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P* TYPE MODELS: EVALUATION AND FORECASTS

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ABSTRACT

This paper critically evaluates the Federal Reserve's p* model of inflation, and develops a model of national income determination implicit in the p* formulation. We use this model to forecast the future paths of key macroeconomic variables and investigate its behavior under a variety of deterministic monetary policy rules. These forecasts and policy simulations suggest a dynamic economic behavior inconsistent with stylized facts, and lead us to question the underlying structure of the p* formulation.

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The recent study by Hallman, Porter and Small (1989) of the inflation process has received considerable attention including citation in Chairman Greenspan's testimony on the Federal Reserve's Monetary Policy Objectives.[1] The purpose of this paper is to review critically the assumptions that are the basis of that inflation model, the so called P^* model, and to present the complete model of nominal income, real output and inflation that is <u>implicit</u> in the assumptions of that model given the behavior of the monetary base and potential real output. This macroeconomic model is then used to forecast in-sample and post-sample economic behavior and to evaluate the efficiency of various monetary base rules.

The dynamic behavior of real output growth and inflation implied by the P^* model of income determination appears suspect as an accurate depiction of the behavior of the U.S. macroeconomy. In this model real output growth and inflation are strongly negatively correlated and both exhibit damped oscillatory responses to exogenous shocks. Nominal income growth thus remains essentially constant: McCallum-Metzler style rules can affect only small improvements in this environment. Our analysis leads us, therefore, to question both the practical applicability and the theoretical foundation of the P^* model.

The paper proceeds as follows. In Section I we examine the fundamental assumptions of the P^* model in light of the existing literature on the behavior of M2 velocity. Specifically, we analyze the empirical support for the main proposition of the P^* model, that the velocity of M2 exhibits mean reversion, and discuss the corollary that the price level also exhibits mean reversion. Finally, we critically evaluate the P^*

[1]Testimony of Allan Greenspan on 1989 Monetary Policy Objectives, February 21, 1989, Board of Governors of the Federal Reserve System, p. 8; Carlson (1989), Humphrey (1989), Kuttner (1990), Christiano (1989).

reversion hypothesis. In Section II we discuss some of the implicit assumptions of the P^* model about the monetary base and the base-M2 multiplier, and from these implicit assumptions develop the complete model of income determination. This model is then estimated. We examine the historical performance and forecasts of this model in Section III, and in Section IV we analyze and compare alternative policy regimes. The paper concludes in Section V.

I. Critical Assumptions of the P^* Model.

A. Mean Reversion

The fundamental proposition of the P^* model of Hallman, Porter and Small (HPS) is a hypothesis about the type of shocks that drive the behavior of the velocity (V2) of M2. This hypothesis is an implementation of a long-run classical quantity theory of money with respect to US M2. Their analysis contributes to the recent discussions in macroeconomics that focus on how different kinds of shocks affect the economy. In particular, researchers are interested in distinguishing between transitory shocks to the level of a variable, permanent shocks to its level and permanent shocks to its growth rate. As an example the working hypothesis that the velocity of the monetary base is a random walk is frequently adopted; i.e. that its behavior is predominately affected by permanent shocks to its level. Many other economic variables appear to share this characteristic (Nelson and Plosser [1982]). Such variables do not exhibit reversion to a trend or mean.[2]

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[2]In more technical terms such variables are said to be nonstationary in levels, to possess unit roots, to be drift stationary, or to be integrated of order ≥ 1.

HPS assume that shocks to V2 are <u>transitory shocks to the level of V2</u>, and that V2 eventually reverts to an unchanged mean (the V2 series is stationary).[3] The mean to which V2 reverts is V^*, the sample mean of V2 over the period 55,1 through 88,1 (quarterly) of 1.6527. The hypothesis about the mean reverting behavior of V2 is crucial to the entire P^* model. If it is false, then V^* does not exist![4]

The properties of V^* are critical to the P^* model because P^* is defined in terms of V^*:

$$(1) \quad P^* = (M2 \times V^*)/QPOT,$$

where QPOT is a measure of potential real output[5] and M2 is <u>actual</u> M2. In terms of logs

$$(2) \quad \ln P^*_t = \ln M2_t + \ln V^* - \ln QPOT_t$$

Since

$$(3) \quad \ln P_t = \ln M2_t + \ln V2_t - \ln Q_t$$

where Q_t is actual real GNP, the "Price-Gap" which is the driving variable in the HPS inflation model is

$$(4) \quad (\ln P_t - \ln P^*_t) = (\ln V2_t - \ln V^*) - (\ln Q_t - \ln QPOT_t).$$

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[3]HPS leave open the possibility that the mean of V2 may have changed since 1981.

[4]Gould and Nelson (1974) and Nelson and Plosser (1982) find that annual velocity of M2 cannot be distinguished from a random walk. However their measure of M2 differs from that used by HPS. Both Gould and Nelson and Nelson and Plosser use M2 as defined by Friedman and Schwartz (1963). The official Federal Reserve concepts used by HPS are closer to what Friedman and Schwartz call M3. Engle and Granger (1989) find marginal evidence that lnM2 (current definitions) and lnGNP are cointegrated using quarterly data from 59.1-81.2. Cointegration of these variables is a necessary (but not sufficient) condition for V2 to exhibit mean reversion.

[5]Potential real output is estimated following the procedure in Braun (1990).

By assumption the first term of the right hand side of equation (4) is the deviation of V2 from its sample mean. The second term is the deviation of actual real output from real potential output. The typical construction of real potential output imposes the condition that real output reverts to real potential output over the course of one or more business cycles, or alternatively that $(\ln Q_t - \ln QPOT_t)$ reverts to zero. Hence $(\ln Q_t - \ln QPOT_t)$ is forced to exhibit transitory shocks to its level. Since $(\ln P_t - \ln P^*_t)$ is just the sum of these two terms under the critical V^* assumption, deviations of $\ln P_t$ from $\ln P^*_t$ revert to zero. Thus the V^* hypothesis establishes P^* as the equilibrium price level in the economy towards which the actual price level reverts.

Unfortunately a definitive answer to the question of the stationarity of lnV2 cannot be achieved. Tests for unit roots, such as the Augmented Dickey-Fuller (ADF) Test, produce values that are on the margin of rejection (of the unit root hypothesis) at the 5% level. However, it is well known that such tests have low power against the alternative hypothesis of a near unit (but stationary) root. For example, (Eichenbaum and Christiano [1989]) discuss the difficulty of testing stationarity versus nonstationarity and are dubious as to whether the imposition of a unit root has any real policy or forecasting implications. Under these circumstances in section II we test an implication of the stationarity of V2 that is important in understanding the effects of monetary policy on inflation, nominal income and real output.

B. The Reversion Process

HPS model the dynamics of the reversion process by their "Price-Gap" hypothesis:

$$(5) \quad \Delta INF_t = \alpha(\ln P_{t-1} - \ln P^*_{t-1}) + \sum_{i=1}^{4} \beta_i \Delta INF_{t-i} + \epsilon_t$$

where $INF_t = \Delta \ln P_t$ measures the inflation rate. They attempt to justify this equation as a model of economic behavior based upon an inflation expectation mechanism. These assumptions are reminiscent of the attempts in the early 1960s to construct an economic theory of the empirical Phillips curve relationship. This relationship broke down with the emergence of inflation in the late 1960s and 70s because it was not based on economic behavior, but rather a reduced form that was specific to the inflationary experience of the 1950s and early 60s. We should be concerned that the "Price-Gap" model may suffer a similar fate.

The "Price-Gap" equation (5) may be interpreted, alternatively, as a model of the time series properties of $\ln P_t$ (or a reduced form model) rather than as a model of economic behavior. It is known that the time series behavior of the quarterly GNP deflator is well described by an ARIMA (0,2,1) model where the estimated value of the single moving average parameter is on the order of -.65 (Rasche, [1987], Table IV.1). This process is

$$(6) \quad \Delta INF_t = (\mu_t - .65\mu_{t-1}),$$

or in autoregressive representation

$$(7) \quad \sum_{i=0}^{\infty} (.65)^i \Delta INF_{t-i} = \mu_t,$$

or

$$(8) \quad \Delta INF_t = -.65\Delta INF_{t-1} - .42\Delta INF_{t-2} - .27\Delta INF_{t-3} - .18\Delta INF_{t-4}$$

$$- \sum_{i=5}^{\infty} (.65)^i \Delta INF_{t-1} + \mu_t \ .$$

The estimated lag coefficients in the "Price-Gap" equation are quite similar to the first four lag coefficients in equation (8). Since 88 percent of the coefficient weights in an infinite geometric distributed lag with a coefficient of .65 is achieved by lag 4, the fourth order autoregressive structure assumed in the "Price-Gap" equation will be an extremely close approximation to the infinite order AR structure of the ARIMA (0,2,1) model of lnP. The only other difference is the inclusion of $(\ln P_{t-1} - \ln P^*_{t-1})$ in equation (5) but not in equation (7). However, this term is an appropriate addition to the time series model in equation (7) under the V^* hypothesis. Under this hypothesis deviations of $\ln P_t$ from $\ln P^*_t$ are only transitory as argued above. If in addition $\ln P_t$ is an ARIMA (0,2,1) model, then $\ln P_t$ and $\ln P^*_t$ satisfy the properties of cointegrated variables (Granger and Engle [1987]). Under these conditions the Granger Representation Theorem establishes that the time series (reduced form) of $\ln P_t$ (and $\ln P^*_t$) is described by an error correction model of the form[6]

$$(9)\quad \Delta INF_t = \sum_{i=1}^{n} \beta_i \Delta INF_{t-i} + \sum_{i=1}^{n} \alpha_i \Delta^j \ln P^*_{t-i} + \theta(\ln P_{t-1} - \ln P^*_{t-1}) + \epsilon_t$$

If we assume $\alpha_i = 0$ for all i, the error correction model for $(INF_t - INF_{t-1})$ has the identical form to the "Price-Gap" equation.

There is no difficulty in using a correctly specified time series model of the inflation rate for forecasting purposes. However, some caution is required in using such an equation to investigate the outcomes of monetary policies that differ significantly from the way that the Federal Reserve has historically conducted monetary policy. Under such conditions the time series properties of the inflation rate can change

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[6]The order of differencing of $\ln P^*$ (j) is that required to achieve stationarity of this time series. From equation (2) it is likely that lnM2 will determine the degree of integration of $\ln P^*$.

considerably, and a model such as equation (9) or equation (5) will be inappropriate and inaccurate. The "Price-Gap" equation can be seriously affected by the "Lucas Critique" problem under such conditions.

II. The V^* -- Price-Gap Model as a Model of Income Determination.

At first glance, it is not apparent that the V^* -- Price-Gap model offers an explanation of the impact of monetary policy on the economy. Clearly it is capable of predicting the impact of changes in M2 on nominal income, the price level and real output, but as Levy [1989] notes in Congressional testimony, the model does not appear to link any of the important macro aggregates to anything directly under the control of the monetary authorities.

Upon close examination a link between the monetary base and economic activity is implicit in the assumptions of the V^* -- Price-Gap model. It is well-documented that the velocity of the monetary base is driven by permanent shocks to the level of the base, does not exhibit a tendency to revert to trend, and as a first approximation it is well characterized as a random walk (e.g. Rasche [1987, 1988]).

Base velocity and M2 velocity are related by the identity:

(10) $\ln VB - \ln MULT2 = \ln V2$

where VB is base velocity and MULT2 is the base multiplier for M2. It is also likely that the M2 multiplier is dominantly affected by permanent shocks to its level (Rasche and Johannes, [1987]). If the hypothesis that V2 reverts to its mean is correct, then lnVB and lnMULT2 satisfy the definition of <u>cointegrated time series</u>.[7] The Granger Representation

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[7]This is true regardless of whether lnMULT2 is driven by permanent or transitory shocks to its level. The monetary base is used here to represent a whole class of potential policy instruments. The causal linkage between the monetary base, M2, and

Theorem (Engle and Granger [1987]) shows that there is a reduced-form error correction model for all cointegrated variables. Hence the assumption that lnV2 reverts to its mean is an assumption of the existence of a reduced-form bivariate error correction model in lnVB and lnMULT2. Estimates of such an error correction model are given in Table 1 over the 55,1-88,1 sample period.[8] Note that the maximum lag in the VAR in the log differences of VB and MULT2 is 2. A preliminary estimation was constructed with a lag length of 4, but all estimated coefficients of lags greater than 2 were insignificant. When the lag length is truncated at 2 all autocorrelation coefficients up to order 4 are less than .09 in absolute value, and Ljung-Box tests reject the hypothesis of serial correlation in the residuals of each equation. The error correction model includes both a constant and a dummy variable (D82) which is zero through 81,4 and 1.0 thereafter. The latter variable is included because of the strong evidence of a "shift in the drift" of base velocity in late 1981 (Rasche, [1987, 1988]). The equations for lnVB and lnMULT2 are estimated by seemingly unrelated regression (SUR) and the cross equation differences of the estimated constants and the estimated coefficients of D82 are constrained

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thus prices implicit in the P^* model is empirically valid for the US, but may not be so in other countries, such as the UK. This leads one to doubt the general applicability of the model. However, it may be in other countries that a P^* type relationship holds for some other measure of money, such as M1.

[8]The Monetary Base data are those from the St. Louis Federal Reserve Bank Adjusted Monetary Base as published in August 1989. The GNP and M2 data are those described above. It is not necessary to assume that the monetary base is the monetary policy operating variable that will be set by the Federal Reserve. Alternative error correction models could be specified from the net monetary base (net of seasonal and adjustment borrowings), total reserves, or nonborrowed reserves. It is likely that such alternative error correction structures will exhibit larger variances for both the operating variable velocity (and hence nominal income) and multipliers (and hence M2) than the variances for the monetary base that are estimated here.

to zero. This is required to be consistent with the assumption that lnV2 reverts to its mean and does not exhibit any trend (Engle and Yoo, [1987]). The estimated constant and the estimated coefficient on D82 are constrained to sum to zero since there is substantial evidence that there is no drift in base velocity after 1981. These three restrictions are not rejected by the data.[9]

In the absence of definative evidence that lnV2 is not stationary it is unlikely that these specifications are subject to serious "spurious regression" problems. The significance of the estimated coefficients on the lagged deviations of lnV2 from its sample mean [$lnV2_{t-1}-lnV^*$] in both of the equations in Table I is an additional important test of the hypothesis that V2 reverts to its mean. If these coefficients are not significantly different from zero, then there is no evidence for a relationship between the <u>price level</u> and a policy instrument subject to Federal Reserve control. If the hypothesis that these coefficients equal zero cannot be rejected, then the model becomes a simple VAR in the log first difference of VB and MULT2 and the data do not support the assumption that the difference in these variables is stationary. Since this difference is just the log of V2, such a conclusion is inconsistent with the hypothesis that V2 reverts to its mean. The computed Likelihood-Ratio test statistic for the test that the two coefficients on ($lnV2_{t-1}-lnV^*$) are jointly equal to zero is 12.82 and has an asymptotic $\chi^2_{(2)}$ distribution.

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[9]SUR estimation is used for three reasons: 1. the cointegration hypothesis requires cross equation restrictions on the estimated coefficient of the error correction model; 2. we want to test additional joint restrictions on other coefficients that appear in separate equations and 3. an estimate of the covariance matrix of the several error terms which appear in the model is required to determine the stochastic properties of real output, inflation and nominal income.

Thus, the hypothesis that lnVB and lnMULT2 are cointegrated with cointegration vector (1,-1) is not rejected at .015 level.

For a given path of the monetary base established (explicitly or implicitly) by the Federal Reserve, the error correction model of lnVB determines the resulting path of nominal GNP. For the same path of the monetary base, the error correction model for lnMULT2 determines the resulting path of M2. Both paths are constrained by the assumption that V2 reverts to its mean by the structure of the error correction process. Thus the V^* model is implicitly a model of nominal income driven by the monetary base. The error correction model is an explicit reduced form representation of that implicit model.

The forecast of M2 from the M2 multiplier model and the path of the monetary base can be combined with the estimate of V^* and the value of potential output to produce a forecast of P^*. This forecast of P^* can be used with the HPS "Price-Gap" hypothesis to generate forecasts of the path of the price level and inflation. The forecast path of real GNP is then determined as the residual component of nominal GNP through the identity

(11) $\ln Y - \ln P = \ln Q$

where Y is nominal GNP and Q is real GNP.[10]

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[10]The structure, though not the equations, of the V^* -- Price-Gap model is closely related to the structure of the old "St. Louis Model" (Anderson and Carlson, [1970]). An alternative model of inflation proposed by Kuttner (1990) (equation (4)) also can be combined with the error correction model for base velocity and the M2 multiplier to produce a complete model of inflation, real output, and nominal income, driven by assumed paths of the monetary base and real potential output. The behavior of nominal income in the two models is identical. Kuttner's analysis of the inflation dynamics of his alternative model ([1990], pp. 13-16) is incorrect because he fails to realize that real output is determined endogenously in his model, and sets it exogenously at real potential output.

Our data closely replicate the estimates of the OLS "Price-Gap" equation reported by HPS. For the sample period 55,1-88,1 our estimates are:

$$(12)\quad \underset{}{\Delta INF_t} = \underset{(.008)}{-.030}[\ln P_{t-1} - \ln P^*_{t-1}] - \underset{(.085)}{.643}\Delta INF_{t-1} - \underset{(.098)}{.445}\Delta INF_{t-2}$$

$$- \underset{(.097)}{.262}\Delta INF_{t-3} - \underset{(.080)}{.077}\Delta INF_{t-4} \qquad \bar{R}^2 = .307 \quad se = .00392$$

We estimate all three stochastic equations of the income determination model (the two error correction equations and the "Price-Gap" equation) by seemingly unrelated regression (SUR). The resulting estimates are given in Table II.

The estimates of the equations for base velocity and the M2-Base multiplier in Table II are quite similar to those in Table I. In addition the estimated coefficients in the inflation rate equation are almost the same as the OLS estimates for this equation reported in equation (12). The estimated covariance matrix of the disturbances indicates a sizable positive correlation (.38) between the disturbance in the base multiplier equation and the disturbance in the inflation equation.

We examine the stability of these specifications in the subsamples 55-73 and 74-88:1. This is not an equal split of the sample period, but it isolates the pre- and post- 1973 oil shock experience in the two subsamples. We add to each equation interaction terms for each of the (17) regressors with a dummy variable that is zero through 73:4 and 1.0 thereafter. We then test the joint hypothesis that the coefficients on all these interaction variables are zero. (This test is equivalent to a Chow test.) The computed value of the test statistic is 8.99 which is distributed as χ^2 with 17 degrees of freedom.

Thus, the tests do not reject the hypothesis that the coefficients are stable between the pre- and post-oil shock sample periods.

We also estimate the model in Table II recursively for samples beginning with 55:1 and ending with 74:1 through 88:1. We focus on estimates of the three error correction coefficients. Recursive estimates of these parameters are plotted in Figure I against their respective end of sample dates. The behavior of these estimates as the sample size is changed reflects the relative precision of the estimates in Table II. The estimated error correction coefficient in the inflation equation is the most stable; the estimated error correction coefficient in the real output equation is the least stable. The major instability in the latter series is from the middle of 1980 to late 1981. This period immediately follows the credit control experience in the U.S. and we believe the unusual behavior of GNP at that time is responsible for the transitory variability of this parameter estimate. On the whole, we believe that the recursive regressions substantially confirm the stability of the reduced form error correction models.

III. Historical Accuracy and Forecast Accuracy of a V^* -- Price-Gap Income Determination Model.

There are two practical applications for the P^* model of income determination: forecasting and the evaluation of policy. In this and the following section we ignore the caveats we and others have articulated for using this model in either of these ways, and generate forecasts and policy evaluations. These exercises give some insight into the reactions of the U.S. economy should the Fed implement monetary policy using this model.

We carry out these analyses under the assumption that the structure and parameters of the model are invariant to the choice of policy rules for the monetary base, and thus recognize that any results derived are suspect. But, we believe that the exercise has merit, since, while the Lucas Critique is of theoretical importance, many economists contend that it is of little practical importance (McCallum, 1988), for a range of experiments like that considered below.

Some characteristics of the model's accuracy during the recent post-sample period is presented in the attached Figures 2-6 for the growth rate of the monetary base, nominal GNP, the inflation rate, the price level, and real GNP. These Figures show the (one-period ahead) model errors for the post-sample period (88,2-89,2).

The important characteristic of these graphs is that the base velocity equation does not catch the upward drift that is currently estimated to have occurred in 88-89. During the five post-sample quarters, the mean error in base velocity growth is 2.5 percent (annual rates) which is significantly different from zero (t ratio = 3.16). The errors of the inflation equation on the other hand, are not significantly different from zero.

The important question for forecasters is how accurate are the forecasts that can be produced from such a model assuming that the paths of the exogenous variables can be projected without error. Fortunately, the log- linear structure of this model permits the derivation of the exact formula for the estimated h-step (h>0) ahead forecast errors. This formula is provided by Engle and Yoo [1987] (equation (10)) in terms of the moving average representation of the time series model.[11] This is easily determined by (1) rewriting the model in the form of an autoregressive

[11]The problem is to obtain the required polynomials (C(B), where B is the lag operator) for the moving average representation from the error correction representation in Table II.

model in the (log) levels of the endogenous variables, (2) computing the determinental polynomial of the autoregressive polynomial matrix in the lag operator B, (3) factoring the determinental polynomial into a residual polynomial times a factor with a unit root, and (4) writing the moving average representation of the (log) differences of the endogenous variables in terms of the inverse of this residual polynomial matrix and the adjoint matrix of the autoregressive polynomial matrix.[12] The implicit autoregressive and moving average representations of the four variables lnVB, lnMULT2, lnP and lnQ are given in lag operator notation in Table III.

Several important characteristics of the model are readily observed in the moving average representation in this Table. First, base velocity and the M2-Base multiplier are affected only by shocks to the first two equations in Table II, and are not affected by shocks to the "Price-Gap" equation. Hence nominal income is not affected by "Price-Gap" shocks. Second, the ultimate effect of a maintained change in the growth rate of the monetary base is an equal change in the maintained rate of growth of inflation and no change in the growth rate of real output (since $a_{33}(1)^{-1} = - \phi^{-1}$). Third, for a predetermined path of the monetary base, any "Price-Gap" shock has an equal and opposite effect on (the log of) real output, since the path of nominal income is independent of such shocks.

The impulse response functions of inflation, real output growth and nominal income growth for each of the three shocks (ϵ_{1t}, ϵ_{2t}, and ϵ_{3t}) are shown in Figures 7-9 respectively. In each case nominal income growth reaches the zero steady state response quickly (the impact and steady-state responses to the "price-gap" shock are zero as discussed above) while the inflation and real growth responses exhibit very slowly damped oscillatory

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[12]See Engle and Yoo [1987], equations (15) - (17)

behavior. The cumulative response function of the inflation rate to a maintained change in base growth is shown in Figure 10. The response exhibits slowly damped oscillations around the steady state value of 1.0. This is exactly the response found by Kuttner [1990] (Figure 1).

A particularly useful property of this type of model is that the exact forecast errors can be computed for the (log) level of each of the endogenous variables, assuming that the exogenous variables follow deterministic paths. The moving average representation in Table III has exactly the form of the model in equation (1) of Engle and Yoo [1987]. The h-step ahead forecast error variance is computed for this model in their equation (10). Of particular interest are the one period and four period (one year) ahead forecast error variances. The standard error of the one quarter ahead forecast of base velocity (and hence nominal GNP) is 4.03 percent at <u>annual rates</u>. The standard error of the one year ahead forecast of base velocity (and nominal GNP) is 2.20 percent. The first of these error statistics merely reflects the annualized standard error of the error correction model for lnVB in Table II. Base velocity has considerable quarter to quarter noise which is not well forecasted by either its own history or the history of lnMULT2 as indicated by the low R^2 of this estimated equation. For short-term forecasting, the model is only marginally better than the random walk model of base velocity.[13]

IV. Policy

Under the assumption that the structure and parameters of the model are invariant to the choice of policy rules for the monetary base,

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[13]The one-quarter ahead and one-year ahead standard errors of the base-M2 multiplier forecasts are 2.19 and 1.87 percent respectively at annual rates.

conditional expectations and variances (based on time t information) can be computed. From Table III.B it can be seen that as h goes to infinity

$$(13)\quad E_t[(1-B)\ln P_{t+h}] = [-\theta_1 a_{21}(B)+\theta_2 a_{11}(B)]\phi B a_{33}{}^{-1}(B)R^{-1}(B)\ln V^* - \phi B a_{33}(B)E_t[(1-B)\ln BASE_{t+h}] + \phi B a_{33}(B)E_t[(1-B)\ \ln QPOT_{t+h}] = E_t[(1-B)\ln BASE_{t+h}] - E_t[(1-B)\ln QPOT_{t+h}]$$

and:

$$(14)\quad E_t[(1-B)\ln Q_{t+h}] = [\theta_1 a_{21}(B)-\theta_2 a_{22}(B)]\phi B a_{33}{}^{-1}(B)R^{-1}(B)\ln V^* + [-\theta_1 a_{22}(B) + \theta_2 a_{12}(B)]R^{-1}(B)\ln V^* + (\phi B a_{33}(B)+1)E_t[(1-B)\ln BASE_{t+h}] - \phi B a_{33}(B)E_t[(1-B)\ln QPOT_{t+h}] = E_t\ [(1-B)\ln QPOT_{t+h}].$$

Under a constant base growth rule, $(1-B)\ln BASE_{t+h} = \tau$. Assuming that $\ln QPOT_{t+h} = \mu + \delta t$, then $E_t[(1-B)\ln P_{t+h}] = \tau - \delta$ and $E_t[(1-B)\ln Q_{t+h}] = \delta$ as h goes to infinity.

The conditional variances $E_t[(1-B)\ln P_{t+h} - \tau - \delta]^2$ and $E_t[(1-B)\ln Q_{t+h} - \delta]^2$ are given by the variance of $e_{t+h|t} - e_{t+h-1|t}$ in Engle and Yoo [1987]. The estimates of these variances for h = 250 (by which point they are close to the asymptotic estimates) for this model are 1.52×10^{-4} and 2.25×10^{-4} for $(1-B)\ln P_{t+h}$ and $(1-B)\ln Q_{t+h}$ respectively. Thus from the historical estimates we would ultimately expect a constant base growth rule to produce a standard deviation of the inflation rate about 1.2 percent around $\tau-\delta$ and a standard deviation of real output growth of about 1.5 percent around δ. The two are strongly negatively correlated; the estimated variance of nominal income around τ is only 1.08×10^{-4} or a standard deviation of only one percent.

These estimates can be used as a benchmark against which to judge alternative deterministic monetary policy rules with feedback such as those proposed by Meltzer [1987] and McCallum [1988]. The representation of this model augmented by the McCallum feedback rule is given in Table IV. The model specified in Table IV includes a judgmental interpretation of McCallum's "target path value of nominal income" ($\ln X^*_t$). Here we have interpreted this (in logs) as $\ln X^*_t = \ln\bar{P}_t + \ln QPOT_t$ and assume that the constant term in McCallum's policy rule is the growth rate of "target GNP"; i.e. $(1-B)(\ln\bar{P}_t + \ln QPOT_t)$.

The impulse response functions for nominal income growth in Figures 7-9 suggest that the McCallum (or Meltzer) type feedback rules have little stabilization potential with this model. The impulse response function for base velocity growth given a constant base growth is just the nominal income impulse response function for each shock. These functions suggest that base velocity will exhibit very little autocorrelation in response to either ϵ_{1t} or ϵ_{2t} shocks; i.e. it exhibits behavior close to a random walk. If base velocity is exactly a random walk, then any feedback rule which introduces variance into base growth will necessarily <u>increase</u> the variance of nominal income, since it cannot reduce the variance of base velocity.

Under the McCallum feedback rule the one-period ahead forecast error for nominal GNP is equal to that of the constant base growth rule, 4.03% at annual rates, since the feedback rule is only effective with a one period lag. The standard error of the one year ahead forecast of nominal GNP under the McCallum rule is 1.45 percent or about 35 percent smaller than the 2.20 percent under the constant base growth rule. The conditional variances of the inflation rate, nominal and real GNP growth as the forecasting horizon (h) goes to infinity are identical under this rule compared to the constant base growth rule.

The optimal response function for the monetary base can be investigated for policies that seek 1) to minimize the expected mean squared error (MSE) of real output growth around real potential output growth, 2) to minimize the expected MSE of inflation around zero and 3) to minimize the expected MSE of nominal income growth around real potential output growth. The derivation of the base growth rules (relative to real potential output growth) is outlined in Appendix A. The first conclusion that emerges from this analysis is that optimal control of real output growth is not feasible in this model. The implied response functions for base growth to each of the three shocks are rational polynomials in the lag operator, B, with the denominator polynomial in all three cases equal to $[a_{33}^{-1}(B)\phi B+1]$.[14] However $a_{33}(1) = -\phi$, so $[a_{33}^{-1}(1)\phi+1] = 0$ and the denominator polynomial has a unit root. Hence the expected mean square error of real output growth around real potential output growth in this model using the growth of the monetary base as the policy instrument fails because of instrument instability.

In contrast, the minimization of the expected MSE of nominal income growth around real potential output growth implies very simple response functions for monetary base growth. These response functions are just the negative of the polynomials in the first three columns of the first row of the moving average matrix in Table III.B, excluding the zero order term in each polynomial. These polynomials are the impulse response functions for nominal income plotted in Figures 7-10, which damp to zero.

Finally, minimization of the expected MSE of inflation around zero is feasible in this model. The optimal response functions for base growth are rational polynomials in the elements of the third row of the moving average

[14]The polynomials are defined in Table III.

polynomial matrix in Table III.B. While these response functions appear complicated, they damp quickly to zero given the estimated parameters of the model as in Figures 11-13.

IV. Conclusions

This study examines the implications of P^* type models of inflation. Under the maintained hypothesis that M2 velocity is a stationary random variable, it is shown that the P^* models imply a complete model of income determination conditional upon the behavior of the monetary base (and perhaps alternative monetary policy instruments). The forecast error statistics for the income determination models are computed for both constant base growth and McCallum feedback, monetary policy rules. Finally, it is shown that optimal control of real output growth is not feasible in such models using the monetary base, but that reasonably simple response functions are implied for the optimal control of nominal income growth or inflation using the monetary base.

Our findings highlight many troubling features of the P^* model. But, the model maintains its strong visceral appeal. Future research directed at improving this approach to income determination models must explore carefully alternative price reversion processes such as those suggested by Kuttner (1990).

Table I

DEPENDENT VARIABLE ΔlnQVB
FROM 55: 1 UNTIL 88: 1

VARIABLE	COEFFICIENT	STAND. ERROR	T-STATISTIC
CONSTANT	.3480809E-02	.7200136E-03	4.834365
D82	-.3480809E-02	.7200136E-03	-4.834365
$\Delta \ln VB_{t-1}$	.2280595	.8279808E-01	2.754406
$\Delta \ln VB_{t-2}$	-.3910711E-01	.8475201E-01	-.4614298
$\Delta \ln MULT2_{t-1}$	-.1151299	.1443930	-.7973374
$\Delta \ln MULT2_{t-2}$	.3590652	.1452109	2.472715
$\ln v2_{t-1} - \ln V^*$	-.4124354E-01	.2921858E-01	-1.411552

R^2 .181 $\bar{R}^2$ .142 SEE .103E-01

Estimated Residual Autocorrelations

1	2	3	4
-.045855	-.008085	-.039172	-.029918

$Q = .61\ [\chi^2_{(4)};\ \rho = .96]$

DEPENDENT VARIABLE ΔlnMULT2
FROM 55: 1 UNTIL 88: 1

VARIABLE	COEFFICIENT	STAND. ERROR	T-STATISTIC
CONSTANT	.3480809E-02	.7200136E-03	4.834365
D82	-.3480809E-02	.7200136E-03	-4.834365
$\Delta \ln VB_{t-1}$	-.6333032E-01	.4684530E-01	-1.351904
$\Delta \ln VB_{t-2}$	-.1086692	.4794321E-01	-2.266624
$\Delta \ln MULT2_{t-1}$	.4387845	.8181721E-01	5.362986
$\Delta \ln MULT2_{t-2}$	.1836402	.8159270E-01	2.250694
$\ln V2_{t-1} - \ln V^*$	.4888616E-01	.1616503E-01	3.024193

R^2 .42 $\bar{R}^2$ .40 SEE .562E-02

Estimated Residual Autocorrelations

1	2	3	4
.011620	-.014840	.056297	-.072790

$Q = 1.18\ [\chi^2_{(4)};\ \rho = .88]$

Restrictions: CHI-SQUARE(3) = 5.22 SIGNIFICANCE LEVEL .156

Table II

DEPENDENT VARIABLE $\Delta lnVB$
FROM 55: 1 UNTIL 88: 1

VARIABLE	COEFFICIENT	STAND. ERROR	T-STATISTIC
CONSTANT	.3400168E-02	.718057E-03	4.734912
D82	-.3400168E-02	.7181057E-03	-4.734912
$\Delta lnVB_{t-1}$	.2203893	.7867077E-01	2.801413
$\Delta lnVB_{t-2}$	-.1445296E-01	.8050591E-01	-.1795268
$\Delta lnMULT2_{t-1}$	-.4979653E-01	.1361809	-.3656645
$\Delta lnMULT2_{t-2}$	.3225544	.1366344	2.360711
$lnV2_{t-1}-lnV^*$	-.4910907E-01	.2848073E-01	-1.724291

R^2 .18 $\bar{R}^2$ .14 SEE .103E-01

Estimated Residual Autocorrelations

1	2	3	4
-.034685	-.022528	-.025436	-.029907

$Q = .43\ [\chi^2_{(4)},\ \rho = .98]$

DEPENDENT VARIABLE $\Delta lnMULT2$
FROM 55: 1 UNTIL 88: 1

VARIABLE	COEFFICIENT	STAND. ERROR	T-STATISTIC
CONSTANT	.3400168E-02	.7181057E-03	4.734912
D82	-.3400168E-02	.7181057E-03	-4.734912
$\Delta lnVB_{t-1}$	-.6101330E-01	.4675357E-01	-1.304998
$\Delta lnVB_{t-2}$	-.1099130	.4784931E-01	-2.297065
$\Delta lnMULT2_{t-1}$	.4341700	.8162360E-01	5.319172
$\Delta lnMULT2_{t-2}$	.1898260	.8139773E-01	2.332080
$lnV2_{t-1}-lnV^*$	.4974711E-01	.1614885E-01	3.080536

R^2 .42 $\bar{R}^2$ .40 SEE .562E-02

Estimated Residual Autocorrelations

1	2	3	4
.015675	-.018003	.054273	-.072068

$Q = 1.16\ [\chi^2_{(4)}\ \rho = .88]$

Table II, Continued

DEPENDENT VARIABLE ΔINF
FROM 55: 1 UNTIL 88: 1

VARIABLE	COEFFICIENT	STAND. ERROR	T-STATISTIC
$lnP_{t-1}-lnP^*_{t-1}$	-.3088223E-01	.7363251E-02	-4.194103
ΔINF	-.6766859	.7875914E-01	-8.591840
ΔINF	-.4735397	.9071340E-01	-5.220174
ΔINF	-.2904600	.8908796E-01	-3.260373
ΔINF	-.1189892	.7326262E-01	-1.624146

R^2 .33 $\bar{R}^2$ .31 SEE .393E-02

Estimated Residual Autocorrelations

1	2	3	4
.013686	.002958	-.007391	-.057763

$Q = .48\ [\chi^2_{(4)}\ ;\ \rho - .98]$

Restrictions: CHI-SQUARE(3) = 3.17 SIGNIFICANCE LEVEL .37

COVARIANCE MATRIX OF RESIDUALS

VARIABLE	ΔlnVB	ΔlnMULT2	ΔINF
ΔlnVB	.10136E-03	.15977	.38219
ΔlnMULT2	.87926E-05	.29881E-04	-.75613E-01
ΔINF	.14818E-04	-.15917E-05	.14830E-04

TABLE III

A. Autoregressive Form of Error Correction Model

$$\begin{bmatrix} a_{11}(B) & a_{12}(B) & 0 & 0 \\ a_{21}(B) & a_{22}(B) & 0 & 0 \\ 0 & \phi B & a_{33}(B) & 0 \\ -1.0 & 0 & 1.0 & 1.0 \end{bmatrix} \begin{bmatrix} \ln VB_t \\ \ln MULT2_t \\ \ln P_t \\ \ln Q_t \end{bmatrix} = \begin{bmatrix} -\theta_1 \ln V^* + \epsilon_1 \\ -\theta_2 \ln V^* + \epsilon_2 \\ -\phi B[\ln BASE_t + \ln V^* - \ln QPOT_t] + \epsilon_3 \\ \ln BASE_t \end{bmatrix}$$

Where From Table 2 it can be determined that the estimated values of the various polynomials are:

$a_{11}(B) = 1-1.1713B+.2349B^2-.0145B^3$; $a_{12}(B) = .0007B-.3724B^2+.3226B^3$; $\theta_1 = -.0491$

$a_{21}(B) = .0113B+.0489B^2-.1099B^3$; $a_{22}(B) = 1-1.3845B+.2444B^2+.1898B^3$; $\theta_2 = .0497$

$a_{33}(B) = 1-1.2925B+.1201B^2+.0201B^3+.0117B^4+.0525B^5+.1190B^6$; $\phi = -.0309$; $a_{33}(1) = .0309 = -\phi$

$(1-B)\ R(B) = a_{11}(B)*a_{22}(B)-a_{12}(B)a_{22}(B)$ so $R(B) = 1-1.5558B+.545B^2+.113B^3-.0172B^4-.0328B^5$; $R(1) = .0522$

B: Moving Average Form of Error Correction Model

$$(1-B)\begin{bmatrix} \ln VB_t \\ \ln MULT2_t \\ \ln P_t \\ \ln Q_t \end{bmatrix} = \begin{bmatrix} a_{22}(B)R^{-1}(B) & -a_{12}(B)R^{-1}(B) & 0 & 0 \\ -a_{21}(B)R^{-1}(B) & a_{11}(B)R^{-1}(B) & 0 & 0 \\ \phi B a_{21}(B)a_{33}^{-1}(B)R^{-1}(B) & -\phi B a_{11}(B)a_{33}^{-1}(B)R^{-1}(B) & a_{33}^{-1}(B)(1-B) & 0 \\ [-\phi B a_{21}(B)a_{33}^{-1}(B)+a_{22}(B)]R^{-1}(B) & [\phi B a_{11}(B)a_{33}^{-1}(B)-a_{12}(B)]R^{-1}(B) & -a_{33}^{-1}(B)(1-B) & (1-B) \end{bmatrix} \begin{bmatrix} -\theta_1 \ln V^* + \epsilon_1 \\ -\theta_2 \ln V^* + \epsilon_2 \\ -\phi B[\ln BASE_t + \ln V^* - \ln QPOT_t] + \epsilon_3 \\ \ln BASE \end{bmatrix}$$

Table IV

A. Autoregressive Form of Error Correction Model with McCallum Rule

$$\begin{bmatrix} a_{11}(B) & a_{12}(B) & 0 & 0 & 0 \\ a_{21}(B) & a_{22}(B) & 0 & 0 & 0 \\ 0 & \phi B & a_{33}(B) & 0 & \phi B \\ -1.0 & 0 & 1.0 & 1.0 & 1.0 \\ a_{51}(B) & 0 & a_{54}B & a_{54}B & (1-B) \end{bmatrix} \begin{bmatrix} \ln VB \\ \ln MULT2 \\ \ln P \\ \ln Q \\ \ln BASE \end{bmatrix} = \begin{bmatrix} -\theta_1 \ln V^* + \epsilon_1 \\ -\theta_2 \ln V^* + \epsilon_2 \\ -\phi B \ln V^* + \phi B \ln QPOT_t + \epsilon_3 \\ 0 \\ [(1-B)+a_{54}B][\ln QPOT_t + \ln \bar{P}_t] \end{bmatrix}$$

Where $a_{11}(B)$, $a_{12}(B)$, $a_{21}(B)$, $a_{22}(B)$, $a_{33}(B)R(B)$, θ_1, θ_2, and ϕ are as in Table III.

And $a_{51}(B) = (.0625B - .0625B^{17})$; $a^{54} = .25$

B. Moving Average Form of Error Correction Model with McCallum Rule

$$(1-B) \begin{bmatrix} \ln VB_t \\ \ln MULT2_t \\ \ln P_t \\ \ln Q_t \\ \ln BASE_t \end{bmatrix} = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \end{bmatrix} \begin{bmatrix} -\theta_1 \ln V^* + \epsilon_{1t} \\ -\theta_2 \ln V^* + \epsilon_{2t} \\ -\phi B \ln V^* + \phi B \ln QPOT_t + \epsilon_{3t} \\ \{1 -(1-a_{54})B\}\{\ln QPOT_t + \ln \bar{P}_t\} \end{bmatrix}$$

Table IV Continued

C. Column Vectors of Moving Average Representation

$$C_1 = \begin{bmatrix} a_{22}(B)R^{-1}(B) \\ -a_{12}(B)R^{-1}(B) \\ \{1-(1-a_{54})B\}^{-1}a_{33}^{-1}(B)R^{-1}(B)\phi B\{a_{22}(B)[a_{54}B+a_{51}(B)] + a_{21}(B)[1-(1-a_{54})B\} \\ \{1-(1-a_{54})B\}^{-1}R^{-1}(B)[a_{22}(B)[1-B-a_{51}(B)]-a_{33}^{-1}(B)\phi B\{a_{22}(B)[a_{54}B+a_{51}(B)]+a_{21}(B)[1-(1-a_{54}B)]\}) \\ -\{1-(1-a_{54})B\}^{-1}R^{-1}(B)a_{22}(B)\{a_{54}B+a_{51}(B)\} \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -a_{12}(B)R^{-1}(B) \\ a_{11}(B)R^{-1}(B) \\ -\{1-(1-a_{54})B\}^{-1}a_{33}^{-1}(B)R^{-1}(B)\phi B\{a_{21}(B)[a_{54}B+a_{51}(B)]+a_{11}(B)[1-(1-a_{54})B]\} \\ -\{1-(1-a_{54})B\}^{-1}R^{-1}(B)[a_{12}(B)\{1-B-a_{51}(B)\}-a_{33}^{-1}(B)\phi B\{a_{12}(B)[a_{54}B+a_{51}(B)]+a_{11}(B)[1-(1-a_{54})B]\} \\ \{1-(1-a_{54})B\}^{-1}R^{-1}(B)a_{12}(B)\{a_{54}B+a_{51}(B)\} \end{bmatrix}$$

$$C_3 = \begin{bmatrix} 0 \\ 0 \\ a_{33}^{-1}(B)(1-B) \\ -a_{33}^{-1}(B)(1-B) \\ 0 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} 0 \\ 0 \\ -\{1-(1-a_{54})B\}^{-1}\ a_{33}^{-1}(B)\phi B(1-B) \\ \{1-(1-a_{54})B\}^{-1}\ a_{33}^{-1}(B)\{a_{33}(B)+\phi B\}(1-B) \\ \{1-(1-a_{54})B^{-1}(1-B) \end{bmatrix}$$

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Appendix A

This appendix outlines the solution process for determining the optimal base growth rule. It follows Sargent (1987) chapter 17 closely, and the reader is referred to that source for a more complete presentation. It should be noted at the outset that these rules are optimal only if the economy can be described by a model with parameters that are invariant across the possible feedback rules the monetary authority may use. While this is a very strong assumption, the same analysis will apply if the monetary authority has operated a single policy rule for a long time and is committed (perhaps by law) to continue to use it forever.

Consider, for example, the policy which seeks to minimize the expected mean squared error of inflation around zero. (All other policies are determined by a like method and are left to the reader). First, from Table III.B, inflation is (gathering terms)

$$(1-B)\ln P_t = \Delta \ln P_t = k + W(B)\epsilon_{1t} + X(B)\epsilon_{2t} + Y(B)\epsilon_{3t} + Z(B)\Delta \ln Base_t^*$$

where k =

$$[[\phi B a_{21}(B) a_{33}^{-1}(B)] R^{-1}(B)(-\theta_1) - \phi B a_{11}(B) a_{33}^{-1}(B) R^{-1}(B)[-\theta_2] - a_{33}^{-1}(B)(1-B)\phi B] \ln V^* \quad (A.1)$$

a constant

$$W(B) = \phi B a_{21}(B) a_{33}^{-1}(B) R^{-1}(B) = c_{31}(B)$$

$$X(B) = -\phi B a_{11}(B) a_{33}^{-1}(B) R^{-1}(B) = c_{32}(B)$$

$$Y(B) = a_{33}^{-1}(B)(1-B) = c_{33}(B)$$

$$Z(B) = a_{33}^{-1}(B)\phi B$$

and

$$\Delta \ln Base^*_t = (1-B)[\ln Base_t - \ln QPOT].$$

Define the rule governing the base as

$$\Delta \ln Base^*_t = b + \sum_{i=0}^{\infty} \tau_i B^i \epsilon_{1t-1} + \sum_{i=0}^{\infty} \nu_i B^i \epsilon_{2t-1} + \sum_{i=0}^{\infty} \Psi_i B^i \epsilon_{3t-1}$$

$$= b + T(B)B\epsilon_{1t} + U(B)B\epsilon_{2t} + V(B)B\epsilon_{3t} \quad (A.2)$$

Combine (A.1) and (A.2)

$$\Delta \ln P_t = [b + Z(1)B] + [W(B) + \zeta(B)]\epsilon_{1t} + [X(B) + \xi(B)]\epsilon_{2t} + [Y(B) + \rho(B)]\epsilon_{3t} \quad (A.3)$$

where $\varsigma(B) = \sum_{i=1}^{\infty} \varsigma_i B^i = BZ(B)T(B)$

$$\xi(B) = \sum_{i=1}^{\infty} \xi_i B^i = BZ(B)U(B)$$

$$\rho(B) = \sum_{i=1}^{\infty} \rho_i B^i = BZ(B)V(B)$$

$\varsigma(B)$, $\xi(B)$ and $\rho(B)$ are all rational polynomials in the lag operator B.

The optimal policy rule (the values of b, the τ_i, v_i and Ψ_i are defined by minimizing the mean squared error (MSE) of inflation around zero. That is, since the MSE is defined as $E[\Delta lnP]^2$, the monetary authority

minimize $E[\Delta lnP_t]^2$

$\Rightarrow$

minimize $E([\Delta lnP_t - E\Delta lnP_t]^2) + (ElnP_t)^2$ (A.4)

which is composed of the bias squared, $[\Delta lnP_t - E\Delta lnP_t]^2$, and the variance $(E\Delta lnP_t)^2$. These two terms can be minimized independently.

Recall that

$E(\epsilon_{it}) = 0 \ \forall i$

$E(\epsilon_{it}\epsilon_{js}) = 0 \ \forall i \neq j, \ t \neq s$

$E(\epsilon_{it}\epsilon_{js}) = \Omega_{ij} \ t = s, \ \forall i,j.$

Then, from (A.3) the bias is

$E\Delta lnP_t = k + Z(1)b.$

To minimize the bias squared, set

$E\Delta lnP_t = 0$

$\Rightarrow$

$$b = -\frac{k}{Z(1)}$$

$$= -k$$

since $Z(1) = 1$.

From (A.3) the variance is

$$\text{var}\,\Delta\ln P_t = [w_0^2 + \sum_{j=1}^{\infty} (w_j + \zeta_j)\text{var}\epsilon_1 + [x_0^2 + \sum_{j=1}^{\infty} (x_j + \xi_j)^2]\text{var}\epsilon_2$$

$$+ [y_0^2 + \sum_{j=1}^{\infty} (y_j + \rho_j)^2]\text{var}\epsilon_3$$

$$+2[w_0x_0 + \sum_{j=1}^{\infty} (w_j + \zeta_j)(x_j + \xi_j)]\text{cov}\,(\epsilon_1\epsilon_2)$$

$$+2[w_0y_0 + \sum_{j=1}^{\infty} (w_j + \zeta_j)(y_j + \rho_j)]\text{cov}\,(\epsilon_1\epsilon_3)$$

$$+2[x_0y_0 + \sum_{j=1}^{\infty} (x_j + \xi_j)(y_j + \rho_j)]\text{cov}\,(\epsilon_2\epsilon_3).$$

Then, the authority

$$\underset{\zeta_i,\ \xi_i,\ \rho_i}{\text{minimize}} \quad \text{var}\,\Delta\ln P_t$$

The first-order necessary conditions are

$$2\text{var}\epsilon_1\ [w_i+\zeta_i] + 2\text{cov}(\epsilon_1\epsilon_2)\ [x_i + \xi_i] + 2\ \text{cov}(\epsilon_1\epsilon_3)\ [y_i + \rho_i] = 0 \quad \forall i > 0$$

$$2\text{var}\epsilon_2\ [x_i+\zeta_i] + 2\text{cov}(\epsilon_i\epsilon_2)\ [w_i + \zeta_i] + 2\ \text{cov}(\epsilon_2\epsilon_3)\ [y_i + \rho_i] = 0 \quad \forall i > 0$$

$$2\text{var}\epsilon_3\ [y_i+\rho_i] + 2\text{cov}(\epsilon_1\epsilon_3)\ [w_i + \zeta_i] + 2\ \text{cov}(\epsilon_2\epsilon_3)\ [x_i + \xi_i] = 0 \quad \forall i > 0$$

Solving yields

$$\zeta_i = -w_i$$

$$\xi_i = -x_i$$

$$\rho_i = -y_i$$

which implies

$$BZ(B)T(B) = -\sum_{i=1}^{\infty} w_iB^i \Rightarrow T(B) = \frac{-\sum_{i=1}^{\infty} w_iB^i}{a_{33}^{-1}(B)\phi B^2}$$

$$BZ(B)U(B) = -\sum_{i=1}^{\infty} x_iB^i \Rightarrow U(B) = \frac{-\sum_{i=1}^{\infty} x_iB^i}{a_{33}^{-1}(B)\phi B^2}$$

$$BZ(B)V(B) = -\sum_{i=1}^{\infty} y_iB^i \Rightarrow V(B) = \frac{-\sum_{i=1}^{\infty} y_iB^i}{a_{33}^{-1}(B)\phi B^2}$$

Since $Z(B) = a_{33}^{-1}(B)\phi B$

Thus, from the optimal control rules we have

$$BZ(B)T(B) + W(B) = w_0$$

$$BZ(B)U(B) + X(B) = x_0$$

$$BZ(B)V(B) + Y(B) = y_0.$$

Substituting into (A.3) yields

$$\Delta lnP_t = w_0\epsilon_{1t} + x_0\epsilon_{2t} + y_0\epsilon_{3t}.$$

The same method can be used for deriving the other optimal control rules mentioned in the text. Specifically, if the base rule is defined as

$$\Delta lnBase_t^* = b' + BM(B)\epsilon_{1t} + BN(B)\epsilon_{2t} + BS(B)\epsilon_{3t}$$

then the optimal control parameters for minimizing the MSE of real output growth around potential are defined by

$$BM(B) = \frac{c_{41}(B)}{[a_{33}^{-1}(B)\phi B+1]}$$

$$BN(B) = \frac{c_{42}(B)}{[a_{33}^{-1}(B)\phi B+1]}$$

$$BS(B) = \frac{c_{43}(B)}{[a_{33}^{-1}(B)\phi B+1]}$$

However, since $a_{33}^{-1}(1) = -\phi^{-1}$, $a_{33}^{-1}(1)\phi+1 = 0$. Therefore, the denominator of the control rules has a unit root, and the optimal control of real output growth around potential growth is not feasible because of instrument instability.

Figure 1

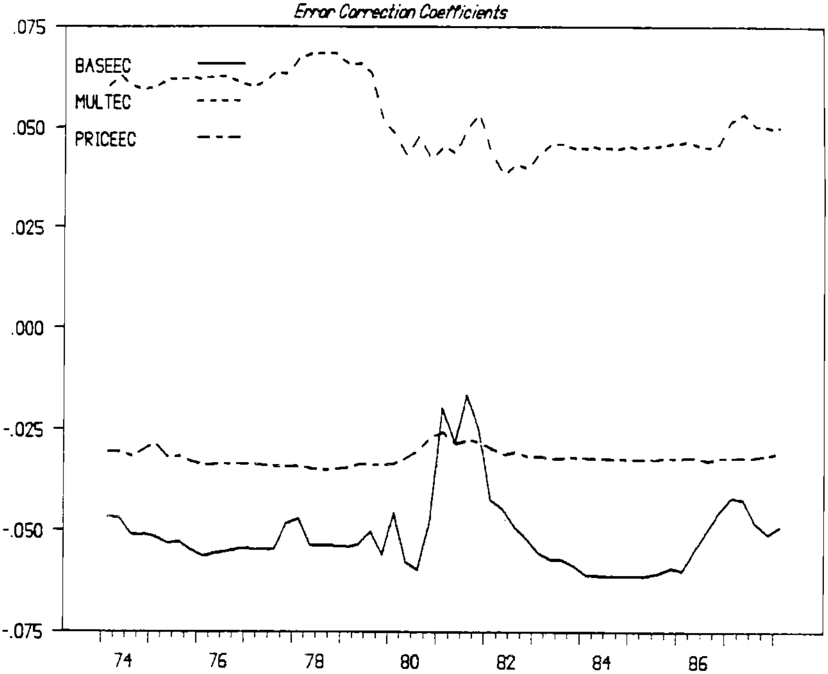


Figure 2

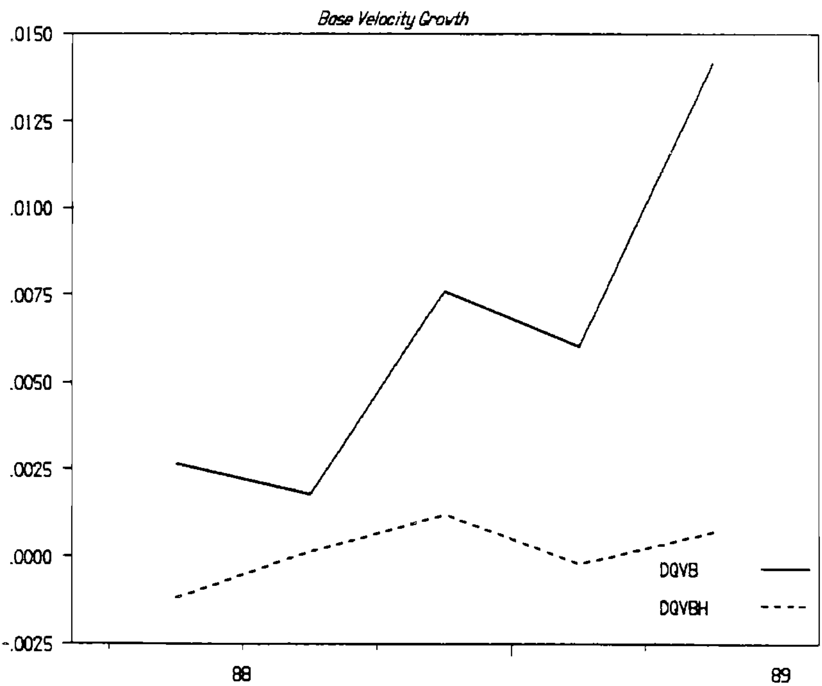
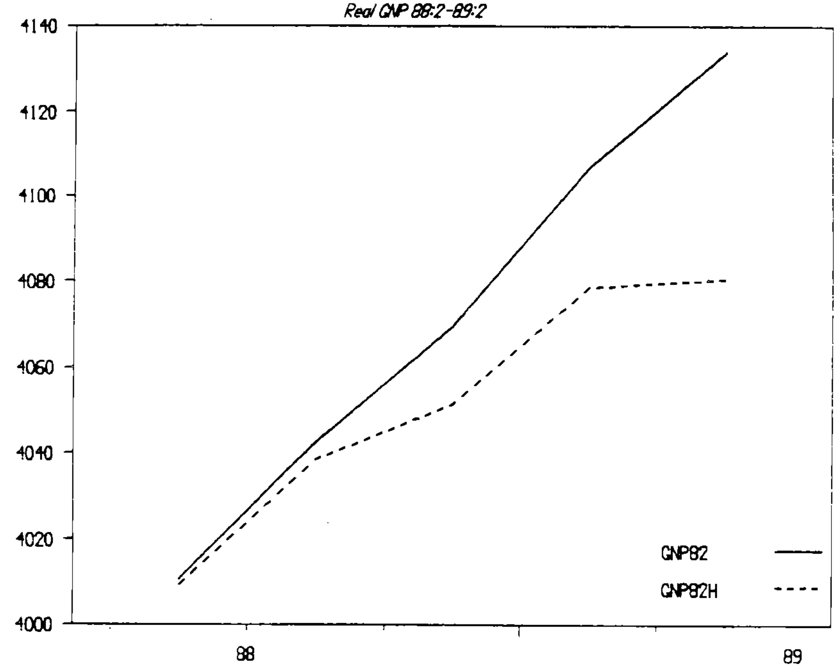


Figure 3



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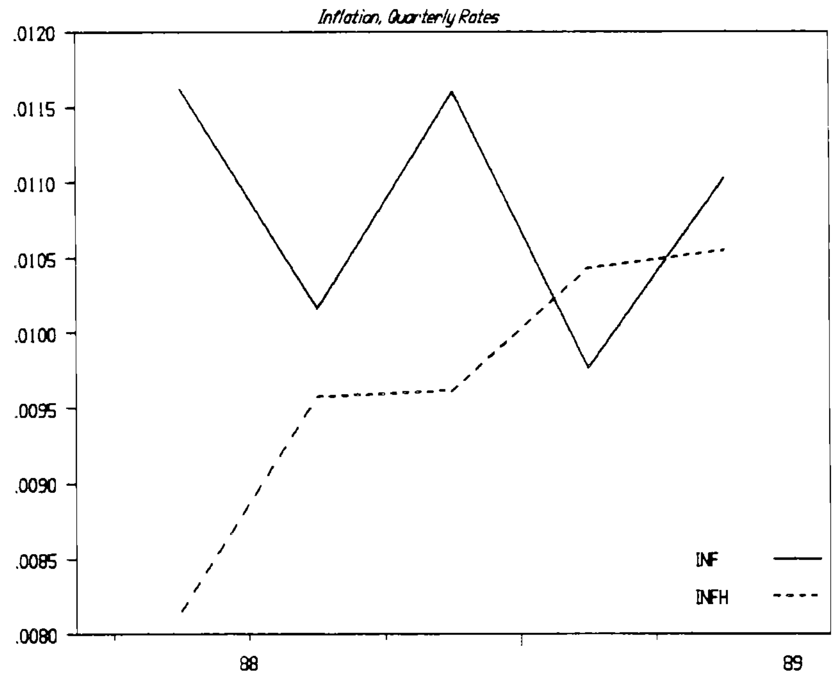

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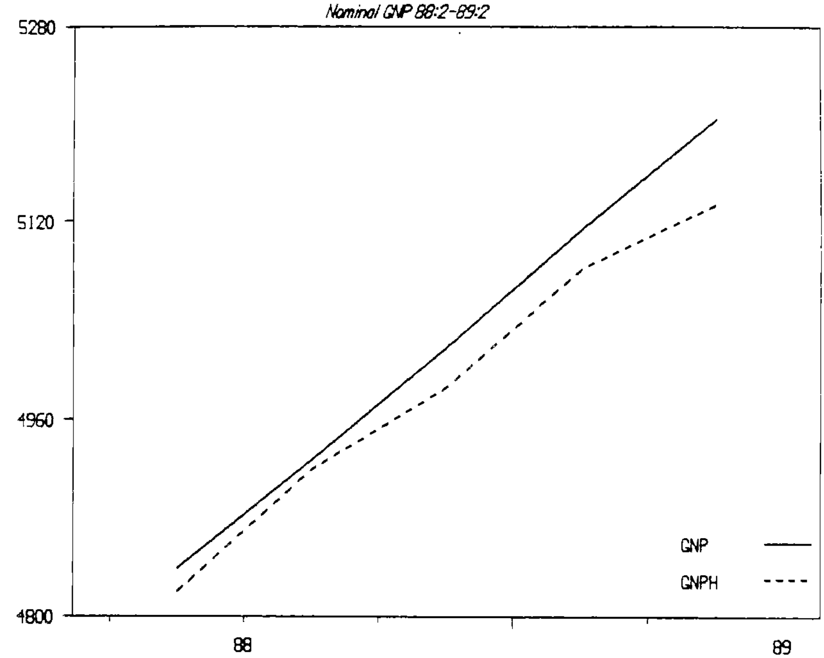

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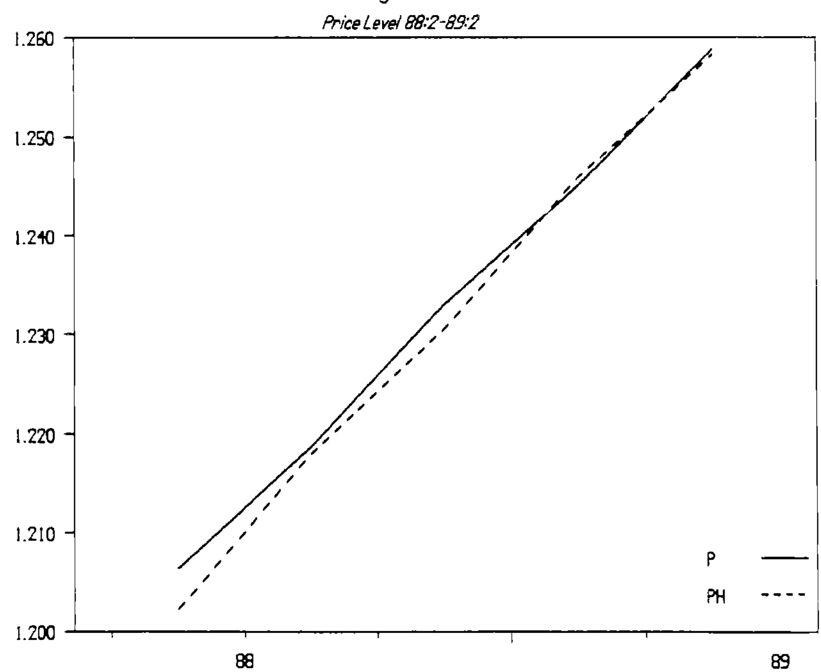

Figure 7

Impulse Responses, Velocity Shock

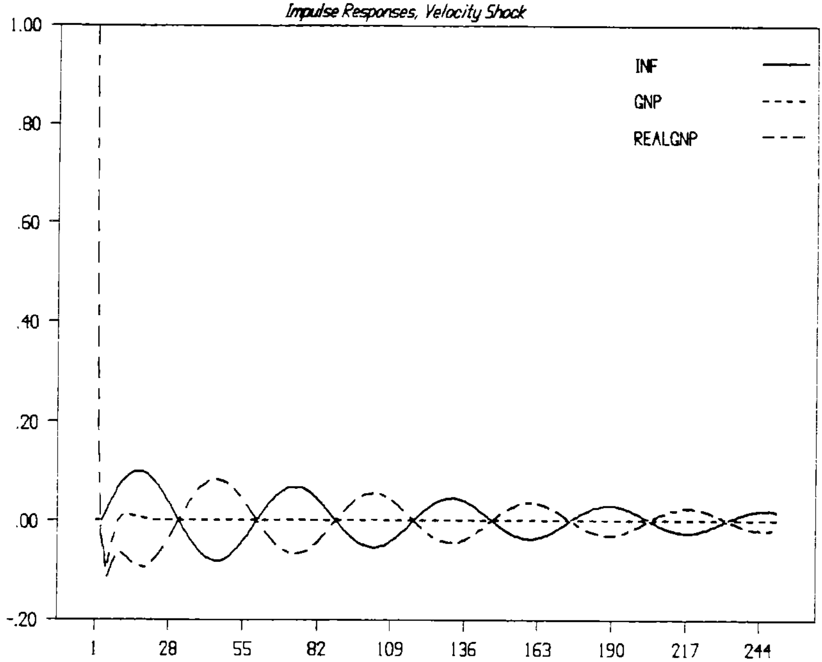

Figure 8

Impulse Responses, Multiplier Shock

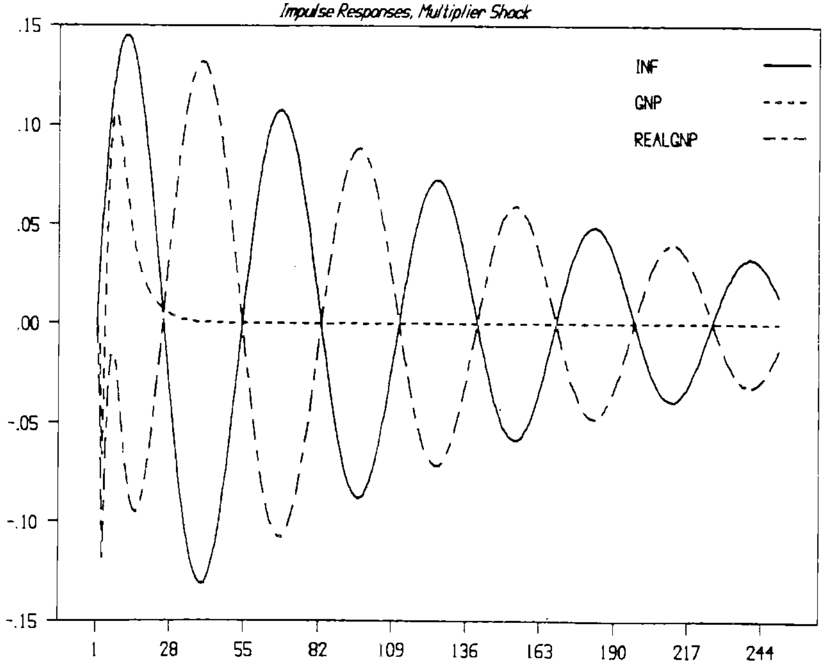

Figure 9

Impulse Responses, P-Gap Shock

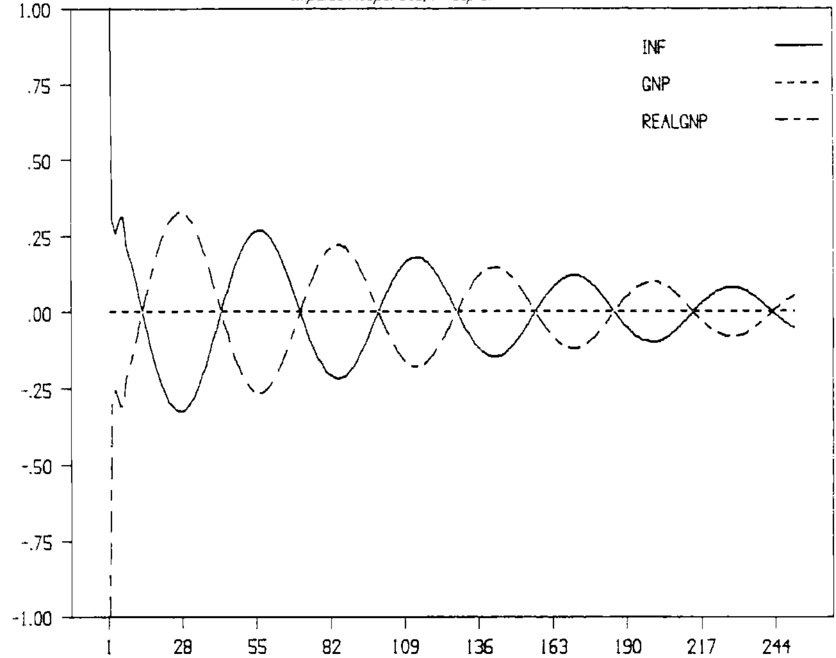

Figure 10

Cumulative Response, Base Growth

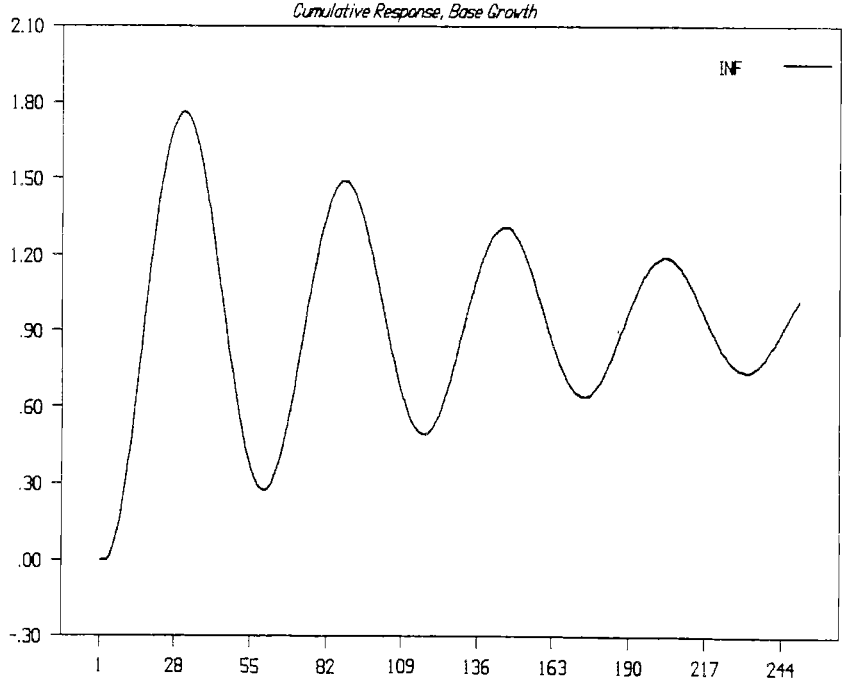

Figure 11

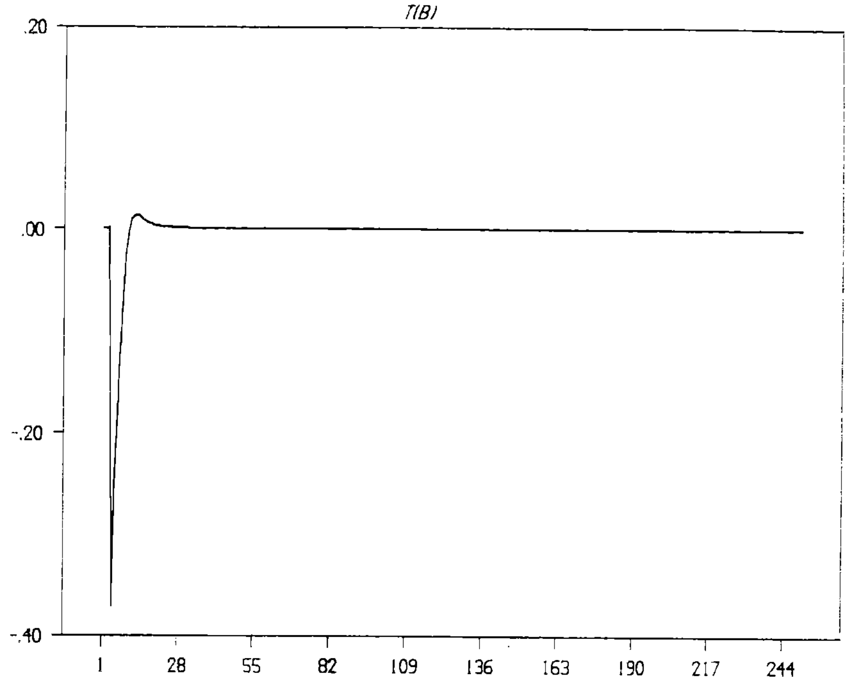

Figure 12

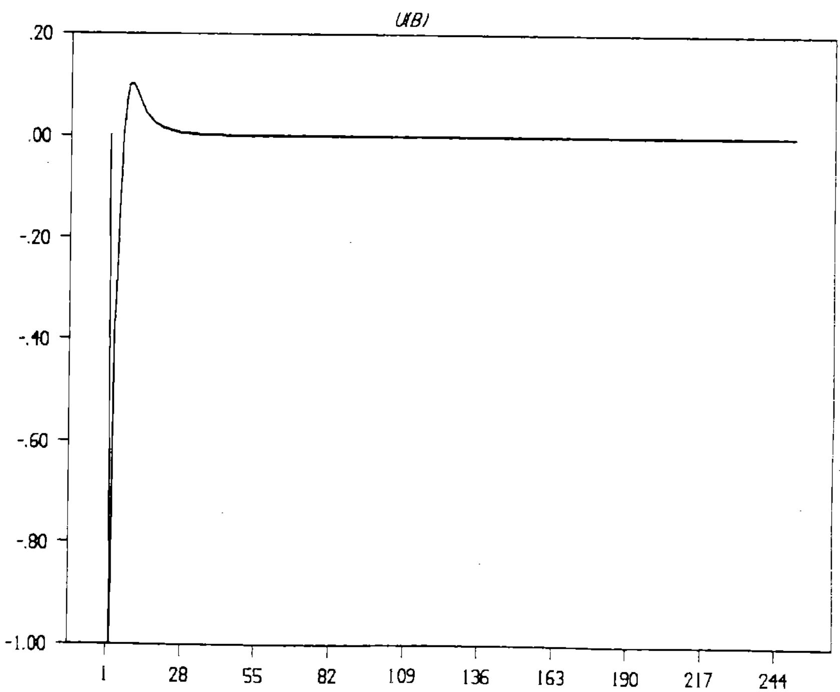

Figure 13