evenly than pre-tax income, which in turn is distributed more equally than tax liabilities. We will make use of this equivalence in developing our framework.

In the same way as we derived the Gini-coefficient from the Lorenz curve, we can also define an area measure of the extent to which the concentration curve deviates from the 45 degree line. This is known as the *Concentration coefficient* C^x_z. Formally the Concentration coefficient can be written as :

[7] Atkinson (1970) and Shorrocks (1983) derive a series of results that show that the ability to rank distributions in terms of welfare depends on the relative positions of the respective Lorenz curves. These results are summarised in Lambert (1983).

$$C_Z{}^X = 1 - 2\int_0^1 CC_Z{}^X(p)dp$$

Within this framework any change in inequality can now be viewed as a two step process. The first step measures the amount of redistribution of post-tax income from the position attainable after a distributionally neutral equal-yield taxation. It can be easily shown that this involves comparing the Lorenz Curve for pre-tax income with the concentration curve of post-tax income. Expressed in this way progressivity is viewed in terms of the relationship between pre and post tax income distributions, keeping fixed an individuals relative ranking. The second component compares the concentration curve of post-tax income with the Lorenz curve for post-tax income. As noted earlier these curves will differ (with the Lorenz curve exhibiting more inequality) in circumstances where the tax schedule results in a reranking of individuals over the two distributions.

Formally we can write this decomposition as :

$$\Delta G = G_X - G_{X-T} = (G_X - C^X{}_{X-T}) - (G_{X-T} - C^X{}_{X-T}) = DP + R$$

The first term measures the redistributive impact of progressivity using only the rankings from the initial distribution. This term is often referred to as the Reynolds-Smolensky index of vertical equity. The second term uses the final distribution of income and measures the increase in inequality due to reranking. In this way we can identify the relative importance of both these processes on the overall change in inequality, ΔG.

The above decomposition can be generalised to settings that utilise the generalised *S-Gini* coefficient ($G_x(v)$).[8] This coefficient allows for a parameter of inequality aversion, v, when calculating the summary measure of dispersion. The formal definition is :

$$G_x(v) = 1 - v(v-1)\int_0^1 (1-p)^{v-2} L_X(p)dp$$

where $1<v<\infty$.

[8] For a more detailed discussion see Lerman and Yitzhaki (1984)

Intuitively the S-Gini allows for different weights to be attached to different income ranges when integrating over the Lorenz curve. The regular Gini is obtained by setting v=2. When v is set less than 2 relatively more weight is given to incomes at the top of the distribution. At the extreme value of v=1, G(v)=0, so that irrespective of the distribution inequality will be coded as zero, a form of inequality neutrality. On the other hand a value of v>2 attaches relatively greater weight to differences at the bottom of the distribution. As $v \to \infty$ G(v) tends to $1-\frac{x_{\min}}{\mu}$ so that reductions in inequality is disproportionally driven by the agent with the lowest income – an extreme form of inequality aversion. Our earlier decomposition carries over to this extended measure and can be written as :

$$\Delta G(v)=G_X(v)-G_{X-T}(v)=(G_X(v)-C^X_{X-T}(v))-(G_{X-T}(v)-C^X_{X-T}(v))=DP(v)+R(v)$$

2.2: *Progressivity, Reranking, β-convergence and σ-convergence.*

In earlier research Benabou and Ok (2001) and Jenkins and Van Kerm (2002) adapt these concepts to study mobility within the distribution of individual incomes. In our paper we apply these techniques to a study of income convergence across countries. We show how the concepts developed above can be exploited so as to obtain a better understanding of income dynamics in the growth literature. To do this we simply let X from the previous section denote initial income, $Y=X-T$ denote final income and T represents a country's income losses or gains over this period. ΔG denotes the change in income dispersion over time and is therefore a direct measure of *σ-convergence*. The extension to the S-Gini allows us to examine the sensitivity of trends in *σ-convergence* to different specifications of inequality aversion. The Progressivity term, $DP(v)$, captures the extent to which income inequality is reduced over time as a result of higher growth rates among lower income countries. Measuring the progressivity of the tax system in terms of the redistributive impact of the tax system is completely analogous to examining the impact of variations in growth rates across countries on income inequality in the convergence setting. In particular, *β-convergence,* defined as situation where "*poor economies tend to grow faster than* *rich ones"* is nothing more than progressive income growth. Thus the first term in our decomposition measures the contribution of *β-convergence* towards the overall reduction in income dispersion. The second term in the decomposition measures the negative impact of positional mobility on income inequality. In the growth context this captures the notion of leapfrogging.

We can use Figures 1, 2 and 3 to illustrate our decomposition. Figure 1 illustrates a situation where both *β-convergence* and *σ-convergence* coexist without any leapfrogging/reranking. In our approach the *σ-convergence* would be captured by a fall in the Gini-coefficient. For this example all of this reduction would be attributed to the progressivity of income growth, so that $\Delta G=DP$. The absence of reranking would be reflected in a measure of $R=0$.

Figure 2, illustrates a situation where there is *β-convergence* but no *σ-convergence*. In this example inequality has not changed over time – so there has been no *σ-convergence* $(\Delta G=0)$. On the other hand there has been substantial *β-convergence* – the poor country has grown faster than the richer country. However this is masked in the overall inequality figure by the complete reranking of the two countries. Our approach will identify the redistributive contribution of *β-convergence* to inequality in these data but this will be entirely offset by the contribution of the leapfrogging component, so that $-DP=R>0$. Not only will our framework identify the tendency of poor countries to grow faster but it simultaneously quantifies the extent to which this is offset by reranking in the data.

Finally, Figure 3 illustrates another process for which there is no *σ-convergence*. However this case differs from that in Figure 2 in that this new process is static. Again $\Delta G=0$ but for this process our decomposition would result in $DP=R=0$. Our decomposition would identify this as a growth process without either *β-convergence* or *leapfrogging*.

These examples help clarify an important point. Sala-I-Martin (1996b) begins his paper by defining *β-convergence* in the traditional way by noting that "*there is β-convergence if poor economies tend to grow faster than rich ones"*. However later in the paper he suggests that *β-convergence* studies the mobility of income within the same distribution. As a result, some researchers (Boyle and McCarthy (1997)) have drawn parallels between *β-convergence* and measures of rank mobility, defining indices of rank concordance as direct measures of *β-convergence*. Clearly for a distribution to exhibit *β-convergence* without *σ-convergence* it must be the case that countries are changing ranks (Figure 2). However as Figure 1 shows it is possible to have *β-convergence* without any positional mobility. It is also possible to have rank mobility without *β-convergence*. The definition of *β-convergence* simply requires poor countries to grow faster than rich countries, *irrespective of whether or not there is leapfrogging*. Both a Barro-regression approach and our redistributive approach would indicate a strong role for *β-convergence* for the process illustrated in the example in Figure 1, measures based on rank correlations would not. While the issue of positional mobility is interesting, it is captured by our measure R, which in turn measures *reranking/leapfrogging* and not *progressivity*/*β-convergence*. In this example R would contribute nothing to the change in income inequality.

We believe that these examples illustrate the potential that our framework offers to provide a coherent approach that integrates the three important features of the growth process: *σ-convergence, β-convergence* and *leapfrogging.* The next section provides an empirical illustration of this approach. We apply our decomposition to data on cross-country income dynamics taken from the latest release of the Penn-World tables. We briefly discuss the data before applying our approach to data on a full sample of 98 countries and also a restricted sample of 25 OECD countries. We conclude with a discussion of the welfare implications of our analysis.

3: Data and Results

3.1 Data

In this section of the paper we analyse income convergence between 1960 and 2000, taken from the latest version of the Penn-World Tables.[9] The Penn World Table provides purchasing power parity and national income accounts converted to international prices for 168 countries for some or all of the years 1950-2000. In this paper we use data for a sample of 98 countries that provided complete data over the period 1960-2000. We also look at income dynamics for a restricted set of 25 OECD countries. Income is measured as real per-capita gross domestic product in 1996 international prices. These data have been used extensively in previous studies of

[9] Alan Heston, Robert Summers and Bettina Aten, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania (CICUP), October 2002.

convergence and are described in more detail in Summers and Heston (1991). The countries used in the analysis are presented in Tables 1 and 2.

Figures 4 and 5, provide a useful graphical summary of the evolution of income inequality over the period 1962-1998. Figure 4 summarises the data for the OECD sample, while Figure 5, provides the results for the full sample. We focus first on the results for the OECD countries.

For the purposes of constructing this graph we use a 5-year moving average of incomes. The income for 1962 is thus an average of that country's income from 1960-1964. Likewise 1998 income measure is an average of incomes from 1996-2000. Incomes are expressed relative to the overall mean for that year, so that values above 1 correspond to high-income countries and values below 1 represent low-income countries.

The North East (NE) and South West (SW) quadrants of Figure 4 are simple transformations of the empirical distribution functions of incomes in the two periods. This establishes a relationship between income and rank in the two years. The estimated line in the North West (NW) quadrants maps the relationship between incomes in these two marginal distributions. We can interpret the numbers on the horizontal axis as showing the income a country would have received in 1998 if they had maintained their 1962 ranking. The 45-degree line corresponds to a situation of constant relative incomes. The results in this quadrant show that in the absence of *leapfrogging* countries with below average income would have expected their income to rise faster than those with above average income over this period. These graphs therefore provide a simple graphical framework for describing the precise nature of the *sigma convergence* that occurred for the OECD countries over this time period.

The South East (SE) quadrant introduces the concept of *leapfrogging* to the analysis Deviations from the 45-degree line in this quadrant show the extent of re-ranking, with countries above the 45 degree having increased their rank over time and vice versa. The results show that almost every country changed rank over this period, with countries such as Ireland, Japan and Norway moving up the distribution, while countries such as Sweden, New Zealand and Britain moving down. To examine the consequences of this *leapfrogging* on inequality we return to the NW quadrant. The easiest way to visualise the impact of leapfrogging on relative incomes is to look at the nature and composition of income clusters in both distributions. Looking at the NW quadrant for 1962 (along the horizontal axis) we can identify approximately 3 clusters of countries: a low-income cluster, consisting of Korea, Turkey, Mexico, Greece, Portugal, Spain, Japan and Ireland, a high-income cluster involving Luxembourg, USA and Switzerland and a cluster of middle-income countries made up of the remaining OECD members. Switching axis to look at 1998, there still appears to be a low income, middle-income and high-income cluster. However our framework allows us to look at compositional changes within and between these groups. We notice that Switzerland has fallen out of the high-income group into the middle-group. New Zealand, which was initially at the upper end of the middle-income group, is now at the lower end of this group. The big movers out of the low-income group were Ireland and Japan who have now joined the middle-income countries.

Figure 5 provides the same information for the full sample of 98 countries used in the extended analysis. Although identifying individual countries becomes more difficult it is again apparent that although leapfrogging was widespread over this timeframe much of the re-ranking resulted in countries changing positions within groups, with relatively few countries changing groups. The growth in relative incomes for this full-sample tends to be concentrated in the middle of the distribution, with relative incomes at the very top of the distribution falling. In the next section we use the decomposition presented earlier to look at these changes more formally.

3.2 σ-convergence, β-convergence **and** *leapfrogging 1960-2000.*

The results of our analysis are presented in Tables 3-6. Tables 3 and 5, refer to the OECD sample while Tables 4 and 6 refer to the full sample. We begin with sample Table 3 which reports the Generalised Gini coefficient for the set of OECD countries at 20-year intervals over the period 1960-2000. We report the Gini for three values of the inequality aversion parameter equal to 1.5, 2, 2.5, respectively. The first places relatively more weight on incomes at the top of the distribution, the second corresponds to the regular Gini-coefficient, while the third gives relatively more weight to inequality at the low end of the distribution.

The overall trend is similar for all three coefficients and indicates a substantial reduction of income dispersion over the period 1960-2000. For all three parameters the majority of this reduction took place between 1960 and 1980, with a significant slowdown in convergence after this period.[10] However precisely how we interpret the trend in the last 20 years depends on the relative weight given to inequality at the top end of the distribution. When we weight heavily income differences at the top end of the distribution we find that inequality has increased substantially in the last 10 years. In fact according to this measure inequality in 2000 is higher than it was in 1980.[11] In contrast the measure which places more weight on inequality at the low end of the distribution, while showing a slowdown in convergence, continues to display a trend towards a more equal distribution of income. These contrasting results suggest that the slowdown in income convergence over this period is driven by a small number of the richest countries pulling away from the rest of the distribution, so that the share of the wealthiest countries has increased substantially. In contrast table 4 reiterates the well-known result that for the world as a whole over this period incomes diverged substantially, though as Figure 5 suggests much of this is driven by income increases for middle-income countries..

The results in Table 5 and 6 decompose these changes in income inequality using the framework outlined in section 2. This approach allows us to determine the redestributive impact of income growth for both samples. The results are provided for the traditional Gini coefficient. The rows of the tables refer to different time periods while the columns refer to various components of the convergence process. The first column shows the change in the Gini-Coefficient over the relevant time period and measures *σ-convergence* as discussed above. Columns 2 and 3, present the relative contribution of progressive income growth *(β-convergence)* and reranking (leapfrogging) towards these trends.

Focusing on the OECD sample the main feature that emerges over this period is that of the two prevailing forces the tendency towards *β-convergence* is the dominant force driving cross-country income dynamics.[12] For example in the early

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[10] This slowdown in convergence among developed countries was discussed in O'Neill (1996). The analysis in that paper suggests that the slowdown may be related to changes in the rate of human capital accumulation.

[11] This is not evident in Figure 4 which plots the evolution of income inequality over the entire 40 years. However if we repeat the analysis for the same sub-periods as reported in Table 1, the same trend of rising relative income at the very top of the distribution between 1978-88 and 1988-1998 becomes apparent. In the interests of brevity we have omitted the graphical summary for each of the sub-periods, though these are available from the authors upon request.

[12] Using measures of rank correlations Boyle and McCarthy (1997) also concluded that "positional mobility" was relatively unimportant over this time. However, our approach differs in two ways. Firstly we can determine precisely the contribution of this component to income dynamics and secondly we do not equate positional mobility with β-convergence.

period during which much of the decline in income inequality is observed leapfrogging contributed almost nothing to income dynamics. Furthermore, it is evident that the stable income distribution observed throughout the last 10 years reflects a static distribution with neither *leapfrogging* nor *β-convergence* contributing anything towards a change in income inequality. The last column presents our estimates of the traditional measure of *β-convergence* from growth regressions. The results are reported in such a way that a positive β indicates convergence. By and large the results using the traditional approach is consistent with our analysis. The early periods are characterised by significant *β-convergence*, which is absent in the later years. However, it is worthwhile making two observations at this stage. Firstly, we notice that in every period under consideration we observe both leapfrogging and values of $\beta<1$. Thus we need to be careful interpreting claims that values of $\beta<1$ rule out leapfrogging. This claim applies only to "deterministic" leapfrogging where poor economies are *systematically* predicted to get ahead of rich economies at future dates. A value of $\beta<1$ says nothing about positional mobility in general. An alternative way of seeing this is to note that it is possible for observed inequality to rise even if the estimated β-coefficient from a Barro-regression is less than 1. We will return to this issue when we discuss the welfare implications of our analysis.

Secondly, it is interesting to compare the full period from 1960-2000, with that from 1980-1990. For both these periods the estimated β coefficient from the growth equations are almost identical. However when you look at our decomposition in columns 2 and 3, we see that over for the two periods in question the dynamics underlying the income distribution differed substantially. For the overall period progressive income growth had a significant redistributive effect as a result of which income inequality declined substantially. In the later period however total income inequality did not change. Furthermore our decomposition shows that neither of the forces underlying convergence was particularly important over this period, so that effective *β-convergence* fell substantially over this period. Relying on the Barro-regression to identify *β-convergence* would miss these differences.

Table 5, shows the results for the world as a whole. In contrast to the OECD sample we find little evidence of *β-convergence*. For almost every period considered both the regressive nature of income growth and any leapfrogging which occurred resulted in greater income inequality. Again the results are for the most part consistent with the traditional Barro-regression, although the periods 1970-80 and 1980-1990 show different trends in effective *β-convergence*, while the estimated coefficient from a Barro-regression is equal in these two periods.

3.3 Conditional β-Convergence

Following Mankiw, Weil and Romer (1992) and Barro and Sala-I-Martin (1995) we can adapt our framework to deal with the distinction between *absolute β-convergence* and *conditional β-convergence*. The analysis presented in the above section focused on whether or not poor countries growing faster than richer countries. This concept has been labelled as *absolute β-convergence* in the literature. In keeping with earlier research our approach finds evidence of *absolute β-convergence* among the OECD but not among the full sample. It is important to realise that models such as the Solow growth model do not actually predict that poor countries grow faster than rich countries. Instead it predicts that countries that are furthest away from their steady states will grow faster than countries closer to the steady state. In the event that all countries share the same steady state this will manifest itself in poorer countries growing more quickly. However if we allow for countries to have different steady states these is no longer the case and we must modify our approach to consider *conditional β-convergence*. Conditional convergence examines the relationship between growth and initial income having controlled for differences in steady state incomes.

The traditional test for *conditional β-convergence* involves regressing growth on initial income holding constant a number of additional variables that determine steady state income. If the partial correlation between growth and income is negative then we say that the set of countries exhibit *conditional β-convergence*. Such a distinction may not be important among groups of countries that are relatively homogenous, such as those members of the OECD. However a number of researchers have sown that this distinction can be important when looking at a more heterogeneous set of countries. With this in mind we modify our approach so as to examine the relative importance of *conditional β-convergence* among the full set of countries considered in our study. To partial out differences in steady states across countries we run the following regression for each year in the sample:

$$Ln(RGDP_i)=\alpha+\delta_1\, ln(S_{Ki})+\, \delta_2 ln(n_i)+\varepsilon_i$$

where $RGDP_i$ is real GDP per capita for country i, S_{Ki} is the average share of real investment in real GDP for country i, and n_i is the average rate of growth of the working age population for country i.[13] The savings rate and population growth rate are key determinants of steady state income in exogenous growth models and using the residuals from the above regression as a measure of income should eliminate much of the heterogeneity arising from differences in steady state incomes.[14] Having obtained the residuals for each country and each year we rescale the annual residuals so as to have the same mean as the raw GDP series for that year.[15] These rescaled residuals are then used to examine income convergence using the framework outlined in section 2. The results are presented in Table 7. The contribution of the re-ranking component (column 4) does not differ much depending on whether we examine absolute or conditional convergence However, in keeping with earlier work when we condition on differences in steady states we now find evidence of *conditional β-convergence* using both the traditional Barro measure and also our measure which focuses directly on the redistributive impact of progressive income growth. Furthermore, although there is some evidence leapfrogging has caused inequality to rise, especially when we take a 40-year horizon, this component is again dominated by the inequality reducing effect of *β-convergence*.

3.4 β-convergence, Welfare and Average Rate Progressivity

In this section we discuss the potential welfare implications of our analysis. In doing so we will also look a little more closely at the relationship between our measure of *β-convergence* and that obtained from a Barro-Regression. In the absence of reranking any income growth (whether redistributive or not) is welfare improving

[13] The averages are taken over the full sample period 1960-2000.

[14] Mankiw, Weil and Romer (1992) also use the average savings and population growth rates to partial out differences in steady states in their analysis of conditional convergence.

[15] It is necessary to rescale the residuals before applying our framework since the raw residuals have a mean zero. As a result the Gini coefficient is not defined for this series. Formally the Gini coefficient can be written as $G=\dfrac{\sum_i\sum_j |y_i-y_j|}{N^2\mu}$. Rescaling the residuals as we have ensures that the differences estimated Gini for the raw and residualised GDP series only reflects differences in the average absolute deviations of both series. This seems a reasonable way to proceed though we should note that different

(relative to the initial distribution) according to any individualistic, symmetric, additively separable and inequality averse social welfare function (by definition $GLC(x+t(x)) > GLC(x)$ (see Lambert (1993) page 152 for a discussion in the context of taxes)).[16] However we can show further that having β-convergence (progressive income growth) is welfare improving, not only in relation to the initial income distribution, but also relative to an equal-yield proportional income growth counterfactual. The following theorem establishes this result.

Theorem: *For every individualistic, symmetric, additive separable and inequality averse social welfare function, progressive income growth (β-convergence) over the full range of incomes, without leapfrogging, increases social welfare more than an equal yield proportional growth rate applied to the same pre-growth income distribution.*

Proof:

If the growth rate is proportional then the Lorenz curve for final income (y) coincides with the Lorenz curve for initial income (x): $L_{prop}(p)=L_x(p)$ for all $p\in [0,1]$

By definition average final income is given by $\mu_{y=}\mu_x(1+t)$, where t is the overall average growth rate. Hence the Generalised Lorenz Curve for final income after progressive growth at rate t can be defined as : $GLC_y(p)=\mu_x(1+t)L_y(p)$.

From the Jakobbson-Fellman theorem (See Lambert page 150) and our assumption of no reranking we can conclude that :

$GLC_y(p)=\mu_x(1+t)L_y(p)\geq \mu_x(1+t)L_x(p)= \mu_x(1+t)L_{prop}(p)$ all $p\in [0,1]$.

The last equality follows from step 1 of the proof.

By definition this implies that : $GLC_y(p)\geq GLC_{prop}(p)$ all $p\in [0,1]$.

Referring to Shorrocks' theorem (Lambert page 59) completes the result.!

It is worth emphasising that the concept of *β-convergence* is the key convergence force underlying this theorem. A reduction in *σ-convergence* is not sufficient to generate this result. It is possible for inequality as measured by say the Gini coefficient or the coefficient of variation to fall and for there to be no reranking

rescaling constants will led to different values for the residualised Gini and also to different values for the estimated components of inequality.

[16] *GLC(x)* denotes the generalised Lorenz curve and is derived by multiplying the original Lorenz Curve by mean income.

and yet for the Generalised Lorenz curves to cross so that unambiguous welfare comparisons may not be possible. This reflects the fact that the Gini-coefficient can fall even when income growth is not progressive over the entire range of incomes.

Although our earlier results clearly highlight the redistributive effect of observed growth patterns for the OECD countries being studied, the analysis up to now has used an overall index of effective progressivity. In order to apply the above theorem we need to be able to measure income progression along the entire income scale. That is we need to switch from an index of effective progression to a measure of local progression. The progressivity index we have discussed so far is based on the difference between the initial Lorenz curve and the concentration curve for final income. As such it measures the redistributive effect of the income growth and is associated with *Residual Progression* measure of local progression (Lambert page 161). In a study of income convergence this seems an obvious approach to take.

An alternative way of measuring local progression which is easy to implement and which turns out to be important in linking the growth and tax literature is to focus directly on the behaviour of the average growth rate : $\frac{Y-X}{X}=\frac{t(X)}{X}$, where t(X) is the change in income over the two periods and *X* is initial income. An equivalent way to measure progression is to examine whether $\frac{d(\frac{t(x)}{x})}{dx}\leq 0$. This forms the basis for *Average Rate Progression.*[17] It is clear that a declining growth schedule, deviations from proportional growth and the redistributive effect of observed growth patterns are very closely connected. Proportional income growth implies a flat growth schedule and no change in inequality. With progressive growth on the other hand a disproportionate share of the benefits is received by low-income countries, the growth schedule is downward sloping and the redistributive effect of the change is positive.

To check whether or not $\frac{d(\frac{t(x)}{x})}{dx}\leq 0$ for *all values of X* we first sorted the data by income level and then plotted $\frac{t(X)}{X}$ against X for the period 1960-2000. The results are given in Figure 6. From this we can see that there are a number of observations

[17] In the same way as the tax yield from equiproportionate increases in initial income is larger the more progressive is the tax schedule (Lambert 1993 page 206) we can also show that the total income yield from am equiproportionate increase in initial income is larger the greater the degree of Average Rate Progression observed in the growth process.

that violate our progressivity condition (that is countries for which the growth rate in income was larger than the next lowest ranked country). These countries are represented by a hollow circle. This makes the application of theorem 1 difficult when applied to realised outcomes.

However, those familiar with the recent literature on growth will quickly recognise this way of presenting the results. It is nothing more than a plot of the data underlying the standard Barro-regression approach to measuring *β-convergence*.[18] The solid line on the graph denotes the OLS fit from a Barro-regression. In order to look at the contribution of *β-convergence* and leapfrogging to observed changes in income inequality we have defined progressivity in terms of realised outcomes. It would also be possible to use the framework developed above to examine the progressivity of the underlying *growth process*. This would simply involve comparing the distribution of initial incomes with the distribution of conditional *expected* incomes rather than actual incomes.[19] The same decompositions and theorems as outlined earlier would apply. The only difference being that the results would now be interpreted in terms of the *opportunities afforded by the process* rather than in terms of *realised outcomes*. Since we can view the estimated Barro-regression as providing the best fit of (X, $\frac{t(X)}{X}$) we can implement this approach using the *average growth schedule* as predicted by a Barro-regression. For the OECD countries that we have analysed the expected average tax schedule is downward sloping. This implies a globally progressive growth process which welfare dominates a equal yield proportional growth process. In this context the violations of "local" progression that we outlined in the outcomes-based approach may be interpreted as simply reflecting stochastic deviations from the *systematic* component of the growth process.[20] When viewed in this light the Barro-regression measure of convergence and the redistributive measure that we adopt in this paper differ only in the principles used to

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[18] To see this formally we note that an equivalent unit free measure of average rate progression can be derived as $ARP^* = \frac{d(\frac{t(x)}{x})}{dx}.x = \frac{d(\frac{t(x)}{x})}{d\ln(x)} \approx \frac{d\ln\left(\frac{Y}{X}\right)}{d\ln(x)}$. This latter term corresponds to the β-parameter from a standard Barro-regression.

[19] This is the approach adopted by Benabou and Ok (2001) in their study of individual income dynamics behaviour.

[20] Aronson, Johnson and Lambert (1994) extend the decomposition that we use in this study to formally allow for randomness in the tax/growth schedule. This enables them to identify the effect of horizontal

measure progressivity. The first focuses on the rate at which the average growth rate changes with initial income (*Average Rate Progression),* while the latter focuses on the elasticity of terminal income to initial income (*Residual Progression).*

Irrespective of whether one is interested in interpreting realised growth patterns and the subsequent changes in inequality or in analysing the underlying growth process, our analysis lends support to the view that both *β-convergence* and *σ-convergence* are important features of the growth process that should be studied together. Our paper provides an integrated framework for this analysis.

4. *Conclusion*

The results presented in this paper are consistent with earlier studies that found that the growth process among OECD countries in the last 40 years has resulted in a reduction in income inequality over this time period. While these results are in line with those presented in earlier research we believe that the approach adopted in this paper represents a significant development in the analysis of cross-country income dynamics. The techniques we use allow us to “marry” the approaches advocated by Friedman and Quah to study income dynamics, on the one hand, and those suggested by Barro and Sala-I-Martin on the other hand. In doing so we develop a coherent integrated framework involving concepts which up to now have often been viewed as competitors in the analysis of income dynamics. We do this by adapting earlier work analysing progressivity in the tax system and applying it to cross-country income dynamics. This allows us to separately examine the contribution of non-proportional income growth and reranking to changes in income inequality.

We also develop a welfare interpretation for the concept of β-convergence. We show that the typical Barro-Regression approach to identifying *β-convergence,* is equivalent to *Average Rate Progression* measures in the taxation literature, whereas our redistributive approach is based on *Residual Progression* measures. In developing the link between the tax and growth literature we feel we have provided an integrated framework for studying income dynamics, through which the connections between the various sources of convergence can be better understood and evaluated.

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inequity (unequal treatment of equals) on redistribution. Unfortunately the relatively small sample sizes available across countries prohibits the use of their decomposition in our setting.

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Table 1: Full Sample of 98 countries included in the analysis

Argentina	Costa Rica	India	Malawi	Sweden
Australia	Denmark	Ireland	Malaysia	Switzerland
Austria	Dominican Republic	Iran	Niger	Seychelles
Burundi	Algeria	Iceland	Nigeria	Syria
Belgium	Ecuador	Israel	Nicaragua	Chad
Benin	Egypt	Italy	Netherlands	Togo
Burkina Faso	Ethiopia	Jamaica	Norway	Thailand
Bangladesh	Finland	Jordan	Nepal	Trinidad and Tobago
Bolivia	France	Japan	New Zealand	Turkey
Brazil	Gabon	Kenya	Pakistan	Tanzania
Barbados	Ghana	Korea	Panama	United Kingdom
Canada	Guinea	Sri Lanka	Peru	Uganda
Chile	Gambia	Lesotho	Philippines	Uruguay
China	Guinea-Bissau	Luxembourg	Portugal	United States of America
Cote d’Ivoire	Equatorial Guinea	Morocco	Paraguay	Venezuela
Cameroon	Greece	Madagascar	Romania	South Africa
Congo, Republic of	Guatemala	Mexico	Rwanda	Zambia
Colombia	Hong Kong	Mali	Senegal	Zimbabwe
Comoros	Honduras	Mozambique	Spain	
Cape Verde	Indonesia	Mauritius	El Salvador	

Table 2: OECD Countries included in the analysis*

Australia	Finland	Italy	Netherlands	Sweden
Austria	France	Japan	Norway	Switzerland
Belgium	Greece	Korea	New Zealand	Turkey
Canada	Ireland	Luxembourg	Portugal	United Kingdom
Denmark	Iceland	Mexico	Spain	United States

* Of the 30 countries currently listed as members of the OECD, the Czech Republic, Slovakia, Poland, Hungary and Germany did not have consistent data for the period 1960-2000.

Table 3: Relative Trends in Income Inequality for the OECD countries with alternative degrees of Inequality Aversion

Time Period	*G(1.5)*	*G(2)*	*G(2.5)*
1960	.163	.253	.318
1970	.132	.205	.260
1980	.108	.174	.226
1990	.105	.169	.218
2000	.114	.171	.214

Table 4: Relative Trends in Income Inequality for the Full-Sample (N=98) with alternative degrees of Inequality Aversion

Time Period	*G(1.5)*	*G(2)*	*G(2.5)*
1960	.327	.483	.572
1970	.337	.503	.600
1980	.336	.510	.612
1990	.358	.538	.641
2000	.370	.553	.659

Table 5: Income Convergence Dynamics for 25 OECD Countries: 1960-2000

Time period	σ-Convergence ΔG(2)	β-convergence DP(2)	Reranking R(2)	β Barro-Regression (s-errors in brackets)
1960-2000	.171-.253 = -.083	-.116	.033	.012** (.0025)
1960-1970	.205-.253 = -.048	-.056	.008	.016** (.005)
1970-1980	.174-.205 = -.031	-.045	.014	.013** (.004)
1980-1990	.169-.174 = -.005	-.013	.008	.012** (.006)
1990-2000	.171-.169= .002	-.009	.011	.005 (.006)

Table 6: Income Convergence Dynamics for the full-sample of 98 countries: 1960-2000

Time period	σ-Convergence ΔG(2)	β-convergence DP(2)	Reranking R(2)	β Barro-Regression (s-errors in brackets)
1960-2000	.553-.483 = .07	.017	.053	-.004** (.0015)
1960-1970	.503-.483 = .02	.012	.008	-.006** (.002)
1970-1980	.510-.503 = .007	-.003	.01	-.003 (.002)
1980-1990	.538-.510 = .028	.02	.008	-.003 (.002)
1990-2000	.553-.538= .015	.01	.005	-.007** (.002)

Table 7: Conditional Income Convergence Dynamics for the full-sample of 98 countries: 1960-2000

Time period	σ-Convergence ΔG(2)	β-convergence DP(2)	Reranking R(2)	β Barro-Regression (s-errors in brackets)
1960-2000	.345-.386 = -.041	-.113	.072	.005** (.0016)
1960-1970	.362-.386 = -.024	-.039	.014	.002 (.003)
1970-1980	.339-.362 = -.023	-.037	.015	.005** (.0028)
1980-1990	.345-.339 = .005	-.01	.015	.008** (.002)
1990-2000	.345-.345= 0	-.011	.011	-.0027 (.003)

Figure 1:
β-convergence, σ-convergence, no leapfrogging

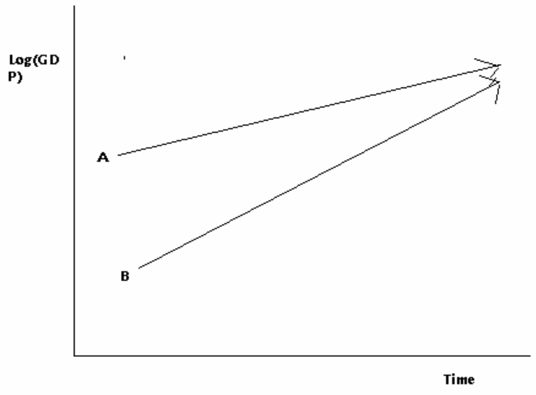

Figure 2:
β-convergence, No σ-convergence, Leapfrogging

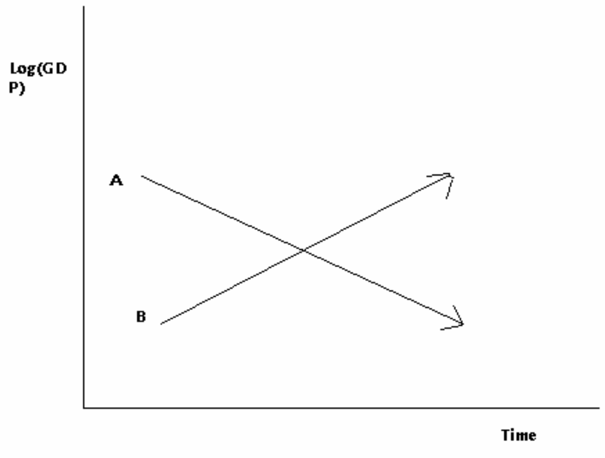

Figure 3
No β-convergence, No σ-convergence, No Leapfrogging

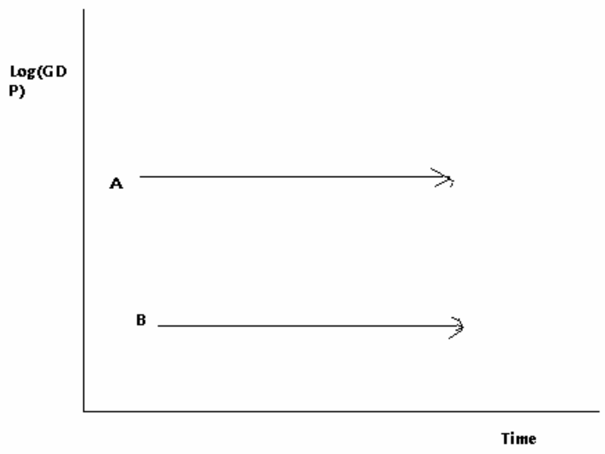

Figure 4:Graphical Summary of the Evolution of Income Inequality among OECD countries 1962-1998.

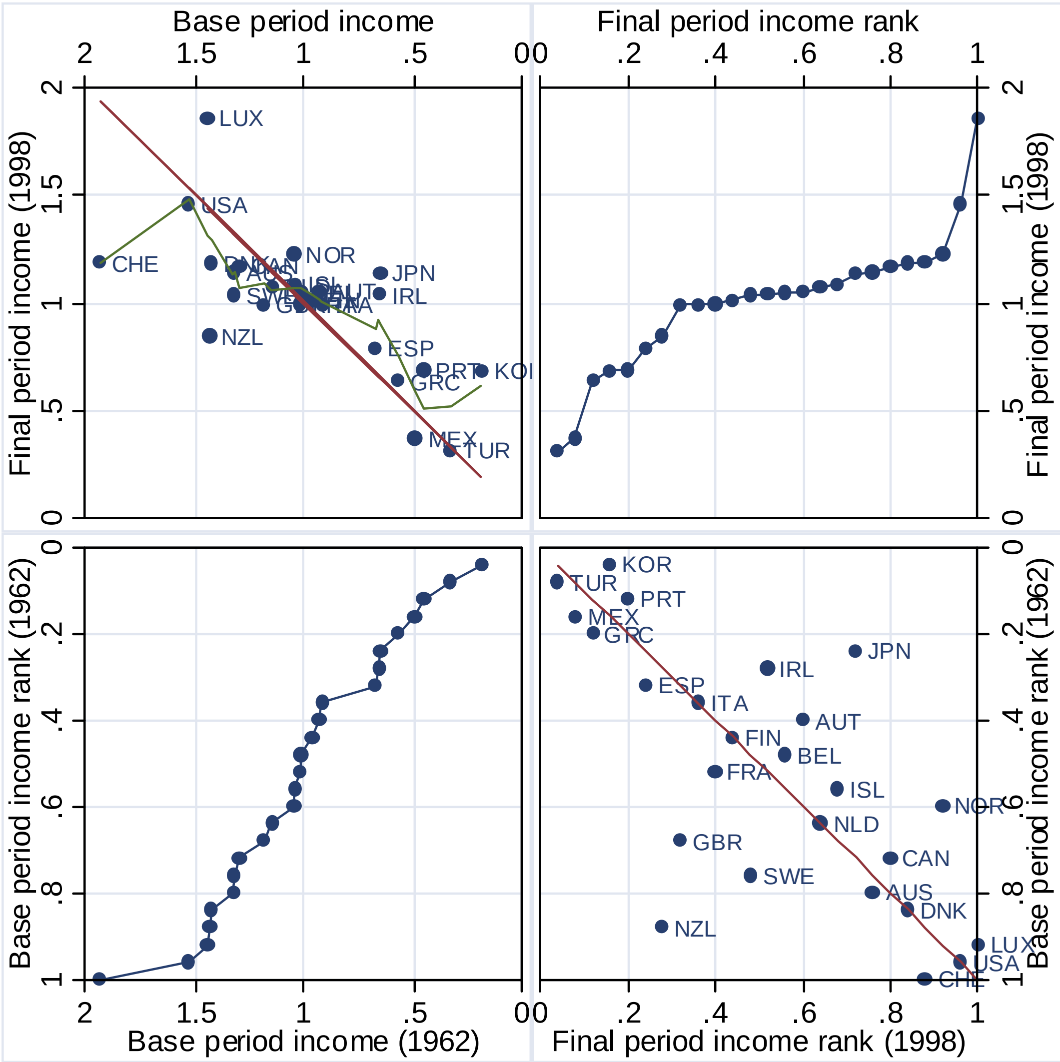

Figure 5: Graphical Summary of the Evolution of Income Inequality among All countries 1962-1998.

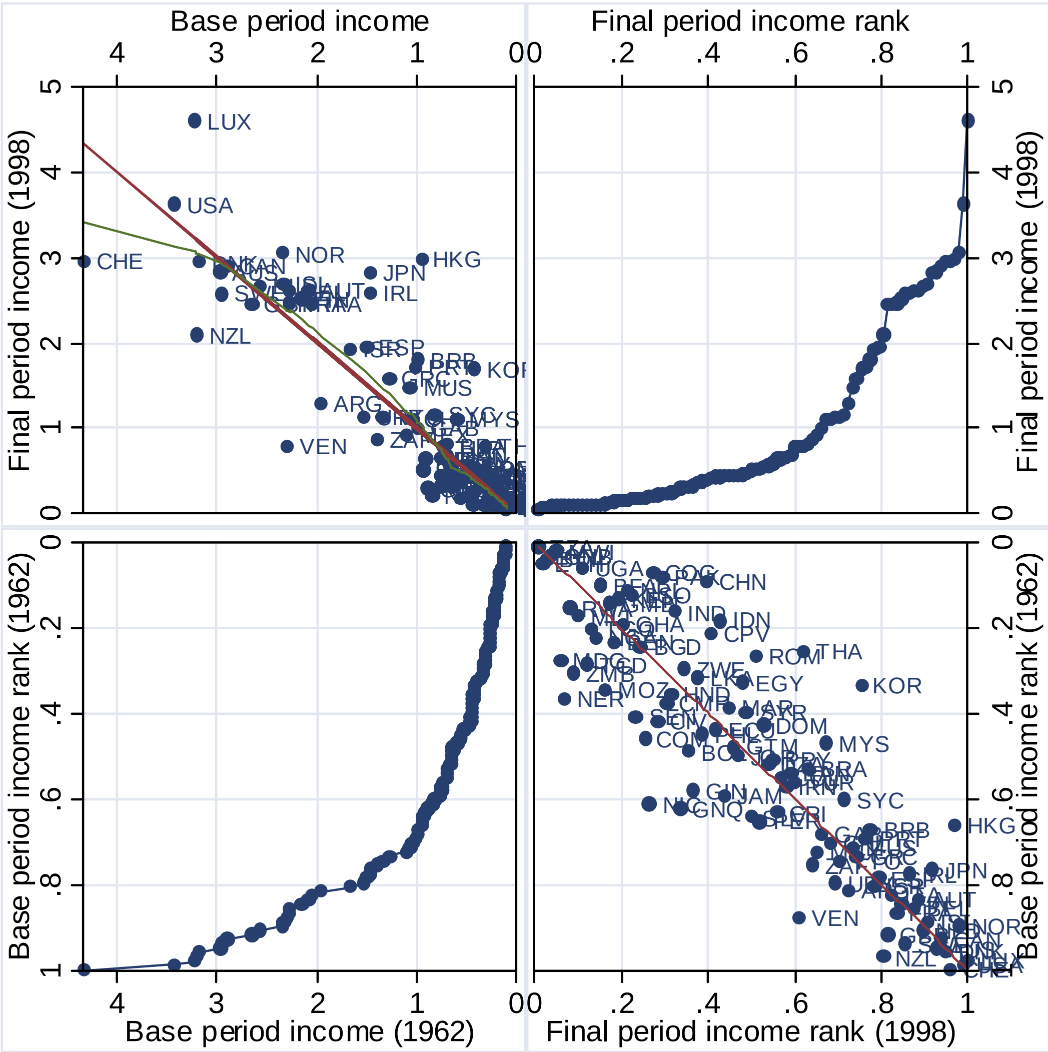

Figure 6: Income Growth Across the OECD Countries 1960-2000