DYNAMIC QUANTIZATION FOR MULTI-SENSOR ESTIMATION OVER BANDLIMITED FADING CHANNELS WITH FUSION CENTER FEEDBACK [1]

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Abstract: This paper considers the state estimation of hidden Markov models (HMMs) in a network of sensors which communicate with the fusion center via finite symbols by fading channels. The objective is to minimize the long term mean square estimation error for the underlying Markov chain. By using feedback from the fusion center, a dynamic quantization scheme for the sensor nodes is proposed and analyzed by a Markov decision approach. The performance improvement by feedback, as well as the effect of fading, is illustrated.

Keywords: Sensor networks, Markov sources, Hidden Markov models, quantization, sensor signal processing, feedback, Markov decision.

1. INTRODUCTION

In recent years there has been enormous research effort on sensor networks due to their wide range of current and potential applications in environment surveillance, detection and estimation, and location awareness services, etc. (Chong and Kumar, 2003). In such networks, geographically scattered sensors send data to a fusion center (rather than communicating with each other) (FC) equipped with a higher computation capability than the sensors themselves. Due to their limited on board battery power the sensors not only have little computational capacity but also possess limited communication capability as data processing and transmission both require energy, the energy required for data transmission usually being the dominant component (Wang and Chandrakasan, 2002). The channel between each sensor and the fusion center is usually bandwidth limited (e.g., a wireless link), and hence only a quantized output can be transmitted where the number of quantization levels is limited by the data rate constraints of the channel. The fusion center combines the data received from all sensors to form an estimate or make a final decision.

Within the context of statistical signal processing, an important application of sensor networks is state estimation of random processes, since in reality sensor networks operate in a time-varying environment and the sensor measurements provide partial information only of such random processes usually modelled by dynamical systems (Fletcher *et al.*, 2004). See (Ishwar *et al.*, 2005) for estimation of i.i.d. sources in an unreliable bandwidth-limit sensor network. In certain applications of interest, the underlying random process may be modelled as a Markov chain and the resulting measurements modelled by hidden Markov chains. See (Shue *et al.*, 2001) for near optimal quantizer design for hidden binary Markov chains for a single sensor. In general the optimal quantizer design problem is difficult, even when the Markov chain has only a few states, primarily due to the associated nonconvex optimization problems.

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In this paper we consider the estimation of finite state Markov chains via sensor networks with quantized sensor measurements. For computational tractability, we start by analyzing binary quantization at the sensor nodes. In general, such a quantization scheme can only transmit very coarse information, and traditionally the network performance is improved by increasing the number of sensors. Recent applications of binary sensors for target tracking can be found in (Aslam *et al.*, 2003; Mechitov *et al.*, 2003). In fact, binary sensors are useful for tracking partial motion information a moving object, or the change trend of certain natural phenomena (Aslam *et al.*, 2003).

Instead of improving the estimation by increasing the number of sensors, we adopt another approach by establishing feedback from the fusion center to the sensors so that a certain coordination of the sensors may be maintained. For our current model, when the statistics of the observation conditioned on the Markov process is time varying, a static quantization scheme is no longer adequate. The consequence of the feedback is that the usual static quantization scheme is then replaced by a dynamic one. Concerning the communication and computational capability in such a sensor network, we make a few basic assumptions. First, we assume that the quantized output at each sensor node is sent to the fusion center via fading channels. This work differs from the dynamic quantization considered in (Huang and Dey, 2005) where it is assumed the fusion center can receive quantized sensor outputs without error. Second, we assume that the feedback channels between the fusion center and the sensors allow error-free transmission of the computed quantizer parameters, which is plausible since the fusion center can transmit at high powers to ensure the probability of error negligible. Although the communication pattern between the fusion center and the sensors is more complicated compared to unidirectional sensor networks, this approach has the potential to reduce the network complexity from another point of view, i.e., in order to achieve a prescribed performance, one only needs to implement fewer sensor nodes compared to the case without feedback. This kind of feedback information pattern has been employed for performance improvement in the sensor network literature, but mainly in the context of hypothesis testing (Pados *et al.*, 1995; Alhakeem and Varshney, 1996), and is referred to as decision feedback.

A List of Notation:

- X_t and P: underlying finite state Markov chain and transition matrix,
- Y_t and W_t: sensor measurement and additive noise,
- f_w: joint probability density function of W_t,
- M: number of sensor nodes,
- Y_t^q: sensor output after quantization,
- Z_t and P^z: wireless channel state and transition matrix,
- $Q(c)$: channel input output transition matrix,
- Y_t^f: received symbol at the fusion center,
- $\mathcal{F}_t$: σ-field (i.e., the set of all events) generated by the observation history $Y_i^f, Z_i, i \leq t$,
- r_t: quantization parameter,
- θ_t: information state,
- $c(\theta)$: one stage cost in terms of the information state.

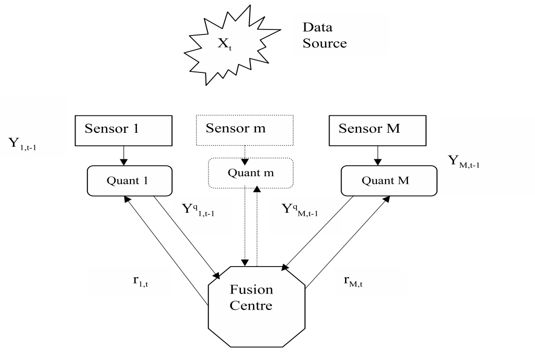

Fig. 1. The network model

2. SYSTEM MODEL

The sensor fusion problem involves an underlying Markov chain describing a certain activity of interest, sensor nodes and the fusion center. The network structure is illustrated in Fig. 1.

2.1 The Markov chain and sensor measurements

Let $\{X_t, t \geq 1\}$ be a discrete time Markov chain with state space $S = \{s_1, \cdots, s_n\}$ and transition matrix $P = (p_{ij})_{n\times n}$, where $p_{ij} = P(X_{t+1} = s_j | X_t = s_i)$. Let the measurement of the M sensors be specified by

$$Y_{m,t} = X_t + W_{m,t} \qquad 1 \leq m \leq M. \qquad (1)$$

Write (1) in the vector form $Y_t = AX_t + W_t$, where $Y_t = [Y_{1,t}, \cdots, Y_{M,t}]^T$, $A = [1, \cdots, 1]^T$ and $W_t = [W_{1,t}, \cdots, W_{M,t}]^T$. The noise $\{W_t, t \geq 1\}$ is a sequence of i.i.d. vector random variables. Denote the joint probability density function for W_t by f_w.

2.2 Sensor quantization

For a set of M binary sensors, any given quantization scheme is specified by M sequences of constants $\{r_{m,t}, t \geq 1\}$, $1 \leq m \leq M$, where $r_{m,t}$ is used to partition the range space of $Y_{m,t}$ measured by the m-th sensor node. Let $r_t = (r_{1,t}, \cdots, r_{M,t})$, and write $\{r_t, t \geq 1\} = \{(r_{1,t}, \cdots, r_{M,t}), t \geq 1\}$. r_t is called the quantization parameter. At time t, let the data (also to be called message) that is transmitted from the m-th sensor to the fusion center be denoted by $Y^q_{m,t}$. One may take any two distinct symbols a_1 and a_2 such that the events $\{Y_{m,t} < r_{m,t}\}$ and $\{Y_{m,t} \geq r_{m,t}\}$ are equivalent to $\{Y^q_{m,t} = a_1\}$ and $\{Y^q_{m,t} = a_2\}$, respectively. Hence the output symbol is

$$Y^q_{m,t} = \begin{cases} a_1 & Y_{m,t} < r_{m,t} \\ a_2 & Y_{m,t} \geq r_{m,t}. \end{cases} \quad (2)$$

Let $Y^q_t = [Y^q_{1,t}, \cdots, Y^q_{m,t}]^T$ and denote $Y^q_t = \mathcal{Q}(r_t, Y_{1,t}, \cdots, Y_{M,t})$, where the map $\mathcal{Q} : \mathbb{R}^M \times \mathbb{R}^M \to \{a_1, a_2\}^M$ is determined from (2) in an obvious manner. Here $\{a_1, a_2\}^M$ denotes the M-fold Cartesian product of the set $\{a_1, a_2\}$, which is the code book for all sensor nodes.

2.3 Wireless transmission of symbols

The quantized output at each sensor is transmitted at a fixed power level via a Markovian fading channel. Denote the m-th sensor's channel state by $Z_{m,t}$, and write $Z_t = (Z_{1,t}, \cdots, Z_{M,t})^T$. All these channels are i.i.d. with state space $S_c = \{c_1, \cdots, c_l\}$ and transition matrix $P^z = (p^z_{ij})$ where

$$p^z_{ij} = P(Z_{m,t+1} = c_j | Z_{m,t} = c_i), \ 1 \leq m \leq M. \quad (3)$$

Each c_i may be used to represent – but not necessarily identical to – a value of the channel gain. In the estimation problem, the channel states are assumed to be known at the fusion center, but not at the sensor nodes. This kind of channel information pattern has been employed in decentralized detection problems (Chamberland and Veeravalli, 2004). To simplify our further derivation of the a posteriori probability of X_t, we assume $p^z_{ij} > 0$ for all i, j.

The received symbol at the fusion center is denoted by $Y^f_{m,t}$ and described by

$$P(Y^f_{m,t} = a_j | Y^q_{m,t} = a_i, Z_{m,t} = c) = q_{ij}(c) \quad (4)$$

where $c \in S_c$ is the channel state. The m i.i.d. channels have the same input-output (I/O) transition relationship for a given channel state. Hence $q_{ij}(c)$ does not depend on the sensor index m. Write $Y^f_t = [Y^f_{1,t}, \cdots, Y^f_{m,t}]^T$ and the resulting I/O transition matrix by $Q(c) = (q_{ij}(c))$ conditioned on the channel state c. The off-diagonal entries in $Q(c)$ are called the crossover probability. Under binary quantization, $Q(c)$ is a 2×2 matrix.

2.4 State estimation and mean square error

For each sequence $\{r_t, t \geq 1\}$, the long term mean square error for the state estimation is given as

$$J(r) = \limsup_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E|X_t - \widehat{X}_t|^2 \quad (5)$$

where $\{r_t, t \geq 1\}$ is simply indicated as r and the estimate $\widehat{X}_t$ is a function of the sequence $\{Y^f_k, Z_k, k \leq t\}$. In further analysis we may also use r to denote a vector in $\mathbb{R}^M$. The determination of r as a sequence or a vector should be clear from the context. In this paper, we define the vector norm $|x| \triangleq \sum_{i=1}^n |x_i|$ for $x \in \mathbb{R}^n$.

3. THE EQUIVALENT CONTROL PROBLEM

The dynamic quantization problem may be treated as a generalized control problem in which r_t affects the observation Y^f_t at the fusion center, but the state variable X_t is autonomous. Since the fusion center is generally equipped with a high computational capability and storage capacity, we assume the parameters r_t, $t \geq 1$, are computed at the fusion center as a function of $(Y^f_1, \cdots, Y^f_{t-1}, Z_1, \cdots, Z_{t-1})$, i.e., r_t is adapted to $\mathcal{F}_{t-1} \triangleq \mathcal{F}(Y^f_i, Z_i, i \leq t-1)$ which is the σ-algebra generated by the past observations. We make the convention $\mathcal{F}_0 = \{\varnothing, \Omega\}$ where $\varnothing$ is the empty set and Ω the probability sample space. Once r_t is computed in the epoch $(t-1, t]$, the entry $r_{m,t}$ is sent by the fusion center to the m-th sensor so that it can be used by that sensor at t. In this framework, the distributed nature of the network is preserved in the sense that the data is preprocessed at the sensor level based upon which the fusion center forms a final estimate, and no direct communication exists between the sensors.

3.1 The information state and its recursion

Define the so-called information state (Kumar and Varaiya, 1986) $\theta_t = [\theta_{1,t}, \cdots, \theta_{n,t}]^T$ where $\theta_{i,t} = E[1_{(X_t = s_i)} | \mathcal{F}_t]$, $1 \leq i \leq n$, $t \geq 1$. The entry $\theta_{i,t}$ provides a measure of likelihood, as learned at the fusion center, that X_t is staying at s_i, given the observations Y^f_i, Z_i, $i \leq t$. We set $F(s_i, r_t, y^q_t) = P(Y^q_t = y^q_t | X_t = s_i, r_t)$, where y^q_t denotes a value for Y^q_t, and we have $F(s_i, r_t, (a_{i_1}, \cdots, a_{i_M})) = \int_{\mathcal{A}} f_w(y_1 - s_i, \cdots, y_M - s_i) dy_1 \cdots dy_M$, where $\mathcal{A} \triangleq \{y \in \mathbb{R}^M, \mathcal{Q}(r_t, y) = (a_{i_1}, \cdots, a_{i_M})\}$, f_w is the joint probability density function for $W_t = (W_{1,t}, \cdots, W_{M,t})^T$. In the case of two sensors, i.e., $M = 2$, then $F(s_i, r, (a_1, a_1)) = \int_{-\infty}^{r_1} \int_{-\infty}^{r_2} f_w(y_1 - s_i, y_2 - s_i) dy_1 dy_2$, etc.

Using the expression (4), we denote

$$P(Y^f_t | Y^q_t, Z_t) = \Pi_{m=1}^{M} q_{i_m j_m}(Z_{m,t}) \quad (6)$$

when $Y^q_{m,t} = a_{i_m}$ and $Y^f_{m,t} = a_{j_m}$, $1 \leq m \leq M$. Let $p(Z_t, Z_{t+1})$ stand for $\Pi_{m=1}^{M} p^z_{i_m j_m}$ (the product of M transition probabilities) when $Z_{m,t} = c_{i_m}$, $Z_{m,t+1} = c_{j_m}$, $1 \le m \le M$. Let

$$\hat{Q}_i(s_i, r_t, y_t^f, z_t) = \sum_{y_t^q \in \{a_1, a_2\}^M} F(s_i, r_t, y_t^q) P(y_t^f | y_t^q, z_t),$$

where the lower case variables y_t^q, y_t^f and z_t denote a value for the corresponding upper case vector random variables. Let

$$\hat{Q}(s_1, \cdots, s_n, r_t, y_t^f, z_t) = \mathrm{Diag}\left(Q_i(s_i, r_t, y_t^f, z_t)\right)_i$$

Proposition 1. θ_t is recursively given as

$$\begin{aligned}\theta_{t+1} &= \frac{p(Z_t, Z_{t+1})}{\alpha'_{t+1}} \\ &\quad \times \hat{Q}(s_1, \cdots, s_n, r_{t+1}, Y_{t+1}^f, Z_{t+1}) P^T \theta_t \\ &\triangleq \frac{1}{\alpha_{t+1}} T(s_1, \cdots, s_n, r_{t+1}, Y_{t+1}^f, Z_{t+1}) \theta_t \quad (7)\end{aligned}$$

where P is the transition matrix of X_t, α'_{t+1} and α_{t+1} are normalizing factors. □

Note that the term $p(Z_t, Z_{t+1}) > 0$ since it is assumed in Section II that $p_{ij}^z > 0$ for all i, j. Since $p(Z_t, Z_{t+1})$ is a common factor for all entries in θ_{t+1}, it vanishes after normalization. Hence the right hand side of (7) does not explicitly involve Z_t when θ_t is given. The matrix $T(s_1, \cdots, s_n, r_t, y_t^f, z_{t+1})$ may be simply written as $T(r_t, y_t^f, z_{t+1}) = \hat{Q} P^T$.

3.2 Markov decision with complete information

Given $\mathcal{F}_t$, the conditional expectation of X_t is

$$\widehat{X}_t = E[X_t | \mathcal{F}_t] = \sum_{i=1}^{n} s_i \theta_{i,t}. \quad (8)$$

In fact, for any quantization sequence $\{r_t\}$, $E|X_t - \widehat{X}_t|^2 = \inf_{\xi_t} E|X_t - \xi_t|^2$, where ξ_t is a random variable adapted to $\mathcal{F}_t$. By this fact, in future analysis $\widehat{X}_t$ in (5) is always taken as the conditional expectation (8). Set the conditional cost

$$c(\theta_t) = E[|X_t - \widehat{X}_t|^2 | \mathcal{F}_t] = \sum_{i=1}^{n} [s_i - \sum_{j=1}^{n} s_j \theta_{j,t}]^2 \theta_{i,t}.$$

For the case $n = 2$, $c(\theta_t)|_{n=2} = (s_1 - s_2)^2 \theta_{1,t} \theta_{2,t}$.

Now the optimal estimation problem associated with (5) may be equivalently expressed as

$$(\mathbb{P}) \text{ minimize } J(r, z, \theta) = \limsup_{N \to \infty} \frac{1}{N} \sum_{t=1}^{N} E[c(\theta_t) | z, \theta] \quad (9)$$

where (z, θ) is the initial condition at $t = 1$ and r denotes $\{r_t, t \ge 1\}$ with r_t adapted to $\mathcal{F}_{t-1}$.

Although one can apply dynamic programming for the optimization of r_t in the overall space $\mathbb{R}^M$, this leads to considerable difficulty for both computation and implementation. First, such an optimization involves non-convex minimization with respect to r in the resulting dynamic programming equation, which is difficult to solve. Second, once the fusion center has selected the value for r_t, it needs to transmit it by a back channel. Due to bandwidth limitation, it is unrealistic to transmit a quantity varying in a continuum.

For the above reasons, we restrict the range of r_t to be a discrete subset of $\mathbb{R}^M$. For notational and computational simplicity, the same finite subset of $\mathbb{R}$ is employed for optimizing each entry r_m in $r \in \mathbb{R}^M$, $1 \le m \le M$. Now, let the range space of $r_{m,t}$ be $L_d = \{\gamma_1, \cdots, \gamma_d\} \subset \mathbb{R}$. Hence r shall be chosen from the product set L_d^M.

Notice that the fusion center cannot directly minimize the cost (5) since it does not have exact knowledge of X_t. However, it can solve the problem $(\mathbb{P})$ since θ_t may be recursively computed using Y_i^f, Z_t, $i \le t$. Indeed, $(\mathbb{P})$ is a standard Markov decision problem with full information, and its associated dynamic programming (Bellman) equation is given as

$$\begin{aligned}\lambda + h(z, \theta) &= \min_{r \in L_d^M} \Big[c(\theta) + \sum_{y^f, z'} p(z, z') \\ &\quad \times |T(r, y^f, z')\theta| h\big(z', \frac{T(r, y^f, z')\theta}{|T(r, y^f, z')\theta|}\big) \Big] \\ &\triangleq \min_r \Phi(z, \theta, r) \quad (10)\end{aligned}$$

where $y^f \in \{a_1, a_2\}^M$ and $z \in \{c_1, \cdots, c_l\}^M$. The function $h(\theta)$ is called the differential cost. Define the simplex $S_1 \triangleq \{\alpha \in \mathbb{R}_+^n, |\alpha| = 1\}$, which is the range space of θ_t.

3.3 Solution to the Bellman equation

For establishing the solvability of the Bellman equation (10), we introduce the assumption:

(H1) For any $r \in L_d^M$, $y^f \in \{a_1, a_2\}^M$ and $z \in \{c_1, \cdots, c_l\}^M$, the matrix $T(r, y^f, z)$ is nonsingular and strictly positive. □

(H1) holds for nonsingular and positive P combined with very mild conditions for the noise.

Proposition 2. Under (H1), there exist λ and a bounded function h satisfying (10). □

Remark: By the verification theorem (Fernandez-Gaucherand *et al.*, 1991), the constant λ in Proposition 2 may be interpreted as the minimum for $J(r)$ in (5) when each r_t is restricted to be in L_d^M and adapted to $\mathcal{F}_{t-1}$. In Proposition 2, (H1) provides a sufficient condition and may be relaxed

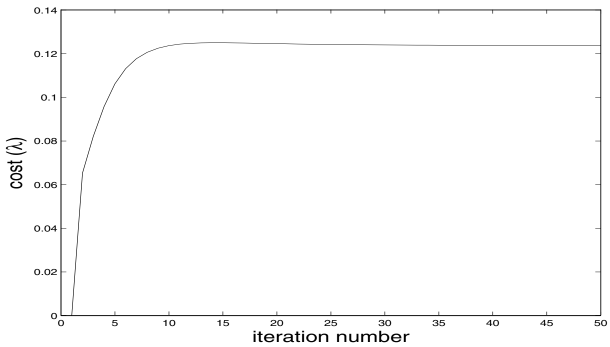

Fig. 2. Convergence of the optimal cost.

such that $T(r, y^f, z)$ is only primitive and invertible (Fernandez-Gaucherand *et al.*, 1991) for any $r \in L_d^M$, $y^f \in \{a_1, a_2\}^M$ and $z \in \{c_1, \cdots, c_l\}^M$. □

3.4 Discretization of the Bellman Equation

For notational simplicity, the numerical procedure for solving (10) is described for the case of $n = 2$, i.e., $\theta \in \mathbb{R}^2$. The same procedure can be employed for the case $n > 2$. Taking $n = 2$, let the range space S_1 of θ be discretized with a step size $\frac{1}{N}$. Let $S_{1,N} = \{[\frac{k}{N}, 1 - \frac{k}{N}]^T,\ k = 0, \cdots, N\}$. Take $\theta \in S_{1,N}$ for the left hand side of (10). However, due to the linear transformation and normalization inside the function h, the right hand side of (10) involves values of h at points outside $S_{1,N}$. Hence this cannot induce an equation only in terms of values of h on the grid $S_{1,N}$. To overcome this difficulty, we consider an approximation by rounding off $\theta' = \frac{T\theta}{|T\theta|}$ to the closest point θ'' in $S_{1,N}$, and then we simply replace $h(\theta')$ by $h(\theta'')$. This leads to a fully discretized equation:

$$\lambda + h(z, l_k) = \min_{r_i \in L_d} \Big[c(l_k) + \sum_{y^f, z'} p(z, z') \times |T(r, y^f, z') l_k| h\big(z', [\frac{T(r, y^f, z') l_k}{|T(r, y^f, z') l_k|}]_{rnd}\big)\Big] \quad (11)$$

where $l_k \in S_{1,N}$, and for $\theta = [\beta_1, \beta_2]^T \in S_1$, $[\beta]_{rnd} = ([\beta_1]_{rnd}, 1 - [\beta_1]_{rnd})^T$ with $[\beta_1]_{rnd}$ given as (i) $\frac{k}{N}$, for $\beta \in (\frac{k}{N} - \frac{1}{2N}, \frac{k}{N} + \frac{1}{2N}]$, (ii) 0, for $\beta \in [0, \frac{1}{2N}]$, and (iii) 1, for $\beta \in (1 - \frac{1}{2N}, 1]$.

Equation (11) is equivalent to the Bellman equation for a standard finite state Markov decision problem and can be solved by the relative value iteration method (see (Bertsekas, 1995) for details).

4. NUMERICAL EXAMPLES

4.1 Tracking multiple state slow Markov chains

We consider a three state Markov chain $\{X_t, t \geq 1\}$ and two sensors. Suppose X_t has state space $\{s_1 = 0, s_2 = 1, s_3 = 2.5\}$ and transition matrix

$$P = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.15 & 0.85 \end{bmatrix}.$$

The two sensors have i.i.d. Gaussian measurement noise $W_{i,t}$, $i = 1, 2$, with variance $\sigma^2 = 0.3$. The two sensors' messages are transmitted by independent Markovian fading channels with states c_1 and c_2, and transition matrix P^z. Following definition (4), denote the channel I/O relationship by $Q(c_1)$ and $Q(c_2)$. We take

$$P^z = \begin{bmatrix} 0.85 & 0.15 \\ 0.2 & 0.8 \end{bmatrix}, \quad Q(c_i) = \begin{bmatrix} 1 - k_i\varepsilon & \varepsilon \\ \varepsilon & 1 - k_i\varepsilon \end{bmatrix},$$

where $k_1 = 1$ and $k_2 = 10$. We will analyze for ε in the range $[0, 0.02]$. A larger crossover probability 10ε in $Q(c_2)$ indicates that state c_2 produces a higher error probability in transmission than c_1. For the trivial case $\varepsilon = 0$, this example reduces to the error-free transmission scenario considered in (Huang and Dey, 2005), which leads to a simpler dynamic programming equation.

4.2 Iteration of the discretized Bellman equation

The relative value iteration algorithm is employed to solve equation (11) for which θ is three dimensional, and l_k has a step size 0.01 for each of its components. The quantization parameter is optimized from the set $L_d = \{0.4, 0.8, 1.2, 1.6, 2.0\}$. For $Q(c_1)$ and $Q(c_2)$, the parameter is taken as $\varepsilon = 0.001$. The solution is computed by 50 iterates. The convergence of the optimal cost J, given by $\lambda = 0.123844$, is illustrated in Fig. 2.

4.3 Performance improvement by feedback

As noted in (Huang and Dey, 2005), for a static quantization scheme, i.e, all r_t take the same fixed value, we can also formally write a trivial dynamic programming equation where r_t is chosen from a singleton. Then one can also compute the associated estimation performance by the relative value iteration algorithm. We consider two different static quantization schemes with $\varepsilon = 0.001$.

4.3.1. Homogenous sensors We compute the cost when the scalar quantization parameters r_1 and r_2 for the two sensors take identical values. In Table I, the lowest cost $J = 0.155913$ is attained by $r_1 = r_2 = 1.4$.

4.3.2. Heterogenous sensors We take $r_1 = 0.5$ and $r_2 = 1.75$. The sensors are used in a complementary manner such that the two quantization parameters are each located at the middle point of two adjacent states of the Markov chain X_t. The cost is $J = 0.164329$ obtained by 50 iterates.

Table 1. Costs computed by 50 iterates

value for $r_1 = r_2$	cost J with $\varepsilon = 0.001$
0.6	0.258244
0.7	0.233841
0.8	0.214568
0.9	0.197525
1.0	0.181089
1.1	0.165935
1.2	0.162217
1.3	0.157508
1.4	0.155913
1.5	0.160901
1.6	0.163224
1.7	0.171080
1.8	0.181078

Table 2. Two types of costs (50 iterates)

ε	J_d dynam.	J_s static	$\lvert J_d - J_s \rvert / J_s$
.0000	.116248	.143506 ($r = 1.4$)	18.99%
.0001	.116986	.145734 ($r = 1.4$)	19.73%
.0005	.119414	.150916 ($r = 1.4$)	20.87%
.0010	.123844	.155913($r = 1.4$)	20.57%
.0050	.159924	.189665 ($r = 1.4$)	15.68%
.0100	.195775	.230424 ($r = 1.5$)	15.04%
.0200	.279314	.315798 ($r = 1.4$)	11.55%

By dynamic quantization, the obtained optimal cost $\lambda = 0.123844$ is lower than the previous costs obtained by the two types of static quantization.

4.4 Effect of channel reliability on performance

To illustrate the effect of the channel's crossover probability on the optimal cost, the numerical solution of the Bellman equation is computed with ε taking different values in $S_\varepsilon = \{0, 0.0001, 0.0005, 0.001, 0.005, 0.01, 0.02\}$.

The optimal cost J_d with dynamic quantization for different ε is listed in the second column in Table II. As expected, when ε increases, the estimation error J_d increases. In the third column J_s denotes the lowest attainable cost (with the associated parameter in the parentheses) for homogenous sensors with static quantization where $r_1 = r_2 = r$ is selected from $\{0.6, 0.7, \cdots, 1.8\}$. When the channel error rate is low, the profit of feedback is especially high. It is interesting to note that as ε increases, the effectiveness of feedback, as measured by the relative improvement between J_d and J_s, is weakened as shown by the last column in the table. An intuitive interpretation is that a less reliable channel makes it more difficult for the fusion center to extract information from the received messages for guiding quantization threshold setting. This consequently limits the effectiveness of fusion feedback.

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