# Fusion of Functional Brain Imaging Modalities via Linear Programming




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## Abstract


Proposed method makes a number of simplifying assumptions which convert the EEG/FMRI integration problem into optimization of a convex function, of a form amenable to efficient solution as a very sparse linear programming (LP) problem. The assumptions made in doing this are, surprisingly, in general somewhat more robust than those generally used to cast EEG/FMRI integration as optimization of a non-convex function not amenable to efficient global optimization. This is because the $L_1$ norm used here corresponds to a more robust statistical estimator than the $L_2$ normal generally used For this reason, even though this technique results in a tractable global optimization, it is more robust to non-Gaussian noise and outliers than approaches that make the Gaussian noise assumption [1].

Current poster presents formulation of the problem together with results obtained on artificial data.


## Introduction to Fusion

Fusion algorithms are employed in an attempt to construct a spatio-temporal estimate of neuronal activity using data gathered from multiple functional brain imaging modalities. Here, the estimate is built by placing a dipole in each voxel of the modality with highest spatial resolution, and estimating the time course of each dipole without constraining dipoles' orientation. The solution space thus consists of a matrix $\mathbf{S}$ of dimensionality $[3N\times T]$, which actually consists of 3 $[N\times T]$ matrices. Each such sub-matrix corresponds to the projection of the dipole to the specific axis [2]: $\mathbf{S}=\left[\mathbf{S}_x\mathbf{S}_y\mathbf{S}_z\right]^T$

| Modality | Matrix | Size | Estimate |
|---|---|---|---|
| EEG | $\mathbf{E}$ | $[M\times T]$ | $\hat{\mathbf{E}}=\mathbf{AS}$ |
| FMRI | $\mathbf{F}$ | $[N\times U]$ | $\hat{\mathbf{F}}=\tilde{\mathbf{S}}\mathbf{B}$ |

*Forward EEG/FMRI equations, where $\tilde{\mathbf{S}}$ is a $[N\times T]$ matrix which holds the dipoles strengths without orientation information and $\tilde{s}_{ij}=||(s_{ij}^{(x)},s_{ij}^{(y)},s_{ij}^{(z)})||_2$*

## General LP Formulation

Using defined abbreviations we formulate an initial LP problem as follows

$$\hat{\mathbf{E}}+\Delta_{\mathbf{E}}=\mathbf{E} \quad \text{Constraints} \quad (1)$$
$$\hat{\mathbf{F}}+\Delta_{\mathbf{F}}=\mathbf{F} \quad (2)$$
$$\tilde{s}_{ij}\geq 0 \quad \text{Region} \quad (3)$$
$$C=\alpha\|\Delta_{\mathbf{E}}\|_1+\beta\|\Delta_{\mathbf{F}}\|_1 \quad \text{Objective} \quad (4)$$

where $\alpha$ and $\beta$ are used to check different trade-offs between two modalities as well as to normalize their influence in the optimization criteria.

## General LP transformations

Next we redefine each $|x|$, which are present in computation of $C$ (4) and $\tilde{s}_{ij}$ (6), in a form suitable for LP through usage of slack variables $x^+$ and $x^-$. This transformation leads to a side effect - minimization of the sum of absolute values of $|s_{ij}|$, so we need to add another term $\gamma\|S\|_1$ to the objective function (4) . This side effect could be considered a desired result - the minimization of $L_1$ norm of the solution results in its increased sparseness.

## EEG Equation in LP form

We can represent (1) in a form suitable for LP by using Kronecker product $\otimes$.

$$(\mathbf{A}\otimes\mathbf{I}_T)\bar{\mathbf{S}}=\bar{\mathbf{E}}, \quad (5)$$

where $\mathbf{I}_Z$ is the identity matrix of size $[Z\times Z]$.

## FMRI Equation in LP form

First we need to encode the definition of $\tilde{\mathbf{S}}$ into LP constraints matrix through presenting it as

$$\tilde{\mathbf{S}}=l(|\mathbf{S}_x|,|\mathbf{S}_y|,|\mathbf{S}_z|), \quad (6)$$

where $l(\ldots)$ is a linear formulation to approximate $L_2$ norm.

In a similar to (5) way we present product $\tilde{\mathbf{S}}\mathbf{B}$ in the form suitable for LP

$$\hat{\mathbf{F}}=(\mathbf{I}_N\otimes\mathbf{B}^T)\bar{\tilde{\mathbf{S}}} \quad (7)$$

## $L_2$ norm in LP

We need a way to approximate vector norm $e=||\mathbf{m}||$ within an LP framework. Our solution is to note that the $\min(\cdot,\cdot)$ and modulus $|\cdot|$ functions can be used freely in a LP and then reduced to canonical form using standard transformations.

For our method, let $\{\mathbf{R}_i\}$ be a set of rotation matrices. To approximate $||\mathbf{m}||$ we let

$$e_i=||\mathbf{R}_i\mathbf{m}||_1 \qquad e=\min_i e_i \quad (8)$$

where $||\cdot||_1$ denotes the $L_1$ norm. These can simply be added to the linear programming problem, enforcing the relation $e\approx||\mathbf{m}||$. We can increase the number of matrices in the set to improve the accuracy of this approximation, at the expense of computational efficiency.

## Final LP form

Finally we group all the constraints and the objective function together into an extended canonical form for LPs,

$$(\mathbf{A}\otimes\mathbf{I}_T)\bar{\mathbf{S}}+\Delta_{\bar{\mathbf{E}}}=\bar{\mathbf{E}} \quad (9)$$
$$(\mathbf{I}_T\otimes\mathbf{B}^T)\bar{\tilde{\mathbf{S}}}+\Delta_{\bar{\mathbf{F}}}=\bar{\mathbf{F}} \quad (10)$$
$$\tilde{\mathbf{S}}-l(|\mathbf{S}_x|,|\mathbf{S}_y|,|\mathbf{S}_z|)=0 \quad (11)$$
$$\tilde{\mathbf{S}}\geq 0 \quad (12)$$
$$C=\alpha||\Delta_{\mathbf{E}}||_1+\beta||\Delta_{\mathbf{F}}||_1+\gamma||S||_1 \quad (13)$$

## Simulation Data

To check the method artificial data was created. Brain volume is simulated as a half-sphere with 9 voxels in diameter, which gives us 132 voxels total to be considered. Simple single sphere model was used to generate gain matrix for EEG 11 sensors distributed across the half-sphere surface. We've generated random activation map $\mathbf{S}$ consisting of 5 voxels firing within 600ms interval after $t_0$=2 sec from the beginning of the timecourse ( 1 voxel per each 0ms, 200ms, 400ms and 2 voxels at 600ms after $t_0$) with the same amplitude but in different locations and with arbitrary orientation. Using this map clean EEG and FMRI were constructed through the forward equations. EEG was sampled at 10sps and FMRI at 1sps, so EEG time resolution in the experiment was 10 times higher than slow FMRI.

Additive noise was used to corrupt EEG signal: Gaussian noise with SNR=-5dB, which due to sparsity constituted equivalently %RMS≈23%, where

$$\%RMS=\frac{\sigma_{noise}}{max(x)}\times 100\% \quad (14)$$

for clean signal $x$. SNR for EEG was fixed across all experiments. An FMRI signal was also corrupted by additive Gaussian noise with variable SNR to do some noise sensitivity analysis (for FMRI SNR 2dB $\approx$ 35% %RMS).

## Data Conditioning

Before analysis, both data sets (EEG $E$ and FMRI $F$) and corresponding matrices ($A$ and $B$) were normalized by estimated noise standard deviation in order to properly scale error terms as well as to remove difference between units of EEG and FMRI. Then weights in error terms were used to remove dimensionality effect by assigning $\alpha=1/MT$ and $\beta=1/NU$.

As multiple tests with different SNR levels have shown, best estimates of activations were achieved when $\gamma=\frac{1}{3NT\sigma_S}$, where $\sigma_S$ is standard deviation of simulated activation map. It can't be known for real data but we're speculating here by using artificial data. Robust method to estimate $\gamma$ is the next goal to achieve in future research.

## Results

Obtained solutions for FMRI SNR $>3$ dB returned all 5 original activations as 5 highest obtained activations for duration of the experiment with $\approx$ 50% of energy spread through the rest of the volume. Lowering SNR down to 1dB lead to a stable detection of 3 out of 5 activations.

## Future Work

**Constrained Orientation:** It is a common practice to use an anatomical MRI to constrain the inverse EEG solution to the dipoles with orientation normal to the cortical surface [3]. In this case we don't need $L_2$ approximation to present $\tilde{\mathbf{S}}$ in a form suitable for LP since the gain matrix for EEG would be based on values of dipole magnitude.

**Experiments:** Real data experiments are desired and will be handled after efficient large-scale sparse LP solver with 'warm' start possibility is found. Experiments will be constructed in the way to get good estimates of HRF for each subject due to high variability of HR among different subjects [4].

## Discussion

Presence of the weight factors $\alpha$, $\beta$ and $\gamma$ makes proposed method less attractive while no efficient and robust method for their estimation is suggested. Even if $\alpha$ and $\beta$ factors are relatively easy to estimate with proper estimation of the noise component present in both modalities, estimation of 'good' $\gamma$ might require robust heuristic and can be computationally challenging. There is a method suggested in [1] for weighting $L_1$ factor in estimating sparse convolution kernel, but it is ad-hoc and very computationally demanding when dealing with big data arrays.

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