OLLSCOIL NA HÉIREANN MÁ NUAD

THE NATIONAL UNIVERSITY OF IRELAND

MAYNOOTH

Evaluation Report on the Algebra Project in Ireland

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Abstract

The *Algebra Project in Ireland* (API) study was a year-long initiative funded by SFI through the SFI Discover Call 2015 scheme and sponsored by the NCCA and the Kildare Education Centre. Additional support for the project came from the PDST and KWETB. The initiative took place in 2016 and continued into 2017, and comprised of three parts - at primary level, post-primary level and at university level. The aim of the project was to introduce the curriculum and pedagogy of the US Algebra Project to Irish teachers (both in-service and pre-service) and students in order to develop mathematical teaching and learning. In this report, we describe the project in detail, and present data from the evaluation carried out by researchers at Maynooth University.

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Glossary of Terms and Acronyms

AP	Algebra Project
API	Algebra Project Ireland
CPD	Continuing Professional Development
DES	Department of Education and Skills
JC	Junior Cycle - Lower Secondary
KEC	Kildare Education Centre
NCCA	National Council for Curriculum and Assessment
PSTs	Pre-Service Teachers
SFI	Science Foundation Ireland
KWETB	Kildare and Wicklow Education and Training Board
PDST	Professional Development Service for Teachers
MLW	Maths Literacy Worker
PMDT	Project Maths Development Team
TY	Transition Year
MU	Maynooth University
DCU	Dublin City University
WSE	Whole School Evaluation
NS	National School
ETBI	Education and Training Boards Ireland

1. Introduction

The *Algebra Project in Ireland* (API) study was a year-long initiative funded by SFI through the SFI Discover Call 2015 scheme and sponsored by the NCCA and the Kildare Education Centre. Additional support for the project came from the PDST and KWETB. The initiative took place in 2016 and continued into 2017, and comprised of three parts - at primary level, post-primary level and at university level. The aim of the project was to introduce the curriculum and pedagogy of the US Algebra Project to Irish teachers (both in-service and pre-service) and students in order to develop mathematical teaching and learning. Exposing teachers and young people to mathematically rich tasks in a fun and engaging way has the potential to empower them to see the value of mathematics in their lives and also to develop mathematical fluency. In this report, we will describe the project in detail, and present data from the evaluation carried out by researchers at Maynooth University.

It was hoped that exposure to the evidence-based proven methods of the Algebra Project curriculum would have a positive impact on mathematics education in Ireland and on students' attitude towards mathematics and their belief that they can do mathematics.

In particular, the project aimed to

1. Establish the Flagway game as a knowledge-based sport in the participating primary schools.
2. Establish the Road Colouring Problem as Transition Year module in KWETB schools
3. Train a cohort of Mathematics Literacy Workers (MLWs) in DCU to work with primary school children in the local community.

The project involved a series of workshops to introduce the curriculum and pedagogy of the Algebra Project.

1.2. The Algebra Project: Background Information

The Algebra Project (see http://www.algebra.org/) is a non-profit US organisation which was founded in 1982 by the renowned Civil Rights' leader Dr Bob Moses. Dr Moses was awarded a McArthur Fellowship in that year and chose to spend his award by setting up an organisation to help under-represented children, especially those performing in the lowest quartile, to achieve mathematical literacy and to train teachers, parents and community workers to create the conditions for this to take place. Dr Moses saw mathematics as a 'gate-keeper' subject, in that in the

US system students could not attend university if they were not able to take calculus. However, many students have problems with algebra which is a pre-requisite of calculus, and so he chose to direct his efforts into helping students move from arithmetical thinking to algebraic thinking.

Many initiatives have aimed to improve students' algebraic thinking. The Algebra Project (AP) has developed its own methods and curricular materials to do this, as well as giving much attention to the professional development of teachers as well as community and youth workers. It has been well documented that, while it is not the only factor, there is a strong correlation between socio-economic status and achievement in mathematics for elementary and secondary school students (Dubinsky and Wilson, 2013). The move from arithmetic to algebraic thinking involves students moving from using numbers as physical quantities to working with abstract variables, while simultaneously generalising operations such as addition and subtraction. A fundamental tenet of the Algebra Project is that this move from the physical experience to abstraction can be facilitated by using a five-step programme which begins with a physical event or experience and takes students through to a formal symbolic representation of this event. AP's curriculum combines inquiry and experiential learning, which is mathematics emerging from human experience. Mathematics is also made accessible by using real-life situations that embody rich mathematical concepts. Through the process of mathematizing these situations or events, students are encouraged to actively engage in mathematical discourse by using their everyday language for talking about mathematical concepts. This discourse leads to a focus on important mathematical features about the event and to the process of symbolization. By actively engaging in the mathematics discovery process, students encounter complex mathematical ideas that they learn to work through. The idea is that this controlled movement from the concrete to the abstract allows students to build their own meanings for algebraic objects as well as helping them see that algebra is not just a collection of mysterious symbols and operations. The five steps are:

1. Physical event
2. Picture or model of this event
3. Intuitive (idiomatic) language description of this event
4. A description of this event in regimented English
5. Symbolic representation of the event (Moses, Kamii, McAllister Swap, Howard, 1989)

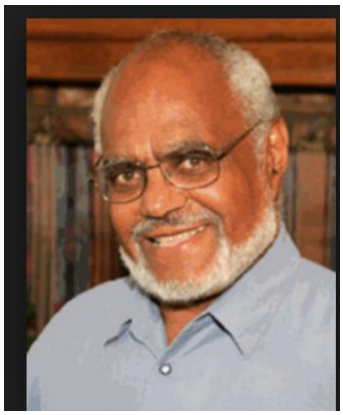
The Algebra Project has developed several modules which have a basic mathematical concept in algebra at their core; these modules have been refined over decades and are the outcome of many

years of practical work and research. They follow the five-step programme above, and so they begin with a real physical experience which involves the students, then gradually the students are encouraged to draw, model, or describe the event in a progressively more formal and rigorous way until they are able to appreciate the underlying concept as well as the language that describes it.

The Algebra Project in Ireland chose to focus on two of these modules, namely the Flagway game and the Road Colouring Problem. The Flagway game (which will be described below) and various types of related games allow students to explore different facets of factorisation and eventually lead them to naturally represent numbers by symbols. This includes factorisation of numbers, including notions such as prime numbers, unique prime decomposition, highest common factor, lowest common denominator, etc. The hope is that students not only improve their mental arithmetic skills through playing the games, but that they are given opportunities to discover important properties of numbers themselves and eventually also see the need for different ways of representing numbers (i.e. as variables).

The Road Colouring Problem is based on an actual research problem in mathematics based on network theory which has many applications in the real world. The implementation of this phase of the project has been delayed and is not the focus of this evaluation report.

The US Algebra Project Team



Dr Bob Moses is a legendary civil rights activist and MacArthur award-winning educator. He is President and founder of the Algebra Project Inc., a national non-profit dedicated to improving the mathematical achievement of historically underrepresented students and communities.

Dr Greg Budzban is Dean of the College of Arts and Sciences in Southern Illinois University Edwardsville. Formally he was Chair of the Mathematics Department and Chair of the STEM Education Research Centre at Southern Illinois University. Greg has been a keen developer of Algebra Project High School curriculum materials since 2001 and leads SIU's effort to create a new teacher certification programme in collaboration with the Algebra Project and Young People's Program.

Maisha Moses came into the story when she took over as Executive Director of the Young People's Project, a movement set up in Mississippi in 1996 by former Algebra Project students. The model was for a youth-led organisation that centred around young people teaching each other mathematics. It developed into a series of national workshops and other events, and reached Chicago in 2002.

Marquis Lowe is the site director for Young Peoples Project (YPP) Jackson. He was born and raised in Jackson, and has been part of YPP and the Algebra Project since 1995, starting with the Algebra Project in 6th grade and going on to become a YPP high school Maths Literacy Worker, Instructor, Trainer and then Program Coordinator before becoming Director. He graduated from Tougaloo College in Jackson Mississippi with a Bachelor's degree in business.

1.2.1. The Flagway Game

There are many different ways to create a cultural context in which mathematics emerges naturally from students' experience. One method used in the Algebra Project is to create mathematically rich games and experiences. Bob Moses developed the Flagway Game in 1995 and patented it in 1996 (Moses, U.S. Pat #5520542 & 5704790). The game involves skill and speed – both mathematically and physically.

The game is based on the Mobius function; this function assigns to each positive whole number one of three possible outputs. In the Flagway game these outputs are the colours red, blue and yellow, and so each natural number is either a red, blue or yellow number. To begin with the players are only told the colours of a few numbers (say the numbers 1-6) and are asked to try to guess the colours of other numbers. The students make conjectures based on the information about the colours of numbers they know and these conjectures are tested when new information about the colours of other numbers is given to them. In this way, a spirit of experimentation is encouraged as well as a reliance on reasoning to check conjectures. In practice, figuring out the rules governing how colours are assigned can take some time but simple versions of the game can be played straight away.

One such version might involve a set of number cards on a table at the top of a hall or playground (maybe with cards for the numbers 1-20 or 1-100) and three buckets at the other end (one for each colour of number). The pupils are divided into teams, and the game is a form of relay race – a student from each team runs to the table, takes a number and runs to deposit the card in the appropriate bucket. Points are awarded for accuracy and speed. A more complicated version of the game has the students taking three cards from the table and running to a circle in the middle of the play area, this circle should have a network of coloured paths leading from it (See Figure 1 below for pictures). Each path from the centre is made up of three portions, each coloured red, blue or yellow. Players need to arrange their cards in a sequence and then follow the path dictated by their cards (for example if they have a sequence of 5-6-4 they need to take the red path leading from the centre (for 5), then the blue path (for 6) from that node, and lastly the yellow path (for 4)).

The colours of the numbers are assigned using the Mobius function, and thus depend on the prime factorisation of the number. The three colours correspond to the three possible categories of prime factorisations: the case where the number is divisible by the square of a prime; the case where the number has an odd number of prime factors (with none repeated); the case where the number has an even number of prime factors (with none repeated). Thus, in order to play the Flagway game well (in particular, in order to be quick), it is important to be able to factor numbers quickly and to be

able to decide quickly to which category they belong. Pupils are encouraged to represent the factorisations using letters and notice for example that 12 and 18 are both of the form a^2b (and so are both the same colour). Thus the use of variables is introduced in a natural setting where pupils can appreciate the need for them. For more details see Appendix E and for information on how Flagway could become a knowledge-based sport see http://www.typp.org/flagway.

Figure 1.1: Flagway game in progress

Figure 1.2: Students taking part in a Flagway game

1.2.2. The Road Colouring Problem

The Road Colouring Problem was a long-standing conjecture in Graph Theory until it was proved in 2009 by Avraham Thratman. It concerns the existence of synchronised instructions for a network which enable you to reach a specified point in the network no matter where you start by using the

given directions. For example, if the network is a map of streets in a city, then the problem is to find a single set of instructions that will get you to the City Hall no matter where you started from.

Professor Greg Budzban described this research problem to Dr Bob Moses during a visit to Southern Illinois University. Dr Moses realised the potential of this problem and the scope to use the five-step model to employ this problem as an introduction to functions. The ideas are introduced to students by first asking them to build cities with 3 or 4 buildings with the constraints that each building needs to have exactly one red road and exactly one blue road leaving from it, and that it must be possible to reach every building from every other building using a combination of roads. The students are asked to find synchronising instructions for their cities (maybe blue-red-blue) such that if people standing in any of the buildings follow these instructions, then they will all end up at the same building at the end of the set of instructions. Students are asked to physically build the cities and to walk the routes called. They are then asked to represent the cities and moves on paper and describe what is happening (firstly in informal language and then more precise formal mathematical language). Finally the students represent the event using symbols and diagrams; this usually involves arrow diagrams and then ordered pairs. At this point the notion of functions is introduced formally and students realise that the red roads define a function on the set of buildings, as do the blue roads. They see that the synchronising instruction corresponds to finding a composition of the red and blue functions which is a constant function. Vocabulary such as domain and range are also introduced in this natural setting. Finally, students see how the red and blue functions could be represented by matrices and discover the link between the composition of functions and multiplication of matrices. Thus, we see that some advanced mathematics can be introduced in the context of the physical activity of building cities with certain properties. This module is aimed at secondary school children. See http://www.algebra.org/curriculum/unit/RC-Road-Coloring/ for more details.

1.2.3. The Young Peoples' Project

The Young Peoples' Project (YPP) is a sister project of the Algebra Project in the US. It is responsible for recruiting and training high-school and university students to act as Math Literacy Workers (MLWs) in schools and communities. The training was partly developed in the course of an NSF-funded project. Since its foundation in 1996, the YPP has trained 5000 MLWs and now runs workshops for over 5,000 children each year. The MLWs organise after-school programmes and summer schools and through the use of games, especially the Flagway game and its variants, teach mathematics and mathematical thinking skills to children and sometimes their parents. Details of

their programmes can be found at http://www.typp.org/. The YPP is also involved in running leagues and tournaments for the Flagway game.

1.3. Structure of the Report

In the remainder of this report we discuss the implementation of the Algebra Project in Ireland. Section 2 describes the history of the project and its aims in Ireland, and also details the events held in order to facilitate the project. Section 3 describes our methods of data collection and analysis. Section 4 evaluates the success and impact of the project on teaching and learning by analysing the responses of PSTs who took part in the project, while in Section 5 we consider the views of students and teachers at Primary Level who participated in the Algebra Project. Section 6 concludes the report with a summary of our findings and recommendations for further implementations of the project based on these.

2. The Algebra Project in Ireland

2.1. The History of the Project

The Algebra Project in Ireland (API) has its roots in a sponsored visit to the US in the Summer of 2014 by Rachel Linney of the NCCA. Along with 5 teachers, she investigated the AP curriculum and pedagogy in Miami, Florida. In May 2015, this was followed by a visit by Dr Moses, Professor Greg Budzban, and Marquis Lowe to Maynooth University to run a series of workshops for primary and secondary teachers focusing on the Flagway Game (for the primary teachers) and the Road Colouring Problem (for the secondary teachers). Some of these teachers (especially the ones at primary level) went on to try out the Algebra Project modules with their own classes. All who engaged with the curriculum highlighted the value of the approach and the potential for the modules in the Irish system; however, they also highlighted the need for more long-term, structured

support for teachers to confidently implement this approach in their classrooms. Later that year, the Kildare Education Centre was granted SFI funding to study the potential of the Algebra Project to impact positively on the teaching and learning of mathematics in Ireland.

2.2. Aims of the Algebra Project in Ireland

The focus of the project was to support teachers in understanding the mathematical potential of the Flagway game and the Road Colouring Problem and to trial these modules in a selection of schools, and to collaborate with DCU to train some pre-service teachers as MLWs.

In particular, the project aims were to

1. Establish the Flagway game as a knowledge-based sport in the participating primary schools.
2. Establish the Road Colouring Problem as a TY module in KWETB schools as well as in the PE and Mathematics Teaching degree in DCU.
3. Train a cohort of MLWs in DCU to work with primary school children in the local community.

2.3. Training Workshops and Implementation

2.3.1. Primary level

At Primary Level the Algebra Project in Ireland aimed to:

• Establish the mathematical Flagway game in the curriculum of the 15 participating schools.

• Develop the game into a knowledge-based sport with participating schools taking part in an inter-county competition league.

• Generate national interest for the sport by hosting a high profile National Final where community groups and other schools are invited to spectate and find out more about participation.

• Train Flagway facilitators capable of disseminating Flagway at a summer course through the Education Centre network.

Teachers from nine primary schools in and around the KEC region took part in a day-long workshop in the Kildare Education Centre in February 2016, following an information session with Principals. At this workshop the teachers were introduced to the Flagway game and to the methodology of the

Algebra Project. The teachers then introduced these methods into their classrooms over the next few months, under the guidance of the project team. See Table 2 for an outline of the work with teachers and the Kildare Education centre team.

In May 2016, members of the US Algebra Project team returned to Ireland, and ran a series of intensive workshops over three days at Maynooth University. The Primary level workshops were facilitated by Maisha Moses, Marquis Lowe, Kessens Green from AP and Professor Nell Cobb and Charlenne De Leon from the YPP along with Professor Greg Budzban on the Road Colouring Problem. The mornings were spent on working through the variants of the Flagway game and discussing how they could be used in the classroom, while the afternoons were spent working with children from the Maynooth Boys' National School using the Flagway game. Attendance at this workshop is detailed below in Table 1.

Table 1: Attendance at the three-day workshop in Maynooth University in May 2016

Sector	*Number in attendance*	*Number of institutions*
Primary	13	6
Post Primary	3	2
Third-Level	4 lecturers and 12 PSTs	2 [MU and DCU]
Support Services	3	Project Maths
Funding Body	1	SFI

Table 2: Algebra Project in Ireland – timetable of events with Kildare Education Centre

Date	*Participation*	*Details*
04/02/2016	Principals of 7 primary schools	Information session on the Algebra Project with an invitation to participate in Algebra Project Ireland.
16/02/2016	12 Teachers from 8	Introduction to the Algebra Project and workshop on the Flagway

	primary schools	Game.
15/04/2016	Teachers from 8 primary schools	Feedback on progress to date and workshop on how to develop materials to support schools
23-25/05/2016	Teachers from 6 primary schools and from post-primary schools	3-day workshop in Maynooth University for pre-service, primary and post-primary schools.
26/09/2016	Teachers from 7 primary schools	Feedback on progress to date and workshop on how to develop Algebra Project in new academic year and planning for Maths Week.
Maths Week Oct 2016	Schools based Flagway events	The lead teacher in each school hosted a mini event involving 1. Demonstration of children playing Flagway 2. Exploring the pre- Flagway games with students from other classes Mini Jamboree hosted by Muireann and AnneMarie in Scoil Mhuire Gan Smál Carlow. Three schools travelled to take part Clonagadoo 4th Class, Halverstown 6th Class, Kildangan Principal and Teacher Note: Following their attendance at the Jamboree, Klidangan NS have since joined the network.
13/2/2017	14 Teachers from 9 primary schools	Feedback on progress to date and workshop on how to develop materials to support schools and pre-planning for Flagway Event in May.
Various		Meetings to prepare for Flagway Jamboree.
25 & 26 April 2017	16 teachers from KWETB schools attend 2 day workshop	Workshop based on Road Colouring Problem facilitated by Dr Gregory Budzban in Athy College.
28 April 2017	Mini-Flagway Jamboree	Flagway event hosted in St Brigid's School, Kildare Town with pupils from St Brigid's school participating with pupils from Kildangan, Halverstown and Robertstown schools.
8/05/2017	School visit	Maisha Moses and colleagues from US visit Halverstown NS
09/05/2017	School visit	Maisha Moses and colleagues from US visit Scoil Mhuire gan Smál, Carlow (see appendix F for attached article from local paper, The Nationalist)
10/05/2017	Flagway Jamboree	9 schools (and over 100 pupils) participate in inaugural Flagway Jamboree event in Kildare Town Community School. Day long activities centring on the Flagway Game were held in front of

		invited guests. US team assist in running of the event with Irish teachers.
11/05/2017	Future planning	Working with US team to review progress to date and to look at possibilities for future implementation of the Algebra Project in Ireland.
03-07/06/2017	Summer Course	Algebra Project Summer Course, approved by DES, for primary teachers organised by Kildare Education Centre. This course is also pre-approved for Summer 2018 and can be run in any of the 21 Education Centres nationwide, with an approved tutor.
Currently	Collaboration and peer support	Schools are collaborating with each other and planning peer support trips. Teachers from schools where Flagway is well established in the curriculum are travelling to those at an earlier stage of implementation to team teach Flagway lessons
Currently	Pre-planning	Working with KWETB to plan for autumn implementation of Road Colouring module in Transition Year.

2.3.2. Second Level

At Post-Primary level the Algebra project in Ireland aims to:

- *Establish the Road Colouring Problem in the TY curriculum of KWETB schools*
- *Encourage and support the participating teachers to disseminate the Road Colouring Problem to colleagues in their schools*
- *Establish a network to disseminate to other schools in Kildare*

KWETB, impressed by the feedback from the initial workshops involving their teachers and students, agreed that there were synergies between the Instructional Leadership initiative currently running in ETBI schools and the Algebra Project.

The timing of this collaboration did not fit with the report schedule and as mentioned earlier is not the focus of this report, however, in April 2017 teachers from 12 KWETB schools attended a successful 2-day workshop with Prof Budzban in Athy College. KWETB, in conjunction with the API

team, will support the implementation of the Road Colouring Problem as a TY module in these schools in Autumn 2017.

2.3.3. Third Level

At Third Level the Algebra Project in Ireland aims to:

• *Use Mathematical Literacy as a tool to develop young leaders' capacity to radically change the quality of education and life in their communities, so that all children have the opportunity to reach their full human potential by training undergraduate mathematicians and mathematics educationalists as MLWs who, as part of their undergraduate course will work with students in local schools and communities helping them learn mathematics in a unique way, with a focus on experiential learning and on a multiple-intelligences perspective.*

Twelve students from the DCU Mathematics Education degree programme took part in the workshops in Maynooth in May 2016. These students were broken into three groups of four. One student from each of the three groups attended workshops given by Professor Greg Budzban on the Road Colouring Problem. The other nine students worked with Professor Nell Cobb on workshops concerning the YPP. These students worked on ways of introducing Algebra Project activities into communities, especially using after-school clubs for primary students.

During the first semester of the 2016/17 academic year, these 12 students undertook a project which aimed to develop activities using the AP and YPP ideas and materials for use in Irish classrooms. The students also used these activities in lessons and using a Lesson Study approach they had opportunities to observe, reflect on, and improve these lessons over the course of the semester. They used modules from the YPP (on averages in statistics) with primary school children in a local school, and the Road Colouring module with 5th year secondary school pupils. They also had opportunities to collaborate and hold discussions with students from DePaul University in the USA who were also training as MLWs.

3. Project Evaluation

The evaluation team was contracted to evaluate the activities of the Algebra Project in Ireland in the calendar year 2016. This is what we will report on here. We would like to point out that further activities have taken place in 2017, as can be seen in Table 2, however evaluation of these activities will not be included in this report.

3.1. Data Collection

The project was evaluated using qualitative and quantitative data. There were two parts to the evaluation. Part one involved analysing questionnaires that were distributed to all participants before they participated in the professional development events organised by Kildare Education Centre (KEC) and the National Council for Curriculum and Assessment (NCCA). In addition, three case studies were developed: one each in the Educate Together School in Maynooth and Scoil Mhuire Gan Smál in Carlow. These focused on the implementation of the Flagway game with two classes in the primary school setting, with the remaining case study focusing on the training of MLWs in DCU. In addition, a focus group was held with the team from Kildare Education Centre. Ethical approval was received for this evaluation study from Maynooth University.

Prior to the initial workshops, all participants were asked to fill out a questionnaire designed by Swan and Swain (2010); they were then asked to complete the same questionnaire at the end of the project in early December 2016. Since the response rate of the post-intervention questionnaire was so low, we will not report on it here. The purpose of the questionnaire was to measure the impact of the professional development workshops and the experience of using the methodologies of the Algebra Project on their teaching practices as well as their beliefs about mathematics, about teaching, and about learning. Swan and Swain's questionnaire included 28 questions which asked participants to rate the frequency (from almost never to almost always) of certain teaching strategies in their lessons. These strategies were classified as either predominantly teacher-centred or predominantly learner-centred. Using the data collected through these questions, we were able to test if the participants reported more frequent use of teacher or learner-centred practices.

Five participants were interviewed during the initial workshops; all five were primary teachers who took part in the Flagway module. There were follow-up interviews in December 2016 with four teachers in the case study schools (i.e. after the participants had used the Algebra Project methodologies in their own teaching). In addition, a focus group interview with six MLWs from DCU was held in December 2016. The interviews were analysed to identify any changes to the teaching practices and beliefs of participants as a result of this project.

Feedback on the impact of the project from pupils in the case study schools was gathered using focus groups, where the pupils spoke to one of the research team about the project. The researchers also visited schools to observe the methodologies in action [the Flagway game being played].

3.2. Qualitative Data Analysis

Analysis of the qualitative interview data (both student focus groups and teacher interviews) followed a general inductive approach as advocated by Thomas (2006). This approach allows "research findings to emerge from the frequent, dominant, or significant themes inherent in raw data, without the restraints imposed by structured methodologies" (p.238). Qualitative data was cleaned and put into a common format where analysis began with a close reading of text. Coding then ensued to develop a set of categories that summarised the raw data and illustrated the key segments of the interviews. The key features of the categories were as follows:

1. *Category label:* a word or short phrase used to refer to the category.
2. *Category description:* a description of the meaning of the category, including key characteristics, scope, and limitations.
3. *Text or data associated with the category:* examples of text coded into the category that illustrate meanings, associations, and perspectives associated with the category.
4. *Links:* Each category may have links or relationships with other categories. Links are likely to be based on commonalities in meanings between categories or assumed causal relationships.
5. *The type of model in which the category is embedded:* The category system may be subsequently incorporated into the research theory, and/or framework.

(Thomas, 2006)

In the next two sections, we will report on the results of our data analysis. We will consider the Third Level study first and then consider the impact of the work done at Primary Level.

4. Impact of the Algebra Project: Results from the Third Level Study

4.1. Impact of Module/Training on MLWs in DCU

The twelve DCU students responded to the questionnaire at the beginning of the workshops in Maynooth in May 2016. Their answers to the Likert scale questions indicated that their classrooms tended to be more teacher-centred than learner-centred (Paired t-test, $t=-5.581$, $df=11$, $n=12$, $p<0.001$).

A focus group (with six of the twelve students) was held in DCU in December 2016 at the end of the MLW module. We analysed the transcript of this conversation and will now describe the themes that

emerged. The pre-service teachers (PSTs) all reported having a very positive experience with the aspects of the AP and YPP that they encountered. They spoke about new ways of teaching and learning to which they were introduced through the AP, along with the impact on learners and teachers, implications for teacher preparation and professional development and how the AP might be implemented in Irish schools.

4.2 Themes from the MLW Focus Group

4.2.1 New Ways of Teaching and Learning

The importance of the physical aspect of the AP task (either physically moving cubes in averages, or physically building the cities in the Road Colouring Problem) was often referred to by the PSTs. They realised that because they themselves had physically engaged with the Road Colouring Problem in May in Maynooth, they were still able to recall the details of the problem and the understanding that came with it several months later. As one PST said:

> *We all remembered it even though it was so long ago because we had done it, physically done it, so it was good yeah, and we were all able to make the connections between functions.*

They also appreciated that pupils in their classes had the same kind of learning experience and as a result were more likely to retain their understanding. They also expressed the view that different learning styles (such as visual or kinaesthetic) could be catered for by using the five-step model. It seems that the students in the PSTs' classes also enjoyed the physical aspects of the AP lessons and liked the activity that is usually absent from their mathematics classes. The PSTs felt that the AP methodologies could and should be used in classes and especially as a way to introduce new concepts and develop deeper understanding:

> *There should be an introduction to the topic, just to develop their understanding before you introduce the formulas because when I went through school you did not know why you were doing the squares, if you actually sit down and understand a thing it just makes it easier to answer questions in the Leaving Certificate.*

They also expressed views which showed that they appreciated that the physical experiences built into the AP lessons allowed students to construct their own understanding and move at their own pace. They saw this as something which was of benefit to students at all ability levels:

> *YPP is good because it made a less able student understand where averages and means were coming from so instead of being told this is averaging and means, the less able students actually physically experienced the stacking and got to see how they count up the average in that so it was more beneficial with the less able student because they*

were more getting physical experience behind the Maths instead of being told "This is how you do it"

Another important aspect of the AP classes that the PSTs spoke about was the difference in assessment methods compared to what they were used to. In particular, they spoke about having more peer and self-assessment and more formative assessment. For example, one PST said:

We kind of had to change the way we were teaching fundamentally like, we are all used to Leaving Cert, there is a right answer, there is a wrong answer… the YPP is kind of different, there is not a lot of right answer, wrong answer in the YPP so we had to kind of change the way we were going to assess our students and we had to kind of give them more comments instead of grades.

They felt that these assessment methods were very effective, especially the peer-assessment aspects of the methodologies which involved students giving each other grades and comments on how to improve.

4.2.2 Impact on Learners

As we have seen from the section above, the PSTs reported on how the AP methodologies impacted the students in their class. We have already seen how they thought that students could develop deeper understandings of concepts which stayed with them over time by allowing them to create meaning for themselves instead of being told 'this is what you do'. They also spoke about the AP as being challenging and said that it was important for both teacher and student to think about what they are doing. In addition, they spoke about the impact of the AP on students' confidence, reporting that it helped increase the confidence of some of the less able students, and also on students' engagement. The PSTs seemed to appreciate that the AP methodologies could reach students who had turned off or who felt left behind. For example, one focus group participant said:

One school we went into we were warned about the TY students but they ended up being the students who asked you lots of questions because they were engaged. They are people the normal school system does not suit.

The PSTs also felt that the pupils in their classes learned about working in groups through the AP activities by collaborating with their peers.

4.2.3 Impact on Teachers

It seems that the PSTs' own confidence as teachers was enhanced by taking part in the project, and that both their subject content knowledge and pedagogical content knowledge were strengthened. They also saw benefits in having more than one teacher in the room

You could do it with two teachers in a room because the one to one time that you spend with a group is invaluable

and in discussing the teaching episode with colleagues later

> *Definitely, the fact that you get sit down and de-brief and say what worked and what did not work, that would definitely work in school, you are sitting in a canteen after each lesson talking about what went on, what worked in that class. Liam turns around and tells me "This needs to be adjusted" and then I try that and go into my class and see if that works and go back to him and you re-plan the whole structure.*

Recall that these PSTs were also engaged in a process of Lesson Study and so were able to reflect on and refine their teaching plans as the module developed.

4.2.4 Preparation of Teachers

The PSTs felt strongly that the kinds of methodologies employed in the AP would be of major benefit to mathematics education in Ireland. They felt that it would be good to build a core group of teachers who were knowledgeable about these methodologies and who could help other teachers to become involved. They saw university teacher education courses as a way of developing such a core group and that this group would grow over time. In tandem, they expressed the view that not enough attention was being given to courses that linked mathematics and pedagogy; furthermore, they questioned relevance of 'Pure Maths' courses to secondary teaching.

4.2.5 The Algebra Project in Irish Schools

The PSTs felt that the AP could link to the proposal for more project work in Junior Cycle, perhaps with an AP module making up a six-week course. They felt that this would work well with the new curriculum and would involve students collaborating, assessing each other and developing their key skills. However, they caution that the AP methodologies require quite a lot of time and necessitate longer classes so students can discover and build meaning. In their trial run of the Road Colouring Problem, they found that they did not have time to let students' understanding develop naturally and had to intervene sometimes:

> *If we started on the three building city and then went into the four building city that might run for the whole class, and say the class was ended after the four building city we might end and they might not have experience to link the functions with it so we were changing it up and sort of tried to include bits that were normally at the end of the Algebra Project and sort of put them in between the two tasks, so I said teach earlier, that is not the right word, I was more facilitating the link to function straight away after they got the three building city.*

They realised that it would be better to have more time and appreciated the value of students working things through for themselves. Clearly, the AP methodologies rely on students having the

opportunity to make meaning and connections for themselves, and so it is important for teachers to be able to set aside enough time for this to happen. The PSTs in this focus group thought that one way around the problem with time would be to use the AP methods in a TY module:

> *It would actually be good to implement it in fourth year [TY] in ways that are not structured, so students get a deeper understanding of what they have covered so when they go into fifth year and meet this topic again they would be like "I have a deep understanding", no bother to them like.*

They conjectured that such a module might influence people to continue with Higher Level Mathematics at Leaving Certificate level. Another suggestion was to use the AP ideas in after-school clubs (as is done in YPP).

4.3 Conclusion

We can see from the analysis of the focus group transcripts that the PSTs saw benefits for both teachers and learners in using the AP methodologies. In particular, they saw the benefit of students making meaning and connections for themselves through the five-step process. There is some evidence that the project may have forced the PSTs to change their own teaching style a little since their conversation in the focus group gives a picture of student-centred classrooms while their responses to the questionnaire in May suggested that their preference was for more teacher-centred classes.

5 Primary Level Teacher and Pupil Feedback on the Algebra Project - The Flagway Module

We report here on the themes that emerged from the analysis of the interview and focus group data collected from teachers and pupils who have been involved in the Flagway module. The overall impression from both groups was very positive. For example, one pupil said:

> *It was not like forced Maths; it was just being able to do Maths but yet feeling kind of free*. (T2_CS1_SFG)

The themes that emerged concerned the model of professional development used, the implementation of AP methods in schools (benefits and challenges), skills development, opportunities for across school collaboration and fitting Flagway into the Irish curriculum.

5.1. The Model of Professional Development and Up-scaling within Schools and between Schools

It was evident from the interviews with teachers that they appreciated the model of training used by the AP team. They emphasised the importance of the initial training which allowed the them to experience the Flagway module as a pupil would, and in particular they saw the benefits of having to figure out the 'rules' themselves. The upscaling of this will take a big-time commitment because, as this teacher explains, it is all about letting teachers have the time together to figure it out for themselves so they can then set up the task in such a way as to let their students muddle through the process and develop conceptual understanding:

> *You need to go through the same journey as the children so it is no use you being handed a set of lesson plans and saying this is what you do. You actually have to do the same thing as the children. That is why it is a struggle, it is the struggle that makes you love it more.* (T2_CS2_T2)

CS2_T1 had six days of working with the AP team and he indicated that it was day five of the process before he felt he really was competent to go on to teach it.

> *What happened was the first year we went we were three days there, I came away the third day and I still did not know how to explain it. I still did not know what the solution was and even the second time we went back for three days, and it was the second day, so kind of day five of the thing and I thought "Now, I have it, I understand it" but then I had to go away and think "How am I going to teach this to the 30 kids I have in my room".* (T2_CS2_T1)

After the three-day session one teaching Principal commented that her lightbulb moment was when she realised that it had taken the teacher group until the end of day two to realise that they needed to strategize in order to win the game, while the student group had it *'copped in 20 seconds'*. She went on to say

> *Then I just realised that children do it all the time and we don't give them credit for it or we tend to stop them, you can't do that, you can't do that, maybe just let them do it.* (T1_Interview_T3)

This participant also commented that there was more time for reflection at the AP Continuous Professional Development (CPD) days compared to her experience at other in-services where the emphasis was on covering material.

The model of the CPD used by the AP team was a success for the teachers involved, especially in the afternoon when the team played the game with local school children. The structure of the training sessions meant that the teachers were working together on the module in the morning and in the afternoon they were able to work with children from a nearby school. The immediacy of being able

to try out the teaching techniques learned straight away was powerful and mitigated against teachers' beliefs that the methods might not work in practice or that the material was too difficult for pupils.

The processes of taking time to work with school children on the CPD days and of seeing the game in action also added to teachers' confidence in implementing the game in their schools. In addition, the experience gained by teachers in playing the games for themselves enhanced their own understanding and ability to think about how to implement them in their own teaching. What was very evident from these data was how the scaffolding of the process through the team at KEC and the AP team coming to Maynooth University for two three-day training sessions was necessary to help teachers take on leadership roles in their schools and bring along other teachers in the process. CS1_T1 spoke of the cascade effect of having some participants who were on day five of training when she was on her day two.

> *I loved the morning sessions where we were brainstorming and doing the activities ourselves and you know sharing, the people who have been here last year, I think that's been good so for instance I'm new this year but the people who were here last year I think it progressed the rate of learning for everybody else, so I suppose it's a cascading effect isn't it? You know you can see that happening in schools then that I'll go back hopefully and pass it on to theory teachers in my school and it will filter out hopefully that way.* (T1_Interview_CS1_T1)

The kind of collaboration mentioned above was evident in the two case study schools. The fact that this ongoing collaboration was supported by the principal was very evident when we visited the CS2 school to observe the AP being implemented in both classrooms. In interview CS2_T2 spoke of the support her colleague gave her when she first tried to implement the AP with her class. For example:

> *It's so good for us because like me today I could not remember why 30 was a red number, you know, and [CS2_T1] told me, and like that we just bounce ideas off one another.* (T2_CS2_T2)

The teachers in this school worked together to adapt the game for different classes:

> *We are not the experts here, but we brought some other classes out just so we could show them and the other teachers were coming up with ideas and it was actually a Fourth Class teacher who said "Why not make a game of operations?" and we took Second Class then and we played basically an orienteering game so I had a clipboard and the pathway might be Red, Yellow, Blue, they would walk the pathway Red, Yellow, Blue, they had a blank analogue clock and inside the Hoop was a digital clock and they had to show the time, so straightaway there you are*

> *bringing it down to another level and they are excited because they are getting to play and they are getting to move around.* (T2_CS2_T2)

This need to *'bounce ideas off one another'* was also evident in the interviews in CS1. CS1_T1 talked about the need for time and for support of another teacher in the school. As Learning Support teacher, she had the time and worked with her colleagues to set up the game. From these two champions in the school the AP learning was spreading to other teachers and classes. In both case study schools the ability to adapt the game and to get all classes involved was alluded to. There is evidence here that if schools could have two champion teachers who were afforded the time for training and for planning, the AP could have a very significant effect on mathematics learning in the schools. From the teacher interviews at T1 and T2 it was evident that they felt empowered by the fact that they could adapt the game to suit their needs. By being able to discuss their own learning through the active process in the workshops they developed more ownership over the techniques and were positively inclined to use them and to share their learning with colleagues. These changes appear to persist over a long period of time; case study school two was in its third year of implementation.

The teachers spoke about how the AP methodologies could be introduced to other schools. They felt that teacher-to-teacher training would be most effective and that the group of teachers who have experience with the AP methodologies would be in a good position to contribute to this effort:

> *I think if you are going to roll it out, you need in-service training and you need like in an ideal world you need a cohort of teachers to sit down in the Education Centre with teachers and say "Right, here is what I did for the last two years, this is what worked, this is what did not work; you can adapt this for Fourth Class, change it in this way".* (T2_CS2_T1)

Interviews at T1 with teachers who were on their second round of AP professional development highlighted many issues for scaling up this project. Teachers spoke of the need for developing understanding through working with other teachers and then using what they learned with the students in the afternoon of the sessions. They also talked about how the second CPD event built on the first one by adding more games to their repertoire.

> *…because last time we learned the game, the Flagway game and that was pretty much it, this time we learned add on games. We're after getting quite a few games … you come to these courses you want something that you can actually bring back into the classroom and teach the kids and that's what we got. I've so much to go back to school with now and even to show the other teachers.* (T1_Interview_T2)

We have seen that the teachers were positive about the AP workshops and liked the fact that they had to work on the mathematics themselves to build their understanding and confidence and then

were quickly given an opportunity to put their learning into action in a real classroom situation. We have also seen that cooperation between teachers is vital to the success of the initiative and that the teachers themselves would like to see this element expanded and exploited in any upscaling of the project.

5.2. Implementing the Algebra Project in Schools: Doing Maths

The teachers spoke about how they implemented the AP methodologies in their own schools. The implementation of the AP in classrooms needs an enthusiastic teacher to drive the process. As CS2_T1 put it,

> *It needs an awful lot of drive and it needs an awful lot of commitment to it and it needs an awful lot of organisation but I think it definitely can work.* (T2_CS2_T1)

It seems that they all took time to prepare their classes for the mathematics involved. Teachers talked about how they worked ahead of playing the game and the need to work on number sense with students:

> *I did an awful lot of work on the numbers even before I introduced the colours into the game. We did a lot of groundwork about "Write down all you know about numbers 2 to 30, each number, what can you say about number 3, it is an odd number, it is a Prime number, it is the number that comes after 2, just getting them familiar with every aspect of the number and then after a couple of weeks of that "What can you notice about these numbers when you break them down?" A lot of concrete activities, can you break 6 down into 2 or more groups with more than one cube in it, and even that took a long time for the kids like, just the number 6; and then we did a lot of work, like we were probably six weeks doing that kind of stuff before I gave them the Mathematician problem.* (T2_CS2_T1)

They all took trouble to introduce the Flagway game in the way that it was introduced to them in order not to diminish the learning involved. Stein at al. (2000) advises that teachers can limit the level of cognitive demand of tasks in their attempt to make expectations clear to learners. This can result in the learner following a prescribed set of steps rather than engaging in meaning making. However, building on the CPD described earlier, teachers needed to set up the task in such a way so that the learners got time and space to engage with the problem and construct meaning and understanding for themselves. This teacher describes how they introduced the game to their classes:

> *What I explained to them was the mathematician Mobius has come up with a way to categorise from 2 to infinity into three groups and those*

> *three groups are red, yellow and blue. What the children came up with first of all was they found a way to make the numbers fit which was a pattern which was work that they had come up with which they could explain which was brilliant; we initially did it with the numbers 2 to 10 so then I would test them by saying "OK, where would 11 go, where would 12 fit in your pattern?" and see if they could explain to me and see if it could work and then from that step then I told them that they needed to find what Mobius function was; so then I would give them another number and see if they could work out the pattern, see if they could work out the system between them ….so they wanted to do it because it was a challenge and because I would not give in and I would not tell them.* (T2_CS2_T2)

The pupils from this school responded to the challenge. They liked the way that the problem was introduced to them and the freedom that this gave them to explore:

> *The way our teacher explained it to us was really good, the way he is like a mathematician, he questions us about it, he would not just tell us.* (T2_CS2_SFG)

This exploration gave opportunities for the pupils to engage in sophisticated mathematical thinking:

> *Because it is not about them working on Mobius Theorem, it is actually better them having their own idea of the maths, so even though some of them did not get to it they still learned an awful lot and it was a different way of thinking about Maths where they had to try and come up with their own theorems, it was not just me regurgitating information and them learning it.* (T2_CS2_T2)

The teachers also spoke about how the AP encouraged independent thinking and problem-solving in their pupils and how it can change pupil's perception of mathematics as being about getting the right answer and nothing else.

The time needed for the learners to work through the process of problem solving and the need for the teacher not to offer clarifying help and to have the confidence to allow the process work was evident in the commentary on implementing the game in class in both case study schools. The Flagway game and the card games created affordances for the activity of the class by structuring the kinds of mathematical knowledge that learners got to use and build. The way the game was played was determined by the individual teachers in interaction with the learners in their classrooms. In the next quote from CS1_T2, we see the comments of the teachers from CS2 echoed:

> *We had card games, we had three people in a group and they all get a little pack of cards and they each have to pull out one and they are written in their prime number names, let's say 2, 3, 7 for 42 and then whatever 2-2 might be next for 4 and 2-3 might be next for 6 and they have to work out which is the highest number, whoever has the highest number gets to*

> *take all the cards; I suppose they are thinking all the time while they are doing that and even if there is a child who is not as able in that group, they are able to be helped on by the other people so I suppose it is peer tutoring in a way and they are definitely all working together.* (T2_CS1_T2)

Just as in CS2, the pupils in CS1 enjoyed the experience of working with the Flagway game and were able to see the benefits they were reaping:

> *Yes, it was a really good experience because it was all to do with prime numbers. It will help us definitely in the future for secondary school.* (T2_CS1_SFG)

> *We did not really think of it as Maths, we kind of thought of it as a game but in our heads, we were kind of doing the strategies and we were learning but we did not really realise it but we did.* (T2_CS1_SFG)

This ability to do mental arithmetic was also highlighted by teachers and pupils. This teacher talked about how she developed songs to help students memorise tables but that with the AP games the children were learning tables in a fun way.

> *I think in general children find tables very difficult, like I've fifth class now and a lot of them are still very weak at their tables whereas with this game, it's nearly that they don't even get that they're learning their tables and this game for me I think it's all about tables,… if you want to play the game you need to think quick and you need to know what three fours are, two threes, three times four you know so I think it's a fun way to learn tables whereas at the moment kids are just rote learning them* (T1_Interview_T2)

Another benefit of the AP methodologies was evident in the fact that both teachers and students found the AP to be inclusive and that the task allowed for all to get involved in the learning. The Flagway game seems to offer opportunities to engage pupils who might otherwise encounter difficulties:

> *I have a child in my class who would be very very weak and by the end she is smiling and she is running and she is looking involved and it is lovely to see everyone getting involved* (T2_CS1_T2).

At T1 when teachers were asked about a typical maths lesson, they referred to the workbooks and how the learning was very teacher-centred. A typical response was:

> *It starts out teacher led … and then they would be working a lot with their maths workbooks so it is, they do work individually* (T1_Interview_T2)

The descriptions of how the AP methodologies were implemented in the case study schools paint a very different picture; the classes seem to be student-centred and involve inquiry-based learning.

The teachers spoke about two main challenges with the implementation of the AP in schools. One was the physical space needed for the setting up of the game, and the second was the time needed to play the game. CS2 had tried to adapt the game and to do it inside on the whiteboard. CS2_T1 lamented the lack of a hall in his school, however, he had improvised and the day the researcher visited the students were very engaged in 'doing mathematics'.

> *Just trying to do it as much as you can in the small space, clearing the tables and make them walk the game rather than run it, which kind of defeats the active side of the game, and as well as setting it up outside takes time*. (T2_CS2_T1)

CS1 had a hall which meant that they got to play the AP Flagway game more often. They also got to host other schools coming to play the game, however the teachers explained that the Irish weather hampers the availability of space for the game.

The pupils expressed very similar opinions; it is difficult to play the game outside unless the weather is good and you need a big hall to play it properly inside.

> *I prefer to do it outside rather than in the hall because sometimes inside it was really squished because it is smaller. I like doing it outside because we have lots more room to run around.* (T2_CS1_SFG)

> *It was good that we actually had a hall because the two schools that came they did not actually have a hall and certainly one did not have a hall and it is a weather permitting game when you do not have a hall because the fact that if it is raining outside you cannot do it and sometimes if it is windy the cards blow away… The hall would be better if it was just a bit bigger.* (T2_CS1_SFG)

The time needed to give students the opportunity to problem solve and 'do maths' was a common theme in both teacher and student interviews. CS1 talked about how they had got the setting up time for the game down to a minimum but they also lamented that to really use the game to its potential takes time.

> *I think there is an awful lot of Maths in it, there is an awful lot of benefit to it but you have to invest in it, we are doing a lot of work, investing time in it, we are able to do it here because we have it in our policy that we do Maths games once a week so I do it on a Friday.* (T2_CS2_T1)

It seems that the teachers have not yet got to introduce the link to Algebra using the game. In fact, only one teacher interviewed mentioned this link, even though it is the point and underlying aim of the Flagway game:

> *There is room for challenging further in relation to Algebra and each of the numbers has a different algebraic form, like the number 4 would be 2-2 as a prime number name so it would be A squared or whatever and*

> *[CS1_T1] and I played a game, I don't know if you saw that game in Maynooth, they had each of the algebraic numbers in form around the walls like AB you put the number 6 in there because it was 2-3 and we had to find out where the number belonged and we really enjoyed that and they would love that as well, so we have to get on to that.* (T2_CS1_T2)

Interestingly CS1_T1 talked at T1 about the perceived challenges she anticipated for implementing the AP in her school. At T1 she talked about physical challenges such as the size of the hall, the time table and so on. At T2 she talked about how to integrate the AP more with the curriculum. We would contend that this represents a move in this school to embedding the AP as a part of the curriculum. It is moving from AP being seen as a game to being seen as doing maths. This need to time to engage with the key ideas was also emphasised by the AP Team at KEC:

> *What we found just us in working with this was that we needed to invest heavily with the primary teachers. We could see that they needed more than what we envisaged in the early days and we spent that first year, after Bob had come, we spent that year keeping them on board, going out to the schools* (AP Team in KEC_Focus_Group)

Summary

The participants (both teachers and pupils) referred to the benefits of this type of learning for the development of mathematical understanding and self-efficacy, and also for the encouragement of group-work and peer tutoring. They felt that the game was inclusive, with opportunities for engagement of pupils of all abilities. They also spoke about the challenge of finding a good space to run the Flagway game and the teachers worried about the amount of time that needed to be spent on the activity. We noted that all but one teacher seemed unaware of the aim of introducing algebraic notation (as a precursor to studying Algebra) through the Flagway game.

5.3. The Development of Skills

Deborah Ball (in an interview with Bob Moses) spoke about the need to make mathematics interesting for learners:

> *Kids decide they don't like math because they have had a diet of math that's like eating cardboard. It's not delicious, so they don't like it. There are tons of problems that are fascinating to little kids, so you need to give them a diet of those things. They need to see that math is much broader than what school is causing them to think math is. (Richardson, 2009)*

The AP methodologies seem to do just that and this was evident from the way that pupils spoke about their experiences. The development of skills such as working with others, peer teaching, empathy and communication was very obvious from all the student focus groups when they talked

about playing the game. They talked about how they taught pupils from the visiting schools, delegated work within teams and encouraged all to get involved. In some of the responses the learning of the mathematics was almost seen as insignificant in comparison to the development of skills.

> *I liked it also because it was team work and if you made a mistake nobody was going to give out to you or anything because you just like, it was all to do with your team, one girl might be taking up sets of cards that have not been taken and you might have one girl trying to run around. I liked it because it was pretty fair.* (T2_CS1_SFG)

In CS1, the teachers delegated the checking of each other's entries to the pupils, and it seems that the pupils liked the extra responsibility. The work with other teams from schools also enhanced the skills development of this group, especially that of peer teaching.

> *Sometimes the other schools were not that confident in going up so you kind of had to encourage them to run up, like one girl was in my group and she did not want to run but like if you just helped her, I helped her … and she kept going because she liked it, she did not really know what it was at first but then she enjoyed it after she had gone once.* (T2_CS1_SFG)

> *Some of the time they did not know how to check so it was fun to help them and help them learn how to do it as well and it was also fun because you had to wait and say if they got it wrong you have to put up your hand so it was like being a teacher for a while and we got to take it in turns so we were not just doing it, the other class were doing it as well.* (T2_CS1_SFG)

Peer tutoring came up in all teacher interviews; the AP was seen as a vehicle for peer tutoring and teachers spoke about how this enhanced the development of skills such as working with others and communication.

> *I think more effective use of peer tutoring in that if you got the strong kid in with a couple of weak kids some of the games are set up so that it has to be a team answer so the whole team has to stand up and shout the answer, so they've got, it's in their interest to make sure that the weak guy knows what's happening.* (T1_CS2_Interview_T1)

The teachers described how students were engaged with the game and how they became more independent in their learning through the game. They did not look to the teacher for the answer and they self-organised.

> *It is funny because they took on those roles and jobs themselves, we did not say right, you are this person and you are that person. It kind of naturally happened and they use different strategies, naturally not with any help from us or even any encouragement from us to come up with strategies, so you might have someone who takes on the role of figuring*

> *out what numbers are left or figuring out who should be doing the running or who should be checking out the numbers.* (T2_CS2_T2)

The way the teachers set up the learning was central to the skills development:

> *A big part of that today for me was challenge, don't just accept that this guy who you think is a genius at Maths is saying it is a yellow number. Test it, call it out, this mathematician might like to cheat so pull him up on it, challenge him, have a bit of confidence; those are all Maths plus other skills in the game, aside from getting good at multiplication and division you know.* (T2_CS2_T1)

> *Trying to get them to come up with all the possible combinations so then that they had ownership and then trying to get them to sketch out their own variations. There is a lot of problem solving in that, that then brings in your oral language skills, it brings in your presentation skills, it brings in your team work, it brings in your group and collaborative learning. We then even decided, because we are doing procedure writing, let's get the kids to write instructions for their games and present stuff. Then they made PowerPoints about them.* (T2_CS2_T2)

The ability to build self-efficacy in maths while also doing physical activity was significant for some of the learners in both case study schools. For some, combining the two activities made learning mathematics more enjoyable:

> *I thought it was good because it gave us an opportunity to get active and to enjoy the Maths, because some people don't enjoy Maths but in a way, we were kind of doing PE with it and we were still learning.* (T2_CS1_SFG)

While for others it was the other way around:

> *I am not that fast as everyone else, I am horrible at running so that is why Flagway, I like the Flagway, it gave me a reason to be good at PE. You need to be fast to win.* (T2_CS2_SFG)

Summary

We have seen evidence that the Flagway game can contribute to the development of skills such as working in groups, collaborative learning, language skills, and presentation skills. In addition, the combination of mathematics and PE seems to be popular with teachers and pupils.

5.4. Curriculum Issues

The Irish primary curriculum for mathematics comprises five strands: Number, Algebra, Shape and Space, Measures and Data. The aims for mathematics education at primary level listed in the Irish curriculum documents are as follows:

1. To develop a positive attitude towards mathematics and an appreciation of both its practical and its aesthetic aspects
2. To develop problem-solving abilities and a facility for the application of mathematics to everyday life
3. To enable the child to use mathematical language effectively and accurately
4. To enable the child to acquire an understanding of mathematical concepts and processes to his/her appropriate level of development and ability
5. To enable the child to acquire proficiency in fundamental mathematical skills and in recalling basic number facts.

The skills as outlined in the primary school curriculum are as follows:

1. Apply mathematical concepts and processes, and plan and implement solutions to problems, in a variety of contexts
2. Communicate and express mathematical ideas, processes and results in oral and written form
3. Make mathematical connections within mathematics itself, throughout other subjects, and in applications of mathematics in practical everyday contexts
4. Reason, investigate and hypothesise with patterns and relationships in mathematics
5. Implement suitable standard and non-standard procedures with a variety of tools and manipulatives
6. Recall and understand mathematical terminology, facts, definitions, and formulae

(Government of Ireland, 1999, p.12)

Given these aims and skills, it would seem that the Flagway game fits easily into the Irish curriculum; for example, it is easy to make a case that the game fulfils all of the aims above (given the evidence

that we have seen in our data and reported on in previous sections above) and develops all of the skills. However, the teachers in this study frequently expressed worries on this front. Two main themes relating to the curriculum arose in the interviews with teachers. One was the need for the AP to be more closely aligned with the current curriculum and the other was the time it takes to allow for pupils to problem solve. One teacher, CS1_T1, talked about the need for the AP to be more closely aligned with the primary curriculum and other teachers felt that the AP classes were an add-on to the written curriculum. Why this is the case would merit further exploration with the project teachers ahead of any upscaling of the project.

> *The curriculum is jam packed, it is really. It fits into some areas of the curriculum but not as much as the time that it takes up … it takes a lot of time to set up and then you have the time taking it away and I suppose you can only use it for certain areas.* (T2_CS1_T2)

It may be that the teachers are worried about assessments such as the Drumcondra Test or secondary school entrance examinations:

> *I suppose the fact that it takes so much time away from the maths curriculum that you are trying to cover for the standardised test at the end of the year which is not ideal either.* (T2_CS1_T2)

The affirmation offered by external inspection was seen as a powerful driver for the inclusion of the AP in the enacted curriculum. This could also be a positive factor when looking to scale up the AP.

> *We had a WSE there a few weeks ago, and I purposely did a Flagway lesson for it and two Department Inspectors were in there for it; they loved it, they were blown away and never asked me at all to justify it to the curriculum because anyone who knows what they are about can see when they see it in operation you know it does not take a genius to figure out this is all Maths, this is all competency and skills, you know being able to work with a team; being able to challenge.* (T2_CS2_T1)

Another interesting factor was how involvement in the AP prompted the teachers to look at how they could use problem solving and games in the more junior classes.

> *Because the concept of the game can work right the way through the school, you can go to Junior Infants and play it with just the colours and you can have classifying animals and birds and things like that; an animal with wings is a green animal, something like that, so that they can get from Junior, Senior, First and Second, they are used to the structure. They are used to run in combinations, they are used to run in patterns and the when you get up to Third or Fourth they start throwing the numbers into it. That was our first time playing the Flagway game, all the other times it was just mapping and swapping cards and things like that. I think it is*

> *very useful, very beneficial and there is a lot to recommend it, but it needs to be implemented efficiently.* (T2_CS2_T1)

The tension of moving from a very tightly controlled curriculum to the discovery learning of the AP caused some teachers to question how the AP might be integrated into their concept of curriculum.

> *Well I mean it is up to the teacher to you know, to set out the curriculum, to, you know to be very clear what the teacher is teaching, give verbal explanations, check the children have understood these, correct what they haven't understood, see what they've failed to grasp and then work on from there do you know, so it's very important to be I suppose prepared, structured.* (T1_Interview_T2)

CS2_T1 described a typical maths lesson when he was interviewed at T1, this was after five days with the AP team. The tension between the curriculum and the value of the AP way of learning is very evident.

> *I'm doing a lot of talking, yeah what I like about Flagway and the Algebra Project is there's a lot of activity based stuff, there's a lot of tangents off it and there's a lot of, a great phrase Dominic uses 'value in the struggle', you know that there's more value in five wrong answers than one right answer you know there's more learning going on there, but with the curriculum the way it is particularly in sixth class where you're trying to get them ready for secondary school, my maths lessons unfortunately have to be "right here's the chapter we got to get this done in a week, I'd love to give you loads of time to explore all the concepts and things like that but we have five days so let's get it done, let's start with dividing two numbers into three numbers … because you would love to get them thinking more about things … So, there's an awful lot of pressure in the curriculum, that doesn't really allow for the struggle.* (T1_Interview_CS2_T1)

An aspect of this pressure to cover the course to 'do long multiplication' is that the AP may not be implemented with fidelity and may be adapted to suit the shorter maths time slots and in doing so loose the real discovery learning for the student. A typical quote would be:

> *The real challenge I think with the Flagway is trying to adapt it now to be most effective within the Irish curriculum; I think that's the biggest challenge.* (T1_Interview_CS2_T1)

Summary

We have seen that on paper the AP, and the Flagway game in particular, seems to fit very well into the Irish primary school curriculum, however the teachers do not always see this potential. This may be because of a worry about covering content areas and about preparation for various tests at the end of primary education. It is also indicative of their conceptions of mathematics as a bundle of

isolated facts that need to be 'covered' by the teacher, and would indicate a need for a reconceptualization of mathematical teaching and learning. Some of the teachers expressed regret at not having more time to allow pupils to experiment with mathematics.

5.5. Opportunities for Across-School Collaboration

The teachers and pupils in CS1 spoke about their experience of collaborating with other schools as part of the AP. This school hosted two other schools and played the Flagway games with them. The lead teacher in each school hosted a mini in-school event involving:

- Demonstration of children playing Flagway
- Exploring the pre- Flagway games with students from other classes

One teacher spoke about how the visits began:

> *We had two other schools come to visit, and the children from those schools were very nervous; when we started those games initially, they were taking a back seat and letting our school take the lead but in the end, they were all running and they were all confident; again, no matter what their ability level there was something for them all to do.* (T2_CS1_T2)

The pupils in CS1 enjoyed their role as hosts

> *I liked it because you got to sit back and relax and watch them and you could see the focus, the concentration on the faces and you could see how competitive they were and it was really good to see how much the other class enjoyed it and I am sure they thought the same about us, because we could all see if one person is that happy, you are that happy yourself because it kind of rubbed off.* (T2_CS1_SFG)

The pupils saw the benefit of learning to mix with new people, and saw it as good practice for when they go to secondary school, again emphasising the development of skills such as collaborative learning and communication:

> *I like mixing with the schools because if they do not have a hall you could see that they were happy to be running around in school … it is like you made new friends for those few hours and it was kind of like if you are going into secondary school, it is nice to mix with other people so know how to make friends.* (CS1_SFG)

Summary

There are opportunities for collaboration (and competition) between schools in the implementation of the Flagway game. In our interviews pupils and teachers from CS1 expressed positive feelings

about such collaborations and it seems that further work in this area would be beneficial. In a previous section, we saw that teachers would like to see across school collaboration in the training of teachers and the upscaling of the AP.

There was a second event held in May 2017. This involved over 100 pupils from 9 primary schools working on activities related to the Flagway game. The event was organised and run by the participating teachers with help from the US Algebra Project team. During the interview with the AP Team from KEC part of the event was described as follows:

> *One of the things they did was that the, the students got cards with numbers on them and then we had around the wall with sheets with… A squared, A squared B. They had to decide which of the categories the cards went into. But even on the day when they were doing that, the role of the teachers was to go around and check afterwards to see were they in the correct location. But even the conversation began to, to evolve, how can we get the children to correct, to check that they were in the correct location rather than…. the teacher being the person with the knowledge going around checking.*

Figure 2.1: Students participating in the Jamboree event in May 2017

Thus, it seems that by this stage the teachers involved were in a position to use the Flagway game as a stepping stone to Algebra and the representation of numbers by variables. Figure 2.1 shows two students categorising numbers according to their type of prime factorisation, and we can see that the link to Algebra is evident. The pupils involved clearly enjoyed the day, as can be seen from their feedback which is displayed in Figure 2.2.

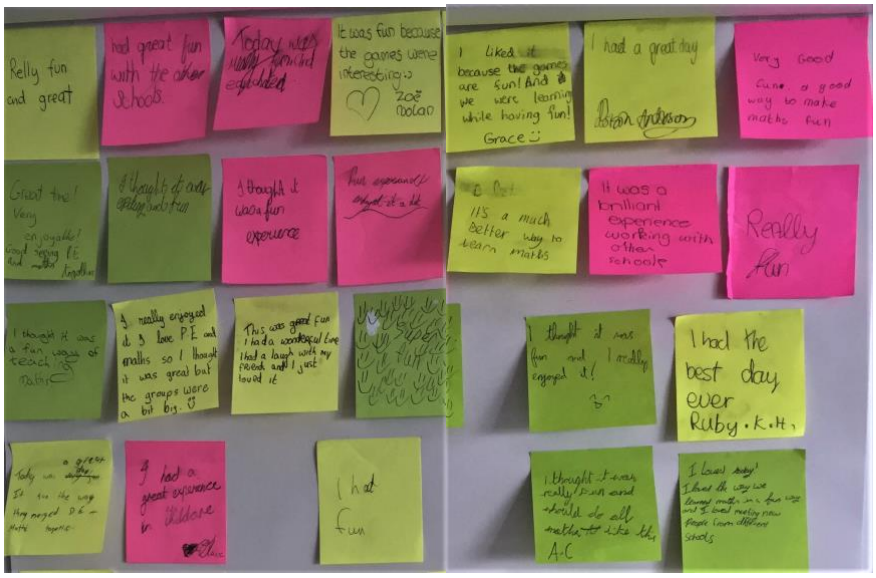


Figure 2.2: Some student responses after participating in the Jamboree event in May 2017

6 Conclusions

6.1. Summary of Findings

We have seen the themes that emerged from the interview and focus group data for both the Flagway module and the MLW module. Overall, pupils, MLWs and teachers were very positive about the AP and the Flagway game. The training was very well received by the teachers and PSTs, and from our analysis and site visits we saw that they put a lot of effort into the implementation of their modules. The effort paid off, in that both the teachers and the pupils spoke eloquently about the various benefits that they observed. The AP five-step process, with its emphasis on moving from a physical experience to an abstract concept, seems to lead to deeper and more persistent understanding. Furthermore, we have seen evidence that the methodologies used can lead to the development of student and teacher confidence, as well as key skills such as group work and communication. Some teachers worried about the amount of class time needed and about whether the aims of the curriculum were being achieved. Others clearly saw the benefits of this approach and justified the time spent in terms of student learning. Our only concern was that the link to Algebra was not evident in most of the primary classrooms. This may be because of the timing of our visits to the schools. The games the teachers engage with clearly provide for this progression, as was seen at the Jamboree in May 2017.

All participants spoke of the importance of having a good location to run the Flagway game and also of the benefits of combining PE and mathematics. As one pupil in CS2 said:

If a student does like PE and they don't like Maths they might learn to like Maths and enjoy it more. (T2_CS2_SFG)

6.2. Recommendations

1. The professional development model (i.e. having sessions focused on methodologies followed immediately by sessions where teachers can try out these in real classrooms) used in this project seems to have been very successful. We recommend that this model be incorporated in other professional development initiatives where possible.

2. There is evidence that the Algebra Project methodologies have real benefits in the teaching of mathematics at both primary and post-primary levels. We recommend that these methodologies are introduced to mathematics teachers around the country as soon as possible.

3. Teachers need to be supported when implementing these methodologies. A peer-support system should be developed which would allow the teachers and pre-service teachers who have experience of the programme to help teachers new to the ideas.

4. Concepts and methods from the Algebra Project should be incorporated into Initial Teacher Education programmes.

5. The Algebra Project methodologies (and the Flagway game in particular) provide an excellent opportunity to promote the link between Mathematics and Physical Education in the school curriculum. This link should be further exploited in order to improve the wellbeing of pupils.

6. The mathematical goals of the Algebra Project curriculum and methodologies should be made clear to participants in Professional Development workshops. The potential of the participants to miss the mathematical goals as they focus on the games should not be underestimated by facilitators.

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Appendix A

Questionnaire given to all participants at the three-day workshop in May 2016

Name

School

Number of years of teaching experience

Number of years teaching in current school

Have you used Algebra Project methods or resources in your classroom? Please comment.

From Swan and Swain (2010) 5 point Likert scale on frequency of use (1-5) in their maths classrooms: (student or teacher centred)

1 almost never, 2 occasionally 3 half the time 4 most of the time 5 almost always

Q1. I draw links between topics and move back and forth between topics.

Q2. Pupils start with easy questions and then move on to harder questions.

Q3. I encourage pupils to make and discuss mistakes.

Q4. I teach each pupil differently according to their individual needs.

Q5. Pupils work collaboratively in pairs or small groups.

Q6. I know exactly what maths the lesson will contain.

Q7. Pupils compare different methods for doing questions.

Q8. I teach the whole class at once.

Q9. I am surprised by the ideas that come up in a lesson.

Q10. I jump between topics as the need arises.

Q11. Pupils learn by doing exercises.

Q12. Pupils work on substantial tasks that can be worked on at different levels.

Q13. Pupils work on their own, consulting a neighbour from time to time.

Q14. I tend to teach each topic separately.

Q15. I tell pupils which questions to tackle.

Q16. Pupils invent their own methods.

Q17. I avoid pupils making mistakes by explaining things carefully first.

Q18. I encourage pupils to work more slowly.

Q19. Pupils choose which questions they tackle.

Q20. I find out which parts pupils understand and I don't teach those parts.

Q21. Pupils only use the methods I teach them.

Q22. I try to cover everything in a topic.

Q23. I teach each topic from the beginning assuming they know nothing.

Q24. I only cover the important ideas in a topic.

Q26. I tend to follow a textbook or worksheets closely.

Q27. I find myself encouraging pupils to work more quickly.

Q28. I only go through one method to do each question.

Section 2

1. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Mathematics is...	Percentage agreement:
a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to learners.	
a creative subject in which the teacher should take a facilitating role, allowing learners to create their own concepts and methods.	
an interconnected body of ideas which the teacher and the learner creates together through discussion.	

Please add your own definition of *Mathematics* if you like:

2. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Learning is...	Percentage agreement:
an individual activity based on watching, listening and imitating until fluency is attained.	
an individual activity based on practical exploration and reflection.	
an interpersonal activity in which learners are challenged and arrive at understanding through discussion.	

Please add your own definition of *Learning* if you like:

3. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Teaching is...	Percentage agreement:
structuring a linear curriculum for the learners; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners fail to 'grasp' what is taught.	
assessing when a learner is ready to learn; providing a stimulating environment to facilitate exploration; and avoiding misunderstandings by the careful sequencing of experiences.	
a non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	

Please add your own definition of *Teaching* if you like:

Appendix B

Algebra project – Interview protocol for teachers: These are the questions asked of a sample (n=5) of the participants at the three-day workshop in May 2016.

Name

School

Number of years of teaching experience

Number of years teaching in current school

1. How did you hear about the Algebra Project?
2. Why did you become involved?
3. Have you every participated in an Algebra Project workshop previously? Is so, when, where?
4. (If answer to the question above is yes then ask…) Have you used Algebra Project methods or resources in your classroom before now? If so, what do you do?
5. Describe your typical maths lessons. What are you doing? What are the children doing?
6. How would you normally introduce the topic of functions/factorisation in your classes?
7. What difficulties do pupils have with this topic? How do you tackle these difficulties?
8. Would you change the way you introduce this topic as a result of the Algebra Project workshops? If so, how and why?
9. Do you anticipate any difficulties in implementing the Algebra Project methodologies? Please explain?
10. What did you expect from the Algebra Project workshops? Did they meet your expectations?
11. What aspects of the workshops did you find most helpful?
12. Do you think that the workshops will have an impact on your teaching methods or on your pupils' learning? How? Why?
13. Can you pick out one 'light bulb' moment for you over the last three days? Please elaborate?
14. How does this workshop compare to previous professional development you have taken part in over the last three years?

Plus we will give teachers the tables overleaf in the interviews and ask them to fill them in and explain their choices.

15. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Mathematics is...	Percentage agreement:
a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to learners.	
a creative subject in which the teacher should take a facilitating role, allowing learners to create their own concepts and methods.	
an interconnected body of ideas which the teacher and the learner creates together through discussion.	

Please add your own definition of *Mathematics* if you like:

16. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Learning is...	Percentage agreement:
an individual activity based on watching, listening and imitating until fluency is attained.	
an individual activity based on practical exploration and reflection.	
an interpersonal activity in which learners are challenged and arrive at understanding through discussion.	

Please add your own definition of *Learning* if you like:

17. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Teaching is...	Percentage agreement:
structuring a linear curriculum for the learners; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners fail to 'grasp' what is taught.	
assessing when a learner is ready to learn; providing a stimulating environment to facilitate exploration; and avoiding misunderstandings by the careful sequencing of experiences.	
a non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	

Please add your own definition of *Teaching* if you like:

Appendix C

Algebra project – Interview protocol for teachers at T2:

Name

School

Number of years of teaching experience

Number of years teaching in current school

1. How did you hear about the Algebra Project?
2. Why did you become involved?
3. Describe your typical maths lessons. What are you doing? What are the children doing?
4. How would you normally introduce the topic of factorisation in your classes?
5. What difficulties do pupils have with this topic? How do you tackle these difficulties?
6. Have you changed the way you introduce factorisation as a result of the Algebra Project workshops? If so, how and why?
7. Did you experience any difficulties in implementing the Algebra Project methodologies? Please explain?
8. Can you please comment on the impact of the Algebra Project methodologies on your pupils' interest and engagement with maths learning?

Plus we will give teachers the tables overleaf in the interviews and ask them to fill them in and explain their choices.

9. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Mathematics is...	Percentage agreement:
a given body of knowledge and standard procedures. A set of universal truths and rules which need to be conveyed to learners.	

a creative subject in which the teacher should take a facilitating role, allowing learners to create their own concepts and methods.	
an interconnected body of ideas which the teacher and the learner creates together through discussion.	

Please add your own definition of *Mathematics* if you like:

10. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Learning is...	Percentage agreement:
an individual activity based on watching, listening and imitating until fluency is attained.	
an individual activity based on practical exploration and reflection.	
an interpersonal activity in which learners are challenged and arrive at understanding through discussion.	

Please add your own definition of *Learning* if you like:

11. Please give each of the following statements a weighting (which should add to 100%) reflecting how much you agree with the views.

Teaching is...	Percentage agreement:
structuring a linear curriculum for the learners; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when learners fail to 'grasp' what is taught.	
assessing when a learner is ready to learn; providing a stimulating environment to facilitate exploration; and avoiding misunderstandings by the careful sequencing of	

experiences.	
a non-linear dialogue between teacher and learners in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on.	

Please add your own definition of *Teaching* if you like:

Appendix D

Summary of responses to the questions on the KEC questionnaire given to the Flagway teachers in Maynooth 2016

Have you found this series of workshops...

Very helpful - all 10 said this.

Which sessions did you find most useful?

All sessions

Games to use for background work before playing the actual Flagway game.

I found them all very useful, but especially each morning session where we had to play the games ourselves and then got to discuss our observations and share ideas. Loved that we then got to implement what we learned immediately with children that afternoon, unlike for example summer courses.

I loved getting the chance to teach the games that we had learned in the morning to the children in the afternoon.

Morning session of each.

New games to play with the group. Different ideas of how you can reinforce concepts. Talking to different teachers was very informative.

Practical teaching activities and opportunities to teach primary school students.

Teacher to teacher discussion. Mid-morning sessions demonstrating and exploring games.

The activities - organising and playing same both as teachers and with pupils.

The practical elements - playing the games.

What aspect of that session most appealed to you?

Activities/games around prime numbers.

Games that we can play with the children.

Games were great and I was amazed at the depth of maths being shared.

Observing children engaged with Flagway activities

Physically playing the game and using strategies.

Reflections - how we could bring various activities back to our schools.

Seeing how the games could work in the classroom.

The games

The scope of work that can be done with the game. New ideas/more detailed explaining of the mathematics.

Which sessions did you consider least useful to you?

All sessions were most useful.

All useful.

I found all the sessions very useful.

None

Some of the discussion.

Do you need further information or practice on any aspect of these workshops?

Cluster meetings to share games and lessons with fellow teachers.

I'd like to explore developing supplementary games specific to the Irish curriculum.

I'd love to have the materials/resources even emailed to me - this would make starting a much quicker process.

More work on the scope of Algebra Project outside of Flagway game.

Need more practice on Flagway game with numbers.

No

No.

No. I have the contact details of the relevant people.

Possibly next academic year.

The materials for games.

What have you learned during these workshops that you will definitely be putting into practice?

All activities and methodologies.

All of it hopefully!

All of the games.

Card games along with how to further develop the game in the school.

Card games and physical games

Games, activities.

Games and Flagway game.

I’m looking forward to starting the prime number games and activities. I've also realised I need to allow the children more time to think and spoon-feed them less.

More theory behind the Mobius function.

Number games using prime number names - adding competitive element.

Further comments or suggestions

A group of teachers given release time to create scheme of work and lesson plans to be given to other schools.

I found the workshops fantastic. Very practical ideas and very clearly demonstrated. I can't wait to do all the activities with the children and I am looking forward to seeing if their attitude to maths changes.

It would be very useful to have all of the equipment/cards etc. provided.

Really enjoyed the workshop this year. There were lots of ideas and activities that we haven't come across before. Made the Flagway game make more sense and provides more scope for learning/reinforcement.

The course was very well run and the facilitators were excellent. Very approachable and looked after everybody. They wanted to help everybody. I have started to put a pack together myself in school. If you want to look at this I would photocopy it and give it to the centre.

Thoroughly useful and enjoyable workshop. Looking forward to using games with my own class.

Very good course. I'm looking forward to expanding further back in school.

Would like access to resources e.g. cards so we could print them off. Would like for group to meet a couple of times annually to share ideas.

Appendix E

Overview of Flagway Game (downloaded from

https://mailman.stanford.edu/pipermail/liberationtech-jobs/.../attachment.pdf)

THE FLAGWAY™ GAME OVERVIEW

Background
The Flagway Game, invented and patented by Robert Moses, is based on the Mobius Function, which categorizes the natural numbers into three mutually exclusive categories. YPP has created board games and physical games that can be played in classrooms, gymnasiums, and playgrounds that capitalize on young students propensity for running, the galvanizing energy of team competition, and the intrinsic sense of achievement they gain from figuring out correctly which category a given number belongs to. YPP specializes in developing math literacy workers, young people age 13 - 22, who teach and play the games with elementary and middle school students. YPP has used the games, particularly the interactive, physical nature of the games, as a motivating factor for elementary students to study the first 150 natural numbers.

The Game
An example of a simple Flagway game is one where players on teams line up at one end of a room. Three buckets are spaced out at the other end of the room – each bucket is a different color (Red, Yellow, or Blue) and represents one of the three categories in the Mobius Function. Players take turns running to a judges table to select a number, back to consult with their team, then running across the room to place their number in the correct bucket. Variations of the game involve playing on a large 2 dimensional lattice (tree graph) structure having a central starting point and a plurality of ending points (see diagram 1). In these variations, students select two or three numbers at a time, determine the correct categorization of each number, and arrange the numbers in a sequence to walk a pathway on the lattice, starting from the starting point, and ending at one of the ending points. Three color-coded legs (Red, Yellow, or Blue), corresponding to the Mobius function categories, extend from each node of the structure, including the starting point. At each node of the structure, students choose one of the three legs to walk or follow. This "choice" is determined by the next number in the sequence. For example a student who arranges the sequence of numbers 12 – 7 - 6 will, beginning at the start, choose the Yellow leg (for 12), then at the end of the Yellow leg choose the Red leg (for 7), then at the end of the Red leg choose the Blue leg (for 6).

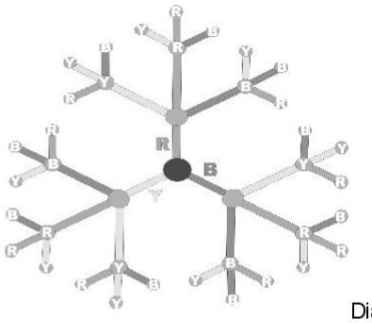

Diagram 1: Flagway Structure

In addition to reinforcing basic math (multiplication facts, prime numbers, factorization) the Flagway game provides a basis for deepening elementary students understanding of the natural numbers and for the exploration of many interesting math concepts. For example students can learn that every number has a unique prime factorization, including numbers that are not in their multiplication tables (like 51, 87, 93); and that numbers can be thought of as having algebraic forms ("A" numbers are prime numbers, "AB" numbers are the product of two different prime

factors, “A^2” numbers are the product of two repeating prime factors, etc.). Students can also explore more complex problems like how to determine the number of divisors of a number, and whether there is an even distribution of Red, Yellow, and Blue numbers.

Pedagogical Approach

Initially students are not told how or why the numbers are categorized as they are. Rather they are asked to form and revise their own conjectures about the rules that are behind the “Flagway” categorization. This process engages students in applying what they already know about numbers and arithmetic operations to the task of “cracking the Flagway code” and challenges them to develop their reasoning skills. Students are helped to make better and better conjectures in three ways. (1) Their facilitators help them to consistently check that their rules for grouping the numbers are such that a rule for a given color works for every number in the group and do not apply to any number in any other group. (2) At first they work (and play) with small sets of numbers in sequence (for example the numbers 2 - 10), and gradually increase the cardinality of the set of numbers. (3) They play games and do activities that help them to explore and learn properties of number that are relevant to the game and to cracking the code.

We envision that over time many students will learn and play this game, which presents the challenge of how we will we prevent people from just telling the rules to new players of the game, and thus undermine the process of the students being able to form their own conjectures. Forming conjectures is an important part of the game because it develops students’ critical thinking skills. It also opens the door to playing the game with families of mathematical functions that utilize broader areas of mathematics. One can envision that as students become mature players and math students, they might show up to a flagway game and have to quickly determine, based on the data given to them, which function they are playing the game with on that particular day.

Appendix F

Article from the Carlow Nationalist (23/5/17) about the Algebra Project in Scoil Mhuire gan Smál.

Pioneering maths project puts fun into learning at Scoil Mhuire gan Smál

IT'S a far cry from having multiplication tables drummed into us in school because youngsters in Scoil Mhuire gan Smál are learning maths through an innovative programme that was devised in America.

Dr Bob Moses, a mathematician and a civil rights campaigner back in the 1960s, devised *The Algebra Project* to help children understand such concepts as patterns in figures. He firmly believes that education and maths in particular are keys to economic and civil equality.

The project is being piloted in a small number of schools, with Scoil Mhuire gan Smál one of the lucky few selected. This came about through the doctoral work of their deputy principal Ann Marie Gurhy. Ann Marie was seconded by the NCCA (National Council for Curriculum and Assessment) last year to work on the new primary maths curriculum.

The children took to the programme immediately because it turned algebra into a fun, interesting and energetic game. Last week, Bob Moses's daughter Maisha Moses, along with Shauna Swindell and Yvelene Logistee, flew to Ireland to support the programme and dropped into the Carlow town school to see how they were getting on. Teachers, visitors and children alike had a brilliant time together, working out the solutions and finding numerical patterns in the game.

"I realise how fortunate our school is to be involved. Through various games and activities, the pupils in fifth and sixth class become proficient in expressing practical aspects of daily life in algebraic equations," principal Bernie Murphy told *Around Carlow Town*.

"Crucially, they are actively engaged in maths in a fun way. Fortunately, we have a large hall, which allows the programme to be implemented indoors as well as outdoors. I truly hope all schools will, in time, experience the benefits of this programme. There is also the additional benefit that pupils have an increased rate of physical activity during the school week," said Ms Murphy.

Paulina, Akasa and Leah

Learning the Algebra project

The Algebra prroject in full swing at Scoil Mhuire gan Smál

Maisha Moses and pupils taking part in the flagway game

Casia, Rebecca, Erin, Emma, Arielda and Nina taking part in the flagway games

All the fun of the Algebra Project at Scoil Mhuire gan Smál

Pupils at Scoil Mhuire gan Smál with US maths teachers Maisha Moses, Shauna Swindell and Yvelena Logistee

Photos: Karl McDonough

Teacher Ann Marie Gurhy and Principal Bernie Murphy with American maths teachers Shauna Swindell, Maisha Moses (centre) and Yvelene Logistee at Scoil Mhuire gan Smál

Shauna Swindell with pupils at Scoil Mhuire gan Smál

Sahima, Mia, Kate, Sara and Ali