



Discussion Paper

Tail Systemic Risk and Banking Network Contagion: Evidence from the Brazilian Banking System

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Tail Systemic Risk And Banking Network Contagion: Evidence From the Brazilian Banking System

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Abstract

In this study the tail systemic risk of the Brazilian banking system is examined, using the conditional quantile as the risk measure. Multivariate conditional dependence between Brazilian banks is modelled with a vine copula hierarchical structure. The results demonstrate that Brazilian financial systemic risk increased drastically during the global financial crisis period. Our empirical findings show that *Bradesco* and Itaú are the origin of the larger systemic shocks from the banking system to the financial system network. The results have implications for the capital regulation of financial institutions and for risk managers' decisions. *Keywords:* Systemic Risk, Brazilian Banking System, Banking Network, Financial Contagion, Financial Crisis

JEL Classification: G01, G21, G32, G38.

1. Introduction

The recent financial crisis drew the attention of both regulators and investors, as it exposed the financial system's fragility and the potential risks arising from bank defaults. Since the collapse of *Lehman Brothers* in mid September 2008, the assessment of systemic risk turned out to be crucial for decision makers, since regulators started to evaluate the impacts of small, fragile, and seemingly isolated portfolios from one financial institution in compromising the safety and soundness of other institutions. In consequence, systemic risk is necessary to determine the amount of regulatory capital of financial institutions (Sanjiv Ranjan Das, 2004; Rosenberg and Schuermann, 2006). The Financial Stability Oversight Council (*FSOC*; Dodd–Frank, 2010) remarked that financial institutions that are systemically important must be forced to have a greater capacity for risk absorption, given their larger contribution to the systemic risk of the global financial system.

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Our research defines and calculates systemic risk as (i) the risk contribution from one bank to another, and (ii) the risk contribution from each bank to the Brazilian financial system (*BFIndex*). We describe a hierarchical model with copulae for the banks' dependence. The hierarchical copula modelling considers bank interconnectedness, conditional systemic dependence, and specific peer-to-peer tail dependence. Using this model, we calculate the effects from the vulnerability of one bank to another measuring the Value-at-Risk (VaR) of the recipient institution, and disentangling direct and indirect effects. In addition a bivariate copula dependence model is used to measure the contribution of each bank within the *BFIndex*. Previous research by Silva et al. (2016) found evidence of the strong network interconnectedness in the Brazilian banking system; nevertheless they used complex mathematical measures that are based on topology, while we use a conditional quantile that can easily be compared to the traditional industry adopted measures such as the unconditional quantile (VaR) and Conditional VaR.

VaR continues to be the most widely used measure of risk which assesses the potential losses in time using a probabilistic model with a defined interval of confidence under normal market circumstances. VaR was proposed during the 1990s by JP Morgan and adopted as the standard market risk measure for individual institutions; nevertheless, it neglects the collateral effects of defaults over other institutions. The academic literature on macroprudential policies has thus focused on examining the individual risk contributions from one institution to another, and from one institution to the financial system, developing different risk measures (see, for example, Bisias et al., 2012; Bernal et al., 2014).

In this study we measure the systemic impact of the financial crisis on listed banks¹ of the Brazil stock index (*Bovespa*), using bank and index daily prices from 1 January 2007 to 18 January 2016. The results demonstrate that the multivariate dependence structure of Brazilian banks is given by a *C*-vine copula hierarchical structure, on which *Bradesco* is the main influence in determining the conditional dependence structure of the financial system. The seven banks presented significant mean dependence changes during the whole period, revealing samples of changing tail dependence. The resulting conditional α -quantile shows that systemic risk evolved during the sample period, increasing during the world financial crisis of 2007/2008. The results show that *Bradesco* and *Banco do Brasil* have a systemic risk effect on all other banks and vice versa, *Itaú* having the lower systemic risk effect over *Bradesco* and *Banco do Brasil*. While mean dependence between banks was high in comparison to industry standards, study results demonstrate that tail dependence is fundamental in determining systemic risk effects and their asymmetry in bear/bull markets: as a result systemic effects from institution A into B are not reflected from institution B into A. Considering the effects of listed banks on the modified *BFIndex* (ex-bank in comparison), the bivariate copula test exhibit a high level of covariance between banks and the index, with the exception of *Banco do Brasil* which exhibits tail independence. Our systemic risk estimations show that the key systemic risk impacts over the financial

¹Bovespa listed banks are: ABC, Banco do Brasil, Bradesco, PanAmericano, Banrisul, Itaú and Paraná.

system from Brazilian banks originated from *Bradesco* and *Itaú*, with *Banco do Brasil* as the bank with lower transmission effect.

Barnhill and Souto (2009) proposed a new portfolio simulation methodology to test the Brazilian banking system strength during the 2007/2008 financial crisis; they found that once sovereign risk is included in the model, the Brazilian banking system is fragile to the effects of the financial crisis. Huang et al. (2009) developed a systemic risk index that measures financial fragility using the prices of credit default swaps (*CDS*). Likewise, Segoviano and Goodhart (2009) applied *CDS* to develop an index of financial stability that measured interbank dependence during extreme events. Rodríguez-Moreno and Peña (2013) found empirical evidence of the adequacy of *CDS* in estimating systemic risk. Acharya et al. (2010) used the expected shortfall (*ES*) and the marginal expected shortfall (*MES*) as measures for quantifying risks in extreme situations, and for estimating financial institutions' contributions to system risk. Brownlees and Engle (2012) defined a systemic risk measure (*SRISK*), that is the required capital in demand to restore the minimum level of mandatory regulatory capital. Allen et al. (2012) proposed a systemic risk measure (*CATFIN*) to predict the decline in aggregated loan activity within six months. Billio et al. (2012) tested five systemic risk measures that capture contagion and exposure in financial institutions' relationships. Engle and Manganelli (2004) developed a model of conditional autoregressive VaR (*CaViaR*) using quantile regression to capture the conditional distribution of the returns in the tails of the distribution.

More recently, Adrian and Brunnermeier (2011) proposed CoVaR (VaR conditional on financial distressed institutions) as a new systemic risk measure. CoVaR captures the spillover risks between financial institutions, providing more information about the VaR of the financial system conditional on the instability or default. In CoVaR, the systemic risk contribution from an institution is measured as the difference between the $\Delta CoVaR$ of an institution and the benchmark CoVaR, where $\Delta CoVaR$ is the linear sensitivity of the institution CoVaR.² Girardi and Ergün (2013), extended the CoVaR measure, including returns below the VaR of the conditional distress event, in comparison to Adrian and Brunnermeier (2011) that considered only returns equal to the VaR of the conditional distress event. Girardi and Ergün (2013) defined a systemic risk CoVaR calculation different from that of Adrian and Brunnermeier (2011)'s quantile regression; this new method adjusted the financial system joint returns density to a multivariate generalised autoregressive conditional heterocedastic (MGARCH), calculating the CoVaR by numerical methods from this adjusted distribution. More recently,

The bank's systemic risk dependence structure in this study is modelled using a multivariate hierarchical tree structure, a vine copula (Joe, 1996), from which we calculate the conditional α -quantile. This hierarchical dependence structure prices the risk that one institution represents to another institution. Since the hierarchical structure can be decomposed into sets of bivariate copulae (pair-copulae) which capture

²López-Espinosa et al. (2012) identified the determinants of systemic risk for a large set of international banks, applying CoVaR as in Adrian and Brunnermeier (2011).

dependence between two variables, the changes in the bivariate copula specification can be informative about diverse dependence characteristics, such a mean, tail, symmetric, and asymmetric dependence. The vine copula specification let us model the marginal distributions an the dependence structure separately, and as a result, independently of the dependence structure we can capture the specific volatility dynamic and asymmetry (volatility smirk)³ of the univariate bank return series. We study the bank individual risk contribution to the financial system, modelling the dependence between each bank and a modified *BFIndex* that removes the bank under consideration; we use bivariate copulas for this, that result in a conditional α -quantile.

The structure of this paper is as follows: Section 2 presents the notation and definitions of systemic risk in terms of vine copulae and bivariate copulae. Section 3 presents the data used for the study and in Section 4 presents the empirical findings. Finally, Section 5 provides concluding remarks and suggestions for further extensions of the work.

2. Notation and methodology

Different risk measures have been proposed in the literature to assess the impact of a risky financial institution over the system or over other individual financial institutions. In our research we define the systemic risk as the individual institution conditional on the distress α -quantile of the returns distribution effect over the financial system α -quantile, and the effect over other individual institutions.

2.1. Conditional α -quantile

The conditional α -quantile of a financial institution is the α -quantile conditioned on distress (low α quantile) of other financial institutions. Let X_t^1 be the returns of bank 1 and X_t^2 the returns of bank 2, the α -quantile of bank 1 return distribution is $P(X_t^1 \leq q_{\alpha,t}^{X_t}) = \alpha$, and can be calculated as:

$$q_{\alpha,t}^{X_t^1} = F_{X_t^1}^{-1}(\alpha), \tag{1}$$

where $F_{X_t^1}^{-1}(\alpha)$ is the inverse cumulative distribution function (cdf) of X_t^1 . Value-at-Risk (VaR) at level α is defined as the unconditional α -quantile for $\alpha = 0.05, 0.025, 0.01$. Applying (1) for bank 2 we will obtain the β -quantile inverse cdf of X_t^2 , defined as $F_{X_t^2}^{-1}(\beta)$. To calculate the conditional α -quantile of bank 1 at t for certain β -quantile of bank 2 we define $Pr(X_t^1 \leq q_{\alpha,\beta,t}^{X_t^1|X_t^2} | X_t^2 \leq q_{\beta,t}^{X_t^2}) = \alpha$ that can be calculated as:

$$q_{\alpha,\beta,t}^{X_t^1|X_t^2} = F_{X_t^1|X_t^2 \leqslant q_{\beta,t}^{X_t^2}}^{-1}(\alpha),$$
(2)

³Volatility dynamics and asymmetry are fundamental for a precise estimation of the vine α -quantile, see Avramidis and Pasiouras (2015).

where $F_{X_t^1|X_t^2 \leqslant q_{\beta,t}^{X_t^2}}^{-1}(\alpha)$ is the inverse cdf of X_t^1 conditioned on $X_t^2 \leqslant q_{\beta,t}^{X_t^2}$. In our methodology we calculate the quantile of a distressed conditional distribution, that includes the conditional bivariate dependence between X_t^1, X_t^2 , and the multivariate dependence with the financial system since the condition $X_t^1|X_t^2 \leqslant q_{\beta,t}^{X_t^2}$ will include indirect effects from any institutions of the system over X_t^1 . Considering the financial system is composed of n banks, we define the conditional quantile as $Pr(X_t^1 \leqslant q_{\alpha,\beta,t}^{X_t^1|X_t^2}|X_t^2 \leqslant q_{\beta,t}^{X_t^2},...,X_t^n) = \alpha$ that can be calculated by:

$$q_{\alpha,\beta,t}^{X_t^1|X_t^2,X_t^3,\dots,X_t^n} = F_{X_t^1|X_t^2,\leqslant q_{\beta,t}^{X_t^2},X_t^3,\dots,X_t^n}^{-1}(\alpha).$$
(3)

Calculation of the conditional α -quantile in (3) consists in finding the quantile of the distribution considering the financial distressed and dependence conditions of other banks with bank 1.

2.2. Models for marginal distributions

In this section unconditional quantiles of returns distribution are calculated. Let X_t have a time variant mean (μ_t) and variance zero such that:

$$X_t = \mu_t + \epsilon_t,\tag{4}$$

where $\mu_t = \phi_0 + \sum_{j=1}^p \phi_j y_{t-j} + \sum_{h=1}^q \varphi_j \epsilon_{t-h}$, with ϕ_0 , ϕ_j and φ_j denote a constant, an autoregressive (AR), and a moving average (MA) parameter, respectively, considering that p and q are non-negative integers. $\epsilon_t = \sigma_t z_t$ is a stochastic variable, with σ_t being the conditional standard deviation and z_t a stochastic variable with zero mean and standard deviation equal to one. The variance of X_t is given by the variance of ϵ_t , which has a truncated generalised autoregressive conditional heterocedastic (*TGARCH*) dynamic proposed by Zakoian (1994) and Glosten et al. (1993):

$$\sigma_t^2 = \omega + \sum_{k=1}^r \beta_k \sigma_{t-k}^2 + \sum_{h=1}^m \alpha_h \epsilon_{t-h}^2 + \sum_{h=1}^m \lambda_h 1_{t-h} \epsilon_{t-h}^2,$$
(5)

where ω is constant; β and α are the *GARCH* and the autoregressive conditional heterocedastic (*ARCH*) parameters, respectively. λ captures the asymmetry effect such that a negative shock has more impact in the variance than a positive shock when $\lambda > 0$. Note that if $\lambda = 0$ we have a simple *GARCH* model. Let z_t have a Student's *t*-distribution with mean zero and unitary variance, then:⁴

$$f(z_{i,t};\nu) = \frac{\Gamma[\frac{1}{2}(\nu+1)]}{\pi^{\frac{1}{2}}\Gamma(\frac{1}{2}\nu)} [(\nu-2)\sigma_t^2]^{-\frac{1}{2}} \left[1 + \frac{z_{i,t}^2}{(\nu-2)\sigma_t^2}\right]^{\frac{1}{2}(\nu+1)}, \tag{6}$$

where ν are the degrees of freedom $(2 < \nu < \infty)$.

⁴Student's t assumption let us model heavy tails of the returns distributions (Bollerslev, 1987)

2.3. Unconditional and conditional α -quantile estimation with copulae and vine copulae

From the mean and the variance of X_t , we can calculate the unconditional α -quantile of the returns distribution as:

$$q_{\alpha,t}^{X_t^1} = \mu_t + F_{\nu}^{-1}(\alpha)\sigma_t,$$
(7)

where $F_{\nu}^{-1}(\alpha)$ denote the unconditional α -quantile from a Student's *t*-distribution in (3).

For calculating the conditional α -quantile, we use copula functions (Joe, 1997; Nelsen, 2006). Note that $Pr(X_t^1 \leq q_{\alpha,\beta,t}^{X_t^1|X_t^2} | X_t^2 \leq q_{\beta,t}^{X_t^2}) = \alpha \ e \ Pr(X_t^1 \leq q_{\alpha,\beta,t}^{X_t^1|X_t^2,X_t^3,\dots,X_t^n} | X_t^2 \leq q_{\beta,t}^{X_t^2,X_t^3,\dots,X_t^n}) = \alpha \ \text{can be written as:}$

$$\frac{F_{X_t^1,X_t^2}\left(q_{\alpha,\beta,t}^{X_t^1|X_t^2}, q_{\beta,t}^{X_t^2}\right)}{F_{X_t^2}\left(q_{\beta,t}^{X_t^2}\right)} = \alpha,\tag{8}$$

$$\frac{F_{X_t^1, X_t^2 | X_t^3, \dots, X_t^n} \left(q_{\alpha, \beta, t}^{X_t^1 | X_t^2, X_t^3, \dots, X_t^n}, q_{\beta, t}^{X_t^2} \right)}{F_{X_t^2 | X_t^3, \dots, X_t^n} \left(q_{\beta, t}^{X_t^2} \right)} = \alpha.$$
(9)

Then, to calculate the conditional quantiles we need the joint distribution of $X_t^1 \in X_t^2$, $F_{X_t^1,X_t^2}(.)$. Considering the Copula theorem of Sklar (1959) let us express a distribution function in terms of a copula C, where $C(F_X(x), F_Y(y)) = F_{XY}(x, y)$, (8) and (9) can be expressed as:

$$C_{X_t^1, X_t^2}\left(F_{X_t^1}\left(q_{\alpha, \beta, t}^{X_t^1 | X_t^2}\right), F_{X_t^2}\left(q_{\beta, t}^{X_t^2}\right)\right) = \alpha\beta,\tag{10}$$

$$C_{X_{t}^{1},X_{t}^{2}|X_{t}^{3},\dots,X_{t}^{n}}\left(F_{X_{t}^{1},X_{t}^{2}|X_{t}^{3},\dots,X_{t}^{n}}\left(q_{\alpha,\beta,t}^{X_{t}^{1}|X_{t}^{2}}\right),F_{X_{t}^{2}|X_{t}^{3},\dots,X_{t}^{n}}\left(q_{\beta,t}^{X_{t}^{2}}\right)\right) = \alpha\beta.$$
(11)

We can describe the values of the conditional α -quantile in terms of a bivariate or multivariate copula. Applying a bivariate copula as in (10), we can calculate $F_{X_t^1}\left(q_{\alpha,\beta,t}^{X_t^1|X_t^2}\right)$ with the inverse copula function having the values α and of $F_{X_t^2}\left(q_{\beta,t}^{X_t^2}\right) = \beta$, denoted as $\hat{F}_{X_t^1}\left(q_{\alpha,\beta,t}^{X_t^1|X_t^2}\right)^5$. Inverting the marginal distribution of X_t^1 the conditional α -quantile yields:

$$q_{\alpha,\beta,t}^{X_t^1|X_t^2} = F_{X_t^1}^{-1} \left(\hat{F}_{X_t^1} \left(q_{\alpha,\beta,t}^{X_t^1|X_t^2} \right) \right).$$
(12)

The use of copula functions in a multivariate setting derived from (11) allows us not only to model the direct contagion of financial distress from Bank 2 to Bank 1, but also allows us to explain the indirect effects of the impact that Bank 2 can create over other banks. For the multivariate dependence setting in n dimensions

⁵Note that bivariate copulae relate two distributions $F_X(x)$ and $F_Y(y)$, with a copula function. Once we have the specific form of the copula function (given by $\alpha\beta$) and the numerical value of $F_Y(y)$ when $F_Y(y) = \beta$, the problem is reduced to an equation with one unknown, and solving this equation by numerical methods we find the value of $F_Y(y)$.

we consider vine copulae that allow us to disentangle the multivariate density in the product of marginal densities, and a multivariate copula which is generated with a hierarchical structure decomposed in cascade bivariate copulae defined as pair-copulae. We considered three types of vine copula: C-vine, D-vine, and R-vine with different hierarchical structures and trees. The multivariate density for a C-vine copula is given by:

$$f(x_1, x_2, ..., x_n) = \prod_{k=1}^n f_k(x_k) \prod_{h=2}^n c_{1,h} \left(F_1(x_1), F_2(x_2) \right)$$

$$\prod_{j=2}^{n-1} \prod_{i=1}^{n-j} c_{j,j+1|1,...,j-1} \left(F(x_j|x_1, ..., x_{j-1}), F(x_{j+1}|x_1, ..., x_{j-1}) \right),$$
(13)

where $c_{j,j+1|1,...,j-1}$ is the conditional copula on which the conditional distribution function of x_i , on x_j is given by Joe (1997):

Figure 1 represents the C-vine copula using a hierarchical structure, on which the first level of the tree n nodes are connected by edges that represent the dependence between two variables. In the next levels, the nodes are derived from the set of edges of the previous level. Each tree (T) has a star structure, where each variable is fundamental for the system. The dependence is measured between the central variable with the remaining variables of the first tree using bivariate copulae, as in the second term of (13), or using conditional bivariate copulae in the remaining trees, as in the third term of (13). Once dependence on each tree is modelled, the tree is expanded recursively such that the nodes of the trees are configured by the edges of the previous trees, as shown in Figure 1. On each tree, the central variable which regulates dependence is identified as the one that maximises the sum of pair-dependences measured by Kendall's τ .

The D-vine copula has a different hierarchical dependence structure, given by:

$$f(x_{1}, x_{2}, ..., x_{n}) = \prod_{k=1}^{n} f_{k}(x_{k}) \prod_{h=1}^{n-1} c_{h,h+1} \left(F_{h}(x_{h}), F_{h+1}(x_{h+1})\right)$$

$$\prod_{j=2}^{n-1} \prod_{i=1}^{n-j} c_{i,i+1|i+1,...,i+j-1} \left(F(x_{i}|x_{i+1}, ..., x_{i+j-1}), F(x_{i+j}|x_{i+1}, ..., x_{i+j-1})\right).$$

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Figure 2 represents the hierarchical dependence structure of the D-vine copula, where all the variables that were treated similarly in the first tree determine the bivariate dependence in the remaining trees.

Finally, an n-dimensional density function of R-vine copula is given by:

$$f(x_1, x_2, ..., x_n) = \prod_{k=1}^n f_k(x_k)$$

$$\prod_{j=1}^{n-1} \prod_{e \in E_i} c_{j(e), k(e)|D(e)} \left(F(x_{j(e)}|x_{D(e)}), F(x_{k(e)}|x_{D(e)}) \right),$$
(16)

where E_i denote the nodes and $x_{D(e)}$ is a sub-vector of x, indexed by the conditional set D(e). The R-vine structure is calculated applying the minimal expansion tree that solves the following optimisation problem for each tree:

$$\max \sum_{\text{nodes } e = \{i, j\} \text{ generating tree}} |\hat{\tau}_{ij}|, \qquad (17)$$

where $\hat{\tau}_{ij}$ denotes empirical pairs of Kendall's τ , and a generating tree is a tree on each node.

From this information on the vine hierarchical structure, we can calculate the conditional distribution $F_{X_t^1,X_t^2|X_t^3,...,X_t^n}\left(q_{\alpha,\beta,t}^{X_t^1|X_t^2}\right)$ from (11). Once we have obtained that value and using the conditional copula model, we can derive information from $\hat{F}_{X_t^1}\left(q_{\alpha,\beta,t}^{X_t^1|X_t^2}\right)$. Finally, inverting the marginal cdf of X_t^1 the conditional α -quantile is yielded from (12).

The estimation of the conditional α -quantile by using copula functions yields some advantages. First, copulae offer flexibility, letting us to model marginals and the dependence structures separately. This is important when there exist differences in the quantile dependence and the joint *cdf* is not elliptical, or when the data have special characteristics (heteroscedasticity). Second, the calculation of conditional α -quantiles by copula means is computationally straightforward, as only the copula definition, the returns marginal distribution of an institution, and the cumulative probability of the quantiles of a second institution, are required.

In our empirical research, different static copula specifications were used, to try to capture different dependence characteristics: non-tail dependence (Gaussian, Plackett e Frank, see Joe, 1997; Nelsen, 2006), tail symmetric dependence (Student's t), and tail asymmetric dependence (Gumbel, Rotated Gumbel e Symmetric Joe Clayton (SCJ), see Joe, 1997; Nelsen, 2006). Their principal characteristics are summarised in Table 1.

Using an inference function for the marginal distribution (Joe and Xu, 1996), we estimate marginal parameters applying the maximum likelihood method, and then we estimate the copula parameters by

pseudo-sampling of the probability integral transform of marginal standardised residuals. The number of differences on the mean and variance equations for each series was selected in accordance with the Akaike information criterion (AIC), and the different copula models were compared with the AIC adjusted for small sample bias, such in Breymann et al. (2003) and Reboredo (2011).

3. Data

In this research we tested the systemic impact of the default of *Bovespa* stock listed banks with other banks of the Brazilian financial system (*BFIndex*). We used daily data bank prices from 1 January 1 2007 to 18 January 2016. The bank dataset comprised seven banks: *ABC*, *Banco do Brasil*, *Bradesco*, *Pan-Americano*, *Banrisul*, *Itaú*, and *Paraná*. Additionally, we considered daily data from the Brazilian financial index (*BFIndex*) as the reference for the financial system's behaviour. When analysing the systemic impact from a specific bank over the *BFIndex*, we excluded the share of the bank in the index, removing any direct effects of the bank price fluctuations into the index (see, for example, López-Espinosa et al., 2012). Data were collected from Bloomberg and returns were calculated with continuous compounding.

Figure 3 shows a time series of stock prices for the Brazilian banks and the *BFIndex*. On the first section of the plot we observe a minor decline during the beginning of the 2007/2008 financial crisis, and at the end we observe a minor decline due to the political crisis in Brazil. Table 2 shows that the banks' returns have similar characteristics; they do not present any significant trend, and the standard deviations were larger than mean returns. All the banks exhibit similar volatility patterns. We detect the presence of tail returns that is verified when the calculated excess kurtosis is higher than three. The Jarque and Bera (1980) test (*J–B*) rejected the normality null hypothesis. The autoregressive conditional heteroscedastic–Lagrange multiplier (*ARCH–LM*), and *Ljung–Box* (*L–B*) statistics (see Lawless, 2003) for square returns indicate that all series present *ARCH* effects.

4. Empirical results

4.1. Marginal distribution estimation

The results of the marginal estimation (4)–(6) for each bank are presented in Table 3 and for the *BFIndex* excluding the bank tested in Table 4. Different combinations of p,q,r, and m parameters were considered;

differences from zero to a maximum of two were tested and the most adequate value was selected by AIC and BIC methods. In Table 3, we observe temporal dependence for all bank series except Bradesco, Banrisul, and Itaú, and the returns volatility is persistent when testing a calibrated GARCH(1, 1) volatility specification, except that of ABC bank that is persistent for a GARCH(2, 1). Leverage effects were found in all series except of ABC and PanAmericano banks. The degrees of freedom of the estimated Student's t-distribution confirm that the error terms are not normal. There is no temporal dependency in the mean returns from the different indices shown in Table 4. The estimated volatility dynamic has a GARCH(1, 1) specification.

The last items in Tables 3 and 4 reveal the quality of the selected marginal models. The ARCH and L-B statistics exhibit no serial autocorrelation nor ARCH effects in marginal residuals. We tested the null hypothesis that the model residuals are UNIFORM(0,1) distributed, comparing the empirical distribution with theoretical distributions (Kolmogorov-Smirnov (KS), Cramer-von Mises (C-vM) e Anderson-Darling (AD), see Lawless, 2003). The resulting p-values from these tests (the last items of Tables 3 and 4) show that for any of the marginal models tested, the correct specification of the distribution function can not be rejected at the 5% significance level. In general, tests of goodness of fit show that the marginal distributions are properly specified.

4.2. Copulae models

Different copulae models were estimated (see Table 1), applying the integral of standard residuals from each of the marginals, and pseudo-sampling methods. First, we present the results of the estimated vine copula for each Brazilian bank paired with the *BFIndex*.

We estimated three vine copula models (13), (15), and (16), using static bivariate copulae. According to the AIC values, the C-vine copula provides the best fit, as shown in Figure 4. The results show that Bradesco dominates the multivariate dependence structure, and Itaú has the lower influence in the dependence structure of the dependent banks in the financial system, with lower dependence than Banco do Brasil, ABC, PanAmericano Banrisul, and Paraná. These results are consistent with some fundamental facts: (i) *Banco do Brasil* is a mix of a public–private partnership (*PPP*) where the exposure is guaranteed by the government; (ii) *Bradesco* is as large as *Banco do Brasil* but privately owned; and (iii) *Itaú* has less commercial activity with smaller banks. We analyse the systemic risk implications of the resulting network for the financial system.

Tables 6, 7, and 8 show the estimated parameters for the bivariate static copulae, into the different hierarchical C-vine tree structures:

- The first tree corresponds to the *Bradesco* dependence with the remaining banks, and estimated parameters confirm a positive and high level of dependence. Comparisons of different copula models tested reveal that: (i) the Student's t copula had the best fit for pairs: *Bradesco-ABC*, *Bradesco-Itaú*, and *Bradesco-Paraná*, were in favour of symmetric tail dependence; (ii) the Frank copula produced the best fit for *Bradesco-Banco do Brasil* and *Bradesco-Pan* with no tail dependence between them; (iii) the BB1 copula yields the best fit for the *Bradesco-Banrisul* pair that revealed an asymmetric tail dependence, with larger dependence in the upper tail.
- The second tree in the C-vine copula model corresponds to the pairs Banco do Brasil-ABC, Banco do Brasil-PanAmericano, Banco do Brasil-Banrisul, Banco do Brasil-Itaú, and Banco do Brasil-Paraná. The estimated parameters of the empirical pair-copula show mild positive dependence. According to the AIC test; conditional dependence from Banco do Brasil with ABC, Itaú, and Paraná was characterised by a static Student's t copula, revealing symmetric tail dependence. For Banco do Brasil with PanAmericano and Banrisul we find evidence of asymmetric tail dependence given the results of the BB7 copula parameters where the lower tail has larger dependence.
- The third tree in the C-vine copula model corresponds to the dependence of ABC-PanAmericano, ABC-Banrisul, ABC-Itaú, and ABC-Paraná. The best copula fit models were: (i) the Student's t for ABC-Banrisul and ABC-Paraná, (ii) the Gumbel for ABC-PanAmericano with larger upper tail dependence and (iii) the Gaussian for ABC-Itaú.
- The fourth tree in the C-vine copula model corresponds to the conditional dependence between (i) *PanAmericano–Banrisul*, (ii)*PanAmericano–Itaú*, and *PanAmericano–Paraná*. The best fit copula models for each of the three pairs are in pairs-order: (i) Gaussian (symmetric dependence), (ii) Frank (non-bias dependence), and (iii) Gumbel (asymmetric upper tail dependence).
- The fifth tree represents the dependence of *Banrisul–Itaú* e *Banrisul–Paraná*, where the best fit copula are given by a Gaussian for the first pair and by a Student's t for the second pair.
- The sixth and last tree is the resulting conditional dependence model between *Itaú* and *Paraná* with a Frank copula, with tail independence.

The evidence about multivariate dependence structure derived from the C-vine copula model reveals that there exists a rich diversity in bank dependency that provides information that can not be derived with any mean dependency model. Tail dependency findings have important implications for the assessment of systemic risk.

Table 5 presents the results of the bivariate copulae models when each bank is paired with the *BFIndex*. Our findings show that there is strong dependence in the both tails (upper and lower) as the results fit into a Student's t copula. An exception was found in the dependence of the *Banco do Brasil* with the system, for which we found tail independence as determined by the fitted Frank copula. These results have implications for the systemic risk, as we show in the next section.

4.3. Results of α -quantile

Conditional and unconditional α -quantile values were calculated for the 95% confidence level ($\alpha = 0.05, \beta = 0.05$)⁶, using univariate marginal distributions and best fit pair-copula estimated with the C-vine hierarchical copula structure as in Tables 6, 7, 8, and 5. Figure 5 shows a map of the network relationships and size of normalised systemic risk, i.e.,

$$\frac{\text{Vine conditional } \alpha \text{-quantile}}{\text{unconditional } \alpha \text{-quantile}},$$

which represents systemic risk measure that individual institutions receive from the other seven banks of the network during the sampling period. Each of the edges defines the normalised systemic risk received or transmitted. The color represents the size of the systemic risk - red for very high risk, blue for high

 $^{^{6}}$ Results with a confidence level of 99% were produced and are available from the correspondent author.

risk, and green for medium risk. The size of the node that represents each of the institutions illustrates the relative total systemic risk generated by the institution in comparison with the other institutions. Descriptive statistics are presented in Tables 9 and 10.

Figure 5 reveals that *Bradesco* is the institution that transmits more systemic risk for the other banks of the system, in particular to *Itaú*. The second institution that generates more systemic risk to *Itaú* is *Banco* do Brasil. Banco do Brasil also generates high levels of systemic risk for others bank of the system, with the exception of *Banrisul. Itaú* represents high levels of systemic risk, in particular to big banks (*Bradesco* and *Banco do Brasil*). Banrisul, PanAmericano, and ABC's levels of systemic risk for the system are very low.

Our results on the systemic risk effects reveal that mean dependence between banks was relatively high, and the tail dependence – fundamental to assess the effects of systemic risk – was asymmetric: a bank 1 can represent a high systemic risk for a bank 2, but not necessarily the opposite. The results demonstrate the influential role of *Bradesco* in aggregating systemic risk to the system, by receiving or transmitting. *Banco do Brasil* was a less influential institution in receiving or transmitting systemic risk, consistent with the support that the bank receives from the government. *Itaú*, a large institution by asset size, was not influential in aggregating systemic risk to the network, although it was still relevant for the big banks (*Bradesco* and *Banco do Brasil*). Minor banks by asset size – *Banrisul*, *Pan*, and *ABC* – are not fundamental in aggregating systemic risk to the network, they only aggregate risk between them.

In relation to the banks' contribution to the systemic risk of the Brazilian financial system, Table 11 shows the descriptive statistics, and Figure 6 presents dynamics and size of the conditional and unconditional α -quantile during the sample period, exhibiting peaks during 2007/2008 financial crisis. The empirical estimations of the conditional and unconditional α -quantile reveal some consistency with pair-institution results, *Bradesco* and *Itaú* having the largest impact in the Brazilian financial system. Banks of smaller asset size such as *ABC*, *PanAmericano*, and *Banrisul*, had a minor role in transmitting risk, and this transmission role was even lower for the *Paraná* bank. Finally, *Banco do Brasil* was the institution that had the lowest transmitting risk role for the index.

Please, insert Table 11 here.

Please, insert Figure 6 here.

Our results have three important implications. First, systemic risk estimation should consider tail dependence, conditional during financial distress. The multivariate dependence structure suggested in this study reveals that unconditional dependence in the lower tail has a significant impact on systemic risk effects. Second, our results have important conclusions for regulators, as the necessary capital due to systemic risk changes will vary dynamically in time according to the financial institution, and in particular during a time of financial crisis. Consequently, this study aims to produce a solution for the assessment and the inclusion of systemic risk into regulatory capital. Third, results have implications for investors in terms of the risk management of the portfolio. Notwithstanding that, the positive mean dependence shows low hedge probabilities for investors of bank assets, our findings reveal that some banks have no systemic risk effects over other banks, with important implications for asset pricing of the portfolio.

5. Concluding remarks

The 2007/2008 financial crisis raised regulators' concerns about the systemic risk impact of defaulted financial institutions. Systemic risk assessment is fundamental for a safe and sound regulation of financial risks and the reduction of the impact of the shocks over the financial system.

We measured the systemic risk resulting from a default of a bank's obligations and its influence for (i) other banks of the Brazilian financial system, and (ii) over the entire financial system, using the conditional α -quantile as a measure of systemic risk. For modelling the multivariate dependence structure between the banks, we used a hierarchical dependence model, the vine copula, which can model the connections, the conditional dependence, and the specific characteristics of tail dependence between banks. We applied a bivariate copula model for each bank pair, and between each bank and the *BFIndex* with the purpose of measuring the risk contribution of each bank to the Brazilian financial system.

Our empirical results – in relation to the period 1 January 2007 to 18 January 2016 – show that multivariate dependence between the banks is given by a C-vine hierarchical structure, on which *Bradesco* is the predominant bank in determining the conditional dependence structure. All bank pairs exhibit covariance, on average, during the sample period, demonstrating different tail dependence evidence. The contribution of the systemic risk from one bank to other is similar during the sample period analysed. *Bradesco* was the key influential institution receiving and transmitting risk to the other banks of the network. *Itaú* had a minor influence. *Banco do Brasil* was the bank that was the least influential in transmitting systemic risk to the network. Suggestions for future research includes the extension of our study in the search of the systemic impact of financial institutions to other countries and regions, such as, the United States, Latin America, Europe, and Asia.

References

Acharya, V. V., Pedersen, L. H., Philippon, T., Richardson, M. P., 2010. Measuring systemic risk.

- Adrian, T., Brunnermeier, M. K., 2011. Covar. Tech. rep., National Bureau of Economic Research.
- Allen, L., Bali, T. G., Tang, Y., 2012. Does systemic risk in the financial sector predict future economic downturns? Review of Financial Studies 25 (10), 3000–3036.
- Avramidis, P., Pasiouras, F., 2015. Calculating systemic risk capital: A factor model approach. Journal of Financial Stability 16, 138–150.
- Barnhill, Jr, T. M., Souto, M. R., 2009. Systemic bank risk in brazil: A comprehensive simulation of correlated market, credit, sovereign and inter-bank risks. Financial Markets, Institutions & Instruments 18 (4), 243–283.
- Bernal, O., Gnabo, J.-Y., Guilmin, G., 2014. Assessing the contribution of banks, insurance and other financial services to systemic risk. Journal of Banking & Finance 47, 270–287.
- Billio, M., Getmansky, M., Lo, A. W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of Financial Economics 104 (3), 535–559.
- Bisias, D., Flood, M. D., Lo, A. W., Valavanis, S., 2012. A survey of systemic risk analytics. US Department of Treasury, Office of Financial Research (1).
- Bollerslev, T., 1987. A conditionally heteroskedastic time series model for speculative prices and rates of return. Review of Economics and Statistics 69 (3), 542–547.
- Breymann, W., Dias, A., Embrechts, P., Jan 2003. Dependence structures for multivariate high-frequency data in finance. Quantitative Finance 3 (1), 1–14.
- Brownlees, C. T., Engle, R. F., 2012. Volatility, correlation and tails for systemic risk measurement. Tech. rep., Becker Friedman Institute, University of Chicago.

Dodd–Frank, 2010. Dodd–Frank Wall Street Reform and Consumer Protection Act. Congress of the United States of America.

- Engle, R. F., Manganelli, S., 2004. Caviar: Conditional autoregressive value at risk by regression quantiles. Journal of Business & Economic Statistics 22 (4), 367–381.
- Girardi, G., Ergün, A. T., 2013. Systemic risk measurement: Multivariate garch estimation of covar. Journal of Banking & Finance 37 (8), 3169–3180.
- Glosten, L. R., Jagannathan, R., Runkle, D. E., 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. Journal of Finance 48 (5), 1779–1801.
- Huang, X., Zhou, H., Zhu, H., 2009. A framework for assessing the systemic risk of major financial institutions. Journal of Banking & Finance 33 (11), 2036–2049.
- Jarque, C. M., Bera, A. K., Jan 1980. Efficient tests for normality, homoscedasticity and serial independence of regression residuals. Economics Letters 6 (3), 255–259.
- Joe, H., 1996. Families of *m*-variate distributions with given margins and m(m-1)/2 bivariate dependence parameters. Lecture Notes-Monograph Series, Institute of Mathematical Studies Lecture Notes, 120–141.
- Joe, H., 1997. Multivariate models and multivariate dependence concepts. CRC Press.
- Joe, H., Xu, J. J., 1996. The estimation method of inference functions for margins for multivariate models. Tech. rep., University of British Columbia, Vancouver.
- Lawless, J. F., 2003. Statistical Models and Methods for Lifetime Data, 2nd Edition. John Wiley & Sons, New York:.
- López-Espinosa, G., Moreno, A., Rubia, A., Valderrama, L., 2012. Short-term wholesale funding and systemic risk: A global covar approach. Journal of Banking & Finance 36 (12), 3150–3162.
- Nelsen, R., 2006. An introduction to copulas. New York: SpringerScience Business Media.

Reboredo, J. C., 2011. How do crude oil prices co-move?: A copula approach. Energy Economics 33 (5), 948–955.

Rodríguez-Moreno, M., Peña, J. I., 2013. Systemic risk measures: The simpler the better? Journal of Banking & Finance 37 (6), 1817–1831.

Rosenberg, J. V., Schuermann, T., 2006. A general approach to integrated risk management with skewed, fat-tailed risks. Journal of Financial Economics 79 (3), 569 – 614.

Sanjiv Ranjan Das, R. U., 2004. Systemic risk and international portfolio choice. The Journal of Finance 59 (6), 2809–2834.

Segoviano, M. A., Goodhart, C., 2009. Banking stability measures.

Silva, T. C., de Souza, S. R. S., Tabak, B. M., Mar 2016. Network structure analysis of the Brazilian interbank market. Emerging Markets Review 26, 130–152.

Sklar, M., 1959. Fonctions de répartition à n dimensions et leurs marges. Université Paris 8.

Zakoian, J.-M., 1994. Threshold heteroskedastic models. Journal of Economic Dynamics and Control 18 (5), 931–955.

Name	Copula	Parameter	Dependence structure
Gaussian	$C_N(u, v; \rho) = \phi(\phi^{-1}(u), \phi^{-1}(v))$	d	Tail independence: $\lambda_U = \lambda_L = 0$
Student-T	$C_{ST}(u,v; ho, u) = T(t_{ u}^{-1}(u),t_{ u}^{-1}(v))$	ρ, ν	Symmetric tail dependence: $\lambda_U = \lambda_L = 2t_{\nu+1}(-\sqrt{\nu+1}\sqrt{1-\rho}/\sqrt{1+\rho})$
Gumbel	$C_G(u,v;\delta) = \exp\left(-((-\log u)^{\delta} + (-\log v)^{\delta})^1/\delta ight)$	$\delta \geqslant 1$	Asymmetric tail dependence: $\lambda_U = 2 - 2^{1/\delta}, \lambda_L = 0$
Rotated Gumbel	$C_{RG}(u,v;\delta) = u + v - 1 + C_G(1-u,1-v;\delta)$	$\delta \geqslant 1$	Asymmetric tail dependence: $\lambda_U = 0, \ \lambda_L = 2 - 2^{1/\delta}$
Plackett	$C_P(u,v;\theta) = \frac{1}{2(\theta-1)}(1+(\theta-1)(u+v)) - \sqrt{(1+(\theta-1)(u+v))^2 - 4\theta(\theta-1)uv}$	$\theta \geqslant 0, \theta \neq 1$	Tail independence: $\lambda_U = \lambda_L = 0$
Frank	$C_F(u,v;\theta) = -\frac{1}{\theta} log(1 + [(\exp^{-\theta u} - 1)(\exp^{-\theta v} - 1)/(\exp^{-\theta} - 1)])$	$0 < \theta < \infty$	Tail independence: $\lambda_U = \lambda_L = 0$
BB1	$C_{BB1}(u,v;\delta,\theta) = \{[(u^{- heta}-1)^{\delta}+(v^{- heta}-1)^{\delta}]^{1/\delta}+1\}$	$\theta > 0, \delta \geqslant 1$	Asymmetric tail dependence: $\lambda_U = 2 - 2^{1/\delta}, \lambda_L = 2^{-1/\theta\delta}$
BB7	$C_{BB7}(u,v;\delta, heta) = 1 - \Big(1 - [(1-(1-u)^{ heta})^{-\delta} + (1-(1-v)^{ heta})^{-\delta} - 1]^{-1/\delta}\Big)^{1/ heta}$	$\theta \geqslant 1, \delta i 0$	ASymmetric tail dependence: $\lambda_U = 2 - 2^{1/\theta}, \lambda_L = 2^{-1/\delta}$

Table 1: Copula specifications.

	ABC	BB	Bradesco	Pan	Banrisul	itaú	Paraná	System
Mean	0.000	0.000	0.000	-0.001	0.000	0.000	0.000	0.000
Std. Dev.	0.026	0.028	0.023	0.030	0.028	0.024	0.024	0.021
Maximum	-0.188	-0.189	-0.122	-0.369	-0.142	-0.129	-0.187	-0.128
Minimum	0.182	0.188	0.200	0.235	0.160	0.210	0.152	0.190
Skewness	-0.029	0.018	0.411	-1.177	0.073	0.500	-0.365	0.461
Kurtosis	8.471	7.676	8.983	29.085	5.777	9.650	11.600	11.018
$J–B^1$	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
$L–B^1$	0.000	0.032	0.000	0.301	0.041	0.000	0.000	0.000
ARCH-LM ¹	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 2: Descriptive statistics for Brazilian banks and BFIndex

 ^{1}p -values lower than 0.05 demonstrate a null hypothesis rejection of the statistical tests with 5% confidence level.

Notes: Daily data prices for the period from 1 January 2007 until 18 January 2016. The table reports the basic descriptive statistics for the prices returns series, including mean, standard deviation, skewness and kurtosis. J-B represents the empirical statistics from Jarque-Bera normality test based on skewness and kurtosis. L-B represents the empirical statistics for the Ljung-Box test of serial autocorrelation of the return series calculated with 20 differences. ARCH refers to the empirical statistics of the autoregressive conditional heteroscedastic of tenth order.

	ABC	BB	Bradesco	Pan	Banrisul	Itaú	Paraná
mean							
ϕ_0	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(-0.482)	(-0.413)	(-0.460)	(-0.688)	(-0.882)	(-0.524)	(-0.734)
ϕ_1	0.802^{*}	0.064^{*}		0.983^{*}			
	(5.159)	(2.634)		(230.215)			
$arphi_1$	-0.804*			-0.976*			-0.145*
	(-5.225)			(-1302.43)			(-6.093)
Variance							
ω	0.000	0.000	0.000	0.000*	0.000*	0.000^{*}	0.000^{*}
	(1.590)	(0.753)	(1.060)	(2.037)	(1.968)	(3.403)	(2.267)
α	0.042	0.042	0.017	0.367^{*}	0.034^{*}	0.011	0.157^{*}
	(1.731)	(1.500)	(0.970)	(2.944)	(2.005)	(1.308)	(2.050)
β_1	0.714^{*}	0.911^{*}	0.929^{*}	0.551^{*}	0.888^{*}	0.924^{*}	0.733^{*}
	(31.440)	(74.062)	(79.610)	(3.891)	(22.227)	(131.948)	(8.606)
β_2	0.194^{*}						
	(8.023)						
γ	0.036	0.076^{*}	0.079^{*}	-0.032	0.067^{*}	0.100*	0.148^{*}
	(1.581)	(3.104)	(2.705)	(-0.332)	(2.431)	(4.091)	(2.468)
Tail	6.989^{*}	13.118^{*}	11.785^{*}	3.651^{*}	8.819*	11.016*	3.603^{*}
	(6.688)	(2.991)	(3.745)	(10.484)	(4.929)	(4.916)	(10.142)
\log Lik	4564.579	4511.857	4871.937	4532.925	4304.213	4809.994	4933.897
L–B	29.596	20.844	23.277	28.327	18.472	26.492	18.211
/ >	[0.08]	[0.41]	[0.28]	[0.10]	[0.56]	[0.15]	[0.57]
L– $B(2)$	20.564	17.800	18.289	30.393	15.484	22.621	5.292
	[0.42]	[0.60]	[0.57]	[0.06]	[0.75]	[0.31]	[1.00]
ARCH	20.135	19.714	18.226	30.891	15.358	23.064	5.537
<i>R G</i>	[0.45]	[0.48]	[0.57]	[0.06]	[0.76]	[0.29]	[1.00]
K-S	[0.55]	[0.48]	[0.53]	[0.49]	[0.45]	[0.46]	[0.46]
C-vM	[0.51]	[0.44]	[0.49]	[0.45]	[0.41]	[0.42]	[0.42]
A-D	[0.58]	[0.50]	[0.56]	[0.51]	[0.47]	[0.48]	[0.48]

Table 3: Maximum likelihood estimation for the Brazilian banks' marginal distributions parameters.

Notes: This table presents the estimated maximum likelihood coefficients (ML) and z statistic for the marginal distribution parameters. LogLik is the log-likelihood value. L-B represents the empirical statistics for the Ljung-Box test of serial autocorrelation of return series calculated with 20 differences. L-B(2) represents the empirical statistics for the Ljung-Box test of serial autocorrelation of squared errors calculated with 20 differences. K-S, C-vM, and A-D denote the Kolmogorov-Smirnov, $Cram\acute{er}-von-Mises$ and Anderson-Darling tests for Student's t distribution model adequacy. The p-values (in brackets) below 0.05 indicate a rejection of the null hypothesis.

	System ex- ABC	System ex- BB	System ex- Bradesco	System ex- Pan	System ex- Banrisul	System ex- Itaú	System ex- Paraná
mean							
ϕ	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(-0.077)	(-0.078)	(0.052)	(-0.079)	(-0.070)	(0.037)	(-0.078)
Variance							
ω	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
	(3.296)	(2.919)	(3.073)	(3.294)	(3.299)	(3.300)	(3.293)
α	0.014	0.009	0.012	0.014	0.014	0.023^{*}	0.014
	(1.227)	(1.000)	(1.242)	(1.229)	(1.220)	(2.030)	(1.224)
β	0.915^{*}	0.925^{*}	0.923^{*}	0.915^{*}	0.916^{*}	0.903^{*}	0.915^{*}
	(48.034)	(48.695)	(53.324)	(48.007)	(48.177)	(45.648)	(48.026)
γ	0.108*	0.104^{*}	0.107^{*}	0.108*	0.108*	0.108^{*}	0.108*
	(3.654)	(3.516)	(3.816)	(3.649)	(3.653)	(3.686)	(3.653)
Tail	9.994^{*}	8.531^{*}	9.226^{*}	10.006*	10.054*	9.589^{*}	10.008*
	(4.296)	(4.765)	(4.308)	(4.284)	(4.255)	(4.456)	(4.280)
logLik	3747.390	3747.172	3734.221	3746.358	3744.236	3813.004	3746.082
LJ	21.460	25.131	22.751	21.473	21.334	19.390	21.438
	[0.371]	[0.196]	[0.301]	[0.370]	[0.378]	[0.497]	[0.372]
L– $J(2)$	12.640	7.716	14.321	12.627	12.607	12.965	12.615
	[0.892]	[0.994]	[0.814]	[0.893]	[0.894]	[0.879]	[0.893]
ARCH	12.180	7.685	13.488	12.166	12.150	12.755	12.155
	[0.910]	[0.994]	[0.855]	[0.910]	[0.911]	[0.888]	[0.911]
K – S	[0.46]	[0.86]	[0.62]	[0.44]	[0.91]	[0.39]	[0.11]
C–vM	[0.42]	[0.75]	[0.57]	[0.40]	[0.76]	[0.36]	[0.15]
A-D	[0.48]	[0.88]	[0.66]	[0.46]	[0.91]	[0.40]	[0.11]

Table 4: Maximum likelihood estimation for the Brazilian financial index.

Notes: This table presents the estimated maximum likelihood coefficients (ML) and z statistic for the marginal distribution parameters. LogLik is the log-likelihood value. L-B represents the empirical statistics for the Ljung-Box test of serial autocorrelation of return series calculated with 20 differences. L-B(2) represents the empirical statistics for the Ljung-Box test of serial autocorrelation of squared errors calculated with 20 differences. K-S, C-vM, and A-D denote the Kolmogorov-Smirnov, Cramér-von-Mises and Anderson-Darling tests for Student's t distribution model adequacy. The p-values (in brackets) below 0.05 indicate a rejection of the null hypothesis.

	System	System	System	System	System	System	System
	ABC	BB	Bradesco	Pan	Banrisul	Itaú	Paraná
Gaussian Copula							
ρ	0.442	0.689	0.859	0.352	0.508	0.830	0.243
	(0.01)	(0.00)	(0.01)	(0.03)	(0.01)	(0.01)	(0.02)
AIC	-346.212	-277.304	-2593.729	-191.658	-579.452	-2263.353	-56.987
Student-t Copula							
ρ	0.613^{*}	0.621^{*}	0.896^{*}	0.561^{*}	0.573^{*}	0.869^{*}	0.497^{*}
	(0.01)	(0.02)	(0.00)	(0.02)	(0.01)	(0.00)	(0.03)
u	100.000*	100.000	5.665^{*}	100.000*	11.722^{*}	7.308*	100.000*
	(3.21)	(165.33)	(0.41)	(7.73)	(2.81)	(0.65)	(4.87)
AIC	-434.451	-315.878	-2858.491	-274.584	-607.422	-2441.654	-121.041
Gumbel Copula							
σ	1.590^{*}	1.585^{*}	3.174^{*}	1.467^{*}	1.555*	2.775^{*}	1.332^{*}
	(0.03)	(0.04)	(0.06)	(0.04)	(0.03)	(0.05)	(0.04)
AIC	-341.331	-216.189	-2659.524	-192.620	-542.955	-2217.239	-71.182
Rotated Gumbel Co	pula						
σ	1.625^{*}	1.617^{*}	3.199^{*}	1.510^{*}	1.580^{*}	2.880^{*}	1.360*
:	(0.04)	(0.04)	(0.06)	(0.04)	(0.03)	(0.05)	(0.04)
AIC	-380.470	-241.530	-2680.354	-236.391	-574.904	-2343.086	-85.263
Plackett Copula							
heta	5.445*	6.139*	38.431*	4.588*	5.433*	27.197*	3.689*
	(0.36)	(0.40)	(1.86)	(0.32)	(0.33)	(1.39)	(0.30)
AIC	-390.699	-327.182	-2775.275	-250.752	-546.390	-2321.147	-107.117
Frank Copula							
θ	4.165*	5.047*	11.615*	3.698*	3.821*	9.948*	3.325*
	(0.18)	(0.19)	(0.33)	(0.19)	(0.16)	(0.23)	(0.21)
AIC	-414.535	-401.432	-2700.269	-265.032	-542.125	-2298.052	-120.347
Clayton Copula		0.044*	8 00 1*		0.010*		0 510*
θ	0.977^{*}	0.844^{*}	3.084*	0.798*	0.919^{*}	2.735*	0.516^{*}
AIG	(0.06)	(0.06)	(0.09)	(0.06)	(0.05)	(0.08)	(0.06)
AIC BB1 Gamala	-324.609	-167.502	-2224.267	-201.967	-489.266	-1988.517	-60.161
	0 447*	0.260*	0 600*	0 129*	0.411*	0.757*	0.000*
θ	(0.447)	(0.360°)	$(0.00)^{\circ}$	(0.438°)	$(0.411)^{\circ}$	(0.757)	(0.289)
2	(0.07)	(0.00)	(0.00)	(0.07)	(0.00)	(0.07)	(0.00)
0	(0.04)	1.307	2.000	(0.04)	1.337	2.120°	(0.04)
AIC	(0.04)	(0.05)	(0.07)	(0.04)	(0.04)	(0.07)	(0.04)
BB7 Copula	-991.199	-200.110	-2119.490	-242.000	-005.520	-2301.212	-90.050
	1 405*	1 277*	2.070*	1 981*	1 429*	4 720*	1 948*
U	(0.06)	(0.06)	2.310	(0.05)	(0.05)	4.120	(0.05)
δ	0.787*	0.007	2 270*	0.670*	0.708*	3 071*	0.03
0	(0.07)	(0.062)	(0.11)	(0,06)	(0.05)	(0.10)	(0.06)
AIC	-368 745	-215 442	_2623 100	-228 588	-594 199	-1548 909	-87 657
	-000.140	-210.442	-2020.199	-220.000	-034.133	-1040.332	-01.001

Table 5: Estimation of the bivariate copula model for the dependency between each of the banks and the BFIndex.

Notes: This table reports the maximum likelihood (ML) estimation for the different copula models for *BFIndex* (excluding the series in the column) and the indicated series in each column. Standard errors (in parenthesis) and AIC values adjusted for small samples are provided for different copula models. The minimum value of the AIC (**bold**) indicates the best fit copula. An asterisk (*) indicates a significance level of 5%.

	(2,1)			(2.5)		
	(3,1)	(3,2)	(3,4)	(3,5)	(3,6)	(3,7)
Gaussian Copula						
ρ	0.420	0.677	0.316	0.484	0.881	0.221
	(0.02)	(0.03)	(0.00)	(0.02)	(0.01)	(0.02)
AIC	-305.182	-207.238	-150.080	-517.544	-2910.823	-44.903
Student-t Copula						
ρ	0.586^{*}	0.583^{*}	0.521*	0.542^{*}	0.901^{*}	0.466^{*}
	(0.02)	(0.05)	(0.02)	(0.02)	(0.00)	(0.03)
ν	100.000*	100.000*	100.000	10.055^{*}	11.174^{*}	100.000*
	(1.42)	(31.73)	(146.62)	(3.50)	(1.57)	(3.88)
AIC	-379.865	-259.093	-202.172	-545.052	-2955.830	-96.407
Gumbel Copula						
σ	1.552^{*}	1.508*	1.412*	1.515^{*}	3.263^{*}	1.301^{*}
	(0.03)	(0.04)	(0.03)	(0.03)	(0.06)	(0.04)
AIC	-308.052	-174.947	-156.704	-491.174	-2812.151	-60.167
Rotated Gumbel Co	pula					
σ	1.558^{*}	1.506^{*}	1.426^{*}	1.520^{*}	3.240^{*}	1.298^{*}
	(0.03)	(0.04)	(0.03)	(0.03)	(0.06)	(0.04)
AIC	-318.479	-172.058	-172.128	-509.134	-2778.641	-58.686
Plackett Copula						
θ	5.087^{*}	5.283^{*}	3.952^{*}	4.946^{*}	37.727^{*}	3.320^{*}
	(0.34)	(0.36)	(0.29)	(0.31)	(2.14)	(0.28)
AIC	-351.485	-266.851	-193.507	-485.012	-2826.452	-84.500
Frank Copula						
θ	3.923^{*}	4.616*	3.275^{*}	3.540^{*}	11.913^{*}	3.042^{*}
	(0.18)	(0.19)	(0.15)	(0.16)	(0.08)	(0.21)
AIC	-369.285	-334.398	-204.323	-476.428	-2809.653	-95.470
Clayton Copula						
θ	0.851^{*}	0.659^{*}	0.646^{*}	0.825^{*}	3.110^{*}	0.403^{*}
	(0.05)	(0.06)	(0.05)	(0.04)	(0.09)	(0.06)
AIC	-261.756	-111.803	-140.698	-437.068	-2288.187	-37.145
BB1 Copula						
θ	0.332*	0.252*	0.314*	0.364*	0.542*	0.208*
	(0.06)	(0.06)	(0.06)	(0.05)	(0.11)	(0.06)
δ	1.370*	1.372*	1.260*	1.318*	2.655*	1.222*
110	(0.05)	(0.05)	(0.04)	(0.04)	(0.12)	(0.04)
AIC	-338.223	-194.314	-184.627	-545.587	-2918.388	-73.150
BB7 Copula	1 401*	1 050*	1 001*	1 410*	0.0FF*	1 0 1 0 *
Ø	1.421*	1.356*	1.291*	1.413*	3.055*	1.246*
5	(0.06)	(0.05)	(0.05)	(0.05)	(0.10)	(0.05)
0	0.648*	0.521^{*}	0.527^{*}	0.626*	2.252*	0.347*
	(0.06)	(0.06)	(0.06)	(0.05)	(0.13)	(0.06)
AIC	-313.297	-159.220	-171.847	-539.250	-2756.593	-65.382

 Table 6: Bank pairs vine copula model estimation: first branch.

Notes: This table reports the maximum likelihood (ML) estimation for the different Brazilian banks pair copula models. T in the first line represents the level of the tree. In the second line, numbers represent the banks names 1.- ABC; 2.- BB; 3.- Bradesco; 4.- Pan; 5.- Banrisul; 6.- Itau; 7,- Parana. AIC minimum values (**bold**) indicate the best fit copula. An asterisk (*) indicate a significance level of 5%.

			T_2				7	3	
	(1,2-3)	(2, 4-3)	(2, 5 - 3)	(2, 6-3)	(2,7-3)	(1,4-2,3)	(1,5-2,3)	(1,6-2,3)	(1,7-2,3)
Gaussian	Copula								
ρ	0.563	0.645	0.115	0.084	0.843	0.166	0.134	0.086	0.248
	(0.00)	(0.02)	(0.01)	(0.02)	(0.00)	(0.04)	(0.02)	(0.01)	(0.01)
AIC	-737.910	-1041.904	-21.695	-10.194	-2287.866	-52.336	-33.147	-12.420	-121.876
Student-t	Copula								
ho	0.785^{*}	0.850^{*}	0.190^{*}	0.120^{*}	0.936^{*}	0.192*	0.147^{*}	0.087^{*}	0.270^{*}
	(0.01)	(0.00)	(0.04)	(0.03)	(0.00)	(0.07)	(0.02)	(0.02)	(0.02)
u	100.000	100.000*	10.184^{*}	9.011^{*}	100.001	8.296	19.850	61.751	6.907^{*}
	(117.86)	(3.22)	(1.06)	(1.63)	(154.37)	(11.18)	(16.82)	(71.41)	(2.20)
AIC	-1057.873	-1515.666	-30.937	-16.855	-2844.030	-58.237	-34.422	-10.749	-131.158
Gumbel (Copula								
σ	2.148^{*}	2.618*	1.141*	1.086^{*}	4.038^{*}	1.141*	1.094*	1.046^{*}	1.198^{*}
	(0.04)	(0.05)	(0.03)	(0.02)	(0.08)	(0.02)	(0.02)	(0.01)	(0.02)
AIC	-939.453	-1389.911	-30.003	-13.134	-2680.389	-59.708	-33.372	-10.271	-121.609
Rotated 0	Jumbel Copu	la				1			
σ	2.169^{*}	2.647^{*}	1.138^{*}	1.094^{*}	4.056^{*}	1.135*	1.089*	1.049*	1.220^{*}
1.7.0	(0.04)	(0.05)	(0.03)	(0.02)	(0.08)	(0.02)	(0.02)	(0.02)	(0.02)
AIC	-962.958	-1430.170	-28.976	-15.454	-2692.874	-54.370	-26.695	-8.960	-131.010
Plackett (Copula		1 001*	1 000*	10.00.1*		1 5104	1 2 2 2 *	2 1 2 1 4
θ	9.496*	15.777*	1.631*	1.392*	40.984*	1.674*	1.519*	1.262*	2.101*
110	(0.55)	(0.82)	(0.14)	(0.13)	(2.04)	(0.11)	(0.11)	(0.09)	(0.14)
	-799.562	-1239.805	-26.449	-11.320	-2481.743	-53.812	-33.392	-9.800	-112.388
Frank Co	pula	F F 40*	0.000*	0.004*	10.007*	1.020*	0.000*	0 150*	1 470*
θ	5.807^{+}	(0.20)	0.968^{+}	0.624^{+}	13.067*	1.032*	0.822^{*}	0.459^{+}	$1.4(3^{+})$
ATC	(0.19)	(0.20)	(0.18)	(0.11)	(0.14)	(0.14)	(0.14)	(0.14)	(0.14)
AIC Classier C	-833.783	-12(8.412	-20.138	-10.487	-2008.480	-53.300	-32.495	-9.018	-109.180
Clayton C		9 609*	0.997*	0.159*	1 677*	0.925*	0.146*	0 000*	0 200*
0	1.000	(0.08)	(0.257)	(0.04)	4.077	(0.235)	(0.140)	(0.080)	0.398
AIC	(0.07)	(0.06)	(0.05)	(0.04)	(0.11)	(0.04)	(0.03)	(0.03)	(0.04)
BB1 Con	-000.0 <i>99</i>	-1287.300	-24.900	-11.004	-2595.719	-40.200	-21.230	-0.150	-115.950
	0.655*	0.807*	0.114	0.084	0 799*	0.043	0.045	0.047	0.208*
0	(0.000)	(0.10)	(0.06)	(0.06)	(0.10)	(0.043)	(0.040)	(0.04)	(0.208)
δ	1 695*	1 959*	1.098*	(0.00) 1 057*	3 030*	1 1 1 2 3*	1.076*	1 031*	1 116*
0	(0.06)	(0.08)	(0.03)	(0.03)	(0.12)	(0.03)	(0.02)	(0.02)	(0.03)
AIC	-1017 194	-1487 796	-31 976	-13 558	$-2765\ 494$	-58 416	-32717	-10 175	-134 894
BB7 Con	10111101 11a	11011100	011010	10.000	21001101	001110	021111	101110	1011001
θ	2.109*	2.571^{*}	1.132^{*}	1.075^{*}	4.329^{*}	1.160*	1.091^{*}	1.037^{*}	1.161^{*}
-	(0.08)	(0.09)	(0.04)	(0.04)	(0.14)	(0.04)	(0.03)	(0.02)	(0.03)
δ	1.549^{*}	2.356*	0.174*	0.118*	4.756*	0.119*	0.090*	0.066*	0.292*
	(0.09)	(0.12)	(0.05)	(0.05)	(0.25)	(0.05)	(0.04)	(0.03)	(0.05)
AIC	-1043.938	-1518.338	-32.432	-13.549	-2734.663	-58.631	-31.257	-10.006	-138.269

Table 7: Bank pairs vine copula model estimation: second and third branch.

Notes: This table reports the maximum likelihood (ML) estimation for the different Brazilian banks pair copula models. *T* in the first line represents the level of the tree. In the second line, numbers represent the banks names 1.- *ABC*; 2.- *BB*; 3.- *Bradesco*; 4.- *Pan*; 5.- *Banrisul*; 6.- *Itaú*; 7.- *Paraná*. AIC minimum values (**bold**) indicate the best fit copula. An asterisk (*) indicate a significance level of 5%.

		T.		Т	<u>.</u>	T_{c}
	4,5-1,2,3	4,6-1,2,3	4,7-1,2,3	5,6-1, 2,3,4	5 6,7—1,2,3,4	5,7-1,2,3,4,6
~ . ~ .			, , ,		, , , , ,	, , , , ,
Gaussian Copula	0.000	0.000	0.100	0.100	0.01	0.004
ho	0.069	0.002	0.128	0.120	-0.017	-0.004
410	(0.02)	(0.01)	(0.04)	(0.03)	(0.02)	(0.02)
AIC	-7.249	1.992	-30.007	-26.020	1.451	1.968
Student-t Copula	0.075*	0.000	0 1 5 4 4	0.115*	0.015	0.000
ho	0.075^{*}	0.003	0.154^{*}	0.117*	-0.015	-0.003
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.01)
u	84.568	500.000	9.021*	58.795	28.396*	275.634
410	(208.48)	(3,242.59)	(3.08)	(153.57)	(0.54)	(424.76)
AIC	-5.486	4.145	-35.851	-24.823	-0.284	3.913
Gumbel Copula	1.00	1 000*			1 00.0*	1 000*
σ	1.037*	1.000*	1.105*	1.055*	1.006*	1.000*
	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.02)
AIC	-3.914	2.002	-35.890	-18.817	1.152	2.002
Rotated Gumbel C	opula	1 0014			1 000*	1 000*
σ	1.040*	1.001*	1.105*	1.059*	1.000*	1.000*
	(0.02)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
AIC	-5.392	2.000	-31.146	-17.591	2.002	2.002
Plackett Copula	1 100*	1 0 1 0 4		1 20.4*	0.040*	1.0054
θ	1.196*	1.046*	1.544*	1.394*	0.943*	1.035*
	(0.09)	(0.07)	(0.11)	(0.10)	(0.06)	(0.06)
AIC	-4.243	1.560	-34.283	-21.340	1.085	1.681
Frank Copula	0.050%	0.001	0.000*			
heta	0.353*	0.091	0.860*	0.665*	0.000	0.070
	(0.14)	(0.14)	(0.15)	(0.09)	(0.00)	(0.10)
AIC	-4.132	1.556	-35.679	-21.221	2.002	1.678
Clayton Copula	0.050%		0.100%			
heta	0.072*	0.016	0.182*	0.108*	0.000	0.004
	(0.03)	(0.03)	(0.04)	(0.03)	(0.03)	(0.02)
AIC	-4.634	1.641	-23.760	-17.347	2.010	1.973
BB1 Copula			0.040		0.001	
heta	0.047	0.015	0.043	0.072*	0.001	0.003
	(0.04)	(0.62)	(0.05)	(0.03)	(0.91)	(0.60)
δ	1.020*	1.001	1.088*	1.032*	1.006*	1.001
	(0.02)	(0.55)	(0.03)	(0.02)	(0.12)	(0.54)
AIC	-3.721	3.892	-34.589	-23.146	3.269	4.305
BB7 Copula	1 00 04	1 0 0 1 14	a a o ok	1 1 0 0 1 14		
θ	1.024*	1.001*	1.106*	1.034*	1.011*	1.001*
_	(0.03)	(0.44)	(0.03)	(0.02)	(0.09)	(0.43)
δ	0.059	0.015	0.104^{*}	0.091*	0.001	0.003
	(0.03)	(0.21)	(0.04)	(0.03)	(0.96)	(0.20)
AIC	-3.544	3.879	-33.726	-22.409	2.597	4.304

 ${\bf Table \ 8:} \ {\rm Bank \ pairs \ vine \ copula \ model \ estimation: \ fourth, \ fifth, \ and \ sixth \ branch.$

Notes: See notes in Table 6

.

	Vine Structure		Unconditional α -quantile	Conditional Vine α -quantile
Bradesco		mean	-0.034	
		std.dev.	(0.01)	
Bradesco - BB	(2,3)	mean		-0.103
		std.dev.		(0.05)
Bradesco - ABC	(1,3)	mean		-0.126
		std.dev.		(0.06)
Bradesco - Pan	(4,3)	mean		-0.097
		std.dev.		(0.05)
$Bradesco - Ita \acute{u}$	(6,3)	mean		-0.141
		std.dev.		(0.07)
Bradesco - Banrisul	(5,3)	mean		-0.133
		std.dev.		(0.06)
Bradesco - Paraná	(7,3)	mean		-0.116
		std.dev.		(0.05)
BB		mean	-0.042	
		std.dev.	(0.02)	
BB - Bradesco	(2,3)	mean		-0.103
		std.dev.		(0.04)
BB - ABC	(1,2-3)	mean		-0.114
		std.dev.		(0.05)
BB - Pan	(2, 4-3)	mean		-0.132
		std.dev.		(0.06)
BB — Itaú	(2, 6-3)	mean		-0.146
		std.dev.		(0.07)
BB - Banrisul	(2, 5 - 3)	mean		-0.098
		std.dev.		(0.05)
BB - Paraná	(2,7-3)	mean		-0.137
		std.dev.		(0.06)
ABC		mean	-0.040	
		std.dev.	(0.01)	
ABC - Bradesco	(1,3)	mean		-0.121
		std.dev.		(0.05)
ABC - BB	(1,2-3)	mean		-0.135
		std.dev.		(0.06)
ABC - Pan	(1, 4-2, 3)	mean		-0.041
		std.dev.		(0.02)
ABC — Itaú	(1, 6-2, 3)	mean		-0.039
		std.dev.		(0.02)
ABC - Banrisul	(1, 5-2, 3)	mean		-0.044
		std.dev.		(0.02)
ABC - Paraná	(1,7-2,3)	mean		-0.128
		std.dev.		(0.06)
Pan		mean	-0.047	
	<i></i>	std.dev.	(0.02)	
Pan - Bradesco	(4,3)	mean		-0.094
		std.dev.		(0.04)
Pan - BB	(2, 4 - 3)	mean		-0.142
		std.dev.		(0.06)
Pan - ABC	(1, 4-2, 3)	mean		-0.036
		std.dev.		(0.02)
Pan — Itaú	(4, 6-1, 2, 3)	mean		-0.062
		std.dev.		(0.03)
Pan - Banrisul	(4, 5-1, 2, 3)	mean		-0.032
		std.dev.		(0.02)
Pan — Paraná	(4, 7 - 1, 2, 3)	mean		-0.036
		std.dev.		(0.02)

Table 9: Descriptive statistics for the Unconditional α -quantile and Vine Conditional α -quantile.

	Vine Structure		Unconditional α -quantile	Vine Conditional α -quantil
Itaú		mean	-0.036	
		std.dev.	(0.01)	
$Ita \'u - Bradesco$	(6,3)	mean		-0.132
		std.dev.		(0.06)
$Ita \acute{u} - BB$	(2, 6-3)	mean		-0.045
		std.dev.		(0.04)
$Ita \acute{u} - ABC$	(1, 6-2, 3)	mean		-0.040
		std.dev.		(0.03)
Itaú — Pan	(4, 6-1, 2, 3)	mean		-0.072
		std.dev.		(0.04)
$Ita \acute{u} — Banrisul$	(5, 6-1, 2, 3, 4)	mean		-0.040
		std.dev.		(0.03)
$Ita \acuteu — Paran \acutea$	(6, 7 - 1, 2, 3, 4)	mean		-0.074
		std.dev.		(0.06)
Banrisul		mean	-0.045	
		std.dev.	(0.01)	
Banrisul - Bradesco	(5,3)	mean		-0.128
		std.dev.		(0.05)
Banrisul - BB	(2,5-3)	mean		-0.087
		std.dev.		(0.04)
Banrisul - ABC	(1, 5-2, 3)	mean		-0.091
		std.dev.		(0.04)
Banrisul - Pan	(4, 5-1, 2, 3)	mean		-0.062
		std.dev.		(0.03)
Banrisul — Itaú	(5, 6 - 1, 2, 3, 4)	mean		-0.067
		std.dev.		(0.03)
Banrisul - Paraná	(5, 7 - 1, 2, 3, 4, 6)	mean		-0.070
		std.dev.		(0.04)
Paraná		mean	-0.038	
		std.dev.	(0.02)	
$Paran{\acute{a}} - Bradesco$	(7,3)	mean		-0.112
		std.dev.		(0.05)
Paraná - BB	(2,7-3)	mean		-0.126
		std.dev.		(0.05)
Paraná - ABC	(1,7-2,3)	mean		-0.043
		std.dev.		(0.02)
$Paran{\acute{a}} - Pan$	(4, 7 - 1, 2, 3)	mean		-0.023
		std.dev.		(0.01)
Paraná — Itaú	(6, 7 - 1, 2, 3, 4)	mean		-0.024
		std.dev.		(0.01)
Paraná — $Banrisul$	(5, 7 - 1, 2, 3, 4, 6)	mean		-0.025
		std.dev.		(0.02)

Table 10: Descriptive statistics for the Unconditional α -quantile and Vine Conditional α -quantile (cont.).

	Unconditional α -quantile	Conditional α -quantile
System	-0.062	
	(0.03)	
System $-ABC$		-0.123
		(0.05)
System	-0.062	()
5	(0.03)	
System $-BB$		-0.105
v		(0.04)
System	-0.063	
v	(0.03)	
System — Bradesco		-0.140
·		(0.07)
System	-0.062	
·	(0.03)	
System — Pan		-0.120
·		(0.05)
System	-0.062	
-	(0.03)	
System — Banrisul		-0.124
-		(0.05)
System	-0.060	
·	(0.03)	
System — Itaú		-0.131
·		(0.06))
System	-0.062	
-	(0.03)	
System — Paraná	. ,	-0.115
		(0.05)

Table 11: Descriptive statistics for the Unconditional α -quantile and Vine Conditional α -quantile.



Figure 1: C-Vine structure.



Figure 3: Time series of the Brazilian stock prices and BFIndex prices from BM&FBOVESPA.





Figure 4: Systemic risk network structure of the Brazilian financial system.

1.- ABC, 2.- BB, 3.- Bradesco, 4.- Pan, 5.- Banrisul, 6.- Itaú, 7.- Paraná



Figure 5: Map of the systemic risk network of the Brazilian banks.



Figure 6: Time series of conditional and unconditional α -quantile for Brazilian banks and for the Brazilian financial system.