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The Optimum Level of Reserves in an Exchange Rate Target Zone*

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Abstract: The target zone exchange rate literature, initiated by Krugman (1991), has relevance for the question of what is the optimal level of reserves. In particular, Krugman and Rotemberg (1990) show that there is a well defined boundary between a target zone that is subject to speculative attack and one which is sustainable. This boundary is defined by whether reserves are sufficient or not. This paper reviews briefly both the target zone and speculative attack literature. The appropriate level of reserves for Ireland is then calculated and discussed.

I INTRODUCTION

Why does a country need foreign exchange reserves? One answer to this question focuses on the fact that if monetary authorities want to defend a particular exchange rate stance, a stock of reserves is required. The quantity of reserves that is required to defend an exchange rate policy will depend on a number of factors, most importantly the magnitude and type of shocks that impact on the exchange rate. If the holding of foreign exchange reserves is costly, then the derivation of the optimal level of reserves becomes important; this will be the level just sufficient for effective exchange rate

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defence. If the level of reserves is small, then investors know that government attempts to defend an exchange rate policy are doomed to failure; the policy lacks credibility, and a speculative run on the currency will be triggered by any attempt to defend the policy.

In this paper we examine this recurring question of the optimal level of reserves, in particular looking at the quantity of reserves that is required if the exchange rate is to be maintained within a band such as the EMS. We make use of some results from recent work on exchange rate regimes, in particular taking from Krugman and Rotemberg (1991) a derivation of the boundary between an exchange rate target zone that is subject to speculative attack, and one which can credibly be defended. The difference between the two exchange rate regimes is the level of reserves; using Irish data, we calculate the stock of reserves required to make the defence of a target zone feasible.

We start the paper by outlining the theory of the behaviour of exchange rates that are maintained within boundaries by Government intervention. This, relatively new, theory has been developed by Krugman (1991) and Froot and Obstfeld (1991). We also look at the theory of exchange rate regimes that are subject to speculative attack. The linkage between target zone exchange rate theory and speculative attack theory has been studied by Flood and Garber (1991), and leads to the definition of required foreign exchange reserves derived by Krugman and Rotemberg (1990, 1991).

The magic number for reserves is a proportion of domestic credit; this proportion depends on the semi-elasticity of money demand and the time series pattern of velocity shocks which are assumed to be the only shocks to impinge on the exchange rate. We estimate the required parameters, and thus estimate required foreign exchange reserves. We plot and discuss required reserves and compare it to an often used import cover measure of adequate reserves. We conclude by pointing out the shortcomings and strengths of the approach as a method of calculating optimal reserves.

II TARGET ZONES, SPECULATIVE ATTACKS AND LIMITED RESERVES

We start our review of the target zone literature by looking at exchange rates in a very simple flex-price monetary model. This model, which assumes purchasing power parity, a continuous time version of uncovered interest rate parity and stable, comparable money demand functions, gives the exchange rate as being determined by fundamentals and the expected changes in exchange rates, i.e.:

$$x(t) = f(t) + \alpha \, E[dx(t) \mid \phi(t)] \quad (1)$$

where

x is the exchange rate
f is fundamentals
α is the semi-elasticity of money demand
ϕ is the relevant information set on which expectations are based.

The fundamental is assumed to consist of two parts; a component under the control of the government, m(t), and exogenous shocks to relative velocity, v(t). Shocks are assumed to hit the system via a stochastic process, that of Brownian motion with drift:

$$dv(t) = \mu dt + \sigma dz \tag{2}$$

where z is a Wiener process.

In the free float case, governments never intervene, and therefore velocity shocks are the only variable affecting fundamentals. The relevant information set, if investors find the no intervention rule credible, is simply the current level of the fundamental. In this simple case, the solution to the above system is:

$$x(t) = \alpha\mu + f(t). \tag{3}$$

A target zone is defined as an exchange rate corridor in which the exchange rate is allowed to float. The government intervenes to stop the exchange rate moving outside this pre-announced corridor. The basic target zone model is derived (following Krugman, 1991 and Svensson, 1991) under the assumptions that governments only intervene when the edges of the band are hit, that they then intervene by an infinitesimally small amount, and that they are never hindered from intervening by inadequate reserves. The model has the same set of equations as those above; here, however, the information set includes the fact that the government will intervene at the edges of the band. The exchange rate solution to the model is the now familiar S-shape derived by Krugman; the expectation of a prospective government intervention is sufficient to bend the exchange rate/fundamental relationship below the free float solution "close" to the upper edge of the target zone and to bend the relationship above the free float solution in the lower half of the band. The exchange rate equation has two components; the linear part which is the free float solution, and extra terms which capture the non-linearities. It is given by:

$$x = G(f) = f + \alpha\mu + A_1 \exp\{\lambda_1 f\} + A_2 \exp\{\lambda_2 f\} \tag{4}$$

where λ_1 and λ_2 are the roots of the equation

$$(\alpha\lambda^2\sigma^2)/2 + \alpha\mu\lambda - 1 = 0 \tag{5}$$

and A_1 and A_2 are constants which are determined by the "smooth pasting" conditions — the requirement that the exchange rate equation be tangent to the band edges, i.e. $G'(\bar{f}) = 0$, $G'(\underline{f}) = 0$.

The above analysis assumes that the Government's announced policy of defending the target zone is fully credible. One reason why such a policy may be incompletely credible is that the authorities may not have sufficient reserves to defend a particular exchange rate policy. The exchange rate/ fundamentals relationship will therefore be different if the market participants' information set includes a finite stock of reserves. Krugman and Rotemberg (1990) have studied such a scenario. Their main result is that the announcement of a target zone will still have a volatility reducing effect on exchange rates even if reserves are insufficient; the exchange rate path will lie somewhere between the free-float solution and the fully credible target zone path. However, once the authorities are called upon to defend their exchange rate stance, a speculative attack occurs which depletes reserves and the exchange rate free floats.[1]

Speculation is defined as any private acquisition of reserves in anticipation of a possible exchange rate regime collapse. A speculative attack on a government's reserves can be viewed as a process by which investors change the composition of their portfolios, reducing the proportion of domestic currency and raising the proportion of foreign currency. This change in composition is then justified by a change in relative yields.

Most of the speculative attack literature looks at fixed exchange rate regimes, where government attempts to maintain the exchange rate peg leads to a depletion of reserves and thus to a balance of payments crisis. Krugman (1979) describes a standard crisis in the following manner. A country pegs its exchange rate solely through direct intervention in the foreign exchange market. At the pegged rate the monetary authorities' reserves gradually decline, because of the balance of payments position. At some point, usually well before reserves would have been exhausted by the gradual depletion, there is a sudden speculative attack that rapidly eliminates the last of the reserves. The exchange rate peg can thereafter no longer be defended.

1. Another branch of the target zone literature (Bertola and Carballero (1992) and Miller and Weller (1989) looks at the situation when the stochastic process switch is from target zone to new target zone; i.e. a realignment of the exchange rate peg rather than an abandonment of the peg. For such a new target zone to be credible, foreign exchange reserves are required; the realignment will have to occur before the speculative attack depletes reserves.

However, there may be some source of secondary reserves (gold, or credit lines) which the authorities may be able to use to temporarily stave off the speculative attack. Krugman shows in this case, even if the Government's policy is uncertain, there may be a whole sequence of temporary speculative attacks and recoveries of confidence before the attempt to maintain the exchange rate is finally abandoned. Buiter (1987) shows that borrowing to defend the exchange rate regime is usually counter-productive; borrowing both increases the likelihood of eventual collapse, and increases the magnitude of the speculative attack. However, if the time won by borrowing is used to initiate a policy of fiscal correction that makes the exchange rate viable, then borrowing will have positive impact.

Flood and Garber (1984) determine the collapse time of a fixed exchange rate regime, by calculating a "shadow" floating exchange rate. This rate is the present discounted value of future expected fundamentals and s(t) would be the shadow rate at time t if it floated freely for all future time with reserves at their critical zero level. At the moment of an anticipated attack on foreign exchange reserves s(t) equals the fixed rate; this condition is used to time a collapse. An increase in initial reserves delays a collapse and an increase in the rate of domestic credit growth hastens the collapse. Blackburn (1988) questions a monetary authority's control of capital mobility and he feels that control measures may actually hasten a regime collapse if prices are sufficiently sluggish. There are many factors which make the timing of a speculative attack extremely complicated, including market speeds of adjustment, stock-flow interactions and future exchange rate policy.

There are two main results in the speculative attack literature that are relevant to our current problem. First, in a fixed exchange rate regime, speculative activity provides a one-way option. Except for transactions costs investors do not lose by speculating against the currency even if predictions of abandonment of fixed rates prove unjustified. In that case the portfolio can simply be readjusted back to its original allocation at the same exchange rate. This is not precisely true in a target zone. Exchange rates have freedom of movement within the zone, and portfolio readjustments after an unsuccessful speculative attack may not be at the same exchange rate. The incentive to speculate decreases with the width of the band. The second result is an arbitrage condition. In equilibrium, there can be no foreseeable jumps in exchange rates; the pre-attack and post-attack exchange rate must be the same. This condition is used as a definition of equilibrium in this literature.

The scenario is as follows; the Government announces a target zone ($\overline{x}$, $\underline{x}$), and is prepared to use all its foreign reserves in unsterilised intervention if the exchange rate threatens to breach the upper limit. We assume that Government behaviour is certain, and we rule out the recourse to secondary reserves discussed above. If reserves are small, this leads to a speculative attack at the moment a velocity shock takes the exchange rate to the upper barrier, after which the exchange rate will free float. Investors know that the Government will continue to intervene until reserves run out; this fuels the speculative attack, and reserves will suddenly be exhausted at the moment the exchange rate hits the upper boundary. The exchange rate equation post attack is a free float straight line as derived above; pre-attack the relationship is the non-linear path:

$$G(f) = f + \alpha\mu + A_1 \exp(\lambda_1 f) + A_2 \exp(\lambda_2 f) \tag{6}$$

Again, A_1 and A_2 need to be tied down by the economics of the situation. Insufficient reserves do not affect the lower boundary; here defending the target zone means adding to rather than depleting reserves. For simplicity, we set the lower boundary to minus infinity, giving a one-sided target zone with only one constant to be determined. This constant is not determined by smooth pasting, but by the more familiar arbitrage condition that there are no foreseeable jumps in exchange rates.

The initial money supply is defined by:

$$m = \ln(D + R) \tag{7}$$

where D is domestic credit and R foreign exchange reserves. Following the speculative attack, the money supply will fall to:

$$m^{*} - \ln(D). \tag{8}$$

The change in the money supply at the moment of the speculative attack is given by:

$$m^{*} - m = -\ln(1 + R/D). \tag{9}$$

The exchange rate post-attack is given by the free float equation:

$$x = m^{*} + v + \alpha\mu. \tag{10}$$

At moment of the speculative attack, when a shock to velocity occurs that pushes the exchange rate to its upper limit, the exchange rate equation is given by:

$$\bar{x} = m + \bar{v} + \alpha\mu + A \exp \lambda_1 \bar{f}. \tag{11}$$

The moment after the attack, when the exchange rate is free-floating the relationship is:

$$\bar{x} = m^{\bullet} + \bar{v} + \alpha\mu. \tag{12}$$

The exchange rate is the same directly before and directly after the speculative attack and one can solve for A. The relationship between the exchange rate and fundamentals is non-linear and depends on the reserves/deposits ratio. A is negative and therefore the path bends away from the free-float line.

However, the above is not the complete story. As reserves get larger, A becomes increasingly negative, and the curved path becomes flatter. A curved path must turn downward at some point; for sufficiently large reserves the above equation gives an exchange rate path that has its maximum below $\bar{x}$. This would indicate no possibility of government intervention or a speculative attack; since the above equation was derived on the logic of a possible attack, the above equation is not valid in these cases.

This implies that the analysis above is only valid for small reserves. For large reserves the exchange rate path is derived as in a fully credible target zone. Reserves are defined to be large if they lead to an exchange rate path that has a maximum on or below $\bar{x}$. There will be a unique amount of reserves that is just large enough; this is the level that gives an exchange rate path with a maximum at $\bar{x}$, that is, the exchange rate path that "smooth pastes" the upper boundary. This gives an intuitive rationale for the smooth pasting boundary condition used to derive the original target zone relationship.

The critical level of reserves can be derived as follows: with smooth pasting at $x = \bar{x}$, (and $A_2 = 0$) taking the first derivative of the general exchange rate fundamentals relationship (6) and setting it to zero gives:

$$A \exp \lambda_1 \bar{f} = -\frac{1}{\lambda_1}. \tag{13}$$

Substituting this above in (11) gives

$$\bar{x} = m + \bar{v} + \alpha\mu - \frac{1}{\lambda_1} \tag{14}$$

and therefore by comparing (11) and (12) we have

$$m^{\bullet} - m = -\ln\left(1 + \frac{R}{D}\right) = -\frac{1}{\lambda_1}. \tag{15}$$

The critical level of reserves is given by:

$$R = D\,[\exp(1/\lambda_1) - 1]. \qquad (16)$$

For reserves smaller than this level, the exchange rate target zone will be subject to speculative attack; for large reserves, the target zone will be defendable.

III REQUIRED RESERVES IN IRELAND

To calculate the threshold level of reserves for a particular economy a number of parameters are required. The first parameter is the semi-elasticity of money demand (α); also required are parameters determining the drift (μ) and variability around drift (σ) of the shocks to velocity. We first looked at the order of magnitudes of these parameters used in the target zone literature. Smith and Smith (1990) require the same set of parameters for their study of the return to the gold standard in Britain; they assign the semi-elasticity the value {0.5, 6.5, 13} the drift term the values {0.0001 0.01} and the magnitude of variation term {.0032, .32}. Svensson (1991) draws a series of target zone diagrams with $\mu = 0$, $\sigma = .1$ and $\alpha = 1$, 3, and 5. Bertola and Carballero calibrate their model of realignments with the parameters $\mu = 0.5$, $\alpha = 1$ and $\sigma = 0.5$. This gives an idea of the appropriate order of magnitude for the required parameters.

The semi-elasticity of money demand was extracted from a simple Cagan style estimated money demand function over the years 1979-1992. The monetary aggregate used was M3, the scale variable used was retail sales and the interest rate used was the exchequer bill rate.[2] The estimated semi-elasticity was 2.48.

To find appropriate values for the shocks that impact on the exchange rate we simply looked at the time series pattern of M3 velocity. We regressed the change in velocity on a constant trend and took μ to be the estimated constant coefficient the trend, and σ to be the standard error of the residuals from the regression. We found:

$$\mu = .0022$$
$$\sigma = .187$$
$$\alpha = 2.48.$$

These parameter values give a required reserves/domestic credit ratio of 23.48 per cent. Optimal reserves calculated by this method are graphed in Figure 1.

2. See Hurley and Guiomard (1989) for a discussion of the appropriate variables for an Irish money demand study.

We also briefly looked at the sensitivity of our required reserves ratio, by plugging in a number of other parameters in our calculations. This is shown in Table 1. Most of the sets of parameters give ratios in the region of 15 per cent to 30 per cent of domestic credit; our calculated value is in the middle of this range.

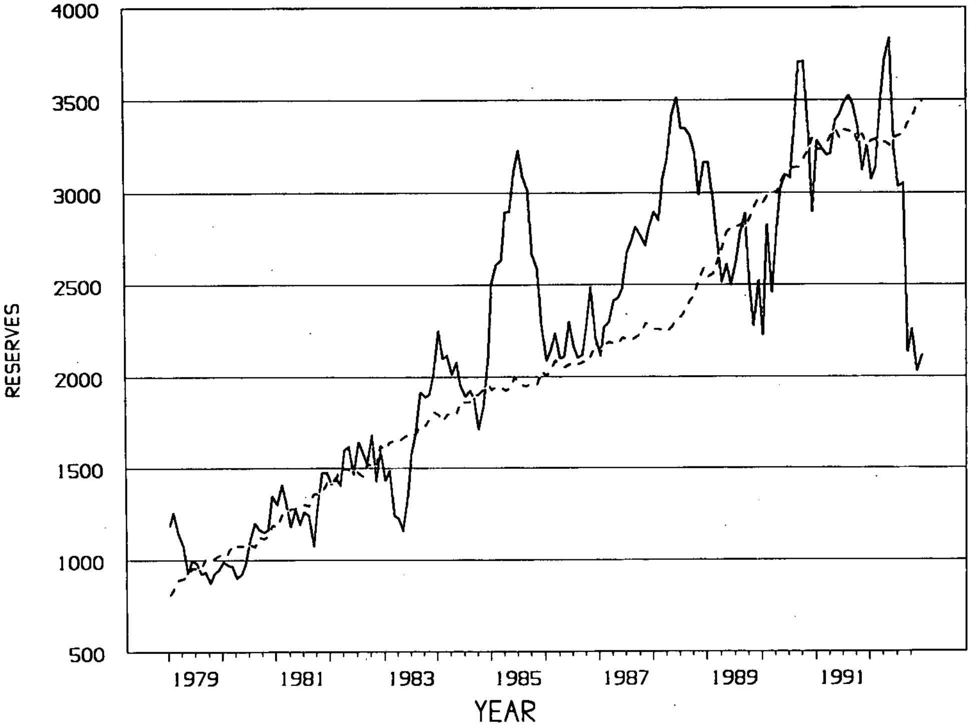

Figure 1: *Actual and Optimal Reserves: Levels*

Figure 1 plots the actual and optimal level of reserves. On average, reserves appear to be approximately optimal during the early years of our sample (1989-1992), above optimal in the mid-eighties, dipping below optimal during 1989/1990 and showing up as dramatically suboptimal in the final months of the sample. However, it is worth emphasising that having reserves that are optimal on average is not sufficient; any point where the reserves are below our optimal level could lead to the triggering of a speculative attack. It is also interesting to point out that around the time of the 1986 devaluation reserves were above our measured optimal level. Below optimal reserves in 1989/1990 are due to outflows of capital; the reason for these outflows is unknown. Actual reserves fell below our optimal level in mid 1992 (Figure 2) and were well below optimal when Britain pulled out of the ERM.

Table 1

mu	*sigma*	*alpha*	*lamda*	*ratio*
0	.1	1	14.14214	.0732707
0	.2	1	7.071068	.1519099
.1	.1	1	7.320508	.1463724
.1	.2	1	5	.2214028
0	.1	2	10	.1051709
0	.2	2	5	.2214028
.1	.1	2	4.142136	.2730573
.1	.2	2	3.090170	.3821038
0	.1	3	8.164966	.1302903
0	.2	3	4.082483	.2775561
.1	.1	3	2.909944	.4100838
.1	.2	3	2.287136	.5484093
.0011	.11	1.69	9.799114	.1074389
.0022	.187	2.4783	4.741427	.2347975

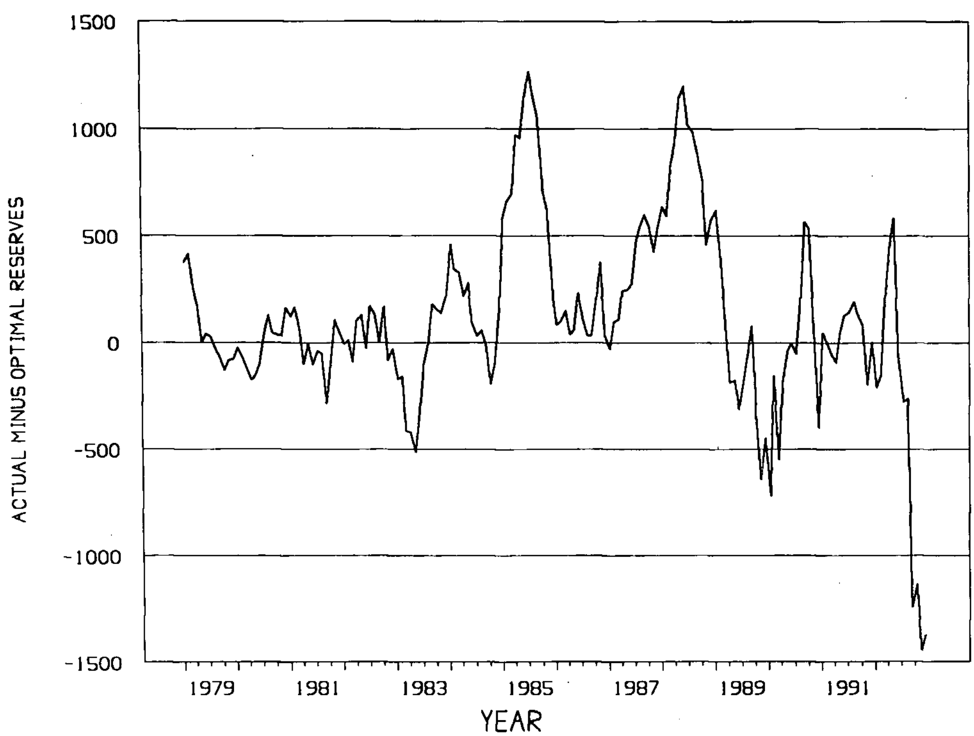

Figure 2: *Actual and Optimal Reserves: Differences*

A rule of thumb estimate of adequate reserves (of unknown origin) is 2.5 months import cover. Figure 3 plots this level of import cover against actual reserves. This measure of calculating optimal reserves gives similar results to our measure in the early years of the sample; in later years the import cover measure underestimates required reserves. In particular, our measure of optimal reserves shows reserves to be dangerously low relatively early in 1992; the import cover floor is only breached in September 1992.

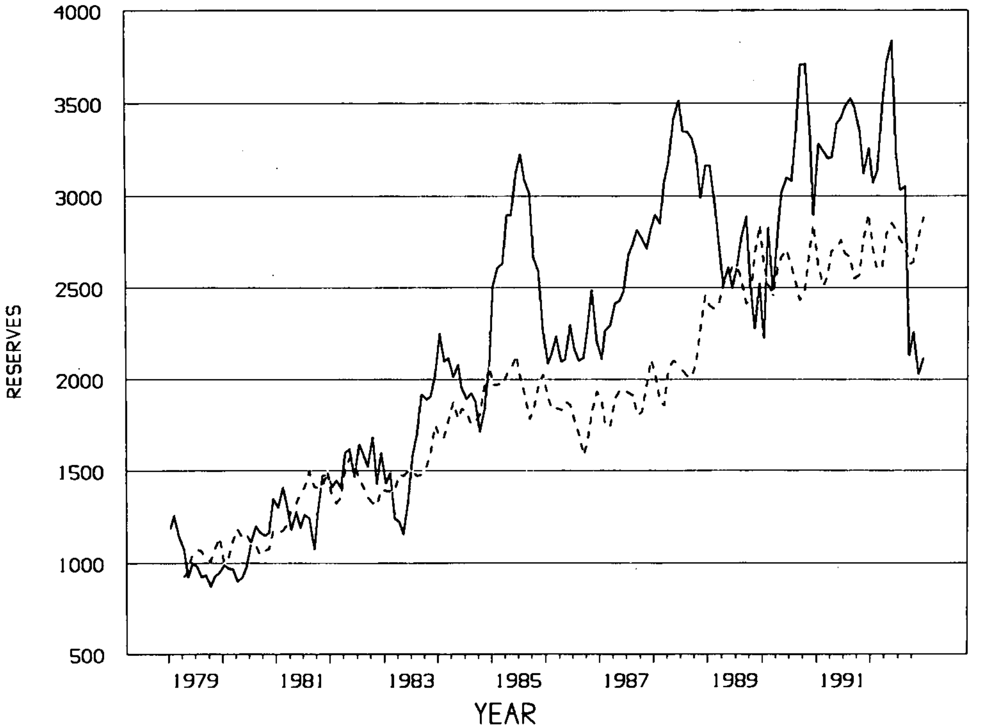

Figure 3: *Reserves and Import Cover*

IV CONCLUSIONS

We have derived a simple measure of optimal reserves, based on the need to have sufficient reserves for credible defence of an exchange rate target zone. For Irish data, reserves were approximately optimal for most of the 1980s but significantly below optimal during 1989 and again during 1992 from June onwards. Our results suggest that foreign exchange reserves should be kept *above* about 25 per cent of domestic credit.

Our measure of reserves has some shortcomings. It shares with much of the optimal reserves literature the neglect of lines of credit as a means of exchange rate regime defence. In theory, if credit lines are sufficiently large (and credible), a target zone could be defended without reserves. The rules of the EMS are such that members can call on unlimited financing for the purposes of marginal interventions. However, as Gros and Thygesen (1992) have documented, this type of financing has been small to non-existent in recent years. Moreover, the continued perception of reserves as an important economic indicator suggests that the lines of credit are insufficient for exchange rate policy credibility.

Secondly, our model looks only at the need for reserves for the defence of the exchange rate at the margin; in reality, most of EMS intervention takes place intermarginally. It appears, however, that our estimate should still form a lower bound for the stock of reserves required for target zone defence. If the monetary authorities choose to intervene intermarginally, thereby using up reserves, then reserves need to be higher; sufficient both for the intermarginal intervention and to cover the possibility of exchange rate defence at the margin. Krugman and Rotemberg's derivation is independent of the current position of the exchange rate in the band.

Finally, our modelling of shocks to the system is highly simplified. In particular, we look only at shocks to domestic money demand and ignore external shocks that impact on the exchange rate. The stochastic process governing fundamentals is assumed to be Brownian motion with drift; in more complex target zone models this process mean reverts.

The strength of approach lies first in the simplicity and neatness of its conclusion; reserves should be maintained above a constant fraction of domestic credit. Second, the approach is firmly based on the defence of a particular exchange rate regime; reserves are required for exchange rate defence and the minimum quantity of reserves required depends on the exchange rate regime being defended.

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