Switching Stability of Automotive Roll Dynamics Subject to Interval Uncertainty

Selim Solmaz*, **Robert Shorten,** and **Oliver Mason**

Hamilton Institute, National University of Ireland-Maynooth, Co. Kildare, Ireland

In this paper we apply some recent results on the stability of switched systems with interval uncertainty for analyzing the roll stability of automotive vehicles subject to bounded parametric uncertainties and switching. The application is motivated by the fact that the roll dynamics of an automotive vehicle is affected in a significant and a nonlinear fashion due to possible sudden switches in the vehicle's center of gravity (CG) height, as well as the uncertainty in the suspension parameters. Utilizing a recent stability result, it is possible to model such parametric variations as bounded interval uncertainties for this problem, and obtain easily verifiable conditions for analyzing the stability of the resulting switched/uncertain system.

1 Introduction

The problem of determining whether or not a family of switched uncertain linear systems is stable has been the subject of a considerable amount of research in the recent years. One such result was proposed recently in [1, 2] for the case of interval matrix families in companion form. Specifically, this result is about checking the existence of a common quadratic Lyapunov function (CQLF) for two families of Hurwitz companion matrices, each of which is independently subject to interval uncertainty. We first state this recent result and then give a practical application, which utilizes the result for checking the stability of automotive roll dynamics subject to uncertainty in the suspension parameters and switching in the CG height.

We denote $\mathscr{A}$ and $\mathscr{B}$ as interval companion matrix families in $\mathbb{R}^{n\times n}$, each of which consist of companion matrices with bounded uncertain elements as described below

$$\mathscr{A} = \{C(a_0,\ldots,a_{n-1}) \in \mathbb{R}^{n\times n} : \underline{a}_i \leq a_i \leq \overline{a}_i\},\ \ \mathscr{B} = \{C(b_0,\ldots,b_{n-1}) \in \mathbb{R}^{n\times n} : \underline{b}_i \leq b_i \leq \overline{b}_i\} \quad \text{for} \quad 0 \leq i \leq n-1 \tag{1}$$

where $C(a_0,\ldots,a_{n-1})$ denotes the n^{th} order companion matrix. In analogy with the Kharitonov polynomials [2], we construct the following four Kharitonov matrices corresponding to the interval matrix family $\mathscr{A}$

$$A_1 = C(\underline{a}_0,\underline{a}_1,\overline{a}_2,\overline{a}_3,\ldots),\ \ A_2 = C(\underline{a}_0,\overline{a}_1,\overline{a}_2,\underline{a}_3,\underline{a}_4,\ldots),\ \ A_3 = C(\overline{a}_0,\underline{a}_1,\underline{a}_2,\overline{a}_3,\overline{a}_4,\ldots),\ \ A_4 = C(\overline{a}_0,\overline{a}_1,\underline{a}_2,\underline{a}_3,\ldots) \tag{2}$$

The matrices B_1,B_2,B_3,B_4 can be defined in the same manner for the family $\mathscr{B}$. The following result is useful for checking the stability of switched systems, where the constituent system matrices belong to certain subsets of $\mathscr{A}$, $\mathscr{B}$.

Theorem 1.1 [1, 2] *Consider the interval matrix families $\mathscr{A}$, $\mathscr{B}$ as described in (1), and assume that all the matrices belonging to $\mathscr{A}$, $\mathscr{B}$ are Hurwitz. Then for every pair of LTI systems of the form $\dot{x} = Ax$, $\dot{x} = Bx$ with $A \in \mathscr{A}$, $B \in \mathscr{B}$ to have a CQLF, it is necessary and sufficient that none of the eight matrix products*

$$A_1B_2\ ,\ A_1B_3\ ,\ A_2B_1\ ,\ A_2B_4\ ,\ A_3B_1\ ,\ A_3B_4\ ,\ A_4B_2\ ,\ A_4B_3,$$

has a negative real eigenvalue.

2 Switching automotive roll dynamics subject to interval uncertainty

Here we demonstrate how the Theorem 1.1 can be utilized to check whether the roll dynamics of an automotive vehicle is stable under switching, when also subject to parametric uncertainties in the suspension parameters. The example is motivated by the fact that the roll dynamics of a road vehicle can change significantly as a result of sudden switches in its center of gravity (CG) height. Also, the suspension parameters and/or the roll center can change depending on many factors (e.g., changes in vehicle speed, aerodynamic forces, suspension geometry, tire pressure, temperature etc.), which also affect roll dynamics stability. As these factors are difficult to model in a comprehensive model, one can instead use a simple linearized roll plane model with a linear spring and damper, and further assume that these linear parameters have bounded uncertainty.

Assuming that the sprung mass of the vehicle rolls about a fixed horizontal roll axis along the centerline of the vehicle body relative to the ground, and also that all angles are small, the equations describing the roll plane motion of an automotive vehicle can be expressed in the following state space form with reference to Figure 1

$$\begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k-mgh}{J_{xeq}} & -\frac{c}{J_{xeq}} \end{bmatrix} \cdot \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{mh}{J_{xeq}} \end{bmatrix} a_y, \tag{3}$$

* Corresponding author: e-mail: selim.solmaz@nuim.ie, Phone: +353 1 7084536, Fax: +353 1 7086269

Table 1 Second order linear roll plane model parameter definitions and their representative numerical values.

Parameter	Numerical Value	unit	definition
m	1200	$[kg]$	vehicle mass
J_{xx}	500	$[kg \cdot m^2]$	roll moment of inertia of the vehicle about the CG
$h_1,\ h_2$	0.5, 0.8	$[m]$	CG height measured over the ground (corresponding to configuration 1 & 2, respectively)
$\overline{k},\ \underline{k}$	40000, 30000	$[kg \cdot m^2/s^2]$	roll spring stiffness (upper & lower bounds, respectively)
$\overline{c},\ \underline{c}$	8000, 4000	$[kg \cdot m^2/s]$	roll damping coefficient (upper & lower bounds, respectively)

where a_y is the lateral acceleration, ϕ is the roll angle, and $J_{x_{eq}} = J_{xx} + mh^2$ denotes the equivalent roll moment of inertia.

Now consider a scenario where the CG height can randomly switch between the two values $h = \{h_1, h_2\}$. Further assume that the uncertainties in the linear suspension stiffness k, and the linear damping coefficient c can be expressed as bounded interval uncertainties such that $k \in [\underline{k}, \overline{k}]$ and $c \in [\underline{c}, \overline{c}]$. We are interested in stability of the roll dynamics subject to the switching in the CG height h, and uncertainties in the linear suspension parameters k, c. Under these assumptions, the roll dynamics evolve according to the following two matrix families $A \in \mathscr{A}$ and $B \in \mathscr{B}$ depending on the CG height configuration

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k-mgh_1}{J_{x_{eq,1}}} & -\frac{c}{J_{x_{eq,1}}} \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 \\ -\frac{k-mgh_2}{J_{x_{eq,2}}} & -\frac{c}{J_{x_{eq,2}}} \end{bmatrix}, \tag{4}$$

where $J_{x_{eq,i}} = J_{xx} + mh_i^2$ for $i = \{1, 2\}$. Further we define the following auxiliary parameters

$$a_0 = \frac{k-mgh_1}{J_{x_{eq,1}}}, \qquad a_1 = \frac{c}{J_{x_{eq,1}}}, \qquad b_0 = \frac{k-mgh_2}{J_{x_{eq,2}}}, \qquad b_2 = \frac{c}{J_{x_{eq,2}}} \tag{5}$$

where $k \in [\underline{k}, \overline{k}]$ and $c \in [\underline{c}, \overline{c}]$. Then, we can cast the resulting family of system matrices from (3) into the notation given in (1). This results in the following two 2^{nd} order interval matrix families in companion form

$$\mathscr{A} = \{C(a_0, a_1) : a_0 \in [\underline{a}_0, \overline{a}_0], a_1 \in [\underline{a}_1, \overline{a}_1]\} \qquad \mathscr{B} = \{C(b_0, b_1) : b_0 \in [\underline{b}_0, \overline{b}_0], b_1 \in [\underline{b}_1, \overline{b}_1]\}. \tag{6}$$

Utilizing the numerical values given Table 1 and making use of the expressions (2), one can obtain the following 8 Kharitonov matrices that belong to the Hurwitz matrix families $\mathscr{A}$, and $\mathscr{B}$

$$\begin{array}{llll} A_1 = C(30.1425, 5), & A_2 = C(30.1425, 10), & A_3 = C(42.6425, 5), & A_4 = C(42.6425, 10), \\ B_1 = C(16.2322, 3.1546), & B_2 = C(16.2322, 6.3091), & B_3 = C(24.1186, 3.1546), & B_4 = C(24.1186, 6.3091). \end{array}$$

The eigenvalues of the eight matrix products of Theorem 1.1 is shown in Figure 2. As none of the matrix products have negative real eigenvalues, there exists a CQLF for any pair of matrices $A \in \mathscr{A}$, $B \in \mathscr{B}$. Thus, the described switching 2^{nd} order automotive roll dynamics model subject to bounded uncertainty in the suspension parameters is stable by Theorem 1.1.

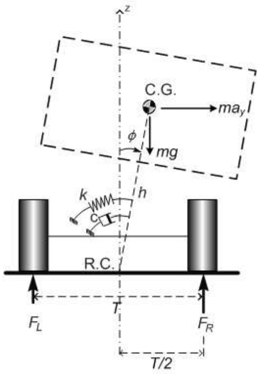

Fig. 1 2^{nd} order roll plane model.

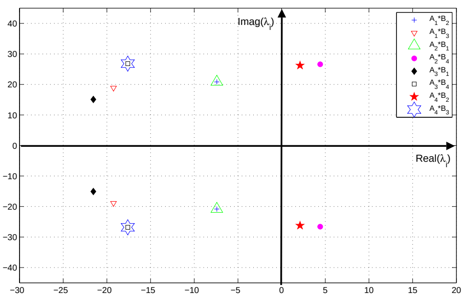

Fig. 2 Eigenvalues of the 8 matrix products of Theorem 1.1 for the switched/uncertain automotive roll dynamics model.

Acknowledgements This work was jointly supported by SFI Grant 04/IN3/I478 and EI Grant PC/2007/0128.

References

[1] E. Zeheb , O. Mason, S. Solmaz, and R. Shorten, Some results on quadratic stability of switched systems with interval uncertainty, *International Journal of Control*, Vol. 80, No. 6, Page(s):825-831, (2007).

[2] S. Solmaz. *Topics in automotive rollover prevention: robust and adaptive switching strategies for estimation and control*, PhD thesis, Hamilton Institute, National University of Ireland-Maynooth (2007). Available online at the following url: *www.hamilton.ie/publications/PhD_thesis_SelimSolmaz_Dec2007.pdf*